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Thermomagnetic effects of nondegenerate Kane semiconductors under the conditions of mutual electron–phonon drag in high electric and arbitrary magnetic fields

M M Babaev, T M Gassym, M Tas and M Tomak

1 Institute of Physics, Academy of Sciences of Azerbaijan, 370143 Baku, Azerbaijan
2 Physics Department, Bilkent University, 06800 Ankara, Turkey
3 Physics Department, Middle East Technical University, 06531 Ankara, Turkey

E-mail: tasm@fen.bilkent.edu.tr

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Abstract
The thermopower and Nernst–Ettingshausen (NE) effect of nondegenerate Kane semiconductors are investigated by taking into account the electron and phonon heating and their arbitrary mutual drag. The electron spectrum is taken in the Kane two-band form. The electric and magnetic field dependences of the electronic and phonon parts of the thermoelectric and NE coefficients and voltages are obtained in analytical forms. It is shown that the mutual drag of electrons and phonons and degree of nonparabolicity of the electron spectrum strongly influence the thermoelectric and thermomagnetic properties of semiconductors under high electric and magnetic fields.

1. Introduction
In recent years considerable efforts have been devoted to the theoretical and experimental investigation of thermoelectric and thermomagnetic effects in mesoscopic quantum dots, quantum wires, heterojunction and quantum well structures as well as bulk materials [1–13]. There are also some theoretical investigations of thermoelectric and thermomagnetic effects in semiconductors under high external electric and nonquantizing magnetic fields [14–22]. A long survey of the literature and some common misunderstandings in this field are given in our recent papers [23, 24]. In addition, there are some review articles devoted to these subjects [25–27]. In [25], the thermopower and Nernst–Ettingshausen (NE) effect of hot electrons are investigated by taking into account mainly the electron–optical phonon drag and size effects. In [26], a comprehensive review of works on two-dimensional systems until 1992 is provided. The more recent developments in the magnetolectric properties of semiconductors mainly concerned with two-dimensional degenerate unheated electron or hole gases, along with some recent results on three-dimensional systems, are given in [27].
It is well known that the drag of electrons by phonons, or so-called thermal drag, was predicted for the first time by Gurevich [28, 29] in 1946, and was revealed experimentally in 1953 by Frederikse [30] (see also [31, 32]). As a result of studying many aspects of these effects both theoretically and experimentally until the early 1980s, it became a common belief that the electron–phonon drag plays an important role in heat transfer whereas it can be neglected in the charge transport processes [33].

Technological developments achieved at the beginning of 1980s stimulated the study of transport phenomena in high external electric, magnetic and electromagnetic fields. Indeed, they led to the revision of many established concepts, like the mutual and thermal drag, and to the prediction of a substantial range of new drag mechanisms and physical effects. Between 1967 and 1970, Gurevich and Gassymov investigated the phonon heating in high external electric and classically high and quantizing magnetic fields in semiconductors [34–38]. They showed that the heating of phonons leads to major changes and new effects in the transport phenomena and in the interpretation of experimental data.

In our recent paper [23] and in this work we show that if the external electric field is aligned in the \(x\) direction and \(\nabla T\) (or the external electric field gradient \(\nabla E\)) is in the \(z\) direction, then the ratio of the mutual drag term to the thermal drag term in the thermoelectric and thermomagnetic expressions becomes \(\gamma(\theta)/[1 − \gamma(\theta)]\). This ratio is smaller than unity if \(\gamma(\theta) < 1/2\), equal to unity if \(\gamma(\theta) = 1/2\) and larger than unity if \(1/2 < \gamma(\theta) < 1\). It tends to infinity as \(\gamma(\theta) \to 1\). Therefore, at high electric fields the mutual drag becomes more important. Here \(\gamma(\theta) = [v_i(\theta)/v(\theta)][\beta_{ph}(\theta)/\beta(\theta)]\), with \(v(\theta)\) and \(\beta(\theta)\) the average total collision frequencies of electrons and phonons, respectively, with scatterers.

Experimentally, the drag effect and heating of carriers and phonons are usually investigated at low temperatures. In weak external electric fields, the electrons transfer their momentum to the impurity ions. On the other hand, in high external electric or magnetic fields, or in strong electromagnetic fields, the ratio of the collision frequencies of hot electrons with charged impurities and that of hot nondegenerate electrons with phonons is \(v_i/v_{ph} \sim (T_e/T)^{-3}\). This ratio decreases sharply and becomes unity at some critical value of external electric field \(E = E_{cr}\). For \(E > E_{cr}\) the electrons and phonons are scattered from each other and, therefore, the mutual drag becomes important.

The thermoelectric and thermomagnetic effects of semiconductors taking into account the heating and mutual drag of electrons and phonons were studied in [22–25]. But, in [22, 24, 25] the electron dispersion was assumed to be parabolic. The influence of the nonparabolicity of the electron spectrum on the thermoelectric power of semiconductors under the mutual drag conditions is investigated in [23]. In this paper, we investigate the thermomagnetic effects of nondegenerate Kane semiconductors in high electric and nonquantizing magnetic fields. In the calculations we take into account the heating of electrons and phonons and their mutual drag. It is shown that the nonparabolicity leads to a significant change of dependence of NE field upon the electron temperature \(T_e\), as well as upon the electric field \(E\). Under the conditions of strong mutual drag for the semiconductors with parabolic spectrum the phonon part of the NE coefficient \(Q_p = 0\), but in the nonparabolic case \(Q_p \neq 0\) and \(Q_p\) is larger than the electron part \(Q_e\), i.e. under these conditions in the semiconductors with nonparabolic spectrum the NE field consists mainly of the phonon part.

2. Theory

The two-band Kane spectrum for electrons is assumed to be [16]

\[
p(\varepsilon) = (2m_p\varepsilon)^{1/2} \left(1 + \frac{\varepsilon}{E_g}\right)^{1/2},
\]
where $m_e$ is the effective electron mass at the bottom of the conduction band, $\varepsilon_g$ is the bandgap and $p$ and $\varepsilon$ are the electron momentum and energy, respectively.

