Extending the Applicability of the Combined-Field Integral Equation to Geometries Containing Open Surfaces

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***Abstract*—In this letter, we consider the solution of large electromagnetics problems of composite structures with coexisting open and closed conductors. By modifying the construction of the combined-field integral equation (CFIE), we demonstrate a significant improvement in the iterative solution of these problems compared to the conventional electric-field formulation.**

***Index Terms*—Integral equations, iterative methods, multilevel fast multipole algorithm.**

I. Introduction

CONSIDERING the iterative solutions of various integral-equation formulations of electromagnetic scattering and radiation problems, the combined-field integral equation (CFIE) is preferred mainly due to its improved convergence characteristics and low iteration counts [1]. However, for the solution of problems containing open surfaces, the traditional CFIE is not applicable since it contains the magnetic-field integral equation (MFIE), which can be applied only to closed geometries. If the overall geometry is composed of open surfaces either completely or even to a minor extent (i.e., even if the majority of the geometry is closed), then the electric-field integral equation (EFIE) becomes the inevitable choice in spite of its ill-conditioned nature. As the problem sizes get larger and the dimensions of the matrix equations grow, the solutions of these problems with EFIE become extremely difficult, even when iterative techniques are used with accelerated methods, such as the multilevel fast multipole algorithm (MLFMA) [2] for the matrix-vector multiplications.

II. Hybrid Technique

In this letter, we extend the applicability of CFIE to geometries containing open surfaces, such as the composite structure shown in Fig. 1, where a cross-shaped open surface is placed over a closed ellipsoid surface. While EFIE is still applied on the open parts of the geometry, CFIE is applied on the closed parts to improve the convergence of the iterative solution. Due to the significant improvement provided by this hybrid technique, a solution can be obtained with a reasonable number of iterations.

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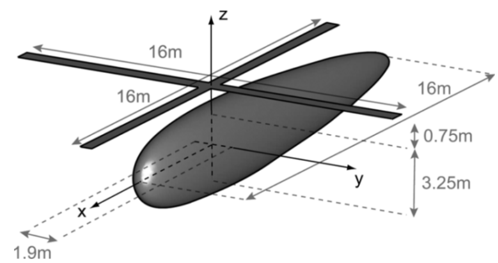

Fig. 1. Composite problem with coexisting closed and open conducting surfaces. Both the cross-shaped open surface and the closed ellipsoid surface are about 27 wavelengths long, which give rise to 131,631 unknowns with $\lambda/10$ triangulation.

Discretizations of the integral equations lead to $N \times N$ matrix equations as

$$\sum_{n=1}^{N} Z_{mn}^{E,M,C} a_n = v_m^{E,M,C}, \quad m = 1, \ldots, N \tag{1}$$

where

$$Z_{mn}^{E} = \int_{S_m} d\boldsymbol{r} \boldsymbol{t}_m(\boldsymbol{r}) \cdot \boldsymbol{E}_n^{sca}(\boldsymbol{r}) \tag{2}$$

and

$$Z_{mn}^{M} = \int_{S_m} d\boldsymbol{r} \boldsymbol{t}_m(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}} \times \boldsymbol{H}_n^{sca}(\boldsymbol{r}) \tag{3}$$

represent the matrix elements for EFIE and MFIE, respectively. In (1) and (2), the scattered fields from nth radiating element $\boldsymbol{E}_n^{sca}(\boldsymbol{r})$ and $\boldsymbol{H}_n^{sca}(\boldsymbol{r})$ are projected onto the testing functions $\boldsymbol{t}_m(\boldsymbol{r})$, each of which has the outward normal vector $\hat{\boldsymbol{n}}$ and the spatial support of S_m. In the conventional CFIE, these matrix elements are linearly combined as

$$Z_{mn}^{C} = \alpha Z_{mn}^{E} + (1-\alpha)\frac{i}{k} Z_{mn}^{M} \tag{4}$$

where k is the wavenumber and α is a constant between 0 and 1. In other words, CFIE is the convex combination of EFIE and MFIE with a fixed parameter for each testing case, i.e., $m = 1, \ldots, N$. This definition can be generalized as

$$Z_{mn}^{C} = \alpha_m Z_{mn}^{E} + (1-\alpha_m)\frac{i}{k} Z_{mn}^{M} \tag{5}$$

where a variable combination parameter (α_m) is used depending on the index of the row (m) in the linear system, i.e., the index of the testing function. The idea of extending the constant α to variable α_m was also used in a previous study [3] to

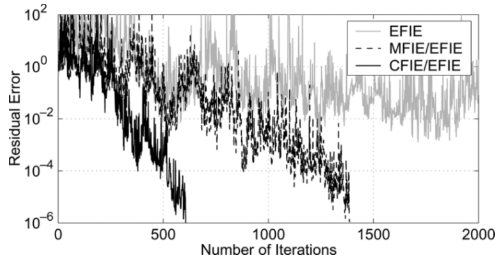

Fig. 2. Convergence characteristics of different formulations for the solution of the scattering problem in Fig. 1 when a preconditioner is not employed in CGS. The hybrid CFIE/EFIE (solid black) offers significantly faster convergence compared to hybrid MFIE/EFIE (dashed black) and the conventional EFIE (gray).

