MARKETS AS INSTITUTIONS

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF ECONOMICS
AND THE INSTITUTE OF ECONOMICS AND SOCIAL SCIENCES
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

By

Erdem Başçı

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I certify that I have read this dissertation and in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy in Economics.

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Abstract

MARKETS AS INSTITUTIONS

Erdem Başçı

Ph. D. Thesis in Economics

Supervisor: Professor Semih Koray

May 1995

This dissertation investigates resource allocation via institutions. A unifying framework for studying various kinds of institutional structures is provided. After the introductory Part 1, Part 2 presents the general model (Chapter 1) and studies existence of equilibria (Chapter 2). Part 3 provides applications to general equilibrium models under complete markets (Chapter 3), public goods and Lindahl prices (Chapter 4), generalized price systems and sales taxes (Chapter 5), lemons type quality problems (Chapter 6), adverse selection and money (Chapter 7), and a model with Markov technologies and freedom effects on utilities (Chapter 8). Welfare implications of most applications are investigated. Chapter 9 discusses further applications and possible future research topics.

Keywords: Societies, institutions, economic systems, social systems, abstract economies, abstract economic systems, private ownership economies, public goods economies, markets for lemons, adverse selection, price system, generalized price system, semicentralized resource equilibrium, non-cooperative equilibrium, social equilibrium, competitive (Walrasian) equilibrium, sales tax equilibrium, sustainable Walrasian equilibrium, equilibrium existence, financial structure, neutrality of money, second best.

Özet

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Tez Yöneticisi: Profesör Dr. Semih Koray

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Bu araştırma kurumlar vasıtası ile kaynak dağılımı ile ilgilidir. Çesitli türlerden kurumsal yapıların incelenebileceği birleştirici bir çerçeve model önerilmektedir. Giriş bölümünden sonra ikinci bölüm genel modeli tanıtmakta ve dengenin varlığı için yeter koşullar bulmaktadır. Üçüncü bölüm ise genel modelin uygulamalarından oluşmaktadır. Tam özel mal piyasalarında rekabetçi denge, kamu malı bulunduğunda Lindahl dengesi, genelleştirilmiş fiyat sistemi ve katma değer türü dolaylı vergiler altında genel denge, bozuk mal problemi, ters seleksiyon ve para, Markov teknolojiler ile özgürlük etkileri altında durağan Walras dengesi bu bölümde incelenen konulardır. Uygulamaların birçoğunda kurumsal yapıların refah üzerine etkisi de gözlemlenmektedir. Olası diğer bazı uygulamalar ve potansiyel araştırma konuları son kısımda tartışılmaktadır.

Anahtar Sözcükler: Toplumlar, kurumlar, ekonomik sistemler, sosyal sistemler, soyut ekonomiler, soyut ekonomik sistemler, özel mülkiyet ekonomileri, kamu mallı ekonomiler, bozuk mal piyasaları, ters seleksiyon, fiyat sistemi, genelleştirilmiş fiyat sistemi, yarı merkezi kaynak dengesi, işbirliksiz denge, sosyal denge, rekabetçi (Walrasgil) denge, satış vergisi dengesi, sürdürülebilir Walras dengesi, dengenin varlığı, mali yapı, paranın nötr olması, ikinci en iyi.

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I am especially thankful to my wife Sıdıka who has provided me with constant support of highest value during all my studies.

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Part I

Introduction

This dissertation is an attempt to formalize the notion of an *institution*. The benefits of having a formal framework under which one can carry out analysis and design of institutions for a given society would be immense. This would also facilitate the task of comparing economic systems. Most of the economics profession is involved in either analysis, design or comparison of economic systems.

Earlier work in social or economic systems has used the mathematical constructs of *games* and *pseudo games* (also called abstract economies) on the one hand and *economies* with price taking agents on the other. The approach of separating the concept of a social or economic system into two parts, namely the *society* that is described by the list of preferences of its members and the *institution* that is the set of rules under which the society operates is not new. The notion of a *game form* introduced by Hurwicz (1960, 1972, 1979) is an example of an institution which is meant to be decentralized. North (1990) similarly sees institutions as rules that not only limit freedom of choice but also describe the consequences of choices in terms of social outcomes. In this sense institutions guide individual choices for attaining certain goals.

> Institutions reduce uncertainty by providing a structure to everyday life. They are a guide to human interaction, so that when we wish to greet friends on the street, drive an automobile, buy oranges, borrow money, form a business, bury our dead, or whatever, we know (or can learn easily) how to perform these tasks. We would readily observe that institutions differ if we were to try to make the same transaction in a different country - Bangladesh for example. In the jargon of the economist, institutions define and limit the set of choices of individuals.[1]

The notion of a game form, being decentralized by definition, is not sufficient to stand for the more general notion of an institution, since an institution may be centralized or semicentralized as well. That is the possibility of the presence of a central authority as a part of an institution which at least guides the system by changing the nature of the game form (semicentralization) if

[1] North(1990), pp.3-4

not determining completely the actions to be taken by the agents in the society (centralization) is excluded from a game form. Efforts to formalize institutions in the more general sense have increased especially after the collapse of the Soviet Union and the desire to transform it to a *market economy*. One of the leading steps to that end came from Hurwicz (1994). In accordance with the ideas of North above Hurwicz (1994) firstly, sees an institution as a class of game forms. Secondly, an institution is generated by human effort, possibly through an extensive form game. Thirdly, the resulting behaviour is either *internalized* or *enforced*. And fourthly, the rules of the institution should apply to categories of people rather than to people at the individual level.

In this dissertation we will likewise see an institution as a class of (pseudo-)game forms which is indexed by a parameter set. There will be a center that picks a parameter from this set to determine the (pseudo-)game form that the agents will face. However, to make life a little bit difficult for the center, there will also be a set of institutionally feasible choices. Then the task of the center is twofold:

1. Make sure to select a game form which has a physically feasible equilibrium;
2. Pick among the game forms with feasible equilibria, the one that is most preferred by the center.

In the first task, only *equilibrium* outcomes are required to be physically feasible. So out of equilibrium behavior could possibly be not viable under the given set of rules. This makes some real life situations, that could not possibly be modeled by a game form, be modeled by an institution. A typical example to this is the pure exchange economy, where the auctioneer is the center and sets prices to clear markets.

A parameter-choice pair that satisfies the first criterion will be called a *semicentralized resource equilibrium* (SECERE) and a pair that satisfies the second criterion will be called a *center's optimum equilibrium* (COE).

This definition of an institution incorporates as special cases a pseudo-game form (by eliminating all the power of the center), a game form (by eliminating the power of the agents to affect strategy sets of others), a private goods economy with a Walrasian price system (Chapter 3), a public goods economy with a Lindahl price system (Chapter 4) and the other examples considered in Part 3.

The equilibrium concept SECERE corresponds to the Walrasian and Lindahl equilibria in

Chapters 3 and 4. The equilibrium concept COE on the other hand is a generalization of the Stackelberg solution concept in many ways to situations in which there is one leader and many followers acting simultaneously. An economic example for COE is provided in Chapter 5.

The dissertation is organized in three parts. Part one is introduction. Part two builds the general framework and provides existence results at this general level. Part three considers several applications. In Chapter 1, we formalize the notions of a society, an institution and an (abstract) economic system, then we introduce the semicentralized resource equilibrium concept. In Chapter 2 after presenting notation and some background results from the literature, we provide two existence theorems. One of these theorems is based on a result by Shafer and Sonnenschein (1975) on the existence of social equilibria of abstract economies. The other existence theorem follows the so called Market Equilibrium approach first introduced by Gale (1955).

In Chapter 3, we prove the existence of a Walrasian equilibrium of a private ownership economy based on a theorem in Chapter 2. Using the same theorem, in Chapter 4, we give a proof of existence for a Lindahl equilibrium of a public goods economy. This proof differs from the existing ones in the literature.

Chapter 5 considers the case of generalized pricing systems. An existence result based on the generalization of the market equilibrium approach in Chapter 2 is presented there. As an exemplary application, a selfish government that has access to only sales taxes in a private ownership economy is investigated. An equilibrium in such an economy is nothing but a COE.

A satisfactory general equilibrium formulation of the so called Lemons problem as introduced by Akerlof (1970) has been missing in the literature. For that reason, the existence of equilibrium in economies with asymmetric quality between purchased and sold commodities has not been studied extensively. Chapter 6 tries to fill this gap to some extent.

Chapter 7 presents a simple model where money is introduced. The function of money is to (imperfectly) resolve a contract enforcement problem. In this model, economic agents face nonlinear programming problems. The equilibrium is derived and comparative static exercises are carried out to see the importance of the quantity of money in the economy. This model also is a special case of the framework of Chapter 1.

Chapter 8, which is written jointly with Murat R. Sertel and is forthcoming in Journal of Mathematical Economics, introduces and discusses an economic application of a result of

Prakash and Sertel (1974b). The example is a general equilibrium model with technological externalities and preferences that are allowed to depend on the individuals' own and others' budget sets. A *sustainable Walrasian equilibrium* is defined for this economy and its existence is established in this chapter.

Chapter 9 provides concluding remarks with emphasis on future work. Desirability aspects of equilibria, the first and second welfare theorems and their possible generalizations are discussed there. The concept of implementation via institutions is also introduced.

Part II

Theory

1 A Framework for Analysis and Design of Economic Institutions

1.1 Societies, Institutions, Economic Systems

To keep the generality level of the framework as high as possible, we have chosen in this chapter not to impose any algebraic or topological structure on the mathematical objects introduced. Elementary set concepts will be enough for the definitions.

We first define a society as a collection of individuals with consumption sets and preferences over these. Note that this definition allows for consumption externalities.

Definition 1 *A* **society** $S = (I, C, (B_i)_{i \in I})$ *is a nonempty* **set of individuals** I*, a nonempty* **outcome set** C*, and for each individual* $i \in I$*, a* **better outcomes correspondence** $B_i : C \to C$.

In the above definition, $B_i(c)$ should be interpreted as the set of strictly preferred outcomes by individual i to outcome c.

North (1990) views institutions as guidelines that describe the options available to individuals for attaining certain goals. The definition below incorporates these options as choice sets. It also describes how the choice sets change with changes in administrator's and other individuals' actions, the outcome and resource implications of the choices of agents, and the set of available resources.

Just as a game form describes only the institutional aspects of a game without reference to preferences, an *institution* as defined below describes the institutional aspects of an abstract economy (Debreu (1952)). In addition, there is a *parameter set*, a *resource implications function* and a *resource set*.

We will denote by $\mathcal{W}$ the *resource space* in which the resource set will lie.

Definition 2 *An* **institution** *for a society S is a list*
$\mathcal{N} = (P, (X_i, \tau_i)_{i\in I}, c, F, W)$ *where P is a nonempty set (***the parameter set***), and for each $i \in I$,*

- *X_i is a nonempty set (***choice set***) ($X = \prod_{i\in I} X_i$),*
- *$\tau_i : P \times X \to X_i$ is a correspondence (***constraint correspondence***),*

*$c : P \times X \to C$ is a function (***outcome function***), $F : X \to \mathcal{W}$, is a function (***resource implications function***), and $W \subset \mathcal{W}$ is a nonempty set (***resource set***).*

Notice that the definition of an institution does not include the individuals' preferences.

Now we are ready to define an economic system simply as a society with an institution.

Definition 3 *An* **economic system** $\mathcal{E}_S = (\mathcal{S}, \mathcal{N})$ *consists of:*

- *a society $\mathcal{S} = (I, C, (B_i)_{i\in I})$ and*
- *an institution $\mathcal{N} = (P, (X_i, \tau_i)_{i\in I}, c, F, W)$ for the society $\mathcal{S}$*

Private ownership economies with Walrasian prices and public goods economies with Lindahl prices are all economic systems. This will be seen clearly in Chapters 3 and 4.

Here preferences are not allowed to depend on the institution, e.g. on *parameters* determined by the central authority. The following definition of an abstract economic system allows for this as well. In an abstract economic system, preferences depend directly on the parameter selected by the administrator and the choices of all the individuals.

Definition 4 *An* **abstract economic system** $\mathcal{A}_S = (I, P, (X_i, B_i, \tau_i)_{i\in I}, F, W)$ *is defined by: a nonempty* **set of agents** *I, a nonempty* **set of parameters** *P, for each agent $i \in I$,*

- *X_i is a nonempty set (***choice set***) ($X = \prod_{i\in I} X_i$),*
- *$\tau_i : P \times X \to X_i$ is a correspondence (***constraint correspondence***),*
- *$B_i : P \times X \to X_i$ is a correspondence (***better choices correspondence***).*

a **resource implications function** *$F : X \to \mathcal{W}$, and a* **resource set** *$W \neq \emptyset$.*

In an abstract economic system, the notions of a society and institution are not separated. In this sense it is a direct generalization of an abstract economy.

Given an economic system $\mathcal{E}_S = (I, C, (B_i)_{i\in I} P, (X_i, \tau_i)_{i\in I}, c, F, W)$, one can obtain the corresponding abstract economic system $\mathcal{A}_S(\mathcal{E}_S) = (I, P, (X_i, \widetilde{B}_i, \tau_i)_{i\in I}, F, W)$ by translating preferences on outcomes to preferences on the individual choice sets according to the rule,

$$\widetilde{B}_i(p, x) \equiv \{\bar{x}_i \in X_i \mid c(p, \bar{x}_i, x_{-i}) \in B_i(c(p, x))\}$$

for all $i \in I$.

Conversely, given an abstract economic system $\mathcal{A}_S = (I, P, (X_i, \widetilde{B}_i, \tau_i)_{i\in I}, F, W)$, the corresponding economic system $\mathcal{E}_S(\mathcal{A}_S) = (I, C, (B_i)_{i\in I} P, (X_i, \tau_i)_{i\in I}, c, F, W)$, can be obtained by setting

$$C = P \times X,$$

$$c(p, x) = (p, x) \ \ for\ all\ \ (p, x) \in P \times X,$$

and for any $c = (p, x)$,

$$B_i(c) = B_i(p, x) \equiv \{(p, \tilde{x}_i, x_{-i}) \in C \mid \tilde{x}_i \in \widetilde{B}_i(p, x)\}$$

for all $i \in I$.

1.2 Market-Type Institutions and Economic Systems

The models considered in the economics literature usually impose a special structure for the set of individuals, I and the function F . A typical example is that of a pure exchange economy where the set of consumers is finite and an allocation is defined as a list of consumption bundles whose *sum* is equal to the total resources in the economy. This observation motivates the following.

Let $\mathcal{W} \subset \mathcal{R}^l$.

Definition 5 *An institution $\mathcal{N} = (P, (X_i, \tau_i)_{i\in I}, c, F, W)$ for a society $\mathcal{S}$ is said to be of* **market type** *if, $(I, \mathcal{I}, \nu)$ is a measure space of agents and there exists $f : I \times X \to \mathcal{W}$ such that*

$$F(x) = \int_I f(i, x) d\nu(i).$$

In that case, we write the market type institution as, $\mathcal{M} = (P, (X_i, \tau_i)_{i\in I}, c, f, W)$. A **market type economic system** *is an economic system which has a market type institution.*

If the set of consumers is finite, then we may use a counting measure and the integral then reduces to a sum.

A pure exchange economy is clearly of market type. In a private ownership economy (Arrow and Debreu (1954)) however, there are firms which maximize profits. Therefore their preferences depend also on prices. The definition below therefore will be useful (see Chapter 3 for an example with finite agents).

Definition 6 *An abstract economic system $\mathcal{A}_S = (I, P, (X_i, B_i, \tau_i)_{i\in I}, F, W)$ is said to be of* **market type** *if, $(I, \mathcal{I}, \nu)$ is a measure space of agents and there exists $f : I \times X \to \mathcal{W}$ such that*

$$F(x) = \int_I f(i, x) d\nu(i).$$

In that case, we write the market type system as, $\mathcal{A}_M = (P, (X_i, B_i, \tau_i)_{i\in I}, f, W)$.

1.3 Semicentralized Resource Equilibrium (SECERE)

The equilibrium here will consist of a parameter chosen by the center and a list of choices by the individuals such that two conditions are met. Firstly the choices should be feasible as far as resources are concerned. Secondly the choices should be consistent with individual maximization. One can argue that this equilibrium concept has a lot of informational requirements on behalf of the center. Such arguments regarding the empirical relevance of the equilibrium concepts introduced here are postponed to the last chapter.

Definition 7 *A* **semi-centralized resource equilibrium (SECERE)** *of an abstract economic system $\mathcal{A}_S = (I, P, (X_i, B_i, \tau_i)_{i\in I}, F, W)$ is a pair $(p^*, x^*) \in P \times X$ such that*

(i) $F(x^*) \in W$, (resource feasibility)

(ii) $x_i^* \in \tau_i(p^*, x^*)$ and $B_i(p^*, x^*) \cap \tau_i(p^*, x^*) = \emptyset$, $i \in I$,

(non-cooperative maximization).

A SECERE of a market type abstract economic system is called **SECEREM**.

The above definition of SECERE has some generality. For example, it can incorporate Lindahl equilibria as well as Walrasian equilibria, as we will see in Chapters 3 and 4.

Definition 8 *A* **semi-centralized resource equilibrium (SECERE)** *of an economic system $\mathcal{E}_S = (I, C, (B_i)_{i\in I}, P, (X_i, \tau_i)_{i\in I}, c, F, W)$ is a pair $(p^*, x^*) \in P \times X$ such that,* *i* $F(x^*) \in W$,

ii $x_i^* \in \tau_i(p^*, x^*)$ *and* $B_i(c(p^*, x^*)) \cap c(p^*, \tau_i(p^*, x^*), x_{-i}^*) = \emptyset$, $i \in I$.

A SECERE of a market type economic system is called **SECEREM**.

A SECERE (p^*, x^*) of an economic system $\mathcal{E}_S$, is also a SECERE of the corresponding abstract economic system $\mathcal{A}_S(\mathcal{E}_S)$ as clearly seen from the construction given in Section 1.1. Likewise, a SECERE (p^*, x^*) of an abstract economic system $\mathcal{A}_S$, qualifies as a SECERE of the corresponding economic system $\mathcal{E}_S(\mathcal{A}_S)$.

Examples of economic systems and their SECEREs in Part 3 are an economy with sales taxes (Chapter 5), an economy with a market for lemons (Chapter 6), an economy operating with money (Chapter 7) and their Walrasian equilibria.

1.4 Center's Optimality and Pareto Efficiency

There may of course be multiplicity of equilibria (SECERE). In such a case, it may be reasonable to think that the center will select its favorite equilibrium. This leads to a generalization of the Stackelberg solution concept to economic systems, where there is one leader who knows the reaction functions of the followers, where the followers act non-cooperatively among themselves taking as given the leader's action.

First we give an objective function to the center which represents its preference.

Definition 9 *Given an abstract economic system* $\mathcal{A}_S = (I, P, (X_i, B_i, \tau_i)_{i \in I}, F, W)$ *a function* $G : P \times X \to \mathcal{R}$ *is called the* **administrators' objective function**.

A desirable property for the center's objective function is that of Samuelson - Bergson.

Definition 10 *An administrators' objective function* G *has the* **Samuelson-Bergson property** *if*

$$G(p, x) > G(p', x')$$

whenever $(p', x') \notin B_i(p, x)$ *for all* $i \in I$ *and* $(p, x) \in B_i(p', x')$ *for some* $i \in I$.

Now we will give a name to an abstract economic system and an administrators' objective function.

Definition 11 *An* **administered abstract economic system** *is a pair* $(\mathcal{A}_S, G)$ *where,*

- $\mathcal{A}_S = (I, P, (X_i, B_i, \tau_i)_{i \in I}, F, W)$ *is an abstract economic system, and*
- $G : P \times X \to \mathcal{R}$*is an administrators' objective function.*

We are ready to define the center's optimum equilibrium.

Definition 12 *A parameter-choice pair* $(p^*, x^*) \in P \times X$ *for an administered abstract economic system* $(\mathcal{A}_S, G)$ *is called* **center's optimal equilibrium (COE)** *if,*

- (p^*, x^*) *is a SECERE of* $\mathcal{A}_S$*, and*
- $G(p^*, x^*) \geq G(p', x')$ *for any SECERE* $(p^{'}, x^{*'})$ *of* $\mathcal{A}_S$.

In Chapter 5, an equilibrium of a sales tax system with private ownership and a merchant government is an example of a COE, where the government cares only about its own consumption.

If one would like to work with an economic system instead of an abstract economic system, a definition of (relative) Pareto efficiency can be made with reference to a society only.

Definition 13 *Given a society* $S = (I, C, (B_i))$, *an* **outcome possibility set** C_f *is a subset of* C. *A* **consumption allocation**, $z \in C_f$, *is* C_f**-Pareto efficient** *if there exists no other consumption allocation,* $z' \in C_f$, *with* $z \notin B_i(z')$ *for all* $i \in I$ *and* $z' \in B_i(z)$ *for some* $i \in I$.

The above definition could be simplified if there is a Samuelson-Bergson welfare function as defined below.

Definition 14 *Given a society* $S = (I, C, (B_i))$ *a function* $U : C \to \mathcal{R}$ *is called a* **Samuelson-Bergson welfare function** *if,*

$$U(z) > U(z')$$

whenever $z' \notin B_i(z)$ *for all* $i \in I$ *and* $z \in B_i(z')$ *for some* $i \in I$.

In Chapters 6 and 7, a government with a Samuelson-Bergson (SB) objective function would be quite helpful to attain restricted Pareto optimal allocations.

2 Existence of Equilibrium

2.1 Notation and Preliminaries

Except for Chapter 8 which is self contained, we will adhere closely to the notation and definitions of Debreu (1959)- throughout the dissertation. The most commonly used ones will be reviewed here.

For a nonempty set S, we denote by $\preceq \subset S \times S$ a complete preordering (a transitive, reflexive and complete relation) on that set and call $\preceq$ a *preference preordering.* When S is a convex subset of a linear space, we call $\preceq$ *semi-strictly convex* whenever the following is satisfied: if s_1 and s_2 are two points of S, and if $t \in (0,1)$, then $s_2 \succ s_1$ implies $ts_2 + (1-t)s_1 \succ s_1$. When S is a topological space, we call $\preceq$ *continuous* if for every s' in S, the sets $\{s \in S \mid s \succeq s'\}$ and $\{s \in S \mid s \preceq s'\}$ are closed in S.

For points $x, y \in \mathcal{R}^l$, we use the following convention $x \geq y$ means $x_i \geq y_i$ for all i, $x > y$ means $x \geq y$ but $x \neq y$, $x \gg y$ means $x_i > y_i$ for all i.

Let $S \subset \mathcal{R}^m$ and T be a compact subset $\mathcal{R}^n$. A correspondence $\eta : S \rightarrow T$ will be called *upper semicontinuous* if its graph is closed in $S \times T$. One can contrast this with the definition of upper semicontinuity in e.g. Ichiichi (1983), to see that a closed correspondence is both upper semicontinuous (Ichiichi's sense) and compact valued (since T is compact). A correspondence $\eta : S \rightarrow T$ will be called *lower semicontinuous at* $x^0 \in S$ if: $x^q \rightarrow x^0$, $y^0 \in \eta(x^0)$ implies there is a sequence (y^q) such that $y^q \in \eta(x^q)$, $y^q \rightarrow y^0$. η is said to be *lower semicontinuous* if it is lower semicontinuous over its domain. η is called *continuous* if it is both upper and lower semicontinuous.

The Theorem of the Maximum, often attributed to Berge (1963) (see also Sertel (1971)) is frequently used in equilibrium analysis. The version here is adapted from Ichiichi (1983, p. 37).

Theorem 1 (Theorem of the Maximum) *Let X, Y be metric spaces with Y compact, let $f : X \times Y \rightarrow \mathcal{R}$ be a continuous function, and let $F : X \rightarrow Y$ be a non-empty-valued and continuous correspondence. Then*
(i) The function $\phi : X \rightarrow \mathcal{R}$, $x \mapsto max\{f(x,y) \mid y \in F(x)\}$ is continuous in X; and
(ii) The correspondence $\Phi : X \rightarrow Y$, $x \mapsto max\{y \in F(x) \mid f(x,y) = \phi(x)\}$ is upper semicontinuous.