The coupled Boltzmann transport equations for electrons and phonons are the basic equations of the problem. We consider the quasi-elastic scattering of electrons from acoustical phonons. In this case, the distribution functions of electrons and phonons, $f(p, r)$ and $N(q, r)$ respectively, may be written as

$$f(p, r) = f_0(\varepsilon, r) + f_1(\varepsilon, r) \frac{p}{p},$$

$$N(q, r) = N_0(q, r) + N_1(q, r) \frac{q}{q},$$

where $f_0 (N_0)$ and $f_1 (N_1)$ are the isotropic and anisotropic parts of the electron (phonon) distribution functions, respectively. We will assume that $|f_1| \ll f_0$ and $|N_1| \ll N_0$, i.e. the so-called diffusion approximation applies.

If interelectronic collision frequency $\nu_{ee}$ is much larger than the collision frequency of electrons for energy transfer to lattice $\nu_t$, then $f_0(\varepsilon, r)$ is the Boltzmann distribution function with an effective electron temperature $T_e$. Note that hereafter all temperatures are in energy units.

We assume that in the lattice there is a thermal reservoir of short-wavelength (SW) phonons for long-wavelength (LW) phonons interacting with electrons. The maximum momentum of LW phonons interacting with electrons satisfies the condition $q_{\text{max}} = 2\tilde{p} \ll T/\hbar$. Here, $q_{\text{max}}$ is the maximum quasi-momentum of LW phonons. Under these conditions LW phonons are heated. Hence, we assume that the isotropic part of the distribution function of phonons has the form [22]

$$N_0(q) = \left[ \exp\left(\frac{\hbar \omega_q}{T_{\text{ph}}} - 1\right) \right]^{-1} \approx \frac{T_{\text{ph}}}{\hbar \omega_q}.$$ 

(4)

In accordance with [22], the distribution function of phonons has the form of equation (4) only in two cases: in the first case the frequency of LW phonon–electron collisions $\beta_e$ is much smaller than the frequency of LW phonon–SW phonon collisions $\beta_{ph}$. In this case $T_{\text{ph}} = T$ if

$$\frac{N(T_e)}{\beta_e} \approx \frac{T_e \beta_e}{\beta_{ph}} \ll 1.$$ 

(5)

In the second case, $\beta_e \gg \beta_{ph}, \beta_{ph}^{(e)}$, where $\beta_{ph}^{(e)}$ is the collision frequency of phonons with crystal boundaries connected with energy transfer to outside. In this case, the temperature of LW phonons becomes equal to the temperature of electrons ($T_{\text{ph}} = T_e$), and LW phonons are in a nonequilibrium state.

The anisotropic parts of the distribution functions of electrons and phonons are obtained by solving the coupled system of Boltzmann equations:

$$\frac{p}{m(\varepsilon)} \nabla f_0(\varepsilon) - eE_\varepsilon \frac{p}{m(\varepsilon)} \left( \frac{\partial f_0(\varepsilon)}{\partial \varepsilon} \right) - \Omega(\varepsilon) [\hat{h} \cdot f_1] + v(\varepsilon)f_1$$

$$+ \frac{2\pi m(\varepsilon)}{(2\pi \hbar)^3} \left( \frac{\partial f_0(\varepsilon)}{\partial \varepsilon} \right) \int_0^{\nu_{ph}} N_1(q) W(q) \hbar \omega_q q^2 \, dq = 0,$$

(6)

$$s_0 \nabla N_0(q) + \beta(q) N_1(q) - \frac{4\pi m(\varepsilon)}{(2\pi \hbar)^3} W(q) N_0(q) \int_{q/2}^{\infty} f_1 \, dp = 0,$$

(7)

where $e$ is the absolute value of the electronic charge, $E_\varepsilon = E + E_T$ with $E_T$ being the thermoelectric field, $\Omega = eH/m(\varepsilon)c$ is the cyclotron frequency, $\hat{h} = H/H$, $\hbar \omega_q = s_0 q$ is the phonon energy, $W(q) = W_0 q^2$ is the square matrix element of the electron–phonon
interaction ($\ell = 1$ for deformation and $\ell = -1$ for piezoelectric interactions) and $\beta(q)$ and $v(\epsilon)$ are the total phonon and electron momentum scattering frequencies, respectively.

For the Kane semiconductors with electron spectrum given by equation (1), $m(\epsilon)$ and $v(\epsilon)$ have the following forms [21]:

$$m(\epsilon) = m_n \left( 1 + \frac{2\epsilon}{\epsilon_g} \right),$$

$$v(\epsilon) = v_0(T) \left( \frac{T_{ph}}{T} \right)^{\frac{\ell}{2}} \left( 1 + \frac{\epsilon}{\epsilon_g} \right) \left( 1 + \frac{\epsilon}{\epsilon_g} \right)^{-\frac{\ell}{2}},$$

where $r = 3/2$, $\ell = 0$, for the scattering of electrons from impurity ions and $r = -\ell/2$, $\ell = 1$, for the scattering of electrons from acoustical phonons. When LW phonons are scattered by SW phonons or crystal boundaries, $\beta(q)$ does not depend on the type of electron spectrum and has the form [18]

$$\beta_{ph}(q) = \frac{T^4 q}{4\pi \rho \hbar^2 s_0^2}, \quad \beta_s(q) = \frac{s_0}{L},$$

where the indices ph and b denote the scattering from SW phonons and crystal boundaries; $\rho$ and $L$ are the density and minimum size of the specimen, respectively. When the LW phonons are scattered by electrons, $\beta(q)$ depends on the spectrum of electrons and for the spectrum defined by equation (1) it is given by

$$\beta_e(q) = \left( \frac{m_n s_0^2}{8\pi T_e} \right)^{1/2} \frac{N W_0}{T_e} \left( 1 + \frac{2T_e}{T} \right)^{2} \left( 1 + \frac{3T_e}{2T_g} \right)^{-3/2} q^4,$$

where $N$ is the electron concentration.

