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To link to this article: https://doi.org/10.1080/13504850410001674867

Published online: 21 Aug 2006.

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Testing Marshall-Lerner condition: 
a non-parametric approach

SYED F. MAHMUD, AMAN ULLAH† and ERAY M. YUCEŁ*

Department of Economics, Bilkent University, Ankara 06800, Turkey
and
Department of Economics, University of California, Riverside, USA

In this study, non-parametric kernel estimation technique has been employed to estimate import and export price elasticities for six developed countries. Based on the estimates of these elasticities Marshall-Lerner condition has been examined. In general the condition is only partially satisfied in the sub-sample periods. The results also suggest that the condition is more likely to be satisfied under fixed exchange rate regime.

I. INTRODUCTION

In formulating commercial policy or exchange rate policy, responsiveness of trade flows to relative price changes is one of the important considerations. The effects of currency depreciation on a country’s trade balance are traditionally analysed by examining Marshall-Lerner Condition (MLC). The condition suggests that devaluation of a currency will improve a country’s trade balance in the long-run if the sum of absolute values of import and export demand price elasticities exceeds unity. See for example, Bahmani-Oskooee (1986, 1998), Bahmani-Oskooee and Niroomand (1998), Khan (1974), and Houthakker and Magee (1969). Some other studies, such as Shirvani and Wilbratte (1997), Arize (1994), Bahmani-Oskooee (1985, 1991), Rose (1991), Himarios (1985, 1989) and Miles (1979) have constructed a direct link between the exchange rate and the trade balance. Both groups of these studies normally employ linear parametric models with constant coefficients. However, it is well known that if the parametric model is not correctly specified then the estimates become biased and it may give misleading results regarding MLC. In addition the assumption of constancy of the coefficient across observations may be neither true nor helpful in getting the examination of MLC across observations.

A modest aim of this study is to estimate import and export demand elasticities using non-parametric kernel estimation technique. Use of a non-parametric technique has several advantages. First, the estimates of elasticities are not constrained by any a priori assumption about the functional forms of the estimating equations. Second, it is possible to obtain point estimates of elasticities for each time period in the sample and therefore a more eloquent analysis is possible, see Fan and Gijbels (1996) and Pagan and Ullah (1999).

II. METHODOLOGY

Consider the stochastic process \( \{y_t, x_t\}, \ t = 1, 2, \ldots, n; \) where \( y_t \) is a scalar and \( x_t = (x_{t1}, x_{t2}, \ldots, x_{tq}) \) is \((1 \times q)\) vector which may contain the lagged values of \( y_t \). The regression model is \( y_t = m(x_t) + u_t \), where \( m(x_t) = E(y_t | x_t) \) is the true but unknown regression function and \( u_t \) is the error term such that \( E(u_t | x_t) = 0 \).

If \( m(x_t) \) is a correctly specified family of parametric regression then one can construct the ordinary least squares (OLS) estimator of \( m(x_t) \). For example, if \( m(x_t) = \alpha + x_t'\beta = X_t\delta \), where \( \delta = (\alpha \ \beta') \) and \( X_t = (1 \ x_t) \), is linear one can obtain the OLS estimator of \( \delta \) by minimizing \( \sum u_t^2 = \sum (y_t - X_t\delta)^2 \) as

\[
\hat{\delta} = (X'X)^{-1}X'y. \tag{1}
\]
However, it is well known that if the specified regression $X_i \delta$ is incorrect then the OLS $\hat{\delta}$ and hence $\hat{m} = X_i \hat{\delta}$ are inconsistent and biased and they may provide misleading results.

An alternative approach is to use the consistent non-parametric regression estimation of the unknown $m(x)$ by the local linear least squares (LLLS) method. For obtaining the LLLS estimator one first writes Taylor series expansion of $m(x_i)$ around $x$ so that

$$y_i = m(x_i) + u_i = m(x) + (x_i - x)m'(x) + v_i$$

$$= \alpha(x) + x_i \beta(x) + v_i = X_i \delta(x) + v_i,$$

where $\alpha(x) = m(x) - x \beta(x)$, $\delta(x) = [\alpha(x) \beta(x)]'$, and $\beta(x) = m''(x)$. Then solving the problem:

$$\min \sum_{i=1}^n v_i^2 K_{x_i} = \min \sum_{i=1}^n (y_i - X_i \delta(x))^2 K_{x_i}$$

with respect to $\delta(x)$ gives the LLLS estimator is obtained as:

$$\tilde{\delta}(x) = (X'K(x)X)^{-1}X'K(x)y$$

where $K(x)$ is a diagonal matrix of the kernel (weight) $K_{x_i} = K((x_i - x)/h)$ and $h$ is the window width. The LLLS estimators of $\alpha(x)$, $\beta(x)$ and $m(x)$ are calculated as $\tilde{\alpha}(x) = [1 0] \tilde{\delta}(x)$, $\tilde{\beta}(x) = [0 1] \tilde{\delta}(x)$ and $\tilde{m}(x) = \tilde{\alpha}(x) + x \tilde{\beta}(x)$. These LLLS estimators are consistent, for further details on properties see Fan and Gijbels (1996) and Pagan and Ullah (1999).

