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## A hierarchical model for the cell loading problem of cellular manufacturing systems

M. SELIM AKTURK<sup>†</sup>\* and GEORGE R. WILSON<sup>‡</sup>

A hierarchical cell loading approach is proposed to solve the production planning problem in cellular manufacturing systems. Our aim is to minimize the variable cost of production subject to production and inventory balance constraints for families and items, and capacity feasibility constraints for group technology cells and resources over the planning horizon. The computational results indicated that the proposed algorithm was very efficient in finding an optimum solution for a set of randomly generated problems.

### 1. Introduction

A recent change in the customers' sense of values has forced many companies to manufacture products in a specified period, with very short notice, and with the production volume for each product very low. This market environment must be accommodated by a classic batch-type production (BP). BP accounts for 60–80% of all manufacturing activities. Group technology (GT) is an innovative approach to BP which seeks to rationalize small-lot production by capitalizing on the similarities that exist among component parts and/or processes. The central theme of GT, when applied to component parts, is the formation of part families on the basis of design or manufacturing, or both. Once formed, these part families can be used to achieve efficiencies in, primarily but not exclusively, (1) product design, (2) manufacturing engineering and (3) cellular manufacturing (CM). CM, which is a subset and derivative of GT, is the physical division of the manufacturing facilities into production cells, representing the basis for advanced manufacturing systems such as just-in-time, flexible manufacturing systems and computer integrated manufacturing as discussed in Gunasekaran *et al.* (1994). In CM, each cell is designed to produce a part family or families efficiently.

There are many studies related to the part-family and machine-cell formation (PFMCF) problems in the context of the CM systems. In the literature, these studies can be categorized into two major groups: the classification and coding (CC) systems and the clustering methods. Offodile *et al.* (1994) provide a comprehensive review of the CM literature and present an extensive bibliography of the PFMCF problems by citing more than 100 GT related works. Furthermore, an overview of similarity and distance measures for solving the cell formation problem can be found in Shafer and Rogers (1993). Hyer and Wemmerlov (1989) reported the findings of a survey of 53 US users of GT. Thus, as an approach to increasing the productivity of BP, GT's importance is growing. But the literature on GT is not specific with regard to how

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the production plan is actually obtained. Rather, it seems to suggest that economic production plans will be easy to find once a production operation is decomposed into machine groups and part families. Morris and Tersine (1989, 1990) and Shafer and Charnes (1995) performed simulation experiments to investigate several factors that might influence the loading problem in CM systems. They have shown that a direct conversion from a process layout to a cellular layout by itself was not able to bring about all the stated advantages suggested in the literature. It would appear that a new cellularly divided shop must be controlled with efficient production planning systems so as to benefit from the advantages of GT. Therefore, the thrust of this paper is the development of a hierarchical cell loading approach to solve the production planning problem in CM systems.

Cell loading, or production planning, in a CM environment is a decision activity that determines the kind of items and the quantities to be produced in each cell in the specified time period, subject to the production capacity and demand forecast. Two distinct approaches for the cell loading problem in a BP environment have appeared in the literature. The first approach, termed the monolithic approach, formulates the cell loading problem as a large mixed-integer linear programming (MILP) problem at an individual item level and heuristic procedures are sought to solve it, such as Ham *et al.* (1985). The second approach is the hierarchical approach, which partitions the overall problem into a hierarchy of smaller problems. The earliest contribution in the area of hierarchical production planning (HPP) is attributed to Hax and Meal (1975). Hax and Meal's HPP approach defines three levels of aggregation for products. The top level derives from an aggregate planning model using linear programming for the variables corresponding to 'types', which are sets of items that are similar in terms of seasonal demand patterns and production rates. The middle level considers a heuristic disaggregation of the types into 'families' which are sets of items that have similar setup costs. Third level decisions consist of disaggregating families into items based on equalizing runout times. The underlying idea of their approach is to make decisions sequentially starting from the highest level. The decision at each level then becomes a constraint for the next lower level. The major drawback of this approach is that constraints imposed by higher levels are based on type level calculations only. This might create empty feasible solution spaces and otherwise unnecessarily limit the number of alternatives possible at the lower levels. Furthermore, the original HPP procedure is based on the assumption that setup costs are of secondary importance and magnitude; therefore, they do not consider its cost impact in the model, and their approach lacks any feedback mechanism.

The initial work of Hax and Meal has been extended by several authors. Bitran *et al.* (1981) reformulated the family and item disaggregation plan of Hax and Meal as a knapsack problem. They showed that whenever setup costs are low, the results approached optimality and remained insensitive to forecast errors. Graves (1982) considered a different approach to the HPP. He first formulated the overall problem as a monolithic MILP assuming an infinite production capacity, then used a Lagrangean relaxation procedure to solve the dual to the MILP. The linear problems obtained through Lagrangean relaxation were the aggregate planning model and a set of uncapacitated lot-size models for each product type. His approach also includes a feedback mechanism between subproblems. Most of the HPP approaches assume infinite production capacity and, therefore, ignore capacity constraints. But their results might easily become infeasible if there is a bottleneck workstation which

governs the production rate in the system. A more detailed discussion of the HPP approaches can be found in Bitran and Tirupati (1993) and McKay *et al.* (1995).

The underlying philosophy of the proposed hierarchical cell loading approach has some similarities to the HPP approach developed by Hax and Meal, and extended by Graves. The proposed approach is directed toward extending and enhancing the HPP in several ways utilizing knowledge of GT-based manufacturing systems. The first enhancement is the formulation of the production planning problem. In the proposed approach, the capacity constraints are added to the problem formulation, as a result, the production rates are determined to be within the current capacity of the system. Furthermore, the advantages of a CM shop configuration are used to simplify the problem by allowing the inclusion of spatial decomposition, where the manufacturing system is divided into a set of GT cells, and there is a structured product-based aggregation/disaggregation (A/D) scheme based on GT oriented CC systems. Another enhancement is to one of the central ideas of the HPP approach which is to making decisions sequentially starting from the highest level. In this top-down constrained approach, solutions to higher levels become 'hard' constraints to the lower levels as discussed above. By contrast, in the proposed approach the higher levels do not dictate bounds to the lower levels, but rather provide guidance, or 'soft' constraints, which are priced out by a set of dual variables, that focus lower level searches in areas most likely to contain good solutions. Consequently, the dual and feedback information are passed between the levels to ensure internally consistent decisions.