Solving the coupled equations (6) and (7) in the same way as in [22] and using the condition $J_x = J_z = 0$ ($E \parallel \mathbf{H} \parallel \mathbf{y}$, $\nabla T_{e,ph} = 0 \parallel \mathbf{z}$), the thermoelectric field $E_{T_e}$ and transverse NE field $E_{Ts}$ are obtained as

$$E_{T_e} = \frac{1}{e} \nabla \zeta(T_e) = \alpha_e \nabla T_e + \alpha_{ph} \nabla T_{ph}, \quad \alpha_{(e,ph)} = -\frac{\sigma_{(e,ph)}^{11} \sigma_{12}^{(e,ph)} + \sigma_{11}^{(e,ph)} \sigma_{12}^{12}}{\sigma_{11}^{(e,ph)} + \sigma_{12}^{(e,ph)}},$$

$$E_{Ts} = -H \left( Q_e \nabla T_e + Q_{ph} \nabla T_{ph} \right), \quad Q_{(e,ph)} = \frac{1}{H} \frac{\sigma_{(e,ph)}^{11} \sigma_{12}^{(e,ph)} - \sigma_{11}^{(e,ph)} \sigma_{12}^{12}}{\sigma_{11}^{(e,ph)} + \sigma_{12}^{(e,ph)}},$$

where $\zeta(T_e)$ is the chemical potential of hot electrons, $\alpha_{(e,ph)}$ are the electronic (e) and phonon (ph) parts of the thermoelectric power, $Q_{(e,ph)}$ are the same parts of the NE coefficient, and

$$\sigma_{ii} = \int_0^\infty a(x) \left[ \frac{\Omega(x)}{v(x)} \right]^{\frac{1}{2}} \left[ 1 + b_1(x) \right] dx, \quad x = \frac{\epsilon}{T_e},$$

$$\beta_{(e)} = \frac{1}{e} \int_0^\infty a(x) \left[ \frac{\Omega(x)}{v(x)} \right]^{\frac{1}{2}} \left\{ x - \frac{\zeta(T_e)}{T_e} + \left[ 1 - \frac{\zeta(T_e)}{T_e} \right] b_1(x) \right\} dx,$$

$$\beta_{(ph)} = \frac{1}{e} \int_0^\infty a(x) \left[ \frac{\Omega(x)}{v(x)} \right]^{\frac{1}{2}} \left[ \lambda(x) + \lambda(1) b_1(x) \right] dx, \quad \varphi_e = \frac{T_e}{T}, \quad \varphi_{ph} = \frac{T_{ph}}{T},$$

with

$$a(x) = \frac{e^2}{3\pi^2 \hbar^3} \frac{m(x) [\Omega^2(x) + v^2(x)]}{m(x) [\Omega^2(x) + v^2(x)] + \Omega(x)} \exp \left[ \frac{\zeta(T_e)}{T_e} - x \right],$$

$$b_1(x) = \frac{\gamma(x) v(x)}{\Omega^2(1) + v^2(1) [1 - \gamma_0]^2} \left[ \frac{v(1)}{1 - \gamma_0} - \frac{\Omega(x) \Omega(1)}{v(1)} \right].$$
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\[ b_2(x) = \frac{\gamma(x)v(x)}{\Omega(1 + \nu^2(1)v(x)) m(x)} \left[ \frac{m(x)}{m(1)} \nu(1) [1 - \gamma_0] + v(x) \frac{m(x)}{m(1)} \right] \],

\[ \gamma(x) = \frac{3 + t}{(2p)^{3r}} \frac{v_{ph}(x)}{\nu(x)} \int_0^{2p} \frac{\beta(q)}{\nu(q)} q^{2s+1} dq, \]

\[ \lambda(x) = \frac{3 + t}{(2p)^{3r}} \frac{m(x)}{T_{ph}} v_{ph}(x) \int_0^{2p} \frac{1}{\beta(q)} q^{2s+1} dq, \]

where \( v_{ph}(x) \) is the scattering frequency of electrons by phonons. The coefficient \( \lambda(x) \) characterizes the efficiency of thermal drag, whereas \( \gamma(x) \) describes the same for mutual drag.

Because the general analysis of equations (11)–(15) is very difficult, we will take the energy dependence of the electron momentum in the form

\[ p(\epsilon) = (2m_n e)^{1/2} \left( \frac{\epsilon}{\epsilon_g} \right)^{s}. \]

For the Kane spectrum given by equation (1), this relation corresponds to the parabolic spectrum if \( T_e \ll \epsilon_g, s = 1/2 \), and corresponds to a strongly nonparabolic spectrum if \( T_e \gg \epsilon_g, s = 1 \). Then, \( m(\epsilon), \nu(\epsilon) \) and \( \beta(q) \) may be written in the following forms:

\[ m(\epsilon) = 2s m_n \left( \frac{\epsilon}{\epsilon_g} \right)^{s}, \]

\[ \nu(\epsilon) = 2s v_0(T) \left( \frac{\epsilon}{\epsilon_g} \right)^{(2s-1)(1-r)} \left( \frac{\epsilon}{T} \right)^{-r}, \]

\[ \beta(q) = \beta(T) \left( \frac{T}{\epsilon_g} \right)^{n(s-1/2)} \left( \frac{s_0 q}{T} \right)^{k}. \]

In equation (25) the variables \( n \) and \( k \) take different values depending on the scattering mechanism: \( n = 1, k = t \), for the scattering of LW phonons from electrons, \( n = 0, k = 0 \), for the scattering of LW phonons from the crystal boundaries and \( n = 0, k = 1 \), for the scattering of LW phonons from SW phonons.

For the spectrum given by equation (21), the chemical potential of hot nondegenerate electrons with density \( N \) is given by

\[ \zeta(T_e) = T_e \ln \frac{3\pi^2 \hbar^3 N}{(1 + 3s)(2m_n T_e)^{3/2}} \left( \frac{T}{\epsilon_g} \right)^{3(s-1/2)} \epsilon_g^{-3s}. \]

