Spatio-temporal interpolation of total electron content using a GPS network

M. N. Deviren,[1] F. Arikan,[1] and O. Arikan[2]

Received 24 July 2012; revised 27 March 2013; accepted 17 April 2013; published 19 June 2013.

[1] Constant monitoring and prediction of Space Weather events require investigation of the variability of total electron content (TEC), which is an observable feature of ionosphere using dual-frequency GPS receivers. Due to various physical and/or technical obstructions, the recordings of GPS receivers may be disrupted resulting in data loss in TEC estimates. Data recovery is very important for both filling in the data gaps for constant monitoring of ionosphere and also for spatial and/or temporal prediction of TEC. Spatial prediction can be obtained using the neighboring stations in a network of a dense grid. Temporal prediction recovers data using previous days of the GPS station in a less dense grid. In this study, two novel and robust spatio-temporal interpolation algorithms are introduced to recover TEC through optimization by using least squares fit to available data. The two algorithms are applied to a regional GPS network, and for a typical station, the number of days with full data increased from 68% to 85%.

Citation: Deviren, M. N., F. Arikan, and O. Arikan (2013), Spatio-temporal interpolation of total electron content using a GPS network, *Radio Sci.*, *48*, 302–309, doi:10.1002/rds.20036.

1. Introduction

[2] Ionosphere is a key player in monitoring Space Weather (SW). The major observable feature of ionosphere is total electron content (TEC), which is defined as the line integral of electron density distribution on given ray path. The variability of TEC directly reflects the variability in electron density profile, which is a complicated function of position, height, and time. In recent decades, the worldwide, dual-frequency GPS receivers provide a cost-effective means in estimating TEC [*Coster et al.*, 1992; *Komjathy*, 1997; *Hajj et al.*, 2000; *Nayir et al.*, 2007]. GPS receivers can be used in Continuously Operating Reference Station (CORS) networks to increase the accuracy and reliability for positioning and surveying applications. CORS network receivers are generally distributed to a large region, and they can be placed at remote locations [*Steigenberger et al.*, 2006]. Due to various physical or operational disturbances, such as temporary antenna obstructions, power cuts, remote login problems, and geophysical or geomagnetic disturbances, GPS-TEC can be disrupted for a certain period during the day or the GPS receiver may cease to operate for a certain number of days. Services in navigation, positioning, surveying, and monitoring of SW require continuous operation of GPS receivers and uninterrupted TEC estimation for 24 h. The continuous data sets are used in modeling of ionosphere, TEC mapping, computerized ionospheric tomography (CIT), within-the-hour statistical analysis, ionospheric earthquake precursor studies, and prediction of SW events such as those provided in *Erturk et al.* [2009]; *Karatay et al.* [2010]; *Turel and Arikan*, [2010]; and *Foster and Evans* [2008]. Thus, it is an important task to interpolate the missing TEC values within a day or for a whole day. Ionosphere is a magnetoplasma; an anisotropic, inhomogeneous, time and space variable, and time and space dispersive channel. Therefore, spatial and temporal correlation structure of ionosphere has to be utilized in any interpolation scheme. As shown in previous studies such as [*Sayin et al.*, 2010], the temporal wide-sense stationarity (WSS) period of ionosphere is about 7.5–15 min for a quiet day. WSS reduces to 5 min for ionospheric conditions including geomagnetic storms and sunrise/sunset periods. Typical spatial correlation distances roughly correspond to 80 km to 150 km in midlatitude regions [*Komjathy*, 1997; *Karatay et al.*, 2010; *Foster and Evans*, 2008]. In order to complete the TEC data gaps, both the geophysical structure and the space-time correlation of ionosphere have to be taken into account [*Orús et al.*, 2005; *Hernández-Pajares et al.*, 2006].

[3] Another important problem is the prediction of spatio-temporal variability in TEC. It may be necessary to estimate the TEC of a GPS station from its neighbors for 1 day and then compare it to the station's own data to observe the spatial differences. Such a study is very useful to predict local disturbances that affect only a few stations in a dense grid. The temporal variability over a station can be observed by

[1]Department of Electrical and Electronics Engineering, Hacettepe University, Ankara, Turkey.
[2]Department of Electrical and Electronics Engineering, Bilkent University, Ankara, Turkey.

Corresponding author: F. Arikan, Department of Electrical and Electronics Engineering, Hacettepe University, Beytepe, Ankara, 06800 Turkey. (arikan@hacettepe.edu.tr)

0048-6604/13/10.1002/rds.20036

comparing the station's own data with the predictions from the previous days of the same station in a less dense grid. In this study, two different interpolation algorithms that join spatial and temporal properties of ionosphere are introduced. Both algorithms can be used for both filling in the TEC data gaps and prediction of spatio-temporal variability of TEC over a station. The two algorithms make use of optimization by least squares fit to available data. Spatio-temporal interpolation can be applied for data gaps as short as a few minutes to 24 h. The algorithms are applied to interpolate in the missing GPS-TEC values for Turkish National Permanent GPS CORS Network (TNPGN) for the years of 2001 to 2011 with great success. In section 2, the two novel spatio-temporal interpolation algorithms are provided. In section 3, the results are presented.

2. The Two Spatio-Temporal Interpolation Algorithms: STI-TEC1 and STI-TEC2

[4] TEC can be interpreted as the total number of electrons in a cylinder of 1 m^2 cross-section on a ray path. The unit of TEC is TECU and 1 TECU is equal to 10^{16} el/m^2. In this study, TEC values in the direction of local zenith over a GPS station u, for a day d are denoted by a vector $\mathbf{x}_{u;d}$ as

$$\mathbf{x}_{u;d} = [x_{u;d}(1) \ \ldots \ x_{u;d}(n) \ \ldots \ x_{u;d}(N_{u;d})]^T \tag{1}$$

where $N_{u;d}$ is the number of TEC values for GPS station u and day d. The superscript T is the transpose operator. If there were no data loss, the number of TEC estimates for a complete data day would be N. For example, a typical commercial GPS receiver provides measurement recordings every 30 s. If TEC estimates are obtained for every 30 s, then the number of TEC values (without any data loss for that day) would be $N = 2 \times 60 \times 24 = 2880$ samples/day. If the number of TEC data for 1 day, $N_{u;d}$ is less than N, then there are missing TEC values in $\mathbf{x}_{u;d}$.