For another important result, namely the Kakutani's Fixed Theorem, we refer the reader to Debreu (1959, p. 26).

Two of the classical approaches in proving existence of competitive equilibria are worth mentioning here. One of these is the so called Market Equilibrium approach or the so called Gale-Nikaido-Debreu (GND) Lemma, and the other is the Arrow and Debreu (1954) Lemma.

Regarding the Market Equilibrium approach, Gale (1955) was the first one to prove this lemma by using the KKM theorem, and to use it in showing the existence of a competitive equilibrium. Nikaido (1956) and Debreu (1956) have proven it independently by using the Kakutani's fixed point theorem. This approach works directly on the aggregate excess demand correspondence which satisfies Walras Law and which is upper semicontinuous. The question is whether we can intersect the image of such a correspondence with the non-positive orthant by changing prices. We refer the reader to Debreu (1959, p.82) for a version of this lemma.

Regarding the Arrow and Debreu (1954) approach which is based on existence of equilibrium in abstract economies, we see worthwhile to recall the result by Shafer and Sonnenschein (1975) on the existence of social equilibria in abstract economies.

An abstract economy and its social equilibria differ from a normal form game and its Cournot-Nash equilibria only in that, in the former, the players' action sets may depend on the other players' actions.

Definition 15 *(Shafer and Sonnenschein (1975)) An* **abstract economy** *consists of a list* $\mathcal{A} = \{I, (X_i, A_i, \beta_i)_{i \in I}\}$, *where* I *is a nonempty* **set of agents**, *for agent* i, $A_i : X \to X_i$ *is a nonempty valued* **constraint correspondence** *and* $\beta_i : X \to X_i$ *is a nonempty valued* **preferred correspondence** *where* $X = \prod X_j$. *A* **social equilibrium** *for* $\mathcal{A}$ *is an* $x^* \in X = \prod X_j$ *satisfying for each* i, $x_i^* \in A_i(x^*)$ *and* $\beta_i(x^*) \cap A_i(x^*) = \emptyset$.

The below statement and its proof are from Shafer and Sonnenschein (1975). It is a powerful result in that the preferences need not be transitive or complete, and the better than sets need not be convex. Its proof is also given here since it is short and it shows the use of the Theorem of the Maximum in a creative way.

Theorem 2 *Let* $\mathcal{A} = \{I, (X_i, A_i, \beta_i)_{i \in I}\}$ *be an abstract economy with* I *finite. Assume that for each* $i \in I$,

(a) X_i *is a non-empty compact and convex subset of* $\mathcal{R}^l$,

(b) A_i *is a continuous correspondence (closed graph and lower semicontinuous),*

(b′) for each $x \in X$, $A_i(x)$ *is non-empty and convex,*

(c) β_i *has open graph in* $X \times X_i$,

(c′) for each $x \in X$, $x_i \notin H(\beta_i(x))$, *where* $H(A)$ *denotes the convex hull of* A.

Then $\mathcal{A}$ *has an equilibrium.*

Proof. (Shafer and Sonnenschein (1975)) For each $i \in I$, let $U_i : X \times X_i \rightarrow \mathcal{R}_+$ be such that $U_i(y, x_i)$ is the Euclidean distance between (y, x_i) and the complement of the graph of β_i. Clearly, U_i is a continuous function and $U_i(y, x_i) > 0$ if and only if $x_i \in \beta_i(y)$. For each $i \in I$, define $F_i : X \rightarrow X_i$ by $F_i(y) = argmax_{x_i \in A_i(y)} U_i(y, x_i)$. Then, since U_i is a continuous function and A_i is a continuous, nonempty-, compact-valued correspondence, $F_i(y)$ is nonempty for each y and F_i has a closed graph by the Theorem of the Maximum. Define $G : X \rightarrow X$ by $G(y) = \prod_{i \in I} H(F_i(y))$. Then G is a nonempty and convex valued correspondence with closed graph (Nikaido (1968, Theorems 4.5 and 4.8)). By Kakutani's fixed point theorem there exists $x^* \in X$ such that $x^* \in G(x^*)$; that is, $x_i^* \in H(F_i(x^*))$ for all i. x^* is an equilibrium of $\mathcal{A}$ as verified below.

Since $F_i(x^*) \subset A_i(x^*)$ and $A_i(x^*)$ is convex, $H(F_i(x^*)) \subset A_i(x^*)$. Thus $x_i^* \in A_i(x^*)$. It remains to show that $\beta_i(x^*) \cap A_i(x^*) = \emptyset$. If $z_i \in \beta_i(x^*) \cap A_i(x^*)$, then $U_i(x^*, z_i) > 0$, so $U_i(x^*, y_i) > 0$ for all $y_i \in F_i(x^*)$. Thus $F_i(x^*) \subset \beta_i(x^*)$, which yields $x_i^* \in H(F_i(x^*)) \subset H(\beta_i(x^*))$, a contradiction to (c′). □

2.2 Abstract Economies and Existence of SECERE

Here, we use the result of Shafer and Sonnenschein (1975) in order to obtain an existence result for semicentralized resource equilibria. This approach has the advantage of allowing for possibly incomplete, non-transitive and nonconvex preferences with externalities.

The following two propositions provide sufficient conditions for the existence of a central parameter that will make the decentralized choices of agents lie in a given set X_f. The third proposition is a result on existence of SECERE. The three propositions are decreasing in power but increasing in practical usefulness.

Theorem 3 *Let* $I = \{1, .., m\}$ *be a finite set of agents and let* P, X_i $(i \in I)$ *be nonempty, convex and compact subsets of* $\mathcal{R}^l$, *and* $X_f \subset X = \prod_i X_i$. *Let* $A_i : P \times X \rightarrow X_i$ *and* A_P :

$P \times X \to P$ be nonempty-, convex- valued and continuous correspondences. Also for each $i \in I$ let $\beta_i : P \times X \to X_i$ and $\beta_P : P \times X \to P$ be correspondences with open graphs and with, $x_i \notin H(\beta_i(p,x))$ and $p \notin H(\beta_P(p,x))$ for all $(p,x) \in P \times X$. If

(*) for all $(p,x) \in P \times X$ such that $x \notin X_f$, $p \in A_P(p,x)$ and $x_i \in A_i(p,x)$ for all i, there is $\bar{p} \in A_P(p,x) \cap \beta_P(p,x)$,

then there exists $(p^*, x^*) \in P \times X$ such that,

(i) $p^* \in A_P(p^*, x^*)$,

(ii) $x_i^* \in A_i(p^*, x^*)$ for all i,

(iii) $A_i(p^*, x^*) \cap \beta_i(p^*, x^*) = \emptyset$ for all i,

(iv) $x^* \in X_f$.

Proof. Consider the abstract economy $\mathcal{A} = \{I', (X_i, A_i, \beta_i)_{i \in I'}\}$ where $I' = \{1, ..., m, m+1\}$, $X_{m+1} = P$, $A_{m+1} = A_P$, and $\beta_{m+1} = \beta_P$. $\mathcal{A}$ satisfies all the hypotheses of the Shafer-Sonnenschein (1975) theorem. Then there exists $(p^*, x^*) \in P \times X$ such that,

(1) $p^* \in A_P(p^*, x^*)$,

(2) $x_i^* \in A_i(p^*, x^*)$ for all i,

(3) $A_i(p^*, x^*) \cap \beta_i(p^*, x^*) = \emptyset$ for all i,

(4) $A_P(p^*, x^*) \cap \beta_P(p^*, x^*) = \emptyset$.

(i) follows from (1), (ii) from (2) and (iii) from (3). Suppose (iv) does not hold. Then (4) is a contradiction to (*). □

If β_P is representable by a function $V : P \times X \to \mathcal{R}$ then we can write the following.

Corollary 1 *Let I, P, X_i, A_i $(i \in I)$, A_P, X_f and β_i be as in the above theorem. Let the function $V : P \times X \to \mathcal{R}$ be such that,*

(1) V is continuous,

(2) for $p, p_1, p_2 \in P$ and $x \in X$ such that $V(p_1, x) > V(p,x)$ and $V(p_2, x) > V(p,x)$, we have $p \neq \lambda p_1 + (1-\lambda)p_2$ for all $\lambda \in [0,1]$,

(3) if $p \in P$, $x_i \in A_i(p,x)$ for all i and $x \notin X_f$, then for some $\bar{p} \in A_P(p,x)$,

$$V(\bar{p}, x) > V(p,x).$$

Then there exists $(p^*, x^*) \in P \times X$ *such that,*

(i) $x_i^* \in A_i(p^*, x^*)$ *for all* i, $p^* \in A_P(p^*, x^*)$,

(ii) $A_i(p^*, x^*) \cap \beta_i(p^*, x^*) = \emptyset$ *for all* i,

(iii) $x^* \in X_f$,

Proof. Let β_P be defined by

$$\beta_P(p, x) = \{p' \in P \mid V(p', x) > V(p, x)\}.$$

By *(1)*, β_P has open graph. *(2)* implies $p \notin H(\beta_P(p, x))$. *(3)* implies (*) in Theorem 3. Therefore, by Theorem 3, the desired result follows. □

Theorem 4 *Let* $\mathcal{A}_S = (I, P, (X_i, B_i, \tau_i)_{i \in I}, F, W)$ *be an abstract economic system where,* I *is a finite set of agents,* P, X_i $(i \in I)$, *are nonempty, convex and compact subsets of* $\mathcal{R}^l$, $\tau_i : P \times X \to X_i$ *are nonempty-, convex-, compact- valued and continuous constraint correspondences,* $B_i : P \times X \to X_i$ *are better choice correspondences with open graphs and with* $x_i \notin H(B_i(p, x))$. *Let* $\emptyset \neq T \subset I$, *and the functions* $v_i : P \times X \to \mathcal{R}$ *for* $i \in T$ *be such that,*

(1′) v_i *is continuous for all* i,

(2′) v_i *is concave on* P *for all* i,,

(3′) if $x_i \in \tau_i(p, x)$ *for all* $i \in I$, *then* $v_i(p, x) \leq 0$ *for all* $i \in T$,

(4′) if $x_i \in \tau_i(p, x)$ *for all* $i \in I$ *and* $F(x) \notin W$, *then for some* $\bar{p} \in P$,

$$\sum_{i \in T} v_i(\bar{p}, x) > 0.$$

Then there exists a semicentralized resource equilibrium of this abstract economic system.

Proof. Let $V(p, x) = \sum_{i \in T} v_i(p, x)$, and $A_P(p, x) = P$. Also let $X_f = \{x \in X \mid F(x) \in W\}$. Comparing (1′)-(4′) with (1)-(3) of corollary 1, we observe the following. Clearly (1′) implies (1). By (2′), V as a sum of concave functions is a concave function on P, so that (2) is satisfied. Also (3′) and (4′) together imply (3). Then, all the hypotheses of the corollary 1 are satisfied and the desired result follows. □

Remark. An immediate corollary to this last result is the existence of competitive equilibrium for a pure exchange economy. This was recognized in Shafer and Sonnenschein (1975) and a sketch of proof was provided there. In such a case, we could write P as the price simplex, $v_i(p, x) = px_i$, W would be $\mathcal{R}^l_-$, and $F(x) = \sum_i (x_i - \omega_i)$.

2.3 Market Equilibrium and Existence of SECERE

2.3.1 An Extension of the GND Lemma: Without Externalities

As a first step, we state and prove Lemma 1 as an immediate generalization of the Gale-Nikaido-Debreu Lemma.

Let $\mathcal{Z} = \mathcal{R}^l$ denote the commodity space and $\mathcal{P} = \mathcal{R}^s$ denote the price space.

Lemma 1 *Let $P \subset \mathcal{P}$ be nonempty, convex and compact. Let $Z \subset \mathcal{Z}$ be nonempty and compact. Let $W \subset \mathcal{Z}$ be nonempty. Let $V : P \times \mathcal{Z} \to \mathcal{R}$ be a continuous function. Let $\zeta : P \to Z$ be a nonempty-, compact-, convex- valued and upper semicontinuous correspondence.* [2] *If*

i. $V(\cdot, z)$ is quasiconcave on P, for all $z \in \mathcal{Z}$,

ii. $\bigcap_{p \in P}\{z \in \mathcal{Z} : V(p,z) \leq 0\} \subset W$,

iii. $V(p,q) \leq 0$, for all $p \in P$, all $q \in \zeta(p)$,.

then there exists a p^* in P such that

$$\zeta(p^*) \cap W \neq \emptyset$$

Any function that satisfies (i)-(ii) above will be called a **value function**. If (iii) holds for a correspondence ζ, it will be said to satisfy the **V-Walras Law**. The proof below follows closely Debreu (1959) , pp. 82-83.

Proof: Z can be replaced by any compact subset Z' of $\mathcal{Z}$ containing it; Z' is chosen to be convex. As Z is non-empty, so is Z'.

Given z in Z', let $\mu(z)$ be the set of p in P which maximize $V(p,z)$ on P, i.e. $\mu(z) = \{p \in P \mid V(p,z) = \max_{\pi \in P} V(\pi, z)\}$. Since P is nonempty, compact, and since V is continuous, $\mu(z)$ is nonempty and compact, and the correspondence μ from Z' to P is upper semicontinuous on Z' by Berge's theorem of the maximum.

Since P is convex and $V(., z)$ is quasi-concave on P for all $z \in Z'$, $\mu(z)$ is also convex. Consider now the correspondence φ from $P \times Z'$ into itself defined by $\varphi(p,z) = \mu(z) \times \zeta(p)$. The

[2] Recall from section 2.1 that the definition of upper semicontinuity in Debreu (1959, p.17) will be used here. Ichiichi (1983) calls such a correspondence *closed*.

set $P \times Z' \subset \mathcal{P} \times \mathcal{Z}$ is nonempty, compact, and convex for P and Z' are. The correspondence φ is upper semicontinuous for μ and ζ are.

Finally, for all (p, z) in $P \times Z'$ the set $\varphi(p, z)$ is nonempty, convex and compact, for $\mu(z)$ and $\zeta(p)$ are. Therefore all of the conditions of Kakutani's fixed point theorem are satisfied, and φ has a fixed point (p^*, z^*). Thus $(p^*, z^*) \in \mu(z^*) \times \zeta(p^*)$ which is equivalent to

(1) $p^* \in \mu(z^*)$

(2) $z^* \in \zeta(p^*)$

(1) implies that for every p in P, one has $V(p, z^*) \leq V(p^*, z^*)$. (2) implies that $V(p^*, z^*) \leq 0$. Hence for every p in P, one has $V(p, z^*) \leq 0$. By property (ii) of V, this implies $z^* \in W$. This with $z^* \in \zeta(p^*)$ proves that p^* has the desired property. □

Remark: The GND lemma follows as a corollary if $l = s$, $P = \Delta^{l-1}$, $V(p, z) = p.z$, and $W = \mathcal{R}^l_-$.

In a classical Arrow-Debreu model, there is no difficulty in finding a value function where the aggregate demand correspondence satisfies V-Walras law (iii). $V(p, z) = p.z$ always qualifies as a value function since each consumer's choice has to come from a classical budget set.

2.3.2 An Extension of the GND Lemma: With Externalities

Below is a further generalization of lemma 1 to allow for externalities. It also emphasizes that the *equilibrium price-choice set* is compact. The proof is similar to that of **Lemma 1**.

Let $\mathcal{X} = \mathcal{R}^n$ denote the choice space, $\mathcal{W} = \mathcal{R}^l$ denote the resource space and $\mathcal{P} = \mathcal{R}^s$ denote the price space.

Lemma 2 *Let $P \subset \mathcal{P}$ be nonempty, convex and compact. Let $X \subset \mathcal{X}$ be nonempty, convex and compact. Let $W \subset \mathcal{W}$ be nonempty and closed. Let $V : P \times \mathcal{X} \to \mathcal{R}$ be a continuous function. Let $\zeta : P \times X \to X$ be a nonempty-, convex- valued and upper semicontinuous correspondence. Let $f : \mathcal{X} \to \mathcal{W}$ be a continuous function. If*

i. $V(\cdot, x)$ is quasiconcave on P for all $x \in \mathcal{X}$,

ii. $f(x) \notin W$ implies $V(p, x) > 0$ for some $p \in P$, *iii.* $V(p,x) \leq 0,$ *for all* $p \in P$ *andfor* $x \in \zeta(p,x)$,

then the set

$$\{(p,x) \in P \times X \mid x \in \zeta(p,x), f(x) \in W\}$$

is nonempty and compact.

Proof: Given x in $\mathcal{X}$, let $\mu(x)$ be the set of p in P which maximize $V(p,x)$ on P, i.e. $\mu(x) = \{p \in P \mid V(p,x) = \max_{\pi \in P} V(\pi,x)\}$. Since P is nonempty, compact, and since V is continuous, $\mu(x)$ is nonempty and compact, and the correspondence μ from X to P is upper semicontinuous on X by Berge's theorem of the maximum.

Since P is convex and $V(.,z)$ is quasi-concave on P for all $z \in \mathcal{Z}$, $\mu(x)$ is also convex. Consider now the correspondence φ from $P \times X$ into itself defined by $\varphi(p,x) = \mu(x) \times \zeta(p,x)$. The set $P \times X \subset \mathcal{P} \times \mathcal{X}$ is nonempty, compact, and convex for P and X are. The correspondence φ is upper semicontinuous for μ and ζ are.

Finally, for all (p,x) in $P \times X$ the set $\varphi(p,z)$ is nonempty, convex and compact, for $\mu(x)$ and $\zeta(p,x)$ are so. Therefore all of the conditions of Kakutani's fixed point theorem are satisfied, and φ has a fixed point (p^*,x^*). Thus $(p^*,x^*) \in \mu(x^*) \times \zeta(p^*,x^*)$ which is equivalent to

(1) $p^* \in \mu(x^*)$

(2) $x^* \in \zeta(p^*,x^*)$

(1) implies that for every p in P, one has $V(p,x^*) \leq V(p^*,x^*)$. (2) and (iii) imply that $V(p^*,x^*) \leq 0$. Hence for every p in P, one has $V(p,x^*) \leq 0$. By (ii) this implies $f(x^*) \in W$. This with $x^* \in \zeta(p^*,x^*)$ proves that (p^*,x^*) is in the equilibrium set. Hence the equilibrium set is nonempty.

For compactness, observe that ζ has compact graph. The set of points $(p,x_1,x_2) \in P \times X \times X$, such that $f(x_2)$ is in W, is compact since f is continuous. Likewise, the set with the property $x_1 = x_2$ is compact. Hence the intersection of all three sets is also compact. But then the projection of this intersection on the domain of ζ, which is nothing but the equilibrium set, is compact as well. □

2.4 Existence of Semicentralized Resource Equilibria

In this section, we consider abstract economic systems $\mathcal{A}_S = (I, P, (X_i, B_i, \tau_i)_{i \in I}, F, W)$, economic systems $\mathcal{E}_S = (I, C, (B_i)_{i \in I}, P, (X_i, \tau_i)_{i \in I}, c, F, W)$ and provide some existence results for their equilibria based on the extensions of the GND Lemma provided in Section 2.3.

As in Section 2.2, we continue to assume that the set of agents as well as the dimensionality of the choice sets are finite. Therefore we let C, the outcome set, P, the parameter set, X_i ($i \in I$), the choice sets, W, the resource set, be nonempty subsets of Euclidean spaces with possibly different dimensionalities. In addition, we introduce the following assumptions for later reference.

(Z) The outcome set C is closed and convex as a subset of $\mathcal{R}^m$.

(U) For each $i \in I$, the better outcomes correspondence $B_i : C \to C$ is derived from a utility function, $u_i : C \to \mathcal{R}$, which is continuous and quasi-concave.

(P) P is compact and convex as a subset of $\mathcal{R}^s$.

(X) For each $i \in I$, the choice set X_i is compact and convex as a subset of $\mathcal{R}^k$, $i \in I$.

(O) The outcome function $c : P \times X \to C$ is affine on $P \times X$.

(R) For each $i \in I$, the constraint correspondence τ_i is continuous and nonempty-, convex-valued on $P \times X$.

(W) The resource set W is nonempty and closed as a subset of $\mathcal{R}^l$.

(F) The resource implications function $F : \mathcal{R}^n \to \mathcal{R}^l$ is continuous, where n is k times the cardinality of the set of agents I.

(RE) There exist $\emptyset \neq N \subset I$ and, for each $i \in N$, a continuous function $e_i : P \times X \to \mathcal{R}$ that is concave on P, such that $e_i(p, x) \leq 0$ for all $x \in X$ with $x_i \in \tau_i(p, x)$, and if $x \in X$ is such that $F(x) \notin W$, then $\sum_{i \in N} e_i(\bar{p}, x) > 0$ for some $\bar{p} \in P$.

(V) In an abstract economic system, for each $i \in I$, the better choices correspondence B_i : $P \times X \to X_i$ is derived from a utility function $v_i : P \times X \times X_i \to \mathcal{R}$ as given by the formula, $B_i(p, x) = \{y_i \in X_i \mid v_i(p, x, y_i) > v_i(p, x, x_i)\}$, where v_i is continuous on its domain and quasi-concave on X_i.

(G) The administrators' objective function:$G : P \times X \to \mathcal{R}$ is continuous.

(GP) $G : P \times X \to \mathcal{R}$ is continuous and has the Samuelson-Bergson property.

First we prove a statement which says that the set of all semicentralized resource equilibria of every abstract economic system is nonempty and compact.

Theorem 5 *Under P, X, R, V, W, F, RE every abstract economic system has a nonempty and compact set of SECERE.*

Proof: In view of (V), the best response correspondence $\zeta_i : P \times X \to X_i$ for individual i can be written as,

$$\zeta_i(p, x) = argmax_{y \in \tau_i(p,x)} v_i(p, x, y).$$

By (R) and (X), τ_i is nonempty- and compact-valued and continuous. Hence by the theorem of the maximum, ζ_i is nonempty-, compact-valued and upper semicontinuous. Since v_i is quasi-concave on X_i and τ_i is convex-valued on $P \times X$, ζ_i is also convex-valued. Let $\zeta : P \times X \to X$ be defined by,

$$\zeta(p, x) = \prod_{i \in I} \zeta_i(p, x)$$

Then clearly ζ is nonempty-, convex-, compact- valued and upper semicontinuous.

By using the functions e_i in hypothesis (RE), construct the function $V : P \times \mathcal{R}^n \to \mathcal{R}$ as

$$V(p, x) = \sum_{i \in N} e_i(p, x),$$

where n is k times the cardinality of I. Clearly, V is continuous, concave on P as a sum of concave functions and hence quasi-concave on P. Again by (RE), $F(x) \notin W$ implies $V(p, x) > 0$ for some $p \in P$, and $V(p, x) \leq 0$, for $x \in \zeta(p, x)$, $p \in P$. Therefore all the hypotheses of Lemma 2 are satisfied and the set of SECERE is nonempty and compact. □

Next we provide a result on the existence of equilibria of economic systems and compactness of the set of equilibria.

Theorem 6 *Under P, Z, U, X, R, O, W, F, RE, every economic system has a nonempty and compact set of SECERE.*

Proof: From the utility function $u_i : C \to \mathcal{R}$, we first obtain the utility function $v_i : P \times X \times X_i \to \mathcal{R}$ by,

$$v_i(p, x, y) = u_i(c(p, x_{-i}, y)).$$

Now, since c is affine on its domain, and by (U), v_i is continuous and quasi-concave on X_i for each $i \in I$. Therefore the abstract economic system constructed by preference correspondences derived from v_i has a nonempty and compact set of SECERE. It is immediate to check that this set is exactly the set of SECERE of our economic system (See section 1.3 for this connection). □

2.5 Existence of Center's Optimal Equilibria

Theorem 7 *Under P, X, V, R, RE, W, F, G, every administered abstract economic system has a nonempty and compact set of COE.*

Proof: By Theorem 5, we know that the set of SECERE is compact. Since G is continuous on $P \times X$ by (G), it attains a maximum on the set of SECERE of the underlying abstract economic system. A maximizing SECERE, (p^*, x^*), is an administrators' optimum by definition. □

Part III

Applications

3 Walrasian Equilibrium of a Private Ownership Economy

For the case of a classical private goods economy, the first theorem of welfare economics tells us that, a Walrasian allocation is always Pareto efficient[3]. Similarly for a convex economy with public goods, it has been shown that the Lindahl allocations are always in the core[4]. If Pareto and core allocations are desirable, we then know that Walrasian and Lindahl allocations are desirable too. Therefore the existence of these equilibria for a general class of preferences and technologies bears importance.

