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Managerial Motivation Dynamics and Incentives

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Firms can increase profitability by appropriately motivating managers. We investigate drivers of managerial motivation, and propose how firms can use performance pay to alter motivational patterns. We focus on the agent's optimal effort decision in trading off compensation utility with effort cost in a static and dynamic setting. Surprisingly, we find that lower risk aversion or increased pay are not necessarily motivating factors, and identify the relevant effort drivers underlying the agent's utility and compensation plan. We characterize properties of agents' preferences for output lotteries (risk aversion, aggressiveness, prudence) that trigger systematic motivational patterns with respect to a variety of factors, such as the agent's productivity and past performance, time to evaluation, the firm's capabilities, and market factors. Our insights are robust, holding under very general modeling assumptions on preferences, rewards, and the stochastic effort-output function.

Key words: decision analysis; risk preferences; risk aversion; incentives; salesforce compensation

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1. Introduction

According to a new Hewitt¹ global study, 2006 will mark an "increased activity in the area of variable pay, as companies rely more on bonuses as a primary means of attracting, motivating, and retaining key talent" (*Business Wire* 2005). For example, the Bank of America offers performance-based contracts to all employees because "incentive pay makes people work harder," just as Alparma Inc. puts more money into incentive compensation because they "have seen a connection between company performance and employee incentive" (*Wall Street Journal* 2004, p. D1).²

Bonus pay is typically tied to corporate profitability, either in an egalitarian structure, or a "meritocracy" that differentiates employees based on their quarterly or annual performance. According to Hewitt, "companies are putting more focus on the notion of *performance*, and they're willing to spend [...] more on bonus pay when the results justify it." To determine performance, firms look at a wide array of measures, ranging from objective sales targets, to customer satisfaction ratings, all the way to subjective assessment of interpersonal effectiveness.

Recognizing the value of top performing employees, firms are "making special efforts to [...] keep them engaged in their work and the company, as well as appropriately rewarded," Hewitt reports. In the

effort to better understand and manage the firm's human resources, this research proposes to investigate, from a rational agent perspective, the drivers of managerial motivation, defined as the "willingness to exert effort to achieve the organization's goals" (Coulter and Robbins 1999). Specifically, we focus on the optimized decisions of a manager, so-called agent, whose motivation is measured by the level of effort expended into his work activity, when the firm offers a particular performance-based contract. We analyze what intrinsic and extrinsic factors influence the managers' effort, and what properties of the agent's preferences and reward structure lead to specific motivational patterns.

The managerial compensation literature has studied these issues primarily in the context of a principal-agent framework.³ Agency models focus on designing optimal compensation from the firm's profit perspective, with the agent's motivational drivers as a by-product. In contrast, this paper focuses on the agent's response problem: we zoom on the individual manager and describe his optimal behavior under a given (but general) contract.

Our investigation is relevant to organizations seeking to better understand how employees react to existing or proposed compensation plans, and how their motivation is affected by various factors (e.g., the

¹ Hewitt Associates is a global HR consultancy.

² Subsequent quotes are from this article, unless otherwise stated.

³ See Salanié (1997) for a comprehensive treatment and Prendergast (1999) for a review of empirical findings.

agent's attitude toward risk, market factors, a projected change in compensation, etc.). For example, a firm may believe that by increasing salary or total compensation by five percent, it will motivate its managers to work harder. Our results (§4) show that the opposite may be true, and explain when this is the case. We do not prescribe, however, the optimal package to offer.

In reality, firms offer contracts that are not always optimal or responsive to the environment. This is in part because optimal contracts (as prescribed by agency theory) are overly complex, and need to adjust (optimally) in response to the myriad of factors affecting the firm's profits. While this is theoretically desirable, it is not always practical. Even if a firm offered optimal contracts, these would typically be set for extended periods (e.g., one year). Our model is particularly relevant during this time frame, when changes in the working environment, market factors, or interim performance affect the agent's motivation, but not his contract.

Surprisingly, little is known about the agent's effort response under given types of contracts. According to Ross (2004, p. 208): "Unfortunately, the effort to characterize [contract] optimality—often in highly specific and parametric models—has crowded out the study of the agent given specific contract forms of the sort that are commonly observed in practice."

This gap in the literature prompts the focus of our work on the agent's problem, within the classical principal-agent model. We analyze how the agent's preferences, the firm's policies, and market conditions affect effort and profits, under given contract structures. Focusing on the agent's problem allows for tractable analysis and strong comparative statics results under very general modeling assumptions on preferences, rewards, and the effort-output function. This leads to robust insights characterizing motivational patterns in response to given (but general) incentive structures.

Our results are different from the principal-agent literature, where the agent's behavior is studied in response to optimal compensation. In the agency framework, changes in a given factor affect the agent's effort level not only directly (as in our model), but also indirectly, by altering the agent's contract (optimally for the firm). Focusing on the agent's problem allows us to separate these effects and obtain new, interesting insights that hold under very general conditions. In contrast, we find that insights arising from existing (parametric) principal-agent models are not robust, hence should be applied with caution to specific needs of an organization.

A common myth in managerial, particularly sales-force, compensation is that motivation is related to

the agent's degree of risk aversion. Indeed, previous results indicate that less risk averse agents exert more effort when the agent's risk aversion is wealth independent (Lal and Srinivasan 1993). We show that this relationship is not robust in the general case and can be reversed, for example, when agents have linear plus exponential utility. In general, no systematic effect of risk aversion on effort can be claimed. This may partially explain inconsistent results on the relationship between risk aversion and optimal reward structure found in empirical studies (e.g., Joseph and Kalwani 1995). Our results indicate that, all else equal, agents with higher marginal utility for wealth will work harder, but those with lower risk aversion may not. In particular, a comparison of marginal utilities can predict who will work harder.

Our analysis of how reward plans impact effort shows that a salary raise does not stimulate managers to work harder. This may explain why "companies are moving away from the traditional annual pay raise in favor of beefing up the amount of money earmarked for employee bonuses" (*Wall Street Journal* 2004, p. D1). Interestingly however, we find that increasing variable pay may not be a good motivator either, unless agents exhibit so-called *aggressive* preferences, i.e., their marginal utility is inelastic to changes in wealth. There are no benefits, however, in offering higher rewards to *conservative* agents (the terms aggressive and conservative are formalized in Definition 2). It is therefore important to elicit these properties of the agent's preferences before designing compensation plans. In general, firms can induce rational agents to work harder by offering lower but steeper compensation plans (these need not convexify the agent's value function). This condition is also necessary to induce motivation from a diverse workforce, when performance, also referred to as output, is unpredictable by the firm.

Factors that influence output have an indirect impact on effort; these include the agent's productivity and past performance, the firm's capabilities, as well as market factors, such as price or risk. In this context, we investigate which properties of the agent's (*reward-induced*) preferences for output (see Definition 1) are relevant triggers of motivational patterns. We find that agents with aggressive preferences for output are motivated by their own and the firm's productivity, contrary to conservative ones. Agents with risk-seeking preferences for output expend more effort in bigger markets, or when prices are lower (the opposite holds in the risk-averse case). An increase in market risk motivates agents with *prudent* output preferences and demotivates *imprudent* ones (see Definition 3). We also show that the corresponding trigger properties of the agent's preferences (risk seeking, aggressive, prudent) are also necessary to elicit a

robust motivational pattern. Finally, we provide conditions on the compensation plan to induce such preferences for output.

The last part of this paper investigates how past performance and evaluation horizon affect the agent's effort level in a dynamic setting, where variable compensation is delivered based on cumulated output at the end of a multiperiod horizon (year, quarter). We identify the agent's induced risk aversion for output as the trigger property of consistent effort behavior. Specifically, managers with risk-averse output preferences (e.g., linear compensation plans) are unmotivated by past successes (i.e., expend less effort the better their achievements). Longer evaluation horizons are not motivating for such agents, who tend to procrastinate at the beginning of the evaluation period and undertake more effort closer to bonus time. Interestingly, these patterns can be reversed by changing the reward function in a way that induces risk seeking preferences for output.

1.1. Literature Review and Positioning

Our work is related to three main streams of literature in marketing (salesforce compensation), decision sciences (economic agent models), and finance (executive compensation). A seminal paper in the salesforce compensation literature is Basu et al. (1985), who characterize the optimal contract in a static principal-agent setting, with power utility and gamma/binomial sales. They provide comparative statics with respect to market uncertainty, salesforce effectiveness, and production cost under linear contracts. A wide range of variations and extensions have been subsequently proposed, but few are related to our work.⁴ Lal and Srinivasan (1993) extend Basu et al. (1985) to a dynamic setting; their dynamic problem is elegantly reduced to a static one due to the exponential utility assumption. Dynamics are also considered by Tapiero and Farley (1975) in a multiproduct deterministic setting, and by Dearden and Lilien (1990) in a two-period production learning model. Reviews on salesforce compensation are due to Coughlan and Sen (1989) and Coughlan (1993). The insights from these parametric principal-agent models are contrasted with the robust, nonparametric results derived from our analysis of the agent's problem.

In the decision analysis literature,⁵ a stream of work related to ours considers the problem of an agent who controls a risky distribution of losses by exerting effort. Dionne and Eeckhoudt (1985) show that

⁴ Most recently, a game theoretic model of salesforce compensation was proposed by Misra et al. (2005), who provide empirical tests of their theoretical hypotheses.

⁵ Sarin and Winkler (1980) provide empirically verifiable conditions on the agent's (multiattribute) preferences that induce certain sound properties of managerial incentives.

the agent's risk aversion has an ambiguous effect on effort under a Binomial loss model. Jullien et al. (1999) provide sufficient conditions on the risk distribution for higher risk aversion to induce higher effort. Eeckhoudt and Gollier (2005) determine prudence as the key determinant of the agent's optimal effort behavior, in line with our results in §5. We provide a general, in-depth analysis of how the agent's preferences affect effort in §3.

Besides effort, another important aspect of the agent's output is risk. Optimal response to risk, under various compensation schemes, is investigated from an agent's perspective by Gaba and Kalra (1999) and Gaba et al. (2004), and, in a principal-agent model, by Godes (2004). The financial literature has extensively investigated the influence of nonlinear contracts (typically convex options) on the agent's risk-taking behavior. In particular, Ross (2004) focuses on the agent's problem to derive conditions on utility and reward plans to induce more or less risk-averse behavior.

