

Corollary 4: At any SNR, for the scalar MAC with K users the sum capacity improvement due to feedback can not be more than $\frac{1}{2} \log(K)$ bits/channel use.

Notice that this bound is valid for any SNR. In [9], it has been shown that feedback cannot increase the sum capacity of the K user scalar MAC by more than a multiplicative factor of K . In other words, $C_{\Sigma}^{\text{fb}} - C_{\Sigma} \leq (K - 1)C_{\Sigma}$. Note that at high SNR, C_{Σ} is unbounded and therefore our bound of $\frac{1}{2} \log(K)$ is stronger. In fact, in the previous section, we have shown directly that the bound is in fact tight for $K = 2$. At low SNR on the other hand, the multiplicative bound may be stronger.

VI. EXTENSIONS TO VECTOR BROADCAST CHANNELS

While our focus in this work is the vector MAC, the topological duality of the multiple access and the broadcast channels allows some results to be directly extended from the multiple access channel to the broadcast channel. In this section we point out examples of such direct extensions. First, in the preceding sections, we have shown that the vector MAC with single antenna at the base station and multiple antennas at the users is equivalent to a scalar MAC obtained by a unitary transformation at each users' input. The same argument can be used to show that a vector BC with a single antenna at the base station and multiple antennas at each user is also equivalent to a scalar BC with maximum ratio combining (which corresponds to a unitary transformation as well) at each user. Second, we showed that on the vector MAC there is no sum capacity benefit of feedback at high SNR if the number of antennas at the base station is at least as large as the number of single antenna users. For the BC, the same argument can be made as follows. Allowing the receivers in a BC to cooperate we again have a point-to-point MIMO channel whose capacity can not be further improved by feedback. However, it is known that for a vector BC, there exists a noise correlation (worst case noise correlation) for which cooperation between users does not increase sum capacity. Therefore, there is always a noise correlation structure for the BC for which feedback does not increase sum capacity. In particular, it can be shown that for the two user nondegenerate vector BC, the worst case noise is also uncorrelated at high SNR. Therefore, for users with uncorrelated noise terms, feedback does not improve capacity at high SNR.

VII. CONCLUSION

We explore the feedback capacity region of the vector Gaussian multiple-access channel (MAC) with either multiple antennas at the base station and a single antenna at each user or multiple antennas at each user and a single antenna at the base station. We show that the vector MAC (and BC) with a single antenna at the base station and multiple antennas at each user are degenerate vector channels as they are equivalent to a scalar MAC (and BC). In the limit of high signal-to-noise ratio (SNR), we show that for the scalar Gaussian MAC (and for the degenerate vector Gaussian MAC), the difference between the sum capacity with and without feedback approaches a positive constant. We explicitly calculate this constant and show that it is no more than $\frac{1}{2}$ bit/channel use. For the nondegenerate two user vector Gaussian MAC we apply the Kailath Schalkwijk coding scheme to determine the capacity region with feedback. Unlike the scalar Gaussian MAC, we show that for a nondegenerate vector Gaussian MAC the entire capacity region with feedback becomes the same as the capacity region without feedback in the limit of high SNR.

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Scaling Results on the Sum Capacity of Cellular Networks With MIMO Links

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Abstract—Scaling results for the sum capacity of the multiple access, uplink channel are provided for a flat-fading environment, with multiple-input-multiple-output (MIMO) links, when there is interference from other cells. The classical MIMO scaling regime is considered in which the number of antennas per user and per base station grow large together. Utilizing the known characterizations of the limiting eigenvalue distributions of large random matrices, the asymptotic behavior of the sum capacity of the system is characterized for an architecture in which the base stations cooperate in the joint decoding process of all users (macrodiversity). This asymptotic sum capacity is compared with that of the conventional scenario in which the base stations only decode the users in their cells. For the case of base station cooperation, an interesting "resource pooling" phenomenon is observed: in some cases, the limiting performance of a macrodiversity multiuser network has the same asymptotic behavior as that of a single-user MIMO link with an equivalent amount of pooled received power. This resource pooling phenomenon allows us to derive an elegant closed-form expression for the sum capacity of a new version of Wyner's classical model of a cellular network, in which MIMO links are incorporated into the model.

Index Terms—Asymptotic sum capacity, base station cooperation, large system analysis, multiple-access channel (MAC), multiple-input-multiple-output (MIMO) channel, random matrices.

Manuscript received June 18, 2004; revised June 22, 2005. This work was supported by the Australian Research Council (ARC). The material in this correspondence was presented in part at the 2004 International Symposium on Information Theory and Its Applications, Parma, Italy, October 2004. CUBIN is an affiliated program of National ICT Australia (NICTA).

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Communicated by M. Médard, Associate Editor for Communications.

Digital Object Identifier 10.1109/TIT.2006.876241

I. INTRODUCTION

Information theoretic studies in [1] and [2] have demonstrated the remarkable capacity gains of single-link multiple-input–multiple-output (MIMO) systems (microdiversity) and ignited enormous interest in multiple antenna systems for wireless applications and the design of low-complexity transmission and reception schemes (such as space–time coding) that achieve these promised gains. Compared to a single antenna system with the same transmit power, the capacity of a single-link MIMO system is shown to scale linearly with the minimum of the numbers of transmit and receive antennas assuming an independent and identically distributed (i.i.d.) flat-fading channel between each transmit and receive antenna. Thus, in theory, arbitrarily large capacities are possible.

Until recently, analyses of MIMO systems until recently have mainly focused on the single point-to-point link scenario where the interference from other users is ignored. In practice, the dominant impairment in a multiuser wireless system is typically the cochannel interference from other users. Thus, the potential gains of a multiple antenna system in a multiuser setting need to be reevaluated. Recently, there have been efforts to characterize the capacity of a MIMO single-cell system with many users. A review of results on the capacity of single-cell multiuser MIMO channels can be found in the paper by Goldsmith *et al.* in [3]. Attention has now shifted to characterizing the capacity of MIMO cellular networks, where there are many cell sites and mobiles, each of which may be equipped with an antenna array. Antenna arrays can be used in beamforming from the transmitting node, and in interference suppression at the receiving node.

In this correspondence, we consider the issue of interference in the uplink of a cellular network in two different ways: 1) where the base stations can cooperate to jointly decode all the users' data (macrodiversity); and 2) the more traditional architecture where base stations do not cooperate and only decode the users in their respective cells. Thus, in the first case, we dispense with the cellular structure altogether and consider the entire network of base stations and users as a multiple-access channel (MAC). Each user has a link to each base station and the base stations cooperate to jointly decode all users' data. Since base stations are typically not mobile, base stations can share information using high speed reliable connections over wireline (e.g., optical fiber) links or wireless (e.g., line-of-sight microwave) links. Such links exist in current systems in order to handle the control and signaling required between base stations.

Alternatively, one can consider the sum capacity (strictly speaking, achievable sum rate) under the more traditional cellular approach, where each user and base station is equipped with an antenna array (microdiversity) and a base station only decodes the users in its cell. The base station can attempt suppression of cochannel interference, but only using the known or measurable statistical structure of the interference from other cells.

One difficulty in making meaningful, or insightful, comparisons between these two architectures is the complexity of the capacity formulas, and the large number of random parameters that need to be realized to numerically compute the associated capacities via Monte Carlo simulation of the random parameters. This is particularly problematic when one considers that the capacities themselves will depend on many design variables, such as the transmit powers used by the mobiles, the choice of base stations that they can connect to, or the relative sizes of antenna arrays in different parts of the network, and these parameters need to be optimized in order to provide the fairest comparisons between different architectures.

Fortunately, a large body of work on the limiting eigenvalue distributions of large random matrices now exists, and has been applied in a variety of different scenarios involving wireless multiple antenna systems [4]. We find this theory a useful way to provide capacity formulas

that can be computed numerically relatively quickly, and used in further network optimization and simulation. Thus, we take the large system point of view. Most of the mathematical results we need are already in the literature; our purpose is to show how these results apply to the given problem, to show that the results do indeed apply in nonasymptotic settings, and to make some preliminary conclusions concerning the issue of base station cooperation.

Our main new result, from a theoretical point of view, is the identification of a resource pooling phenomenon for the macrodiversity architecture that occurs when certain symmetry conditions in the network are met. In this case, the formulas for sum capacity have a closed form, identical to that in the single point to point MIMO link studied in [1].