3. Results

We will investigate the behaviour of \( \alpha \) and \( Q \) in weak and strong magnetic fields. Defining \( \tilde{\Omega} \) and \( \tilde{\nu} \) as the average values of \( \Omega \) and \( \nu \), respectively, the weak magnetic fields satisfy \( \tilde{\Omega} \ll \tilde{\nu} \), and the strong magnetic fields satisfy \( \tilde{\Omega} \gg \tilde{\nu} \). In weak magnetic fields, in a first approximation on \( \Omega/\tilde{\nu} \), we obtain

\[ \alpha_e = -\frac{1}{e} \left( 1 + C_1 \frac{\gamma_0}{1 - \gamma_0} \right)^{-1} \left\{ 3 - s + 2sr - \frac{\zeta(T_e)}{T_e} + \left[ 1 - \frac{\zeta(T_e)}{T_e} \right] C_1 \frac{\gamma_0}{1 - \gamma_0} \right\}, \]

\[ \alpha_{ph} = -\frac{1}{e} \left( C_2 + (C_1 - C_2)\gamma_0 \right) \lambda(1), \]

\[ \alpha_{ph} = -\frac{1}{e} \left( C_2 + (C_1 - C_2)\gamma_0 \right) \lambda(1), \]
defined by equation (22),

\[ C = \frac{(2 - 2s + 2sr)C_3 + [(4 - 5s + 4sr)C_1C_3 - (2 - s + 2sr)C_3]}{1 - \gamma_0 - (2 - s + 2sr)C_1(1 - \gamma_0)^2}. \]  

(29)

\[ Q_{\text{ph}} = \frac{1}{ec \mu_0} C_6 \left( 1 + C_1 \frac{\gamma_0}{1 - \gamma_0} \right)^{-2} \lambda(1) \mu_s(T_e) \left\{ C_5 - C_2 C_3 + (C_4 + C_1 C_5 - C_1 - C_3 - C_2 C_3) \frac{\gamma_0}{1 - \gamma_0} + C_1 (1 - C_2) \frac{\gamma_0}{(1 - \gamma_0)^2} \right\}, \]

(30)

where \( \mu_s(T_e) \) is the mobility of hot electrons. For nondegenerate electrons with a spectrum defined by equation (22), \( \mu_s(T_e) \) is given by

\[ \mu_s(T_e) = \frac{\Gamma(3 - s + 2sr)}{4 \pi^2 \Gamma(1 + 3s) m_n v_0(T) / \hbar} \left( \frac{T}{\hbar} \right)^{(1 - 2s)(2 - r)} T_e^{2 - 4s + 2m_0 \gamma} \frac{1}{\mu_s(T_e)}. \]

(31)

It is obvious from equations (27)–(30) that in weak magnetic fields the mutual drag essentially influences the electronic and phonon parts of the thermoelectric power and NE coefficient.

In high magnetic fields, on the other hand, from equations (11) and (12), we obtain

\[ \alpha_e = - \frac{1}{e} \left( 1 + 3s - \frac{T(T_e)}{T} \right), \]

(32)

\[ \alpha_{\text{ph}} = - \frac{1}{e} C_7 \lambda(1), \]

(33)

\[ Q_e = \frac{c}{e H^2 C_6} \left( 1 - 2s + 2sr \right) C_8 - 3s \gamma_0 C_9 \frac{1}{\mu_s(T_e)}, \]

(34)

\[ Q_{\text{ph}} = \frac{c}{e H^2 C_6} \left[ C_7 C_8 - C_6 C_{10} + C_6 (C_6 - C_7) \gamma_0 \right] \frac{1}{\mu_s(T_e)} \lambda(1). \]

(35)

As is seen in equations (32)–(35), in high magnetic fields the thermoelectric power does not depend on the mutual drag coefficient \( \gamma_0 \) explicitly. However, both \( Q_e \) and \( Q_{\text{ph}} \) depend linearly on \( \gamma_0 \). In the above equations

\[ C_1 = \frac{\Gamma(1 + 3s + 2sr + 2st - sk)}{\Gamma(3 - s + 2sr)}, \]

\[ C_2 = \frac{\Gamma(1 + 3s + 2sr + st - sk)}{\Gamma(3 - s + 2sr)}, \]

\[ C_3 = \frac{\Gamma(5 - 5s + 4sr)}{\Gamma(3 - s + 2sr)}, \]

\[ C_4 = \frac{\Gamma(3 - s + 4sr + 2st - sk)}{\Gamma(3 - s + 2sr)}, \]

\[ C_5 = \frac{\Gamma(3 - s + 4sr + st - sk)}{\Gamma(3 - s + 2sr)}, \]

\[ C_6 = \frac{\Gamma(1 + 3s)}{\Gamma(3 - s + 2sr)}, \]

\[ C_7 = \frac{\Gamma(7s - 1 + st - sk)}{\Gamma(3 - s + 2sr)}, \]

\[ C_8 = \frac{\Gamma(7s - 1 + 2st - sk)}{\Gamma(3 - s + 2sr)}, \]

\[ C_9 = \frac{\Gamma(7s - 1 + 2st - sk)}{\Gamma(3 - s + 2sr)}, \]

\[ C_{10} = \frac{\Gamma(11s - 3 - 2rs + st - sk)}{\Gamma(3 - s + 2sr)}, \]

\[ \gamma_0 = \frac{(3 + t)^2 (r-k)^{3(k-3)/2}}{3 + 2t - k} \left( \frac{m_n v_0^2}{T} \right)^{(r-k)/2} \frac{\beta_e(T)}{\beta(T)} \frac{1}{\gamma_0}, \]

(36)

\[ \lambda(1) = \frac{(3 + t)^2 (3-3(k-3)/2)}{3 + t - k} \left( \frac{m_n v_0^2}{T} \right)^{(r-k-3)/2} \frac{\beta_e(T)}{\beta(T)} \frac{1}{\gamma_0}, \]

(37)
Equations (27)–(35) are valid for any value of $\gamma_0$ (0 $\leq\gamma_0 < 1$). The thermoelectric power and thermomagnetic effects of hot carriers in the absence of mutual drag ($\gamma_0 = 0$) were studied in [20]. Therefore, here we consider only the strong mutual drag regime. This regime occurs when electrons and phonons are mainly scattered from each other, i.e. $k = t$, $n = 1$, $r = -t/2$, $\ell = 1$, $\theta_{ph} = \theta_e$, $\gamma_0 = [\beta_e(T)/\beta(T)][(v_{ph}(T))/v_0(T)] \rightarrow 1$. Under these conditions, equations (27)–(30) and (32)–(35) can be transformed to the following forms in weak and high magnetic fields.

