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Local-Pair Superconductivity in Very High Magnetic Fields

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Abstract

Superconductivity of narrow-band systems with local, short-range attractive interaction in very high magnetic fields is discussed. By examining the excitation spectra of both type-II superconductors with BCS like interaction and local-pair superconductors with negative-U type interaction, it is concluded that gapless single particle energy spectrum is a characteristic feature of superconductivity in very high magnetic fields.

In narrow-band systems local, short-range attractive interactions can result in formation of real space electron pairs which leads local-pair superconductivity[1]. Many of the models proposed for high temperature superconductors involve real space pairing. In this paper, electromagnetic properties of local-pair superconductors in very high magnetic fields will be discussed. Characteristic features of superconductivity in narrow-band systems with local nonretarded attractive interactions will be reviewed in Sec.1. In Sec. 2, theoretical aspects of superconductivity in very high magnetic fields will be summarized. Finally, in Sec. 3 behavior of local-pair superconductors in high fields will be studied.

1. Local Pair Superconductivity

Strong electron-phonon coupling or local electronic excitations coupling to electrons can lead to an effective short range attraction of electrons. Instability of the system can result in a superconducting or charge-ordered state. The simplest model to understand the physics of local-pair superconductivity is the negative-U or attractive Hubbard Hamiltonian given by

$$H = -t \sum_{<ij>\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} - U \sum_{i} n_{i\uparrow} n_{i\downarrow}. \qquad (1)$$

The parameters of the system are U/t and the filling of the band $\nu = N_e/2N$ where N_e and N are the number of electrons and the number of lattice sites, respectively.

At zero temperature, pairing correlations can be treated with BCS-like mean field approach. In the weak coupling limit, $U/T \ll 1$, the gap Δ is exponentially small and the size of the Cooper pairs is very large in comparison to the lattice spacing. This is the usual BCS picture where electrons do not feel the presence of the lattice. On the other hand, in the strong coupling limit, $U/t \gg 1$, lattice structure is essential. Electrons form local singlet pairs which can move from site to site via virtual ionization. Since these bosonic pairs can not occupy the same site simultaneously they behave as hard core bosons. In this case superconductivity is due to Bose-Einstein condensation of these composite particles. Due to hard core nature of bosons density of electrons plays an important role. At low densities, $\nu \ll 1$, since bosons do not interact very frequently, the system behaves like an ideal Bose gas. Around half filling, $\nu \sim 1/2$, bosons interact strongly. BCS wave function can explain the behavior of the system in both the weak and strong coupling regimes. In the weak coupling limit, transition temperature is controlled by breaking of pairs, while in the strong coupling case it is associated with the center of mass motion of electron pairs. Thus, in BCS case, normal state is degenerate Fermi liquid, and formation of Cooper pairs and their condensation occur simultaneously. In Bose-Einstein condensation, below a critical temperature, bosons formed at a high temperature, occupy a single quantum state. Some of the earliest attempts to understand superconductivity in metals were involving this kind of mechanisms.

2. Superconductivity in very high magnetic fields

In the past few years, after the pioneering work of Rasolt, Tesanovic and collaborators[2], it has been realized that due to Landau quantization, superconductivity can survive to very strong magnetic fields. In the absence of a magnetic field, time – reversed electron states are paired to form superconducting state. Since magnetic field breaks time – reversal – invariance symmetry, it frustrates pairing and hence decreases the critical temperature T_c. The mixed state of type – II superconductors is usually investigated by Ginzburg – Landau method which can be derived from Gorkov's microscopic theory. Semiclassical theory of Gorkov for the behavior of a superconductor in weak external magnetic fields breaks down when the temperature is sufficiently low and the disorder is sufficiently weak so that the Landau quantization of motion becomes important. The behavior of the system becomes qualitatively different from the semiclassical case. Superconductivity can coexist with arbitrarily strong magnetic fields. Dukan *et al.* have pointed out that, within the mean field approximation, there exists gapless quasiparticle excitations since the order parameter vanishes at certain points in the magnetic Brillouin zone[3]. The linear dispersion of the energy spectrum around these points leads to characteristic features in several thermodynamic quantities.