The macrodiversity architecture has been studied in a number of works, beginning with [5]–[8]. Both [5] and [6] compute the Shannon capacities of particularly simple, yet insightful, cellular network models under the macrodiversity assumption, in which base stations are uniformly distributed along a line. In [6], this example is shown to have an elegant capacity formula, which can be studied to determine the effect of different levels of intercell interference. The original work on cellular capacity in [6] was extended in [7] to include fading, but the macrodiversity assumption was dropped. MIMO is not considered there, and the focus is on the capacity of single antenna cell sites, under the architecture 2).

We consider the circular version of Wyner's cellular model, as proposed in [5], (i.e., Wyner's linear cellular array wrapped into a circle), but the new feature in the present work is the MIMO aspect, and the modeling of flat, multipath fading between each transmit and receive antenna element. This example exhibits resource pooling, as the antenna arrays at each user and cell site grow large, and hence its asymptotic sum capacity can be expressed in closed form.

The characterization of the asymptotic sum capacity for architecture 2), when the base stations do not cooperate, under the scaling of antenna arrays at each mobile and cell site, can be obtained directly from the results in [9]. We apply these results, and hence compare and contrast the two different architectures.

There have been several other recent papers to address the capacities of cellular systems with MIMO links, under various scenarios and assumptions that differentiate these results from those in the present correspondence. The potential of using antenna arrays to enhance the system capacity of the downlink of a multicell system was first demonstrated in [10], where the sum capacity of a system in which the users utilize single user detection treating interference from other users as additional noise, is evaluated through simulations. In [11], these numerical studies are extended to other multiuser receiver structures. Dai and Poor [12] have studied the large system asymptotic sum capacities of multicell MIMO systems with different multiuser detectors and verified the numerical results in [11]. In [9], [13]–[16], the capacity of a multiuser MIMO system in the presence of cochannel interference is analyzed, in which the base station utilizes single user detection.

The resource pooling effect observed in the present correspondence is related to a resource pooling effect observed in [17] for a multiple antenna CDMA single-input–multiple-output (SIMO) link. In that case, the resources pooled were the antennas in a microdiversity antenna array at the receiver, and symmetry across antennas was a critical feature of the result. In the present correspondence, we extend the concept to macrodiversity: the resources pooled in this correspondence are the (nonidentical) base stations, and only a weaker form of symmetry is required, as discussed in Section IV.

The correspondence is organized as follows. Section II introduces the system model. The sum capacity of the network of base stations and users is derived in Section III using random matrix theory. Section IV provides the resource pooling result. Numerical results for a special system construction where resource pooling is observed is presented

in Section V. In Section VI, application of the sum capacity results to the design of practical cellular systems is demonstrated. Finally, some concluding remarks are offered in Section VII.

In the correspondence, we use boldface letters to denote vectors and matrices and \mathbb{C} for the field of complex numbers. $\mathbb{E}_X[\cdot]$ denotes expectation with respect to the distribution of random variable X , $\log(\cdot)$ denotes logarithm base e , $|\cdot|$ denotes absolute value and $\Im(\cdot)$ denotes the imaginary part of the argument. For a matrix \mathbf{A} , $\det(\mathbf{A})$ denotes the determinant, $\text{tr}(\mathbf{A})$ denotes the trace, \mathbf{A}^T denotes the transpose and \mathbf{A}^\dagger denotes the complex conjugate transpose. We denote an identity matrix of size M as \mathbf{I}_M .

II. SYSTEM MODEL

We consider the uplink of a multiuser MIMO system where K independent users communicate to M base stations. Base station j is equipped with $N\gamma_j$ antennas and user k is equipped with $N\beta_k$ antennas. The parameter N controls the simultaneous growth of the size of antenna arrays at the users and the base stations. In this correspondence, we are interested in large system analysis where $N \rightarrow \infty$ and γ_j and β_k are fixed. The vector of baseband received signals at base station j is given by

$$\mathbf{y}_j = \sum_{k=1}^K \alpha_{jk} \mathbf{H}_{jk} \mathbf{x}_k + \mathbf{n}_j \quad (1)$$

where $\mathbf{x}_k \in \mathbb{C}^{N\beta_k}$ is the vector of symbols transmitted by user k , \mathbf{H}_{jk} is the matrix of channel gains from user k to base station j and \mathbf{n}_j is the additive white Gaussian noise vector at this base station. The channels between users and base stations are modeled as frequency flat Rayleigh fading processes and the antennas at the base stations (and respectively, users) are located far enough apart such that the fading coefficients are independent. The entries of \mathbf{H}_{jk} are independent and identically distributed circularly symmetric complex Gaussian random variables with zero mean and unit variance and α_{jk} denotes the average channel gain from user k to base station j . The average channel gain may include channel effects such as path loss, antenna gain and lognormal shadowing. The vectors $\mathbf{x}_1, \dots, \mathbf{x}_K, \mathbf{n}_1, \dots, \mathbf{n}_M$ and the channel matrices $\mathbf{H}_{11}, \dots, \mathbf{H}_{MK}$ are independent. The noise vector has independent circularly symmetric zero mean complex Gaussian entries with unit variance. The covariance of the signal transmitted by user k is $\mathbf{S}_k = \mathbb{E}[\mathbf{x}_k \mathbf{x}_k^\dagger]$ with a total power constraint of P_k , i.e., $\text{tr}(\mathbf{S}_k) \leq P_k$.

We assume that base stations have perfect channel state information, whereas the users have no knowledge of the channel realizations. We are interested in the scenario where the base stations can cooperate to jointly decode the transmitting users' data. The system can thus be modeled as a MIMO MAC with macrodiversity where some of the antenna elements of the hyper-receiver are widely separated. The system model can be represented in vector form as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (2)$$

where

$$\mathbf{H} = \begin{bmatrix} \alpha_{11} \mathbf{H}_{11} & \cdots & \alpha_{1K} \mathbf{H}_{1K} \\ \vdots & \ddots & \vdots \\ \alpha_{M1} \mathbf{H}_{M1} & \cdots & \alpha_{MK} \mathbf{H}_{MK} \end{bmatrix} \quad (3)$$

is the overall channel matrix, $\mathbf{y} = [\mathbf{y}_1^T \dots \mathbf{y}_M^T]^T$ is the received signal vector, $\mathbf{x} = [\mathbf{x}_1^T \dots \mathbf{x}_K^T]^T$ is the transmitted signal vector and $\mathbf{n} = [\mathbf{n}_1^T \dots \mathbf{n}_M^T]^T$ is the noise vector.

In this correspondence, we are interested in characterizing the scaling of the ergodic sum capacity of this system as the size of the

antenna arrays at the users and base stations grows asymptotically large (i.e., as $N \rightarrow \infty$). In practice, the sizes of antenna arrays at the nodes of the network are limited. However, as in the single link MIMO case, the convergence to the asymptotic results is observed to be fast as the array size grows large (see Figs. 5 and 6), and therefore, the asymptotic results can be used in the design of practical systems.

III. ASYMPTOTIC SUM CAPACITY WITH BASE STATION COOPERATION

In this section, using the results on the eigenvalue distributions of large random matrices, we will characterize the asymptotic sum capacity of the multiuser MIMO channel when the base stations cooperate.

Lemma 1: The sum capacity of the system in (2) is

$$I(\mathbf{x}; \mathbf{y}, \mathbf{H}) = \mathbb{E}_{\mathbf{H}} \left[\log \det \left(\mathbf{I}_{(N \sum_{j=1}^M \gamma_j)} + \mathbf{H}\mathbf{S}\mathbf{H}^\dagger \right) \right] \quad (4)$$

where the capacity is achieved with a circularly symmetric complex Gaussian transmit signal vector \mathbf{x} with zero mean and block diagonal covariance matrix

$$\mathbf{S} = \frac{1}{N} \begin{bmatrix} \frac{P_1}{\beta_1} \mathbf{I}_{(N\beta_1)} & & \\ & \ddots & \\ & & \frac{P_K}{\beta_K} \mathbf{I}_{(N\beta_K)} \end{bmatrix}. \quad (5)$$

The proof of this lemma follows from [18] and [19] and is omitted for brevity.

We now apply large system analysis to evaluate the limiting sum capacity as the antenna arrays at the users and base stations grow asymptotically large with $N \rightarrow \infty$. A similar approach is applied in [20] for a single user multiple antenna system with polarization and/or pattern diversity. The result that we use is the following theorem due to [4, Th. 2.53].