[5] The goal is to combine spatial and temporal interpolation in a unique way to compensate for the missing values of $\mathbf{x}_{u;d}$ either within a day or for a whole day. In the spatial interpolation part of STI-TEC1 and STI-TEC2, the neighboring GPS stations of the network within the radial distance of R_r of station u are taken into account. In the following equations, N_{u,R_r} denotes the number of GPS stations in the neighborhood of station u within a radial distance R_r. In filling a data gap for station u within a day d, the temporal interpolation part of STI-TEC1 and STI-TEC2 both make use of a mathematical function that can be chosen from various alternatives including, but not limited to, cubic splines (C-splines) or polynomials [*Kahaner et al.*, 1989]. Let N_n denote the number of missing TEC values in $\mathbf{x}_{u;d}$, such that if n_i denotes the last sample that has a TEC value before the missing data sequence and n_s denotes the first sample after the missing data sequence, then $N_n = n_s - n_i - 1$. The temporally interpolated data vector for the missing values between the n_i and n_s can be given as

$$\hat{\mathbf{x}}_{u;d;N_n} = f_p(x_{u;d}(n_i), x_{u;d}(n_s), N_n) \tag{2}$$

where f_p is the temporal interpolation function that generates N_n number of samples for the data gap. Typically a third-degree polynomial between the end points of $x_{u;d}(n_i)$ and $x_{u;d}(n_s)$ can be used and for C-splines, the constraint extends to the point where both the function and the first and second derivatives at the end points have to be continuous [*Kahaner et al.*, 1989]. This constraint guarantees a smooth interpolated section fitting the data at the end points.

[6] STI-TEC1 and STI-TEC2 differ from each other in the spatial interpolation of the data gap as discussed in detail in the following subsections.

2.1. STI-TEC1

[7] In STI-TEC1, the spatial interpolation step primarily takes into account the TEC values of neighbors of station u depending on the radial distance R_r for a number of days prior and/or posterior of day d. Thus, an estimate of $\mathbf{x}_{u;d}$ for station u, in the neighborhood of radius R_r on day d, $\hat{\mathbf{x}}_{u;d;R_r}$, can be obtained as

$$\hat{\mathbf{x}}_{u;d;R_r} = \sum_{v=1}^{N_{u,R_r}} \alpha_{u;d;R_r}(v)\mathbf{x}_{v;d;R_r} \tag{3}$$

where $\alpha_{u;d;R_r}(v)$ is the spatial interpolation coefficient for vth station in the R_r neighborhood of station u for day d. N_{u,R_r} is defined as the number of stations that will be used in the interpolation of TEC that are in the neighborhood of radius R_r of station u. $\mathbf{x}_{v;d;R_r}$ denotes the TEC vector for station v and day d in the neighborhood of radius R_r. The spatial interpolation coefficient $\alpha_{u;d;R_r}(v)$ can be obtained by solving the following minimization problem:

$$\min_{\alpha_{u;d;R_r}(v)} \sum_{d_n=d_i}^{d_s} \left\| \mathbf{x}_{u;d_n} - \sum_{v=1}^{N_{u,R_r}} \alpha_{u;d;R_r}(v)\mathbf{x}_{v;d_n;R_r} \right\|_2^2 \tag{4}$$

for the total number of days $N_{d_s-d_i}$ from day d_i to day d_s prior to the day d. It is assumed that neighboring stations v have complete temporal data for $N_{d_s-d_i}$ number of days. $\|\cdot\|_2$ denotes the $\mathcal{L}_2$ norm corresponding to the metric distance between two vectors. The minimization in equation (4) can be obtained in closed form and the interpolation coefficients can be obtained as

$$\underline{\alpha}_{u;d;R_r} = \left(\sum_{d_n=d_i}^{d_s} \mathbf{X}^T_{u;d_n;R_r}\mathbf{X}_{u;d_n;R_r} \right)^{-1} \left(\sum_{d_n=d_i}^{d_s} \mathbf{b}_{u;d_n;R_r} \right) \tag{5}$$

where $\underline{\alpha}_{u;d;R_r}$ denotes the optimized interpolation coefficient vector for station u, day d and in the neighborhood R_r, and it is given as

$$\underline{\alpha}_{u;d;R_r} = [\alpha_{u;d;R_r}(1) \ \ldots \ \alpha_{u;d;R_r}(v) \ \ldots \ \alpha_{u;d;R_r}(N_{u,R_r})]^T \tag{6}$$

$\mathbf{X}_{u;d_n;R_r}$ in equation (5) is the matrix whose columns are TEC vectors from neighboring stations for day d_n. Specifically, $\mathbf{X}_{u;d_n;R_r}$ can be expressed as

$$\mathbf{X}_{u;d_n;R_r} = \left[\mathbf{x}_{1;d_n;R_r} \ \ldots \ \mathbf{x}_{v;d_n;R_r} \ \ldots \ \mathbf{x}_{N_{u,R_r};d_n;R_r}\right] \tag{7}$$

and the vector $\mathbf{b}_{u;d_n;R_r}$ in equation (5) is given as

$$\mathbf{b}_{u;d_n;R_r} = \mathbf{X}^T_{u;d_n;R_r}\mathbf{x}_{u;d_n}. \tag{8}$$

[8] The temporal interpolation of missing TEC values can be obtained using equation (2). Then,the separate temporally and spatially interpolated data from equation (2) and equation (3) can be combined with smoothing function that favors the temporal interpolation at the end points and spatial