The remainder of this paper is organized as follows. In the following section, a mathematical programming formulation is presented to solve the cell loading problem. We discuss the proposed solution procedure in §3. A full factorial design is developed in §4 to evaluate the effects of several system parameters. Finally, some concluding remarks are provided in §5. Furthermore, a list of abbreviations used throughout the paper is as follows:

GT: Group technology

CM: Cellular manufacturing

A/D: Aggregation/disaggregation

CCS: Classification and coding systems

## 2. Mathematical formulation

Our aim is to allocate production capacity among GT families and items by means of the proposed aggregate planning model. This can be achieved by solving a multi-period optimization problem which minimizes the summation of production, setup, inventory holding, and regular and overtime capacity costs subject to production and inventory balance constraints for families and items, and capacity feasibility constraints for GT cells and resources over the planning horizon. The objective function corresponds to the minimization of the variable cost of production. The set of the parameters and decision variables are given in tables 1 and 2, respectively.

In the proposed cell loading approach we consider the capacity constraints at a more detailed level at the higher levels of the decision making hierarchy. As stated earlier, most of the hierarchical approaches in the literature either assume infinite production capacity or deal with the capacity issues in an aggregated manner at the higher decision making levels, which might lead to an infeasible solution when we consider the detailed capacity constraints of bottleneck resources. Let's look at the

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$C_{ijt}$	: Aver. unit cost for producing one unit of family $i$ by cell $j$ in period $t$
$r_{jt}$	: Aver. cost of one regular time unit for cell $j$ during period $t$
$o_{jt}$	: Aver. cost of one overtime unit for cell $j$ during period $t$
$h_{it}$	: Aver. holding cost for family $i$ in period $t$
$d_{it}$	: Demand for family $i$ in period $t$
$a_{ij}$	: Aver. total time required to produce one unit of family $i$ at cell $j$
$BS_{ij}$	: Setup cost for family $i$ in secondary cell $j$
$BP_{ij}$	: Setup cost for family $i$ in primary cell $j$
$bs_{ij}$	: Setup time for family $i$ in secondary cell $j$
$bp_{ij}$	: Setup time for family $i$ in primary cell $j$
$Q_{ijt}$	: Initial estimate for the lot size of family $i$ in cell $j$ in period $t$
$P_j$	: Set of families which their primary cell is $j$
$S_j$	: Set of families which their secondary cell is $j$
$FS(j)$	: A feasible set of families assignable to cell $j$
$d_{k,tw}$	: Demand for item $k$ in subperiod $w$ of period $t$
$TI(i)$	: A set of items belonging to family $i$
$t$	: $n * w$ , where $n$ is an integer multiple
$PR_{kt}$	: Aver. total time required to produce one unit of item $k$ using resource $l$
$\alpha_t$	: Aver. proportion of time resource $l$ is down in period $t$
$LR(j)$	: Set of resources belonging to cell $j$
$NO(k,j)$	: Number of operations for item $k$ in cell $j$
$NI(i)$	: Number of items belonging to family $i$
$NR(j)$	: Number of resources in cell $j$
$J$	: Number of cells
$N$	: Number of families
$K$	: Number of items
$L$	: Number of resources
$T$	: Planning horizon

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Table 1. Parameters.

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$X_{ijt}$	: Number of units of family $i$ produced by cell $j$ in period $t$
$IF_{it}$	: Inventory of family $i$ at the end of period $t$
$O_{jt}$	: Overtime used by cell $j$ in period $t$
$R_{jt}$	: Regular time used by cell $j$ in period $t$
$Z_{kjt,w}$	: Number of units of item $k$ produced by cell $j$ in subperiod $w$ of period $t$
$I_{k,tw}$	: Inventory of item $k$ at the end of subperiod $w$ of period $t$
$OR_{lt}$	: Overtime used by resource $l$ in period $t$
$RR_{lt}$	: Regular time used by resource $l$ in period $t$

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Table 2. Decision variables.

following example of 3 items and 3 resources with the corresponding demand and processing times per item on each resource as shown in table 3. If we assume that the available capacity for each resource is 40 time units then total available capacity is  $40 * 3 = 120$  time units, and total required capacity is  $2.5 * 10 + 3.5 * 15 + 3 * 10 = 107.5$  time units. If we only apply an aggregated capacity check then we conclude that there is enough capacity so we proceed on. Although we cannot meet total demand requirements by producing exactly the required quantities at the required period with zero inventories since Resource 2 is a bottleneck resource and  $1 * 10 + 2 * 15 + 1 * 10 = 50 > 40$ .

An A/D scheme is applied to reduce the size of the problem, where the decomposition of the manufacturing system proceeds in three dimensions: by floor space

Item	Demand	Resource			Total Processing Time
		1	2	3	
1	10	1	1	0.5	2.5
2	15	1	2	0.5	3.5
3	10	1	1	1	3

Table 3. Aggregate capacity planning problem.

(or resource-based), by product, and by time horizon. In the floor space decomposition, the manufacturing system is divided into a set of GT cells where each cell is designed to produce a GT family or families. In the product-based decomposition, similar items are grouped into GT families, based on their designs or processes, or both. Throughout this research a GT family is defined as a set of items that require similar machinery, tooling, machine operations, jigs and fixtures. Both GT cell formation and the prerequisite product family determinations are assumed to have been done *a priori* to this planning activity, but their impact on the performance of the results are tested in §4. In the time scale decomposition, the levels of the decision hierarchy differ by complexity, scope and time horizon in that higher levels deal with longer range and more aggregated issues, and lower levels deal with short term and more specific issues. The linkage between the different levels is achieved through a feedback mechanism and a set of Lagrange multipliers as discussed in the next section. Our time scale decomposition corresponds to the shop and cell levels of the control structure developed for the automated manufacturing research facility at the National Institute of Standards and Technology in the USA, which decomposes the manufacturing functions into five levels: facility, shop, cell, workstation, and equipment as discussed by Jackson and Jones (1987).