Theorem 1: Let \mathcal{A} be a $[cN] \times [dN]$ random matrix with independent entries which are zero-mean and satisfy the condition

$$N\text{Var}(\mathcal{A}_{ij}) < \mathcal{B} \quad (6)$$

for some uniform bound $\mathcal{B} < \infty$. Moreover, suppose that we define for each N a function $v_N : (0, c] \times (0, d] \rightarrow \mathbb{R}$ by

$$v_N(x, y) = N\text{Var}(\mathcal{A}_{ij}) \quad (7)$$

for i, j satisfying

$$\frac{i}{N} \leq x < \frac{i+1}{N} \quad \text{and} \quad \frac{j}{N} \leq y < \frac{j+1}{N} \quad (8)$$

and that $v_N(x, y)$ converges uniformly to a limiting bounded function $v(x, y)$ as $N \rightarrow \infty$. Then, almost surely, the empirical eigenvalue distribution of $\mathcal{A}\mathcal{A}^H$ converges weakly to a limiting distribution $F_{\mathcal{A}}(\lambda)$ whose Shannon transform

$$\begin{aligned} \lim_{N \rightarrow \infty} \frac{1}{N} \mathbb{E}[\log \det(I + \nu \mathcal{A}\mathcal{A}^H)] \\ \equiv c \int_0^\infty \log(1 + \nu \lambda) dF_{\mathcal{A}}(\lambda) \end{aligned} \quad (9)$$

for a nonnegative real number ν , is given by

$$\begin{aligned} \lim_{N \rightarrow \infty} \frac{1}{N} \mathbb{E}[\log \det(I + \nu \mathcal{A}\mathcal{A}^H)] \\ = \int_0^d \log \left(1 + \nu \left(\int_0^c v(x, y) u(x, \nu) dx \right) \right) dy \\ + \int_0^c \log \left(1 + \nu \left(\int_0^d v(x, y) \omega(y, \nu) dy \right) \right) dx \\ - \nu \int_0^d \int_0^c v(x, y) u(x, \nu) \omega(y, \nu) dx dy \end{aligned} \quad (10)$$

where $u(x, \nu)$ and $\omega(y, \nu)$ are the unique solutions to the fixed point equations

$$u(x, \nu) = \frac{1}{1 + \nu \int_0^d v(x, y) \omega(y, \nu) dy} \quad (11)$$

$$\omega(y, \nu) = \frac{1}{1 + \nu \int_0^c v(x, y) u(x, \nu) dx} \quad (12)$$

for every $x \in (0, c]$ and $y \in (0, d]$, respectively.

Our first Result is an application of Theorem 1 to the macrodiversity channel with cooperating base stations, where we identify the matrix

$$\bar{\mathbf{H}} = \begin{bmatrix} \alpha_{11} \sqrt{\frac{P_1}{N\beta_1}} \mathbf{H}_{11} & \cdots & \alpha_{1K} \sqrt{\frac{P_K}{N\beta_K}} \mathbf{H}_{1K} \\ \vdots & \ddots & \vdots \\ \alpha_{M1} \sqrt{\frac{P_1}{N\beta_1}} \mathbf{H}_{M1} & \cdots & \alpha_{MK} \sqrt{\frac{P_K}{N\beta_K}} \mathbf{H}_{MK} \end{bmatrix} \quad (13)$$

with the matrix \mathcal{A} in Theorem 1.

Result 1: The sum capacity of the system (1), normalized by N , is characterized in the limit of large N by

$$\begin{aligned} C &\equiv \lim_{N \rightarrow \infty} \frac{1}{N} I(\mathbf{x}; \mathbf{y}, \mathbf{H}) \\ &= \sum_{k=1}^K \beta_k \log \left(1 + \sum_{n=1}^M \gamma_n \frac{\alpha_{nk}^2 P_k}{\beta_k} u_n \right) \\ &\quad + \sum_{n=1}^M \gamma_n \log \left(1 + \sum_{k=1}^K \alpha_{nk}^2 P_k \omega_k \right) \\ &\quad - \sum_{k=1}^K \left(\sum_{n=1}^M \gamma_n \left(\alpha_{nk}^2 P_k u_n \right) \omega_k \right) \end{aligned} \quad (14)$$

nats/s/Hz, where u_n and ω_k are the unique solutions to the following equations:

$$u_n = \left(1 + \sum_{k=1}^K \alpha_{nk}^2 P_k \omega_k \right)^{-1} \quad n = 1, \dots, M \quad (15)$$

$$\omega_k = \left(1 + \sum_{n=1}^M \gamma_n \frac{\alpha_{nk}^2 P_k}{\beta_k} u_n \right)^{-1} \quad k = 1, \dots, K. \quad (16)$$

Proof: For all $n \in \{0, 1, \dots, M\}$ and $k \in \{0, 1, \dots, K\}$, let $\tilde{\gamma}_n = \sum_{\ell=0}^n \gamma_\ell$ with $\gamma_0 = 0$ and $\tilde{\beta}_k = \sum_{\ell=0}^k \beta_\ell$ with $\beta_0 = 0$. Defining $v_N(x, y)$ as in Theorem 1, we observe that it converges to a bounded function, $v(x, y)$, where $v(x, y) = \alpha_{nk}^2 \frac{P_k}{\beta_k}$ for $x \in (\tilde{\gamma}_{n-1}, \tilde{\gamma}_n]$ and $y \in (\tilde{\beta}_{k-1}, \tilde{\beta}_k]$. Thus, the assumptions of Theorem 1 hold, and hence the empirical distribution of the eigenvalues of $\bar{\mathbf{H}}\bar{\mathbf{H}}^\dagger$ converge to a deterministic limit, $F_{\bar{\mathbf{H}}}(x)$. Coupled with Lemma 1, this implies that

$$C = \lim_{N \rightarrow \infty} \frac{1}{N} \mathbb{E}_{\bar{\mathbf{H}}} \left[\log \det \left(\mathbf{I}_{(N \sum_{j=1}^M \gamma_j)} + \bar{\mathbf{H}}\bar{\mathbf{H}}^\dagger \right) \right] \quad (17)$$

$$= \sum_{j=1}^M \gamma_j \int_0^\infty \log(1 + \lambda) dF_{\bar{\mathbf{H}}}(\lambda). \quad (18)$$

Since $v(x, y)$ is constant in $x \in (\tilde{\gamma}_{n-1}, \tilde{\gamma}_n]$, for fixed y , and constant in $y \in (\tilde{\beta}_{k-1}, \tilde{\beta}_k]$, for fixed x , it follows that $u(x) \equiv u(x, 1)$, and $\omega(y) \equiv \omega(y, 1)$ (see (11) and (12) for definitions) are also constant in the respective intervals. Defining $u_n \equiv u(x)$ for $x \in (\tilde{\gamma}_{n-1}, \tilde{\gamma}_n]$, and $\omega_k \equiv \omega(y)$ for $y \in (\tilde{\beta}_{k-1}, \tilde{\beta}_k]$, and setting $\nu = 1$ in (10), we obtain

$$\begin{aligned} C &= \sum_{k=1}^K \int_{\tilde{\beta}_{k-1}}^{\tilde{\beta}_k} \log \left(1 + \sum_{n=1}^M \int_{\tilde{\gamma}_{n-1}}^{\tilde{\gamma}_n} v(x, y) u(x) dx \right) dy \\ &\quad + \sum_{n=1}^M \int_{\tilde{\gamma}_{n-1}}^{\tilde{\gamma}_n} \log \left(1 + \sum_{k=1}^K \int_{\tilde{\beta}_{k-1}}^{\tilde{\beta}_k} v(x, y) \omega(y) dy \right) dx \\ &\quad - \sum_{k=1}^K \sum_{n=1}^M \int_{\tilde{\beta}_{k-1}}^{\tilde{\beta}_k} \int_{\tilde{\gamma}_{n-1}}^{\tilde{\gamma}_n} v(x, y) u(x) \omega(y) dx dy \end{aligned} \quad (19)$$

$$\begin{aligned} &= \sum_{k=1}^K \beta_k \log \left(1 + \sum_{n=1}^M \gamma_n \frac{\alpha_{nk}^2 P_k}{\beta_k} u_n \right) \\ &\quad + \sum_{n=1}^M \gamma_n \log \left(1 + \sum_{k=1}^K \beta_k \frac{\alpha_{nk}^2 P_k}{\beta_k} \omega_k \right) \\ &\quad - \sum_{k=1}^K \beta_k \sum_{n=1}^M \left(\gamma_n \frac{\alpha_{nk}^2 P_k}{\beta_k} u_n \right) \omega_k. \end{aligned} \quad (20)$$

Similarly, the (11) and (12) reduce to (15) and (16). Uniqueness of solution to (15), (16) can be demonstrated easily from first principles, but the uniqueness follows directly from the uniqueness for $u(x, \nu)$ and $\omega(y, \nu)$, that holds more generally in Theorem 1. \square

Result 1 characterizes the sum capacity of a general multiuser MIMO system with cooperating receivers and uncorrelated flat Rayleigh fading as the number of antenna elements grows large. Note that it is a scaling result: (14) tells us the linear growth rate in sum-rate capacity, as we scale up the number of antennas in all antenna arrays in the system. Since the units are nats/s/Hz, we call the quantity in (14) the **spectral efficiency** of the system.