The Hamiltonian of the system is given by $H = H_0 + H_1$ where

$$H_0 = \int d\mathbf{r} \sum_{\alpha} \psi^{\dagger}(\mathbf{r}\alpha)[\frac{1}{2m}(\mathbf{p} - \frac{q}{c}\mathbf{A})^2 - E_F]\psi(\mathbf{r}\alpha) \tag{2}$$

and

$$H_1 = \frac{1}{2}\int d\mathbf{r} \sum_{\alpha\beta} \psi^\dagger(\mathbf{r}\alpha)\psi^\dagger(\mathbf{r}\beta)V(\mathbf{r})\psi(\mathbf{r}\beta)\psi(\mathbf{r}\alpha) \quad . \tag{3}$$

Here, it has been assumed that H_1 is independent of spin and point-like $\psi\psi\psi\psi$ t-like i.e., $V(\mathbf{r},\mathbf{r}') = V(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')$. Introducing the pair potential $\Delta(\mathbf{r})$, it is possible to write an effective Hamiltonian of the form

$$H_{eff} = \int d\mathbf{r} \sum_{\alpha} \psi^\dagger(\mathbf{r}\alpha)H_0\psi(\mathbf{r}\alpha) + [\int d\mathbf{r}\Delta(\mathbf{r})\psi^\dagger(\mathbf{r}\uparrow)\psi^\dagger(\mathbf{r}\downarrow) + H.c.] \quad . \tag{4}$$

Equation (4) can be diagonalized by performing a unitary transformation which results in a linear system so called the Bogoliubov–de Gennes (BdG) equations[4, 5]

$$\begin{aligned} Eu(\mathbf{r}) &= H_- u(\mathbf{r}) + \Delta(\mathbf{r})v(\mathbf{r}) \\ Ev(\mathbf{r}) &= \Delta^*(\mathbf{r})u(\mathbf{r}) - H_+ v(\mathbf{r}) \end{aligned} \tag{5}$$

where H_+ and H_- are defined by

$$H_\pm = \frac{1}{2m}(\mathbf{p} \pm \frac{q}{c}\mathbf{A})^2 - E_F \quad . \tag{6}$$

BdG equations must be solved with the self consistency condition for the effective potential $\Delta(\mathbf{r})$ given by

$$\Delta(\mathbf{r}) = -V(\mathbf{r}) \sum_m v_m^*(\mathbf{r})u_m(\mathbf{r})[1 - 2n_F(E_m)] \quad . \tag{7}$$

n_F is the Fermi–Dirac distribution function and m labels the eigenvalues E_m and eigenstates $u_m(\mathbf{r})$ and $v_m(\mathbf{r})$ of Eq. (5).

Defining the Green's functions

$$G_\pm(E) = \frac{1}{E \pm H_\pm}, \tag{8}$$

BdG equations can be combined to give a single eigenvalue equation for $u(\mathbf{r})$ (or $v(\mathbf{r})$)

$$u = G_-\Delta G_+ \Delta^* u. \tag{9}$$

Along with Eq. (7), Eq. (9) describes the mean field solution completely for any magnetic field strength.

When the magnetic field is very high so that all of the carriers are confined into a single, e.g. the lowest, Landau level, Green's functions G_+ and G_- take simple forms in the coordinate space representation. This is a good approximation as long as the energy scales of the system are small in comparison to Landau level splitting $\hbar\omega = \hbar qB/mc$ (it

is assumed that q is positive). Using the symmetric gauge $\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}$ and complex coordinates $z = X + iY = \sqrt{qB/2\hbar c}(x + iy)$ where $\mathbf{r} = (x, y)$, the matrix elements of Green's function $G(z_1, z_2; E) =< z_1|G(E)|z_2 >$ can be written as

$$G_-(z_1, z_2; E) = \exp\left(z_1 z_2^* - \frac{|z_1|^2}{2} - \frac{|z_2|^2}{2}\right) / \pi[E - (\frac{\hbar\omega}{2} - E_F)] \quad . \tag{10}$$