In weak magnetic fields, the electronic part of the thermoelectric power has the same form in both parabolic and nonparabolic spectrum cases. Moreover, it depends on the degree of nonparabolicity only through the chemical potential of electrons,

$$
\alpha_e = -\frac{1}{e} \left[ 1 - \frac{\xi(T_e)}{T_e} \right].
$$

However, the phonon part of the thermoelectric power strongly depends on the degree of nonparabolicity

$$
\alpha_{ph} = -\frac{1}{e} \left[ 1 - \frac{4\sqrt{2}(2s)^2}{3\pi^{3/2}} \left( \frac{T}{e} \right)^{3(s-1)/2} \frac{(m_e T)^{3/2}}{\hbar N} \right].
$$

As follows from equations (26), (27) and (39), $\alpha_e$ depends weakly (logarithmically) on $T_e$. On the other hand, as follows from equations (28), (38) and (40), $\alpha_{ph}$ strongly increases with increasing electron temperature: $\alpha_{ph} \sim T_e^{3/2}$ for parabolic, and $\alpha_{ph} \sim T_e^{5/2}$ for strongly nonparabolic spectra.

The electronic and phonon parts of the NE coefficient $Q$ in weak magnetic fields are found as

$$
Q_e = -(2 - s - st) \frac{\mu_e(T)}{e\epsilon_e} \theta_e^{1-4s-st},
$$

$$
Q_{ph} = 4\sqrt{2}(2s)^2 \left[ 1 - \frac{\Gamma(1 + 3s - st)}{\Gamma(3 - s - st)} \right] \frac{\mu_e(T)}{e\epsilon_e} \left( \frac{T}{k_B} \right)^{3(s-1)/2} \frac{(m_e T)^{3/2}}{\hbar N} \theta_e^{3s}.
$$

where $\mu_e(T)$ is the mobility of cold electrons ($T_e = T$). For the parabolic spectrum, $Q_e$ decreases with increasing $T_e$ for both deformation acoustical (DA) and piezo acoustical (PA) interactions because $1 - 4s - st < 0$ for both scattering mechanisms. However, as is seen in equation (41), $Q_e = 0$ for a strongly nonparabolic spectrum and DA interaction. On the other hand, $Q_{ph} = 0$ in both DA and PA interactions for the parabolic spectrum, and $Q_{ph} \sim \theta_e^{-s}$ for the strongly nonparabolic spectrum. In other words, $Q_{ph}$ increases with increasing $T_e$ in PA and decreases in DA interactions.

In strong magnetic fields, however, we find

$$
\alpha_e = -\frac{1}{e} \left[ 1 + 3s - \frac{\xi(T_e)}{T_e} \right],
$$

$$
\alpha_{ph} = -\frac{1}{e} \left[ 4\sqrt{2}(2s)^2 \right] \frac{\Gamma(7s - 1)}{\Gamma(3 - s - st)} \left( \frac{T}{k_B} \right)^{3(s-1)/2} \frac{(m_e T)^{3/2}}{\hbar N} \theta_e^{3s}.
$$

From equations (32) and (39) it follows that $\alpha_e$ increases in magnetic field as

$$
\Delta \alpha_e = |\alpha_e(H) - \alpha_e(0)| = \frac{1}{e} 3s.
$$

The increase for the nonparabolic spectrum is much bigger than for the parabolic spectrum. Moreover, the dependence of $\alpha_{ph}$ upon $T_e$ in strong magnetic fields is the same as in weak magnetic fields.
In strong magnetic fields, $Q_e$ and $Q_{ph}$ are obtained as

$$Q_e = -(7s - 2 + st) \Gamma(7s - 1 + st) \frac{c}{\Gamma^2(1 + 3s)} eH^2 \mu_s(T) \frac{1}{\mu_s(T)} \frac{\vartheta_e^{4+st-1}}{\vartheta_e^{4+st-1}},$$  \hspace{1cm} (46)

$$Q_{ph} = \frac{4\sqrt{2}(2s)^3}{3\pi^{3/2}} \Gamma(3s - s - st) \frac{(7s - 1 + st) - 11s + st}{\Gamma(7s - 1 + st) - \Gamma(11s - 3 + st)} \frac{c}{\Gamma^2(1 + 3s)} \frac{1}{eH^2 \mu_s(T)} \frac{1}{\mu_s(T)} \frac{(m_nT)^{3/2}}{h^3N} \vartheta_e^{7s-1-st}.$$  \hspace{1cm} (47)

In strong magnetic fields, as follows from equation (46), $Q_e$ increases with increasing $T_e$ (for all cases satisfying $4s + st - 1 > 0$). Equation (47) shows that for the parabolic spectrum $Q_{ph} = 0$, whereas for the strongly nonparabolic spectrum $Q_{ph}$ strongly increases with increasing $T_e$ as $Q_{ph} \sim \vartheta_e^{7s}$. Therefore, $Q_{ph} \sim \vartheta_e^{7s}$ in DA and $Q_{ph} \sim \vartheta_e^{5}$ in PA interactions.

As follows from equation (26), for nondegenerate electrons

$$\left( \frac{T}{\xi_E} \right)^{3(3s-1)/2} \frac{(m_nT)^{3/2}}{h^3N} \approx \exp \left( -\frac{\zeta(T)}{T} \right) \gg 1,$$  \hspace{1cm} (48)

and by the comparison of equation (39) with (40), and equation (43) with (44) we see that under strong mutual drag conditions $|\alpha_{ph}| \gg |\alpha_e|$ in both weak and strong magnetic fields. In other words, the thermoelectric power mainly consists of the phonon part.

Since for the parabolic spectrum $Q_{ph} = 0$, the NE field consists of only the electronic part. However, for the strongly nonparabolic spectrum, from equations (41), (42), and (46), (47), we have $|Q_{ph}| \gg |Q_e|$, i.e. the NE field mainly consists of the phonon part.