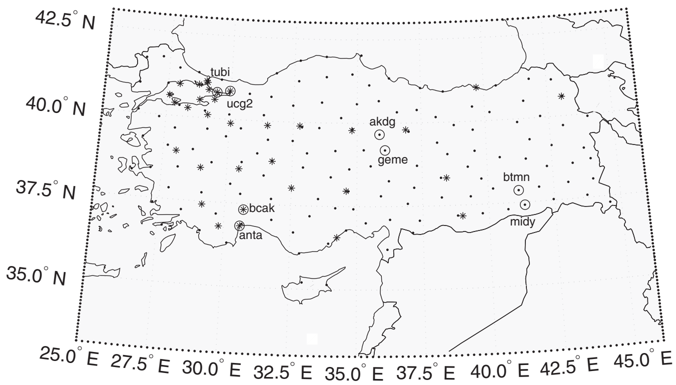

Figure 1. Distribution of TNPGN (asterisk) and TNPGN-Active (dots) GNSS CORS receiver station network. The circles indicate the stations that have been used in the manuscript.

interpolation in between. The joint spatio-temporal interpolation, STI-TEC1, can thus be achieved using a combiner matrix **G** as shown below

$$\hat{\mathbf{x}}_{u;d;c} = \mathbf{G}\hat{\mathbf{x}}_{u;d;R_r} + (\mathbf{I} - \mathbf{G})\hat{\mathbf{x}}_{u;d;N_n} \quad (9)$$

where $\hat{\mathbf{x}}_{u;d;c}$ is the joint spatio-temporal interpolated TEC vector; **I** is the identity matrix, and the combiner diagonal matrix **G** can be chosen as a solution to the following minimization problem:

$$\min_{\mathbf{G}} \sum_{d_n=d_i}^{d_s} \|\mathbf{x}_{u;d_n} - \hat{\mathbf{x}}_{u;d_n;c}\|_2^2 \quad (10)$$

The combiner diagonal matrix **G** can be expressed as $\mathbf{G} = \mathrm{diag}(g_1, \ldots, g_k, \ldots, g_{N_n})$, where $\mathrm{diag}(\cdot)$ is the diagonal matrix operator. The minimization in equation (10) can be rewritten in a simplified form as

$$\min_{g_1,\ldots,g_{N_n}} \sum_{d_n=d_i}^{d_s} \sum_{k=1}^{N_n} [x_{u;d_n}(k) - g_k \hat{x}_{u;d_n;R_r}(k) - (1-g_k)\hat{x}_{u;d_n;N_n}(k)]^2 \quad (11)$$

The optimal g_k can be found independent of each other as the minimizer of the following equation:

$$\min_{g_k} \sum_{d_n=d_i}^{d_s} \left[x_{u;d_n}(k) - \hat{x}_{u;d_n;N_n}(k) - g_k\left(\hat{x}_{u;d_n;R_r}(k) - \hat{x}_{u;d_n;N_n}(k)\right)\right]^2 \quad (12)$$

The solution can be obtained as follows:

$$g_k = \frac{\sum_{d_n=d_i}^{d_s} \left(\hat{x}_{u;d_n;R_r}(k) - \hat{x}_{u;d_n;N_n}(k)\right)\left(x_{u;d_n}(k) - \hat{x}_{u;d_n;N_n}(k)\right)}{\sum_{d_n=d_i}^{d_s} \left(\hat{x}_{u;d_n;R_r}(k) - \hat{x}_{u;d_n;N_n}(k)\right)^2} \quad (13)$$

In a GPS network, where many operational faults occur, it is difficult to find a number of consecutive days with full TEC estimates between days d_i to d_s for both station d and its neighboring stations in a radius of R_r. In most cases, 1 day before the day of interest and one neighbor in a radius of R_r are the available data sources. Therefore, in cases where there is highly sparse data in space and time, equation (13) can be rewritten using an alternate combiner function as

$$g_k = 1 - \frac{e^{-\beta(k-1)} + e^{-\beta(N_n-k)}}{1 + e^{-\beta(N_n-1)}} \quad (14)$$

where β can take values between 0 and 1, where $\beta = 0$ corresponds to only temporal interpolation.

2.2. STI-TEC2

[9] An alternate spatio-temporal interpolation, STI-TEC2, can be performed by giving more weight to temporal data of the station u. In filling the data gaps with STI-TEC2, the spatial interpolation from the neighbors are used only as a multiplying factor that guarantees the spatial homogeneity in ionosphere. Thus, the first step of STI-TEC2 takes into account the TEC data of station u 1 day before and 1 day after the missing day d as the primary interpolator. An estimate of $\mathbf{x}_{u;d}$ for station u, day d from $\mathbf{x}_{u;d-1}$ and $\mathbf{x}_{u;d+1}$, $\hat{\mathbf{x}}_{u;d}$, can be obtained as

$$\hat{\mathbf{x}}_{u;d} = \sum_{\substack{d_n=d-1 \\ d_n \neq d}}^{d+1} \alpha_{u;d_n} \mathbf{x}_{u;d_n} \quad (15)$$