1.2. Structure and Framing

The rest of this paper is organized as follows. The main static model is presented in §2. Section 3 investigates the impact of the agent's preferences on effort. The motivational impact of pay structures is the subject of §4. Section 5 investigates comparative statics with respect to factors affecting output, including the agent's and the firm's productivity, and market factors. Section 6 extends the problem to a dynamic setting and obtains insights with respect to the agent's past performance and time to evaluation. Section 7 concludes the paper.

Our model and results are applicable in contexts where the agent is subject to any form of performance pay, and effort is a strategic decision affecting the agent's performance or output. For simplicity, however, we focus the exposition of this paper on sales as the sole pragmatic measure of the agent's output; the effort-output function refers to the sales response.

2. The Agent's Static Problem

This section sets up the static model, our main assumptions, basic notation. The agent's preferences are captured by a separable bi-criteria utility for wealth $w \in \mathbb{R}$ and effort $x \in [0, 1]$:⁶

$$\mathcal{U}(w, x) = U(w) - C(x).$$

U represents the agent's utility for wealth, assumed increasing and concave (i.e., risk averse).⁷ The disu-

⁶ This can be interpreted as the percentage of effort devoted to the sales activity. Alternatively, and at no loss of generality, x could be defined in absolute terms over $[0, \infty)$.

⁷ We use the terms negative, positive, increasing, and decreasing in their weak sense (a.k.a. nonpositive, nonnegative, nondecreasing, and nonincreasing).

tility or cost of effort C is positive, increasing, and convex, with $C(0) = 0$ and $C(1) = \infty$. Such a separable model is most common in the salesforce literature (see, e.g., Basu et al. 1985, Lal and Staelin 1986).

The agent's effort level decision x controls a random output, also referred to as performance or sales function $\mathbf{S}(x) = s(x, \mathbf{Y})$. The random variable \mathbf{Y} captures exogenous factors (such as market forces) which affect output, but are not under the agent's control. Throughout this paper we denote random variables in bold. The riskless sales response function $s(x, y)$ is assumed to be positive, increasing, and concave in x . Concavity captures the diminishing marginal impact of effort on output. Monotonicity implies that the random sales response $\mathbf{S}(x)$ is increasing in effort in the sense of first-order stochastic dominance,⁸ i.e., higher levels of effort increase the probability of output above any given level. Relevant special cases include additively separable models $\mathbf{S}(x) = f(x) + \mathbf{Y}$, with f increasing, or multiplicative models $\mathbf{S}(x) = x\mathbf{Y}$, where \mathbf{Y} is a positive random variable. In our model, output can refer to either unit or dollar sales, depending on what the agent's compensation is structured around.

The compensation plan, r , also referred to as contract or reward, typically consists of a fixed salary F and a variable, sales dependent component v : $r(s) = F + v(s)$, with $v(0) = 0$.⁹ For example, the variable part may be a linear commission λ per unit sale, $v(s) = \lambda s$, leading to a so-called linear contract. More complex, commonly used (step/piecewise linear) contracts involve a fixed bonus B for output above a target θ_F : $v(s) = B1_{s \geq \theta_F}$, or a commission rate λ for output above the target $v(s) = \lambda(s - \tau)^+$, or combinations thereof. We denote 1_A the indicator of the set A and $x^+ = \max\{x, 0\}$.

Formally, the agent's static problem is modeled as

$$\max_x \mathbb{E}[U(r(s(x, \mathbf{Y}))) - C(x)]. \quad (1)$$

Let $R(x)$ denote the expected return from effort, i.e., $R(x) = \mathbb{E}[U(r(s(x, \mathbf{Y})))]$ and $W(x) = R(x) - C(x)$, the objective value in (1). We denote the optimal effort level $x^* = \arg \max_x W(x)$, where, without loss of generality, the operator $\arg \max$ refers to the largest maximizer (i.e., among equally appealing alternatives, the agent chooses the maximum effort one).

A factor λ is said to be *motivating* (*demotivating*) if the agent's optimal effort $x^*(\lambda)$ is increasing (decreasing) in λ . A given control parameter λ is motivating whenever the agent's marginal return from effort

$R_x(x, \lambda)$ is increasing in λ (partial derivatives are denoted by corresponding subscripts). This *marginal return effect* is the key driver of motivational patterns. The next result follows from Topkis (1998, Theorem 2.8.2).

LEMMA 1. *The agent's optimal choice of effort $x^*(\lambda)$ is increasing in the parameter λ , provided that his return from effort, $R(x, \lambda)$, is supermodular in λ and x .¹⁰*

Throughout this paper, we identify trigger conditions for the agent's marginal return to increase in response to a change in a given control factor λ , such as the agent's risk aversion, reward plan, productivity, past performance, or market factors. Lemma 1 ensures systematic motivational patterns. The following concept will be relevant for characterizing systematic effects.

DEFINITION 1. The agent's (*reward-induced*) utility for output is defined as the value function $V = U \circ r$. This value function captures the agent's (*reward-induced*) preferences for output.

The agent's preferences for wealth, captured by his utility function U , induce via the reward plan r , preferences for sales/output, captured by V . Faced with the direct choice between two output (e.g., sales) lotteries \mathbf{S}_1 and \mathbf{S}_2 , the agent will prefer the one that gives higher expected value under V , i.e., higher compensation utility. It is not that the agent cares for sales per se, but he cares for sales in as much as they determine his wealth. While the agent's preferences for money are intrinsic, preferences for output are induced by compensation. For example, the agent is naturally risk averse for wealth (U concave), but may exhibit risk-seeking preferences among output lotteries if the compensation offered by the firm induces V to be convex.

2.1. The Firm's Profit Problem

While this paper is primarily concerned with the agent's effort decision under a given contract, it is important to understand how this indirectly affects the firm's profitability. In particular, when does a higher level of effort translate into higher net profits for the firm? In our setup, this is true as long as overall profits increase with the agent's output level, i.e., if the marginal return from output exceeds the marginal cost of compensating the agent.

Assume for simplicity that output $\mathbf{S}(x)$ is measured by contribution margin (similar results can be derived for other output measures, such as volume). The firm's net profit from the agent expending effort

⁸ \mathbf{X} dominates \mathbf{Y} in the first order if $\mathbb{E}[\varphi(\mathbf{X})] \geq \mathbb{E}[\varphi(\mathbf{Y})]$ for all increasing functions φ .

⁹ See Peck (1982) for a survey of compensation plans offered in practice.

¹⁰ A bi-variate function $g(x, y)$ is supermodular if for all $x_2 \geq x_1$, $y_2 \geq y_1$, $g(x_2, y_2) - g(x_2, y_1) \geq g(x_1, y_2) - g(x_1, y_1)$. If g is differentiable, this amounts to a positive cross-partial derivative $g_{xy}(x, y) \geq 0$.

level x , under the contract r , is $\Pi(x) = S(x) - r(S(x))$. The next result insures that, for this given contract, a higher level of effort increases the probability of firm profits above any given level, and, in particular, increases expected profits. All proofs are provided in the appendix.

REMARK 1. If $s - r(s)$ is increasing in s , in particular, if $r' \leq 1$, then the firm's net profit $\Pi(x)$ is stochastically increasing in the agent's effort level x . The condition is also necessary for this to be true for any increasing sales function s .

This condition states that compensation should not be steeper than output itself, which is practically not very restrictive. In particular, under a salary plus commission contract, this means that commission is not to exceed product margin, a necessary condition for profitable compensation.

Remark 1 identifies a simple design condition that aligns managerial effort and firm profits. In particular, under such contracts, all our comparative statics for effort translate into analogous results with respect to firm profits. The condition, however, is not necessary (hence not assumed) for the rest of the results in this paper.

3. Impact of Agent's Preferences on Motivation

In this section, we investigate the impact of the agent's preferences on effort. In an agency framework, Lal and Srinivasan (1993) show that the agent's optimal effort decreases in the degree of risk aversion for an exponential utility model with linear contracts and normally distributed sales.¹¹ We first confirm these results in the context of the agent's problem under exponential and power utility models with general contract and sales functions. However, we show that the insights are nonrobust, relying essentially on the parametric form of the utility function, and, in particular, on the independence of risk aversion and wealth. With general utility functions, we show that more risk-averse agents do not necessarily work less (in fact, the opposite can be true), and we determine the appropriate property of the agent's preferences that acts as a systematic motivational driver.

We first remind some key concepts of risk aversion (see Pratt 1964, Arrow 1965). An agent with utility U is more risk averse than an agent with utility u if $U = g \circ u$ for a certain concave function g . An agent exhibits increasing absolute risk aversion

¹¹ In their agency framework, the profit maximizing firm seeks to balance marginal net revenue from sales and marginal compensation cost. Under their parametric model, the latter increases in both effort and risk aversion, whereas the former is independent of both. So, at higher levels of risk aversion, the firm has to decrease marginal compensation cost, which is achieved by inducing a lower effort (via a lower commission level).

(IARA) if he is more risk averse at higher wealth levels, i.e., if for any $\delta \geq 0$, there exists a concave transformation g_δ such that $U(w + \delta) = g_\delta(U(w))$. If U is twice differentiable, this amounts to the absolute risk aversion coefficient $A_U(w) = -U''(w)/U'(w)$ being increasing. Decreasing/constant absolute risk aversion (DARA/CARA) are defined similarly. Relative risk aversion is a measure of risk aversion weighted by the level of wealth, $R_U(w) = wA_U(w)$. Equivalently, R_U is the elasticity of the agent's marginal utility, $U'(w)$. The concepts of monotone relative risk aversion (IRRA/DRRA/CRRA) are defined by the monotonicity of $R_U(w)$ in w .

3.1. Insights from Special Utility Classes

We investigate the effect of the agent's degree of absolute and relative risk aversion on optimal effort for some relevant parametric utility classes. Assuming compensation consists of a nonzero salary $F > 0$, plus variable pay on sales, $r(s) = F + v(s)$, the agent's problem is

$$\max_x \mathbb{E}[U(r(s(x), \mathbf{Y}))] - C(x). \quad (2)$$

3.1.1. CARA Utility. To study the impact of the degree of absolute risk aversion on optimal effort, consider an agent with exponential (CARA) utility function, $U(w) = 1 - e^{-w/\rho}$, where $\rho = A_U^{-1}(w)$ measures risk tolerance. The next result shows that the optimal effort level $x^*(\rho)$ is increasing in the risk tolerance $\rho \in [0, 1]$.

REMARK 2. The optimal choice of effort for an agent with CARA utility decreases in his degree of absolute risk aversion.