A mathematical formulation of the problem is as follows:

$$\text{Minimize } \sum_{t=1}^T \sum_{j=1}^J \left( \sum_{i \in S(j)} C_{ijt} \cdot X_{ijt} + \sum_{i \in \Theta_j} (BS_{ij} / Q_{ijt}) \cdot X_{ijt} + \sum_{i \in \Gamma_j} (BP_{ij} / Q_{ijt}) \cdot X_{ijt} + o_{jt} \cdot O_{jt} + r_{jt} \cdot R_{jt} \right) + \sum_{t=1}^T \sum_{i=1}^N h_{it} \cdot IF_{it}$$

subject to

- production and inventory balance equations for each family:

$$\sum_{j=1}^J X_{ijt} + IF_{i,t-1} - IF_{it} = d_{it}, \text{ for } i = 1, \dots, N \text{ and } t = 1, \dots, T \quad (1)$$

- capacity restrictions for each cell:

$$\sum_{i \in S(j)} a_{ij} \cdot X_{ijt} + \sum_{i \in \Theta_j} (bs_{ij} / Q_{ijt}) \cdot X_{ijt} + \sum_{i \in \Gamma_j} (bp_{ij} / Q_{ijt}) \cdot X_{ijt} - O_{jt} = R_{jt} \quad \text{for } j = 1, \dots, J \text{ and } t = 1, \dots, T \quad (2)$$

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$$0 \leq O_{jt} \leq (\text{Upper limit}) \quad \forall j \text{ and } t \tag{3}$$

$$0 \leq R_{jt} \leq (\text{Upper limit}) \quad \forall j \text{ and } t \tag{4}$$

- production and inventory balance equations for each item:

$$\sum_{j=1}^J Z_{kjt看} + I_{kt,w-1} - I_{ktw} = d_{ktw} \quad \forall k \in TI(i), w \text{ and } t \tag{5}$$

- inventory consistency equations:

$$\sum_{k \in TI(i)} \sum_{w=1}^n I_{ktw} - IF_{it} = 0 \quad \forall i \text{ and } t \tag{6}$$

- capacity restrictions for each resource:

$$\sum_{i \in S(j)} \sum_{k \in TI(i)} \left( PR_{kl} \sum_{w=1}^n Z_{kjt看} \right) - OR_{lt} = RR_{lt} \quad \forall l \in LR(j), j \text{ and } t \tag{7}$$

$$0 \leq OR_{lt} \leq (\text{Upper Limit}) \quad \forall l \text{ and } t \tag{8}$$

$$0 \leq RR_{lt} \leq (\text{Upper limit}) \cdot (1 - \alpha_{lt}) \quad \forall l \text{ and } t \tag{9}$$

- resource consistency relations:

$$\sum_{l \in R(j)} OR_{lt} - O_{jt} = 0 \quad \forall j \text{ and } t \tag{10}$$

$$\sum_{l \in R(j)} RR_{lt} - R_{jt} = 0 \quad \forall j \text{ and } t \tag{11}$$

- non-negativity restrictions:

$$X_{ijt}, IF_{it}, O_{jt}, R_{jt}, Z_{kjt看}, I_{ktw}, OR_{lt} \text{ and } RR_{lt} \geq 0 \quad \forall i, j, k, l, t \text{ and } w. \tag{12}$$

The constraint sets (1) and (5) are the inventory balance constraints for families and items, respectively, in which both the amount of inventory left in stock at the end of each period and the demand in each period are supplied by the amount of production in each period and the amount of inventory carried over from the previous period. No backordering is allowed. Moreover, a deterministic, but time-varying, demand for every item in every time period is assumed. Constraint (6), which represents the inventory consistency equations, links the item inventories to the inventory of the associated family. This constraint requires that the inventory for a family equal to the sum of the inventories of the items contained in the family. As a result, individual items are mapped into their corresponding families. Given that  $\sum_{w=1}^n \sum_{k \in TI(i)} d_{ktw} = d_{it}$ , it can be shown that the constraint set, which includes all the resource, production and inventory constraints, implies that  $\sum_{w=1}^n \sum_{k \in TI(i)} Z_{kjt看} = X_{ijt}$  for all  $i, j$  and  $t$ ; that is, for each time period total family production equals the sum of the production quantities for its items.

Constraints (2) and (7) are the capacity feasibility constraints for GT cells and resources. Upper limits on regular and overtime usages are also defined by constraints (3), (4), (8) and (9). For the computational analysis, the upper limits on the overtime are set to 25% of the upper limits on the amount of regular time available. Constraints (10) and (11) link the available time for each GT cell to the

resources comprising that cell. An important assumption concerns the definition of the capacity, which depends on the time scale. Long-term capacity is a statistical average of actual short-term capacity. The available resource times are defined in terms of proportion of time down, such as if a resource is down 10% of the time, and this will be deducted from its available capacity as shown in constraint (9). As a result, the production quantities are determined such that they are much more likely to be within the current capacity of the system as prescribed by chance constrained programming in Charnes and Cooper (1959).

Implicit in constraint (2) is the possibility that each GT family can have more than one feasible cell for its production. A feasible cell is defined as a cell in which a family can be processed entirely within that cell considering feasibility requirements. A more detailed discussion on the formation of primary and secondary cells can be found in Akturk and Balkose (1996). It is assumed that the primary cell of a family is capable of producing the family at the lowest possible cost. Secondary cells are the ones in which the manufacture of the family is possible at a higher cost, due to both increased setup and material handling costs, assuming that all cells are initially tooled for their primary families. An additional cost is incurred when other than the primary families need to be produced at that cell. Therefore, the setup costs and setup times for the families in their secondary cells are assumed greater than the setup costs and setup times in the primary cells. The parameter,  $Q_{ijt}$ , is an initial estimate for lot size allowing the cell resource constraints to approximately account for the total setup time which is directly proportional to the number of setups required to meet the desired production quantities at each cell. Also, the definition of primary and secondary cells for each family allows the production management system to react to the variations in the families' total demand. For example, during a very low demand period, one cell may be completely shut down because of maintenance and that cell's families are assigned to some other cell.