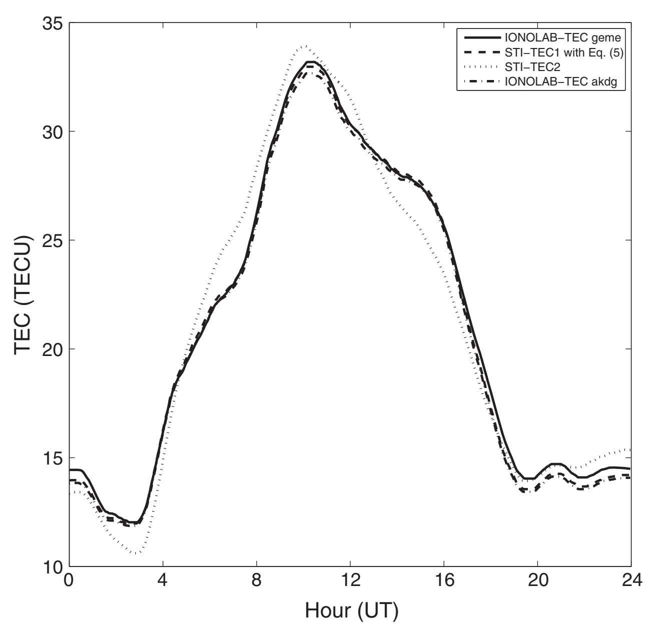

Figure 2. Application of STI-TEC1 with equation (5) (dashed line), and STI-TEC2 (dotted line) to 24 h gap of geme on 30 March 2011, using akdg (dash-dotted line) as a neighboring station. The original IONOLAB-TEC estimate for geme on 30 March 2011 is given with a solid line.

Table 1. Mean and Standard Deviation for Gaussian Distribution of $\alpha_{u;d;R_r}$

Neighbor Distance to Station u	$\hat{\mu}_\alpha$	$\hat{\sigma}_\alpha$
0–80 km	0.859978	0.015186
80–100 km	0.859972	0.016062
100–120 km	0.859981	0.017308
120–150 km	0.859974	0.020952

where $\alpha_{u;d_n}$ is the temporal interpolation coefficient for d_nth day of station u. The temporal interpolation coefficient $\alpha_{u;d_n}(v)$ can be obtained by solving the following minimization problem:

$$\min_{\alpha_{u;d_n}} \sum_{\substack{d_n=d-1\\ d_n\neq d}}^{d+1} \left\| \mathbf{x}_{u;d_n} - \sum_{\substack{d_n=d-1\\ d_n\neq d}}^{d+1} \alpha_{u;d_n}\mathbf{x}_{v;d_n} \right\|_2^2 \quad (16)$$

It is assumed that station u has complete temporal data for days $d-1$ and $d+1$. The minimization in equation (16) can be obtained in closed form as

$$\underline{\alpha}_{u;d} = \left(\sum_{\substack{d_n=d-1\\ d_n\neq d}}^{d+1} \mathbf{X}^T_{u;d_n}\mathbf{X}_{u;d_n} \right)^{-1} \left(\sum_{\substack{d_n=d-1\\ d_n\neq d}}^{d+1} \mathbf{b}_{u;d_n} \right) \quad (17)$$

where $\underline{\alpha}_{u;d}$ denotes the optimized coefficient vector for station u, day d, and it can be given as

$$\underline{\alpha}_{u;d} = [\alpha_{u;d-1} \;\; \alpha_{u;d+1}]^T \quad (18)$$

The data matrix of station u, $\mathbf{X}_{u;d_n}$, excludes the data of station u for day d, and it can be obtained from days $d-1$ and $d+1$ as

$$\mathbf{X}_{u;d_n} = \left[\mathbf{x}_{u;d-1} \;\; \mathbf{x}_{u;d+1}\right] \quad (19)$$

and

$$\mathbf{b}_{u;d_n} = \mathbf{X}^T_{u;d_n}\mathbf{x}_{u;d_n}. \quad (20)$$

When the above minimization problem is solved for one station u and 1 day d, equation (18) becomes

$$\underline{\alpha}_{u;d} = [1/2 \;\; 1/2]^T \quad (21)$$

Although the coefficients in equation (21) produce a reasonable temporal interpolation for quiet midlatitude ionosphere, it cannot represent significant diurnal variations due to ionospheric disturbances. In order to include the daily variability, the spatial modifications can be included using the GPS stations in the neighborhood of u within a radial distance R_r. Equation (15) can be modified to

$$\hat{\mathbf{x}}_{u;d} = \sum_{\substack{d_n=d-1\\ d_n\neq d}}^{d+1} r_{d;d_n}\alpha_{u;d_n}\mathbf{x}_{u;d_n} \quad (22)$$

where

$$r_{d;d_n} = \frac{1}{N_{u,R_r}} \sum_{v=1}^{N_{u,R_r}} \frac{\overline{\mathbf{x}}_{v;d}}{\overline{\mathbf{x}}_{v;d_n}}. \quad (23)$$

In equation (23), the overline denotes the mean of TEC values for station v and day d or d_n. The ratio factor in equation (23) introduces a correction for ionospheric variability from the neighbors of station u.

[10] For the temporal interpolation of a data gap for station u within the day d, the interpolation in equation (2) can be used. The separate temporally and spatio-temporally interpolated data in equations (2) and (22) can be combined with smoothing function as shown below

$$\hat{\mathbf{x}}_{u;d;t} = \mathbf{G_t}\hat{\mathbf{x}}_{u;d} + (\mathbf{I} - \mathbf{G_t})\hat{\mathbf{x}}_{u;d;N_n} \quad (24)$$

where $\hat{\mathbf{x}}_{u;d;t}$ is the joint spatio-temporal interpolated TEC vector, and $\mathbf{I}$ is the identity matrix. The diagonal elements of the combiner matrix $\mathbf{G_t}$, $g_{t;k}$, can be chosen similar to the combiner in equation (14) as

$$g_{t;k} = 1 - \frac{e^{-\beta(k-1)} + e^{-\beta(N_n-k)}}{1 + e^{-\beta(N_n-1)}}. \quad (25)$$

[11] The developed techniques of STI-TEC1 and STI-TEC2 are applied to interpolate the missing data sections of TNPGN as discussed in detail in the following section.