3.1.2. CRRA Utility. To measure the effect of relative risk aversion on effort, consider an agent with a power (CRRA) utility $U(w) = w^\rho$. Higher $\rho \in [0, 1]$ corresponds to lower relative risk aversion. The next result shows that $x^*(\rho) = \arg \max \mathbb{E}[r(s(x), \mathbf{Y})^\rho] - C(x)$ is increasing in ρ .

REMARK 3. The optimal choice of effort for an agent with CRRA utility decreases in his degree of relative risk aversion.

Remarks 2 and 3 imply that the more risk averse the agent is, the less effort he will put into his sales activity. Both results, however, obtain from special parametric utility classes that assume the agent's (absolute, relative) degree of risk aversion is independent of wealth.

3.1.3. One-Switch Utility. To verify the robustness of the above results beyond the above models, consider an agent with a one-switch linear-exponential utility function (see Bell 1988), $U(w) = w - \rho e^{-\beta w}$, $\beta, \rho > 0$.¹² This is DARA with $A_U(w) = \beta^2 \rho e^{-\beta w} / (1 + \beta \rho e^{-\beta w})$ increasing in ρ , i.e., agents with higher ρ are

¹² As wealth increases, the agent with one-switch utility can reverse his preference between two alternatives only once.

more risk averse at any given wealth level w . In sharp contrast with the results of Remarks 2 and 3, we show that, with one-switch preferences, more risk-averse agents actually work harder.

REMARK 4. For agents with linear-exponential utility $U(w) = w - \rho e^{-\beta w}$, at any given wealth level the optimal effort is larger at higher levels of ρ , i.e., for higher absolute risk aversion.

3.2. Motivating Preference Structures

So far we have considered parametric classes of utility functions that insure a monotone (albeit not consistent) relationship between risk aversion and effort. We next investigate the robustness of such a relationship under general utility functions. Contrary to previous results in the literature, we show that lower risk aversion does not necessarily induce the agent to exert more effort, and we identify the appropriate conditions to achieve that.

Consider two sales agents 1 and 2, with utility functions $U_1(w)$ and $U_2(w)$, and denote the corresponding optimal effort levels by x_1^* and x_2^* , respectively. Let $\Omega(r)$ be the space of all reward values achievable with the contract r and any positive sales, i.e., $\Omega(r) = r([0, \infty))$.

THEOREM 1. *If $U_2 - U_1$ is increasing, then agent 2 expends more effort than agent 1, regardless of the reward and sales function.¹³ Conversely, if for any increasing sales function s , agent 2 exerts more effort than agent 1 under the contract r , then $U_2 - U_1$ must be increasing on the achievable rewards set $\Omega(r)$.*

From Lemma 1, the agent with higher marginal return from effort works harder, i.e., $x_2^* \geq x_1^*$. Higher effort corresponds to higher rewards, i.e., more wealth. Hence, agents with higher marginal value for wealth are motivated to work harder. Indeed, the condition of Theorem 1 can be restated as $U_2(w + h) - U_2(w) \geq U_1(w + h) - U_1(w)$ for all w and all $h > 0$. The second part of the result shows that higher marginal utility is also a necessary condition for an agent to work harder, regardless of the sales response. Hence, motivational dominance among preferences is naturally and robustly characterized by higher marginal utility.

A direct consequence of Theorem 1 is that a risk-averse agent with utility U works harder than a risk-neutral agent, provided that $U(x) - x$ is increasing, i.e., marginal utility exceeds unity. In general, consider the transformation g that maps one agent's utility into the other's, $U_2 = g \circ U_1$. Theorem 1 states that $x_2^* \geq x_1^*$ whenever $g' \geq 1$. This does not imply, nor is it

implied by convexity of g (which defines U_2 as being less risk averse than U_1), explaining why the degree of risk aversion has no robust, systematic impact on effort. The following example illustrates a nonmonotone relationship between risk aversion and effort.

EXAMPLE 1. Consider three sales agents with utility functions $U_0(w) = \beta + w$, $U_1(w) = \ln(\beta + w)$, and $U_2(w) = \ln(\beta + w) - e^{-\ln(\beta + w)}$, $\beta \geq 1$, so $A_{U_2} \geq A_{U_1} \geq A_{U_0}$, i.e., agent 2 is more risk averse than agent 1 who is more risk averse than (the risk-neutral) agent 0 at any wealth level. However, both $U_2 - U_1$ and $U_0 - U_1$ are increasing, hence, $x_0^* \geq x_1^*$ and $x_2^* \geq x_1^*$, i.e., agents 0 and 2 both work harder than agent 1.

Our results in this section focus on wealth preferences, but extend naturally for effort preferences. One can similarly show that agents with lower marginal disutility of effort work harder, and this is also a necessary condition for the result to be robust with respect to the sales response. This confirms and generalizes the insights of Lal and Srinivasan (1993), who show that under a linear cost model, $k(C) = kC$, the agent's optimal effort choice decreases in k for $k \geq 1$.

4. The Impact of Compensation on the Agent's Motivation

Given that firms offer performance-based pay to motivate agents, it is important to understand how properties of the compensation plan impact the optimal effort level exerted by the agent. This is investigated in the current section. We identify two systematic, but typically opposed motivational patterns driven by reward structures: the *wealth* and *marginal reward* effects. We further characterize properties of the agent's preferences under which increasing (or decreasing) the agent's variable, respectively, total, compensation is motivating; these are linked to the *aggressiveness* of the agent's utility function.

4.1. Motivating Reward Structures

Fixed salaries, the most common type of compensation, fail to provide motivation for high performance. Indeed, for a risk-averse agent, marginal return from effort decreases as fixed salary increases ($U(F + h) - U(F)$ is decreasing in F for concave U), leading to decreased effort levels (Lemma 1). We refer to this as the *wealth effect*.

PROPOSITION 1. *The agent's optimal choice of effort is decreasing in the level of his fixed salary.*

This suggests that to increase effort, firms should offer the lowest salary acceptable by the agent, and focus on variable compensation schemes to drive motivation. The result is consistent with current efforts by private and governmental organizations to move away from the traditional annual pay raise

¹³ For a given a contract r and a sales function S , it is sufficient to grant monotonicity of $U_2 - U_1$ on the domain of reward values w that are achievable under this contract and sales function.

to performance-based bonuses, as indicated in the introduction.

While salary appears to be a systematic demotivator, we investigate what transformations of the reward plan systematically motivate managers. From Lemma 1, the agent works more under a reward plan, provided that his marginal return from effort is higher. Because effort has a direct impact on sales, the reward plan which ensures higher effort is the one which induces a higher marginal value for output. We refer to this as the *marginal reward effect*. Moreover, this condition is necessary for the result to be robust with respect to the sales response.

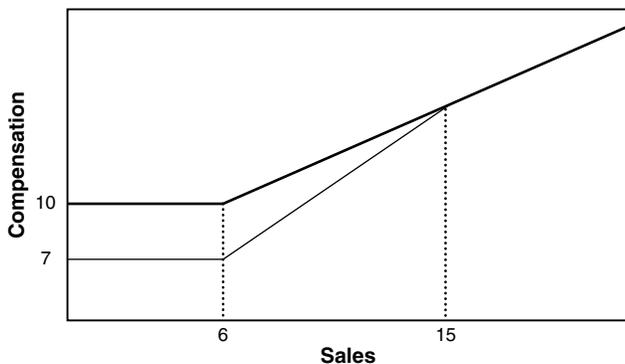
PROPOSITION 2. *Given two reward plans r_1 and r_2 , if $U \circ r_2 - U \circ r_1$ is increasing, then the agent with utility U always expends more effort under r_2 than under r_1 . Conversely, if for any increasing sales function s , the agent exerts higher effort under r_2 than under r_1 , then $U \circ r_2 - U \circ r_1$ must be increasing.*

The conditions of Proposition 2 depend on the agent’s utility function. When this is unknown to the firm, the following conditions on the reward functions induce any risk-averse agent to work more.

THEOREM 2. *If $r_2 \leq r_1$ and $r_2 - r_1$ is increasing, then all risk-averse agents expend more effort under r_2 than under r_1 . Conversely, if for any increasing sales function s , all risk-averse agents exert higher effort under r_2 than under r_1 , then $r_2 \leq r_1$ and $r_2 - r_1$ is increasing.*

Interestingly, the firm can induce the agent to work more by actually paying him less, as long as his new reward plan is steeper. Equivalently, the transformation $g = r_2 \circ r_1^{-1}$ that maps r_1 into r_2 , lies below and is steeper than the identity line, i.e., $g(x) \leq x$ and $g' \geq 1$. In particular, Proposition 1 is a special case of the first part of Theorem 2 with $g(x) = x - F$. An example of such a transformation that also changes the slope of the (piecewise-linear) contract is provided in Figure 1.

Figure 1 The Marginal Reward Effect: Contract 2 Is More Motivating Than 1 at Any Level of Sales Because $r_2 \leq r_1$ and $r_2 - r_1$ Is Increasing; $r_1(s) = 10 + (0.5s)1_{[6, \infty)}$ and $r_2(s) = 7 + (0.7s)1_{[6, 15]} + (0.5s)1_{[15, \infty)}$



The results in this section characterize robust conditions on the shape of the contract that motivate agents to work harder. However, such prescriptions may be difficult to follow in practice, either because they require knowledge of the agent’s utility (Proposition 2) or they imply lowering the agent’s pay (Propositions 1 and Theorem 2). The latter is in contrast with the “common wisdom” that an increase in variable pay should motivate agents. The next sections provide conditions for this to be the case.

4.2. Magnitude of the Agent’s Total Pay

To assess the motivational impact of increasing the magnitude of the agent’s total reward, consider

$$\max_x E[U(\lambda r(s(x), Y))] - C(x),$$

where λ captures the magnitude of the agent’s return. Increasing λ results in a combined wealth and marginal reward effect, which go in opposite directions. From Lemma 1, the agent is motivated by an increase in the magnitude of his total reward, if his marginal return from effort is increasing in λ . The following dual properties of the agent’s preferences formalize this effect; terminology is adapted from Liu (2001).¹⁴

DEFINITION 2. An agent with utility function U is aggressive (conservative) if $U(\delta(w + h)) - U(\delta w)$, $h > 0$, is increasing (decreasing) in δ . If U is twice differentiable, aggressiveness is equivalent to $R_U \leq 1$; conservativeness is captured by the opposite inequality.