The key mathematical result underlying Theorem 1 is a celebrated Theorem in random matrix theory by Girko ([21, Corollary 10.1.2]). Girko's theorem proves the existence of the limiting distribution of eigenvalues of $\mathcal{A}\mathcal{A}^H$, as the dimensions of \mathcal{A} grow large, and characterizes the Stieltjes transform of the limiting distribution. An alternative expression for the spectral efficiency provided in Result 1 can be obtained by using techniques for obtaining the spectral efficiency from the Stieltjes transform developed in [22]. However, the resulting expression is not as suitable for numerical evaluation as that provided in (14)–(16).

The approach taken by Tulino and Verdu in [4] is to build on Girko's result, but instead of directly using the Stieltjes transform of the eigenvalue distribution, they obtain an expression for the Shannon transform of the limiting eigenvalue distribution of $\mathcal{A}\mathcal{A}^H$, which is more useful for evaluating spectral efficiency. The benefit in applying Theorem 1 to the macrodiversity model is that the resulting expression in (14) simply requires the solution of (15) and (16): a fixed point equation in $M + K$ variables which can be evaluated numerically very efficiently.

Girko's theorem has previously been applied to wireless problems in which degrees of freedom are provided by CDMA spreading codes, and linear receivers are employed, rather than optimal Shannon-theoretic processing. In [23], the effective interference in large dimensional asynchronous CDMA systems is characterized, and in [17], the network-layer capacity of a CDMA antenna array system is investigated, both using Girko's result.

We now present an interesting special case where one can obtain a closed-form expression for the spectral efficiency. This special case exhibits a weak form of symmetry, and we show that the spectral efficiency is the same as that of a single user MIMO link whose expression is already known [1].

IV. RESOURCE POOLING

In this section, we present a nontrivial channel scenario for which the asymptotic sum capacity in the case of base station cooperation has a closed-form expression.

Lets assume that $P_k = P$ and $\beta_k = \beta, \forall k \in \{1, \dots, K\}$ and $\gamma_j = \gamma, \forall j \in \{1, \dots, M\}$. In addition, we make the assumption that the $M \times K$ matrix

$$\Omega \equiv \begin{bmatrix} \alpha_{11}^2 & \cdots & \alpha_{1K}^2 \\ \vdots & \ddots & \vdots \\ \alpha_{M1}^2 & \cdots & \alpha_{MK}^2 \end{bmatrix} \quad (21)$$

satisfies the symmetry condition that all row sums are equal, and all column sums are equal. Thus, if we define the row sums and column sums by:

$$\Upsilon_j = \sum_{k=1}^K \alpha_{jk}^2 \quad j = 1, \dots, M \quad (22)$$

$$\Theta_k = \sum_{j=1}^M \alpha_{jk}^2 \quad k = 1, \dots, K \quad (23)$$

the assumption is that $\Upsilon_j = \Upsilon, \forall j$ and $\Theta_k = \Theta, \forall k$, with $\Theta = \frac{M}{K}\Upsilon$. Note that if $M = K$, and gains are suitably normalized, then Ω is a doubly stochastic matrix.

Although the above assumptions are restrictive from a practical point of view, they do arise in theoretical models in which there is a high degree of symmetry. For example, the assumptions are satisfied whenever there is ‘‘cellular symmetry’’: all cells are equivalent, after an appropriate relabeling of base stations and mobiles, although this is not necessary. One example is the Wyner circular cellular model that we consider in Section V, but more general models also satisfy the assumptions of the present section: for example, the MIMO version of the Wyner hexagonal cell model [6], and models in which there are many mobiles per cell, each with their own set of average gains to the base stations, provided the cellular symmetry condition holds. Although theoretical, these models provide insight into the effects of varying system parameters, and have the advantage that the following resource pooling result holds:

Result 2: Under the above assumptions, the spectral efficiency per receive antenna, as measured by asymptotic sum capacity normalized by total number of receive antennas, is given by

$$C_c = \kappa \log[1 + P_0 - \mathcal{F}(P_0, \kappa)] + \log[1 + \kappa P_0 - \mathcal{F}(P_0, \kappa)] - \frac{1}{P_0} \mathcal{F}(P_0, \kappa) \quad (24)$$

where

$$\mathcal{F}(x, z) \equiv \frac{1}{4} \left(\sqrt{1 + x(1 + \sqrt{z})^2} - \sqrt{1 + x(1 - \sqrt{z})^2} \right)^2 \quad (25)$$

$$\kappa \equiv \frac{\beta K}{\gamma M}, \text{ and } P_0 \equiv \frac{P\Upsilon}{\kappa}.$$

Proof: Note that (15), (16) reduce to

$$u_n = \left(1 + \sum_{k=1}^K \alpha_{nk}^2 P \omega_k \right)^{-1}, \quad \text{for } n = 1, \dots, M \quad (26)$$

$$\omega_k = \left(1 + \gamma \sum_{n=1}^M \frac{\alpha_{nk}^2 P}{\beta} u_n \right)^{-1}, \quad \text{for } k = 1, \dots, K. \quad (27)$$

Since these equations have a unique solution, it is enough to exhibit a particular solution. We arrive at such a solution by first supposing that the solution will satisfy the additional conditions $u_n = u^*, \forall n$ and $\omega_k = \omega^*, \forall k$ where u^* and ω^* must then satisfy,

$$u^* = (1 + P_0 \kappa \omega^*)^{-1} \quad (28)$$

$$\omega^* = (1 + P_0 u^*)^{-1}. \quad (29)$$

Substituting (29) into (28), one obtains a quadratic in u^* , and it is therefore easy to obtain the unique positive roots of (28) and (31) as

$$u^* = 1 - \frac{1}{P_0} \mathcal{F}(P_0, \kappa) \quad (30)$$

$$\omega^* = 1 - \frac{1}{\kappa P_0} \mathcal{F}(P_0, \kappa) \quad (31)$$

where $F(x, z)$ is defined in (25).

One can verify directly that $u_n = u^*$, and $\omega_k = \omega^*$ provide the unique solution to (26) and (27).

The following formula for the spectral efficiency per receive antenna then follows from Result 1:

$$C_c = \frac{1}{\gamma M} \left[\sum_{k=1}^K \beta \log \left(1 + \sum_{n=1}^M \gamma \frac{\alpha_{nk}^2 P}{\beta} u_n \right) + \sum_{n=1}^M \gamma \log \left(1 + d \sum_{k=1}^K \alpha_{nk}^2 P \omega_k \right) - \sum_{k=1}^K \left(\sum_{n=1}^M \gamma (\alpha_{nk}^2 P u_n) \omega_k \right) \right] \quad (32)$$

$$= \frac{1}{\gamma M} \left[K \beta \log \left(1 + \gamma \frac{\Theta P}{\beta} u^* \right) + M \gamma \log(1 + \Upsilon P \omega^*) - K \Theta \gamma P u^* \omega^* \right] \quad (33)$$

$$= \kappa \log(1 + P_0 u^*) + \log(1 + \kappa P_0 \omega^*) - \kappa P_0 u^* \omega^*. \quad (34)$$

Finally using (30) and (31), the spectral efficiency per receive antenna can be obtained in closed form as given in (24). \square

Consider a single user link with $NK\beta$ transmit antennas and $NM\gamma$ receive antennas where the channels between transmit and receive antennas are i.i.d. and the received signal power at each antenna is P_r . One can easily show using Result 1 that the capacity per receive antenna of this system as $N \rightarrow \infty$ is given by (24) where $\kappa = \frac{K\beta}{M\gamma}$ and $P_0 = \frac{P_r}{\kappa}$. Thus, we observe that the macrodiversity multiuser system of this section, with the matrix Ω of channel gains satisfying the scaled doubly stochastic condition, curiously behaves like a microdiversity single user system with i.i.d. channels between transmit and receive antennas but with the number of transmit and receive antennas equal to the sum of the number of user antennas and the sum of base station antennas in the multiuser system, respectively, and with the received signal power at each antenna equal to the total received signal power at each antenna in the multiuser system.