3. Results

[12] In this study, novel STI-TEC1 and STI-TEC2 interpolation methods are applied to TNPGN and TNPGN-Active Continuously Operating Receiver Station (CORS) networks. TNPGN consists of 23 stations, some permanent and some mobile, that operated between 2001 to 2008. TNPGN-Active is made up of 146 CORS GNSS stations distributed uniformly across Turkey and North Cyprus Turkish Republic since May 2009. The receiver stations are indicated in Figure 1 for both TNPGN and TNPGN-Active network.

[13] The GPS-TEC values are estimated as IONOLAB-TEC (www.ionolab.org) based on Reg-Est algorithm and IONOLAB-BIAS as given in *Nayir et al.* [2007] and *Arikan et al.* [2004, 2008]. Cycle slip faults and very short duration gaps in pseudorange and phase delay due to momentary antenna obstructions are corrected in IONOLAB-TEC preprocessing algorithm [*Sezen and Arikan*, 2012]. If IONOLAB-TEC gaps are less than 15 min and TEC difference between gap ends is less than 3 TECU, equation (2) is used with C-spline interpolation. Both STI-TEC1 and STI-TEC2 are used to fill the TEC gaps whose duration is longer than 15 min and/or whose TEC difference between gap ends is more than 3 TECU.

[14] In TNPGN-Active, for a typical station, the percentage of days that have full TEC data without any gaps is 68. For some remote stations, this number can get as low as 37%. STI-TEC1 and STI-TEC2 are both applied to all TNPGN-Active stations and the Marmara Region permanent CORS stations of TNPGN separately. With STI-TEC1, on the average, the number of days with full data increased from 68% to 82%. For example, in 2009, the data increase for anrk station is 17%; For fasa station, the increase is 12%; and for tnce station, the increase in data is 25%. In extreme cases, in 2011, snop station has 311 days with full data, and the number of days of complete data increased only by 0.8% to 314 days after the application of STI-TEC1 algorithm. On the other hand, in 2011, vaan station has 64 complete data days and with STI-TEC1, the number of days that have complete data is 314 with 68% increase. Similarly, using STI-TEC2 interpolation algorithm, the number of days that have full data improved from 68% to 75% for a typical station.

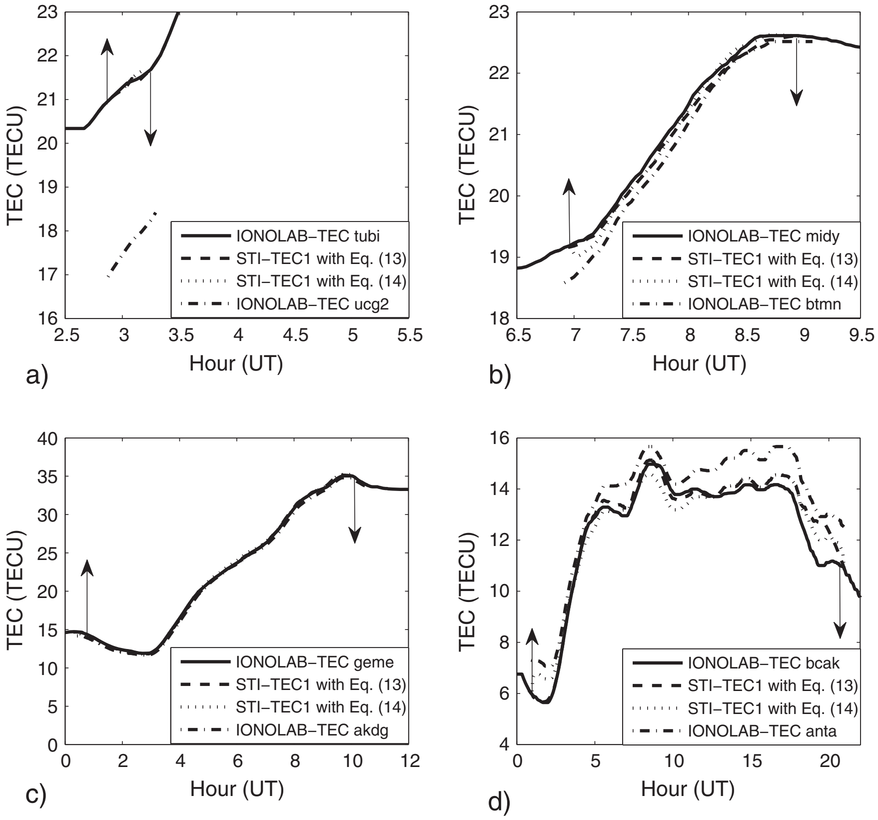

Figure 3. Application of STI-TEC1 with equation (13) (dashed line) and equation (14) (dotted line) (a) 15 min data gap of tubi on 31 March 2001, (b) 2 h data gap of midy on 6 August 2011, (c) 10 h data gap of geme on 31 March 2011, (d) 20 h data gap of bcak on 21 June 2006. The arrows indicate the initial and final samples for interpolation. The original IONOLAB-TEC estimates in each subplot are given in solid line, and neighbors are indicated with dash dotted line.