REMARK 5. Any conservative agent is risk averse. Any risk-seeking agent is aggressive.

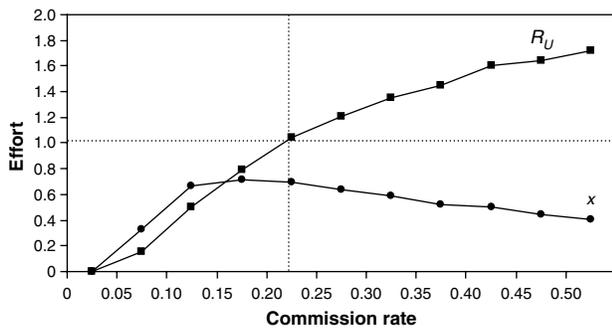
Aggressiveness means that the agent’s marginal utility is inelastic with respect to changes in wealth (relative risk aversion is the elasticity of U). In this case, we show that the marginal reward effect from a percentage increase in reward dominates the corresponding wealth effect. This suggests that increasing pay works as a motivational tool for aggressive agents. The opposite is true for conservative agents. Moreover, these properties are necessary to obtain a robust effort pattern, regardless of the sales response.

THEOREM 3. *The optimal choice of effort for an aggressive (conservative) agent is increasing (decreasing) in the magnitude of reward. Conversely, an agent whose optimal choice of effort is increasing (decreasing) in the reward magnitude for all increasing sales functions s , is aggressive (conservative).*

¹⁴In a dynamic portfolio choice setting, Liu (2001) shows that agents with relative risk aversion less than one invest more in the risky asset, the shorter their investment horizon. He describes this behavior and corresponding property as “aggressive.” The opposite “conservative” behavior holds for investors with relative risk aversion larger than one.

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Figure 2 Optimal Effort Level vs. Magnitude λ of Total Pay for $U(w) = 1 - e^{-w}$, $r(s) = \lambda s$, $S(x) = x + Y$, Where $Y = (0.5, 1; 0.5, 5)$



Several common concave utility functions, such as power, exhibit aggressiveness. In general, however, relative risk aversion is not uniformly less than one and hence the optimal effort level is not always increasing in the magnitude of reward. In a survey of relative risk aversion, Meyer and Meyer (2005) present mixed evidence of aggressive versus conservative behavior. However, they argue that a reasonable property is IRRA, i.e., agents are initially aggressive, and then become conservative as wealth increases. In particular, for our model, this implies that optimal effort is unimodal in the commission rate: it increases as long as the agent is aggressive (relative risk aversion less than one), and decreases once the agent becomes conservative. Such an example is provided in Figure 2 with exponential utility. Discrete distributions are denoted $(p; x)$, where x is the vector of outcomes and p the corresponding probabilities.

4.3. Magnitude of the Variable Pay

Do larger bonuses motivate agents to work harder? We next investigate how a percentage increase in variable (as opposed to total) pay impacts the agent's effort. These results are also contingent on the agent's aggressiveness and conservatism, and are driven by the preservation of these properties with respect to shifts in wealth (induced by the fixed salary F). The following result holds for all types of agents, regardless of risk aversion.

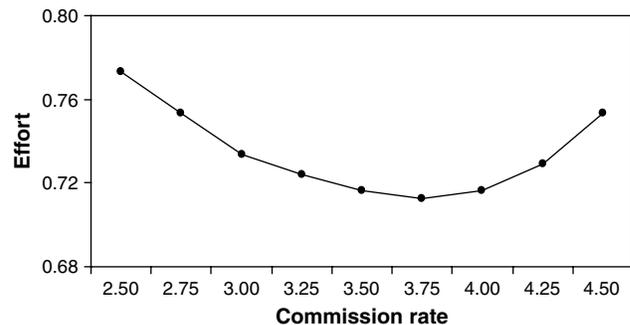
REMARK 6. For any utility function U and $h \geq 0$, consider the shifted utility $\tilde{U}(w) = U(h + w)$.

- (a) If $U(w)$ is aggressive, then $\tilde{U}(w)$ is aggressive.
- (b) If $U(w)$ is conservative and IARA, then $\tilde{U}(w)$ is conservative.

This result allows us to characterize how optimal effort $x^*(\lambda) = \arg \max \mathbb{E}[U(F + \lambda v(s(x, Y)))] - C(x)$ changes with respect to the magnitude λ of variable reward.

PROPOSITION 3. The optimal effort of an aggressive agent increases in response to a percentage increase in the variable portion of his reward. The opposite is true for a conservative agent who exhibits IARA.

Figure 3 Optimal Effort Level vs. Magnitude λ of Variable Pay for $U(w) = w - e^{-w}$, $r(s) = 1 + \lambda s$, and the Same Sales Response as in Figure 2



Intuitively, a percentage increase in variable pay leads to the same marginal effect but a weaker wealth effect than a same order increase in total reward (see the previous section). Hence, aggressiveness remains sufficient for higher variable pay to act as a motivator. On the other hand, conservatism is not enough to counter the reward effect, as illustrated by the non-monotone effort behavior in Figure 3, for one-switch conservative (DARA) utility. IARA strengthens the wealth effect (the agent's utility becomes more concave at higher wealth levels), and in combination with conservatism insures that increasing variable pay is systematically demotivating.

5. Factors Affecting the Agent's Output

This section studies the impact of factors which influence the agent's sales response on his optimal effort choice. An increase in the agent's or firm's productivity, as well as changes in the marketplace, have an influence on sales, and indirectly on the agent's motivation to exert effort. We identify conditions on the agent's induced preferences for output that lead to a systematic effort behavior with respect to such factors.

Formally, consider an abstract performance factor θ that positively affects the sales response $s(x, Y, \theta)$, i.e., s is increasing in θ . The factor θ has an indirect impact on the agent's optimal effort level $x^*(\theta) = \arg \max \mathbb{E}[V(s(x, Y, \theta))] - C(x)$.

Certain structural parameters are consistent in motivating agents regardless of their preferences, such as, for example, the amount of responsibility or workload (e.g., inventory) that the agent is allocated. Given a workload level θ , the agent's effective output is given by $\min(S(x), \theta)$, and marginal return from output increases in workload, hence the following result:

PROPOSITION 4. The agent's optimal effort increases with the amount of workload he is allocated.

Yet few factors influencing output have such a direct and consistent impact on effort. In general, comparative statics results for $x^*(\theta)$ are difficult to obtain because of the compounded marginal effects of the sales response and value function, and the interactions between x and θ . A *productivity factor* θ is one that increases the marginal impact of effort on sales (i.e., s is supermodular in (x, θ)). For such factors, Lemma 1 implies that optimal effort $x^*(\theta)$ increases with θ for V convex. The opposite, however, is not necessarily true if V is concave, unless, e.g., the effect of effort x and θ are additively separable.

Further insights are derived from focusing on separable and multiplicative interactions. We investigate (1) productivity factors that have a multiplicative effect on overall sales, respectively, on the controllable sales component (as controlled by the firm, respectively agent: θ_F, θ_A), and (2) factors that influence the uncontrollable market component in a general stochastic way (θ_U). These effects are jointly formalized in the following sales response model:

$$s(x, \mathbf{Y}) = \theta_F(\theta_A s(x) + \mathbf{Y}(\theta_U)). \quad (3)$$

5.1. Productivity Factors

Several factors can enable the agent to generate more sales at a given effort level. Factors controllable by the firm, such as an improvement in brand or technology or an increase in advertising, have a positive influence on the overall sales response. The parameter θ_F in model (3) captures such “firm productivity” factors.¹⁵ Sales are also conditioned by the agent’s own productivity, as a result of past experience, familiarity with the sales territory, etc. This impact, however, is limited to the component of sales that is effectively under the agent’s control. Such “agent productivity” factors are captured by the parameter θ_A in model (3), with $\mathbf{Y} \geq 0$. As θ_A or θ_F increase, the agent is able to generate higher sales for any given effort level x .

The agent’s aggressiveness/conservatism toward sales (V) captures the relationship between optimal effort and productivity. The mechanics are analogous to §§4.2 and 4.3, with sales preferences V replacing wealth preferences U . The relevant insights can be summarized as follows:

PROPOSITION 5. (a) *If an agent exhibits aggressive preferences for sales, then his optimal effort increases in the firm’s and his own productivity.*

(b) *If an agent exhibits conservative preferences for sales, then his optimal effort decreases in the firm’s productivity. Moreover, if the agent is also increasingly risk averse for sales (V exhibits IARA), then optimal effort also decreases in his productivity.*

Both Lal and Srinivasan (1993) and Basu et al. (1985) consider the impact of the agent’s productivity on the optimal choice of effort from a principal-agent perspective. Basu et al. (1985) model the agent’s productivity as the slope of the deterministic sales response, and his utility with a power function for which $R_U \leq 1$. They show that when the uncertainty in sales is given by a binomial or gamma distribution, the agent’s optimal choice of effort increases in his productivity. Lal and Srinivasan (1993) reach the same conclusion with an exponential utility function.

Firms may fear that increased productivity would allow agents to get lazier, as sales are easier to achieve. However, if the firm offers rewards that induce aggressive, in particular, risk-seeking preferences for sales, then the opposite behavior should be expected. We next show how such behavior can be induced through compensation.

5.1.1. Inducing Aggressive and Risk-Seeking Behavior. With linear contracts $r(s) = F + r \cdot s$, the value function V inherits the properties of the agent’s utility function U . Convex compensation plans, however, can induce risk-seeking behavior, even from risk-averse agents. For strictly increasing, twice differentiable contracts r , define $q(w) = r^{-1}(w)$ as the amount of sales required for the agent to achieve wealth level w . This is an increasing function for which we abstractly define the corresponding measures of risk aversion. If the firm offers a contract such that $q = U$, then the agent exhibits risk-neutral preferences for sales. The following conditions are necessary and sufficient to induce risk-seeking, respectively, aggressive attitudes toward sales.

PROPOSITION 6. (a) *The agent exhibits risk-seeking preferences for sales (i.e., V is convex) if and only if $A_q \geq A_U$; in particular, r needs to be convex.*

(b) *The agent exhibits aggressive preferences for sales if and only if $q'/q + A_q \geq A_U$.*

The conditions $A_U \leq A_q$, respectively, $A_U \leq q'/q + A_q$, are equivalent to q'/U' , and, respectively, q'/qU' being logconcave.¹⁶ In particular, to induce aggressive behavior toward sales, it is sufficient to offer convex compensation plans r with $q'/q \geq A_U$, i.e., $1/qU'$ logconcave.