This Result 1s a generalization of the Result 1n [1] where Telatar showed that in a multiuser system where the MIMO channels for the users are identically distributed, the multiuser system is equivalent to a single user system with the same total number of antennas and the same total transmit power constraint. A similar ‘‘resource pooling’’ phenomenon was observed in [17] but within the context of a microdiversity multiuser CDMA network with random spreading and an antenna array at the base station only.

A similar observation on the simplification of the asymptotic capacity expression to the form above was made in [20] for single link MIMO systems with polarization and/or pattern diversity. The structure of Ω required for this simplification was reported to be double-regularity where every row and every column of Ω have the same empirical distribution. Our resource pooling result generalizes this by allowing a weaker form of symmetry, which is an important generalization in the context of the multiuser macrodiversity model of the present correspondence.

One interesting observation is that the system spectral efficiency expression in (24) is equivalent to the asymptotic optimal spectral efficiency of a CDMA system with random spreading, in the absence of fading, as considered (and initially characterized) in [24] where the number of users is $NK\beta$, spreading gain is $NM\gamma$, the energy per transmitted symbol ($NM\gamma$ chips) is P_0 . This closed-form expression was later rederived in [25] using a more direct approach for a single link MIMO system with i.i.d. channels.

V. WYNER'S CELLULAR MODEL REVISITED

In [6], Wyner proposed an insightful model of a cellular network that allowed the effect of intercell interference to be investigated, using a single parameter to represent the strength of the other-cell interference. The macrodiversity assumption was used, allowing the network to be treated as a multiple access channel, and elegant formulas for the per-user capacity were obtained, for the infinite linear array of base stations, and for the infinite 2-D array of base stations in the plane.

In this section, we revisit Wyner's cellular model of cooperating cell sites, but with each link from mobile to base station replaced by a MIMO link, with the same average gain. As in the recent works on MIMO, the fading between individual antenna elements is modeled as independent and Rayleigh distributed. We show that the circular version of Wyner's linear array (as studied in [5]) satisfies the resource pooling conditions, meaning that we can write down a closed-form expression for the spectral efficiency of this network. Thus, we obtain simple formulas, not by taking the array of cells large, but by taking the size of the MIMO arrays large instead. We also apply the theory from [9] to calculate spectral efficiencies when the cell-sites do not cooperate, and thereby provide a point of comparison between the two different architectures for the circular cellular model.

Consider the special case in which K base stations and K users are arranged in a circle as depicted in Fig. 1. Each user is identified with a particular base station. Each base station is equipped with N antennas and each user has $N\beta$ antennas. The total transmit power constraint for each user is P . The average channel gain between base station m and user k is given as

$$\alpha_{mk} = \begin{cases} 1, & \text{if } k = m \\ \alpha, & \text{if } (k - m)_K = 1 \\ \alpha, & \text{if } (k - m)_K = K - 1 \\ 0, & \text{otherwise} \end{cases} \quad (35)$$

where $(\cdot)_K$ denotes integer modulo K . The squared average channel gain matrix Ω defined in (21) is of the form

$$\Omega = \begin{bmatrix} 1 & \alpha^2 & & & \alpha^2 \\ \alpha^2 & 1 & \alpha^2 & & \\ & & \ddots & \ddots & \ddots \\ & & & \alpha^2 & 1 & \alpha^2 \\ \alpha^2 & & & \alpha^2 & & 1 \end{bmatrix} \quad (36)$$

and is a circulant (and thus a scaled doubly stochastic) matrix. Since this multiuser setting is an example of the special case of resource pooling, we can use the system spectral efficiency results in Section IV. The spectral efficiency per receive antenna with base station cooperation is given as

$$C_c = \beta \log(1 + P_0 - \mathcal{F}(P_0, \beta)) + \log(1 + \beta P_0 - \mathcal{F}(P_0, \beta)) - \frac{\mathcal{F}(P_0, \beta)}{P_0} \quad (37)$$

where $P_0 = \frac{P}{\beta}(1 + 2\alpha^2)$ and $\mathcal{F}(x, z)$ is defined in (25).

Using the results from (21) and (22) in [9], the spectral efficiency per receive antenna without base station cooperation is

$$C_{nc} = \beta \log\left(1 + \frac{P}{\beta}\eta_1\right) + 2\beta \log\left(\frac{1 + \frac{P}{\beta}\alpha^2\eta_1}{1 + \frac{P}{\beta}\alpha^2\eta_2}\right) + \log\left(\frac{\eta_2}{\eta_1}\right) + \eta_1 - \eta_2 \quad (38)$$

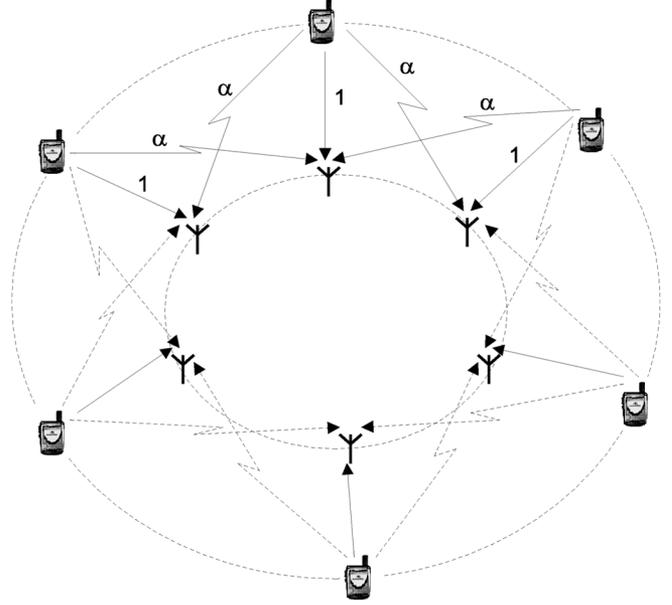


Fig. 1. Circular cellular array.

where $\eta_1 \geq 0$ and $\eta_2 \geq 0$ satisfy

$$\eta_1 + \beta \frac{\frac{P}{\beta}\eta_1}{1 + \frac{P}{\beta}\eta_1} + 2\beta \frac{\frac{P}{\beta}\alpha^2\eta_1}{1 + \frac{P}{\beta}\alpha^2\eta_1} = 1 \quad (39)$$

$$\eta_2 + 2\beta \frac{\frac{P}{\beta}\alpha^2\eta_2}{1 + \frac{P}{\beta}\alpha^2\eta_2} = 1. \quad (40)$$

Note that base station cooperation requires the joint decoding of all the users in the network, whereas the noncooperative case involves only single user decoding by the base stations.

In Fig. 2, we consider the special case of $K = 3$ cell sites and compare the spectral efficiency per receive antenna as a function of β for each user's transmit SNR of 20 dB. This allows us to observe the effect of increasing the number of antennas at the mobile, holding the number of antennas at the cell site fixed, which we will consider in further detail below. This last statement relies on the fact that the asymptotic theory applies very accurately, even for quite small sized systems, such as those with only six antennas; this is well known, but we provide further justification for this statement in Section VI.

The lower four curves in Fig. 2 correspond to the traditional cellular model with no base station cooperation. The upper four curves correspond to spectral efficiency with base station cooperation. As must be the case, the joint decoding of all users via cooperation results in higher spectral efficiency for any given α and β . Furthermore, the spectral efficiency with BS cooperation is an increasing function of α for a given β , whereas for the noncooperative scheme, increasing α decreases the spectral efficiency due to increasing interference from the two neighboring cells.