If $\gamma_0 \ll 1$, the $E$ dependence of $\vartheta_e$ (when $\vartheta_e = \vartheta_{ph} \gg 1$) is given by equation (37) of our recent paper [23] as

$$\vartheta_e \sim E^{2/(8s-1-2st)}.$$  \hspace{1cm} (49)

Therefore, we find

$$\vartheta_e \sim E^{2/(3-st)} \quad \text{for} \quad s = 1/2$$

and

$$\vartheta_e \sim E^{2/(3-st)} \quad \text{for} \quad s = 1.$$  \hspace{1cm} (50)

Under the strong mutual drag conditions ($\vartheta_e = \vartheta_{ph}$ and $\gamma_0 \rightarrow 1$), the effective electron temperature is determined by the energy balance equation

$$\sigma_{11}(\vartheta_e)E^2 = W_{pp}(\vartheta_e),$$  \hspace{1cm} (51)

where $W_{pp}(\vartheta_e)$ is the power transferred by LW phonons to the thermal reservoir of SW phonons. We now consider the following cases:

(i) $\frac{\beta_{ph} + \beta_b}{\beta_e} \ll \frac{v_i}{v_{ph}},$  \hspace{1cm} (ii) $\beta_{ph} \gg \beta_b,$  \hspace{1cm} (iii) $\beta_{ph} \ll \beta_b,$

$$\frac{\beta_{ph}}{\beta_e} \gg \frac{v_i}{v_{ph}}.$$  \hspace{1cm} (52)

The results obtained for $\vartheta_{ph} = \vartheta_e \gg 1$ are given in table 1. As is seen in table 1, the nonparabolicity of the electron spectrum strongly changes the $E$ dependence of electron temperature. Using these results one can easily obtain the dependence of the thermoelectric power and NE coefficient upon the heating electric field in all cases considered. As follows from equations (27), (39) and (43), $\alpha_e$ weakly (logarithmically) depends on $E$ for all cases given in equation (52).

The $E$ dependence of $\alpha_{ph}$, $Q_{ph}$, and $Q_e$ in the cases given in equation (52) are presented in table 2. In using this table one must keep in mind that for the parabolic spectrum $Q_{ph} = 0$ and
Thermomagnetic effects of nondegenerate Kane semiconductors

Table 1. Dependences of $\alpha_e$ on $E$ in the limit $\gamma_0 \to 1$.

<table>
<thead>
<tr>
<th>Case</th>
<th>$s = \frac{1}{2}$</th>
<th>$s = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>$\alpha_e \sim E^{4/3}$</td>
<td>$\alpha_e \sim E^{1/2}$</td>
</tr>
<tr>
<td>ii</td>
<td>$\alpha_e \sim E^{1/3}$</td>
<td>$\alpha_e \sim E^{1/5}$</td>
</tr>
<tr>
<td>iii</td>
<td>$\alpha_e \sim E^{4/11}$</td>
<td>$\alpha_e \sim E^{2/9}$</td>
</tr>
</tbody>
</table>

Table 2. Dependences of $\alpha_{ph}$, $Q_e$ and $Q_{ph}$ on $E$ in DA and PA scattering mechanisms for the parabolic spectrum of electrons ($s = 1/2$) under the strong mutual drag conditions. The results for the PA scattering mechanism are given in brackets.

<table>
<thead>
<tr>
<th>Case (i)</th>
<th>Case (ii)</th>
<th>Case (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{ph}$</td>
<td>$\sim E^2$</td>
<td>$\sim E^{1/2}$</td>
</tr>
<tr>
<td>$Q_e(\Omega \ll \bar{v})$</td>
<td>$\sim E^{-2} [e^{-2/3}]$</td>
<td>$\sim E^{-1/2} [e^{-1/6}]$</td>
</tr>
<tr>
<td>$Q_e(\Omega \gg \bar{v})$</td>
<td>$\sim E^2 [E^{2/3}]$</td>
<td>$\sim E^{1/2} [E^{1/8}]$</td>
</tr>
<tr>
<td>$Q_{ph}(\Omega \ll \bar{v})$</td>
<td>$\approx 0 [\sim E^{4/3}]$</td>
<td>$\approx 0 [\sim E^{1/3}]$</td>
</tr>
<tr>
<td>$Q_{ph}(\Omega \gg \bar{v})$</td>
<td>$\sim E^4 [E^{4/3}]$</td>
<td>$\sim E [E^{2/3}]$</td>
</tr>
</tbody>
</table>

Table 3. The same as table 2 but for the strongly nonparabolic spectrum of electrons ($s = 1$).

<table>
<thead>
<tr>
<th>Case (i)</th>
<th>Case (ii)</th>
<th>Case (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{ph}$</td>
<td>$\sim E^{5/2}$</td>
<td>$\sim E^{3/3}$</td>
</tr>
<tr>
<td>$Q_e(\Omega \ll \bar{v})$</td>
<td>$\sim E^{-2} [e^{-1}]$</td>
<td>$\sim E^{-4/5} [e^{-2/5}]$</td>
</tr>
<tr>
<td>$Q_e(\Omega \gg \bar{v})$</td>
<td>$\sim E^2 [E]$</td>
<td>$\sim E^{4/5} [E^{2/5}]$</td>
</tr>
<tr>
<td>$Q_{ph}(\Omega \ll \bar{v})$</td>
<td>$\sim E^{-1/2} [e^{1/2}]$</td>
<td>$\sim E^{-1/5} [E^{1/5}]$</td>
</tr>
<tr>
<td>$Q_{ph}(\Omega \gg \bar{v})$</td>
<td>$\sim E^{7/2} [E^{5/2}]$</td>
<td>$\sim E^{7/5} [E]$</td>
</tr>
</tbody>
</table>

total NE coefficient $Q = Q_e + Q_{ph} = Q_e$. On the other hand, for the strongly nonparabolic spectrum $|Q_{ph}| \gg |Q_e|$ and $Q \approx Q_{ph}$. For example, under strong magnetic fields if the DA interaction is taking place and case (i) is satisfied, then $Q \sim E^2$ for the parabolic and $Q \sim E^{3/2}$ for the strongly nonparabolic spectra.

Under the strong mutual drag conditions, $|\alpha_{ph}| \gg |\alpha_e|$ and total thermoelectric power $\alpha = \alpha_e + \alpha_{ph} \approx \alpha_{ph}$ for both parabolic and nonparabolic spectra. For example, if case (i) is satisfied, then $\alpha \sim E^2$ for the parabolic, and $\alpha \sim E^{3/2}$ for the strongly nonparabolic spectra. In tables 2 and 3 we observe that for all cases the nonparabolicity of the spectrum essentially changes the $E$ dependence of $\alpha$ and $Q$.