Competitive equilibrium and Lindahl equilibrium have something in common. In both, the agents take certain price parameters as given and solve their own choice problems in their constraint sets, without caring about the rest of the economy, however their choices turn out to be feasible in terms of the resources available to the overall economy. Both are semicentralized because the prices are to be determined and the corresponding constraint set are to be enforced by a third party.

In this chapter and the next, we demonstrate that both of these equilibrium concepts can be obtained as SECEREs of suitably defined economic systems. Then we obtain existence theorems based on some results from Chapter 2.

Here we consider a private goods economy as considered by Arrow and Debreu (1954), and show that it fits to the framework of Chapter 1.

Suppose that there are l commodities, the commodity space therefore is taken as $\mathcal{R}^l$. There are m consumers and n firms. Each consumer has a consumption set and preferences thereover. Each firm has a technology, or set of possible production plans. Also given is the total quantity available of each commodity.

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[3] For example Debreu (1959)

[4] For example Foley (1970b)

Definition 16 (Debreu) *An* **economy** $E = ((Z_i, \preceq_i), (Y_j), \omega)$ *consists of: for each consumer i a non-empty consumption set $Z_i \subset \mathcal{R}^l$ completely preordered by $\preceq_i$ (i=1,..,m); for each firm j a non-empty production set $Y_j \subset \mathcal{R}^l$ (j=1,..,n); and a total resource vector $\omega \in \mathcal{R}^l_+$.*

A **private ownership economy** $\mathcal{E}_p$ *is:*

(i) an economy $((Z_i, \preceq_i), (Y_j), \omega)$;

(ii) for each i, a point $\omega_i \in Z_i$ such that $\sum_{i=1}^m \omega_i = \omega$, the endowment owned by each consumer;

(iii) for each pair (i,j), a non-negative number θ_{ij} such that $\sum_{i=1}^m \theta_{ij} = 1$ for every j, the profit shares of each consumer.

A **Walrasian equilibrium** *of a private ownership economy $\mathcal{E}_p$ is an $(m+n)$- tuple $((z_i^*), (y_j^*))$ of points of $\mathcal{R}^l$ and $p^* \in \mathcal{R}^l_+ \backslash \{0\}$ such that:*

(i) z_i^* is a greatest element of the budget set

$$\{z \in Z_i \mid p^* \cdot (z - \omega_i) \leq \sum_{j=1}^{n} \theta_{ij} p^* \cdot y_j^*\}$$

for $\preceq_i$ according to every i;

(ii) y_j^* maximizes $p^* \cdot y$ on the production set Y_j, for every j;

(iii) $\sum z_i^* - \sum y_j^* \leq \omega$.

Theorem 8 *The private ownership economy*
$\mathcal{E}_p = ((Z_i, \preceq_i), (Y_j), (\omega_i), (\theta_{ij}))$ *has a Walrasian equilibrium if,*
for every i,

(a) Z_i is compact and convex,

(b.1) for every z_i' in Z_i, the sets $\{z_i \in Z_i \mid z_i \succeq z_i'\}$ and $\{z_i \in Z_i \mid z_i \preceq z_i'\}$ are closed in Z_i (continuity),

(b.2) if z_i^1 and z_i^2 are two points of Z_i, and if $t \in (0,1)$, then $z_i^2 \succ z_i^1$ implies $tz_i^2 + (1-t)z_i^1 \succ_i z_i^1$ (semi-strict convexity),

(c.) there is z_i^o in Z_i, such that $z_i^o \ll \omega_i$;

for every j,

(d.1) Y_j is compact and convex for all j,

(d.2) $0 \in Y_j$.

Proof. Consider the corresponding abstract economic system $\mathcal{A}_S = (I, P, (X_i, B_i, \tau_i)_{i\in I}, F, W)$. Here,

- $I = M \cap N$, where $M = \{1, ..m\}$, and $N = \{m+1, ..m+n\}$,
- $P = \Delta^{l-1}$,
- $X_i = Z_i$ *for* $i \in M$ and $X_i = Y_i$ *for* $i \in N$,
- $B_i(p, x) = \{\bar{x} \in X_i \mid x_i \preceq \bar{x}\}$ for $i \in M$ and $B_i(p, x) = \{\bar{x} \in X_i \mid p \cdot x_i < p \cdot \bar{x}\}$ for $i \in N$,
- $\tau_i(p, x) = \{z_i \in X_i \mid p(z_i - \omega_i) \leq max\{\sum_{j\in N} \theta_{ij} p x_j, 0\}\}$ for $i \in M$ and $\tau_j(p, x) = \{z_j \in X_i \mid p \cdot z_j \geq 0\}$ for $j \in N$,
- $F(x) = \sum_{i\in M}(x_i - \omega_i) - \sum_{i\in N} x_i$,
- $W = -\mathcal{R}^l_+$.

By construction and by (d.2), a SECERE of the above abstract economic system is a Walrasian equilibrium of the private ownership economy.

We are going to use Theorem 4. Clearly, I is a finite set of agents and P is nonempty, convex and compact. Under (a) and (d.1) X_i $(i \in I)$ are nonempty, convex and compact subsets of $\mathcal{R}^l$. Under (c) and (d2) $\tau_i : P \times X \rightarrow X_i$ is a nonempty-, convex-, compact- valued and continuous constraint correspondence for all $i \in I$. Under (b.1) and (b.2) $B_i : P \times X \rightarrow X_i$ has open graph and $x_i \notin H(B_i(p, x))$ for all $i \in I$. For each $i \in M \subset I$, define the functions $v_i : P \times X \rightarrow \mathcal{R}$ by,

$$v_i(p, x) = p(x_i - \omega_i) - \sum_{j\in N} \theta_{ij} p x_j.$$

We can easily verify that

(1′) v_i is continuous for all $i \in M$,

(2′) v_i is concave on P for all $i \in M$,

(3′) if $x_i \in \tau_i(p, x)$ for all $i \in I$, then $v_i(p, x) \leq 0$ for all $i \in M$,

(4′) if $p \in P$, $x_i \in \tau_i(p, x)$ for all $i \in I$ and $F(x) \notin W$, then for some $\bar{p} \in P$,

$$\sum_{i\in M} v_i(\bar{p}, x) > 0$$

are satisfied. (1′) and (2′) are obvious. (3′) follows directly from the construction of consumers' and firms' constraint sets (where negative profits are not allowed). To see (4′), suppose $F(x) \notin$

W. Then for some coordinate $k \in \{1, ..., l\}$, we have

$$\sum_{i \in M} (x_{ik} - \omega_{ik}) - \sum_{j \in N} x_{jk} > 0.$$

Choose $\bar{p}$ equal to one for the k th coordinate and zero for other coordinates. Then $\sum_{i \in M} v_i(\bar{p}, x) > 0$.

All the hypotheses of Theorem 4 are satisfied. Hence there exists a SECERE of the abstract economic system A_S, which is a Walrasian equilibrium of the private ownership economy. □

The above proof is very close in spirit to the original work of Arrow and Debreu (1954). Just like consumers, the firms are viewed as agents with preferences monotone in profits. The actions of the firms affect the budget sets of the consumers as constraint set externalities. The slight difference here is that we need to limit the firms' choices to the no-loss production plans in order to maintain (3′).

Theorem 8 can immediately be modified to allow for general preferences that are not necessarily complete or transitive. The proof would be an imitation of the above. The sketch of this application was given by Shafer and Sonnenschein (1975) in the context of a pure exchange economy.

4 Lindahl Equilibrium of a Public Goods Economy

We will, for simplicity, consider an economy with one public good and one private good. The commodity space, then, is $\mathcal{R}^2$, where the first coordinate stands for the private good and the second for the public good. There are m consumers and one firm. Each consumer has a consumption set and preferences over it. Also given, is the consumers' endowments of the private good. The endowments of public goods are zero. The firm has a technology, or set of possible production plans, which allows producing the public good from the private good.

A consumer will face a personalized price vector for the two commodities. The price of the public good may differ from consumer to consumer. All consumers, however, will face the same price for the private good.

The firms will also face a price for each commodity. For the public good, this will be the sum of the personalized prices over the consumers. For the private good, this will be the same price as all the consumers face.

A Lindahl equilibrium with free disposal for this economy is a price-allocation pair where the firm maximizes its profits, the consumers maximize their preferences in their *budget sets* and demands do not exceed supplies in each commodity.

The below definition makes these ideas precise. It is inspired from the framework in Foley (1970b), where there are many public goods, many private goods and the technology is a convex cone which leads to zero profits in equilibrium so that the ownership issue of the firm can be bypassed. Here, we have one public good and one private good, but the firm is allowed to make nonnegative profits and this profit is distributed to consumers.

In what follows, subscripts denote the consumers or the firm and superscripts denote commodities.

Definition 17 *A* **public good economy with private ownership** $\mathcal{E}_{pp} = ((Z_i, \preceq_i, \omega_i, \theta_i), Y)$ *is,*

(i) for each consumer a non-empty consumption set $Z_i \subset \mathcal{R}^2_+$ *completely preordered by* $\preceq_i$ *(i=1,..,m), an endowment point* $\omega_i \in Z_i$ *such that* $\omega_i^2 = 0$ *and a profit share* $\theta_i > 0$ *such that* $\sum_{i=1}^m \theta_i = 1$,

(i) a non-empty production set $Y \subset \mathcal{R} \times \mathcal{R}_+$.

A **Lindahl equilibrium** *of a public good economy with private ownership* $\mathcal{E}_{pp}$ *is an* $(m+1)$*-tuple* $((z_i),(y))$ *of points of* $\mathcal{R}^2$ *and an* $(m+1)$*- tuple* $(p_1,...,p_m,p_y)$ *of points of* $\mathcal{R}^2_+\backslash\{0\}$ *such that:*

(i) z_i *is a greatest element of the budget set*

$$\{z \in Z_i \mid p_i \cdot (z - \omega_i) \leq \theta_i p_y \cdot y, \;\; z^1 \leq \omega_i^1\}$$

for $\preceq_i$ *for every* i*;*

(ii) y *maximizes* $p_y \cdot \bar{y}$ *on the production set* Y*,*

(iii) $z_i^2 = y^2$ *(for the public good) for all consumers* i*, and* $\sum(z_i^1 - \omega_i^1) = y^1$ *(for the private good).*

(iv) $p_y^2 = \sum_{i=1}^m p_i^2$ *(for the public good), and* $p_y^1 = p_i^1$ *(for the private good) for all consumers* i*.*

As in Foley's (1970b) theorem, the below result needs monotonicity of preferences. This is because of the free disposal in our definition of equilibrium. On the production side, we allow for any kind of convex and compact technology. The compactness hypotheses are put here for simplicity. They can be dropped by imposing some other restrictions on consumption and production sets[5].

Theorem 9 *The public good economy with private ownership*
$\mathcal{E}_{pp} = ((Z_i, \preceq_i, \omega_i, \theta_i), Y)$ *has a Lindahl equilibrium if,*
for every i*,*

(a) Z_i *is compact and convex,*

(b.1) for every z_i' *in* Z_i*, the sets* $\{z_i \in Z_i \mid z_i \succeq z_i'\}$ *and* $\{z_i \in Z_i \mid z_i \preceq z_i'\}$ *are closed in* Z_i *(continuity of preferences),*

(b.2) for every $z_i' \in Z_i$*, the set* $\{z_i \in Z_i \mid z_i \succeq z_i'\}$ *is convex (convexity of preferences),*

(b.3) for every $z_i, z_i' \in Z_i$ *with* $z_i' > z_i$*, we have* $z_i' \succ z_i$ *(monotonicity of preferences),*

for every j*,*

(c.1) Y *is compact and convex,*

(c.2) $0 \in Y$*.*

[5]See for example Debreu (1962).

Proof. Consider the corresponding abstract economic system $\mathcal{A}_S = (I, P, (X_i, B_i, \tau_i)_{i\in I}, F, W)$. Here,

- $I = \{M, N\}$, where $M = \{1, ..m\}$, and $N = \{m+1\}$,
- $P = \{(p_1, ...p_m, p_{m+1}) \in \mathcal{R}_+^{2m+2} \mid \sum_{i\in I} p_i^j = 1,\ p_i^1 = p_j^1\ for i, j \in I,\ \sum_{i\in M} p_i^2 = p_{m+1}^2\}$,
- $X_i = Z_i\ for\ i \in$ and $X_{m+1} = Y$,
- $B_i(p, x) = \{\bar{x} \in X_i \mid x_i \preceq \bar{x}\}$ for $i \in M$ and $B_i(p, x) = \{\bar{x} \in X_i \mid p_i x_i < p_i \bar{x}\}$ for $i \in N$,
- $\tau_i(p, x) = \{z \in X_i \mid p_i(z - \omega_i) \leq max\{\ \theta_i p_{m+1} x_{m+1}, 0\},\quad z^1 \leq \omega_i^1\}$ for $i \in M$ and $\tau_i(p, x) = \{z \in X_i \mid p_i z \geq 0\}$ for $i \in N$,
- $F(x) = (F_1(x), F_2(x))$, where $F_1(x) = \sum_{i\in M}(x_i^1 - \omega_i^1) - x_{m+1}^1$, and $F_2(x) = max_{i\in M}\{x_i - x_{m+1}\}$
- $W = -\mathcal{R}_+^2$.

By construction, a SECERE of the given abstract economic system is a quasi-Lindahl equilibrium (i.e. a Lindahl equilibrium with equalities replaced by less or equal signs in condition (iii)) of the public good economy with private ownership.

Imitating the proof of Theorem 8, we are going to use Theorem 4. To that end let, for $i \in M \subset I$, the functions $v_i : P \times X \to \mathcal{R}$ be defined by,

$$v_i(p, x) = p_i(x_i - \omega_i) - \theta_i p_{m+1} x_{m+1}.$$

The conditions (1′), (2′) and (3′) are satisfied as in the proof of Theorem 8. The only difference is in showing why

(4′) if $p \in P$, $x_i \in \tau_i(p, x)$ for all $i \in I$ and $F(x) \notin W$, then for some $\bar{p} \in P$,

$$\sum_{i\in M} v_i(\bar{p}, x) > 0$$

holds. To that end, suppose $F(x) \notin W$. Then we have either

$$F_1(x) = \sum_{i\in M}(x_i^1 - \omega_i^1) - x_{m+1}^1 > 0$$

or

$$F_2(x) = max_{i\in M}\{x_i^2 - x_{m+1}^2\} > 0$$

or both. Suppose that $F_1(x) > 0$ holds. Then choose $\bar{p} \in P$ such that $\bar{p}_i^2$ is zero for all $i \in I$. Then $\bar{p}_i^1 = q > 0$ for all $i \in I$ and

$$\sum v_i(\bar{p}, x) = \sum q(x_i^1 - \omega_i^1) - \theta_i q x_{m+1}^1 > 0.$$

Now suppose that $F_2(x) > 0$ holds. Then choose $\bar{p} \in P$ such that $\bar{p}_a^2 = 1$ for the index $a \in M$ that maximizes $x_i^2 - x_{m+1}^2$. Then,

$$\sum v_i(\bar{p}, x) = x_a^2 - x_{m+1}^2 > 0.$$

All the hypotheses of Theorem 4 are satisfied. Hence there exists a SECERE $(p^*, z^*, y^*) \in P \times \prod_M Z_i \times Y$ of the abstract economic system A_S, which is a quasi-Lindahl equilibrium of the public good economy with private ownership.

Now, from the monotonicity assumption (b.3), we know that the budget constraints will hold as equality for the consumers. Adding the budget equalities over the consumers and using the properties of the price set, we obtain

$$p_y^{1*} \sum_{i \in M} (z_i^{1*} - \omega_i) + \sum_{i \in M} p_i^{2*} z_i^{2*} = p_y^{1*} y^{1*} + \sum_{i \in M} p_i^{2*} y^{2*}$$

which excludes the possibilities of excess supply in either of the commodities. Therefore (iii) is also satisfied and every SECERE of the abstract economic system is also a Lindahl equilibrium of the public good economy with private ownership. □

The above proof is substantially different in spirit from that of Foley (1970b). Foley extends the dimensionality of the commodity space by considering each consumer's public good as a separate commodity. Then he uses a result of Debreu (1962) for this new economy. In our proof we work directly on the original commodity space, by viewing the given construct as a semicentralized resource equilibrium of an abstract economic system introduced in Chapter 1.

5 Generalized Price Systems and Sales Taxes

One of the important problems of general equilibrium theory has been demonstrating the existence of a competitive equilibrium for an economic model under general assumptions. Usually, the price system is assumed to be *linear, complete and anonymous.* [6] Here, linearity means that the value function, which gives a real number for each point in the space of possible trades, is linear. Completeness means that for each commodity there is a nonnegative real number, a *price* for that commodity, so that the value function can be written as an inner product of a given price vector and a given trade, consumption or production plan. Anonymity means that each producer and consumer faces the same value function.

The aim of this chapter is to relax these three requirements in significant ways and still obtain an existence result for an equilibrium concept that has the same spirit as the competitive equilibrium.

One motivating question is the existence of general equilibrium under a sales tax regime. In presence of sales taxes, each consumer and each firm face two sets of prices, one for purchases and another for sales. This means, in a two commodity world, that the budget lines and iso-profit lines are kinked. A linear value function cannot provide such a situation. Hence this case is not handled by the well known results in general equilibrium theory[7] Moreover, as will be clearer later, approaches of Foley (1970) or Hahn (1971) type, that extend the dimensionality of the commodity space and then work with linear price systems, are also not applicable here.

With the sales tax example in mind, an existence result applicable to more general cases is of interest. Such a general economic system and the corresponding equilibrium concept are defined in Section 2, where the main existence result is also given. The proof of the theorem, as well as a generalization of the Gale-Debreu-Nikaido lemma, are presented in Section 3. Section 4 formulates the sales tax example for an economy with a selfish and powerful government and shows how the result in section 2 can be applied to this specific case.

[6] Arrow and Debreu (1954) introduce such a model and provide sufficient conditions for existence of equilibrium

[7] Two exceptions to our knowledge are Mantel (1975) in the context of sales taxes and Vega-Redondo (1987) in the context of public goods. Both approaches however are different from ours.

5.1 Generalized Price Systems and Existence of Equilibria

The definition of an *economy* here follows Debreu (1959, p.75). As such, it is a list of the consumers' consumption sets, their preferences, the available technologies, and *total* resources available. For Sertel (1982), this is a *presystem.* Among all possible resource allocating mechanisms, the *price system* for a private ownership economy has been the focus of attention in the literature. Several other possible ways of allocating resources have been dealt with by Drèze (1975) , Sertel (1982) and others (MC pricing equilibrium, AC pricing equilibrium, etc.). It would be satisfactory to have an existence result applicable to all these models. Here only semi-centralized allocation systems with private ownership will be considered. That is, the economic agents will be parameter takers, and they will solve their own maximization problem in choosing trade plans, without caring about the rest of the economy. The question then is the existence of a parameter that will make the sum of all demands less than or equal to the resources available in the economy. To that end, the following formalization is used.

The number of commodities being l, the commodity space is $\mathcal{R}^l$. There are m consumers and n firms. Each consumer has a consumption set, and preferences thereover. Each firm has a technology, or a set of possible production plans. Also given is the total quantity available of each commodity.

Definition 18 (Debreu) *An* **economy** $E = ((X_i, \preceq_i), (Y_j), \omega)$ *has, for each consumer, a non-empty consumption set* $X_i \subset \mathcal{R}^l$ *completely preordered by* $\preceq_i$ *(i=1,..,m), for each firm, a non-empty production set* $Y_j \subset \mathcal{R}^l$ *(j=1,..,m), and a total resource vector* $\omega \in \mathcal{R}^l_+$.

Ownership structure on commodities or firms is not included in this definition, we will specify that later when we define a *generalized price system with private ownership.*

Given a consumer's consumption set, and preferences on that set, an economic system will impose some restrictions upon the freedom to choose a consumption bundle. The consumer will therefore have only a subset of commodities in the consumption set available for use. The prevailing *trade system* will determine the nature of this set. In the case of a competitive private ownership economy, for instance, this will be a budget set, determined when *prices* and *income* are given. In a system where trade is forbidden and commodities are allocated from the center, this set will consist of the points less than or equal to the endowment point if free disposal is admitted. Similarly, quantity rationing and positive sales taxes impose their own restrictions upon the consumption possibilities of a consumer.

In each case, there are parameters to be set by a central authority, such as prices announced by a Walrasian auctioneer, quotas and sales taxes administered by the government, etc.. We allow for s such parameters, each a real number. In the familiar case where each commodity has a price and there are no further restrictions, we have $s = l$. But $s > l$ and $s < l$ are both possible, depending on the nature of the system. We therefore take the set of possible parameter values that can be chosen by the central agent as a nonempty set $P \subset \mathcal{R}^s$.

We will consider here only systems with private ownership, but possibly with distorted or generalized price systems. In other words, we will take the endowments and profit shares of the consumers as given and fixed. We first introduce a function that gives the *value* of net trades of a consumer.

Definition 19 *A function $e : P \times \mathcal{R}^l \to \mathcal{R}$ is a* **billing function** *if it is continuous and, for all $p \in P$,*

(i) the lower cost set $\gamma(p, w) = \{x \in \mathcal{R}^l \mid e(p, x) \leq w\}$ is convex ($w \in \mathcal{R}$),

(ii) $e(p, 0) = 0$,

(iii) if $x \gg y$ then $e(p, x) > e(p, y)$. [8]

The convexity requirement (i) rules out quantity discounts. In Figure 1 we see a trade possibility set for a consumer with an endowment $\omega \in \mathcal{R}^2$ and an additional income $w \in \mathcal{R}$, induced by a billing function and a given parameter vector $p \in \mathcal{R}^4$. Here, there are sales taxes on the commodities, hence the buying and selling prices are different for each commodity. Therefore there are four different relative prices in four quadrants. Only three of the relative prices are seen in the figure, *i.e.* the case where both of the goods are purchased, and the two cases where one of the goods is purchased and the other sold.

Similarly for the firms, a function that gives the *value* of a production plan is introduced below.

Definition 20 *A function $\pi : P \times \mathcal{R}^l \to \mathcal{R}$ is a* **profit function** *if it is continuous and, for all $p \in P$,*

[8] For vectors: $x, y \in \mathcal{R}^l$, $x \geq y$ means $x_i \geq y_i$ for all i, $x > y$ means $x \geq y$ but $x \neq y$, $x \gg y$ means $x_i > y_i$ for all i.

(i) the upper profit set $H(p,w) = \{x \in \mathcal{R}^l \mid \pi(p,x) \geq w\}$ *is convex for all* $w \in \mathcal{R}$,

(ii) $\pi(p,0) = 0$,

(iii) if $x \gg y$ *then* $\pi(p,x) > \pi(p,y)$.

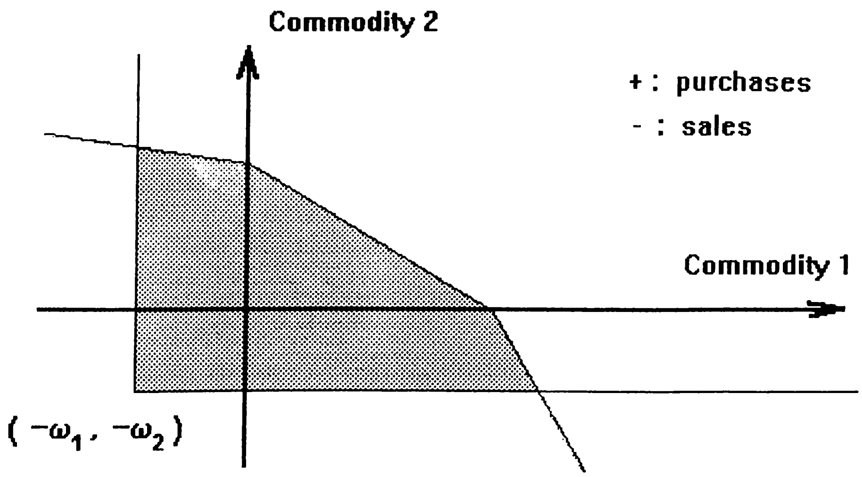

Figure 1. The set of possible trades for given buying and selling prices, income and endowments.