EXAMPLE 2. Consider an agent with exponential utility $U(w) = 1 - e^{-w}$. The convex compensation plan $r(s) = -\ln(1 - s^\rho)$, $\rho \leq 1$, induces aggressive behavior: $V(s) = s^\rho$, so $R_V \leq 1$. For $\rho = 1, 2$, the agent’s induced preferences for sales are risk neutral, respectively, risk seeking.

¹⁵ Our results hold more generally for $s(x, \mathbf{Y}, \theta_F) = \theta_F \mathbf{S}(x)$.

¹⁶ A function f is logconcave if $\log f$ is concave.

5.2. Uncontrollable Market Factors

This section investigates the impact on effort of changing market conditions, which are outside the agent's control, captured by the factor $\theta = \theta_U$ (indexing is omitted henceforth) in (3), i.e.,

$$\mathbf{S}(x, \theta) = s(x) + \mathbf{Y}(\theta). \quad (4)$$

In particular, we investigate the impact of exogenous dynamics in market size, price, and risk on the effort level expended by the agent. Risk aversion and prudence toward output are identified as the relevant triggers of systematic motivational patterns.

5.2.1. Market Size and Market Price. In this section, the random, uncontrollable sales component $\mathbf{Y}(\theta)$ is assumed stochastically increasing under first-order stochastic dominance in the "market size" θ . That is, $\mathbf{Y}(\theta_2) \succeq_{\text{FSD}} \mathbf{Y}(\theta_1)$ for $\theta_2 \geq \theta_1$. Hence, for a given effort level x , sales are stochastically larger as θ increases. For example, $\theta \geq 0$ may control average sales: $\mathbf{Y}(\theta) = \theta + \mathbf{Y}$.

We show that agents with convex value functions V are motivated by a bigger market size θ , while smaller markets motivate agents with concave value functions. This follows from Lemma 1, by observing that concavity/convexity of V drive the monotonicity of the agent's marginal return from effort with respect to θ . Furthermore, these properties are necessary to robustly characterize the motivational impact of an increase in market size.

THEOREM 4. *If the agent exhibits risk-averse (risk-seeking) preferences for output, then his optimal choice of effort decreases (increases) with market size. Conversely, if an agent decreases (increases) his effort level in response to any first-order increase in market size, then V is concave (convex).*

An alternative perspective on this result emerges from interpreting market price as the driving factor of both market size and effort. Let $\mathbf{S}(x, p) = f(x) - g(p) + \mathbf{Y}$, where p is the product's market price, and $f(x)$, $g(p)$ are increasing deterministic functions. $\mathbf{S}(x, p)$ is stochastically decreasing in p under first-order stochastic dominance. Theorem 4 implies that agents with concave value functions V exert more effort, the higher the market price, whereas convex value functions reverse this behavior. Interestingly, while demand for normal goods decreases in price, this effect is countered by the additional impact of price on effort, and the latter's impact on sales. At the extreme, if demand is not very price elastic, and the agent has significant control over sales, sales can theoretically increase in price, as shown by the following example (proved in the appendix).

EXAMPLE 3. For an agent with $V(s) = 1 - e^{-s}$, $C(x) = x$, and $s(x, p, \mathbf{Y}) = e^x - e^p + \mathbf{Y}$, the resulting sales response is increasing in p .

5.2.2. Market Variability. This section investigates which agents are motivated by riskier markets. To capture the effects of market variability on the sales response function (4), we assume that $\mathbf{Y}(\theta)$ is stochastically increasing in θ under the convex order, i.e., $\mathbf{Y}(\theta_2) \succeq_{\text{CX}} \mathbf{Y}(\theta_1)$ for $\theta_2 \geq \theta_1$.¹⁷ In other words, $\mathbf{Y}(\theta_2)$ is riskier than $\mathbf{Y}(\theta_1)$ in the (mean preserving spread) sense of Rothschild and Stiglitz (1970, 1971). As θ increases, sales are more variable at a given effort level x . An example is $\mathbf{S}(\theta, x) = x + \theta\mathbf{Y}$, with $\mathbb{E}[\mathbf{Y}] = 0$ and $\theta \geq 0$.

We find that the agent's motivation from changes in market risk is triggered by his prudence, formally defined by Kimball (1990). A prudent agent increases his precautionary savings in anticipation of future risk. An equivalent, insightful characterization of prudence in terms of risk apportionment is provided by Eeckhoudt and Schlesinger (2005). Intuitively, a prudent agent prefers to disaggregate a sure loss from a zero-mean risk.

DEFINITION 3. An agent with utility function U is said to be prudent (imprudent) if his marginal utility $U(w + \delta) - U(w)$ is convex (concave) in w for any $\delta > 0$. If U is three times differentiable, prudence is equivalent to $U'''(w) \geq 0$ ($U'''(w) \leq 0$ for imprudence).

It is generally assumed that individuals exhibit prudent behavior. Most common forms of utility functions, like exponential, logarithmic, power, or Bell's one-switch utilities, exhibit prudence. Note the difference between prudence and risk aversion. Prudence refers to the propensity of the decision maker to prepare and forearm himself in the face of uncertainty, whereas risk aversion measures how much the decision maker dislikes uncertainty and would turn away from it if possible. In particular, the following relationships hold:

REMARK 7. All DARA agents are prudent. All imprudent agents exhibit IARA.

In Kimball's (1990) context of precautionary savings (see also Eeckhoudt and Gollier 2005), an agent is prudent if an increase in future risk raises the marginal value of wealth. Because higher marginal return is motivating in our setting, the prudent agent will expend more effort in response to an increase in market risk. Moreover, prudence/imprudence are necessary conditions for such behavior to be robust with respect to the sales function.

THEOREM 5. *If the agent has prudent (imprudent) preferences for sales, then his optimal choice of effort increases (decreases) with the variability in the market. Conversely, if for all increasing sales functions s , the agent's optimal*

¹⁷ \mathbf{X} dominates \mathbf{Y} in the convex order ($\mathbf{X} \succeq_{\text{CX}} \mathbf{Y}$) if $\mathbb{E}[\varphi(\mathbf{X})] \geq \mathbb{E}[\varphi(\mathbf{Y})]$ for all convex functions φ . If $\mathbf{X} \succeq_{\text{CX}} \mathbf{Y}$, then $\mathbb{E}[\mathbf{X}] = \mathbb{E}[\mathbf{Y}]$ and $\text{Var}(\mathbf{X}) \geq \text{Var}(\mathbf{Y})$. For more details on stochastic orders, see Müller and Stoyan (2002).

choice of effort is increasing (decreasing) in the market variability, then he is prudent (imprudent).

Interestingly, our results are in contrast with insights previously obtained by Lal and Srinivasan (1993) and Basu et al. (1985) who work with prudent utility functions in a principal-agent setting. In their models, however, a change in market variability leads to a change in the agent's optimal contract, which, in turn, affects the agent's effort behavior.

5.2.3. Inducing Prudence and IARA. From Theorem 5, agents are motivated by an increasingly uncertain environment whenever the firm offers compensation plans which induce a prudent attitude toward output. The following measure of the degree of prudence was recently proposed as a direct analogue of the degree of absolute risk aversion, $D_U = -U'''(w)/U'(w)$ (see Modica and Scarsini 2005).¹⁸ The next result shows how a firm can preserve or reverse the agent's prudent preferences for wealth into preferences for output. Imprudence toward output can be induced by offering a compensation plan which preserves the risk-averse attitude of the agent toward money, but with the inverse reward $q = r^{-1}$ being "more prudent" than the (prudent) utility U .

PROPOSITION 7. Consider an agent with prudent utility U rewarded with compensation plan r .

- (a) If r is concave and prudent, then the agent exhibits a prudent attitude toward sales, V .
- (b) If r is convex, $A_U \geq A_q$, and $D_U \leq D_q$, then the agent's attitude toward sales, V , is imprudent.

From Remark 7, Proposition 7 can also be used to induce IARA.

EXAMPLE 4. Consider an agent with quadratic utility function $U(w) = 4 + w - w^2/2$, $w \in [0, 1]$. By offering a compensation plan $r(s) = -\ln(1 - s)$, the agent's induced value function is prudent, $V'''(s) = -(1 - s)^{-4} \leq 0$. Note that $A_U = (1 - w)^{-1} \geq 1 = A_q$ and $D_q = 1 \geq 0 = D_U$.

Proposition 7 gives only sufficient conditions for compensation plans to induce prudent/imprudent preferences for sales. In special cases, other methods to induce prudence are conceivable, as illustrated in the following example:

EXAMPLE 5. The compensation plan $r(s) = e^s$ induces prudence from an agent with utility function $U(w) = \ln(1 + w)$. Indeed, $V'''(s) = (1 + e^s)e^s(1 - e^s) \leq 0$. However, $A_U = (1 + w)^{-1} \leq w^{-1} = A_q$.

6. The Agent's Dynamic Problem

In the following, we extend the single-period model of §2 to a dynamic setting. The agent chooses in each

period how much effort to exert by trading off an immediate effort cost with the utility of variable compensation. The latter is administered at the end of the sales horizon, contingent on cumulative output. The multiperiod model allows us to obtain motivational patterns with respect to two factors underlying the agent's intertemporal value function, namely, past performance and time to evaluation. We show that agents with concave value functions are demotivated by past successes and longer sales horizons. These predictions are reversed for agents with convex value functions.

Let s_t denote the agent's accumulated output up to time t , calculated recursively as $s_{t+1} = s_t + \mathbf{S}(x_t)$, $t = 1, \dots, N - 1$. Sales are assumed to be i.i.d. over time; all results easily extend for Markovian sales processes.¹⁹ For given accumulated sales s_t at time t , the agent chooses the optimal effort level x_t according to the Bellman equation

$$V_t(s_t) = \max_{x_t} \{ \mathbb{E}[V_{t+1}(s_t + \mathbf{S}(x_t))] - C(x_t) \},$$

$$t = 0, \dots, N - 1, \quad (5)$$

where the terminal valuation is $V_N(s_N) = V(s_N)$. Denote $W_t(s_t, x_t) = \mathbb{E}[V_{t+1}(s_t + \mathbf{S}(x_t))] - C(x_t)$.

Certain properties of the agent's terminal preferences for cumulative sales, captured by V , are preserved over time by the value function at time t , $V_t(s)$.