At the cell loading level, there are three basic ways of responding to changes in demand: holding a relatively constant production rate and using inventory to satisfy demand peaks, using changes in level of production to follow demand closely, or combining these two strategies to meet demand. There are different ways to change the level of production, including overtime and assigning some of the items into their secondary cells with an additional production cost and time. Given a capacity limit, tradeoffs can be made among the costs of inventory, overtime and secondary cells. To further illustrate the mathematical formulation of the cell loading problem, we consider a numerical example involving 4 GT families and 2 manufacturing cells such that  $P_1 = \{\text{Family } 1,2\}$ ,  $P_2 = \{\text{Family } 3,4\}$ ,  $S_1 = \{\text{Family } 3\}$  and  $S_2 = \{\text{Family } 1\}$ , consequently  $FS(1) = \{1,2,3\}$  and  $FS(2) = \{1,3,4\}$ . The planning horizon consists of 4 periods. Furthermore, there are 20 items and 9 resources, and their corresponding families and cells, respectively, are as follows:  $TI(1) = \{\text{Item } 1,2,3,4,5,6\}$ ,  $TI(2) = \{\text{Item } 7,8,9,10,11\}$ ,  $TI(3) = \{\text{Item } 12,13,14,15,16\}$ ,  $TI(4) = \{\text{Item } 17,18,19,20\}$ ,  $LR(1) = \{\text{Resource } 1,2,3,4,5\}$ , and  $LR(2) = \{\text{Resource } 6,7,8,9\}$ . The cost parameters are  $C_{1,1,t} = 0.75$ ,  $C_{1,2,t} = 1.49$ ,  $C_{2,1,t} = 1.11$ ,  $C_{3,1,t} = 1.88$ ,  $C_{3,2,t} = 1.13$ ,  $C_{4,2,t} = 0.92$ ,  $r_{1,t} = 0.58$ ,  $r_{2,t} = 1.17$ ,  $o_{jt} = 2 * r_{jt}$ , and  $h_{it} = (1 + 0.05(t - 1)) * UN \sim [1.5, 2.5]$  for every  $t \in T$ . The corresponding data for each item are given in table 4.

We have created two scenarios. In the first scenario,  $RR_{lt} \leq 120$  and  $OR_{lt} \leq 30$  for every  $l$  and  $t$ . The optimal solution for the cell loading problem is summarized in table 5. In order to simplify the output, we only present the results for families and



Item	Processing Times Resource									Demand Period			
	1	2	3	4	5	6	7	8	9	$t = 1$	$t = 2$	$t = 3$	$t = 4$
1	0.27	0.29	0.34	0.30	0.31	0.49	0.48	0.47	0.52	26	45	42	37
2	0.30	0.30	0.30	0.29	0.31	0.49	0.50	0.54	0.46	37	36	33	31
3	0.30	0.27	0.26	0.31	0.31	0.49	0.53	0.49	0.52	48	35	60	28
4	0.29	0.31	0.35	0.29	0.31	0.47	0.51	0.46	0.52	32	30	31	29
5	0.34	0.29	0.26	0.33	0.35	0.50	0.49	0.47	0.52	27	32	60	56
6	0.31	0.27	0.29	0.27	0.26	0.45	0.51	0.47	0.50	35	24	44	25
7	0.34	0.27	0.34	0.30	0.27	-	-	-	-	59	30	54	30
8	0.29	0.33	0.30	0.28	0.26	-	-	-	-	44	47	35	57
9	0.33	0.29	0.34	0.29	0.33	-	-	-	-	37	55	51	35
10	0.27	0.29	0.27	0.30	0.33	-	-	-	-	27	58	46	50
11	0.25	0.31	0.32	0.26	0.30	-	-	-	-	48	56	46	53
12	0.49	0.49	0.55	0.49	0.50	0.29	0.26	0.29	0.26	28	60	42	34
13	0.45	0.54	0.52	0.50	0.49	0.29	0.29	0.25	0.26	45	58	34	48
14	0.46	0.53	0.51	0.52	0.45	0.26	0.26	0.30	0.25	40	55	35	24
15	0.48	0.50	0.50	0.49	0.52	0.34	0.30	0.29	0.30	30	47	34	34
16	0.48	0.45	0.53	0.51	0.49	0.31	0.28	0.34	0.35	56	59	50	25
17	-	-	-	-	-	0.25	0.29	0.28	0.35	27	30	47	56
18	-	-	-	-	-	0.29	0.30	0.30	0.26	29	47	48	57
19	-	-	-	-	-	0.30	0.34	0.27	0.32	43	48	35	31
20	-	-	-	-	-	0.27	0.31	0.35	0.35	51	42	37	33

Table 4. Item data for numerical example.

$r$	Regular time <i>Cell</i>		Overtime <i>Cell</i>		Inventory <i>Family</i>				Number of units produced <i>Family</i>			
	1	2	1	2	1	2	3	4	1 cell 1	2 cell 1	3 cell 2	4 cell 2
1	514.2	386.4	0	0	0	19	0	0	205	234	199	150
2	545.0	479.6	0	13.25	37	0	0	0	239	227	279	167
3	544.0	401.3	0	0	0	0	0	0	233	232	195	165
4	504.7	379.9	0	0	0	0	0	0	206	225	165	177

Table 5. Optimal solution for scenario.

cells. In this solution, all of the items are assigned to their primary cells, and both overtime and inventory options are utilized to absorb the demand changes. The objective function value is equal to 6654.6. In the second scenario, we decreased the available resource capacities in cell 1 to  $RR_{it} \leq 80$  and  $OR_{it} \leq 16$  for the first two periods. The upper limits on the resource availabilities in each cell are given in table 6. In this case, some of the items of Family 1 are assigned to their secondary cells, i.e. Cell 2, in addition to the overtime and inventory options as shown in table 7. As a result of that the objective function value is increased to 6861.4.

### 3. Solution procedure

Linear programming (LP) is a convenient type of model to use at this level because of the wide availability of LP codes. LP also permits sensitivity and parametric

$t$	Regular time <i>Cell</i>		Overtime <i>Cell</i>	
	1	2	1	2
1	400	480	80	96
2	400	480	80	96
3	600	480	120	96
4	600	480	120	96

Table 6. Upper limits on reesource availabilities for scenario 2

$T$	Regular time <i>Cell</i>		Overtime <i>Cell</i>		Inventory <i>Family</i>				Number of units produced <i>Family</i>							
	1	2	1	2	1	2	3	4	1		2		3		4	
									cell 1	cell 2	cell 1	cell 1	cell 2	cell 2		
1	400	478.6	76.7	0.0	0	0	62	0	192	13	215	-	261	150		
2	400	480	79.0	20.8	0	0	0	0	162	40	246	-	217	167		
3	586.6	401.3	0	0	0	0	0	0	270	-	232	-	195	167		
4	504.7	379.9	0	0	0	0	0	0	206	-	225	-	165	177		

Table 7. Optimal solution for scenario 2.

analysis to be performed quite easily and the information on dual values can be derived at little additional computational cost. It is also important to consider how such a production planning system would be implemented in practice. The planning horizon given by the mathematical model is posed as if all demand is known with certainty and all parameters are to be frozen over the planning horizon. Because of the uncertainties present in the planning process, a rolling horizon method with a lookahead mechanism similar to Maes and Van Wassenhowe (1986) is applied to solve this model in each period in order to deal with either fluctuations or seasonalities in demand or other inputs. The lookahead mechanism anticipates possible capacity shortages and considers the following tradeoffs to minimize the variable production cost: building up sufficient inventory in earlier periods by increasing production rates, or using overtime, or assigning families to their secondary cells with an additional cost of production, or a mixture of these alternatives. Baker (1977) describes, ‘the typical scenario of a rolling horizon procedure is as follows: solve the model and implement only the first period’s decisions; for the following period, update the model to reflect information collected in the interim, re-solve the model, and implement only the imminent decision pending subsequent model runs’. That is, the implementation of rolling horizons requires routinely updating or revising plans taking into consideration more reliable data as they become available. The rolling horizon procedure simply reflects the continuity of the production planning and scheduling process into a non-finite future.