The LLLS estimators of $\delta(x)$ and $m(x)$ are also called the non-parametric kernel estimators which are essentially the local linear fits to the data corresponding to these $x_i$'s which are in the interval of length $h$ around $x$, the point at which $\delta$ is calculated. In this sense the LLLS estimator provides the varying estimates of $\delta$ with changing values of $x$. It depends on the kernel function $K$ and the window width $h$. The function $K$ is chosen to be a decreasing function of the distances of the regressor $x$, from the point $x$, and the window width $h$ determines how rapidly the weights decrease as the distance of $x_i$ from $x$ increases. In our empirical analysis an optimal parabolic kernel and the cross validated window width has been considered, for further details see Pagan and Ullah (1999, ch. 3) and Racine (1999).

III. EMPIRICAL RESULTS AND DISCUSSION

The study employs standard model specification for import and export demand functions in which demand equations, dropping the time subscripts, are in the form of:

$$\log M = m[\log(\text{PM}/\text{PD})], \log Y] + u \quad [\text{import model}]$$

$$\log X = m[\log(\text{PX}/\text{PW})], \log YW] + v \quad [\text{export model}]$$

In the import model, $M$ is the volume of imports. PM and PD are the domestic import price and domestic price level respectively, and $Y$ is the domestic income. In the export model, $X$ is the volume of exports. PX and PW are the domestic and world export prices respectively. YW stands for the world real income. $u$ and $v$ are the error terms associated with each observation.

### Table 1. Point elasticity estimates

<table>
<thead>
<tr>
<th>Country</th>
<th>Price elasticities</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>Australia</td>
<td>-0.30</td>
<td>-0.07</td>
<td>-0.37</td>
<td>-0.18</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.21</td>
<td>-0.59</td>
<td>-0.25</td>
<td>-0.32</td>
</tr>
<tr>
<td>Japan</td>
<td>0.12</td>
<td>-0.54</td>
<td>0.25</td>
<td>-0.40</td>
</tr>
<tr>
<td>Norway</td>
<td>-0.16</td>
<td>-0.87</td>
<td>-0.21</td>
<td>-0.73</td>
</tr>
<tr>
<td>UK</td>
<td>-0.12</td>
<td>-0.44</td>
<td>-0.09</td>
<td>-0.26</td>
</tr>
<tr>
<td>US</td>
<td>-0.42</td>
<td>-0.26</td>
<td>-0.30</td>
<td>-0.23</td>
</tr>
<tr>
<td></td>
<td>Income elasticities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>1.46</td>
<td>2.11</td>
<td>1.71</td>
<td>2.41</td>
</tr>
<tr>
<td>Germany</td>
<td>1.89</td>
<td>1.71</td>
<td>1.99</td>
<td>1.59</td>
</tr>
<tr>
<td>Japan</td>
<td>2.02</td>
<td>2.34</td>
<td>1.29</td>
<td>2.74</td>
</tr>
<tr>
<td>Norway</td>
<td>1.24</td>
<td>2.23</td>
<td>1.13</td>
<td>2.27</td>
</tr>
<tr>
<td>UK</td>
<td>1.74</td>
<td>1.43</td>
<td>1.57</td>
<td>1.24</td>
</tr>
<tr>
<td>US</td>
<td>1.75</td>
<td>2.09</td>
<td>1.80</td>
<td>1.45</td>
</tr>
</tbody>
</table>

A: Local linear least squares estimates, with density weighting.
B: Local linear least squares estimates, with equal weights.
C: Ordinary least squares estimates.
Shaded columns are for export elasticities.
The application of non-parametric techniques is useful for large samples, and availability of data is an important factor in the selection of countries. Quarterly data and estimated trade elasticities for only six developed countries, which are Australia, Germany, Japan, Norway, the United Kingdom, and the United States have, therefore, been used.

Two types of non-parametric estimates are obtained. First the non-parametric LLLS $\hat{\beta}(x)$ from (2.4) has been estimated with respect to each regressor at each sample period.

In each panel, simple sum of the pointwise varying price elasticities of import and export demand functions is plotted against time. The values in parentheses show the percentage of the sample period that the Marshall-Lerner condition is satisfied.

