CURRENCY SUBSTITUTION: A NUMERICAL DYNAMIC PROGRAMMING APPROACH

A Thesis
Submitted to the Department of Economics
and the Institute of Economics and Social Sciences of
Bilkent University
In Partial Fulfillment of the Requirements
for the Degree of

MASTER OF ARTS IN ECONOMICS
by

Engin Volkan
June 1998

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I certify that I have read this thesis and in my opinion it is fully adequate in scope and in quality as a thesis for the degree of Master of Arts in Economics.

Assistant Professor Dr. Erdem Başçı

I certify that I have read this thesis and in my opinion it is fully adequate in scope and in quality as a thesis for the degree of Master of Arts in Economics.

Assistant Professor Dr. Tuvana Pastine

I certify that I have read this thesis and in my opinion it is fully adequate in scope and in quality as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

CURRENCY SUBSTITUTION:

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Engin Volkan

MA in Economics

Supervisor: Assistant Professor Dr. Erdem Başçı

June 1998

This thesis conducts a theoretical study on currency substitution in an infinitely-lived small open financially repressed economy which is subject to stochastic inflation shocks. For this purpose, a dynamic programming model is constructed under the assumption that purchasing power parity holds. The solution of the model through value function iteration shows that under high inflation, and financial repression, the inhibitants of an economy will demand foreign currency to the extend that it provides a better protection of their wealth against inflation.

Key Words: Currency Substitution, Bellman's equation, Value function iteration, Cash-in-advance, Transaction costs

ÖZET

PARA İKAMESİ:

SAYISAL DİNAMİK PROGRAMLAMA YAKLAŞIMI

Engin Volkan

Yüksek Lisans Tezi, İktisat Bölümü

Tez Danışmanı: Yardımcı Doçent Dr. Erdem Başçı

Haziran 1998

Bu tez, stokastik enflasyon şoklarına maruz kalan sonsuz ömürlü küçük, açık ve mali piyasaları az gelişmiş bir ekonomideki para ikamesini teorik olarak çalışmaktadır. Bu amaçla, satın alma paritesinin tuttuğu varsayımıyla bir dinamik programlama modeli yapılmıştır. Model değer fonksiyonu iterasyonuyla çözülerek gösterilmiştirki yüksek enflasyon ve mali az gelişmişlik altında ekonominin sakinleri servetlerini enflasyona karşı koruduğu sürece döviz talebinde bulunacaklardır.

Anahtar Kelimeler: Para İkamesi, Bellman denklemi, Değer fonksiyonu iterasyonu, Ön-ödeme, İşlem maliyeti

To my father

Dad I finished…

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SECTION I

INTRODUCTION

The literature on *demand for foreign monies* is considerably wide. In fact, there have been important theoretical contributions in this area since 1980's. The *overlapping generations* (OLG) model introduced to the literature by Kareken and Wallace (1981), a *two-country model* with a *cash-in-advance constraint* by Lucas (1982), a model with *money-in-the-utility function* by İmrohoroğlu (1994), and many others can be listed as prominent examples.

In my thesis, I attempt to conduct a theoretical study on currency substitution in an infinitely-lived small open financially repressed economy which is subject to stochastic inflation shocks. I find the extend of currency substitution by calculating the proportion of foreign currency in the money holdings of the agents. I construct the model as a one-country model with a cash-in-advance constraint and a transaction cost function. There is a composite good -governing both the domestic and international goods- that is freely traded in the goods market. Thus, for the nominal exchange rate purchasing power parity always holds exactly. There are no interest bearing assets and the only financial asset in the economy is money. There are two different currencies in the economy, namely foreign and domestic currency. I assume that foreign currency dominates the domestic currency in its store of value function. Allowing the agents to have access to foreign exchange during the goods market, but at a high transaction cost, and the cash-in-advance requirement in domestic currency faced during the goods market, provides framework for endogenous currency substitution. The key features of the model are as follows: First, there is a cash-in-advance constraint in the goods market where only domestic money is acceptable. Second, there is an opportunity to convert the foreign currency to the domestic one in the goods market but at a bad rate. Third, the consumption takes place right after the goods market is opened, and before the income is received. Fourth, after the income is received there is an efficient market exchange rate available to convert income to foreign currency before the next period comes. Sixth, I assume that the purchasing power parity holds and foreign currency is a perfect hedge against inflation. Finally, inflation is announced right before the goods market opens - agents know the Markov process of inflation.

Assuming that the domestic price level is specified as a stochastic process and is taken as given by the agents, the model will constitute a partial equilibrium analysis. By using this approach I aim to show that under high inflation, and financial repression, the inhibitants of an economy will demand foreign currency to the extend that it yields a better protection of their wealth against inflation.

While depreciation rate is a direct factor affecting the demand for foreign currency, transaction cost, on the other hand, enters to the model as an indirect factor. This is due to the fact that under a very low transaction cost agents ignore this cost and demand more foreign money as their savings, however, high levels of cost will prevent them from saving their money for liquidity purposes in the form of foreign currency.

After a review of literature in section II, the model will be introduced in section III. Section IV will contain information about the solution methodology, which is one of the numerical dynamic programming solution methods. Section V will contain concluding remarks.

SECTION II

LITERATURE SURVEY ON CURRENCY SUBSTITUTION

Currency substitution has received considerable attention since mid - 1970s during which fixed vs. flexible exchange rates debate was persistent. There has been a broad range of literature of currency substitution[1]: taking currency substitution as the main focus or as a cause of a macroeconomic problem. A closer look to all these studies reveals that the way in which currency substitution is defined, also varies. While some of these studies pose the definition of currency substitution as the use of different currencies as media of exchange, and/or store of value, and/or unit of account, some others define it as a situation in which domestic money demand is influenced by foreign economic variables. Finally, in papers that study currency substitution in Latin America, the concept of dollarization is used semantically similar to currency substitution. Ignoring the semantic differences, it would be convenient to classify the studies according to their contributions to the literature in terms of their theoretical, empirical and econometric content.

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[1] For a detailed discussion of this literature, refer to the survey by Giovannini and Turtelboom(1994)

The theoretical literature captures the concept of currency substitution through different treatments. In fact, such treatment differences, in turn, determine which function(s) of a currency is substitutable. Some of the theoretical studies include foreign currency into their model via a cash-in-advance constraint therefore, consider foreign currencies as substitutes for domestic currency only in transactions[2]. Some others use transaction cost functions in budget constraints[3]. With the help of these transaction costs functions they create an incentive for agents to hold foreign currency as a store of value, since foreign currencies are no more liquid than other financial assets. On the other hand, there are models that prefer capturing the currency substitution idea by a money-in-the-utility function[4]. They consider money as having a value and being held because it provides a return, namely liquidity services. Liquidity services are modeled by a CES production function where inputs are the real balances of domestic and foreign money. The most important of studies that intent to capture the degree of currency substitution

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[2] Lucas (1982)
[3] Poloz (1986)
[4] Imrohoroğlu (1994)

in a country are İmrohoroğlu (1994), and Bufman and Leiderman (1993). İmrohoroğlu, uses a small open economy whose inhibitants are optimizing a money-in-the-utility function where the currency substitution concept is introduced to the model through a CES function for domestic and foreign currency- this function is defined as the money services. He uses the first-order conditions of optimization and Hansen's GMM procedure to directly estimate the elasticity of substitution of currencies between Canadian and U.S. dollars. Bufman and Leiderman (1993) is an extended version of İmrohoroğlu (1994) with a nonexpected utility incorporated to the model. This is done in order to separate the behavior towards risk and intertemporal preferences. There are quite a number of theoretical models that study the implications of currency substitution. These models can be viewed under three major topics: exchange rate indeterminacy, exchange rate volatility, and inflationary finance. Exchange rate indeterminacy implication of currency substitution is shown by Kareken and Wallace (1981). They use an OLG model, which provides costless transaction possibility for currencies that are viewed solely as financial assets. Thus, monies are held for their store of value function.