The size of the problem is an important issue for LP applications, because the time required to find an optimum solution increases with the number of constraints.

On the other hand, recent advancements in microelectronics are making multiprocessor systems more cost-effective than a single processor. Distributing a task over a multiprocessor (or parallel) system can increase system throughput and speed up computation. Decomposition methods allow large scale models to be broken down into manageable sub-models, and then systematically reassembled. These methods show considerable promise for time critical decision support applications, especially when the methods have been adapted for and implemented on parallel computers. Decomposition methods can be inefficient on serial computers when compared to a monolithic approach, unless the subproblems have a special structure that may be exploited.

The optimization of decomposable problems comprised of a number of related subproblems is an important and frequently referred to topic in the literature with an early seminal discussion given by Geoffrion (1970). The two principal types of decomposition methods that have appeared in the literature are price directed decomposition and resource directed decomposition. In price directed decomposition, the separation is accomplished by putting prices, or dual variables, on the joint constraints and placing them in the objective function. The price directed coordination problem is concerned with calculating optimal prices on the shared resources to be used in the subproblems so that an optimal solution to the overall problem is achieved by optimizing separately each of the subproblems. In resource directed decomposition, each of the subproblems is given a portion of the shared resources. The resource directed coordination problem is concerned with effecting an apportionment that permits the overall problem to be optimized by optimizing separately each of the subproblems. Both types of decomposition are aimed at decomposing the overall problem into, essentially,  $k$  separate optimization problems. Making a choice between price and resource directed decomposition is based on which approach leads to a set of subproblems with the most exploitable structure. A discussion on the different decomposition principles is given in detail in Geoffrion (1970) and Shapiro (1993).

The solution procedure proposed for the cell loading problem is an example of a price directed decomposition. It consists of formulating a Lagrangean relaxation of the initial model and solving this dual problem by an efficient, iterative solution procedure. For the problem given in the previous section, the joint constraints, or coupling constraints, which are inventory (6), and resource consistency (10) and (11) equations, are dualized to obtain:

$$\begin{aligned}
 L(\lambda, \mu^1, \mu^2) = \text{Minimize } & \sum_{i=1}^T \sum_{j=1}^J \left( \sum_{i \in \mathcal{S}(j)} C_{ijt} \cdot X_{ijt} + \sum_{i \in \mathcal{S}_j} (BS_{ij}/Q_{ijt}) \cdot X_{ijt} \right. \\
 & \left. + \sum_{i \in \mathcal{E}^j} (BP_{ij}/Q_{ijt}) \cdot X_{ijt} + o_{jt} \cdot O_{jt} + r_{jt} \cdot R_{jt} \right) \\
 & + \sum_{i=1}^T \sum_{i=1}^N h_{it} \cdot IF_{it} + \sum_{i=1}^n \sum_{i=1}^T \lambda_{it} \left( \sum_{k \in \mathcal{I}(i)} \sum_{w=1}^n I_{ktw} - IF_{it} \right) \\
 & + \sum_{j=1}^J \sum_{i=1}^T \mu_{jt}^1 \left( \sum_{l \in \mathcal{R}(j)} OR_{lt} - O_{jt} \right) + \sum_{j=1}^J \sum_{i=1}^T \mu_{jt}^2 \left( \sum_{l \in \mathcal{R}(j)} RR_{lt} - R_{jt} \right)
 \end{aligned}$$

subject to constraint sets 1, 2, 3, 4, 5, 7, 8 and 9.

The dual problem to the original problem is:

$$(D) \max_{\lambda, \mu^1, \mu^2} L(\lambda, \mu^1, \mu^2)$$

Furthermore, the Lagrangean relaxation as given above may be separated into the following two subproblems:

- family/cell aggregation subproblem (FCA)

$$\begin{aligned} \text{Minimize } & \sum_{t=1}^T \sum_{j=1}^J \left( \sum_{i \in \mathcal{S}(j)} C_{ijt} \cdot X_{ijt} + \sum_{i \in \mathcal{B}_j} (BS_{ij} / Q_{ijt}) \cdot X_{ijt} + \sum_{i \in \mathcal{B}_j} (BP_{ij} / Q_{ijt}) \cdot X_{ijt} \right. \\ & \left. + (o_{jt} - \mu_{jt}^1) O_{jt} + (r_{jt} - \mu_{jt}^2) R_{jt} \right) + \sum_{t=1}^T \sum_{i=1}^N (h_{it} - \lambda_{it}) IF_{it} \end{aligned}$$

subject to constraint sets 1, 2, 3 and 4.

- Item/resource disaggregation subproblem for each  $t$  (IRD<sub>*t*</sub>)

$$\text{Minimize } \lambda_{it} \left( \sum_{k \in \mathcal{I}(i)} \sum_{w=1}^n I_{ktw} \right) + \mu_{jt}^1 \left( \sum_{l \in \mathcal{L}(j)} OR_{lt} \right) + \mu_{jt}^2 \left( \sum_{l \in \mathcal{L}(j)} RR_{lt} \right)$$

subject to constraint sets 5, 7, 8 and 9.

The IRD<sub>*t*</sub> model is solved over a shorter horizon,  $t$ , with periods,  $w$ , allowing a finer resolution than period  $t$  used for the subproblem FCA. For instance, items might be scheduled weekly, while the production of families would be planned monthly. The linkage mechanism for these two subproblems is resource and inventory consistency relationships which are priced out by a set of Lagrange multipliers  $\lambda$ ,  $\mu^1$  and  $\mu^2$ , which reflect the cost penalties at the item level due to the requirements set at the family level. The determination of these multipliers provides a feedback process in the hierarchical framework. Furthermore, the separation of the mathematical formulation into the FCA and IRD<sub>*t*</sub> subproblems allows us to solve these optimization problems in parallel as shown in figure 1.