PROPOSITION 8. If the agent's terminal value for sales V is increasing, convex, respectively, concave, then the time- t value function, $V_t(s)$, is also increasing, convex, respectively, concave in s for all t .

Preservation of risk preferences in a dynamic setting, particularly risk aversion, is a nontrivial result. In contrast, other properties such as prudence, aggressiveness, or monotone risk aversion are not robust in a dynamic setting (they are not preserved through the maximization operator; see Smith and McCardle 2002).

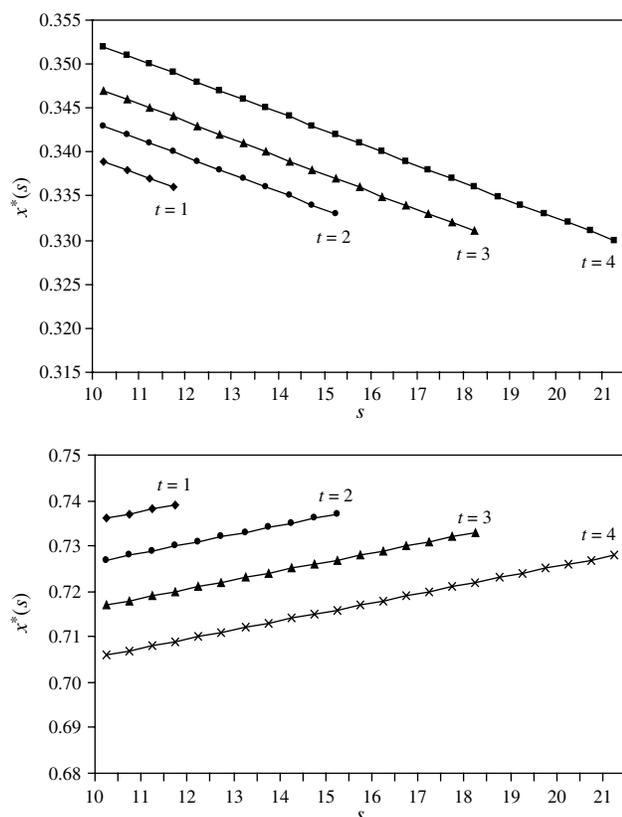
6.1. Past Performance and Time to Evaluation

The agent's cumulative output at any point in time, as well as the time remaining until evaluation, impact his optimal choice of effort, but in opposite ways. For agents with risk-averse preferences, we show that better output makes them lazy. Specifically, the better their achieved performance at any given point in time, the less effort they will undertake, ceteris paribus. On the other hand, getting closer to evaluation time acts as a motivator (these agents procrastinate). The closer the agent is from bonus time, the more effort he undertakes, given a certain output level. Agents

¹⁸ An alternative, widely accepted measure is $P_U = -U'''(w)/U''(w)$.

¹⁹ In this case, $\mathbf{S}_i(x, y) = s(x, \mathbf{Y}(y))$, where y is the previous period realization of \mathbf{Y} .

Figure 4 Optimal Effort vs. Past Performance and Time to Evaluation Under Risk-Averse ($\rho = 0.9$), Respectively, Risk-Seeking ($\rho = 1.1$) Sales Preferences $V(s) = s^\rho$; $C(x) = x^2$, $S(x) = x + Y$, Where $Y \equiv (0.3, 0; 0.7, 2)$



with risk-seeking sales preferences exhibit the opposite behavior: they are motivated by higher output levels and longer horizons.

THEOREM 6. *The optimal level of effort for agents with a concave value function V decreases with accumulated output at any given time, and with the time remaining until evaluation. That is, $x_t^*(s)$ is decreasing in s for any given t and $x_{t+1}^*(s) \geq x_t^*(s)$ for any given level of past performance s and at any time t . The opposite is true for agents with convex value function V .*

The behavior prescribed by this result is commonly observed in practice. Lal and Srinivasan (1993, pp. 777–778) mention that “it is not unusual to hear about sales people spending time playing golf or indulging in other activities if their past efforts have been unusually successful.” The opposite behavior is rational if firms offer convex compensation plans which induce a risk-seeking attitude (see Proposition 6). Figure 4 illustrates the insights of Theorem 6 over a four-period horizon ($T = 4$).

7. Conclusion

This paper characterizes the motivational drivers and patterns of a risk-averse agent who chooses an

optimal level of effort to trade off performance-based payoff with disutility of effort. Our approach is different from the bulk of the compensation literature, which focuses on a principal-agent framework. In this context, the agent’s contract is assumed to change (optimally for the firm) with respect to factors affecting his decision. Such an approach obscures the direct impact of given compensation structures on effort, particularly on the short run when contracts are usually fixed. To isolate this effect, we focused on the agent’s problem in response to fixed contracts. We obtained comparative statics that quantify managers’ rational behavior under very general forms of output, reward, and utility functions.

The insights that we obtained are robust, and surprisingly, different from those obtained under principal-agent frameworks. We identified what properties of the agent’s utility function and the reward plan impact effort. In contrast with previous results, we showed that less risk-averse agents do not necessarily work harder, but those with higher marginal value of wealth do. We also showed that higher rewards do not necessarily motivate, and actually demotivate so-called conservative agents. Firms who expect to use changes in compensation as a lever to motivate managers should employ an aggressive workforce.

In the second part of this paper, we showed how factors that impact output act as indirect drivers of the managers’ effort. These results typically require additional conditions on the agent’s value function for output (the utility of compensation); interestingly, the conditions that we identify are in contrast with results in the salesforce agency literature. For example, we found that the agent’s and the firm’s productivity are motivating factors for agents with an induced aggressive (and, in particular, risk-seeking) attitude. These agents are also motivated by larger market sizes. On the other hand, agents with prudent attitudes toward output are motivated by increased market risk.

In a dynamic setting, we showed that the agent’s induced risk preferences for output are preserved over time through the iterations of the dynamic program. This enabled us to trace the agent’s motivation in response to past performance and time to evaluation, back to his risk preferences for output. Our results are summarized in Table 1.

In particular, our results highlight the importance for firms to identify and elicit various properties of employees’ preferences such as prudence, aggressiveness, and risk aversion to understand motivational patterns. This opens up new directions for empirical investigation. It would be interesting to investigate how these motivational patterns may change when accounting for judgmental biases in the agent’s decision-making process, such as prospect theory valuations.

Table 1 Output Preferences Insuring Systematic (De)motivation from Increasing a Critical Factor

Critical factor	Demotivating	Motivating
Firm’s productivity	Conservative	Aggressive
Agent’s productivity	Conservative and IARA	Aggressive
Past performance	Concave	Convex
Time to evaluation	Concave	Convex
Market size	Concave	Convex
Market variability	Imprudent	Prudent

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Appendix. Proofs

We only prove one part of a proposition when the second part is analogous. For simplicity of exposition, the proofs assume differentiability of the functions involved. However, all the results can be extended, using finite differences, to allow for the nondifferentiability of r , and consequently, V .

PROOF OF REMARK 1. Let $\pi(s) = s - r(s)$, so $\mathbf{\Pi}(x) = \pi(\mathbf{S}(x))$. The first part follows because $\mathbf{S}(x)$ is stochastically increasing in x , so an increasing function thereof (π) will also be stochastically increasing. Conversely, assume by contradiction that there exist $s_1 < s_2$ such that $\pi(s_1) > \pi(s_2)$. Consider the increasing sales function $\mathbf{S}(x) \equiv s_1$ for $x < x_0$ and $\mathbf{S}(x) \equiv s_2$ otherwise. Hence, the firm’s profit is $\mathbf{\Pi}(x) = \pi(s_1)$ for $x < x_0$ and $\mathbf{\Pi}(x) = \pi(s_2)$ for $x \geq x_0$. This is not stochastically increasing in x , a contradiction.

PROOF OF REMARKS 2, 3, AND 4. It is equivalent to prove the results for the agent with utility function $\tilde{U}(w) = U(w/F)$, $F > 0$. In this case, $\tilde{R}(x, \rho) = \mathbb{E}[\tilde{U}(1 + \tilde{v}(s(x, \mathbf{Y})))]$, where $\tilde{v}(s) = v(s)/F$. Monotonicity of $s(x, y)$ and $\tilde{v}(s)$ in x and s , respectively, implies in each case $\tilde{R}_{x\rho}(x, \rho) \geq 0$, as detailed below. From Lemma 1, this shows that $x^*(\rho)$ is increasing in ρ .

For Remark 1: $\tilde{R}_{x\rho}(x, \rho) = \mathbb{E}[e^{-\rho^{-1}(1+\tilde{v}(s(x, \mathbf{Y})))} \rho^{-2} \tilde{v}'(s(x, \mathbf{Y})) \cdot s_x(x, \mathbf{Y})(\rho^{-1}(1 + \tilde{v}(s(x, \mathbf{Y}))) - 1)] \geq 0$.

For Remark 2: $\tilde{R}_{x\rho}(x, \rho) = \mathbb{E}[\tilde{v}'(s(x, \mathbf{Y}))s_x(x, \mathbf{Y})(1 + \tilde{v}(s(x, \mathbf{Y})))^{\rho-1}(1 + \ln(1 + \tilde{v}(s(x, \mathbf{Y}))))] \geq 0$.

For Remark 3: $\tilde{R}_{x\rho}(x, \rho) = \mathbb{E}[e^{-\beta(1+\tilde{v}(s(x, \mathbf{Y})))} \beta \tilde{v}'(s(x, \mathbf{Y}))s_x(x, \mathbf{Y})] \geq 0$.

PROOF OF THEOREM 1. Because $U_2 - U_1$ is increasing, for $\mathbf{Z}_2 \succeq_{\text{FSD}} \mathbf{Z}_1$, we can write

$$\mathbb{E}[U_2(\mathbf{Z}_2) - U_1(\mathbf{Z}_2)] \geq \mathbb{E}[U_2(\mathbf{Z}_1) - U_1(\mathbf{Z}_1)].$$

In particular, for $\mathbf{Z}_j \equiv r(s(x_j, \mathbf{Y}))$, $j = 1, 2$, we have $\mathbf{Z}_2 \succeq_{\text{FSD}} \mathbf{Z}_1$, whenever $x_2 \geq x_1$. This implies supermodularity of $R(x, i) = \mathbb{E}[U_i(r(s(x, \mathbf{Y})))]$ in (x, i) . From Lemma 1, $x_2^* \geq x_1^*$.