Since $H_+ = H_-^*$, the Green's functions are related by $G_+(E) = -G_-^*(-E)$. Therefore, in the high magnetic field limit Eq. (9) takes the form

$$\begin{aligned} \lambda u(z) \quad &= \int dz_1 dz_1^* dz_2 dz_2^* \exp\left(z z_1^* - \tfrac{|z|^2}{2} - \tfrac{|z_1|^2}{2}\right) \Delta(z_1) \\ &\times \exp\left(z_1^* z_2 - \tfrac{|z_1|^2}{2} - \tfrac{|z_2|^2}{2}\right) \Delta^*(z_2) u(z_2) \end{aligned} \tag{11}$$

where

$$\lambda = \pi^2[E^2 - (\frac{\hbar\omega}{2} - E_F)^2] \quad . \tag{12}$$

For the BCS interaction i.e., $V(z) = const = -V$ where $V > 0$, Eq. (12) takes a very special form. Since all the wave functions are in the lowest Landau level, $u(z) = U(z)\exp(-|z|^2/2)$ and $v(z) = V(z)\exp(-|z|^2/2)$ where $U(z)$ and $V(z)$ are analytic functions of z. Thus, the self-consistency condition implies that pair potential is of the form $\Delta(z) = f(z)\exp(-|z|^2)$ which implies that

$$\frac{2\lambda}{\pi} U(z) = \int d\zeta d\zeta^* f(\frac{z+\zeta}{2}) f(\zeta)^* \exp(-2|\zeta|^2) U(\zeta) \quad . \tag{13}$$

As a result, the BdG equations have been reduced to a single integral equation.

In the periodic vortex lattice case, $f(z)$ must have regularly distributed zeros like the theta function defined by

$$\theta_3(z|\tau) = \sum_{n=-\infty}^{\infty} \exp(\pi i \tau n^2) \exp(2n\pi i z) \tag{14}$$

which vanishes at $z = (p + 1/2) + (q + 1/2)\tau$ where p and q are any integers. Therefore, $\theta_3(z/a|b/a)$ will have regularly distributed zeros with lattice vectors a, assumed to be real without loss of generality, and b, whose imaginary part must be $\pi/2a$ to make sure that flux enclosed per unit cell is $hc/2|e|$. Next question is how to construct a function with periodic norm. The answer is

$$f(z) = \theta_3(\frac{z}{a}|\frac{b}{a}) \exp(z^2) \quad . \tag{15}$$

It is easy to show that with this choice of $f(z)$, $|\Delta(z)|$ becomes a two dimensional periodic function and furthermore u and v have also the same form except the lattice

constants due to the fact that this time flux to be enclosed is twice big. Therefore, $U_0(z) = \theta_3(z/a|2b/a)\exp(z^2/2)$ solves the integral equation. It turns out that this is only one of the eigenstates and the most general solution $U_\eta(z)$ is obtained by translating $U_0(z)$ by an amount η, an arbitrary complex number in the unit cell determined by a and b. Since $U_\eta(z) = U_0(z-\eta)\exp(\eta^* z - |\eta|^2/2)$, one gets the very simple result

$$\lambda_\eta = \pi|\Delta(\eta)|^2 \quad . \tag{16}$$

Hence, energy spectrum is of the same form as the order parameter. An immediate consequence is that there are gapless single particle excitations. In fact, it has been shown that the excitation spectrum is gapless for any distribution of zeros of the order parameter[6]. Therefore, the presence of gapless excitations is a topological property of the system.

3. Superconductivity induced by negative-U centers in very high magnetic fields

In this section the excitation spectrum of a two dimensional superconductor with randomly distributed attractive centers in the presence of a high magnetic field will be investigated. Behavior of a similar system, diluted negative–U centers randomly quenched in a host three dimensional metal, in low enough magnetic fields, so that the semiclassical approximation is applicable, has already been studied in reference [7] where the authors have also made statements about the strong field limit. However, in very strong magnetic fields, the semiclassical substitution used to obtain Green's functions is no more valid. It turns out that superconductivity can coexist with very strong magnetic fields not only because of the similarity of the system to Josephson junction arrays, a characteristic feature of local pair superconductors, but also the Landau quantization of the electronic energy levels. In this paper, only two – dimensional case will be considered where it is possible to obtain analytic results and one can use the same method to investigate the properties of three – dimensional system with columnar impurities.