The dual problem is to find  $\lambda$ ,  $\mu^1$  and  $\mu^2$  to maximize the Lagrangean as stated above. For a primal-dual approach to solving the Lagrangean, the Lagrangean is solved for a given  $\lambda$ ,  $\mu^1$  and  $\mu^2$ , and based on this solution a new set of multipliers is calculated. Recognizing that  $\lambda$ ,  $\mu^1$  and  $\mu^2$  may be interpreted as the marginal cost of having to provide additional increments of inventory, overtime and regular time in time period  $t$ , this iterative process continues until the inventory and resource consistency relationships are satisfied within an  $\varepsilon$  range of the best known feasible solution. The revision of the multipliers depends upon the current degree of inconsistency between the FCA and IRD<sub>*t*</sub> subproblems. A subgradient optimization method, similar to Held *et al.* (1974), is used to update the values of Lagrange multipliers. A validation of the subgradient optimization method can be found in Held *et al.* (1974). The  $\lambda_{it}$  values at step  $c$  are updated by the formula:

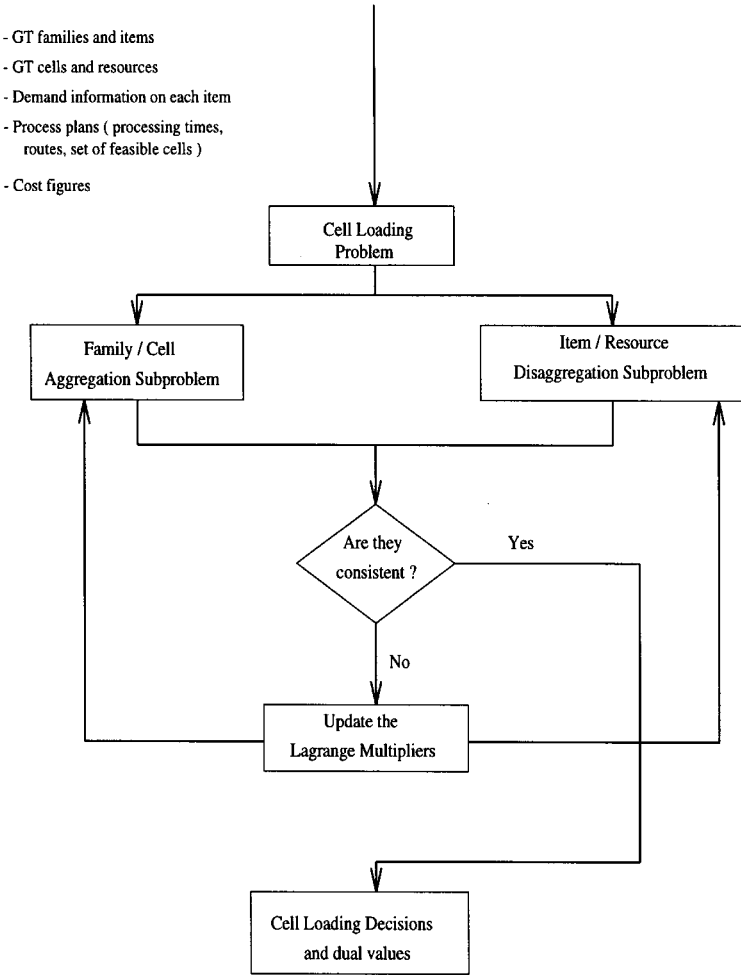


Figure 1. Flow-chart representation of the algorithm's framework.

$$\lambda_{it}^{c+1} = \lambda_{it}^c + \delta_c \left( \sum_{k \in I(i)} \sum_{w=1}^n I_{ktw} - IF_{it} \right).$$

where  $\delta_c$  is a positive scalar step size which is determined by the following formula:

$$\delta_c = \frac{\rho_c(Z^* - L(\lambda, \mu^1, \mu^2))}{\left( \sum_{k \in I(i)} \sum_{w=1}^n I_{ktw} - IF_{it} \right)^2}.$$

In this formula,  $Z^*$  is the objective function of the best known feasible solution to the original problem, which provides an upper bound on the dual problem, and  $\rho_c$  is a scalar between 0 and 2. A description of an upper bounding heuristic is given in the Appendix. The sequence of  $\rho_c$  is determined by setting  $\rho_0$ , initially to 0.01 and increasing by a factor of two whenever the Lagrangean dual has failed to increase in a specific number of iterations. Fisher (1985) has shown empirically that this rule

Factors	Definition	Low	High
A	S/I ratio	0.75	1.25
B	Number of families	15	35
C	Upper limits on resource availability	No idle time	10% idle time
D	Number of GT cells	5	10
E	Set of items within each family	Low variability	High variability

Table 8. Experimental factors.

works well, although it is not guaranteed to satisfy the complementary slackness condition for convergence.

#### 4. Computational analysis

The proposed cell loading algorithm and the matrix generator for the problem formulation are coded in the C language. An optimal solution is found by using the CPLEX optimization package on a Sparcstation 10 under SunOS 5.4. We wish to investigate to what degree the proposed approach is robust in the face of uncertainty and how sensitive it is to the assumptions we have made throughout this research with regard to machine-component groupings, GT cells and families, inclusion of new products to the existing families, and resource availabilities. There are a large number of variables which could have an effect on the performance of the cell loading problem. Within the conceptual framework of the hierarchical procedure, an experimental design is developed with two objectives in mind. The first objective is to generate a set of test problems to calculate the computation time to find an optimal solution. The second objective is to explain the relationships between the variable production cost and the system parameters.