In this paper, we investigate the $E$ dependence of the phonon parts of the NE coefficient $Q_{ph}$ and NE voltage $U_{ph}$ in weak and high magnetic fields for different scattering mechanisms. In our analysis, we consider weak and strong mutual drag cases ($\gamma_0 \ll 1$ and $\gamma_0 \to 1$, respectively) separately.

3.1. The $\gamma_0 \ll 1$ limit

In this limit we will investigate the phonon parts of the NE effects under weak and high magnetic fields in detail.

3.1.1. Weak magnetic field. The NE voltage is defined as $U_{ph} \sim Q_{ph}\phi_E$, where $\phi_E$ is given by equation (49). Equations (30), (31) and (38) yield $Q_{ph} \sim \phi_E^{(N+k+2r-n+2m)}$. Therefore, in weak magnetic fields $Q_{ph}$ and $U_{ph}$ take the following forms for the given scattering mechanisms.
(1) Electrons are scattered by DA phonons and phonons by electrons. The coupled system formed by the mutual drag is scattered by the crystal boundaries. \( t = 1, r = -1/2, \ell = 1, k = 0, n = 0 \) (strong mutual drag):

\[
Q_{\text{ph}} \sim E^{-4/9}, \quad U_{\text{ph}} \sim E^0 \quad \text{for} \quad s = 1/2, \\
Q_{\text{ph}} \sim E^{-2/9}, \quad U_{\text{ph}} \sim E^0 \quad \text{for} \quad s = 1.
\]  

(2) Electrons are scattered by PA phonons and phonons by electrons. The coupled system formed by the mutual drag is scattered by the crystal boundaries. \( t = -1, r = 1/2, \ell = 1, k = 0, n = 0 \) (strong mutual drag):

\[
Q_{\text{ph}} \sim E^{-4/9}, \quad U_{\text{ph}} \sim E^0 \quad \text{for} \quad s = 1/2, \\
Q_{\text{ph}} \sim E^{-2/9}, \quad U_{\text{ph}} \sim E^0 \quad \text{for} \quad s = 1.
\]  

(3) Electrons transfer their momentum to charged impurities and energy to DA phonons, and phonons are scattered from the crystal boundaries. \( t = 1, r = 3/2, \ell = 0, k = 0, n = 0 \) (thermal drag):

\[
Q_{\text{ph}} \sim E^{8/3}, \quad U_{\text{ph}} \sim E^{8/3} \quad \text{for} \quad s = 1/2, \\
Q_{\text{ph}} \sim E^{2}, \quad U_{\text{ph}} \sim E^{5/2} \quad \text{for} \quad s = 1.
\]  

(4) Electrons transfer their momentum to charged impurities and energy to PA phonons, and phonons are scattered from the crystal boundaries. \( t = -1, r = 3/2, \ell = 0, k = 0, n = 0 \) (thermal drag):

\[
Q_{\text{ph}} \sim E^{8/3}, \quad U_{\text{ph}} \sim E^{8/3} \quad \text{for} \quad s = 1/2, \\
Q_{\text{ph}} \sim E^{2}, \quad U_{\text{ph}} \sim E^{5/2} \quad \text{for} \quad s = 1.
\]  

(5) Electrons transfer their momentum to charged impurities and energy to DA phonons, and phonons are scattered by electrons. \( t = 1, r = 3/2, \ell = 0, k = 1, n = 1 \) (thermal drag):

\[
Q_{\text{ph}} \approx 0, \quad U_{\text{ph}} \approx 0 \quad \text{for} \quad s = 1/2, \\
Q_{\text{ph}} \approx E^{2}, \quad U_{\text{ph}} \approx E^{5/2} \quad \text{for} \quad s = 1.
\]  

(6) Electrons transfer their momentum to charged impurities and energy to PA phonons, and phonons are scattered by electrons. \( t = -1, r = 3/2, \ell = 0, k = -1, n = 1 \) (thermal drag):

\[
Q_{\text{ph}} \approx 0, \quad U_{\text{ph}} \approx 0 \quad \text{for} \quad s = 1/2, \\
Q_{\text{ph}} \approx E^{2}, \quad U_{\text{ph}} \approx E^{5/2} \quad \text{for} \quad s = 1.
\]  

3.1.2. High magnetic field. By using equations (30) and (35), \( Q_{\text{ph}} \) is obtained as

\[
Q_{\text{ph}} \sim \delta^4_e(8-2r+\ell-k-n)\ell+2n-4+\ell.
\]  

(1) Electrons are scattered by DA phonons and phonons by electrons. The coupled system formed by the mutual drag is scattered by the crystal boundaries. \( t = 1, r = -1/2, \ell = 1, k = 0, n = 0 \) (strong mutual drag):

\[
Q_{\text{ph}} \sim E^{8/9}, \quad U_{\text{ph}} \sim E^{4/3} \quad \text{for} \quad s = 1/2, \\
Q_{\text{ph}} \sim E^{14/9}, \quad U_{\text{ph}} \sim E^{16/9} \quad \text{for} \quad s = 1.
\]  

(2) Electrons are scattered by PA phonons and phonons by electrons. The coupled system formed by the mutual drag is scattered by the crystal boundaries. \( t = -1, r = 1/2, \ell = 1, k = 0, n = 0 \) (strong mutual drag):

\[
Q_{\text{ph}} \sim E^{0}, \quad U_{\text{ph}} \sim E^{4/7} \quad \text{for} \quad s = 1/2, \\
Q_{\text{ph}} \sim E^{6/7}, \quad U_{\text{ph}} \sim E^{8/7} \quad \text{for} \quad s = 1.
\]
and phonons are scattered from the crystal boundaries. In this paper, we studied the thermomagnetic effects of nondegenerate Kane semiconductors under the conditions of mutual electron–phonon drag in high electric and nonquantizing fields.

Table 4. Dependences of \( U_e \) and \( U_{ph} \) on \( E \) in DA and PA scattering mechanisms for the parabolic spectrum of electrons (\( s = 1/2 \)) under the strong mutual drag conditions. The results for the PA scattering mechanism are given in brackets.

