3.18, 2.06 and 1.56%, respectively. The GMMSVM performances with variance of λ in verification and identification tasks are shown in Fig. 2. Fig. 2*a* shows the equal error rates (false accept equals to false reject) in 50 speakers verification. Fig. 2*b* shows the identification accuracy in 50 speakers identification. Both verification and identification tasks achieve best performance while λ = 0.95, where the highest identification accuracy is 99.25% and the lowest verification equal error rate is 0.5%. It can be seen that by incorporating GMM in SVM output, the performances of speaker identification and verification are greatly improved.

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References

1 FARRELL, K.R., MAMMONE, R.J., and ASSALEH, K.T.: 'Speaker recognition using neural networks and conventional classifiers', *IEEE Trans. Speech Audio Process.*, 1994, **2**, (1), Part II

2 CORTES, C., and VAPNIK, V.: 'Support vector networks', *Mach. Learn.*, 1998, **20**, pp. 273–297

3 JAAKKOLA, M.D.T., and HAUSSLER, D.: 'A discriminative framework for detecting remote protein homologies', *J. Comput. Biol.*, 1998, pp.

4 FARRELL, K., KOSONOCKY, S., and MAMMONE, R.: 'Neural tree network/vector quantization probability estimators for speaker recognition'. Neural Networks for Signal Processing IV, Proc. 1994 IEEE Workshop, 1994

Time-varying repetitive control for better transient response and stochastic behaviour

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A stability proof is given for a time-varying repetitive control strategy. It is illustrated by a simulation example that this time varying strategy can be used to improve the transient reponse and stochastic behaviour of the repetitive control system simultaneously.

Introduction: Repetitive control is the discipline which studies the control strategies developed for tracking/rejection of periodic signals in control systems (see [1]). A prototype discrete-time repetitive controller was developed in [2]. This structure was modified by [3] for better stochastic behaviour and stability robustness. Motivated by these repetitive control strategies, a linear quadratic (LQ) optimal repetitive control structure was developed for linear time-invariant (LTI) discrete-time systems [4]. It was observed in this work that there was a trade-off between the transient reponse and the stochastic behaviour of the LQ-optimal repetitive control system. In this Letter, we consider a time-varying unit in the LTI repetitive controller of [4] and give the condition for bounded-input bounded-output (BIBO) stability. Minor notational preferences are as follows. Systems are denoted by transfer functions of z and input/output relations are expressed in operator notation ($Hx(t)$ is the output of system H when it is excited by x). For a transfer function H, we assume a polynomial description of the form $H(z) = N_H(z)/D_H(z)$, where N_H and D_H are coprime polynomials. The denominator degree is shown as $n_H = \deg D_H$. Finally, $H(z^{-1})$ is expressed as $H^*(z)$ and the z dependency is suppressed wherever appropriate.

Discrete-time LQ-optimal repetitive control: The infinite horizon (or steady-state) frequency weighted LQ cost is defined as

$$J_{LQ} \triangleq \lim_{\tau\to\infty} \frac{1}{\tau} \sum_{t=0}^{\tau-1} [e(t)]^2 + [Fu(t)]^2 \tag{1}$$

where e is the tracking error, u is the control input (see Fig. 1) and F is a stable and minimum phase filter. Minimisation of this LQ cost was considered for periodic reference (r) and (d) disturbance signals in [4]. The LQ-optimal repetitive control structure developed by [4] is shown in Fig. 1. Here the control unit is formed by an arbitrary stabilising controller $C = N_C/D_C$ and a plug-in unit (dashed box) which guarantees the LQ optimality of the overall repetitive control system. All of the blocks denote LTI systems. In the plug-in unit, k is the repetitive control gain, C_r is the delayed positive feedback unit, and F_{ff} and F_{fb} are finite impulse response type filters. C_r is a basic ingredient of repetitive control structures (see [2]) and its transfer function is given by

$$C_r(z) = \frac{1}{1 - z^{-n}} \tag{2}$$

where n is the period of the signals under consideration. However, the filters are constructed as

$$F_{ff}(z) = z^{n-n_C} QMM^* D_F D_F^* N_P^* \tag{3}$$

$$F_{fb}(z) = z^{n-n_C} QMM^* N_F N_F^* D_P^* \tag{4}$$

where M is a design polynomial and Q is the characteristic polynomial of the feedback loop formed by $P = N_P/D_P$ and C. As is well known, Q is given by

$$Q(z) = N_P N_C + D_P D_C \tag{5}$$

To have an implementable system, M should have a degree less than $n - n_P - n_F$. If the controller C stabilises the plant P, the LQ-optimal repetitive control system will also be stable with a k in the range (0, 2), when there is no unstable pole/zero cancellation in the loop and M satisfies [4]

$$\left|M(e^{j\omega})G(e^{j\omega})\right| \le 1 \tag{6}$$

Here G is defined as the solution of the spectral factorisation equation

$$GG^* = N_F N_F^* D_P D_P^* + D_F D_F^* N_P N_P^* \tag{7}$$

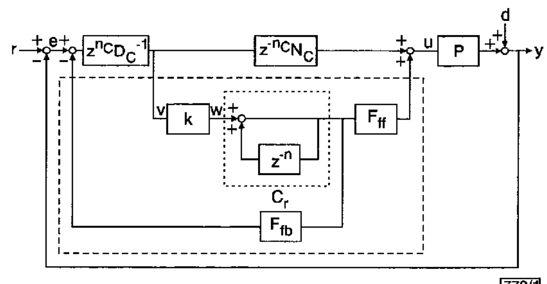

Fig. 1 *Discrete-time LQ-optimal repetitive control system*

Time-varying repetitive control: As is obvious from Fig. 1, the behaviour of the LQ-optimal repetitive control system becomes similar to that of the feedback system formed by P and C, as k gets close to zero. However, if M is chosen to satisfy

$$\left|M(e^{j\omega})\right| \simeq \frac{1}{|G(e^{j\omega})|} \tag{8}$$

tracking/rejection of the periodic signals will be fast with $k = 1$ and slow with $k \simeq 0$ [4]. If the system formed by P and C has a good stochastic behaviour (i.e. good response to stochastic disturbances), it will be useful to keep k close to zero. But in this case the transient response of the system might be undesirable. This problem can be circumvented by a time-varying k (which is unity at the beginning and close to zero in the steady-state). We show below that the system of Fig. 1 is stable even when k is a time-varying gain.