Exchange rate volatility implication of currency substitution stems out from fixed vs. flexible exchange rates debate, and is a very popular problem area. Calvo and Rodriguez (1977), provides a model of exchange rate determination under currency substitution and rational expectations, where a monetary disturbance leads to fluctuations in the real exchange rate associated with currency substitution. Canzoneri and Diba (1993), analyzes the exchange rate volatility implications of currency substitution and capital mobility in the European monetary union framework. Lucas(1982), Boyer and Kingston (1987) both incorporate cash and credit goods and a cash-in-advance constraint imposed on the cash good in order to observe the exchange rate volatility due to currency substitution. Lucas (1982), allows consumers to buy goods with the seller country's currency only. This yields an equilibrium exchange rate that is determined in the goods market and that does not depend on future expectations. An interesting approach to exchange rate volatility is seen in Sibert and Ha (1997). The authors use an OLG model, which has a parameter measuring the degree of portfolio substitutability and in which a locally unique exchange rate exists that is determined in financial markets. Some other studies on exchange rate volatility due to currency substitution are Lapan and Enders (1983), Marquez (1984), Chand and Onitsuka (1985), Daniel (1985), Ratti and Jeong (1994), Agenor (1995), Mahdavi and Kazemi (1996). Finally, it is known that currency substitution has unavoidable effects on inflationary finance. Inflation tax has been a popular policy tool used in developing countries since in contrast to other taxes it has zero collection costs. However, currency substitution in developing countries also has a negative effect on the inflation tax revenue attained. This implication has been studied by Kimbrough (1986), Végh (1989), Sibert and Liu (1993), Guidotti (1993), Chang (1994), Végh (1995), İmrohoroğlu (1996), and Selçuk (1994). Mainly, all these studies analyze the optimality of inflation tax under currency substitution. It is known that as the degree of substitutability increases, deficit financing through monetization becomes difficult. Hence, at any inflation tax the revenue decreases as currency substitution increases. Kimbrough (1986) concludes that under currency substitution an optimal taxation policy requires zero nominal interest rate. Chang (1994), uses an OLG model, where both currency substitution and inflation is explained as endogenous equilibrium outcomes whose coexistence depends on fundamentals of the economy. İmrohoroğlu (1996), concludes that in a low-inflation economy the seigniorage-maximizing inflation rate can be quite high despite a high elasticity of substitution when the share of foreign real balances are low in liquidity services.

In empirical studies, currency substitution has been associated mainly with three macroeconomic issues: hyperinflation, developing countries, and economic and monetary integration[5]. It is well known that during hyperinflation the opportunity cost of holding the national currency is so high that currency substitution is an expected outcome. Abel et al. (1979) analyzes the evidence for currency substitution in Germany during its hyperinflationary period. Currency substitution in a developing country could be seen under two headings:

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[5] EU is the core of the subject

macroeconomic instability and existence of an underdeveloped financial market. Two examples of such studies that are conducted for Latin America, are Canto and Nickelsburg (1987), Savastano (1992).

Finally, it is worth mentioning some major econometric studies on currency substitution. Miles (1978), Bordo and Choudri (1982), İmrohoroğlu (1994), Girton and Roper (1981), Frenkel (1977), Agénor and Khan (1996) are some examples. Among these Miles (1978), Bordo and Choudri (1982), and Girton and Roper (1981), Frenkel (1977) estimate portfolio balance models, İmrohoroğlu (1994) and Agénor and Khan (1996) use the first order conditions from an optimization problem in forming the econometric model.

This thesis is a theoretical study on currency substitution. It is different from other studies in terms of being a partial equilibrium analysis, and using a numerical dynamic programming approach. It improves upon other infinite horizon models since it assumes foreign currencies both as a store of value and a medium of exchange, therefore, uses both cash-in-advance constraints and transaction cost. The infinitely-lived framework is an improvement over OLG models in terms of providing a better explanation for the large numbers of transactions observed between domestic and foreign currencies. Finally, by inserting foreign currency through institutional constraints, it overcomes the deficiencies created by money-in-the-utility function.

SECTION III

THE MODEL

I consider a small open financially repressed economy in which there is a composite good, c, that represents both domestic and international goods. There are infinitely lived agents whose preferences are in the same additively separable form given by $E \sum \beta^t U(c_t)$, where $\beta \in (0,1)$ is the subjective discount rate, and U(.) denotes the instantaneous utility function.

Assumption: *The utility function $U : R \rightarrow R$ is twice continuously differentiable, strictly increasing and strictly concave. Thus, for all $c>0$, $U'(c)>0$ and $U''(c)<0$.*

Each agent is endowed with real income, y, which can be converted to domestic and foreign real balances, immediately. The agent is certain that at each period she will receive the same amount of income. At time 0, a positive amount of money both in domestic and foreign currency is given to the agents. Agents are required to consume a nonnegative amount of their balances in the goods market. The unused amounts and income, on the other hand, are then required to be carried over to the next period as either domestic or foreign money balances after the transactions, in the foreign exchange market. Thus, the portfolio - money holdings only, there are no interest bearing assets - distribution between domestic and foreign currency is made at this final stage. It is worth noting at this point that the interest rates for both currencies are zero. This is due to the purpose of capturing the savings motive of the agents in holding foreign currency in a high-inflation economy, where the nominal return of the foreign currency held will only be the yield acquired from the depreciation of the domestic currency. It is assumed that the goods can be purchased by using the domestic currency only. Nevertheless, agents have access to the foreign exchange even from the goods market at a transaction cost, however. The transaction cost concept that is used here refers to the difference between the equilibrium nominal exchange rate in the exchange market, and the nominal exchange rate charged in the goods market. Unless the transaction cost is too high, the transaction motive of the agents for holding foreign currency would be present. Thus, they would leave a high portion of their portfolio as foreign currency, for the next period. The agents in the economy live in an inflationary environment. I assume that inflation follows a three state Markov process.