[15] The performance of STI-TEC algorithms are measured using Symmetric Kullback-Leibler Distances, $e_{Si}(u;d;N_n)$, and normalized $\mathcal{L}_2$ norms, $e_{Ni}(u;d;N_n)$, as follows

$$e_{Si}(u;d;N_n)=\sum_{n=1}^{N_n}\left[\frac{\hat{x}_{u;d;i}(n)}{\bar{\hat{\mathbf{x}}}_{u;d;i}}\ln\left(\frac{\hat{x}_{u;d;i}(n)/\bar{\hat{\mathbf{x}}}_{u;d;i}}{x_{u;d;N_n}(n)/\bar{\mathbf{x}}_{u;d;N_n}}\right)+\frac{x_{u;d;N_n}(n)}{\bar{\mathbf{x}}_{u;d;N_n}}\ln\left(\frac{x_{u;d;N_n}(n)/\bar{\mathbf{x}}_{u;d;N_n}}{\hat{x}_{u;d;i}(n)/\bar{\hat{\mathbf{x}}}_{u;d;i}}\right)\right] \qquad (26)$$

and

$$e_{Ni}(u;d;N_n)=\frac{\|\mathbf{x}_{u;d;N_n}-\hat{\mathbf{x}}_{u;d;i}\|}{\|\mathbf{x}_{u;d;N_n}\|} \qquad (27)$$

for station u, day d, and gap of N_n samples. i can be c for STI-TEC1 or t for STI-TEC2. Both algorithms are tested on 23 stations between 2001 and 2008, and on 146 stations for 2009 to 2011. When both algorithms are tested for interpolations for different temporal gaps, different ionospheric states, and with different neighbors within 150 km, it is observed that TEC can be robustly and successfully estimated using one neighboring station and using $(d-1)$th day using equation (3).

[16] An example for applications of STI-TEC1 and STI-TEC2 for a 24 h data gap is provided in Figure 2 for station geme (Gemerek, Sivas, Turkey) located at [39.18°N, 36.08°E] on a quiet day of 30 March 2011. The neighboring station is chosen as akdg (Akdağmadeni, Sivas, Turkey) at [39.66°N, 35.87°E]. The distance between geme and akdg is 56 km, and they are both in TNPGN-Active. geme and akdg stations are indicated in Figure 1 with circles. In application of STI-TEC1 for a 24 h gap, the temporal interpolation combiner is not used. STI-TEC1 is implemented only with equation (3) in this case. The interpolation coefficients $\alpha_{u;d;R_r}$ in equation (5) are computed using akdg on 29 March 2011. In Figure 2, the STI-TEC1 interpolation with equation (5) is given with a dashed line. For this case, $e_{Sc} = 1.14 \times 10^{-4}$, and $e_{Nc} = 1.4 \times 10^{-2}$. The application of STI-TEC2 as an interpolator for 24 h gap for geme on 30 March 2011 is also provided in Figure 2 with a dotted line. For this case, akdg station is used as the neighbor. The interpolation coefficients are computed using 29 March 2011 and 31 March 2011, 1 day prior and 1 day after the interpolation day. The SKLD and L2 norm for STI-TEC2 application are $e_{St} = 1.8 \times 10^{-3}$ and $e_{Nt} = 5.6 \times 10^{-2}$. As it can be observed from Figure 2, on a quiet day of ionosphere, the STI-TEC1 with only equation (5) can be used with high reliability. STI-TEC2 is also a good performer on a quiet day and it can interpolate whole 24 h with reasonable accuracy.

[17] STI-TEC1 interpolation is applied to all TNPGN-Active stations using one neighbor and one prior day in the interpolation equation (3). All estimated $\alpha_{u;d;R_r}$ values within a year are grouped with respect to the distance to the neighboring station as 0–80 km, 80–100 km, 100–120 km, and

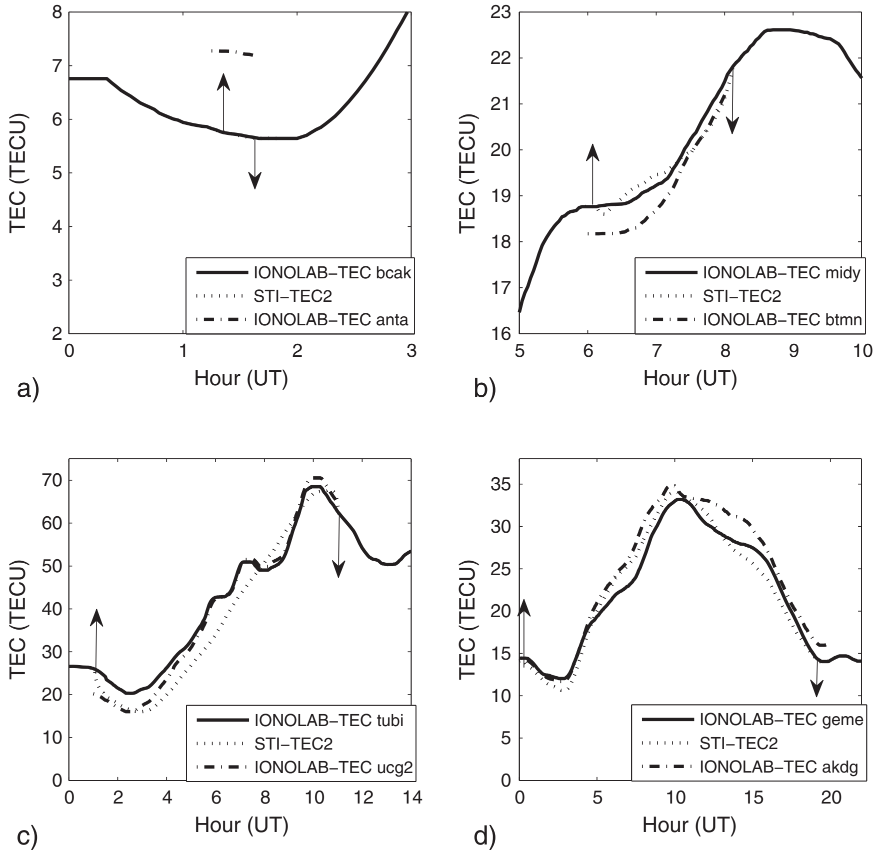

Figure 4. Application of STI-TEC2 in equation (24) with equation (25) (dotted line), (a) 15 min data gap of bcak on 21 June 2006, (b) 2 h data gap of midy on 6 August 2011, (c) 10 h data gap of tubi on 31 March 2001, (d) 20 h data gap of geme on 30 March 2011. The arrows indicate the initial and final samples for interpolation. The original IONOLAB-TEC estimates in each subplot are given in solid line, and neighbors are indicated with dash dotted line.