<table>
<thead>
<tr>
<th>Case (i)</th>
<th>Case (ii)</th>
<th>Case (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_e(\Omega \ll \nu) )</td>
<td>( \sim E^{-2/3} ) [ E^{2/3} ]</td>
<td>( \sim E^{-1/8} ) [ E^{1/8} ]</td>
</tr>
<tr>
<td>( U_e(\Omega \gg \nu) )</td>
<td>( \sim E^{10/3} ) [ E^{2} ]</td>
<td>( \sim E^{2/5} ) [ E^{1/2} ]</td>
</tr>
<tr>
<td>( U_{ph}(\Omega \ll \nu) )</td>
<td>( \approx 0 ) [ \sim E^{8/3} ]</td>
<td>( \approx 0 ) [ \sim E^{5/3} ]</td>
</tr>
<tr>
<td>( U_{ph}(\Omega \gg \nu) )</td>
<td>( \sim E^{16/3} ) [ E^{4} ]</td>
<td>( \sim E^{4/3} ) [ E ]</td>
</tr>
</tbody>
</table>

(3) Electrons transfer their momentum to charged impurities and energy to DA phonons, and phonons are scattered from the crystal boundaries. \( t = 1, r = 3/2, \ell = 0, k = 0, n = 0 \) (thermal drag):

\[
Q_{ph} \sim E^{-4/3}, \quad U_{ph} \sim E^{0} \quad \text{for} \quad s = 1/2, \\
Q_{ph} \sim E^{1/2}, \quad U_{ph} \sim E^{3/2} \quad \text{for} \quad s = 1.
\]  

(4) Electrons transfer their momentum to charged impurities and energy to PA phonons, and phonons are scattered from the crystal boundaries. \( t = -1, r = 3/2, \ell = 0, k = 0, n = 0 \) (thermal drag):

\[
Q_{ph} \sim E^{-8/3}, \quad U_{ph} \sim E^{-4/3} \quad \text{for} \quad s = 1/2, \\
Q_{ph} \sim E^{3/2}, \quad U_{ph} \sim E^{5/2} \quad \text{for} \quad s = 1.
\]  

(5) Electrons transfer their momentum to charged impurities and energy to DA phonons, and phonons are scattered by electrons. \( t = 1, r = 3/2, \ell = 0, k = 1, n = 1 \) (thermal drag):

\[
Q_{ph} \sim E^{5/2}, \quad U_{ph} \sim E^{0} \quad \text{for} \quad s = 1/2, \\
Q_{ph} \sim E^{1}, \quad U_{ph} \sim E^{3/2} \quad \text{for} \quad s = 1.
\]  

(6) Electrons transfer their momentum to charged impurities and energy to PA phonons, and phonons are scattered by electrons. \( t = -1, r = 3/2, \ell = 0, k = -1, n = 1 \) (thermal drag):

\[
Q_{ph} \approx 0, \quad U_{ph} \approx 0 \quad \text{for} \quad s = 1/2, \\
Q_{ph} \sim E^{1}, \quad U_{ph} \sim E^{3/2} \quad \text{for} \quad s = 1.
\]  

3.2. The \( \gamma_0 \rightarrow 1 \) limit

In this limit, electrons and phonons are mainly scattered from each other; therefore, we have strong mutual drag. If electrons are scattered by DA phonons, and phonons by electrons, we must take \( t = 1, r = -1/2, \ell = 1, k = 1, n = 1 \). On the other hand, if electrons are scattered by PA phonons and phonons by electrons we must take \( t = -1, r = 1/2, \ell = 1, k = -1, n = 1 \).

The dependences of \( \alpha_{ph}, Q_e \) and \( Q_{ph} \) upon \( E \) in the present limit for the parabolic spectrum is given in table 2 and for the strongly nonparabolic spectrum in table 3. By using the results given in tables 1–3, one can easily calculate the NE potentials \( U_e \sim Q_e \theta_e \) and \( U_{ph} \sim Q_{ph} \theta_e \). The results for \( U_e \) and \( U_{ph} \) are given in tables 4 and 5.

4. Conclusion

In this paper, we studied the thermomagnetic effects of nondegenerate Kane semiconductors under the conditions of mutual electron–phonon drag in high electric and nonquantizing
magnetic fields. We have found that the nonparabolicity of the electron spectrum and the mutual drag leads to significant changes in the thermoelectric and thermomagnetic effects of hot carriers.

Under the conditions of strong mutual drag for the semiconductor with parabolic spectrum the phonon part of the NE coefficient $Q_{ph} = 0$ and the total NE coefficient $Q = Q_e + Q_{ph} = Q_e$ [22]. We show that in these conditions for the nonparabolic spectrum $Q_{ph} \neq 0$ and $Q_{ph} \gg Q_e$, i.e. in the nonparabolic case the NE field mainly consists of the phonon part and is much higher than the NE field in the parabolic case. In high magnetic fields for the semiconductor with strongly nonparabolic spectrum, the NE coefficient $Q \approx Q_{ph}$ sharply increases with increasing electron temperature as $Q \sim \vartheta_e^2$ in DA and $Q \sim \vartheta_e^3$ in PA interactions of electrons with phonons.

Under the strong mutual drag conditions, $|\alpha_{ph}| \gg |\alpha_e|$ both in weak and strong magnetic fields, i.e. the thermoelectric power mainly consists of the phonon part ($\alpha \approx \alpha_{ph}$) and essentially depends on the degree of nonparabolicity: $\alpha \sim \vartheta_e^{3/2}$ for the parabolic, and $\alpha \sim \vartheta_e^3$ for the strongly nonparabolic spectra.

As is seen in table 1, the nonparabolicity of the electron spectrum strongly changes the $E$ dependence of electron temperature. By using these relations the $E$ dependence of thermoelectric power and NE coefficients are obtained and presented in tables 2 and 3. In constructing these tables we used the fact that for the parabolic spectrum $Q_{ph} = 0$ and the total NE coefficient $Q = Q_e + Q_{ph} = Q_e$, whereas for the strongly nonparabolic spectrum case $|Q_{ph}| \gg |Q_e|$ and $Q = Q_{ph}$.

In summary, we have demonstrated that the mutual drag of electrons and phonons, and the degree of nonparabolicity of the electron spectrum strongly influences the thermoelectric and thermomagnetic properties of semiconductors.

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