The following notational conventions are used:

f_t	proportion of foreign currency converted to domestic currency for consumption in the goods market at time t
$M^i_{\ t}$	nominal money balance at time t, i = h, f (h = home, f = foreign)

P^i_t	price level in the economy at time t, for i = h, f
d	transaction cost, or (1-d) the percentage difference between the equilibrium exchange rate determined at the exchange market and the one charged in the goods market
e_t	nominal exchange rate, P^h / P^f, at time t

The timing of transactions is as follows:

Goods Market Exchange Market

T T+1

Decisions : f_t, c_t, M^h_{t+1}, M^f_{t+1}

Informations : d, P^h_t, P^f_t, e_t

Wealth and income : M^h_t, M^f_t, y_t

At the beginning of the time period t, information of the nominal exchange rate, the foreign money balance, and the domestic money balance in her vault, the price level, and, hence, the inflation rate is available to the representative agent.

First the goods market opens

In the goods market, first the representative agent will decide the proportion of her foreign money balances that she will convert to domestic currency and spend for consumption. The following constraint should be satisfied:

$$(3.1) \qquad 0 \leq f_t \leq 1$$

Since the exchange market is not open at that moment, she has to convert her foreign currency into domestic currency in the goods market, incurring a transaction cost. After the decision for f is given, the representative agent will decide on the consumption level. The agent here faces a liquidity constraint:

$$(3.2) \qquad 0 \leq p_t.c_t \leq M_t^h + (1-d).f_t.e_t.M_t^f$$

After the decisions for f, and consumption are made, the goods market closes.

The exchange market opens

After the goods market is closed and consumption is made, the income transfer is made to the representative agent. The transfer could be seen as the labor income of the agent. Holding the remaining foreign money balances and income, the agent will now decide on her savings for the next period in the form of foreign and domestic nominal money balances. This will give us information about the agent's portfolio distribution between domestic and foreign currencies. While deciding on her foreign currency savings, the agent will be subject to the following constraint:

(3.3) $$0 \le e_t.M^f_{t+1} \le p_t.y + (1 - f_t).e_t M^f_t$$

Having decided on her foreign currency holdings, the slack value will be her domestic currency holdings, that is:

(3.4) $$M^h_{t+1} = p_t.y + (1 - f_t).e_t.M^f_t - e_t.M^f_{t+1}$$

Overall, the agent's budget constraint at the end of the period can be written as:

(3.5) $$M^h_{t+1} + e_t.M^f_{t+1} \le p_t.y + (1 - f_t).e_t.M^f_t + (M^h_t + (1 - d).f_t.e_t.M^f_t - p_t.c_t)$$

It can be argued that the agent is holding domestic currency for liquidity purposes, only. The analysis below will prove this argument.

Since we use a model in which all goods are freely traded, purchasing power parity always holds, so e $\equiv P^h / P^f$, for all t. Let the lower case letters denote the nominal value divided by the appropriate price such that $m^i \equiv M^i / P^i$, for all t, for i = h, f. For notational convenience, also define the variable $\pi^i \equiv P^i / P^i_{-1}$, for all t, for i = h, f, the gross inflation rate of country i. The normalized constraints in the asset market and goods market can be rewritten as :

(3.2′) $$0 \leq c_t \leq m_t^h + (1-d).f_t.m_t^f$$

(3.3′) $$0 \leq \pi_{t+1}^f.m_{t+1}^f \leq y + (1-f_t).m_t^f$$

(3.4′) $$\pi_{t+1}^h.m_{t+1}^h = y + (1-f_t).m_t^f - \pi_{t+1}^f.m_{t+1}^f$$

In these inequalities dividing the nominal variables by the price level has transformed them into real ones.

Assuming that the foreign country's price level is unity and that the initial price level P_o for home country is given, the relevant state vector will consist of (m^h, m^f, π), which are the domestic real balance, foreign real balance and inflation rate, respectively. On the other hand, the representative agent should now decide on (f, c, $m^{f'}$), that are, the proportion of foreign real balance converted to domestic currency for consumption, consumption, and foreign real money saved for next period, respectively.

Thus, the problem of the representative agent takes the following functional equation form. Note that the variables without primes denote current state or choice variables and primed variables are future values.

$$v(m^h, m^f, \pi) = \max_{\{f, c, m^{f'}\}} \left\{ u(c) + \beta . E\left[v(m^{h'}, m^{f'}, \pi') \mid \pi \right] \right\} \tag{3.6}$$

subject to (3.1), (3.2′), (3.3′), and (3.4′).

SECTION IV

SOLUTION METHODOLOGY

In order to solve the problem defined in the preceding section, the *value function iteration* method will be used with a discretized state space. This method is one of the approaches of implementing numerical dynamic programming. Implementation of the problem follows the sequence, given below[6]:

- The preferences of the agents is assumed to be represented by a functional equation form consistent with the assumption in the preceding section.
- The state space is discretized and partitioned into a finite set of points. This procedure should satisfy a presumed growth path of the state variables together consistent with their steady state values. This in turn implies that the control variables will take on a finite set of values.
- A transition probability should be defined for the stochastic variables.
- Parameters, and the discount rate are assigned to appropriate values.

Applying all of the above, the problem can be handled by the following recursion formula on the value function:

(4.1) $$V^{n+1}(.) = \max_{\{.\}}\left[u(.) + \beta \sum prob.V^{n}(.)\right]$$

subject to the relevant constraints. Now, let us define the operator T by:

(4.2) $$TV^{n}(.) = V^{n+1}(.) = \max\left[u(.) + \beta \sum_{\{.\}} prob.V^{n}(.)\right]$$

The operator maps the set of increasing concave functions into itself, is monotone increasing, and satisfies discounting, that is for any constant v, T(V+v) = TV + βv. The following theorem can be written[7] as:

Theorem : *Let C(S) denote the space of continuos, bounded functions. Let T : C(S) → C(S) be defined by equation (4.2). Under the assumption for the utility function, the operator is a contraction with unique fixed point V^*, which is bounded, increasing, and concave.*

A computer program can, then, be written to solve TV=V.

Consistent with the above steps, I assume the preferences to be of the constant relative risk aversion variety, i.e. U (c) = c^{σ} / σ. Moreover, I discretized the state

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[6] Altuğ and Labadie, (1994), pp 151

[7] For the proof see Stokey & Lucas (1989), pp 263 – 264

space of m^f, m^h, and the control space of f into a finite set of points. I limited the interval for money holdings as [0, 2], moreover I partitioned the interval with 0.1 grids. The control space for c is also discretized and partitioned with 0.1 grids. Since f is the portion of the foreign money holdings converted to domestic currency for consumption, it will by default be defined on [0, 1] interval, and the partition will be done with 0.05 grids. Discretized state spaces and control space of f and c will imply a finite set of values for the control variables $m^{f'}$, and $m^{h'}$. Next, I assume that π can take on three possible values, that are {1.0, 1.05, 1.25}. Therefore, the transition probability matrix Φ is 3x3 with elements $\phi_{i,j}$ = Prob $\{\pi_{t+1} = \Theta_j \mid \pi_t = \Theta_i\}$. Notice that this procedure substitutes a discrete Markov chain for the continuous Markov process for π_t. Thus the matrix is :

$$\Phi = \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.2 & 0.6 & 0.2 \\ 0 & 0.4 & 0.6 \end{bmatrix}$$

For the parameter sigma 0.5, for the discount rate 0.9, for income {0.25, 0.4}, and for the transaction cost {0, 0.15, 0.5} are used.