In the case of negative–U centers the interaction takes the form $V(z) = -\sum_i V_i\delta(z-z_i)$ where z_i denotes the position of the i^{th} center. Equation (9) becomes a system of linear equations in $u_i = u(z_i)$ which can be written as $\lambda u_i = \sum_j a_{ij}u_j$ where

$$a_{ij} = \exp(-\frac{|z_i|^2}{2} - \frac{|z_j|^2}{2})\Delta_j^* \sum_k \exp[(z_i + z_j)z_k^* - |z_k|^2]\Delta_k \quad . \tag{17}$$

Here $\Delta_i = \Delta(z_i)$. In order to have real λ values, the matrix elements must satisfy $a_{ij} = a_{ji}^*$. The above equation and the self consistency condition Eq. (7) must be solved simultaneously. It is easy to show that for the following form of the order parameter, hermiticity of matrix **a** and hence reality of eigenvalues λ are realized:

$$\Delta(z) = |\Delta| \sum_j \delta(z - z_j)\exp(i\theta_j) \tag{18}$$

where the phase angles are related to the positions of the impurities by $\theta_j - \theta_k = -2(\mathbf{r}_j \times \mathbf{r}_k)_z$. In other words the amplitude of the order parameter is assumed to be the same for all of the negative–U centers and it is fixed by the BdG equations. The above form of $\Delta(z)$ is a good approximation at low enough impurity concentrations. In the high density limit, where $V(\mathbf{r},\mathbf{r}')$ approaches to the standard BCS type interaction $V(\mathbf{r},\mathbf{r}') = V\delta(\mathbf{r}-\mathbf{r}')$, the assumption must fail because in this limiting case it is known that the order parameter has strong variations in amplitude. In fact, $|\Delta(z)|$ cannot be constant due to the fact that $\Delta(z)$ must lie in the lowest Landau level where all wave functions are of the form (analytic function of z) $\times \exp(-|z|^2)$ and there is no analytic function whose modulus grows like $\exp(|z|^2)$ for all z.

Equation (18) leads to a very important simplification for the eigenvalue problem. Matrix $\mathbf{a}$ can be written as square of another matrix h so that $\mathbf{a} = \mathbf{h}^2$ where

$$h_{ij} = |\Delta| \exp\left(-\frac{|z_i|^2}{2} - \frac{|z_j|^2}{2} + z_i z_j^*\right). \tag{19}$$

Therefore, λ values are obtained by taking the square the eigenvalues of $\mathbf{h}$. It is possible to think of $\mathbf{h}$ as Hamiltonian of a tight binding problem. Returning to vector notation, the exponent in the above equation can be written as $-\frac{1}{2}|\mathbf{r}_i - \mathbf{r}_j|^2 - i\mathbf{r}_i \times \mathbf{r}_j$ which means that h_{ij} is hopping amplitude from site i to site j whose magnitude change in a Gaussian way with distance. The phase $\exp(-i\mathbf{r}_i \times \mathbf{r}_j)$ has also a very simple interpretation. It is nothing but the line integral of the vector potential $\mathbf{A}$ along a straight line. Thus, $\mathbf{h}$ is the Hamiltonian of a two dimensional tight binding system in the presence of a perpendicular magnetic field where the hopping amplitude between randomly distributed sites is a Gaussian function of the distance between them. Now, relaxing the self consistency condition, it can be shown that diagonalization of $\mathbf{h}$ can be mapped onto an exactly solvable problem due to Brezin and Gross[8], namely the one particle spectrum of two dimensional fermion gas in a strong magnetic field in the presence of impurities. They start with the Hamiltonian $H = H_0 + V$ where V is a random potential. For a strong enough field the discussion can be limited to the lowest Landau level since the gap between levels $\hbar\omega$ is much bigger than the perturbing impurity potential. A Poisson model of random impurities corresponds to a uniform density σ of zero – range scattering centers. The probability density to find N impurities at points $\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N$, in an area A is given by

$$P(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N) = \exp(-\sigma A)\sigma^N / N! \tag{20}$$

and the corresponding potential of strength V_0 is

$$V(\mathbf{r}) = V_0 \sum_{i=1}^{N} \delta(\mathbf{r} - \mathbf{r}_i) \quad . \tag{21}$$