To prove the converse, assume by contradiction that there exists $w_2 > w_1$ such that $a_1 = U_2(w_2) - U_2(w_1) < U_1(w_2) - U_1(w_1) = a_2$. Let $x_0 > 0$ such that $C(x_0) \in (a_1, a_2)$; this exists because C spans $[0, \infty)$.²⁰ Let $s_1 = q(w_1) < s_2 = q(w_2)$, where $q = r^{-1}$. Consider the deterministic sales function $s(x, y) = s_2$ if $x \geq x_0$ and $s(x, y) = s_1$ for $x < x_0$. Because C is increasing,

it follows that $x_i^* \in \{0, x_0\}$. Because $C(0) = 0$, and $C(x_0) \in (U_2(w_2) - U_2(w_1), U_1(w_2) - U_1(w_1))$ by definition, we obtain that $W_1(x_0) > W_1(0)$ and $W_2(x_0) < W_2(0)$. Hence, $x_1^* = x_0 > 0 = x_2^*$, a contradiction.

PROOF OF PROPOSITION 1. From Lemma 1, it is sufficient to show that $U''(F + v(s))V'(s)s_x \leq 0$, which follows from the concavity of U , and monotonicity of $s(x, y)$ and $v(s)$.

PROOF OF PROPOSITION 2. Because $U \circ r_2 - U \circ r_1$ is increasing, for $\mathbf{Z}_2 \succeq_{\text{FSD}} \mathbf{Z}_1$, we can write

$$\mathbb{E}[U(r_2(\mathbf{Z}_2)) - U(r_1(\mathbf{Z}_2))] \geq \mathbb{E}[U(r_2(\mathbf{Z}_1)) - U(r_1(\mathbf{Z}_1))].$$

In particular, for $\mathbf{Z}_i = s(x_i, \mathbf{Y})$, we have $\mathbf{Z}_2 \succeq_{\text{FSD}} \mathbf{Z}_1$ whenever $x_2 \geq x_1$. Hence, $R(x, i) = \mathbb{E}[U(r_i(s(x, \mathbf{Y})))]$ is supermodular in (x, i) . From Lemma 1, $x_2^* \geq x_1^*$.

The proof of the necessity part follows the same lines as Theorem 1 by assuming that there exists $w_2 > w_1$ such that $a_1 = U(r_2(w_2)) - U(r_2(w_1)) < U(r_1(w_2)) - U(r_1(w_1)) = a_2$.

PROOF OF THEOREM 2. From the concavity of U , if $r_2 \leq r_1$ and $r_2' \geq r_1'$, then $U'(r_2)r_2' - U'(r_1)r_1' \geq 0$, which implies $U \circ r_2 - U \circ r_1$ is increasing.

Conversely, for risk-neutral agents with $U(x) = x$, the result implies, via Proposition 2, that $r_2 - r_1$ is increasing. It remains to show that $r_2(s) \leq r_1(s)$ for all achievable sales values s . Let $a > 0$ such that $s < q_1(a), q_2(a)$. Consider the agent with piecewise-linear increasing concave utility $U_a(x) = x$ for $x \leq a$ and $U_a(x) = a$ for $x > a$. For $s' = \max(q_1(a), q_2(a))$, Proposition 2 implies

$$\begin{aligned} r_2(s) - r_1(s) &= U_a(r_2(s)) - U_a(r_1(s)) \leq U_a(r_2(s')) - U_a(r_1(s')) \\ &= a - a = 0. \end{aligned}$$

PROOF OF THEOREM 3. The first part follows from Lemma 1 and Definition 2. The proof of the converse follows the same lines as that of Theorem 1 by assuming that $w_2 > w_1$ exist such that $a_2 = U(\lambda_2 w_2) - U(\lambda_2 w_1) > U(\lambda_1 w_2) - U(\lambda_1 w_1) = a_1$.

PROOF OF REMARK 6. (a) If $U''(w + h) \geq 0$, $R_{\tilde{U}}(w) = -wU''(w + h)/U'(w + h) \leq 0 \leq 1$. If $U''(w + h) < 0$,

$$R_{\tilde{U}}(w) = \frac{-wU''(w + h)}{U'(w + h)} \leq \frac{-(w + h)U''(w + h)}{U'(w + h)} \leq 1,$$

where the second inequality follows from $R_U \leq 1$.

(b) Because U conservative implies U concave (Remark 5), for $h \geq 0$ we can write

$$R_{\tilde{U}} = \frac{-wU''(w + h)}{U'(w + h)} \geq \frac{-wU''(w)}{U'(w)} \geq 1, \tag{6}$$

where the first inequality follows from IARA and the second from $R_U \geq 1$.

PROOF OF PROPOSITION 3. Follows from Remark 6 and Theorem 3.

PROOF OF PROPOSITION 4. From Lemma 1, it is sufficient to show that $R(\theta, x) = \mathbb{E}[V(\min(\mathbf{S}(x), \theta))]$ is supermodular in (x, θ) . For arbitrary fixed $\theta_2 \geq \theta_1$, if V is increasing, $\Delta(z) = V(\min(z, \theta_2)) - V(\min(z, \theta_1))$ is increasing in z . Because $\mathbf{S}(x_2) \succeq_{\text{FSD}} \mathbf{S}(x_1)$, for all $x_2 \geq x_1$, $\mathbb{E}[\Delta(\mathbf{S}(x_2))] \geq \mathbb{E}[\Delta(\mathbf{S}(x_1))]$, which is equivalent to the supermodularity of $R(\theta, x)$.

PROOF OF PROPOSITION 5. The results for the firm’s, respectively, the agent’s productivity follow the same lines of proof as Theorem 3, respectively, Proposition 3, with V replacing U .

²⁰ Note that this assumption could be relaxed to C spanning the same domain as U .

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PROOF OF PROPOSITION 6. (a) Letting $w = r(s)$, we obtain

$$U'(w) = V'(q(w))q'(w), \quad (7)$$

$$U''(w) = V''(q(w))q'(w)^2 + V'(q(w))q''(w). \quad (8)$$

From (7) and (8), we obtain

$$V''(q(w))q'(w)^2 = U'(w)(A_q - A_U).$$

Because U is increasing, V is convex iff $A_U \leq A_q$. In particular, if $\log q'/U'$ is concave, then $A_U \leq A_q$.

(b) Letting $r(s) = w$, from (7) and (8), we obtain

$$\frac{-sV''(s)}{V'(s)} = \frac{q(w)}{q'(w)}(A_U - A_q). \quad (9)$$

This is less than unity if $A_U \leq q'/q + A_q$. In particular, $R_V \leq 1$ if r is convex (q is concave, so $A_q \geq 0$) and $A_U \leq q'/q$. Note that $A_U \leq q'/q + A_q$ is equivalent to $(\log q'/qU')' \leq 0$, i.e., q'/qU' logconcave. Furthermore, $A_U \leq q'/q$ if $(\log 1/qU')' \leq 0$.

PROOF OF THEOREM 4. Because V is concave, for any $x_2 > x_1$, $\Delta(z) = V(s(x_2) + z) - V(s(x_1) + z)$ is decreasing in z . Hence, by first-order dominance, $\mathbb{E}[\Delta(\mathbf{Y}(\theta_2))] \leq \mathbb{E}[\Delta(\mathbf{Y}(\theta_1))]$ for all $\theta_2 \geq \theta_1$. This shows submodularity of $R(x, \theta) = \mathbb{E}[V(s(x) + \mathbf{Y}(\theta))]$ in (x, θ) . From Lemma 1, $x^*(\theta)$ is decreasing in θ .

For the necessity part, assume, by contradiction, that there exists $w_2 > w_1$, such that $h(w_2) > h(w_1)$, where $h(w) = U(w + x_0) - U(w)$, $x_0 \geq 0$. Let $\mathbf{Y}_1 \equiv w_1$ and $\mathbf{Y}_2 \equiv w_2$, so that $\mathbf{Y}_2 \succeq_{\text{FSD}} \mathbf{Y}_1$. The proof follows the same construction as Theorem 1, with $a_2 = \mathbb{E}[U(\mathbf{Y}_2 + x_0) - U(\mathbf{Y}_2)] > \mathbb{E}[U(\mathbf{Y}_1 + x_0) - U(\mathbf{Y}_1)] = a_1$.

PROOF OF EXAMPLE 3. Letting $s^*(p, y) = s(x^*(p), p, y)$, from the implicit function theorem, $\partial x^*(p)/\partial p = -e^p/(1 - e^x) \geq 0$. Furthermore,

$$\frac{\partial s^*(p, y)}{\partial p} = s_p(x, p, y) + \frac{\partial x^*(p)}{\partial p} s_x(x, p, y) = \frac{-e^p(1 + e^x)}{1 - e^x} \geq 0,$$

hence, $s^*(p, y)$ is increasing in p .

PROOF OF THEOREM 5. Because $V(s)$ exhibits prudence, $\Delta(z) = V(s(x_2) + z) - V(s(x_1) + z)$ is convex in z for $x_2 \geq x_1$. From $\mathbf{Y}(\theta_2) \succeq_{\text{CX}} \mathbf{Y}(\theta_1)$, for $\theta_2 \geq \theta_1$, it follows that $\mathbb{E}[\Delta(\mathbf{Y}(\theta_2))] \geq \mathbb{E}[\Delta(\mathbf{Y}(\theta_1))]$. This implies supermodularity of $R(x, \theta) = \mathbb{E}[V(s(x) + \mathbf{Y}(\theta))]$ in (x, θ) . From Lemma 1, it follows that $x^*(\theta)$ is increasing in θ .

For the necessity part, assume by contradiction that for $h(w) = U(w + x_0) - U(w)$, $x_0 \geq 0$, there exists w_1, w_2, λ such that $h(w_\lambda) > \lambda h(w_1) + (1 - \lambda)h(w_2)$.²¹ Let $\mathbf{Y}_1 \equiv w_\lambda$ and $\mathbf{Y}_2 \equiv (\lambda, w_1; (1 - \lambda), w_2)$, so that $\mathbf{Y}_2 \succeq_{\text{CX}} \mathbf{Y}_1$. The proof follows a similar construction as for Theorem 1, with $a_2 = \mathbb{E}[U(\mathbf{Y}_1 + x_0) - U(\mathbf{Y}_1)] > \mathbb{E}[U(\mathbf{Y}_2 + x_0) - U(\mathbf{Y}_2)] = a_1$.