Fig. 2 shows that the spectral efficiency per receive antenna is not a monotonic function of β in the noncooperative scenario: there exists an optimum β^* beyond which capacity starts to decay due to lack of available degrees of freedom to cancel the interference. Indeed, the figure illustrates the fact that when $\beta > 0.5$, there are not enough degrees of freedom at any base station to null the interference from the two adjacent base stations. Such nonmonotonicity has been observed in [9]. This effect does *not* occur with macrodiversity, since all users are being decoded by the same global decoder, and the linear growth

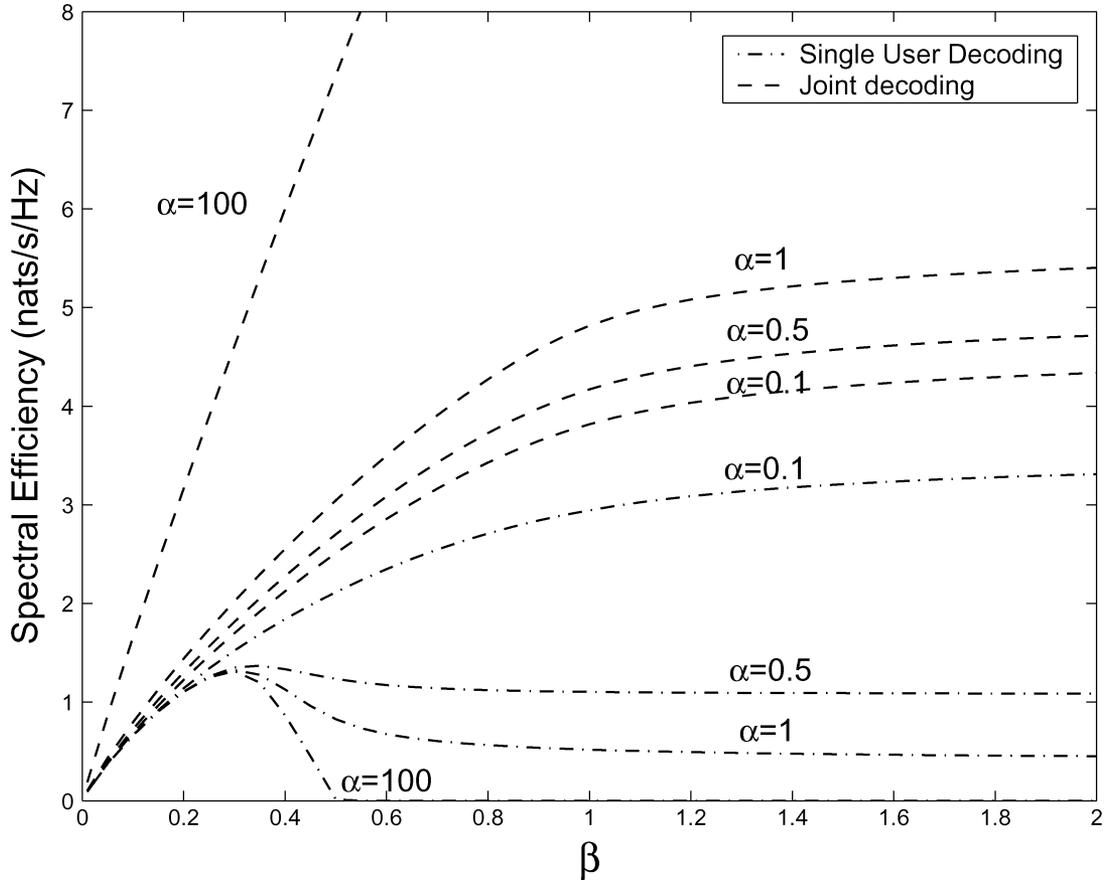


Fig. 2. Comparison of the spectral efficiency per receive antenna of the circular cellular array with and without base station cooperation with each user's transmit SNR equal to 20 dB.

rate increases with α , as depicted in the figure. As such, we see a large gain from macrodiversity for $\beta > 0.5$.

For $\beta < 0.3$, the gain in spectral efficiency due to base station cooperation is not as great, since the noncooperative case is not interference limited in this case. Indeed, the gain of macrodiversity is an increasing function of β . However, at $\alpha = 1$, there is still a discernible gain at moderately small values of β , such as 20% at $\beta = 0.1$, and 30% at $\beta = 0.2$. When α is small, there is very little gain for $\beta < 0.5$. It is likely in practice for the base station to have many more antennas than the mobile devices, and hence for β to be fairly small. The results above suggest that, at best, moderate gains will accrue from base station cooperation when β is small. However, this example is for a high SNR value of 20 dB.

We now illustrate the gain from base station cooperation that occurs at a relatively low signal to noise ratio. In Fig. 3 we plot the corresponding graph for each user's transmit SNR of 0 dB. From Fig. 3, we notice that there is now a more significant gain from base station cooperation at $\alpha = 1$ and relatively low values of β : a 55% gain at $\beta = 0.1$, and a 90% gain at $\beta = 0.2$. On the other hand, the gain from macrodiversity at $\alpha = 0.1$ is less than in the high SNR scenario, uniformly across all β . Presumably, in this case, the weaker signals at the adjacent base stations provide little benefit. Nevertheless, the benefit of base station cooperation increases with β for all parameter values, a general trend in common with the high SNR example.

VI. APPLICATIONS TO THE DESIGN OF CELLULAR SYSTEMS

Although insightful, it might be argued that the Wyner model is not very realistic. It is beyond the scope of the present correspondence to

undertake a detailed study of cellular network optimization, but we do wish to present an alternative example with some additional features that apply in real networks. We restrict attention to the central questions of this correspondence: what is the gain from base station cooperation, and how does this gain vary with the sizes of the antenna arrays at the mobile and base station?

Consider the two-tier hexagonal cellular structure depicted in Fig. 4 with a total of 19 identical cells where the base stations are located in the middle of the hexagonal cells. In each cell, assume that there is only one user transmitting information to the base station at a given time due to an orthogonal multiple access scheme (e.g., TDMA or FDMA) employed within the cell. Each user is randomly located within the cell with a uniform distribution. The base stations are equipped with antenna arrays of the same size and so are the users. The average squared channel gain from user j to base station i is determined by the path loss model

$$\alpha_{ij}^2 = \left(\frac{1}{d_{ij}} \right)^4 \quad (41)$$

where d_{ij} is the distance between user j and base station i , normalized with respect to the maximum distance within a cell and the path loss exponent is assumed to be 4 which is typical for an urban cellular environment.

For simplicity, we will assume power control is not employed, and instead assume that the mobiles transmit with a constant power. We consider two cases corresponding to high and low SNR, respectively. In the first case, the transmit SNR of a mobile is 20 dB, and in the second

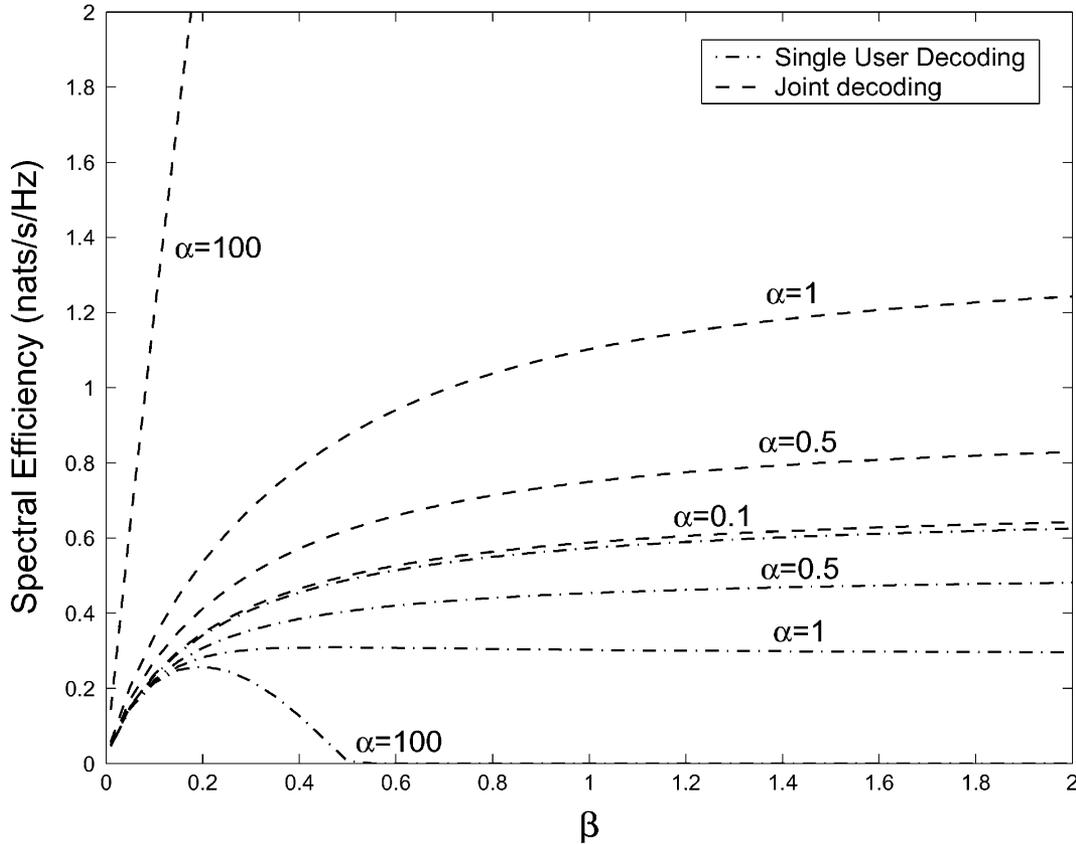


Fig. 3. Comparison of the spectral efficiency per receive antenna of the circular cellular array with and without base station cooperation with each user's transmit SNR equal to 0 dB.