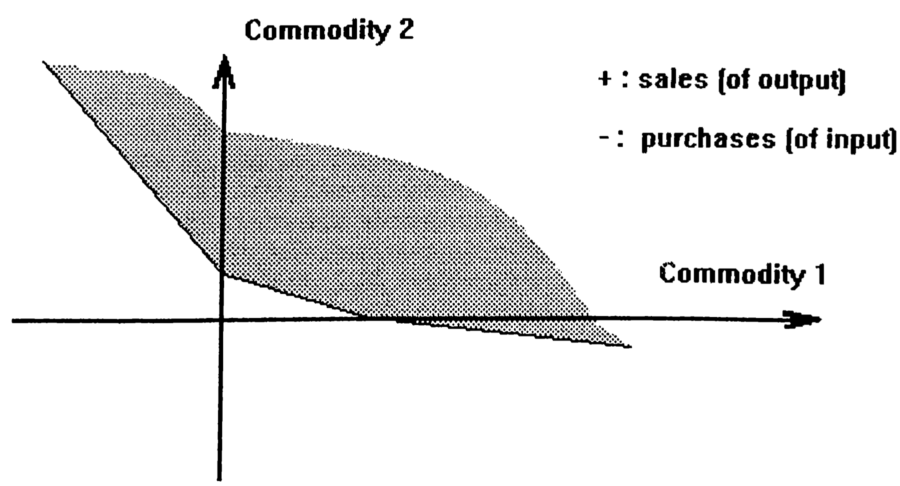

Figure 2. The upper profit set for given buying and selling prices and a certain level of profit.

An example of an upper profit set in $\mathcal{R}^2$, induced by a profit function and a given parameter vector, is seen in Figure 2. Also in this example, there are sales taxes on the trade operations of the firm.

Remark: A linear function on $\mathcal{R}^l$ is both a billing function and a profit function. This means that the analysis below will hold for linear price systems as a special case.

Now we are ready to extend our definition of an economy, so as to have an ownership structure over the commodities and firms, as well as a system of restriction possibilities on the possible trades of both the firms and consumers. The system is described by a list of billing functions and profit functions for each consumer and firm respectively, and a parameter set for an auctioneer-like central agent.

Definition 21 *A* **generalized price system with private ownership** $\mathcal{E}_S$ *is defined by:*

(i) an economy $((X_i, \preceq_i), (Y_j), \omega)$*;*

(ii) for each i, a point $\omega_i \in X_i$ *such that* $\sum_i \omega_i = \omega$*, the endowment of each consumer;*

(iii) for each i,j a non-negative number θ_{ij} *such that* $\sum_i \theta_{ij} = 1$ *for all j, the profit shares of each consumer;*

(iv) a nonempty parameter set $P \in \mathcal{R}^s$*;*

(v) for each i, a billing function e_i*,*

(vi) for each j, a profit function π_j*.*

The behavioral assumption of profit maximization for the firms is retained. Given a parameter vector p, firm j will maximize the profit function π_j over the production set Y_j. We can write the **supply correspondence** η_j from P to $\mathcal{R}^l$ as

$$\eta_j(p) = \{y^* \in Y_j \mid \pi_j(p, y^*) \geq \pi_j(p, y), \quad y \in Y_j\}.$$

The consumers are assumed to maximize preferences over generalized budget sets. The budget set for a consumer depends on the endowment ω_i, billing function e_i and income, coming from the profit shares, w. The **budget set correspondence** for consumer i, $\gamma_i : P \times \mathcal{R} \to X_i$ is specified by means of the billing function as

$$\gamma_i(p, w) = \{x \in X_i \mid e(p, x - \omega_i) \leq w\}.$$

Then the set of preference maximizing consumption points in this set will be demanded from the system by the consumer. Formally, the **demand correspondence** of consumer i, $\xi_i : P \times \mathcal{R} \to X_i$ is defined by

$$\xi_i(p, w) = \{x^* \in \gamma_i(p, w) \mid x \preceq_i x^* \ \ \textit{for all} \ \ x \in \gamma_i(p, w)\}.$$

The definition of equilibrium given below clarifies the main idea. It is a list of consumption and production bundles and a parameter vector such that, given the parameter, each firm maximizes its profit function, each consumer maximizes satisfaction given the profit income, and the total trade is feasible for the economy.

Definition 22 *An* **equilibrium** *of the generalized price system with private ownership $\mathcal{E}_S$ is an $(m+n)$- tuple $((x_i^*), (y_j^*))$ of points of $\mathcal{R}^l$ and $p^* \in \mathcal{R}^s$ such that:*

(i) x_i^* is a greatest element of the budget set

$$\{x_i \in X_i \mid e_i(p^*, x_i - \omega_i) \leq \sum_{j=1}^{n} \theta_{ij} \pi_j(p^*, y_j^*)\}$$

for $\preceq_i$, for every i;

(ii) y_j^* maximizes π_j relative to p^* on the production set Y_j, for every j;

(iii) $\sum x_i^* - \sum y_j^* \leq \omega$.

We will call p^* an **equilibrium parameter** and $((x_i^*), (y_i^*))$ an **equilibrium allocation of system $\mathcal{E}_S$**. One special case would be an *equilibrium price* and a *competitive allocation* in the *private ownership economy* of Debreu (1959).

Two more definitions will be needed in the following section. The first one introduces a special type of function which will replace the value of aggregate trade function in the statement of Walras Law. The second one introduces a special category of functions, which will be said to satisfy a generalization of Walras Law suitable for our case.

Definition 23 *Let $P \subset \mathcal{R}^s$ be nonempty and convex. A function $V : P \times \mathcal{R}^l \to \mathcal{R}$ will be called a* **generalized value function** *if*

(i) V is continuous,

(ii) For all $z \in \mathcal{R}^l$, $V(p, z)$ is quasiconcave on P,

(iii) $\bigcap_{p \in P}\{z \in \mathcal{R}^l : V(p,z) \leq 0\} \subset \mathcal{R}^l_-$.

Definition 24 *Let* $P \subset \mathcal{R}^s$ *be nonempty and convex and* $Z \subset \mathcal{R}^l$. *Let* $V : P \times \mathcal{R}^l \to \mathcal{R}$ *be a generalized value function. A correspondence* $\zeta : P \to Z$ *is said to satisfy* **the V-generalized Walras Law** *if, for any* $p \in P$, $V(p,q) \leq 0$ *for all* $q \in \zeta(p)$.

Let $Z = \sum_i X_i - \sum_j Y_j - \omega$ and let the correspondence $\zeta : P \to Z$ be defined through $\zeta(p) = \sum_i \xi_i(p) - \sum_j \eta_i(p) - \omega$, where ξ_i and η_i are demand correspondence of consumer i and supply correspondence of producer j, respectively, as defined before. Therefore ζ is interpreted as the total excess demand correspondence.

An existence result for a standard economy with convex and compact consumption sets and technologies is given below.

Theorem 10 *A generalized price system with private ownership*
$\mathcal{E}_S = ((X_i, \preceq_i), (Y_j), (\omega_i), (\theta_{ij}); P, (e_i), (\pi_j))$ *has a nonempty and compact set of equilibria, if* P *is convex and compact,*

for every i,

(a) X_i *is compact and convex,*

(b.1) for $x_i' \in X_i$, *the sets* $\{x_i \in X_i \mid x_i \succeq x_i'\}$ *and* $\{x_i \in X_i \mid x_i \preceq x_i'\}$ *are closed,*

(b.2) for x_i^1, $x_i^2 \in X_i$, *and* $t \in (0,1)$, $x_i^2 \succ x_i^1$ *implies* $tx_i^2 + (1-t)x_i^1 \succ_i x_i^1$,

(c.) there exists $x_i^o \in X_i$ *such that* $x_i^o \ll \omega_i$;

for every j,

(d.1) Y_j *is compact and convex,*

(d.2) $0 \in Y_j$;

and,

(e) there is a generalized value function $V : P \times Z \to \mathcal{R}$ *such that* $\zeta : P \to Z$ *satisfies V-generalized Walras Law, where* $Z = \sum_i X_i - \sum_j Y_j - \omega$.

With the exception of the last, these are familiar hypotheses. In (a) and (d.1) compactness could be dropped, by well known methods (e.g. Debreu (1959)), but it is assumed here for simplicity. An allowance for nonconvexities in consumption and production sets under special generalized price systems is possible, but will not be pursued here. (b.1) is the usual continuity requirement on preferences, and (b.2) the semi-strict convexity requirement. Neither monotonicity nor local nonsatiation is assumed, since the definition of equilibrium permits free disposal. (c) is also standard, and used to ensure the continuity of the budget set correspondences of consumers. (d.2) excludes the possibility of negative profits, to make sure that budget sets of consumers will always be nonempty.

The hypothesis (e), which is not standard, is an implicit condition on all the primitives of the economic system. For some types of systems, however, it is possible to find a suitable value function that satisfies this condition. One such example is discussed in the following section.

We will make use of Lemma 1 of Chapter 2 in proving Theorem 10. Before proving Theorem 10, a lemma on the continuity of the budget set correspondences of consumers will be helpful. The definitions, lemma and method of proof below are straightforward extensions of Debreu (1959; pp. 64-65).

Let $P \subset \mathcal{R}^s, X \subset \mathcal{R}^l$ be compact and convex sets, $e : P \times \mathcal{R}^l \to \mathcal{R}$ a billing function. Let S denote the set of price-wealth pairs $(p, w) \in P \times \mathcal{R}$ such that the corresponding budget sets are not empty, i.e. $S = \{(p, w) \in \mathcal{R}^{s+1} \mid \exists x \in X s.t. e(p, x) \leq w\}$. And the correspondence $\gamma : S \to X$ is defined by $\gamma(p, w) = \{x \in X \mid e(p, x) \leq w\}$.

Lemma 3 *Let $(p^o, w^o) \in S$, $w^o \neq \min_{x \in X} e(p^o, x)$. Then γ is continuous at (p^o, w^o).*

Proof: By the continuity of e, the graph of γ is closed and since X is compact, upper semicontinuity of γ follows.

Lower semicontinuity follows from the assumption that avoids a minimum wealth situation, the convexity of the at most as costly as sets, and the continuity of e. □

Proof of Theorem 10: The same arguments as in Debreu (1959; p.86) are applicable here. The continuity of the budget set correspondences follow from, the requirement that each consumer can dispose positive amounts of each commodity, monotonicity of e, and lemma 5. Under the given assumptions, the profit functions are continuous and each firm's supply correspondence is nonempty-, convex-valued and upper semicontinuous. Since the profit functions and the budget correspondences are continuous, each consumer's demand correspondence is similarly nonempty-, convex-valued and upper semicontinuous. Therefore the aggregate demand correspondence $\zeta : P \to Z$ is also nonempty-, convex-valued and upper semicontinuous. By property (e), there exists a generalized value function such that ζ satisfies the V-generalized Walras Law. Then by Lemma 1 with $W = \mathcal{R}^l_-$, there exists a $p \in P$ such that $\zeta(p) \cap \mathcal{R}^l_- \neq \emptyset$. Therefore there exists an equilibrium of the economic system $\mathcal{E}_s$, and hence the set of equilibria is nonempty.

Since the aggregate demand correspondence is upper semicontinuous with a compact domain and range, its graph is compact. But the set of equilibria is nothing but the intersection of this graph with $P \times \mathcal{R}^l_-$. This intersection, clearly, is compact. □

5.2 An Example: Sales Taxes and Existence of Equilibrium

The problem of existence of equilibrium under commodity taxes and subsidies has been studied by Sontheimer (1971), Shoven (1974), Mantel (1975), Shafer and Sonnenschein (1976) The example and the equilibrium concept here are closest those in Mantel (1975). The method of proof, however, is an application of Theorem 10 and is quite different.

We consider a semi-centralized economy where trade is possible only with the presence of a powerful central authority which will maintain property rights and enforce contracts. Such an authority will be called a government in this chapter. [9] Having such a monopoly power over the economy, the *governing* body may want to exploit that power. Suppose that due to some technical reasons taxing the endowments of the consumers is impossible, whereas extracting resources through the traded commodities is possible and costless. For example, this would be the case when monitoring endowments is very difficult compared to monitoring trade. The government has the option of determining buying and selling prices, the difference being perceived as sales taxes, so that the aggregate excess supply of each commodity will be left for her consumption. The government also has preferences over such possible (non-negative) consumption bundles. Therefore we imagine the case of an auctioneer with blood and flesh, who enjoys the commodity

[9] In cases where the government is weak, private bodies like *mafia* emerge in order to profit from the operation of markets. Mafia does not usually aim to totally destroy the trading activity but of course is not socially desirable since it cannot be controlled by a constitution or by a threat of losing elections.

profits arising from his rights to discriminate buyers from sellers.

The analysis of the previous sections will be crucial in showing the existence of an equilibrium where the consumers and firms act as price takers but the government obtains the highest satisfaction she can extract from this economy by setting prices and operating markets. Of course the extent of resources that may be extracted will depend on the *need for trade* in the economy. In the case of no firms and identical consumers, the government would be bound to consume zero from all commodities.

To start with, the trading system will be formally defined. The commodity space, the economy, the endowments and ownership structure are as in section 2. The price space is $\mathcal{R}^{2l}_{+}$ and the price set P is given by

$$P = \{(p^b, p^s) \in \mathcal{R}^l_+ \times \mathcal{R}^l_+ \mid \sum_{i=1}^{n} p^b_i + \sum_{i=1}^{n} p^s_i = 1; \;\; p^b_i \geq p^s_i \;\; for \;\; all \;\; i \in \{1, .., n\}\}.$$

The profit functions are the same for each firm and can be defined by,

$$\pi(p, y) = p^b[y]^- + p^s[y]^+,$$

where $[\cdot]^+$ and $[\cdot]^-$ are operators that replace respectively the negative and positive coordinates with zero, leaving other coordinates unchanged. Similarly billing functions are the same for each consumer and given by

$$e(p, y) = p^b[x]^+ + p^s[x]^-.$$

The preference of the government are given by $\succeq_g \subset (P \times \mathcal{R}^l_+) \times (P \times \mathcal{R}^l_+)$. Although not required for the existence result below, one can think that the government's preference may be taken to be independent of prices.

Now an equilibrium concept that captures the main idea can be defined as follows.

Definition 25 *A* **sales tax system with private ownership and a merchant government** *is the list* $\mathcal{E}_g = ((X_i, \preceq_i), (Y_j), (\omega_i), (\theta_{ij})$. *An* **equilibrium** *of the sales tax system with private ownership and a merchant government* $\mathcal{E}_g$ *is an* $(m + n + 1)$*- tuple* $((x^*_i), (y^*_j), g^*)$ *of points of* $\mathcal{R}^l$ *and* $p^* \in P$ *such that:*

(i) x^*_i *is a greatest element of*

$$\{x_i \in X_i \mid e(p^*, x_i - \omega_i) \leq \sum_{j=1}^{n} \theta_{ij} \pi(p^*, y^*_j)\}$$

for $\preceq_i$ *for every* i*;*

(ii) y_j^* *maximizes* π *relative to* p^* *on* Y_j*, for every* j*;*

(iii) $x^* - y^* + g^* \leq \omega$, $g^* \geq 0$;

(iv) (p^*, g^*) *maximizes governments preferences on* $Graph(-\zeta) \cap (P \times \mathcal{R}^l_+)$*, where* $\zeta : P \to Z$ *is the aggregate excess demand correspondence with* $Z = \sum_i X_i - \sum_j Y_j - \omega$.

The above equilibrium can be visualized as follows. The government knows the total excess demand correspondence ζ. Knowing that the commodity bundle g^* in *iii.* will be left for her consumption, she chooses the best price-allocation pair according to her preferences.

An existence theorem now, can be stated.

Theorem 11 *The sales tax system with private ownership and a merchant government* $\mathcal{E}_g = ((X_i, \preceq_i), (Y_j), (\omega_i), (\theta_{ij}); P, (e_i), (\pi_j); \succeq_g)$ *has a sales tax equilibrium if for every* i*,*

(a) X_i *is compact and convex,*

(b.1) $\preceq_i$ *is continuous,*

(b.2) $\preceq_i$ *is semi-strictly convex,*

(c.) *there is* x_i^o *in* X_i*, such that* $x_i^o \ll \omega_i$*;*

for every j*,*

(d.1) Y_j *is compact and convex for all* j*,*

(d.2) $0 \in Y_j$*; and*

(e) *The government's preference* $\succeq_g$ *is continuous.*

Proof: Clearly, P is a compact subset of $\mathcal{R}^{2l}$. It can be verified that $V : P \times \mathcal{R}^l \to \mathcal{R}$ described by

$$V(p, z) = p^b[z]^+ + p^s[z]^-$$

is a generalized value function. And $\zeta : P \to Z$ satisfies V-generalized Walras Law. Hence, all the hypotheses of Theorem 10 are satisfied and there exists at least one equilibrium of the corresponding economic system $\mathcal{E}_s$. Hence there is at least one price which enables the government to extract nonnegative amounts of each commodity from the economy. Moreover, from Theorem 10, we know that the set of equilibria, $Graph(-\zeta) \cap \mathcal{R}^l_+$, is compact. Since the preference of the government is continuous , there exists a best point in this set, i.e. a best price-allocation pair for the government among all pairs satisfying (i)-(iii). □

5.3 Some Remarks

Various generalizations of Theorem 10, by relaxing the compactness requirement for the consumption and production sets, is possible by well known methods (eg. Debreu (1962)) . It is not done here to keep the exposition and proofs simple, in order not to divert attention from the main point of generalized price systems.

In order to apply the existence result of Theorem 10 to models other than the sales tax example given here, the modeller has to find in each case a generalized value function that will work. The generalized value function has to be such that the aggregate demand correspondence of a defined economy satisfies the V-Walras law under it.

The Foley-Hahn type of transaction cost models with kinked budget lines are in the same spirit from the consumer's point of view. However, these models cannot be readily investigated under the setting here, because their equilibrium concept necessitates a distinction between purchased an sold commodities. By their definition of equilibrium, no transaction is allowed to take place without the presence of intermediary firms. In that case, doubling the dimensionality of the commodity space seems inescapable. And then, the price systems become linear.

On the other hand, the sales tax example given here cannot be analyzed by the approach of Foley-Hahn, because the definition of equilibrium in this case allows for direct trade of any good between any two agents. So one has to work with generalized, thus possibly nonlinear, price systems defined on the original commodity space.

The model of Drèze (1975) with price rigidities and quantity restrictions may be another suitable framework for Theorem 10 to be applied. In that case the parameter set P will include prices as well as quotas on permissible trades for agents. However, it is conjectured here that the set of Drèze equilibria will be a subset of the equilibria of the economic system with arbitrary quantity rationing. For example, total prohibition of trade will be one equilibrium of the economic system with arbitrary rationing. But this is not a Drèze equilibrium, since Drèze allows rationing only if a price constraint for a good is binding.

The most exciting applications of generalized price systems would be obtained if a theorem allowing nonconvexities of production or consumption sets, were available. It would be nice to connect the approach here to the literature on marginal or average cost pricing and other pricing rules considered in the literature. Cornet (1988) and references therein describe the state of the art in this area to a large extent. Such a connection, however, is beyond the scope of this thesis.

6 The Lemons Problem: An Example

6.1 Introduction

The supplier of a commodity usually is well aware of its quality. The potential buyer, however, may not have the opportunity to identify the quality of the commodity before purchase. A typical example is the market for used cars. The fact that a car is a **lemon** can only be identified after a sufficiently long period of use. Hence the seller of a used car would know what he is selling, while the buyer could at best have an idea about the average quality of a car sold in the secondary car market. The observation that this kind of quality externality may lead to serious problems has been made by Akerlof (1970):

> There are many markets in which buyers use some market statistic to judge the quality of prospective purchases. In this case there is incentive for sellers to market poor quality merchandise, since the returns for good quality accrue mainly to the entire group whose statistic is affected rather than to the individual seller. As a result there tends to be a reduction in the average quality of goods and also the in the size of the market.[10]

This problem is also called *adverse selection* in the literature. Akerlof (1970) also presents an example where demand for used cars is zero at all prices whereas supply is positive at all positive prices and is zero at zero price. Hence the equilibrium price and quantity traded are all zero. Whereas in the absence of quality externality, or dishonesty, Pareto improving trade would take place.

In this chapter we provide another example where, depending on the preferences, both equilibria with positive trade and equilibria with zero trademay exist. So it is not the case that lemons problem will necessarily destroy the market completely. In fact, we present a case where the lemons equilibrium is Pareto efficient.

In the following sections, firstly the society is described. Then three alternative institutions are analyzed. The first one is a trivial no-trade institution. The second one is a complete market system, where the quality of all the goods can be detected by the market maker. The third one is a lemons institution, where the market maker cannot distinguish between two qualities of one

[10] Akerlof (1970), p.488

of the goods. It is shown that Pareto ordering of the equilibria of the three institutions can be made for some societies but not for all possible societies.

6.2 The Example

There are three commodities: 1. Apricot Jam, 2. Genuine Honey, 3. Fake Honey. Hence the commodity space is $\mathcal{R}^3$. There are two consumers 1 and 2, $I = \{1, 2\}$. The consumption sets of the consumers are,

$$C_1 = C_2 = \mathcal{R}^3_+$$

and the outcome set is, $C = C_1 \times C_2$. For both consumers apricot jam and genuine honey are perfect substitutes with rates of substitution β_1 and β_2 (jam for honey). Consumer two values genuine honey more than consumer one does. Neither consumer values fake honey at all. There are no consumption externalities. The utility functions $u_i : C \to \mathcal{R}$ then can be written as,

$$u_1(c) = c_{11} + \beta_1 c_{12} \quad 0 < \beta_1 < 1$$

$$u_2(c) = c_{21} + \beta_2 c_{22} \quad \beta_2 > 4,$$

where the first subscript stands for consumer and second stands for commodity. The corresponding better consumptions correspondence $B_i : C \to C$ is given by

$$B_i(c) = \{z \in C \mid u_i(z) > u_i(c)\} \ \ (i \in I).$$

Then our society is $S = (I, C, (B_i)_{i \in I})$.

The endowments of the consumers are designed in such a way that there will be benefit from trade. Also to satisfy the hypotheses of the Arrow and Debreu (1954) theorem, we give positive endowments from all the goods:

$$\omega_1 = (0, 20, 60)$$

$$\omega_2 = (20, 0, 0)$$

The above endowment structure will be common to all institutions to follow. Remember that an institution $N = (P, (X_i, \tau_i)_{i \in I}, g, F, W)$ for a society S is a list consisting of a central parameter set, individual choice sets and constraint correspondences, an outcome function, a resource implications function and a resource set.

6.2.1 Institution 1: No-Purchase

This trivial institution can be expressed in alternative ways in our framework. One possibility is the following.

P is any nonempty compact and convex set. To simplify, take

$$P = \{0\}.$$

The potential choices are trades (+ for purchase, - for sale in each commodity):

$$X_i = \{x \in \mathcal{R}^3 \mid x \leq (60, 60, 60), \; x \geq -\omega_i\}^{11} \quad i \in I.$$

Nevertheless purchases are not allowed. Only sales, as donations to the auctioneer, are possible:

$$\tau_i(p, x) = \mathcal{R}^3_- \quad (p \in P, \; x \in X, \; i \in I)$$

The outcome function consists of two parts:

$$g(x) = (g_1(x), g_2(x))$$

where,

$$g_1(x) = \omega_1 + x_1,$$

and

$$g_2(x) = \omega_2 + x_2.$$

The resource implications function is:

$$F(x) = x_1 + x_2,$$

and the feasible resource set is:

$$W = \mathcal{R}^3_-.$$

The parameter-choice point

$$(p^*, x^*) = (0, 0, 0, 0, 0, 0, 0)$$

[11] To compactify the choice set, a natural upper limit on purchases that could be used is the total endowment of each commodity. Any vector greater or equal to this can be also used, since this constaint will not be binding in equilibrium.

trivially qualifies as a semicentralized resource equilibrium (SECERE). At all SECERE, the consumption bundles and utility levels are equal and given by:

$$g_1(x^*) = \omega_1 \quad u_1^* = 20\beta_1$$

$$g_2(x^*) = \omega_2 \quad u_2^* = 20$$

6.2.2 Institution 2: Complete Walrasian Market

The Complete Walrasian institution can be expressed in a familiar way.