120–150 km. A histogram is drawn for each radius category, and it is observed that the interpolation coefficient $\alpha_{u;d;R_r}$ has a Gaussian distribution with mean, $\hat{\mu}_\alpha$, and standard deviation, $\hat{\sigma}_\alpha$. The parameters of normal distribution are obtained in the maximum likelihood sense from the data. The parameters $\hat{\mu}_\alpha$ and $\hat{\sigma}_\alpha$ for the years of 2010 and 2011 combined are provided in Table 1. As it is observed from Table 1, the distance of neighboring station within 150 km does not affect the mean of the distribution, yet the standard deviation increases slightly as the distance of the neighbor increases. For neighboring stations that are farther than 150 km radius of station u, the ionospheric correlation decreases. Thus, the STI-TEC1 interpolation is not applied for neighbors which are more than 150 km apart. In *Turel and Arikan* [2010], it is stated that for GPS stations that are located in midlatitude that have no more than ±5° latitude difference from each other, the within-the-hour probability density functions of TEC are very similar. Thus, it might be expected that the values in Table 1 can be used for any GPS network located in any midlatitude region to interpolate TEC values from neighboring stations. For the case of a single GPS station, nearest Global Ionospheric Map (GIM) grid point values can be utilized (ftp://igs.ensg.ign.fr/pub/igs/iono).

[18] In section 2.1, two possible combiner coefficients for spatio-temporal interpolation are proposed in equations (13) and (14) to be used in equation (9). In Figure 3, the performance of these two possible combiners is presented for 15 min, 2 h, 10 h, and 20 h gaps, for various ionospheric conditions. In each subplot, the solid line indicates the original IONOLAB-TEC estimate for each station and

Table 2. $e_{Si}(u; d; N_n)$ and $e_{Ni}(u; d; N_n)$ Values for the Interpolations Provided in Figures 3 and 4 for Stations and Dates Given in the Subfigures

	15 min	2 h	10 h	20 h
$e_{Sc}(u; d; N_n)$ with (13)	1.22×10^{-6}	3.05×10^{-6}	1.07×10^{-5}	5.74×10^{-4}
$e_{Sc}(u; d; N_n)$ with (14)	2.40×10^{-3}	8.37×10^{-6}	4.09×10^{-5}	9.73×10^{-4}
$e_{St}(u; d; N_n)$ with (25)	3.11×10^{-6}	7.50×10^{-5}	6.90×10^{-3}	1.70×10^{-3}
$e_{Nc}(u; d; N_n)$ with (13)	1.70×10^{-3}	4.40×10^{-2}	4.40×10^{-3}	3.86×10^{-2}
$e_{Nc}(u; d; N_n)$ with (14)	2.59×10^{-6}	4.80×10^{-2}	7.70×10^{-3}	4.12×10^{-2}
$e_{Nt}(u; d; N_n)$ with (25)	2.90×10^{-3}	1.26×10^{-2}	10.82×10^{-2}	5.58×10^{-2}

for each day. STI-TEC1 using equation (9) with equation (13) is given with dashed line. STI-TEC1 using equation (9) with equation (14) is given with dotted line. The arrows indicate the beginning and end points of interpolation. In Figure 3a, 15 min data gap of station tubi is interpolated using the neighboring station ucg2 (44 km away from tubi), on 31 March 2001, during which there is a negative disturbance in ionosphere in a solar maximum year. In Figure 3b, 2 h data gap of station midy is interpolated using the neighboring station btmn (53 km away from midy), on 6 August 2011, a severe ionospheric storm day. The year 2011 is in the ascending phase of solar cycle 24. In Figure 3c, 10 h data gap of station geme is interpolated using the neighboring station akdg (56 km away from geme), on 31 March 2011, a quiet day. In Figure 3d, 20 h data gap of station bcak is interpolated using the neighboring station anta (56 km away from bcak), on 21 June 2006, summer solstice day for the northern hemisphere in a solar minimum year. All mentioned stations are indicated in Figure 1 with circles. tubi, ucg2, bcak, and anta are TNPGN stations and geme, akdg, midy, and btmn are TNPGN-Active stations. It is observed from Figure 3 that STI-TEC1 performs very well as a spatio-temporal interpolator with various length data gaps and on both quiet and disturbed days of ionosphere. When equations (13) and (14) in equation (9) are compared with each other, the computation of STI-TEC1 with equation (14) as a combiner works without a demand on data of the station for previous days. In all of these examples in Figure 3, β is chosen as 0.35.

[19] An example for application of STI-TEC2 in equation (24) using equation (25) is provided in Figure 4 for 15 min, 2 h, 10 h and 20 h gaps, for various ionospheric conditions. In each subplot, the solid line indicates the original IONOLAB-TEC estimate for each station and for each day. STI-TEC2 using equation (24) with equation (25) is given with a dotted line. The arrows indicate the beginning and end points of interpolation. In Figure 4a, 15 min data gap of station bcak is interpolated using the neighboring station anta, on 21 June 2006, summer solstice day. In Figure 4b, 2 h data gap of station midy is interpolated using the neighboring station btmn, on 6 August 2011, a severe ionospheric storm day. In Figure 4c, 10 h data gap of station tubi is interpolated using the neighboring station ucg2, on 31 March 2001, where there is a negative disturbance in ionosphere. In Figure 4d, 20 h data gap of station geme is interpolated using the neighboring station akdg, on 30 March 2011, a quiet day. In all of these examples in Figure 4, β value in the combiner equation (25) is chosen as 0.35.