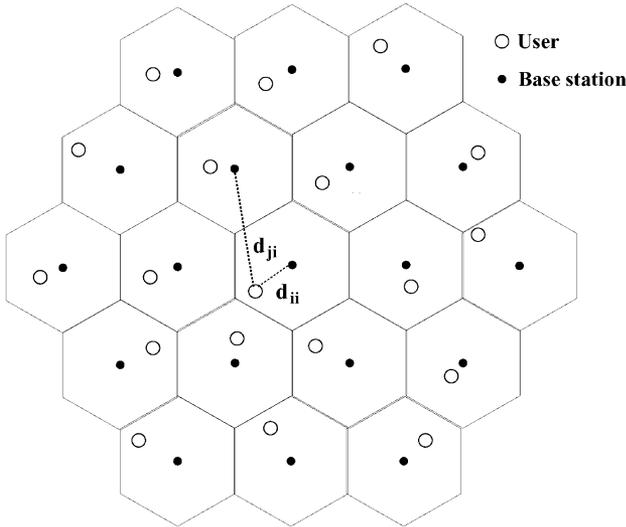


Fig. 4. Two-tier hexagonal cellular system.

case it is 0 dB. Similarly in the second case, the average transmit SNR of a mobile, averaged over the uniform distribution of locations within a cell, is 0 dB.

In both cases, we consider the cellular system in Fig. 4 where each user has six antennas and the number of antennas at each base station is varied from 1 to 60. We take one randomly selected snapshot of the user locations which are distributed uniformly within each cell; how-

ever, we restrict the selected snapshot to be such that no user is within normalized distance of 0.1 from their respective base stations. For the given selection of user locations, we compute the sum capacity of this finite antenna array system by averaging over 500 independent channel realizations. The sum capacities are normalized by the total number of user antennas. The results are depicted in Figs. 5 and 6, respectively.

We condition the randomly selected snapshot of user locations as above in order to obtain a "typical" realization of user locations, i.e., one not dominated by the relatively rare event that several mobiles are very close to their respective base stations and hence obtain a massive (and unrealistic) received SNR. These rare events also make computing the sum capacity averaged over user locations problematic, and we do not attempt to do so. Instead, we randomly select user locations once (as above) and then compute the conditional average sum capacities, where the averaging is performed over the small scale fading parameters only.

Defining β as the ratio of the number of antennas at the user to the number of antennas at its base station, we compare the sum capacity normalized by the total number of antennas at the users (spectral efficiency per transmit antenna) with and without base station cooperation as a function of $1/\beta$. We plot this spectral efficiency, obtained by Monte Carlo simulations as described above, and superimpose on the same graph the results that are obtained by numerical calculations from the large system asymptotics.

The first point to note from both graphs is that the asymptotic theory provides very accurate results, even though there are only six antennas at the mobiles. In both cases, we observe that if we increase the number of base station antennas while keeping the number of user antennas fixed (i.e., we increase $1/\beta$), the spectral efficiency per transmit antenna increases monotonically for both BS cooperation and BS noncooperation.

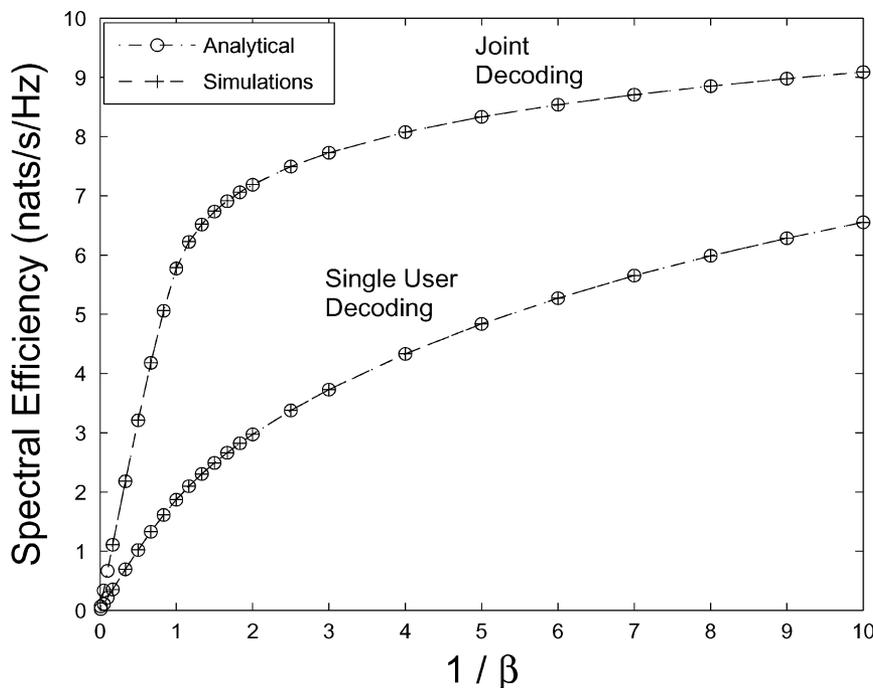


Fig. 5. Spectral efficiency per transmit antenna of the two-tier hexagonal cellular system with average transmit SNR of each user equal to 20 dB.

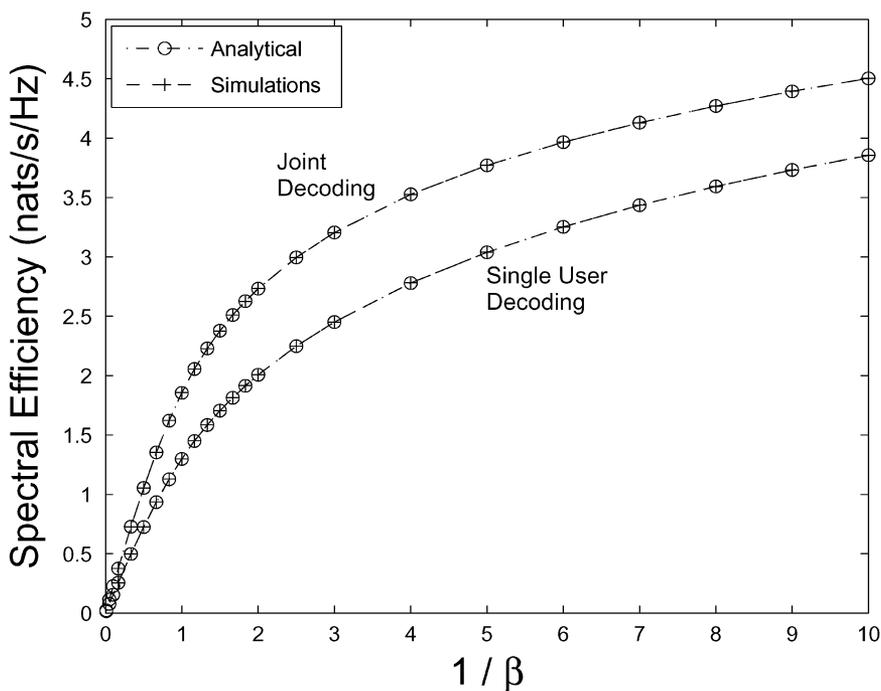


Fig. 6. Spectral efficiency per transmit antenna of the two-tier hexagonal cellular system with average transmit SNR of each user equal to 0 dB.

In both cases, the gain from macrodiversity is highest at small values of $1/\beta$, which is consistent with the trends observed in Wyner's model considered in Section V. At high SNR, the maximum macrodiversity gain is approximately 215%, achieved when $1/\beta = 0.667$, i.e., four antennas at each base station, decreasing to a gain of about 39%, when there are 60 antennas at the base stations. The gain of macrodiversity is much reduced in the low SNR scenario. This is consistent with the results obtained from Wyner's model for small α . In the low SNR scenario, the maximum gain from macrodiversity is about 47%, achieved when there is only one antenna at

the base stations, and this decreases to a gain of about 17%, when there are 60 antennas at the base stations.