PROOF OF PROPOSITION 7. (a) Because $U'''(w) \geq 0$, $r''(s) \leq 0$, and $r'''(s) \geq 0$,

$$V'''(s) = U'''(r(s))r'(s)^3 + 3U''(r(s))r'(s)r''(s) + U'(r(s))r'''(s) \geq 0. \quad (10)$$

²¹ To simplify notation, we denote $x_\lambda = \lambda x_1 + (1 - \lambda)x_2$ for any x_1, x_2 and any scalar λ .

(b) Letting $w = r(s)$, we obtain

$$V'''(q(w))q'(w)^3 + 3V''(q(w))q'(w)q''(w) + V'(q(w))q'''(w) = U'''(w). \quad (11)$$

From (7), (8), and (11), we can write

$$V'''(q(w))q'(w)^3 = U'(w)([D_U - D_q] - 3A_q[A_U - A_q]).$$

From $A_U \geq A_q$, $D_U \leq D_q$, and $A_q \geq 0$ (by convexity of r), we obtain $V'''(s) \leq 0$.

PROOF OF PROPOSITION 8. Monotonicity is trivially preserved by induction. Preservation of convexity is also proved by induction. Convexity of V_{t+1} implies for any s_1, s_2 ,

$$\begin{aligned} & \mathbb{E}[\delta V_{t+1}(s_1 + \mathbf{S}(x)) + (1 - \delta)V_{t+1}(s_2 + \mathbf{S}(x)) - C(x)] \\ & \geq \mathbb{E}[V_{t+1}(s_\delta + \mathbf{S}(x))] - C(x), \end{aligned}$$

which is equivalent to the convexity of $W_t(s_t, x_t)$. Hence, $V_t(s_t) = \max W_t(s_t, x_t)$ is convex in s .

Preserving concavity of V_t is slightly more involved, as it requires joint concavity of W_t . For any $s_i, x_i, i = 1, 2$, we have

$$\begin{aligned} & \delta V_{t+1}(s_1 + s(x_1, y)) + (1 - \delta)V_{t+1}(s_2 + s(x_2, y)) - C_\delta \\ & \leq V_{t+1}(s_\delta + s_\delta(x, y)) - C_\delta \\ & \leq V_{t+1}(s_\delta + s(x_\delta, y)) - C_\delta \\ & \leq V_{t+1}(s_\delta + s(x_\delta, y)) - C(x_\delta), \end{aligned}$$

where the first inequality follows from the concavity of V_{t+1} , the second from the concavity of $s(x, y)$ in x , and the third from the convexity of $C(x)$. Taking expectations, with x_1, x_2 denoting the optimal decisions at time t at sales level s_1, s_2 , we obtain

$$\begin{aligned} & \delta V_t(s_1) + (1 - \delta)V_t(s_2) \\ & = \mathbb{E}[\delta V_{t+1}(s_1 + \mathbf{S}(x_1)) + (1 - \delta)V_{t+1}(s_2 + \mathbf{S}(x_2)) - C_\delta] \\ & \leq \mathbb{E}[V_{t+1}(s_\delta + \mathbf{S}(x_\delta))] - C(x_\delta) \\ & \leq \max_{x_t} \mathbb{E}[V_{t+1}(s_\delta + \mathbf{S}(x_t))] - C(x_t) = V_t(s_\delta). \end{aligned}$$

PROOF OF THEOREM 6. We first prove the results with respect to past performance. Let $s_2 \geq s_1$, and define $\Delta(z) = V_{t+1}(s_2 + z) - V_{t+1}(s_1 + z)$, which is decreasing from the concavity of $V_{t+1}(s)$. Because $\mathbf{S}(x_2) \succeq_{\text{FSD}} \mathbf{S}(x_1)$, for all $x_2 \geq x_1$, $\mathbb{E}[\Delta(\mathbf{S}(x_1))] \geq \mathbb{E}[\Delta(\mathbf{S}(x_2))]$. This implies submodularity of $W_t(s, x) = \mathbb{E}[V_{t+1}(s_t + \mathbf{S}(x_t))] - C(x_t)$ in (s, x) . Hence, $x_t^*(s)$ is decreasing in s .

To prove the results with respect to time, consider

$$\Lambda(z) = V_t(z) - V_{t+1}(z) = \max_x \{\mathbb{E}[V_{t+1}(z + \mathbf{S}(x)) - V_{t+1}(z)] - C(x)\}.$$

Because $\mathbf{S}(x) \geq 0$ and V_{t+1} is concave, it follows that for any action x , $\mathbb{E}[V_{t+1}(z + \mathbf{S}(x)) - V_{t+1}(z)]$ is decreasing in z . Hence, $\Lambda(z)$ is decreasing.

First-order stochastic dominance implies $\mathbb{E}[\Lambda(\mathbf{Y}_1)] \geq \mathbb{E}[\Lambda(\mathbf{Y}_2)]$ whenever $\mathbf{Y}_2 \succeq_{\text{FSD}} \mathbf{Y}_1$. In particular, for $\mathbf{Y}_i = s + \mathbf{S}(x_i)$, $i = 1, 2$, we have $\mathbf{Y}_2 \succeq_{\text{FSD}} \mathbf{Y}_1$ whenever $x_2 \geq x_1$. This shows that $W_t(s, x) = \mathbb{E}[V_{t+1}(s_t + \mathbf{S}(x_t))] - C(x_t)$ is supermodular in (t, x) , hence, $x_t^*(s)$ is increasing in t for given s .

References

- Arrow, K. J. 1965. *Aspects of a Theory of Risk Bearing*. Yrjö Jahnsson Lectures, Helsinki, Finland. Reprinted in *Essays in the Theory of Risk Bearing*. 1971. Markham Publishing, New Haven, CT.
- Basu, A. K., R. Lal, V. Srinivasan, R. Staelin. 1985. A theory of sales-force compensation plans. *Marketing Sci.* 4 267–291.
- Bell, D. E. 1988. One switch utility functions and a measure of risk. *Management Sci.* 34 1416–1424.
- Business Wire. 2005. Hewitt study shows base pay increases flat for 2006 with variable pay plans picking up the slack (August 31).
- Coughlan, A. T. 1993. Salesforce compensation: A review of MS/OR advances. J. Eliashberg, G. L. Lilien, eds. *Handbooks in OR&MS*, Vol. 5. Elsevier Science, Amsterdam, The Netherlands.
- Coughlan, A. T., S. K. Sen. 1989. Salesforce compensation: Theory and managerial implications. *Marketing Sci.* 8 324–342.
- Coulter, M., S. P. Robbins. 1999. *Management*. Prentice Hall, Upper Saddle River, NJ.
- Dearden, J. A., G. L. Lilien. 1990. On optimal sales force compensation in the presence of production learning effects. *Internat. J. Res. Marketing* 7 179–188.
- Dionne, G., L. Eeckhoudt. 1985. Self insurance, self protection and increased risk aversion. *Econom. Lett.* 17 39–42.
- Eeckhoudt, L., C. Gollier. 2005. The impact of prudence on optimal prevention. *Econom. Theory* 26 989–994.
- Eeckhoudt, L., H. Schlesinger. 2005. Putting risk in its proper place. *Amer. Econom. Rev.* 96 280–289.
- Gaba, A., A. Kalra. 1999. Risk behavior in response to quotas and contests. *Marketing Sci.* 18 417–434.
- Gaba, A., I. Tsetlin, R. L. Winkler. 2004. Modifying variability and correlations in winner-take-all contests. *Oper. Res.* 52 384–395.
- Godes, D. 2004. Contracting endogenous risk. *Quant. Marketing Econom.* 2 321–345.
- Joseph, K., M. U. Kalwani. 1995. The impact of environmental uncertainty on the design of salesforce compensation plans. *Marketing Lett.* 6 183–197.
- Jullien, B., B. Salanié, F. Salanié. 1999. Should risk averse agents exert more effort? *Geneva Papers on Risk and Insurance: Theory* 24 19–28.
- Kimball, M. S. 1990. Precautionary saving in the small and large. *Econometrica* 58 53–73.
- Lal, R., V. Srinivasan. 1993. Compensation plans for single and multiproduct sales forces. *Management Sci.* 39 777–793.
- Lal, R., R. Staelin. 1986. Salesforce compensation plans in environments with asymmetric information. *Marketing Sci.* 5 179–198.
- Liu, J. 2001. Dynamic portfolio choice and risk aversion. Working paper, University of California, Los Angeles, CA.
- Meyer, D. J., J. Meyer. 2005. Relative risk aversion: What do we know? *J. Risk Uncertainty* 31 243–262.
- Misra, S., A. T. Coughlan, C. Narasimhan. 2005. Salesforce compensation: Revisiting and extending the agency theoretic approach. *Quant. Marketing Econom.* 3 5–39.
- Modica, S., M. Scarsini. 2005. A note on comparative downside risk aversion. *J. Econom. Theory* 122 267–271.
- Müller, A., D. Stoyan. 2002. *Comparison Methods for Stochastic Models and Risks*. John Wiley & Sons, Chichester, UK.
- Peck, C. A. 1982. Compensating field sales representatives. Report 828, The Conference Board, New York.
- Pratt, J. W. 1964. Risk aversion in the small and large. *Econometrica* 32 122–136.
- Prendergast, C. 1999. The provision of incentives in firms. *J. Econom. Literature* 37 7–63.
- Ross, S. A. 2004. Compensation, incentives and the duality of risk aversion and riskiness. *J. Finance* 59 207–225.
- Rothschild, M., J. Stiglitz. 1970. Increasing risk I: A definition. *J. Econom. Theory* 2 225–243.
- Rothschild, M., J. Stiglitz. 1971. Increasing risk: Its economic consequences. *J. Econom. Theory* 3 66–84.
- Salanié, B. 1997. *The Economics of Contracts: A Primer*. MIT Press, Cambridge, MA.
- Sarin, R. K., R. L. Winkler. 1980. Performance based incentive plans. *Management Sci.* 26 1131–1144.
- Smith, J. E., K. F. McCardle. 2002. Structural properties of stochastic dynamic programs. *Oper. Res.* 50 796–809.
- Tapiero, C., J. Farley. 1975. Optimal control of sales force effort in time. *Management Sci.* 21 976–985.
- Topkis, D. M. 1998. *Supermodularity and Complementarity*. Princeton University Press, Princeton, NJ.
- Wall Street Journal. 2004. Getting a bonus instead of a raise (December 29) D1.