There are five experimental factors that can affect the efficiency of the proposed approach, which are listed in table 8. The initial estimation of lot sizes depends on the direct setup cost for each item, and plays an important role within the context of the trade-offs between inventory holding and overtime costs since it determines the number of setups required. The direct setup cost, to make the results of the research meaningful, must be compared to the inventory holding cost as a ratio, S/I, as suggested by Maes and Van Wassenhowe (1986). The S/I ratios, factor A, are used to find the initial lot size of family  $i$  in cell  $j$  in period  $t$  as  $Q_{ijt} = \sqrt{2 \cdot S/I \text{ ratio} \cdot d_{it}}$  for every  $i \in FS(j)$ . The representative ranges for factor B, the number of families, and factor D, the number of cells, are based on the studies done by Hyer and Wemmerlov (1989), and Wemmerlov and Hyer (1989) on current practices seen in industry for GT and CM systems. In addition, an assignment of the items to the GT families is one of the objectives of the part-family and machine-cell formation problem. These assignments are done depending upon the similarities that exist between the items, and similarity coefficients are calculated using several criteria as discussed in Shafer and Rogers (1993) and Offodile *et al.* (1994). Factor E, the number of items in each family, reflects the fact that the variability within each family could be different depending upon the threshold values used to form the GT families. A high threshold value means a low feature variability and a high similarity among the items in each family. Since the total number of items is a fixed parameter for all runs, factor E is used to measure the impact of variability in each family by varying the size of the each family, and the processing time for each

Parameters	Set of values
Total number of items, $K$	250
Total number of resources, $L$	50
Number of periods, $T$	12
Number of subperiods per period	4
Cost of production, $c_{ijt}$	UN~ [0.75, 1.25] if $i \in P_j$ UN~ [1.5, 2.0] if $i \in S_j$
Cost of regular time, $r_{jt}$	UN~ [1.25, 2.0]
Cost of overtime, $o_{jt}$	$2 * r_{jt}$
Inventory holding cost, $h_{it}$	$(1 + 0.05(t - 1)) * UN~ [1.5, 2.5]$
Setup cost for the families, $BS_{ij}$ and $BP_{ij}$	$0.03 * c_{ijt} * d_{it}$
Processing times, $PR_{kl}$	
(1) Low variability	UN~ [0.25, 0.35] if $i \in P_j$ UN~ [0.45, 0.55] if $i \in S_j$
(2) High variability	UN~ [0.2, 0.4] if $i \in P_j$ UN~ [0.4, 0.6] if $i \in S_j$
Setup times	$(0.1 * a_{ij})$ if $i \in FS(j)$
Number of operations per item	UN~ [3, 5]
Effective demand, $d_{ktw}$	UN~ [6, 15]

Table 9. Fixed parameters.

item  $k$  on resource  $l$ ,  $PR_{kl}$ , is based upon the degree of similarity in each family, as can be seen in table 9. The levels of factor C specify the upper limits on resource availability where the low level corresponds to a congested shop floor, while the high level represents a 10% idle time. Since there are five factors and two levels, our experiment is a  $2^5$  full-factorial design, which corresponds to thirty-two treatment combinations. The number of replications of each combination is taken as five producing 160 different randomly generated runs.

Other variables in the system are treated as fixed parameters and summarized in table 9, where  $UN~ [a, b]$  represents a uniformly distributed random variable in interval  $[a, b]$ . All of the parameters' values are constant throughout the planning horizon, except the inventory holding cost for family  $i$  in period  $t$ ,  $h_{it}$ , which increases over time to approximately account for factors like inflation and time value of money. Furthermore, there are some other parameters, such as  $d_{it}$  and  $a_{ij}$ , that assume fixed parameter values. The effective demand for each item in each subperiod of each period,  $d_{ktw}$ , is fixed. Therefore, the demand for each family in a particular period should be calculated by summing the demands of all of the items belonging to that family corresponding to that period; i.e.

$$d_{it} = \sum_{w=1}^n \sum_{k \in T(i)} d_{ktw} \quad \forall i, t$$

The processing time of each item at each feasible resource,  $PR_{kl}$ , and the number of operations for each item are fixed. So, the average total time required to produce one unit of family  $i$  at cell  $j$ ,  $a_{ij}$ , should be the product of the average processing time of an item belonging to family  $i$  at cell  $j$  and the average number of operations required for an item  $k$  in family  $i$ . A mathematical expression is given below.

Factors					CPU times (seconds)		
A	B	C	D	E	Minimum	Average	Maximum
0	0	0	0	0	12	38.8	86
1	0	0	0	0	15	39.0	81
0	1	0	0	0	14	21.2	31
1	1	0	0	0	13	22.0	32
0	0	1	0	0	17	40.6	80
1	0	1	0	0	16	39.2	80
0	1	1	0	0	15	22.6	32
1	1	1	0	0	14	22.2	30
0	0	0	1	0	111	129.0	146
1	0	0	1	0	100	125.4	138
0	1	0	1	0	45	50.8	57
1	1	0	1	0	40	50.4	60
0	0	1	1	0	106	134.0	161
1	0	1	1	0	8	102.6	149
0	1	1	1	0	42	54.0	67
1	1	1	1	0	39	52.0	65
0	0	0	0	1	16	45.8	119
1	0	0	0	1	15	37.8	80
0	1	0	0	1	13	20.4	29
1	1	0	0	1	14	21.0	32
0	0	1	0	1	16	41.0	82
1	0	1	0	1	15	40.8	79
0	1	1	0	1	16	21.4	28
1	1	1	0	1	13	20.8	27
0	0	0	1	1	100	128.2	156
1	0	0	1	1	110	137.2	153
0	1	0	1	1	47	53.4	57
1	1	0	1	1	44	54.6	68
0	0	1	1	1	104	132.4	159
1	0	1	1	1	103	136.4	166
0	1	1	1	1	43	53.6	61
1	1	1	1	1	43	52.4	65
overall					8	60.7	166

Table 10. Results of the computational experiments.

$$a_{ij} = \frac{(\sum_{k \in I(i)} \sum_{l \in R(j)} PR_{kl})(\sum_{k \in I(i)} NO(kj))}{NI(i)^2 \cdot NR(j)} \quad \forall i \in FS(j).$$

Table 10 summarizes the CPU times (in seconds) to find the optimum solution for each run, along with the minimum, average, and maximum CPU times (based on five random replications) for each factor combination. In this table, low and high levels for each factor are represented by 0 and 1, respectively. For all 160 problems reported in this table, the maximum CPU time was 166 s, whereas the average time was 60.7 s. The maximum CPU time was found for the factor combination of (1 0 1 1 1). In other words, all the factors except the number of families were at their high levels. On the other hand, the minimum average computation time is found for the factor combination of (0 1 0 0 1), where the S/I ratio, upper limit on the resource availability and the number of GT cells were at their low levels. Furthermore, if we would like to use the Lagrangean relaxation procedure discussed in § 3 on a parallel pro-



Replication	Upper Bound	Optimum		% dev.
		Solution	Comp. time	
Rep. 1	210 773·7	208 101·8	73·88	1·33
Rep. 2	209 556·7	204 966·3	57·25	2·28
Rep. 3	234 940·4	230 446·8	53·81	1·95
Rep. 4	233 865·1	231 608·9	65·22	1·04
Rep. 5	237 990·6	233 384·2	53·13	2·11
Overall	225 425·3	221 701·6	60·66	1·68

Table 11. Comparison of computational results.

cessor, then the Lagrange multipliers are updated by a subgradient optimization method, which requires a calculation of an upper bound for the cell loading problem. In table 11, we compare the average results of the upper bound with the average optimum solution for each replication along with the computation time to find the optimum solution and the % deviations. The minimum % deviation was 0·000 006% for the factor combination of (1 0 1 1 0), whereas the maximum one was 6·05% for the factor combination of (1 0 1 0 1). As mentioned above, the average computation time to find the optimum solution was 60·7 s, which indicated that within the scope of our experimental framework these problems can be solved optimally using a commercial optimization package on a serial processor without using a means of decomposition.