By means of anticommuting variables it is possible to evaluate the average spectral density per unit area[8]. For low impurity concentrations ($0 < f < 1$), as $E \to \hbar\omega/2$

the density of states $\rho(E)$ changes as

$$V_0\rho(E) \sim (1-f)\delta(\nu) + c(f)\nu^{-f} + \ldots \quad . \tag{22}$$

Here, $c(f)$ is a constant independent of ν, $\nu = 2\pi(E - \hbar\omega/2)/V_0K^2$ and $f = 2\pi\sigma/K^2$ where $K = eB/\hbar$. Hence, a fraction $(1-f)$ of the states in the lowest Landau level is unaffected by the presence of the scatterers.

The relation between the eigenvalue spectrum of the above problem and that of $\mathbf{h}$ can be seen as follows. Let $G = 1/(E-H)$ and $G_0 = 1/(E-H_0)$. G_0 is nothing but G_- defined by Eq. (10). Using the relation $G = G_0 + G_0VG$, the trace of the Green's function G, which is related to density of states by $\rho(E) = -ImTrG(E^+)/\pi$, can be written as

$$TrG(E) = \frac{A}{\pi\epsilon} + \frac{V_0\sigma A}{\pi^2\epsilon^2} + \frac{V_0^2}{\pi\epsilon^2}\sum_{ij}\exp\left(-\frac{|z_i|^2}{2} - \frac{|z_j|^2}{2} + z_i^* z_j\right) G(z_i, z_j; E) \tag{23}$$

where $\epsilon = E - \hbar\omega/2$. Since $G = G_0 + G_0VG$, for the above form of the potential, one can write a matrix equation $\mathbf{G} = \mathbf{G}_0 + V_0\mathbf{G}_0\mathbf{G}$ where $G_{ij} = G(z_i, z_j; E)$ etc.. Thus, the second term in Eq. (23) can be written as $Tr\mathbf{G}_0\mathbf{G}$ which can be easily evaluated in the basis where $\mathbf{G}_0$ is diagonal. Finally, density of states takes the form

$$\rho(E) = \frac{1-\pi\sigma}{\pi}\delta(E - \frac{\hbar\omega}{2}) + \frac{1}{A}\sum_i \delta(E - \frac{\hbar\omega}{2} - \frac{V_0}{\pi}l_i) \tag{24}$$

where l_i is the i^{th} eigenvalue of $\mathbf{h}$ and comparison with Eq. (22) gives that as $\nu \to 0$, the average spectral density for $\mathbf{h}$ varies as ν^{-f}. Since the eigenvalues of $\mathbf{a}$ and $\mathbf{h}$ are related by $\lambda_i = l_i^2$, the density of states for the superconducting state is given by $\nu^{-f-1/2}$. Therefore, superconducting state induced by the impurities manifests itself by appearance of a tail attached to the usual Dirac delta singularity in the excitation spectrum. Furthermore, absence of an energy gap is true even for higher impurity concentrations as long as the assumption about the functional form of the order parameter is valid.

It must be noted that throughout the discussion, effects of negative–U centers have been introduced by pairing interactions. Missing single particle interaction terms, which describe the interaction of fermions with impurities, are expected to lead to diffusion of Cooper pairs away from the centers resulting in a smoother order parameter. It is possible to think of this phenomenon as a kind of proximity effect. Another issue is the Zeeman splitting effect which can destroy the superconducting state. The candidate materials to observe the effects of Landau quantization are semiconductors or semimetals having low electronic densities for which the quantum limit can be reached in physically accessible range of magnetic fields and it is possible to have very small g–factors in many low carrier density systems. In addition to this, for superconducting state induced by negative–U centers, pairing occurs in real space. Hence, as long as the ratio of the number of electrons

having opposite spins does not deviate too much from unity, Zeeman splitting effect will not be very strong. However, quantitative treatment of the effect requires the solution of the gap equation which is a formidable task. Finally, triplet superconductivity is another possibility to be investigated.