Fig. 1. *Pointwise varying elasticity sums*

point, with their associated standard errors. This is referred to here as varying point estimates. According to our specification, a derivative estimate \( \hat{\beta}_j(x_t) \) is the partial derivative of a trade volume estimate with respect to a price or income variable. Since our variables are defined in logarithms, these partial derivatives are price and income elasticities of trade flows, evaluated at each sample point. Second, our nonparametric point estimates which describe the global behaviour of these elasticities are computed. A point estimate can be obtained by simply averaging the pointwise elasticities using equal weights:

\[
\hat{\beta}_j = \frac{1}{n} \sum_{t=1}^{n} \hat{\beta}_j(x_t)
\]

where \( \hat{\beta}_j(x_t) \) denotes the partial derivative of the conditional mean with respect to \( j \)-th regressor at \( t \)-th observation, \( \hat{\beta}_j \) has the common drawback of being sensitive to extreme values. Therefore, a second way in obtaining a point estimate is to calculate a weighted-average of pointwise elasticities, using the estimated density function of the regressor as weights (Pagan and Ullah, 1999). The point estimate, in this case, is given by:

\[
\hat{\beta}_j' = \sum_{t=1}^{n} \hat{\beta}_j(x_t) \hat{f}(x_t) / \sum_{t=1}^{n} \hat{f}(x_t)
\]

where \( \hat{f}(x_t) \) denotes the density ordinate for regressor \( j \) at observation \( t \). Both methods have been employed in this study.

Using these point estimates of elasticities one can check whether the MLC is satisfied for the countries in our sample. When varying point estimates of elasticities are employed, one can also examine the condition through time. The point estimates of import and export elasticities, using (3.3) and (3.4), are reported in Table 1. Most of the elasticity estimates have the expected sign. The results based on a parametric approach have also been reported in Table 1. These include our OLS estimates (2.1) on the same database and Bahmani-Oskooee and Nirooomand (1998) where different data had been employed. In general our non-parametric results are different from the results based on the parametric approach. For instance, the non-parametric estimate of export elasticity for US is highly inelastic compared to the parametric results. This may be because the parametric estimators can be biased due to the misspecification of linear model. One of the primary objectives of this paper is to examine the MLC using non-parametric estimates of elasticities. One straight way to check for the condition is to take simple sum of point estimates of import and export elasticities and verify if the sum is less than minus unity.\(^2\) Using our non-parametric point estimates of import and export elasticities (Table 1), MLC is only satisfied for Norway.

Next the examination of varying point estimates which have an advantage of providing estimates of the MLC for all periods in the sample is looked at. The simple sums of import and export elasticities is plotted in Figure 1. It is interesting to note that for all countries there are sub-sample periods where the condition is satisfied, the proportions of which are also given in Figure 1. In most of the cases the condition seems to be profoundly satisfied in the 1960s, with the exception of Japan\(^3\) where it is mostly satisfied around the mid-1970s. All these sub-sample periods, where the condition is intensely satisfied, also associate to fixed exchange rate regimes. Therefore it may be cautiously concluded that likelihood of satisfaction of MLC is higher under fixed exchange rate regime. There are other periods where the basic condition is satisfied and one needs to examine various policy shifts of these countries to come up with possible explanations. The fluctuations in key macroeconomic variables have therefore been examined to draw further conclusions. However we could not come up with any other explanation in this regard.

IV. CONCLUSION

In this study we have used, for the first time, non-parametric kernel estimation technique to examine MLC for six developed economies. Both point estimates and estimates through time of import and export elasticities have been examined. Based on our non-parametric point estimates of elasticities it is found that the MLC is only satisfied for Norway. A more elaborate analysis based on varying point estimates reveals that for all countries the condition is satisfied for some sub-sample periods. Furthermore, it has been concluded that fixed exchange rate regime seems to be more conducive in realizing the MLC.

ACKNOWLEDGEMENTS

The authors are indebted to Prof. Fatma Taskin from Bilkent University for her invaluable comments, during the earlier phase of this study. All the views expressed in this paper belong to the authors and do not represent the

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\(^2\) Normally the sum of absolute values of elasticities is taken and if it is greater than one then the condition is satisfied. In our case, some of elasticity estimates are positive and therefore simple sum is more convenient.

\(^3\) Japan shifted to flexible exchange rate regime in 1973.
views of the Central Bank of the Republic of Turkey, or its staff.

REFERENCES


APPENDIX

Data sources and definitions

- All data are taken from IMF/IFS Ver. 1.1.53, with base year 1995.
- Variable definitions are as follows:
  
  M: Import volume expressed as an index
  PM: Unit value of imports expressed as an index
  PD: Index of domestic prices, proxied by CPI
  Y: Domestic real income, GDP volume
  X: Export volume expressed as an index
  PX: Unit value of exports expressed as an index
  PXW: Unit value of exports of the IMF’s industrial country aggregate
  YW: World real income proxied by the industrial production index of the IMF’s industrial country aggregate.