Finally, my algorithm becomes:

$$(4.3) \qquad V^{n+1}(m_i^h, m_i^f, \pi_k) = \max_{\{f_i, c, m_i^f\}} \left[u(c) = +\beta \sum_{s=1}^{3} \phi_{k,s} V^n(m_j^h, m_j^f, \pi_s) \right]$$

subject to

$f_j \in [\,0,1\,]$,

$0 \le c \;\; \le m_i^h + (1-d).f_i.m_i^f$

$m^f_j \in [\,0,\, y+(1\text{-}d).f_j.m^f_i\,] \cap [\,0,\,2\,]$,

$m^h_j = (\,y+(1\text{-}d).f_j.m^f_i\,) - m^f_j$

The computer program used for solving the problem is written along with the following lines:

1. Formulate an initial guess of the function V^0. For example set $V^0 = 0$.
2. For each triplet of state variables (m^h, m^f, π), compute the value function for each point of control variables f, c, $m^{f'}$, $m^{h'}$ in their feasible set that satisfies the constraints (3.1), (3.2′), (3.3′), (3.4′), respectively; this will limit the set over which the value function must be computed. Given the initial guess V^0, choose the points of control variables f^0, $m^{f'0}$ that maximizes the right side of the equation (4.3); call this new value function V^1.
3. Evaluate the value function for all triplets of state (m^h, m^f, π). There are 21x21x3 points in the range of V^1 and the associated policy functions, f, c, $m^{f'}$, and $m^{h'}$.
4. Repeat the steps 2. and 3. until $|\; V^{n+1} - V^n \;| \le \varepsilon$ for all states, where $\varepsilon > 0$ is the convergence criterion.

SECTION V

CONCLUSION

The value function, consistent with the theorem, is bounded, strictly increasing and strictly concave with respect to domestic and foreign real balances. On the other hand, value function turns out to be decreasing with respect to inflation (Figure 1a – 1c).

The consumption policy function is increasing with respect to foreign and domestic real balances. I have observed that for initial wealth levels exceeding income, only part of wealth is spent for consumption. In other words, some of domestic and foreign currency is retained for next period from where on they will be gradually eaten up (Figure 2a-2c). Additionally, when there is no transaction cost the consumption function is symmetric with respect to the two currencies (Figure 6a-6c). As transaction cost increases, consumption out of foreign currency tends to decrease (Figure 8a-8c).

Concerning the savings motive of agents, at higher levels of inflation, the agent substitutes foreign money for domestic money in her savings (Figure 4a – 4c,

Figure 5a – 5c). The foreign real balances function is non-decreasing in initial foreign money holdings. Likewise, the domestic real balances function is non-decreasing in foreign and domestic real balances. When inflation rate is low, at low initial levels of domestic and foreign real balances the agent spends a proportion of her foreign real money for consumption then at the exchange market she converts all her foreign money into domestic. After receiving the income, she makes all her savings in the form of domestic currency. On the other hand, at a low inflation rate, low levels of initial domestic real money and high levels of foreign real money, the agent saves small proportion of her money holdings in the form of foreign currency. This proportion increases when there is high initial levels of domestic and foreign real money holdings under a low inflation state. A decline in transaction cost increases the amount of foreign real money savings (Figure 7a – 7c, Figure 9a – 9c).

In my simulations, I have observed at steady state that (ceteris paribus) while higher transaction cost levels increase the mean rate of currency substitution, (ceteris paribus) higher income levels reduce it (Figure 10, Figure 11a – 11b). On the other hand, an increase in the mean inflation rate from 10 percent to 30 percent level (ceteris paribus) stabilizes the rate of currency substitution considerably, at a higher level (Figure 12a – 12b). While changes in income level have no stabilization effects, changes in transaction cost has stabilization effects on the time fluctuations in the currency substitution rate (Figure 10, Figure 11a – 11b).

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APPENDIX A

FIGURES

Figure 1a – VALUE FUNCTION at LOW INFLATION [illegible]

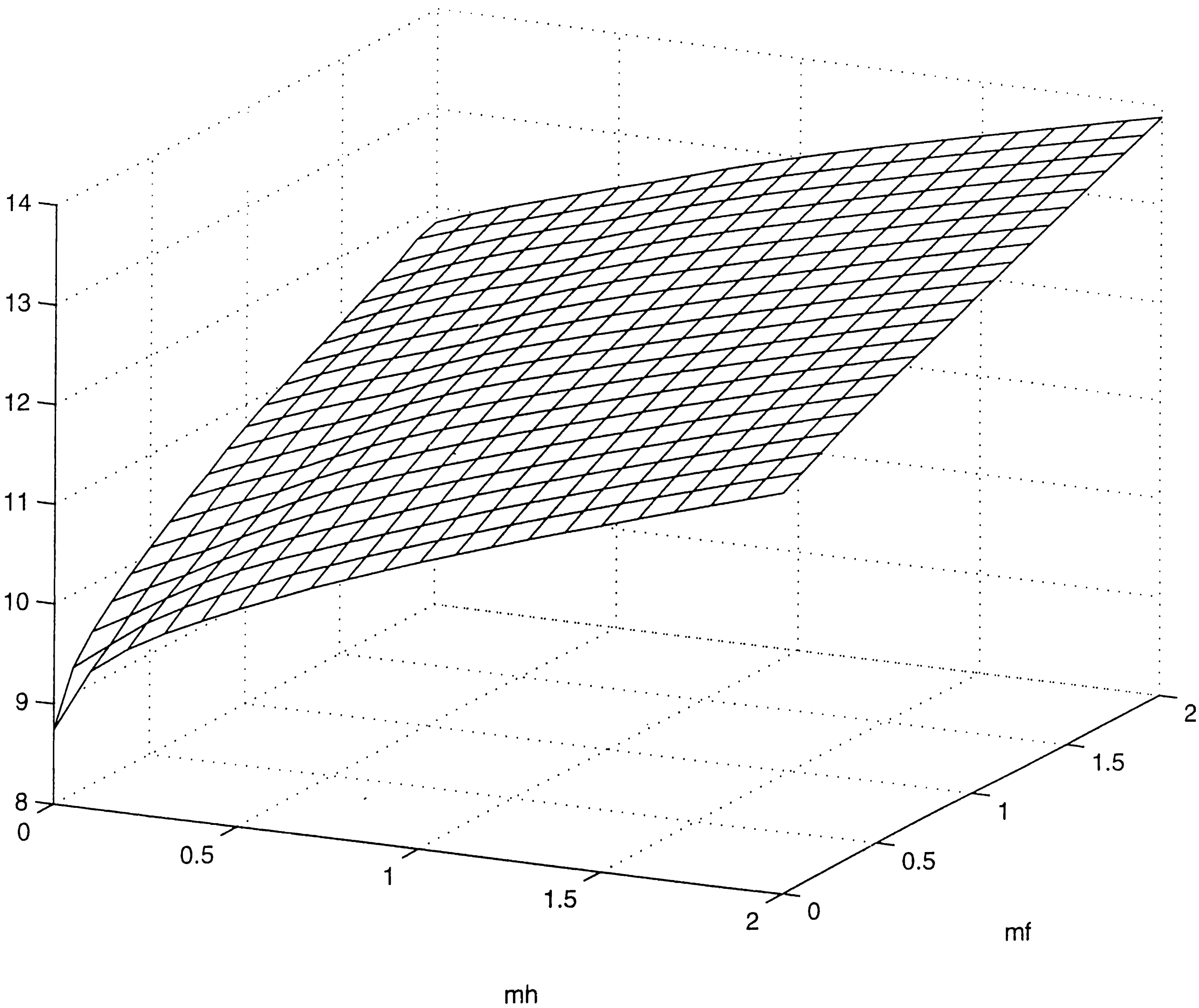


Figure 1b – VALUE FUNCTION at MID INFLATION (d= 0,15)