The central parameter set is the full price simplex:

$$P = \{p \in \mathcal{R}^3_+ \mid \sum p_i = 1\}.$$

The potential choices are trades (+ for purchase, - for sale in each commodity):

$$X_i = \{x \in \mathcal{R}^3 \mid x \leq (60, 60, 60), \ \ x \geq -\omega_i\} \quad i \in I.$$

Purchases are allowed only if at the given price system their value exceeds the value of sales, ie. the usual budget set expressed on the trade space.

$$\tau_i(p, x) = \{y \in X_i \mid py \leq 0\} \quad (p \in P, \ \ x \in X, \ \ i \in I)$$

The outcome function consists of two parts:

$$g(x) = (g_1(x), g_2(x))$$

where,

$$g_1(x) = \omega_1 + x_1,$$

and

$$g_2(x) = \omega_2 + x_2.$$

The resource implications function is:

$$F(x) = x_1 + x_2,$$

and the feasible resource set is:

$$W = \mathcal{R}^3_-.$$

It is clear that an equilibrium price of fake honey cannot be positive. Since the markets only for the remaining two goods will be active, by making use of the Walras law, we can carry out an equilibrium analysis in the genuine honey market only. The simple exercise of finding prices that equate demand and supply in the genuine honey market determines the point

$$(p^*; x^*) = (0.5, 0.5, 0; 20, -20, 0, -20, 20, 0)$$

as the unique equilibrium.

The corresponding consumption bundles and utility levels are:

$$g_1(x^*) = (20, 0, 60) \quad u_1^* = 20$$

$$g_2(x^*) = (0, 20, 0) \quad u_2^* = 20\beta_2.$$

Both consumers are better off as compared to the SECERE of Institution 1. Moreover, we know from the first welfare theorem that this equilibrium is Pareto efficient.

6.2.3 Institution 3: Lemons Market

In this case, the market administrator cannot distinguish between genuine and fake honey. He, nevertheless, can distinguish jam from honey. For him, honey is honey and jam is jam. So he follows the following procedure. He announces two prices, one for jam and one for *honey.* After he collects the trade plans and checks their budgetary feasibility, he wants the consumers to bring their supplies to the market warehouse. There, he mixes homogeneously all the honey that is brought in a container.[12] Similarly, he puts all the jam supplied in another container. Then he tries to meet the demands for jam and honey. The case where he succeeds is called a semicentralized resource equilibrium.

The central parameter set then is the price simplex:

$$P = \{p \in \mathcal{R}_+^2 \mid \sum p_i = 1\}.$$

The potential choices are trades. We will use the convention that interprets the first coordinate as (+) for purchase, (-) for sale of jam, the second coordinate as (+) for purchase of *honey,*

[12] If the consumers are risk neutral, even if he does not mix them in a container but put them randomly on a shelf, the same analysis would follow. The probabilitiy of getting genuine honey in that case would take place of the proportion of genuine honey

(-) for sale of genuine honey and the third coordinate as (-) for sale of fake honey.

$$X_i = \{x \in \mathcal{R}^3 \mid x \leq (60, 60, 0),\ \ x \geq -\omega_i\} \quad (i \in I).$$

Purchases are allowed only if at the given price system their value does not exceed the value of sales. The budget set in this case becomes

$$\tau_i(p, x) = \{y \in X_i \mid p_1 y_1 + p_2(y_2 + y_3) \leq 0\} \quad (p \in P,\ \ x \in X,\ \ i \in I)$$

The outcome function, again, consists of two parts:

$$g(x) = (g_1(x), g_2(x))$$

where,

$$g_1(x) = \omega_1 + (x_{11}, [x_{12}]^- + [x_{12}]^+ \frac{[x_{22}]^-}{[x_{22}]^- + [x_{23}]^- + [x_{13}]^-}, [x_{13}]^- + [x_{12}]^+ \frac{[x_{23}]^- + [x_{13}]^-}{[x_{22}]^- + [x_{23}]^- + [x_{13}]^-}),$$

$$if \quad [x_{22}]^- + [x_{23}]^- < 0;$$

$$= \omega_1 + (x_{11}, [x_{i2}]^-, [x_{13}]^- + [x_{12}]^+), \quad if \quad [x_{22}]^- + [x_{23}]^- = 0;$$

and

$$g_2(x) = \omega_2 + (x_{21}, [x_{22}]^- + [x_{22}]^+ \frac{[x_{12}]^-}{[x_{12}]^- + [x_{13}]^-}, [x_{23}]^- + [x_{22}]^+ \frac{[x_{13}]^-}{[x_{12}]^- + [x_{13}]^-}),$$

$$if \quad [x_{12}]^- + [x_{13}]^- < 0;$$

$$= \omega_2 + (x_{21}, [x_{22}]^-, [x_{23}]^-), \quad if \quad [x_{12}]^- + [x_{13}]^- = 0;$$

Here the operator $[.]^+$ ($[.]^-$ resp.) replaces a negative (positive resp.) real number with zero while keeping it otherwise unchanged. If consumer 1 buys one unit of *honey*, in fact he buys some genuine honey and some fake honey, depending on what the other consumer sells. Hence the choice externality is clearly present under this institution and is exhibited by the outcome function.

The resource implications function $F : X \rightarrow \mathcal{R}^3$ is:

$$F(x) = (x_{11} + x_{21}, x_{12} + x_{13} + x_{22} + x_{23}),$$

and the feasible resource set is:

$$W = \mathcal{R}^2_-.$$

Unfortunately, we cannot use Theorem 6 here because, the outcome function is not affine. The simplicity of the setup however allows us to carry out an analysis in a two dimensional graph from the point of view of the market administrator. The strategy is to first get the best response correspondences of consumers, and then concentrate on the *honey* market to derive the demand and supply curves for honey in Figure 3.

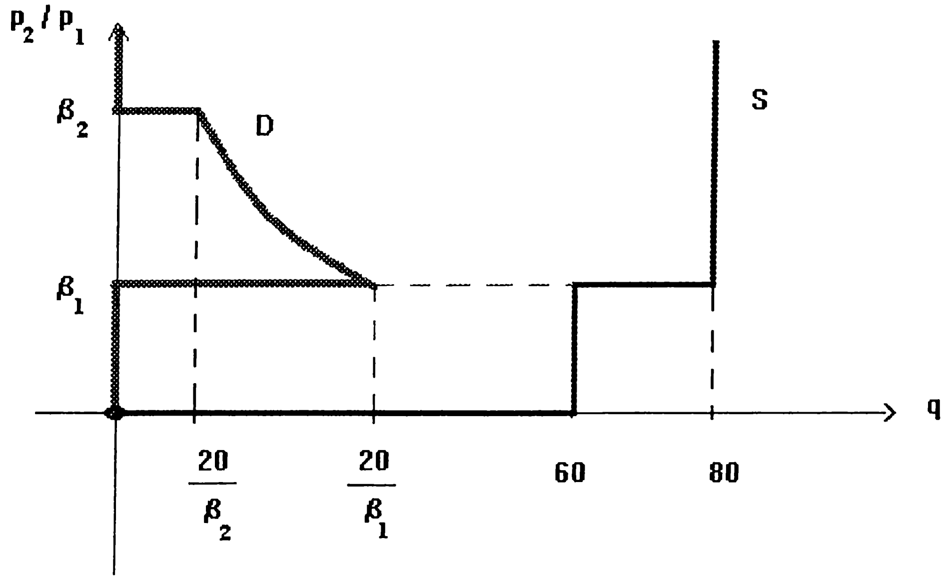

Figure 3. Equilibrium with zero trade and zero price in the lemons market. If β_1 were much lower, there would be another equilibrium with positive trade.

Consumer 1 will sell all of his 60 units of fake honey at any positive p_2. If $p_2 = 0$, he will be indifferent between selling any amount from zero to 60. Moreover, if $p_2/p_1 > \beta_1$ he will also sell his genuine honey. At $p_2/p_1 = \beta_1$ he will be indifferent between selling any amount of genuine honey from zero to 20. He will use the revenue from honey sales in purchasing jam. Since the other consumer can not sell genuine honey, we do not consider the possibility of selling fake honey and buying *honey* for consumer 1.

Consumer 2 will not buy *honey* as long as all of it is fake. This means for $p_2/p_1 < \beta_2$ the honey demand will be zero, since at this range no genuine honey is supplied. Let the proportion of good honey supplied be denoted by,

$$r(x) = \frac{[x_{12}]^-}{[x_{12}]^- + [x_{13}]^-}$$

Therefore if $p_2/p_1 > \beta_2 r(x)$, honey demand will be zero, and if $p_2/p_1 < \beta_2 r(x)$ honey demand will be given by,

$$x_{22} = 20\frac{p_1}{p_2}.$$

If $p_2/p_1 = \beta_2 r(x)$ demand will be between zero and $20\frac{p_1}{p_2}$.

The total demands and supplies for *honey* as a function of p_2/p_1 is drawn in Figure 3. Since the demands also depend on what the other consumer does, the figure should be investigated carefully. Especially at the price ratio $\frac{p_2}{p_1} = \beta_1$, the demand depends on the quantity supplied of honey in the way described above.

The result of the graphical analysis suggests that for preference parameters that satisfy

$$\beta_1 > \frac{\beta_2}{3\beta_2 + 1},$$

the unique equilibrium is the zero trade one. In societies where

$$\beta_1 < \frac{\beta_2}{3\beta_2 + 1},$$

there are also equilibria with positive trade. In particular, if

$$\beta_1 < 0.25,$$

the quantity of honey traded in one of the equilibria is 80, and the welfare of the society at that equilibrium is the same as that of complete Walrasian market equilibrium.

6.2.4 A Comparison

Suppose the market administrator has a Samuelson- Bergson objective function. Then, under Institution 3, he would choose the equilibrium price that leads to the highest quantity of honey trade since this provides a higher utility to both consumers.

Let us fix $\beta_1 = 0.28$, and $\beta_2 = 5$. In this case the administrator's optimum equilibrium under Institution 3 will be (approximately) given by

$$(p^*; x^*) = (0.78125, 0.21875; 20, -11.42857, -60; -20, 71.42857, 0).$$

The corresponding consumption bundles and utility levels are:

$$g_1(x^*) = (20, 8.57143, 0) \quad u_1^* = 22.4$$

$$g_2(x^*) = (0, 11.42857, 60) \quad u_2^* = 57.14$$

Both consumers are better off as compared with the SECERE of Institution 1. Interestingly enough, we see that this equilibrium is also Pareto efficient. If we recall the consumptions and utility levels of the complete Walrasian market,

$$g_1(x^*) = (20, 0, 60) \quad u_1^* = 20$$

$$g_2(x^*) = (0, 20, 0) \quad u_2^* = 100.$$

we see that by making use of this cheating possibility, Consumer 1 becomes better off in the lemons market than in the complete market, while Consumer 2 is hurt by the lemons problem.

This result is, of course, not generic. For instance, for $\beta_1 = 0.25$, and $\beta_2 = 5$, the equilibrium in the lemons market would be

$$(p^*; x^*) = (0.8, 0.2, 0; 20, -20, -60; -20, 80, 0)$$

and the utility levels

$$u_1^* = 20$$

$$u_2^* = 100$$

would be identical to those under the complete market institution. For $\beta_1 = 0.5$ and $\beta_2 = 5$, on the other hand, the only equilibrium in the lemons market would be that of zero trade.

7 Money as a Resolution to Lemons Problem

7.1 Introduction

This chapter presents a simple example of a two period economy with production, where both money and financial structure matters. The economy is such that there is a unique competitive equilibrium for a given money growth rate and financial structure. In view of the framework in Chapter 1, if we let the parameter set P include the policy variables of initial distribution of money (financial structure) and money growth rate as well as prices, we will see that there is a multiplicity of equilibria and moreover, Pareto comparisons between some of the equilibria can be made.

In the model, the need for money arises due to an adverse selection problem resulting from the unverifiable quality of commodities by the contract enforcing agency. Money issued by the government is valued, because as a reliable and storable piece of paper, financial contracts can be written in terms of it. We characterize and investigate the competitive equilibrium of this economy. The results imply that socially desirable money growth is possible, which becomes more emphasized as the extent of aggregate financing by debt as opposed to internal funds increases.

The empirical observations on effects of money on output lead to the rejection of the neutrality of money hypothesis [13]. The failure of theoretical attempts to generate such a result under perfect information and perfect competition has led to two lines of theoretical reasoning. One line has followed Lucas' (1972) model with imperfect information where only the noise (unexpected component) in money supply affects output. The other line has rather remained faithful to the sticky wage-price argument of Keynes, but tried to introduce coherence to Keynes' argument by generating the rigidity through monopolistic competition or staggered labor contracts [14].

What is missing from staggered labor contracts approach [15] is the theoretical justification for the existence of such contracts. This chapter, instead, will concentrate on an already existing form of rigidity, the debt contracts. By its very nature, any debt contract, if properly enforced

[13] Stock and Watson (1989) is a good example of this line of empirical research.

[14] Blanchard (1990) provides a survey.

[15] Taylor (1979 and 1980)

by the law, binds one party to deliver a fixed amount of future resources. The theoretical reasons for the existence of such instruments as opposed to equity contracts have already been explored by Townsend (1979). On the other hand, the corporate finance literature, following Modigliani and Miller (1958) studied the determination of optimal debt to equity ratios of corporations.

In this chapter we explore the conjecture that, given a positive level of aggregate debt to equity ratio, even in the absence of staggered labor contracts, changes in the money stock will have an effect on the output. We focus on the nature and sensitivity of the mentioned effect to the extent of debt financing. In a nutshell, the idea underlying our neutrality result is this: If there is a positive nominal interest rate on monetary debt contracts, a decline in future money stock is likely to have adverse effects on output. Formalization of this intuition, however, turns out to be quite demanding for three reasons.

The first reason is the need for introducing some heterogeneity among agents in order to generate borrowing and lending among agents. Clearly, this cannot be done in a representative agent framework. Nor should it be done in an overlapping generations model for the issue is not an intergenerational one. Within the same generation, agents of one type should be lending to agents of some other type for purposes of financing production. Hence the intra-generational exchange is the critical one. An alternative is to work with at least two types of infinitely lived agents but this complication is more than what we need to illustrate the main result. A two-period model with two types of economic agents is sufficient for our purposes: it contains heterogeneous agents and is dynamic.

The second difficulty is related to the modeling of expectations. Once we allow for nominal debt contracts, expected and unexpected changes in money stock may have effects on output through different channels. Unexpected contractions in money supply may generate bankruptcy or liquidity problems. Under-investment or under-production problems, on the other hand, would be associated with expected money contractions. Effects of money surprises on output have been studied extensively in the literature. In contrast, this chapter works under the perfect foresight assumption to concentrate on the effects of only *anticipated* changes in money supply on output.

The third difficulty in formalizing this issue is related to a deeper theoretical question. Why would the agents write their contracts in terms of money instead of commodities? Further, why would they use money in their sales and purchases of commodities anyway? These questions require a coherent theory of the transactions demand for money.[16] For the purposes of this chapter, I will adopt an adverse selection type of explanation as opposed to a moral hazard explanation by Bryant (1980). If agents with superior technologies could hire other agents' resources in order to use them as inputs in production and simply pay them in terms of the future potential output, there would be no need for money. But if the commodity produced is somehow not standardized and subject to a lemons problem, so that it is impossible to enforce contracts written on them, the producers will have an incentive to pay in terms of the output of the lowest possible quality. This adverse selection problem would deter such intertemporal transactions. Hence the use of money as a standard piece of paper in transactions and contracts could be seen as a second-best solution to an adverse selection problem.

The organization of the chapter is as follows. Section 7.2 describes the first-best world where commodity contracts are enforceable and hence there is no transactions demand for money. In such a world, we consider an experiment in which the government issues currency backed by the second period real tax collections. We show that both the quantity and rate of increase of the supply of money are neutral in this experiment. This result follows from the fact that there is no need for either a store of value or a medium of exchange in the model. Section 7.3 describes the second-best world where we introduce the adverse selection problem. Without money, no exchange can take place between agents since commodity contracts are not operational due to undetectable quality of output. Hence, money issued by the government in the same way as in Section 7.2, this time will have real effects. The nature of such effects and their sensitivity to financial structure are explored in Section 7.4. Conclusions are presented in Section 7.5. The list of variables and parameters that are used in the text and a compact presentation mathematical formulations can be found in the Appendix.

7.2 The Model with Commodity Contracts

With the purpose of illustrating the effects of various financial structures on economic outcomes in a general equilibrium model with perfect foresight, we try to construct the simplest possible model that will exhibit the underlying ideas. There is a price to be paid for any deviation from the representative agent paradigm, however: Even the simplest construction requires the deter-

[16] Ostroy and Starr (1990) provide a survey of literature on this issue.

mination of equilibrium prices and trades for alternative policy experiments, which very quickly becomes analytically messy. However, as indicated in Section 7.1., introducing heterogeneity among agents is necessary to investigate some economic phenomena which may be more important in causing macroeconomic fluctuations than those that can be studied in either overlapping generations models of a single type of agents or in infinitely lived, single representative agent models.

Consider the problem of financing of working capital by the producers from external short term loans. At least two parties have to be involved; one lender and one borrower. The issue definitely is not an intergenerational one. It is rather a very short run phenomenon which spans through the time of one production cycle. Raw materials and labor have to be purchased by some means of financing, then these are to be used in producing outputs, and funds have to be generated to cover the required reimbursement to the creditors. Hence, if the interest rates are positive, and if all debt contracts must be nominated in terms of money, there seems to be the need for an increase in money stock. And to see whether this intuition is correct, one needs a model with agents living in the same period, but being endowed with different abilities or technologies to generate the borrowing-lending relationships. Even if their preferences and resource endowments are the same, differences in know-how will be sufficient to divide the agents into demanders and suppliers of credit. To see this more rigorously, we describe and analyze below a simple two-period economy.

7.2.1 The Economy

There are two commodities, labor and a consumption good. There is a total of n+m agents. There is a common technology which enables the conversion of labor into the consumption good and is available to all the agents. There also is a superior technology available to only m of the agents. These two technologies can be represented by twice continuously differentiable production functions $f_i(L)$, $f_i : \mathcal{R}_+ \rightarrow \mathcal{R}_+$, with $f_i^{'} > 0$ and $f_i^{''} \leq 0$, where $i = 1$ and 2 stand for the inferior and superior technologies, respectively. To be precise about what constitutes a better technology, it will be assumed that $f_2^{'}(L) > f_1^{'}(L)$ for all $L > 0$. Moreover, the production of the good by the second technology takes some time longer than that by the first technology. This will be the only time element in the model and t=1 and 2 will stand for the dates where the first and second production plants provide their output. The agents with the inferior technology are endowed with one unit of labor each and nothing else. The remaining agents have no labor that can be used directly in production.

There is also a fake production technology, available to everyone, which uses no labor input but produces an output, called the *fake good*, that looks like the consumption good but delivers no satisfaction once consumed.

We assume that preferences are defined for the consumption good at two dates only. Hence, leisure is excluded and our agents supply labor inelastically even at zero wage. The preferences are represented by the simplest time separable form $U(c_{i1}, c_{i2}) = c_{i1} + \beta c_{i2}$ where $i = 1$ and 2 stand for agents with inferior and superior technologies respectively and $0 < \beta \leq 1$. From now on, the term *agent 1* will be used to represent the agents with the low technology and n units of labor in total, and the term *agent 2* will be used to represent the agents with the high technology and zero labor endowment.

There also is a government agency endowed with the power to enforce private debt contracts. The bankruptcy law stipulates sufficiently high penalties on those who cannot fulfill contract requirements assuring that no debts will be repudiated in equilibrium. A most natural contract in such a case would be an agreement like agent 1 promising to work L hours for agent 2 and agent 2 promising to deliver q units of consumption good to agent 1 in the next period. The commodity wage rate so described will be denoted by ω. Throughout this section, the maintained assumption will be that both the government agency and the private agents can easily distinguish the quality of the consumption good from the fake good *before* consummating the transaction. This assumption will make the existence of a market for the exchange of labor and goods possible.

7.2.2 The Equilibrium

A competitive equilibrium of this economy is defined as a price-trade pair $(\omega; L, q)$ such that the following properties hold:

1. given ω; L and q are optimal choices for both agents, and
2. the trades L and q are feasible.

Condition 2 means that $L \leq n$ and $q \leq f_2(L)$. Condition 1 requires that given ω, q and L are the choices that maximize

$$U(c_{11}, c_{12}) = c_{11} + \beta c_{12}$$

subject to the constraints:

$$c_{11} = f_1(n - L) \tag{1}$$

$$c_{12} = q \tag{2}$$

$$q = \omega L \tag{3}$$

$$L \leq n \tag{4}$$

$$c_{11}, c_{12}, q, L \geq 0 \tag{5}$$

for agent 1, and maximize

$$U(c_{21}, c_{22}) = c_{21} + \beta c_{22}$$

subject to the constraints:

$$c_{21} = 0 \tag{6}$$

$$c_{22} = f_2(L) - q \tag{7}$$

$$q = \omega L \tag{8}$$

$$q \leq f_2(L) \tag{9}$$

$$c_{21}, c_{22}, q, L \geq 0 \tag{10}$$

for agent 2.

If the values of ω, q and L can be determined from the above two programs, it is clear from (4) and (9) that the feasibility Condition 2 will automatically be satisfied.

Due to our assumptions on the production and utility functions, the objective functions are concave and the feasible sets are convex. The constraint qualification is also satisfied. Hence, the Kuhn-Tucker-Lagrange (KTL) conditions are necessary and sufficient for optimality in the above problems. The appropriate Lagrangeans and KTL conditions can be written as:

$$\mathcal{L} = f_1(n - L) + \beta\omega L + \lambda(n - L)$$

$$\frac{\partial \mathcal{L}}{\partial L} = -f_1'(n-L) + \beta\omega - \lambda \leq 0 \qquad L\frac{\partial \mathcal{L}}{\partial L} = 0 \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = n - L \geq 0 \qquad \lambda\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \tag{12}$$

$$\lambda, L \geq 0 \tag{13}$$

$$\mathcal{L} = f_2(L) - \omega L + \eta(f_2(L) - \omega L)$$

$$\frac{\partial \mathcal{L}}{\partial L} = f_2'(L) - \omega + \eta(f_2'(L) - \omega) \leq 0 \qquad L\frac{\partial \mathcal{L}}{\partial L} = 0 \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial \eta} = f_2(L) - \omega L \geq 0 \qquad \eta\frac{\partial \mathcal{L}}{\partial \eta} = 0 \tag{15}$$

$$\eta, L \geq 0 \tag{16}$$

For the specific case of constant returns to scale technologies, where $f_1(L) = L$ and $f_2(L) = \gamma L$, with $\gamma > \frac{1}{\beta}$, constraints (11), (12), (14) and (15) are binding. Hence also using (8), the price-trade pair:

$$\omega = \gamma \quad L = n \quad q = \gamma n \tag{17}$$

easily verifies as the competitive equilibrium of this economy by satisfying the equations (11) through (16). This is the *first-best* outcome, where all the labor is used in the superior technology, and yet the wage rate is so high that it just sweeps the profits of the type 2 agents to zero. This, however, is due to the constant returns to scale technologies.