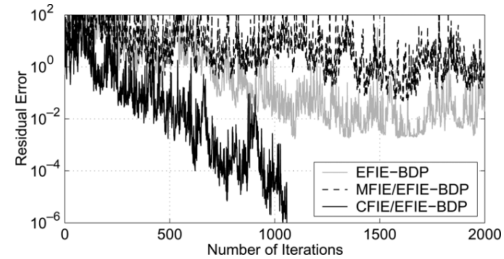

Fig. 3. Convergence characteristics of different formulations when BDP is employed. For all formulations, BDP does not accelerate the convergence. Even so, CFIE/EFIE converges faster than the other two formulations.

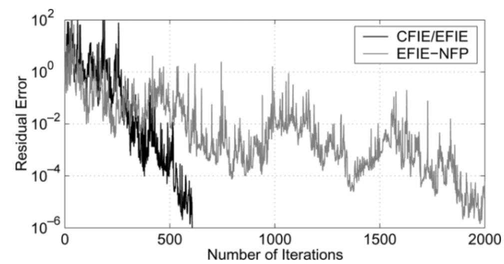

Fig. 4. CFIE/EFIE with no preconditioner converges even faster than EFIE with NFP.

reduce the cost of the matrix filling. Another study successfully used the same idea to properly formulate the electromagnetic problems with composite structures of thin and thick conductors that are completely closed [4]. In the present study, we use the generalized definition of CFIE for the iterative solutions of large problems with composite structures involving both open and closed surfaces, and demonstrate the acceleration of the iterations. Specifically, we take $\alpha_m \neq 1$ (CFIE) on the closed parts, while $\alpha_m = 1$ (EFIE) on the open parts. Physically, this corresponds to imposing the boundary conditions both on the electric and magnetic fields for the closed parts, while only EFIE is applied on the open parts.

III. Results

The composite structure shown in Fig. 1 is solved at 500 MHz. It is illuminated by a plane wave propagating in the $-x$ direction with the electric field polarized in the y direction. Rao–Wilton–Glisson [5] functions defined on planar triangles are employed to expand the unknown surface current density. With the given electrical dimensions, the ellipsoid (closed surface) has 115,023 unknowns and the cross-shaped part (open surface) has 16,608 unknowns when the mesh size is about $\lambda/10$. The problem is solved by a conjugate-gradient-squared (CGS) algorithm and the matrix-vector multiplications are performed efficiently by employing MLFMA.

Fig. 2 demonstrates the convergence characteristics for different formulations of the problem when a preconditioner is not employed in CGS. With the conventional EFIE, normalized residual error does not drop under 10^{-3} within 2000 iterations. When MFIE is applied on the closed part (denoted as "MFIE/EFIE"), the convergence is slightly improved. However, when we apply CFIE with $\alpha_m = 0.2$ on the closed part (denoted as "CFIE/EFIE"), the convergence is significantly accelerated so that the residual error drops under 10^{-6} after about 600 iterations. To our experience, $\alpha_m = 0.2$–0.3 is the optimal range for rapid convergence.

Next, we establish that even powerful preconditioning schemes do not accelerate the convergence of EFIE as well as the CFIE/EFIE hybrid technique. For this purpose, two different preconditioners are implemented. The near-field preconditioner (NFP) is a strong and computationally expensive preconditioner, which is obtained by the exact factorization of the sparse near-field matrix of MLFMA [6]. The block-diagonal preconditioner (BDP) retains only the self interactions of the lowest level clusters of an MLFMA tree. Though weaker than NFP, BDP is the leading preconditioner employed with MLFMA due to its favorable computing cost [2]. Fig. 3 shows that the use of BDP has an adverse effect on the convergences of the hybrid formulations MFIE/EFIE and CFIE/EFIE, while it does not provide a convergence for EFIE. Nevertheless, CFIE/EFIE emerges as the only convergent method in Fig. 3. Finally, in addition to having established that CFIE/EFIE has better convergence properties than both MFIE/EFIE and EFIE for both no-preconditioner and BDP cases, Fig. 4 demonstrates that CFIE/EFIE with no preconditioner converges even faster than EFIE with NFP, which is a very strong preconditioner.

IV. Conclusion

The extension of CFIE to the electromagnetic problems of composite geometries is presented. For thin conductors that are modeled with open surfaces, EFIE is inevitable. However, we employ CFIE on the closed parts to improve the efficiency of the iterative solutions. As the problem size gets larger, the improved convergence provided by this hybrid technique becomes critically important.

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