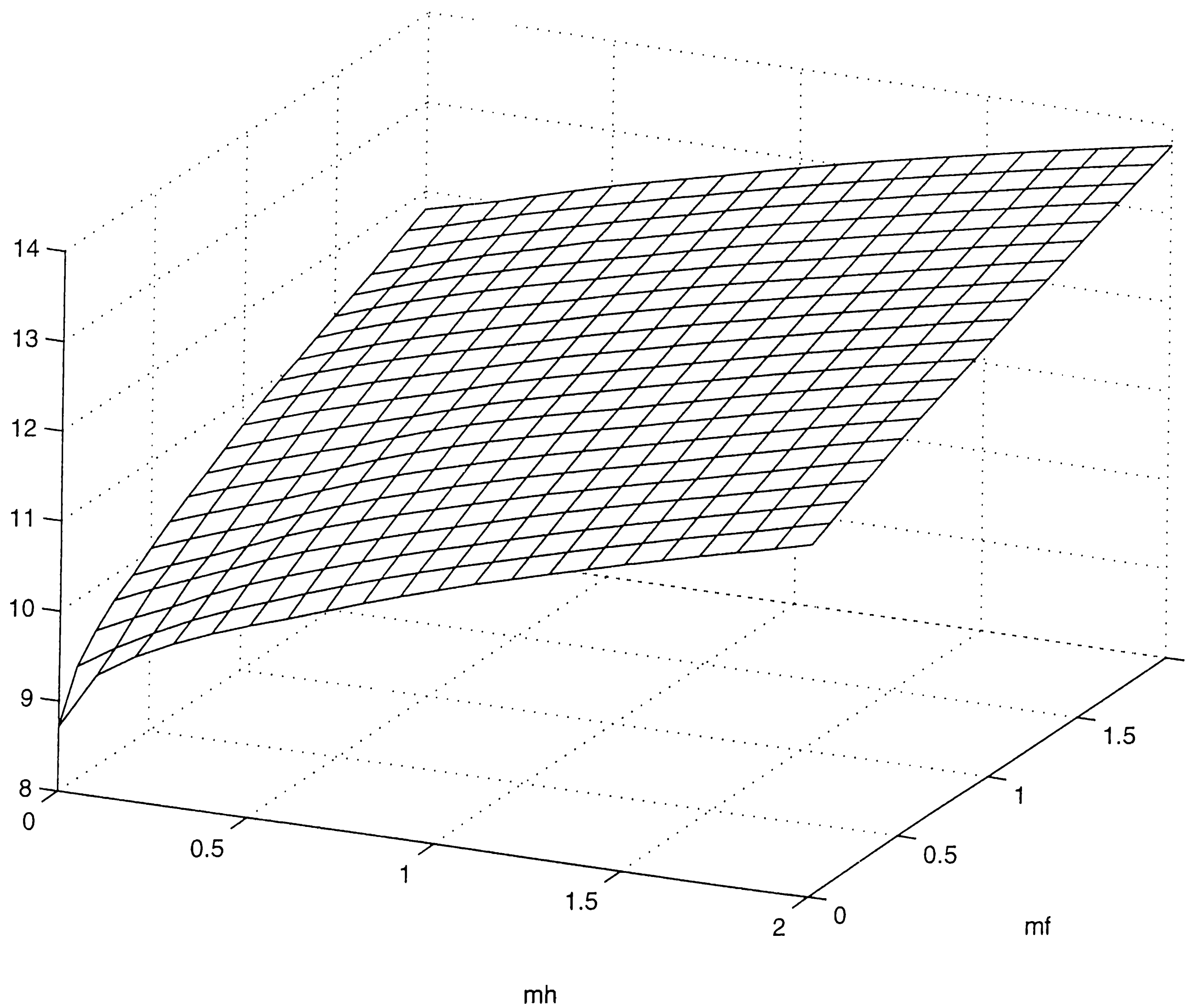


Figure 1c – VALUE FUNCTION at HIGH INFLATION ($d=0,15$)