Theorem 1: The system of Fig. 1 is BIBO stable for a time-varying k with $0 < k(t) < 2$, if there is no unstable pole/zero cancellation in the loop and M satisfies eqn. 6.

Proof of Theorem 1: If k is a time-varying gain, we will have

$$w(t) = k(t)v(t) \tag{9}$$

As all the other blocks in the control system are linear and time-invariant, we can obtain the transfer from w to v (in operator notation) as

$$v(t) = z^{n_C} D_P Q^{-1}(r(t) - d(t)) - z^{-n} C_r MM^* GG^* w(t) \tag{10}$$

With the guarantee that there are no unstable pole/zero cancellations, we can apply the Circle criterion (see [5]) to eqns. 9 and 10 for stability analysis. For a similar application of this criterion, the reader is referred to [6]. According to the discrete-time version of this criterion, when $0 < k(t) < 2$, eqns. 9 and 10 will generate bounded v and w from bounded $r - d$ provided that $z^{n_C} D_P Q^{-1}$ is a stable transfer function and

$$\Re\left\{e^{-j\omega n} C_r(e^{j\omega}) \left|M(e^{j\omega})\right|^2 \left|G(e^{j\omega})\right|^2\right\} \geq -\frac{1}{2} \quad \forall\omega \in [0, 2\pi) \tag{11}$$

is satisfied ($\Re$ denotes the real part). As C is assumed to be stabilising, Q has all its roots inside the unit circle, which means that $z^{n_C} D_P Q^{-1}$ is stable. However, a straightforward manipulation of eqn. 11 leads to the equivalent eqn. 6 and this concludes the stability proof. □

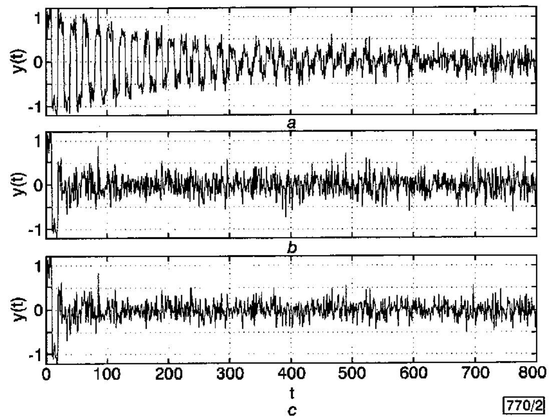

Fig. 2 *Simulation results*

a $k = 0.1$
b $k = 1.0$
c $k = 0.9^{t/30}$

Example simulations: In this Section we present the results of three simulations realised with $P(z) = (z - 2)/(z^2 - 1.4z + 0.45)$. We set $r = 0$ and $d = d_p + d_s$, where d_p is a square wave of variance unity and period 20, and d_s is a white noise of variance 0.03. Simulations are performed by the LQ-optimal control system of Fig. 1 with $N_C = 0$, $D_C = 1$, $N_F = 0$, $D_F = 1$, $M = 0.4706z^3 + 0.2353z^2 + 0.1176z + 0.0588$ for $k = 0.1$, 1.0 and $0.9^{t/30}$. As shown in Fig. 2, the transient response is better with the unity gain, whereas the stochastic behaviour is superior with $k = 0.1$. For the case of time-varying gain, a desirable transient reponse is obtained together with an acceptable stochastic behaviour.

Conclusions: We considered the LQ-optimal repetitive controller of a previous work and showed that the system is BIBO stable with a time-varying repetitive control gain. We illustrated by an example that, if the time variation of the repetitive control gain is adjusted appropriately (to give a gain close to unity at the beginning and close to zero at the steady-state), the transient reponse and stochastic behaviour of the repetitive control system can be improved simultaneously.

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References

1 HILLERSTRÖM, G., and WALGAMA, K.: 'Repetitive control theory and applications – a survey'. Proc. 13th IFAC Triennial World Congress, San Francisco, CA, USA, June/July 1996, Vol. D, pp. 1–6

2 TOMIZUKA, M., TSAO, T.C., and CHEW, K.K.: 'Analysis and synthesis of discrete-time repetitive controllers', *ASME J. Dyn. Syst. Meas. Control*, 1989, **111**, (3), pp. 253–258

3 CHEW, K.K., and TOMIZUKA, M.: 'Steady-state and stochastic performance of a modified prototype repetitive controller', *ASME J. Dyn. Syst. Meas. Control*, 1990, **112**, (1), pp. 35–41

4 KÖROĞLU, H., and MORGÜL, Ö.: 'Discrete-time LQ optimal repetitive control'. Proc. American Control Conf., San Diego, CA, USA, June 1999, pp. 3287–3291

5 DESOER, C.A., and VIDYASAGAR, M.: 'Feedback systems: input-output properties' (Academic Press, New York, 1975)

6 MA, C.C.H.: 'Stability robustness of repetitive control systems with zero phase compensation', *ASME J. Dyn. Syst. Meas. Control*, 1990, **112**, (3), pp. 320–324