Note that there is no need for a medium of exchange in this economy. To see this, consider an experiment where government issues pieces of paper and distributes them to the agents. At the same time, the government levies a tax on agent 1, payable at time t in terms of the consumption good. Each one of the pieces of paper, which will be called *money* from now on, delivers the right to its holder at time 2, to claim a certain amount of consumption good from the government. For simplicity suppose that the government will convert all the taxed goods to money at time 2. In this new setting, there is room for new markets. Money for labor in period one, money for consumption good in periods one and two. But neither of these markets will be operational and an equilibrium will consist of agents holding their money balances till the end of period 2, and of a shadow period-1 value for money in terms of consumption good $1/p = \beta/p_2$ where p and p_2 denote the period 1 and period 2 money prices of the consumption good, respectively. This result can easily be verified.

7.3 The Model with Adverse Selection and Money

The analysis in the previous section is based on the crucial assumption that a market for the exchange of labor and future output exists. Suppose, however, that the government's contract enforcement agency (the courts) cannot distinguish between the fake good and the consumption good. The fact that the agents can distinguish the quality is of no help. At any given positive wage rate, the optimal choice[17] for agent 2 will be to hire as much of labor as he can and use them in production of the consumption good, but when the payment time comes, pay in terms of the fake good. He can therefore enjoy all the output and his labor demand will be infinite. For agent 2, on the other hand, the goods in the market are worthless and fake, hence his labor supply will be zero at any positive wage rate. This market therefore cannot be operational. and the desirable results of Section 7.2 will not obtain. The outcome will be hundred percent of the labor force being employed in the inferior technology.

This economy cries out for a medium of exchange which should have two properties. It should be *reliable* and *storable*. Reliability is used to mean that its quality should easily be distinguished by every agent and especially by the contract enforcement agency. Storability is used to mean that its *value* should not be zero after the production processes are completed. Note that the money issued by the government backed by future real tax collections as in Section 7.2 has both of these properties. Its value at the end is known to be positive and it can easily be distinguished from fake money by the government's contract enforcement agency.

In this case, there will be a total of three markets. The first one will be a market for the exchange of labor and money (*Labor Market*, rate: w). The second one will be for the exchange of goods and money (*Goods Market*, rate: p). Finally the third one will be the market for the exchange of period 1 money and period 2 money (*Money Market*, rate: R).

There will be no adverse selection problem to damage the goods market in this case because

[17] As in Akerlof's (1970) market for lemons, here, fake goods kill the market for genuine goods.

agent 1 simply has the option of not buying the good if he finds out that it is fake. And this does not require the contract enforcement agent since there is no contract to enforce here. Hence in this market only the consumption good will be supplied and demanded. The labor market has no quality problems, so it will be operational. The money market is a potentially the problematic one due to the differences in the dates of executions of transactions, hence it is potentially subject to a adverse selection problem. But now the contract enforcement agency can easily recognize money and therefore can operationalize the bankruptcy law properly.

7.3.1 The Economy and Equilibrium

In order to determine and contrast with Section 7.2 the value of money in period one, and the levels of activity in the labor, goods and money markets the mathematical formalization below will be utilized.

The description of the commodity space, preferences and technologies are the same as in Section 7.2. But the markets are different. Here, we have three distinct markets as opposed to one in Section 7.2, hence three different market prices R, w and p. The price p_2 of goods in the second period is fixed by the government as:

$$p_2 = \frac{M_{11} + M_{21} + \Delta M}{\tau} \tag{18}$$

where M_{11}, M_{21} are the money endowments of agents 1 and 2 respectively, ΔM is the additional money given to agent 1 just before the goods market opens, and τ is the final period taxes payable in consumption goods by agent 1. Hence ΔM is the increase in the nominal stock of money. There is also a hundred percent tax over the eventual money holdings of agent 2.

Given the values of R, w and p, agent 1 will choose the level of money to lend (d), the amount of labor to supply (L) and the quantity of goods to buy (q) in order to maximize:

$$U(c_{11}, c_{12}) = c_{11} + \beta c_{12}$$

subject to,

$$c_{11} = f_1(n - L) \tag{19}$$

$$c_{12} = q + \frac{M_{12}}{p_2} - \tau \geq 0 \tag{20}$$

$$M_{12} = M_{11} - d + wL - pq + Rd + \Delta M \geq 0 \tag{21}$$

$$pq \leq M_{11} + \Delta M - d + wL \tag{22}$$

$$d \leq M_{11} \tag{23}$$

$$L \leq n \tag{24}$$

$$d, L, q \geq 0 \tag{25}$$

Equations (19), (20), (21) and (24) are self evident. Equation (22) expresses the idea that the purchases from the goods market take place before the interest repayment [18]. Equation (23) states that the amount of money lent cannot exceed the initial money balances.

Similarly, given the values of R, w and p, agent 2 will choose the level of money to borrow (d), the amount of labor to demand (L) and the quantity of goods to sell (q) in order to maximize:

$$U(c_{21}, c_{22}) = c_{21} + \beta c_{22}$$

subject to,

$$c_{21} = 0 \tag{26}$$

$$c_{22} = f_2(L) - q \tag{27}$$

$$M_{22} = M_{21} + d - wL + pq - Rd \geq 0 \tag{28}$$

$$wL \leq M_{21} + d \tag{29}$$

$$q \leq f_2(L) \tag{30}$$

$$d, L, q \geq 0 \tag{31}$$

Equation (26) is imposed because there does not exists a market for the goods at time 1 [19]. Equation (27) excludes the possibility of the use of any retained money earnings in purchasing goods from the government. The reason for this is the hundred percent money holding tax levied by the government on agent 2. Equation (28) gives final money balances, (29) states that wage bill cannot exceed the total funds available, and (30) says that sales cannot exceed output.

In the following analysis, using (19),(20),(21), (26),(27) and (28) the size of the problem is reduced so that the only remaining choice variables are the levels of trade in each market.

[18] The borrowing party has to earn the money first from the sales of the goods and make the repayment after the sales.

[19] This is assumed to simplify the exposition. Even if the market were open, no trade would take place here.

Again, due to our assumptions on the production and utility functions, the objective functions are concave and the feasible sets are convex. The constraint qualifications are also satisfied. Hence, the Kuhn-Tucker-Lagrange (KTL) conditions are necessary and sufficient for optimality in the above problems. The appropriate Lagrangeans and KTL conditions can be written for agent 1 as:

$$\mathcal{L} = f_1(n-L) + \beta(q + \frac{M_{11} - d + wL - pq + Rd + \Delta M}{p_2} - \tau) + \lambda_1(M_{11} + \Delta M - d + wL - pq)$$
$$+\lambda_2(M_{11} - d) + \lambda_3(n-L)$$

$$\frac{\partial \mathcal{L}}{\partial d} = \beta\frac{R-1}{p_2} - \lambda_1 - \lambda_2 \leq 0 \tag{32}$$
$$d\frac{\partial \mathcal{L}}{\partial d} = 0 \tag{33}$$
$$\frac{\partial \mathcal{L}}{\partial L} = -f_1^{'}(n-L) + \beta\frac{w}{p_2} + w\lambda_1 - \lambda_3 \leq 0 \tag{34}$$
$$L\frac{\partial \mathcal{L}}{\partial L} = 0 \tag{35}$$
$$\frac{\partial \mathcal{L}}{\partial q} = \beta(1 - \frac{p}{p_2}) - \lambda_1 \leq 0 \tag{36}$$
$$q\frac{\partial \mathcal{L}}{\partial q} = 0 \tag{37}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = M_{11} + \Delta M - d + wL - pq \geq 0 \tag{38}$$
$$\lambda_1\frac{\partial \mathcal{L}}{\partial \lambda 1} = 0 \tag{39}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = M_{11} - d \geq 0 \tag{40}$$
$$\lambda_2\frac{\partial \mathcal{L}}{\partial \lambda_2} = 0 \tag{41}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = n - L \geq 0 \tag{42}$$
$$\lambda_3\frac{\partial \mathcal{L}}{\partial \lambda_3} = 0 \tag{43}$$
$$\lambda_1, \lambda_2, \lambda_3, d, L, q \geq 0 \tag{44}$$

and for agent 2 as,

$$\mathcal{L} = \beta(f_2(L) - q) + \eta_1(M_{21} + d - wL) + \eta_2(f_2(L) - q) + \eta_3(M_{21} + d - wL + pq - Rd)$$

$$\frac{\partial \mathcal{L}}{\partial d} = \eta_1 + \eta_3(1-R) \leq 0 \tag{45}$$

$$d\frac{\partial \mathcal{L}}{\partial d} = 0 \tag{46}$$

$$\frac{\partial \mathcal{L}}{\partial L} = f_2'(L) - \eta_1 w + \eta_2 f_2'(L) - \eta_3 w \leq 0 \tag{47}$$

$$L\frac{\partial \mathcal{L}}{\partial L} = 0 \tag{48}$$

$$\frac{\partial \mathcal{L}}{\partial q} = -1 - \eta_2 + \eta_3 p \leq 0 \tag{49}$$

$$q\frac{\partial \mathcal{L}}{\partial q} = 0 \tag{50}$$

$$\frac{\partial \mathcal{L}}{\partial \eta_1} = M_{21} + d - wL \geq 0 \tag{51}$$

$$\eta_1\frac{\partial \mathcal{L}}{\partial \eta_1} = 0 \tag{52}$$

$$\frac{\partial \mathcal{L}}{\partial \eta_2} = f_2(L) - q \geq 0 \tag{53}$$

$$\eta_2\frac{\partial \mathcal{L}}{\partial \eta_2} = 0 \tag{54}$$

$$\frac{\partial \mathcal{L}}{\partial \eta_3} = M_{21} + d - wL + pq - Rd \geq 0 \tag{55}$$

$$\eta_3\frac{\partial \mathcal{L}}{\partial \eta_3} = 0 \tag{56}$$

$$\eta_1, \eta_2, \eta_3, d, L, q \geq 0 \tag{57}$$

The competitive equilibrium of this monetary economy is a price-trade pair $[R, w, p; d, L, q]$ which satisfies the following two conditions:

1. Given R, w and p; d, L and q are optimal choices for both agents, and
2. The trades are feasible to the economy.

Condition 1 says that equations (32) through (57) should be satisfied. Condition 2 requires $d \leq M_{11}, L \leq n$ and $q \leq f_2(L)$, which will be automatically satisfied if a solution to (32) through (57) exists.

The following section will find out the equilibrium and conduct some comparative statics for the case of constant returns to scale technologies.

7.4 Financial Structure and Some Policy Experiments

Since agent 2 is viewed as a firm manager who has management skills to operationalize a constant returns to scale superior technology $f(L) = \gamma L, \gamma \geq 1/\beta$, the money he initially has, M_{21}, will be called internal funds, owner's equity or simply equity interchangeably. The money he borrows,

d, will be called external funds or debt. The extent of potential debt financing as opposed to equity financing will be measured by δ where,

$$\delta = \frac{M_{11}}{M_{11} + M_{21}} \tag{58}$$

This number, which constitutes an upper limit to financial leverage, will be the key variable that summarizes the financial structure of the economy. By definition, we have $0 \leq \delta \leq 1$. If $\delta = 0$ the economy will be said to be internally financed, and if $\delta = 1$ it will be called an externally financed economy.

For notational convenience, the total final money balances in the economy will be denoted by M_f which is given by

$$M_f = M_{11} + M_{12} + \Delta M \tag{59}$$

and for computational convenience, it will be assumed that

$$M_f = \tau \tag{60}$$

Hence in the following arguments changes in money stock are accompanied by the same amount of changes in future commodity taxes to keep the final period value of money constant at the rate one, or $p_2 = 1$.

Also, taking the constant returns to scale inferior technology as $f_1(L) = L$, for $L \geq 0$, we are ready to proceed with finding the competitive equilibrium. Substituting for the production functions our constant returns to scale forms, $f_1(L) = L$ and $f_2(L) = \gamma L$, (32) to (57) provide a system consisting of 26 equations and inequalities in the 12 variables $[R, w, p; d, L, q; \lambda_1, \lambda_2, \lambda_3; \eta_1, \eta_2, \eta_3]$. I proceed by guessing that for *low rates of money growth and high values of total labor endowment n*, the inequalities (32), (34), (36), (38), (45), (47), (49), (51), (55) will be binding and (40), (42), (53) will not be binding, so that $\lambda_2 = \lambda_3 = \eta_2 = 0$ will hold. These guesses provide us with 12 equations in our 12 unknowns. All equations are linear and luckily enough the explicit derivation of the solution is not so difficult. The solution is given by

$$R = \frac{\gamma}{\gamma - 1}(p_2 + 1) \tag{61}$$

$$w = 1 \tag{62}$$

$$p = \frac{p_2 + 1}{\gamma - 1} \tag{63}$$

$$d = (\gamma - 1)\frac{M_f}{\gamma(p_2 + 1)} \tag{64}$$

$$L = M_{21} + \frac{(\gamma - 1)M_f}{\gamma(p_2 + 1)} \tag{65}$$

$$q = (\gamma - 1)\frac{M_f}{p_2 + 1} \tag{66}$$

$$\lambda_1 = \beta\frac{\gamma p_2 + 1}{(\gamma - 1)p_2} \tag{67}$$

$$\lambda_2 = 0 \tag{68}$$

$$\lambda_3 = 0 \tag{69}$$

$$\eta_1 = \frac{\gamma p_2 + 1}{p_2 + 1} \tag{70}$$

$$\eta_2 = 0 \tag{71}$$

$$\eta_3 = \frac{\gamma - 1}{p_2 + 1} \tag{72}$$

and as it is seen they all satisfy the nonnegativity constraints for $p_2 = 1 > 0$. One should also verify that (40), (42) and (53) are also satisfied. (53) verifies easily since $M_{21} \geq 0$. (40) would verify for low values of money growth

$$\frac{\Delta M}{M_0} < \frac{2\gamma}{\gamma - 1}\delta - 1, \tag{73}$$

where $M_0 = M_{11} + M_{21}$ is the total initial money balances, and for high values of labor endowment

$$n > \beta M_0. \tag{74}$$

But then we obtain a nonneutrality result for money growth. Rewriting (65),

$$L = M_0((1 - \delta) + \frac{\gamma - 1}{2\gamma}(1 + \frac{\Delta M}{M_0})) \tag{75}$$

which shows that labor allocated in the superior technology is an *increasing* function of money growth rate $\Delta M/M_0$, with M_0 kept constant. The effect of this phenomenon on total output y can be seen from

$$y = n + (\gamma - 1)L = n + (\gamma - 1)(1 - \delta)M_0 + \frac{\gamma - 1}{2\gamma}(1 + \frac{\Delta M}{M_0})M_0 \tag{76}$$

Hence, total output clearly increases with increases in money growth. The explanation of this effect is as follows. Equilibrium nominal interest rate is constant at a value greater than one. Hence the firms have to pay back more than the total amount they have borrowed. But suppose money growth were zero. Then to be able to pay back, under perfect foresight equilibrium, they have to demand credit at a level which is less than the total money endowments of the households. But then any increase in money stock just before money market opens will increase the level of borrowed funds, hence the level of employment, hence the level of output.

This of course cannot go forever. There is an upper limit to money growth given by (73). Now we will explore what will happen if money growth exceeds that limit. For extensive money growth, one would expect to see the loanable funds constraint (23) to be binding whereas the spendable funds constraint (22) not to be binding. These require (32), (34), (36), (40), (45), (47), (49), (51), (55) to hold as equalities and (38), (42), (53) to hold as inequalities, so that $\lambda_1 = \lambda_3 = \eta_2 = 0$ will hold. Again, these guesses provide us with 12 equations in our 12 unknowns. In this case, the below set of values verify as a competitive equilibrium.

$$R = \gamma\beta \tag{77}$$

$$w = \frac{p_2}{\beta} \tag{78}$$

$$p = p_2 \tag{79}$$

$$d = M_{11} \tag{80}$$

$$L = \frac{\beta}{p_2}(M_{11} + M_{21}) \tag{81}$$

$$q = \frac{\gamma\beta}{p_2}M_{11} \tag{82}$$

$$\lambda_1 = 0 \tag{83}$$

$$\lambda_2 = \frac{\gamma\beta - 1}{p_2}\beta \tag{84}$$

$$\lambda_3 = 0 \tag{85}$$

$$\eta_1 = \frac{\gamma\beta - 1}{p_2} \tag{86}$$

$$\eta_2 = 0 \tag{87}$$

$$\eta_3 = \frac{1}{p_2} \tag{88}$$

As clear from (81), further increases in $\Delta M/M_0$ will not affect level of industrial employment. Hence total output will be fixed at the level of

$$\bar{y} = n + (\gamma - 1)\beta M_0 \tag{89}$$

The level of total output as a function of rate of growth in money supply is plotted in Figures 4 and 5 for $\beta = 1$. Figure 4 displays the resulting phenomenon for the case of an externally financed economy ($\delta = 1$), and Figure 5 shows what would happen in the case of an internally financed economy. In both figures also plotted is the level of output in a non-monetary economy with and without adverse selection. Without adverse selection, or in the case of enforceable commodity contracts, the first best outcome is attained. In case of adverse selection and unenforceable commodity contracts, due to unverifiable quality, the worst outcome is attained. In this sense money here can be seen as a partial resolution of an adverse selection problem.

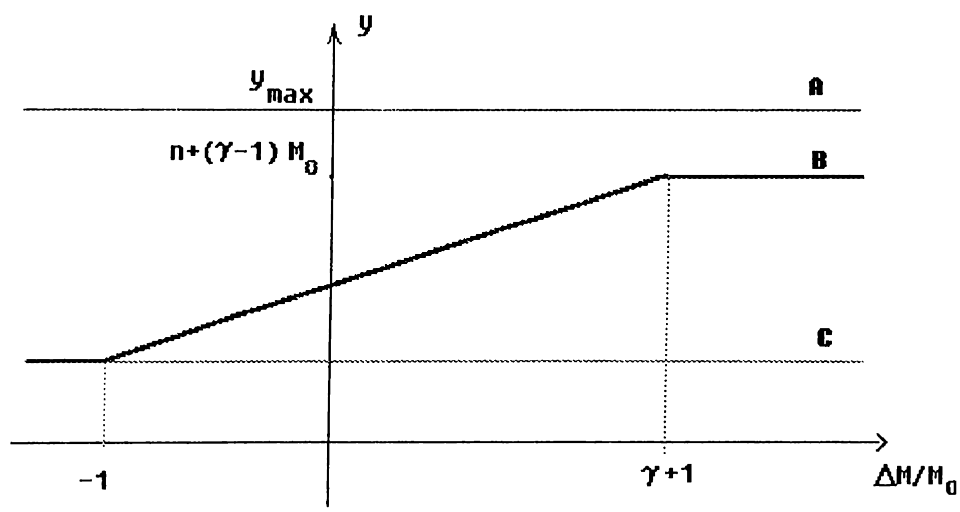

Figure 4. **All debt financed economy: Response of output to money growth (For β =1).**

A : With commodity contracts

B : With adverse selection and money

C : With adverse selection and no money

In the case of an economy with pure external financing (Figure 4B), it is seen that money growth is good for total output. In the case of an economy with pure internal financing (Figure 5B), money growth turns out to be neutral but the output cannot reach the maximum possible level seen in Figure 5A. The hybrid cases can be analyzed as well via equations (76), (89) and the inequality (73). The nonneutrality result will still prevail in these cases. If the utility and production functions were selected to be generalized, the figures might have looked smoother. But then, direct analytical derivations of solutions would not be feasible.

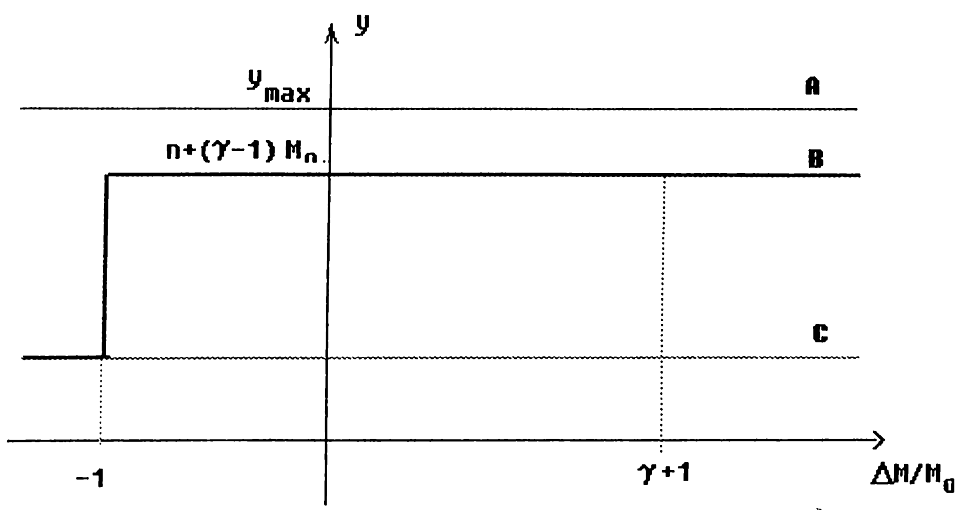

Figure 5. All internally financed economy: Response of output to money growth (For β =1).

A : With commodity contracts

B : With adverse selection and money

C : With adverse selection and no money

7.5 Conclusions and Future Work

This chapter constructed a simple example of a two period economy with production, where both money and financial structure matter. The results imply that there is a certain *need* for money growth and this need becomes more emphasized as the extent of financing by debt as opposed to internal funds increases.

The generalizations of the results in this chapter in several directions seem to be worthwhile. One direction would be to obtain the results for as general representations of technology and preferences as possible. Another direction would be the task of introducing *fiat* money in an infinite period context instead of currency backed by future real taxes in a finite period model. Yet a third direction would be to allow for deviations from perfect foresight of future prices. In all three cases however, the analytical difficulty of having to deal with heterogeneous types of agents would be there.

An issue which has not been analyzed in this chapter is the *optimal* monetary policy. Clearly, the level of output is also affected positively by the level of initial money balances. Hence in the presented model, there is more room for policy experiments than those carried out here.

8 Prakash and Sertel's Theory of Non-Cooperative Equilibrium in Social Systems - Twenty Years Later

by

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8.1 Introduction

The Prakash and Sertel (1974b) (henceforth PS) paper, "Existence of Non-Cooperative Equilibria in Social Systems", which is the cause of the present chapter, has remained unpublished for over two decades, coming to the attention of very few theorists[21] during this period.

Its results in economic theory or game theory had actually formed the main motivation for a certain amount of "background" work (1971, 1974a) by Prakash and Sertel in topological semigroups in the early 1970's. In this work they had formulated their *topological semivector spaces*, developing a fixed point theory for these spaces, all with the direct aim of proving their equilibrium existence theorems (Sertel, 1971; Prakash, 1971) relevant to generalized games and economies. In fact, the mathematical basis of the PS paper causing the present chapter can be found in Prakash and Sertel (1974a), followed by their embedding theory in Prakash and Sertel (1976).

A canonical example of a topological semivector space is the collection $\mathcal{K}\mathcal{Q}[L]$ of compact and convex nonempty subsets of a real topological vector space (tvs) L, where addition is

[20]This chapter which is written jointly with Murat R. Sertel is forthcoming as a paper in *Journal of Mathematical Economics* (Başçı and Sertel, 1995).

[21]Notably, Ichiishi (1983, p.74) has remarked

> Prakash and Sertel (1974) generalized the abstract economy to the situation in which the utility and also the feasible set of a player depend not only on the others' choices of strategy but also on the others' feasible strategy sets (e.g. on enemy's capabilities). Their logic for the existence theorem goes deep (beyond fixed point theorems for topological vector spaces).