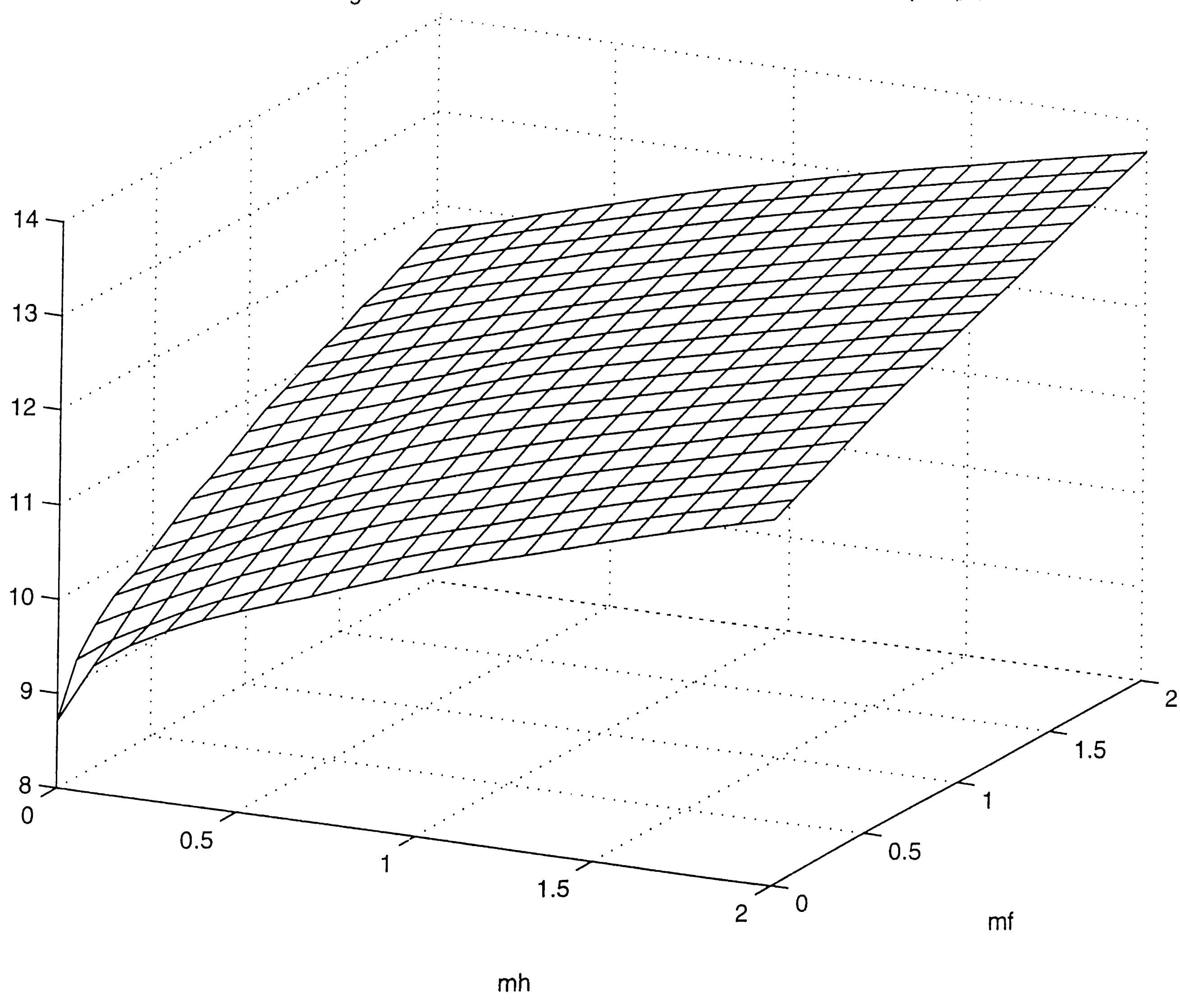


Figure 2a – CONSUMPTION FUNCTION at LOW INFLATION (σ=0.15)

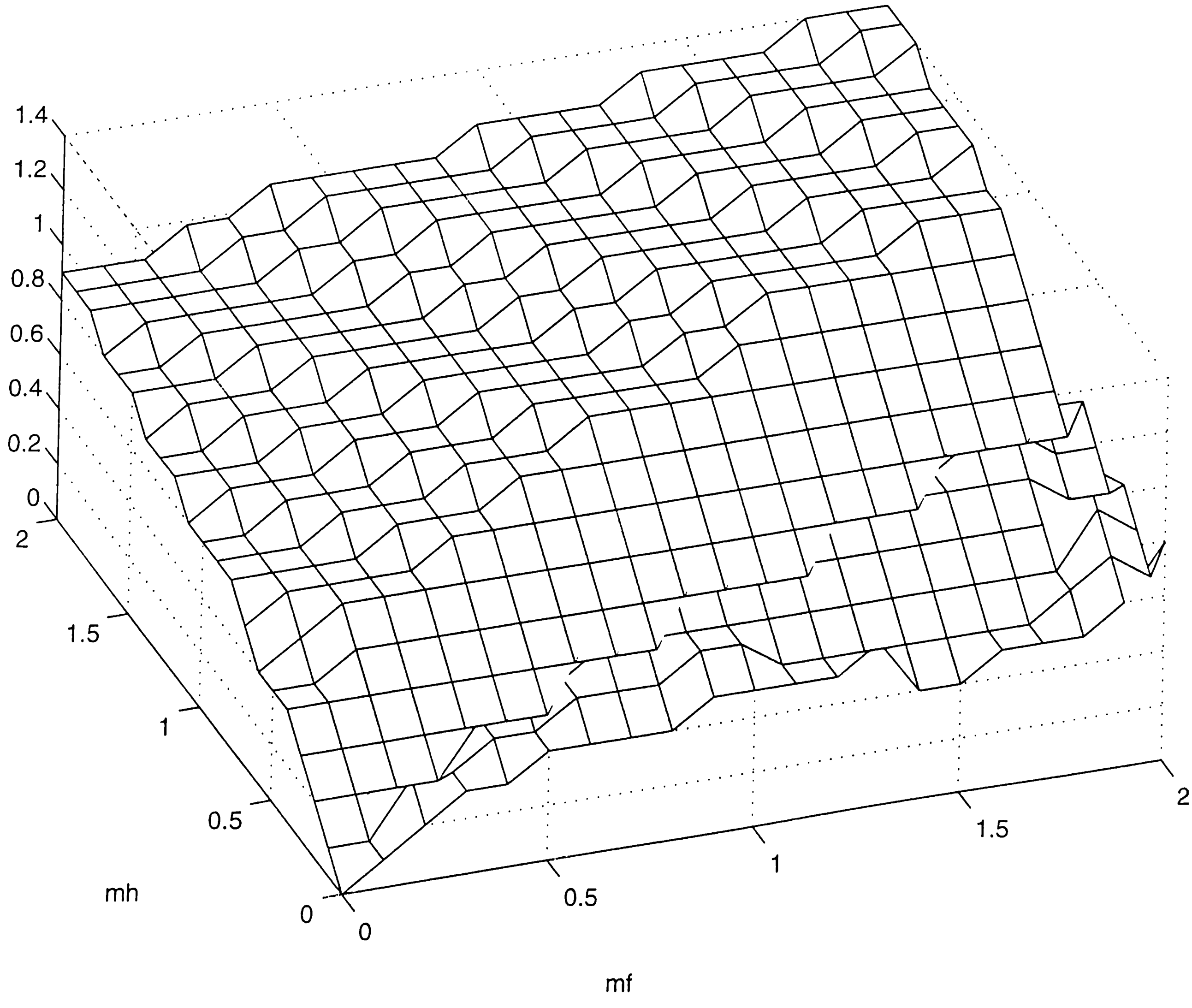


Figure 2b – CONSUMPTION FUNCTION at MID INFLATION (d=0.15)