In the model of the present section, the gains from macrodiversity are moderate, or quite significant, depending on the SNR, and the value of β . These results are preliminary for a number of reasons. First, they are for a single realization of user locations. Secondly, and perhaps more importantly, they are for a log-distance path loss model, with exponent 4, which has the property that the interference a mobile in one cell creates in an adjacent cell has a small average value, when averaged over the mobile's location in the cell.

More realistic cellular models, with shadowing, will have larger intercell interference, and the gains from macrodiversity will almost certainly be higher than those reported in the present section. A recent paper that does include many realistic features is [26], a paper that studies the capacity of cellular networks with MIMO links, and which includes lognormal shadowing, sectorized cells, and a variety of frequency reuse patterns. However, this paper does not consider macrodiversity, and it is for the downlink of a cellular system. In contrast to our approach of harnessing asymptotic theory, it takes a simulation-based approach to measure system spectral efficiency. It will be interesting in future work to see if the theoretical approach of the present correspondence can be fruitfully applied to more complex, cellular scenarios, such as those investigated in [26].

VII. CONCLUSION

In this correspondence, we have applied recent results on the limiting eigenvalue distributions of large random matrices to the design of cellular systems with MIMO links. In particular, we are interested in characterizing the gain that can be accrued from allowing the base stations to cooperate in the joint decoding of all users in the network. Our focus is entirely the mobile to base station uplink.

We show that large systems analysis provides expressions for spectral efficiency that can be evaluated numerically much faster than Monte Carlo simulations, and yet are accurate enough to be used in network simulation and optimization.

We compare the spectral efficiency results for base station cooperation with the corresponding results for a more traditional cellular architecture where each user is assigned to a base station and base stations try to only decode their user data treating interference from other users as random noise. Clearly, we should expect a gain from cooperation, but the question is, how much? The answer depends on the relative sizes of the antenna arrays at the users and cell sites, as well as the SNR, and the relative interference strength from adjacent cells.

A general trend is that the benefit from BS cooperation increases with the relative size of the mobile transmit antenna array compared to the base station antenna array size. However, the precise gain, and the point where returns start diminishing, varies widely with the other parameters that characterize the network scenario. Other parameters that have a strong impact are the SNR, and the level of intercell interference. If the SNR is low, then cooperation gain across base stations becomes an important issue that helps the macrodiversity case. However, the intercell interference factor must be high enough to get significant gain, and the higher it is, the more macrodiversity gains from being able to jointly decode mobiles that would otherwise be interfering with each other.

These comparisons are somewhat preliminary, and a more detailed study of a cellular network might include issues such as shadow fading, which can greatly increase the interference between cells. Including shadow fading to the model is not very complicated. Based on the conclusions of the present correspondence, it should be sufficient to use spectral efficiency formulas obtained in this correspondence, but the numerical results are then random inputs to a Monte Carlo simulation of slow fading parameters. One thing to expect is that the small intercell interference we observed in Section VI will become much larger with lognormal shadowing included, and then gains of macrodiversity will be much greater than those reported here.

An important remark is that for simplicity, and to provide generalization of current results in the literature, we have assumed a single user in each cell, i.e., an orthogonal multiple access scheme is employed within each cell with full frequency reuse over the cells. However, it is more efficient in terms of sum capacity to allow multiple users per cell to share the same channel using space-division multiple-access (SDMA)

or a spatial multiplexing scheme [26]. In fact, one can consider models in which the number of users grow large with the number of antennas at the base station, with the number of antennas at each mobile held fixed. This analysis is beyond the scope of the present correspondence, but clearly provides an opportunity for there to be many transmit antennas in total (much more than at the base stations) without requiring there to be many antennas at each mobile; therefore, we might expect the gains from macrodiversity to be more significant. This is the subject of ongoing research (see [27]).

From a theoretical point of view, our main contribution is to show that when the system has a certain symmetry, a phenomenon of resource pooling occurs whereby the multiuser macrodiversity system behaves like a single user microdiversity system with same total received signal power and same number of transmitting and receiving antennas.

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Power Allocation Over Parallel Gaussian Multiple Access and Broadcast Channels

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Abstract—In this correspondence, we determine the optimal power allocation, that achieves any specified point on the boundary of the capacity region, for sets of parallel Gaussian multiple-access channels (MACs) and sets of parallel Gaussian broadcast channels (BCs). The power allocation is across the parallel channels and, within each channel, across the users. In both cases, there is a single constraint on the total power used. First, the allocation for the parallel MACs is determined, in a simple form, using the Karush–Kuhn–Tucker (KKT) conditions and a simple Lagrangian argument. Using this result, the allocation for the parallel BCs is derived using recent findings on the duality of the MAC and BC.

Index Terms—Broadcast Channel (BC), capacity region, duality, multiple-access channel (MAC), parallel channels, power allocation.

I. INTRODUCTION

The Gaussian multiple access channel (MAC) and the Gaussian broadcast channel (BC) (Fig. 1) are the most common modeling choices whenever there is many-to-one or one-to-many communication over a common channel, notably in the cases of the cellular uplink and downlink. On the other hand, sets of parallel channels appear in the modeling and analysis of various channels, for example, channels with intersymbol interference (ISI), fading, time-frequency code-division

Manuscript received December 17, 2005. This work was supported by *Kplus* funding for the ftw. project IO "Signal and Information Processing."

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Communicated by R. R. Müller, Associate Editor for Communications.

Digital Object Identifier 10.1109/TIT.2006.876250

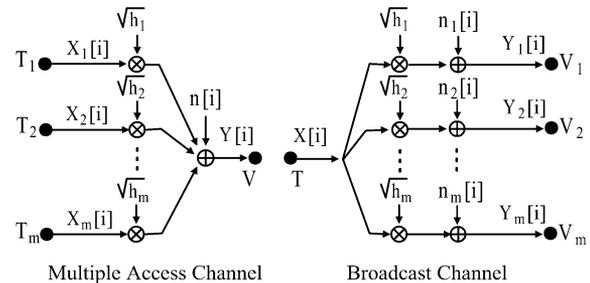


Fig. 1. System models.

multiple access, and so on. Consequently, parallel Gaussian MACs and BCs have attracted significant research interest.

A first study of parallel Gaussian BCs appears in [1]. There, an algorithm is given for calculating the optimal allocation of power among the different parallel channels and among the users of each channel. The power allocation is optimal in the sense that it can achieve any specified point on the boundary of the capacity region. In [2], [3] an alternative greedy algorithm is offered.

In [4], Gaussian BCs with ISI and colored noise are modeled as sets of parallel BCs, each BC corresponding to a different frequency. The authors study the optimal power allocations across different users and frequencies, using the results of [1]. In [5], Gaussian fading BCs are modeled as sets of parallel channels, each channel corresponding to a different fading state. The authors compare the optimal power allocation across different users and states, determined by the method of [1], with a number of suboptimal, but much simpler, power allocations.

Parallel MACs are studied in [6], in the context of MACs with ISI. Similarly to [4], the authors show that such channels can be modeled as sets of parallel MACs, each corresponding to a different frequency. The authors determine graphically the optimal power allocation across different frequencies and users. Parallel MACs are also studied in [7], in the context of channels with fading. In this work, and similarly to [5], each parallel channel corresponds to a different fading state.

In this work, we calculate optimal power allocations, that achieve any point on the boundary of the capacity region, for parallel Gaussian MACs and BCs. Contrary to previous works, [6], [7], we assume a sum-power constraint for the MAC, meaning that there are no constraints on the powers of individual transmitters, but rather a global constraint on the total power across all users and channels.

The sum-power constraint is worth investigating firstly because it appears naturally in various applications. For example, consider a wireless sensor network that consists of a number of sensors T_i , each relaying data with rate R_i to a central site. If the total power that can be allocated to this network, for example in the form of batteries, is fixed, but we are free to distribute it as we like, we would like to know the optimal distribution of batteries. As another example, consider the uplink of a cellular network, in which a constraint has been placed on the sum of transmitter powers, so as to bound the amount of interference caused at nearby cells. Secondly, recent duality results [8] have shown that the capacity region of the MAC with a sum-power constraint is equal to the capacity region of an appropriately defined *dual* BC, and in addition the power allocation that achieves a given point in the capacity region of the BC can be calculated using the power allocation that achieves the same point in the MAC. We use this duality to derive the power allocations of the parallel BC from the power allocations of the parallel MAC, in a very straightforward manner.

The rest of this work is organized as follows: We start in Section II by calculating the optimal power allocation in the case of a single Gaussian