$A + B = \{(a+b) \mid a \in A, b \in B\}$ and scalar multiplication is again as one would expect: $\lambda A = \{\lambda a \mid a \in A\}$ $(A, B \in \mathcal{K}\mathcal{Q}[L], \lambda \in \Re)$. Note that $\mathcal{K}\mathcal{Q}[L]$ is the typical space of "feasible regions" in optimization, games, pseudo-games, economies, etc. Observe that the inverse under $+$ does not exist, e.g. for $L = \Re$ and $A = [0,1]$ (The identity element is $\{\underline{0}\}$, where $\underline{0}$ is the "origin" of L, and there is no $B \in \mathcal{K}\mathcal{Q}[L]$ such that $A+B = \{\underline{0}\}$ or $B+A = \{\underline{0}\}$). Thus, we have, not only a *semi*group under $+$, but in fact a *semi*vector space under $+$ and scalar multiplication (as defined above). The topological semivector spaces developed in Prakash and Sertel (1974a) are abstracted from $\mathcal{K}\mathcal{Q}[L]$, motivated by the importance of this space for economics and game theory. As to Prakash and Sertel (1976), it invents an inverse for each element of a "hyperspace" (collection of subsets) of a tvs, as exemplified by $\mathcal{K}\mathcal{Q}[L]$, and this in a way which jives with the algebra of the semivector space and the continuity of its operations, extending the topological semivector space $\mathcal{K}\mathcal{Q}[L]$ to a topological *vector* space. Indeed, Prakash and Sertel thus embed $\mathcal{K}\mathcal{Q}[L]$ in a tvs $\widehat{L}$ whenever L is a tvs, choosing $\widehat{L}$ locally convex if L is so, etc.

Prakash and Sertel's (1974a) fixed point theory (FPT) of "locally convex" topological semivector spaces generalized the then known FPT in locally convex topological vector spaces (the FPT of Brouwer (1912), Kakutani (1941), Tychonoff (1935), Fan (1952) and Glicksberg (1952)), and so they had a relatively easy time establishing the existence of equilibrium in their 1974b paper (PS). (This is something that could not have been done, one should emphasize, by use of the Fan and Glicksberg FPT, at least directly without necessarily invoking both of Prakash and Sertel (1974a, 1976).) In the equilibrium existence theory of PS, we have a very broad and free "endogeneity," in that feasible regions and preferences as well as the behavior of agents, are endogenous, the adjustment process allowing just about everything to depend on everything else: Agents' preferences are about, not only joint behavior of all agents (the usual thing), but also feasible regions of all agents; so, best responses of agents depend on all the mentioned, witnessing a broad class of externalities in preference. Furthermore, an agent's preferences are about, not just the "current" behavior of all agents, but also pairs of "last" and "current" such behavior. So multistage settings are abstractly included to this extent. On the other hand, the feasible regions of all agents depend through a "feasibility map" on joint behavior of all agents and on themselves, in the sense that a "new" feasible region listing for all agents is a deformation of the "old" such listing as affected by the agents' choice behavior, and the manner in which an individual agent's feasible region is altered may depend, not only on the original feasible region of that agent, but on those of all other agents as well. Thus, we see a rather broad class of externalities in "production" (feasibility) incorporated.

This chapter is organized as follows. The next section discusses the literature since 1974 on abstract economies, generalized games, economies and their equilibria. Section 8.3 provides a class of examples of production economies in which the PS approach has a distinct advantage in establishing the existence of equilibrium. Section 8.4 discusses when the noncooperative equilibria of these economies will also be sustainable Walrasian equilibria and where (e.g. in the presence of switching costs) they may fail the requisite material balances condition. Section 8.5 ends the chapter with some closing remarks.

8.2 A Review of the Literature

The construct of an *abstract economy* and its *social equilibrium*, introduced by the classical paper of Debreu (1952), differ from a normal form *game* and its Cournot-Nash *equilibrium* in that, the feasible sets of the players in an abstract economy are allowed to depend on the actions of others. The social equilibrium, whose existence Debreu (1952) proves, involves a finite number of agents, each with a strategy space lying in $\Re^n$. This construct, besides being of interest in its own right for the purpose of modelling social and economic phenomena, has been a valuable tool in proving the existence of competitive equilibria of economies. The first demonstration of this is in the celebrated Arrow and Debreu (1954) paper.

Several extensions of Debreu's (1952) result have appeared in the literature. Mas-Colell (1974) demonstrated, with a novel method differing from that of Arrow and Debreu (1954), that preferences of consumers need not be transitive or complete for the existence of a competitive equilibrium in finite economies. Shafer and Sonnenschein (1975), inspired by this work of Mas-Colell (1974), and using Kakutani's (1941) fixed point theorem, showed that the preferences of agents need not be transitive or complete for the existence of a social equilibrium of an abstract economy. Borglin and Keiding (1976), using an argument concerning the existence of maximal elements, slightly generalized the Shafer and Sonnenschein result by weakening their requirements on preferences a bit further, keeping however both the number of agents and the dimensionality of their strategy spaces finite.

Another line of research aimed at relaxing the finiteness condition for the dimensionality of strategy spaces and the number of agents. Yannelis and Prabhakar (1983), relying on a continuous selection theorem of Michael (1956), allowed the strategy sets of agents to lie in any metrizable, locally convex topological vector space, permitting the set of agents to be countably infinite. Toussaint (1984) proved an equilibrium existence theorem for an arbitrary set of agents with strategy sets lying in an *arbitrary* topological vector space. Toussaint's method of proof, again (cf. Borglin and Keiding (1976)) based on the existence of maximal elements, used the Browder (1968) fixed point theorem. Khan (1985) surveyed much of this and related work. In all of the above, the maps determining feasible regions were assumed continuous. Tulcea (1988) dropped the lower semicontinuity assumption on these maps. Tarafdar (1991) provided a similar result for Hausdorff topological vector spaces and an arbitrary set of agents. After the important steps of Yannelis and Prabhakar (1983) and Toussaint (1984) in dropping the finiteness assumptions on the set of agents and the dimensionality of strategy spaces, researchers such as Tan and Yuan (1994) also made advances in relaxing certain restrictions on "better than" and "feasible set" correspondences.

A third line of research modelled the set of agents as a measure space, defining abstract economies and their social equilibria accordingly. Khan and Vohra (1984) accomplished this for finite-dimensional strategy spaces, while Yannelis (1987) as well as Kim, Prikry, and Yannelis (1989) allowed strategy spaces to be infinite-dimensional. All three studies used the Fan (1952) - Glicksberg (1952) fixed point theorem. PS does not take the measure theoretic modelling approach.

All these studies, with minor modifications, adhere to the construct of an abstract economy. In contrast, PS introduces a novel object of a *social system*. In addition to dependence of utilities and strategy sets on strategies chosen by all agents, a social system also allows for dependence of utilities and strategy sets on strategy sets of all agents. Moreover, a social system can incorporate phenomena such as addiction or stickiness in behavior (as perhaps due to switching costs). The potential applications of such a construct are numerous, as illustrated by the class of examples presented in the following section. Alternative results on abstract economies or generalized games seem to be less well-suited to be applied to such examples.

PS also proves the existence of a *non-cooperative equilibrium* of such a social system. The existence theorem places no restrictions on the cardinality of the set of agents, and the strategy sets may lie in any locally convex Hausdoff topological vector space (cf. Yannelis and Prabhakar (1983) and Toussaint (1984)). The proof is based on one of the Prakash and Sertel (1974a) fixed point theorems in topological semivector spaces.

Turning now specifically to the topic of *competitive equilibrium*, Arrow and Debreu (1954) used an abstract economy construct in proving their existence theorem for such equilibria. Thereafter, their method was frequently applied. Examples of its application in finite-dimensional and finite-agent settings can be found in Shafer and Sonnenschein (1975) and Shafer (1976). Toussaint (1984) on the other hand, gave an outstanding example, demonstrating how to use the Arrow and Debreu approach when commodity space is infinite-dimensional. This paper also generalizes Bewley's (1972) equilibrium existence result (see the next paragraph) to the case where preferences need not be complete or transitive.

Three other proof strategies in infinite-dimensional commodity spaces are worth mentioning. One approach is based on *market equilibrium*, or the so called Gale-Nikaido-Debreu lemma. Infinite-dimensional extensions of this lemma are provided by Florenzano (1983) and Yannelis (1985), and these are applied by Florenzano (1983) and Khan and Yannelis (1991), respectively, in proving the existence of competitive equilibria. A second approach has its roots in a paper of Negishi (1960), where the existence of competitive equilibrim is established by use of results from welfare economics. Magill (1981) and Mas-Colell (1986) provided infinite-dimensional applications of this approach. A third approach is the by now classical one of Bewley (1972), in $\mathcal{L}_\infty$ commodity spaces, based on a limit argument which makes use of existence results in the finite dimensional case. Khan (1984), Duffie (1986), Yannelis and Zame (1986) are prominent applications of this method.[22]

Before passing on to the next section we want to bring attention to the PS Lemma 7 (See also Sertel (1971, Section 3.1.4 and Theorem 3.3.1)) on the upper semicontinuity of optimization. This is a tidy generalization of a similar result often attributed to Berge (1959), a yet more specific form of which was shown by Debreu (1952, p. 889). The PS optimization lemma directly states that the set of optimal solutions according to a closed complete preorder on a compact space is upper semi-continuous in the feasible region on which the optimization takes place. Their proof of this result, whose typical economic application concerns the upper semicontinuity of demand, is quite compact.

The next section presents a class of examples in a private ownership economy with finitely

[22]The book edited by Khan and Yannelis (1991a) provides a more complete list of references on the subject together with a collection of some recent, as well as some older but previously unpublished, very noteworthy, work.

many agents and just two commodities. They display several sorts of technological externalities, and of preference externalities. Consumers may exhibit addiction or flip-flop tastes, and firms may experience switching costs in production. The question is the existence of a sustainable Walrasian equilibrium, and to answer this question here, we need the results of PS.

8.3 A Class of Examples

As an economic application of the PS existence theorem for non-cooperative equilibrium, in this section we will present a class of genuinely economic examples of social systems which satisfy the PS sufficient conditions and hence possess non-cooperative equilibria of economic interest. We will keep technicalities to a minimum, restricting attention to Euclidean space as our underlying tvs and in fact to a world of two commodities, for instance, but will try to incorporate in our described social systems a rich variety of interactive aspects, which are normally avoided as troublesome in equilibrium theory.

To this end, we first recall some notation of PS, to which we will henceforth adhere. For any topological space Y, $\mathcal{K}[Y]$ will denote the space of nonempty compact, subsets of Y and when Y lies in a real vector space $\mathcal{KQ}[Y]$ will denote the space of compact and convex nonempty subsets of Y.

It will be recalled that, using the terminology and notation of PS, the PS sufficient conditions for the existence of a non-cooperative equilibrium of a social system,

$$S = \{(X_\alpha, \mathcal{F}_\alpha, \preceq_\alpha, \delta_\alpha)_A\},$$

are that

(PS1) X_α be (nonempty) compact and convex in a locally convex Hausdorff topological vector space,

(PS2) $\mathcal{F}_\alpha \subset \mathcal{KQ}[X_\alpha]$ be a closed and convex cover of X_α when $\mathcal{K}[X_\alpha]$ carries the finite topology,[23]

(PS3) $\preceq_\alpha$ be closed as a subset of $(X \times \mathcal{F} \times X_\alpha)^2$, as well as upper semi-convex on X_α,

(PS4) $\delta_\alpha : X \times \mathcal{F} \to \mathcal{KQ}[\mathcal{F}_\alpha]$ be upper semi-continuous.

Throughout, we choose every $X_\alpha \neq \emptyset$ to be compact and convex in $\Re^2$, directly satisfying condition (PS1). Condition (PS3) will also be satisfied quite directly, for in each of our examples

[23] For the *finite topology* see Appendix A.1.1 of PS or directly see Michael (1951).

we will take each $\preceq_\alpha$ to be represented by a real-valued function which we check to be continuous on $X \times \mathcal{F} \times X_\alpha$ as well as quasi-concave on X_α. For the rest, we will check (PS2) and (PS4) for each example as we proceed.

The social systems in our examples will be allowed to exhibit several types of unusually interactive features. One of these will be in the form of technological externalities in the sense that technologies of firms will be allowed to be affected by each other and/or by productive activities, be these one's own or others'. Next, the profit functions of producers will be allowed to suffer switching costs. Likewise, consumers will be allowed to have preferences dependent on past behavior - be this the consumer's own or others' - and even on the feasibility (budget set) available to the consumer himself or on those available to others.

Consider an economy consisting of a sequence of spot markets in each of which, myopically, the consumers are concerned only with maximizing current utility and the firms with maximizing current profit.[24] The question then is the existence of a *sustainable* competitive equilibrium for such an economy: does there exist a list of technologies, budget sets, prices, production and consumption plans which can stay at rest while firms maximize profits, consumers maximize utility and markets clear (in the sense that there is no excess demand)? In considering this, one can also allow for the usual kinds of consumption and production externalities, which can be easily handled in the abstract economy framework of Debreu (1952). Among the extensions of Debreu's existence theorem discussed in the previous section, however, there is no work to our knowledge, whose construct of an abstract economy is rich enough to allow for the presence of the unusual features mentioned above and to be exemplified below.

To avoid introducing new terminology, we present our examples in the language of a social system, resorting also to Arrow and Debreu (1952)'s "auctioneer", a fictitious player who sets

[24] To keep the exposition simple, here we do not allow for the presence of assets or futures markets that give trading opportunities over time. These could be incorporated without much difficulty. For instance there could, on top of apples x and bananas y, be money z too, and in each period z could only be spent or saved for the next decision moment. A consumer $i \in I$ would then have initial endowments $(\bar{x}_i^t, \bar{y}_i^t, \bar{z}_i^t)$ in each period $t = 1, 2, \ldots$, where the money endowment $\bar{z}_i^t = z_i^{t-1}$ is the chosen level of nonnegative saving in the previous period, the consumption decision at t being taken so as to maximize $u_i(x_i^t, y_i^t, z_i^t)$ subject to $p^t x_i^t + q^t y_i^t + z_i^t \leq p^t \bar{x}_i^t + q^t \bar{y}_i^t + \bar{z}_i^t$ and $\bar{z}_i^t = z_i^{t-1}$ $(t = 1, 2, \ldots)$, p^t and q^t standing for the prices of x^t and y^t, respectively. Thus, in this model, z is also numérarire, and there is no borrowing from the future. (It might be instructive to think of x (or y) as labor here.)

prices with the value of excess demand as the objective function. (As a result, at any non-cooperative equilibrium of our social system, aggregate excess demand will be non-positive.)

We take our set A of individuals to be finite and partitioned into $\{\{0\}, I, J\}$, where 0 is our "auctioneer", I is the set of consumers, and J is the set of firms. For our social system $S = \{(X_\alpha, \mathcal{F}_\alpha, \preceq_\alpha, \delta_\alpha)_A\}$, we first describe the characteristics of the firms $j \in J$.

8.3.1 Firms (Producers)

Taking some $c > 0$ sufficiently large, for each firm $j \in J$, we let

$$X_j = \{y \in \Re^2 \mid \ ||y|| \leq c\}$$

be the behavior space of j. Agreeing to give the finite topology[25] to all hyperspaces of a Euclidean space, we take

$$\mathcal{F}_j = \{Y \in \mathcal{KQ}[X_j] \mid \underline{0} \in Y\}, \tag{90}$$

as the feasibility space of the typical firm $j \in J$. Thus, $\mathcal{F}_j$ covers X_j, and is convex. We also check that $\mathcal{F}_j$ is a closed subspace of $\mathcal{K}[X_j]$, the (hyper)space of non-empty compact subsets of X_j. To that end, first we note that $G = \{K \in \mathcal{K}[X_j] \mid \underline{0} \in K\}$ is closed in $\mathcal{K}[X_j]$ (For, taking $U = X_j \setminus \{\underline{0}\}$, $U \subset X_j$ is open, and $\{K \in \mathcal{K}[X_j] \mid K \subset U\} = G^c$ is therefore open, in the upper semifinite topology.). Since $\mathcal{KQ}[X_j]$ is closed by *A.1*, it follows that $\mathcal{F}_j = G \cap \mathcal{KQ}[X_j]$ is also closed in $\mathcal{K}[X_j]$, satisfying (PS2).

The preference $\preceq_j$ of any firm is that represented by its profit function $\pi_j : X \times \mathcal{F} \times X_j \to \Re$, and π_j may be of various forms. One form is that of

$$\pi_j(x, F, z_j) = x_0 \cdot z_j, \tag{91}$$

the inner product of the price x_0 (chosen by the auctioneer, 0) with the production plan z_j of

[25] See footnote 23.

firm j. Alternatively, we can incorporate switching costs[26] in π_j by giving it a form

$$\pi_j(x, F, z_j) = x_{01} min\{x_{j1}, z_{j1}\} + x_{02} z_{j2}. \tag{92}$$

Here (x_{01}, x_{02}) is the price vector chosen by the auctioneer. This reflects switching costs, by penalizing period-to-period decreases in absolute quantities of input and increases in quantities of output of commodity 1. In either form (91) or (92), π_j is readily verified to be a continuous function (so that $\preceq_j$ is closed), and it can easily be checked to be quasi-concave in z_j (so that $\preceq_j$ is upper semi-convex on X_j). Thus (PS3) is satisfied by $\preceq_j$ here.

Regarding the feasibility maps

$$\delta_j : X \times \mathcal{F} \to [\mathcal{F}_j],$$

one can imagine four interesting possibilities. The first possibility is

$$\delta_j(x, F) = \{\rho(x_j)F_j\}, \tag{93}$$

where the function ρ determines a real number $\rho(x_j) \in [0, 1]$ for each production plan x_j of firm $j \in J$. Thus, as a function of productive activity the production set is allowed to shrink but not allowed to swell. An example is where

$$\rho(x_j) = 1 - \frac{||x_j||}{2c}.$$

This captures the idea of *depreciation by use* or a sort of *depletion*. As a second possibility, the production plan of a firm may affect the productivity of the other firms. For instance, if the production of firm $j' \in J$ is detrimental for the technology of firm $j \in J$, we may see the effect of this through a feasibility map δ_j of the form

$$\delta_j(x, F) = \{\rho(x_{j'})F_j\}, \tag{94}$$

where $\rho : X_{j'} \to [0, 1]$ is as above. If instead ρ_j is a function which typically takes values in excess of unity on X_j and we write

$$\delta_j(x, F) = \{(\rho_j(x_j)F_j) \cap X_j\},$$

[26] Switching costs have often been recognized as an important factor affecting economic behavior. A most recent example (although in a different context) is in the work of Banks and Sundaram (1994):

> ...Indeed, it is difficult to imagine a relevant economic decision problem in which the decision maker may costlessly move between alternatives. (Banks and Sundaram (1994), p.687.)

this will reflect a certain type of *learning by doing*, which will preserve constant-returns-to-scale technologies. For a special learning-by-doing effect which favors small experiments in production but regards large production as depletionary, one may regard the functional form

$$\rho_j(x_j) = ||x_j||^{\frac{1}{2}}.$$

Assuming that ρ is continuous, δ_j will be (upper semi-)continuous in both forms (93) and (94) satisfying (PS4).

As a next possibility, firm $j \in J$ can *learn* parts of the technology of firm $j' \in J$ and combine these with its own know-how, to obtain its next period's technology.[27] One such type of *technology spill-over* can be modelled by the feasibility map

$$\delta_j(x, F) = \{h(F_j \cup F_{j'})\}, \tag{95}$$

where, $h : \mathcal{K}[\Re^2] \longrightarrow \mathcal{K}[\Re^2]$ is the (closed) convex hull operator. Now factorization $F \longmapsto \{F_\alpha\}_{\alpha \in A}$ is continuous, by Prakash and Sertel (1977), and the union operator is continuous on $\mathcal{K}[\Re^2] \times \mathcal{K}[\Re^2]$. Since projection $\{F_\alpha\}_{\alpha \in A} \longmapsto (F_j, F_{j'})$ is also continuous, $F_j \cup F_{j'}$ is continuous in F, hence in (x, F). By Sertel (1989), furthermore, the closed convex hull operator h is continuous on $\mathcal{K}[\Re^m]$. Thus, δ_j in (95) is also (upper semi-)continuous, again satisfying (PS4).

The fourth possibility is that firm j sees the production activity $x_{j'}$ of firm j' and, thus, "learns" all convex combinations of $x_{j'}$ with any $x_j \in F_j$. This sort of "learning from one's neighbors" is expressed by the following form of feasibility map:

$$\delta_j(x, F) = \{h(\{x_{j'}\} \cup F_j)\}. \tag{96}$$

This feasibility map is also (upper semi-)continuous, as one can establish by arguments similar to those in the case of (95), once again obeying (PS4).

Now we describe the characteristics of the consumers ($i \in I$).

[27]The firm $j \in J$ in (95) and (96) below can be seen as a bartender who always knows how to make cocktails (convex combinations) of recipes (productions) he knows, in (95) discovering an entire new recipe list $F_{j'}$ and in (96) discovering only some new recipe $x_{j'}$. Once any recipes are "discovered", their convex combinations with all known (previously discovered) recipes become known too.)

8.3.2 Consumers

A consumer $i \in I$ is specified by an ordered quadruplet

$$(X_i, \omega_i, \theta_i, u_i),$$

where picking some $d > 0$ sufficiently large, we always take the "consumption set" to be

$$X_i = \{z \in \Re^2_+ \mid z \leq (d,d)\},$$

while the "initial endowment" $\omega_i \in X_i$ with $\omega_i \gg \underline{0}$, and the "portfolio" $\theta_i \in \Re^J_+$ (with $\sum_{k \in I} \theta_{kj} = 1$ for each firm $j \in J$) are as usual, and the "utility" is some $u_i : X \times \mathcal{F} \times X_i \rightarrow \Re$, where $\mathcal{F}_j$ for $j \in J$ is given by (90), $\mathcal{F}_i$ for each consumer $i \in I$ is given by

$$\mathcal{F}_i = \mathcal{KQ}[X_i],$$

and $\mathcal{F}_0 = \{X_0\}$ with $X_0 = \{x_0 \in \Re^2_+ \mid x_{01} + x_{02} = 1\}$ comprising the price simplex for the auctioneer.

To express the idea of a budget set for consumer $i \in I$, we write

$$B(x_0, x_J; \omega_i, \theta_i) = \{x_i \in X_i \mid x_o \cdot x_i \leq x_0 \cdot \omega_i + max\{0, \sum_{j \in J} \theta_{ij} x_0 \cdot x_j\}\}, \tag{97}$$

where x_0 is the auctioneer's chosen price vector, $x_J \in X_J = \prod_{j \in J} X_j$ stands for the production plan profile of the producers. Now clearly,

$$\mathcal{F}_i \supset \{B(x_0, x_J; \omega_i, \theta_i) \mid x_0 \in X_0, x_J \in X_J\},$$

while $\mathcal{F}_i$ satisfies (PS2) since it is closed in $\mathcal{K}[X_i]$ (see *A.1*), covers X_i and is convex.

For the feasibility map $\delta_i : X \times \mathcal{F} \rightarrow [\mathcal{F}_i]$ we take

$$\delta_i(x, F) = \{B(x_0, x_J; \omega_i, \theta_i)\},$$

whose unique selection is the *budget set map* $\beta_i : X \times \mathcal{F} \rightarrow \mathcal{F}_i$ with

$$\beta_i(x, F) = B(x_0, x_J; \omega_i, \theta_i),$$

actually independent of F and of $x_I \in X_I = \prod_{k \in I} X_k$, depending only on (x_0, x_J). From *A.2* we see that β_i is continuous (since the budget size $b = x_0 \cdot \omega_i + max\{0, \sum_{j \in J} \theta_{ij} x_0 \cdot x_j\}$ is continuous in $(x_J, \omega_i, \theta_i)$) and hence also δ_i is (upper semi-)continuous, thus satisfying (PS4).

So far our consumers are quite classical text-book types; what will distinguish them from the run of the mill will be their preferences. Each consumer i's preference $\preceq_i$ will be represented by the utility $u_i : X \times \mathcal{F} \times X_i \to \Re$, and it is in the specification of this utility that we capture certain possible paradigmatic characteristics which our consumers may exhibit. For this purpose, it will suffice to regard the general Cobb-Douglas-inspired form

$$u_i(x, F, z_i) = z_{i1}^{\mu_i(x,F)} z_{i2}^{1-\mu_i(x,F)}$$

where, $\mu_i : X \times \mathcal{F} \to [0,1]$ is a continuous function determining the elasticity of utility w.r.t. the consumption of the two goods, z_{i1}, z_{i2}. The continuity of μ_i ensures that of u_i, so $\preceq_i$ is always closed. The fact that $\mu_i(x, F) \in [0,1]$ ensures that u_i is concave in z_i, so $\preceq_i$ is certainly upper semi-convex on X_i. Thus, (PS3) is satisfied by $\preceq_i$ here.