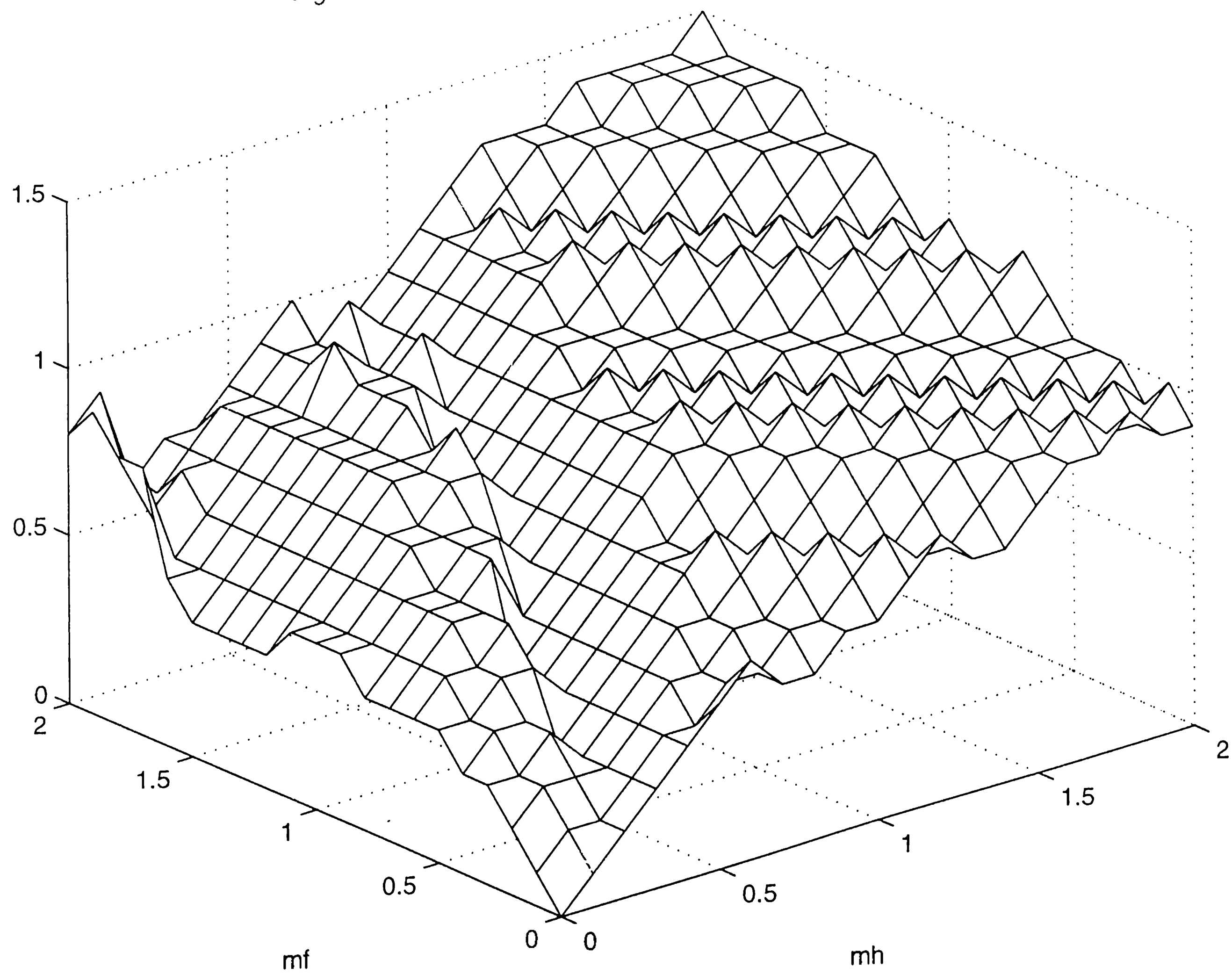


Figure 2c – CONSUMPTION FUNCTION at HIGH INFLATION (d=0.15)

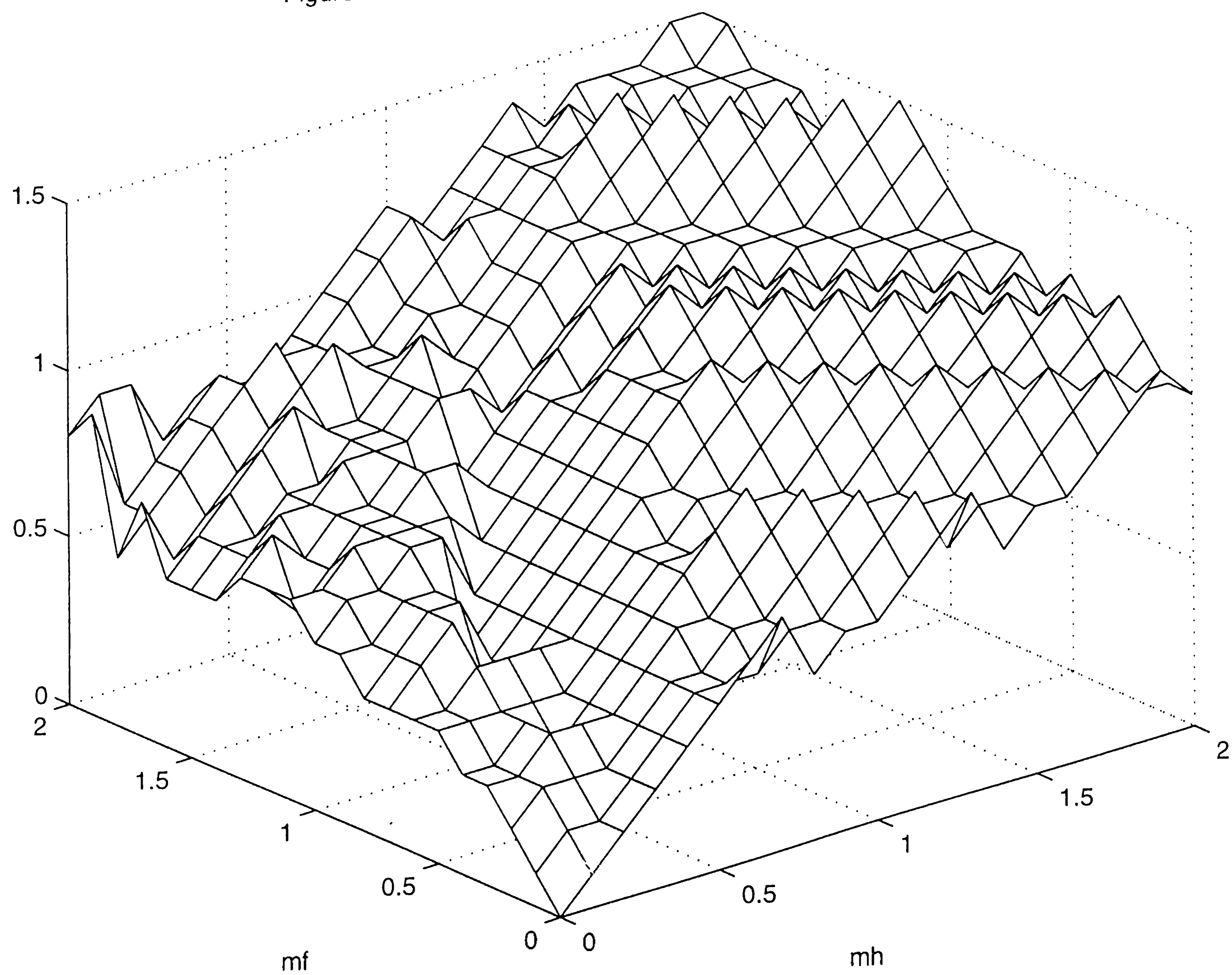


Figure 3a - FUNCTION F at LOW INFLATION (d= 0.15)