[20] The $e_{Si}(u; d; N_n)$ and $e_{Ni}(u; d; N_n)$ values for the interpolations provided in subplots of Figures 3 and 4 are given in Table 2. It is observed that both STI-TEC1 and STI-TEC2 are very successful in interpolation of gaps with different sizes with both error norms for quiet days of ionosphere. For disturbed days of ionosphere and with data gaps that are longer than 6 h, STI-TEC1 must be preferred.

4. Conclusion

[21] Two novel spatio-temporal interpolation algorithms are developed both to fill in the data gaps and to predict spatio-temporal variability of TEC in GPS networks. The algorithms make use of optimization of spatial and temporal correlation of ionosphere by least squares fit to available data. The two algorithms, STI-TEC1 and STI-TEC2, are applied separately to TNPGN between 2001 and 2008 and TNPGN-Active between 2009 and 2011. The missing TEC data for durations longer than 15 min to 24 h are interpolated using both neighboring station TEC values. In the computation of interpolation coefficients and combiners, the data of previous day and 1 day after the interpolation day are utilized. With the application of STI-TEC1, the days with complete data increased from 68% to 85%. With STI-TEC2, the rate of increase is from a typical 62% to 75%. The algorithms are tested using Symmetric Kullback-Leibler distance and normalized $\mathcal{L}_2$ norm. With both norms, it is observed that the interpolated data agrees with the original data of the station with great success for any gap length from a few minutes to 24 h. STI-TEC1 can be used with any data gap length and for any ionospheric condition. The interpolation error of STI-TEC2 increases for gaps longer than 6 h for disturbed days of the ionosphere. The algorithms can be applied to any GPS regional network data in midlatitude regions using the TEC data from neighboring stations within 150 km radius. The spatio-temporal coefficients can be obtained by generating random numbers from a Gaussian distribution whose mean and standard deviations are provided in this study. For single stations that are not located in a network, closest GIM grid point can be substituted to fill in the TEC gaps using the same distribution.

[22] **Acknowledgment.** This study is supported by TUBITAK EEEAG grant 109E055.

References

Arikan, F., C. B. Erol, and O. Arikan (2004), Regularized estimation of VTEC from GPS data for a desired time period, *Radio Sci.*, *39*, RS6012, doi:10.1029/2004RS003061.
Arikan, F., H. Nayir, U. Sezen, and O. Arikan (2008), Estimation of single station interfrequency receiver bias using GPS-TEC, *Radio Sci.*, *43*, RS4004, doi:10.1029/2007RS003785.
Coster, A. J., E. M. Gaposchkin, and L. E. Thornton, (1992), Real-time ionospheric monitoring system using the GPS, *Tech. Rep. 954*, Lincoln Laboratory, Lexington, MA.
Erturk, O., O. Arikan, and F. Arikan (2009), Tomographic reconstruction of the ionospheric electron density as a function of space and time, *Adv. Space Res.*, *43*, 1702–1710.
Foster, M. P., and A. N. Evans (2008), An evaluation of interpolation techniques for reconstructing ionospheric TEC maps, *IEEE Trans. Geosci. Remote Sens.*, *46*, 2153–2163.
Hajj, G. A., L. C. Lee, X. Pi, L. J. Romans, W. S. Schreiner, P. R. Straus, and C. Wang (2000), COSMIC GPS ionospheric sensing and space weather, *TAO*, *11*, 235–272.
Hernández-Pajares, M., J. M. Juan, and J. Sanz (2006), Medium-scale traveling ionospheric disturbances affecting GPS measurements: Spatial and temporal analysis, *J. Geophys. Res.*, *111*, A07S11, doi:10.1029/2005JA011474.
Kahaner, D., C. Moler, and S. Nash (1989), *Numerical Methods and Software*, pp. 100–125, Prentice Hall, Englewood Cliffs, N.J., USA.
Karatay, S., F. Arikan, and O. Arikan (2010), Investigation of total electron content variability due to seismic and geomagnetic disturbances in the ionosphere, *Radio Sci.*, *45*, RS5012, doi:10.1029/2009RS004313.
Komjathy, A. (1997), Global ionospheric total electron content mapping using the global positioning system, Ph. D. dissertation, Department of Geodesy and Geomatics Engineering, University of New Brunswick, Fredericton, New Brunswick, Canada.
Nayir, H., F. Arikan, O. Arikan, and C. B. Erol (2007), Total electron content estimation with Reg-Est, *J. Geophys. Res.*, *112*, A11313, doi:10.1029/2007JA012459.
Orús, R., M. Hernández-Pajares, J. M. Juan, and J. Sanz (2005), Improvement of global ionospheric VTEC maps by using kriging interpolation technique, *J. Atmos. Sol. Terr. Phys.*, *67*, 1598–1609.
Sayin, I., F. Arikan, and K. E. Akdogan (2010), Optimum temporal update periods for regional ionosphere monitoring, *Radio Sci.*, *45*, RS6018, doi:10.1029/2009RS004316.

Sezen, U., and F. Arikan (2012), A Novel algorithm for cycle slip detection and repair, geophysical research abstracts, 14, EGU2012-7586, EGU General Assembly 2012.

Steigenberger, P., M. Rothacher, R. Dietrich, M. Fritsche, A. Rülke, and S. Vey (2006), Processing of a global GPS network, *J. Geophys. Res.*, *111*, B05402, doi:10.1029/2005JB003747.

Turel, N., and F. Arikan (2010), Probability density function estimation for characterizing hourly variability of ionospheric total electron content, *Radio Sci.*, *45*, RS6016, doi:10.1029/2009RS004345.