First we consider the case of μ_i dependent only on x, in particular $\mu_i(x, F) = r_i(x)$, so that

$$u_i(x, F, z_i) = z_{i1}^{r_i(x)} z_{i2}^{1-r_i(x)}$$

where r_i is continuous. Specifically, setting

$$r_i(x) = \frac{x_{i1}}{x_{i1} + x_{i2}}$$

exemplifies a paradigm of ***addiction***. The more the consumer i consumes of alcoholic beverages (x_{i1}), the more partial he becomes toward them in his next consumption decision (z_{i1}). Alternatively, setting

$$r_i(x) = \frac{x_{i2}}{x_{i1} + x_{i2}},$$

we obtain a ***"flip-flop" boredom*** effect, whereby higher consumption of one good renders the consumer more easily bored by this good and tilts him to be more partial towards the other in his next consumption decision.

Furthermore, writing

$$r_{i'}(x) = \frac{x_{i1}}{x_{i1} + x_{i2}}$$

gives us an ***emulation*** effect, where consumer i' becomes more partial toward wearing dark suits ($x_{i'1}$) the more his boss i wears them (x_{i1}). Likewise, writing

$$r_{i'}(x) = \frac{x_{i2}}{x_{i1} + x_{i2}}$$

gives the opposite effect, which we may call ***"negative emulation"*** where consumer i' becomes *less* partial toward wearing dark suits the more his boss' butler i wears them.

The peculiarities of consumer preferences displayed so far are in the genre of externalities which have certainly been discussed both verbally (e.g. by Veblen(1899) in the case of emulation) and theoretically (e.g. by McKenzie (1955),[28] Shafer and Sonnenschein (1975), von Weizsäcker (1971)), and our indicating to such possibilities heralds no news. The next family of possibilities in preference externalities, however, may not be such old hat in general equilibrium existence theory. For now we take μ_i to be of the form $\mu_i(x, F) = s_i(F_i)$, dependent on the feasibility F_i (alone), where the function $s_i : \mathcal{F} \longrightarrow [0, 1]$ is continuous. Thus, we are now looking at the case where

$$u_i(x, F, z_i) = z_{i1}^{s_i(F_i)} z_{i2}^{1-s_i(F_i)},$$

in order to model a certain *freedom effect.* [29] Consider, first, the specific form

$$s_i(F_i) = \frac{1}{\sqrt{2}d} sup\{\ ||z||\ \mid z \in F_i\}, \tag{98}$$

where s_i measures the sup norm of the consumer's own feasibility F_i. In this case, the greater the consumer's freedom in choosing z with large norm, the less his partiality for happiness pills (z_{i2}).[30] Alternatively, we can take either of

$$s_i(F_i) = \frac{1}{d^2} sup\{\ z_1 z_2\ \mid z \in F_i\}, \tag{99}$$

$$s_i(F_i) = \frac{1}{d} max\{sup\{z_1 \mid z \in F_i\}, sup\{z_2 \mid z \in F_i\}\}, \tag{100}$$

$$s_i(F_i) = \frac{1}{d} min\{sup\{z_1 \mid z \in F_i\}, sup\{z_2 \mid z \in F_i\}\}, \tag{101}$$

$$s_i(F_i) = \frac{1}{d^2} sup\{z_1 \mid z \in F_i\} sup\{z_2 \mid z \in F_i\}, \tag{102}$$

as a measure of the consumer's freedom of choice,[31] his partiality for jolly pills (z_{i2}) decreasing

[28]The entry on "externalities" by J. J. Laffont (1987) in the *New Palgrave: A Dictionary of Economics* mentions only McKenzie (1955) regarding equilibrium existence results in the presence of externalities.

[29]One should be cautious not to regard this as a Slutsky-like income (or wealth) effect, as the examples below should clearly show.

[30]The reader may find it entertaining to observe and tabulate (as below) demand on budget sets $B = h(\{(0,0), (d,0), (0,d)\})$, $B' = h(\{(0,0), (\frac{d}{2},0), (0,d)\})$, $\frac{1}{2}B = h(\{(0,0), (\frac{d}{2},0), (0,\frac{d}{2})\})$, $B_n = h(\{(0,0), (\frac{d}{2},0), (0,\frac{d}{n})\})$, $B_\infty = h(\{(0,0), (\frac{d}{2},0), (0,0)\})$.

Budget set	B	B'	$\frac{1}{2}B$	B_n	B_∞
Demand	$(d,0)$	$(\frac{d}{2},0)$	$(\frac{d}{4},\frac{d}{4})$	$(\frac{d}{4},\frac{d}{2n})$	B_∞

[31]Another possible case one might consider for $s_i(F_i)$ is also the normalized Lebesque measure of F_i (i.e. the measure divided by d^2). cf. Kreps (1979), Barberá and Pattanaik (1984), Barberá, Barret and Pattanaik (1984), Pattanaik and Xu (1990).

with freedom. Likewise, we could let $\mu_i(x, F)$ depend instead on other consumers' feasibilities, for instance by setting in each above case $\mu_i(x, F) = s_i(F_{i'})$ for some $i' \in I \setminus \{i\}$. The cases (98) to (102) above would then have their analogous forms (9') to (13') with F_i replaced by $F_{i'}$.

Of course, various combinations and derivatives of these forms -and certainly yet other forms- can be imagined for the functions μ_i. As remarked at the outset of specifying the Cobb-Douglas-like form with elasticities μ_i and $1 - \mu_i$, all of the above considered forms of externality in preference are admissible from the viewpoint of satisfying the PS sufficient conditions for the existence of equilibrium, since preference $\preceq_i$ in each case is closed and on X_i upper semi-convex.

8.3.3 The Auctioneer

Finally, regarding our auctioneer $0 \in A$, we specify

$$X_0 = \{p \in \Re^2_+ \mid p_1 + p_2 = 1\},$$

$$\mathcal{F}_0 = \{X_0\},$$

with $\delta_0 : X \times \mathcal{F} \to [\mathcal{F}_0]$ taken as the constant function with value

$$\delta_0(x, F) = \{X_0\}.$$

The auctioneer's preference is that represented by $u_0 : X \times \mathcal{F} \times X_0 \to \Re$ defined through

$$u_0(x, F, z_0) = z_0 \cdot (\sum_{i \in I}(x_i - \omega_i) - \sum_{j \in J} x_j),$$

Thus, our auctioneer is quite classical, $\mathcal{F}_0$ clearly obeying (PS2), $\preceq_0$ obeying (PS3), and δ_0 trivially satisfying (PS4).

8.4 Non-Cooperative Equilibria as Sustainable Walrasian Equilibria

We have run through a broad class of examples allowing for several sorts of endogeneity in technologies and various kinds of technological externality. Then again we have considered over a dozen types of endogeneity in preferences or of preferential externalities. In each case we have checked that the sufficient conditions PS1-PS4 of Prakash and Sertel for the existence of a non-cooperative equilibrium are satisfied. This means that we could take several different types of producer, each subject to one or another of our various technological externalities or switching costs, and we could mix this motley bunch with a yet motlier bunch of consumers, each exhibiting one or another of the various preferential externalities we have discussed, and we would still be guaranteed the existence of a non-cooperative equilibrium for the social system they comprise. These equilibria would furthermore be genuinely *economic* equilibria and, at least in the absence of switching costs, *Walrasian* in the usual sense of satisfying material balances and price taking individual optimization - in the form of profit maximization subject to production sets for producers and utility maximization subject to budget constraints of consumers - as well as *sustainable* in the sense that the endogenous feasibilities are also at rest.

To be more specific, in the absence of switching costs (e.g. when all the firms have profit functions of form (91)), we can show by standard arguments that at any non-cooperative equilibrium (x^*, F^*), the material balances condition

$$\sum_{i\in I}(x_i^* - \omega_i) - \sum_{j\in J} x_j^* \leq \underline{0} \tag{103}$$

holds. To that end, first we observe that $x_0^* \cdot x_j^* \geq 0$ has to hold for each $j \in J$ since $\underline{0} \in F_j^*$. Hence the feasibility (budget, see (97)) requirement on consumer's choice can be re-expressed as

$$x_0^* \cdot (x_i^* - \omega_i) - \sum_{j\in J} \theta_{ij} x_0^* \cdot x_j^* \leq 0 \tag{104}$$

for each $i \in I$. Adding this inequality over consumers, we obtain

$$x_0^* \cdot (\sum_{i\in I}(x_i^* - \omega_i) - \sum_{j\in J} x_j^*) \leq 0 \tag{105}$$

which shows that the maximum "utility" of the auctioneer is non-positive. Now suppose that in some commodity the aggregate excess demand were positive, permitting the auctioneer higher utility u_0 than that at the fixed point (by adjusting prices suitably). This would contradict that $u_0(x^*, F^*, \cdot)$ is maximized on X_0 at x_0^*, and so we conclude that aggregate excess demand in each commodity is non-positive.

In the presence of switching costs (i.e. when some firms have profit functions of form (92)), however, a non-cooperative equilibrium need not satisfy material balances. To see where the standard argument fails, imagine the possibility of a non-cooperative equilibrium where $x_{1j}^* = -1$, $x_{2j}^* = 0$, and $x_{01}^* = 1$. Now at equilibrium $z_j^* = x_j^*$ and profit $\pi_j^* = x_{01}^* z_{1j}^* + x_{02}^* z_{2j}^* < 0$, but the consumers' limited liability (see (97)) prevents this loss from being reflected in the typical consumer's budget, so consumer i behaves as if the loss of j is not his liability even if he fully owns firm j ($\theta_{ij} = 1$) and owns no other stock ($\theta_{ik} = 0$ for every $k \in J \setminus \{j\}$). But then (104) does not follow from (97). Therefore with switching costs of the type we have supposed (92), we cannot guarantee that non-cooperative equilibria will also be sustainable Walrasian.

8.5 Closing Remarks

To be reader-friendly and as simple as possible in our exposition, we have kept our examples in the realm of a strip-miner, i.e. we have only scratched the surface of the rich domain of imaginable cases covered by the PS sufficient conditions for the existence of equilibrium, and economists will easily find more examples relevant to their special interests. Where the applications of the PS equilibrium existence theorem may be especially fruitful is in the general area of public economics, abundant in externalities. In particular, environmental economists are welcome to apply the theorem in their domain of discourse, just as researchers in adjustment costs, in sustainable growth, in endogenous growth may find it attractive to take advantage of the variety of externalities allowed by the theorem.

Finally, the facility of dealing with hyperspaces in the PS model may be attractive to researchers in extending preferences to hyperspaces, as in the literature on ranking of opportunity sets (e.g. Kreps (1979), Barberá and Pattanaik (1984), Barberá, Barret and Pattanaik (1984), Pattanaik and Xu (1990)). It will have been noted that individuals' preferences in the PS social systems already comprise such extensions as their restrictions to appropriate subspaces.

Appendix to Chapter 8

A.1 THEOREM: Let $X \neq \emptyset$ be a locally compact space lying in a real Hausdorff topological vector space L. Then $\mathcal{K}\mathcal{Q}[X] \subset \mathcal{K}[X]$ is closed, under the finite topology on $\mathcal{K}[X]$.

Proof. Defining $\mathcal{H} = \mathcal{K}[X] \setminus \mathcal{K}\mathcal{Q}[X]$, take any $H \in \mathcal{H}$. We construct an open nbd $\mathcal{U}$ with $H \in \mathcal{U} \subset \mathcal{H}$. To this end, observe that, since H is non-convex, there are points $x, x' \in H$ and $\lambda \in [0,1]$ such that $\bar{x} = \lambda x + (1-\lambda)x' \notin H$. As convex combination with coefficients $(\lambda, 1-\lambda)$ (i.e. the function $\Lambda : L \times L \to L$ defined through $\Lambda(y, y') = \lambda y + (1-\lambda)y'$) is continuous, taking any open nbd $V \subset X \setminus H$ of $\bar{x}$, there are open nbds U and U' of x and x', resp., such that $\lambda y + (1-\lambda)y' \in V$ whenever $y \in U$ and $y' \in U'$ (i.e. $\Lambda(U \times U') \subset V$). As L (hence $X \subset L$) is Hausdorff and X is locally compact, w.l.o.g., V may be assumed compact. Thus, $V^c = X \setminus V$ is an open nbd of H. Now writing $\mathcal{U} = \langle U \rangle_- \cap \langle U' \rangle_- \cap \langle V^c \rangle_+$, where $\langle U \rangle_- = \{K \in \mathcal{K}[X] \mid K \cap U \neq \emptyset\}$, $\langle U' \rangle_- = \{K \in \mathcal{K}[X] \mid K \cap U' \neq \emptyset\}$ and $\langle V^c \rangle_+ = \{K \in \mathcal{K}[X] \mid K \subset V^c\}$, we have $H \in \mathcal{U} \subset \mathcal{H}$, while $\langle U \rangle_-$ and $\langle U' \rangle_-$ are sub-basic open sets in the lower semi-finite topology and $\langle V^c \rangle_+$ is a basic open set in the upper semi-finite topology on $\mathcal{K}[X]$, so $\mathcal{U}$ is open in the finite topology (i.e. the coarsest topology containing both the upper and the lower semi-finite topologies) on $\mathcal{K}[X]$. Thus, $\mathcal{H}$ is open, i.e. $\mathcal{K}\mathcal{Q}[X]$ is closed in $\mathcal{K}[X]$, as to be shown.

A.2 CONTINUITY OF BUDGET CORRESPONDENCES:

Proving the continuity of the "budget correspondences" $B_i : (p, b) \mapsto B_i(p, b) = \{x_i \in X_i \mid p \cdot x_i \leq b\}$ on $X_0 \times (0, \infty)$, where X_0 is the price simplex (here in $\Re^2_+$), is by now a standard exercise for economists.[32] Nevertheless, we would like to give a direct proof.

Using A.1.5 of PS, we first establish the upper semicontinuity of B_i by showing that the graph $G(B_i) = \{(p, b, x) \in X_0 \times (0, \infty) \times X_i \mid p \cdot x_i \leq b\}$ of B_i is closed. For this, simply note that the "deficit" $v(p, b, x_i) = p \cdot x_i - b$ is continuous and that $G(B_i) = v^{-1}((-\infty, 0])$ is therefore closed.

To establish the lower semicontinuity of B_i, we take any open $V \subset X_i$ and show that the

[32] In fact Ichiichi (1983) sets it as Exercise 1 (p.39), suggesting a proof in several (eight) spelled-out steps.

inverse image $B_i^{-1}(\langle V\rangle_-)$ of the (sub-basic) open set $\langle V\rangle_-$ (of the l.s.f. topology) is open. To that end, take any $(\bar{p},\bar{b}) \in B_i^{-1}(\langle V\rangle_-)$. Thus $B_i(\bar{p},\bar{b}) \cap V \neq \emptyset$ and, since $b > 0$, there exists $\bar{x} \in V$ such that $\bar{p} \cdot \bar{x} - \bar{b} < 0$. Since the deficit function v is continuous, there exists a nbd U of $(\bar{p},\bar{b})$ such that for all $(p,b) \in U$, $v(U \times \{\bar{x}\}) \subset (-\infty, 0)$, so that $\bar{x} \in B_i(p,b)$ and hence $B_i(U) \subset \langle V\rangle_-$. Thus, $B_i^{-1}(\langle V\rangle_-)$ is open, completing our proof.

9 Concluding Remarks on Implementation and Welfare Theorems

We have defined a society as a set of individuals, a set of outcomes, and the individuals' preferences over the outcomes. The social choice literature after the path-breaking work of Arrow (1951), has tried to develop with a reasonable way of aggregating these preferences in order to obtain some ordering which deserves the name of a *social preference*. This ordering then could be used to choose, among the possible outcomes, socially the most preferred ones.

The difficulties associated with constructing a *social preference* are not present with a *social choice rule*, which takes the list of preferences as its input and provides a subset of outcomes as *chosen* outputs. Suppose the society unanimously agrees on a reasonable rule and wants to implement it. However, the preferences of all the individuals are not known. Moreover it is not clear that they would tell the truth if they were asked to reveal their preferences. The *mechanism design* literature initiated by Hurwicz (1960, 1972, 1975) points to this problem and aims to solve it in an *informationally decentralized* manner by introducing the idea of a *game form.*

A *game form* is a collection of message spaces, one for each individual, and an outcome function which maps the list of messages to the outcome set. When adjoined with the preferences of the individuals over the outcomes, a game form determines a game. If it turns out that the corresponding outcome to solution of the game agrees with the social choice rule for all possible preferences from a prespecified class, the game form is said to *implement* the social choice rule in a given solution concept. The solution concepts are usually sought among non-cooperative ones. The most non-cooperative and hence the most informationally decentralized one is the dominant strategy equilibrium. The use of this solution concept led to the literature on *implementation in dominant strategies.* Similarly the literature on *implementation in Nash equilibria* has emerged due to difficulties in implementability in dominant strategies.

A game form together with a non-cooperative solution concept is informationally decentralized because there is no need for a central authority to worry about obtaining information related to the preferences of the individuals to make the system function.

An institution is a more general object than a game form in three ways. First of all there is a set of parameters to be determined by the *center*. The parameter selected by the center may cost pricing literature. This literature tries to find suitable extensions of the first and second fundamental theorems of welfare economics to the cases with increasing returns technologies. Here, the center is to determine and enforce a price system at which some of the nonconvex firms may in fact be making losses.

Another interesting question is the optimality properties of equilibria when the price system is not complete. Under incomplete markets it is trivially easy to show that Walrasian equilibria are not (full) Pareto optimal. Moreover, the example given by Hart (1975) shows that even among the Walrasian equilibria of an incomplete market system, Pareto rankings are possible. So even restricted analogs of the welfare theorems may not be available. Clearly a center with a Samuelson-Bergson objective function would help in such a situation to attain a Pareto optimal allocation relative to the equilibrium outcomes (See Section 1.4, Definition 14). This kind of implementation will be called *implementation in COE.* The informational requirements on behalf of the center for implementation in COE would be higher, since the center is required to know the preferences at least over the equilibrium outcomes.

To formalize these ideas, we will use the notation of previous chapters. Let a set of individuals I and an outcome set C be given. Every preference profile $(B_i)_i \in I$ will give us a society. Let the space of permissible preference profiles be denoted by $\mathcal{B}$.

A social choice correspondence is a (nonempty) set-valued map

$$S_c : \mathcal{B} \rightarrow C.$$

Examples in the context of a pure exchange economy are the Pareto efficient map, the individually rational map (with reference to endowments), imputation map, core map, Walrasian from equal division map, etc.

An institution $N = (P, (X_i, \tau_i)_{i \in I}, g, F, W))$ is said to (weakly) implement a social choice correspondence S_c in SECERE if

$$g(p^*, x^*) \in S_c(B)$$

non-vacuously for all SECEREs (p^*, x^*) of the corresponding economic system for all permissible preference profiles B in $\mathcal{B}$.

As an example, the complete markets institution implements the core correspondence (in SECERE). In a public goods economy, the Lindahl price mechanism similarly implements the core correspondence (in SECERE).

These are well known examples. There are many other economic environments where the implementation via institutions issue is yet unresolved. Examples are, presence of production and consumption externalities, nonconvexities in preferences and/or technologies.

Semicentralized resource equilibrium may not sound as a relevant concept at the first glance for the real life economic systems we observe. On a second reflection, however, one can find examples where it could be *the* suitable equilibrium concept. An obvious example is the formation of the legal system. The law makers in fact define the limits of freedom of action (via the constraint correspondence) for each individual as well as the consequences of the actions taken by the society (via the outcome function). These are explicitly announced to the public as written rules. Usually, however, it is the case that *equilibrium* choices are resource feasible, while individually feasible but disequilibrium choices may not be so. In other words the legal system works the way it is announced if individuals pursue noncooperative maximization. Under an alternative social behaviour, the system, because of physical limitations, might not work the way it was announced.

An example to implementation via institutions, then, is the formation of a constitution. You want to provide a constitutionally well described legislatory institution to the potential parliament in such a way that regardless of the (equilibrium) laws that the parliament passes, the equilibrium be acceptable (resource feasible) on social grounds. In such a case, you can sleep well knowing that your constitution will implement your social goals regardless of who wins the elections.

Another familiar example is related to freedom of choosing where to live. The government promises all the citizens of a country this basic freedom. The government also promises to provide them with basic public infrastructure as long as they pay their taxes. Moreover, tax system will not depend on the city you live. Despite these announcements, suppose there is a capacity of each city, after which it is very costly and hence not possible to provide infrastructure under the revenue provided by the announced tax system. Now this is a simple legal institution. It may turn out that in equilibrium, the population among the cities is more or less balanced so that the promised public goods can be provided. In such a case, we would call the equilibrium a SECERE. But suppose all the society suddenly decides to live in one and the same city, say Istanbul. Then this outcome clearly is not feasible to the announced rules. In such a case, a normally expected behavior from a government is to change the announced rules rather than to lose credibility. One possibility is to abandon the freedom to migrate. Another possibility is to give up the promises on infrastructure. Yet a third one is to play with the taxation rules, possibly allowing taxes to vary across cities. It may turn out that each of these policy alternatives will lead to a feasible outcome. Then each one of them will be called a SECERE. But the government may choose among SECEREs, the best one according to its preferences over outcomes. The set of such best equilibria, we call center's optimal equilibrium (COE). Now, both maintaining the feasibility of the outcome and choosing among all feasible equilibria, naturally requires a Stackelberg-leader reasoning from the center. A Cournot-Nash reasoning clearly would not work since it would be like the mistake of a retailer who triples its price with the assumption that the quantity sold will not change. Therefore the SECERE and COE concepts are quite relevant in many circumstances where a *center* can be identified. Moreover, in the absence of a center, we obtain the familiar constructs of either a game or an abstract economy (a generalized game) whose Cournot-Nash equilibria coincide with SECERE.

If we go back to the example of Walrasian equilibrium of a pure exchange economy, the analogy with the migration example above is immediate. Here, the parameters determined by the center are prices. But if the prices are set wrongly, the promises of the market as an institution will not be fulfilled. Moreover, genericly, there are more than one Walrasian equilibria as shown by Debreu (1970). It may be the case that the center has preferences over these equilibria and may select among the best SECERE.

All these examples seem to constitute future research projects that will be partly based on the framework introduced in this dissertation.

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Vita

Erdem Başçı was born in Ankara, on August 9, 1966. After completing his primary and secondary education in T.E.D. Ankara College, he studied Electrical and Electronics Engineering at the Middle East Technical University in Ankara from September 1983 to June 1987, graduating with a high honors B.Sc. degree. Between September 1987 and June 1989, he continued his graduate studies in the field of Business Administration at the Department of Management of Bilkent University with the help of the fellowship provided by Bilkent. His M.B.A. dissertation was on "The Behavior of Stock Returns in Turkey: 1986-1988" supervised by Professor Kürşat Aydoğan. His interest in Economics during the M.B.A. studies attracted him to the field, and in September 1989, he joined the Department of Economics of Bilkent University as a research assistant and started the graduate program there. His Master's thesis in Economics, titled "Rationality of Inflation Expectations in a Financially Repressed Economy" was supervised by Professor Sübidey Togan and was completed by September 1990. After completing the coursework of the Economics Ph.D. program of Bilkent in June 1991, he spent the 1991-92 academic year at Johns Hopkins University Department of Economics with the scholarship provided by Bilkent University, receiving a second M.A. degree in Economics from Johns Hopkins. On his return to Bilkent in June 1992, he has taught the graduate macroeconomics sequence jointly with Süheyla Özyıldım for three academic years. He also served in organizing the Department seminars and in preparation and grading of Ph. D. comprehensive examinations at Bilkent. He has two co-authored papers accepted for publication in *Journal of Banking and Finance* and *Journal of Mathematical Economics.* He is currently working on macroeconomic theory and general equilibrium theory of the second best. Starting from September 1995 he will be appointed as an Assistant Professor at the Department of Economics, Bilkent University. He is married with Sıdıka Başçı and they have one child, Uğur.