Finally, a two-way analysis of variance (ANOVA) test was applied to two performance measures, the optimum value of the total production cost and the computation time, to test the equality of observed responses from the different treatment combinations of the chosen factors. The number of GT cells and families, factors D and B, respectively, were found to be significant at the 0·1% significance level on the computation time criterion, followed by the factor E, the number of items in each family, that was used to represent the variability of the items within each family. For the total production cost criterion, the factor D was the only significant one at the 0·1% significance level. The number of GT cells, factor D, is the most important output of classification and coding systems (CCS) and cell formation techniques discussed in §1. ANOVA tables indicated that the factor D has the most significant effect on both of the performance measures considered. Therefore, an interface between CCS and cell loading decisions becomes a critical issue in any GT based production planning approach. Unfortunately, most of the existing approaches do not consider parameters associated with the cell loading activities during the initial design stage.

Another important question is the sensitivity of cell loading decisions with respect to the inclusion of new products into the existing GT families. Our computational experiments indicate that if there is more feature variability among the items in each family, there will, consequently, be more in-process inventories, which is not desirable. Therefore, if the feature variability of the items in each family is reduced then the in-process inventory levels will tend to be decreased. Welke and Overbeeke (1988) report their experiences at Deere & Co., and argue that GT families and cells are generally constructed and based on the products that are currently being manufactured. In order to maintain the existing GT cell formation valid for a long time,

certain plans must be made so that new products can be designed to fully conform to the existing manufacturing cells, i.e. design for manufacturing. Therefore, the existing GT database should be an influencing factor as to how the new parts will be designed and processed within a CM system.

## 5. Conclusions

In this research, an aggregate planning model of the cell loading problem for CM systems has been developed to minimize the variable production cost. The proposed approach has several advantages over models in the current literature on hierarchical planning and cell loading. First, the proposed approach allows more accurate portrayal of the operation of CM systems by using the capacity constraints to assess the impact of the cell loading decisions on the lower levels. As a result, the production rates are determined such that they are much more likely to be within the current capacity of the system. Another advantage is the enhanced computational tractability which is achieved by incorporating and combining the advantages of a hierarchical planning, a CM shop configuration, and decomposition principles including Lagrangean relaxation and its related pricing mechanism. An aggregation/disaggregation scheme with respect to products, resources, and time horizon is also included in the mathematical formulation. The determination of Lagrange multipliers, which reflect the cost penalties at the item level due to requirements set at the family level, provides a feedback process within the cell loading problem to satisfy the inventory and resource consistency constraints. Furthermore, the ANOVA tables indicated that the number of GT cells and families had a significant effect both on the total production cost and the computation time to find an optimum solution. Therefore, in future CCS, the feedback information from the cell loading level should provide a more significant input to the GT part-family machine-cell formation in addition to the other commonly used factors, i.e. design and processing requirements.

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## Appendix: Calculation of the upper bound

The overall aim of the upper bounding algorithm is to produce all the required items at their primary cells in the amounts that they are demanded in each time period. An algorithmic description of the proposed algorithm is given below.

*Step 1.* Equalize the production level of each item at its primary cell at a particular period to its demand at that period such that  $Z_{kjt} = d_{kt}$   $\forall k \in TI(i)$  and  $i \in P_j$ .

*Step 2.* Calculate the regular time and overtime requirements for each of the resources in each cell. Let

$$TR_{jt} = \sum_{i \in S(j)} \sum_{k \in TI(i)} \left( 1.1 * PR_{kl} * \sum_{w=1}^n Z_{kjt} \right) \quad \forall t, j \text{ and } l \in LR(j).$$

If  $TR_{it} \leq UL_{it}$  then  $RR_{it} = TR_{it}$  and  $OR_{it} = 0$ . Otherwise,  $RR_{it} = UL_{it}$  and  $OR_{it} = TR_{it} - UL_{it}$ , where  $UL_{it}$  is the upper limit on the availability of regular time at resource  $l$  in period  $t$ .

Step 3. Equalize the amount of production for each item at its primary cell in a particular period to the total demand for that family in that period such that

$$X_{ijt} = d_{it} = \sum_{k \in T(i)} \sum_{w=1}^n Z_{kjt_w} \quad \forall i \text{ and } i \in P_j.$$

Step 4. Calculate the regular time and overtime requirements at each cell. Let  $TC_{jt} = \sum_{i \in P_j} a_{ij} * X_{ijt} \quad \forall j \text{ and } t$ . If  $TC_{jt} \leq UL_{jt}$  then  $R_{jt} = TC_{jt}$  and  $O_{jt} = 0$ . Otherwise,  $R_{jt} = UL_{jt}$  and  $O_{jt} = TC_{jt} - UL_{jt}$ , where  $UL_{jt}$  is the upper limit on the availability of regular time in cell  $j$  in period  $t$ .

Step 5. Calculate the upper bound as follows:

$$UB = \sum_{t=1}^T \sum_{j=1}^J \left( \sum_{i \in P_j} (C_{ijt} + BP_{ij}/Q_{ijt}) \cdot X_{ijt} + o_{jt} \cdot O_{jt} + r_{jt} \cdot R_{jt} \right) + \sum_{t=1}^T \sum_{i=1}^N h_{it} \cdot IF_{it}.$$

The upper bounding algorithm utilizes the just-in-time logic in which the overall goal is to produce exactly the required quantities at precisely at the required period with zero inventories. The upper bound on the total production cost, which is based on the local information in each period, will tend to be larger than the optimum cost for the overall planning horizon when there is a high variability in demand and resource availabilities in the planning horizon as shown in table 11.

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