Therefore, it has been shown that two dimensional fermi gas can exhibit superconducting transition mediated by attractive centers even in the presence of very strong magnetic fields perpendicular to plane. For order parameters having constant amplitude at impurity sites BdG equations can be solved exactly and they lead to the prediction that the single particle excitation spectrum is gapless. An interesting problem is the case where the negative–U centers form a regular lattice and diagonalization of $\mathbf{h}$ becomes a generalized Hosftadter problem[9]. It is generalized in the sense that $\mathbf{h}$ contains all possible interactions between lattice sites rather than the nearest neighbor interactions only. Although, the known solutions of the Hofstadter problem suggests that spectrum will be gapless, the correct answer cannot be given without diagonalizing $\mathbf{h}$.

Finally, it is interesting to compare the above system with two dimensional negative-U Hubbard model[10]. The system can be analyzed by means of Ginzburg-Landau theory. At temperature T, the free energy functional up to quadratic terms in on-site parameters $\psi_{\mathbf{r}=(m,n)}$ can be written as

$$F = \sum_{\mathbf{r}\mathbf{r}'} -\frac{2t^2}{U}\psi^*_{\mathbf{r}}\psi_{\mathbf{r}'}\exp(i\phi_{\mathbf{r}\mathbf{r}'}) + \frac{k_BT}{1-2\nu}\ln(\frac{1-\nu}{\nu})\sum_{\mathbf{r}}|\psi_{\mathbf{r}}|^2 \quad . \tag{25}$$

Here, $\mathbf{r}$ and $\mathbf{r}'$ are nearest neighbor sites on the square lattice and $\phi_{\mathbf{r}\mathbf{r}'}$ is the phase due to vector potential $\mathbf{A} = (0, xB, 0)$ for the magnetic field $\mathbf{B} = (0, 0, B)$ and it is given by

$$\phi_{\mathbf{r}\mathbf{r}'} = \frac{2\pi Ba^2m}{\phi_0}\delta_{m,m'}(\delta_{n-1,n'} - \delta_{n+1,n'}) \tag{26}$$

where a is the lattice constant and $\phi_0 = hc/2|e|$ is the flux quantum.

Minimization of the free energy F with respect to $\psi^*_{\mathbf{r}}$ gives the Ginzburg-Landau equations. The upper critical magnetic field is calculated by finding the highest eigenvalue $\tilde{T}$ for the system of equations

$$\psi_{m-1} + 2\cos(\frac{2\pi Ba^2m}{\phi_0})\psi_m + \psi_{m+1} = \tilde{T}\psi_m \quad . \tag{27}$$

The fact that for the highest eigenvalue, the on-site parameters have no n dependence is stressed by using the index m only. Here $\tilde{T}$ is the dimensionless temperature defined by

$$\tilde{T} = \frac{k_BTU}{2t^2}\frac{1}{1-2\nu}\ln\frac{1-\nu}{\nu} \quad . \tag{28}$$

The eigenvalue equation is Harper's type and in general its solutions can be obtained numerically. Magnetic field B enters via a trigonometric function and hence critical

temperature is a periodic function of B. This is a result of Peierles substitution obtained by multiplying hopping matrix elements by associated phase factors. Even if the fact that amplitude of the matrix elements are also modified is taken into account, one ends up with finite critical temperatures for all magnetic fields. In other words, superconductivity survives at arbitrarily large magnetic fields.

The origin of the divergence of the upper critical field below a certain temperature lies at the lattice structure. In this system, vortex currents are entirely Josephson tunneling currents. At a certain temperature coherence length becomes smaller than the lattice constant and therefore the system behaves as a collection of isolated lattice sites. Since the sites are assumed to have zero extension in space, no critical field can destroy superconductivity.

In conclusion, theoretical studies of both weak coupling continuum and strong coupling lattice models of local pair superconductors indicate that superconductivity can persist at arbitrarily large magnetic fields. Space variation of the order parameter and its dependence on the distribution of negative-U centers are subjects of future work.

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