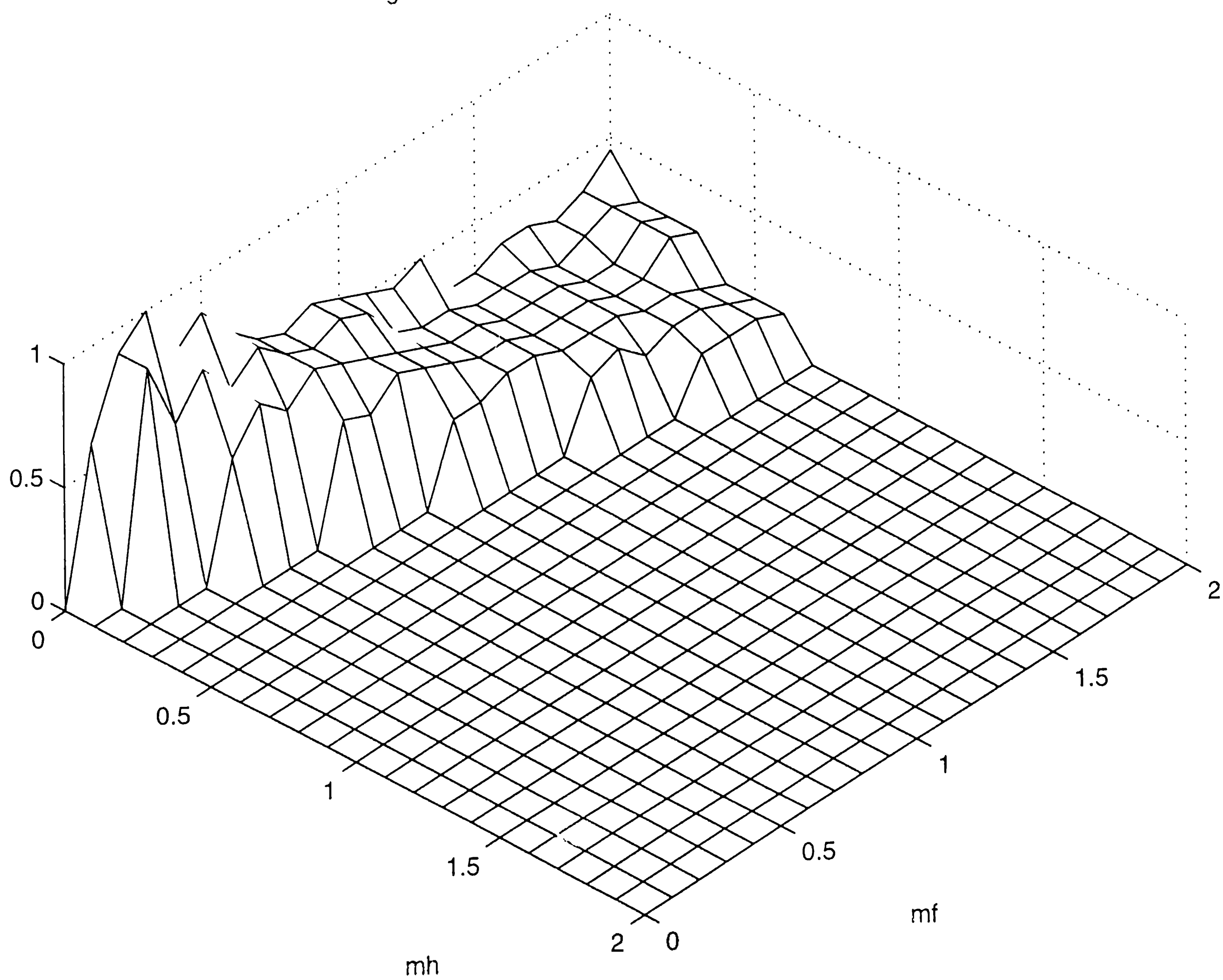


Figure 3b - FUNCTION F at MID INFLATION (d=0.15)

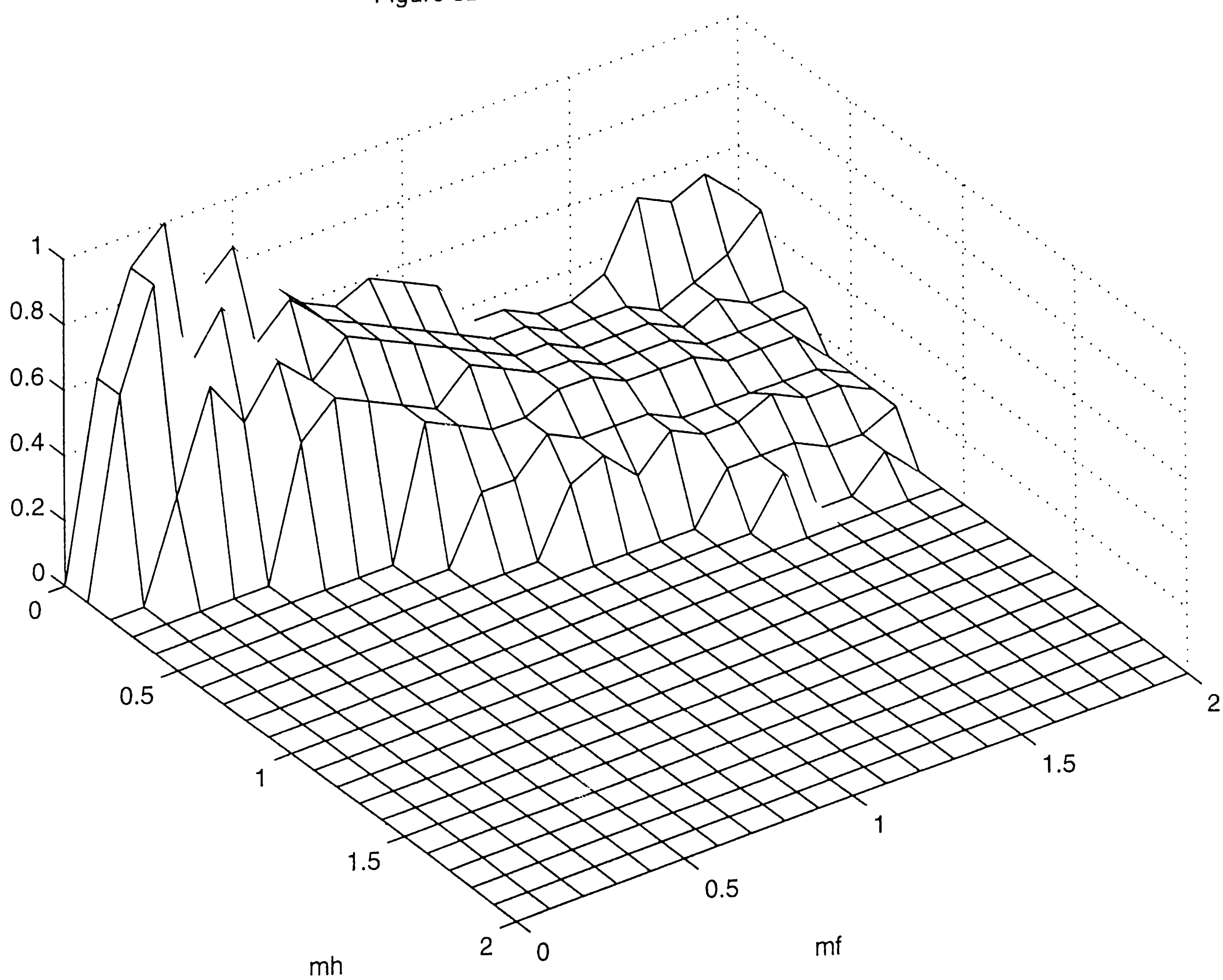


Figure 3c - FUNCTION F at HIGH INFLATION (d=0.15)

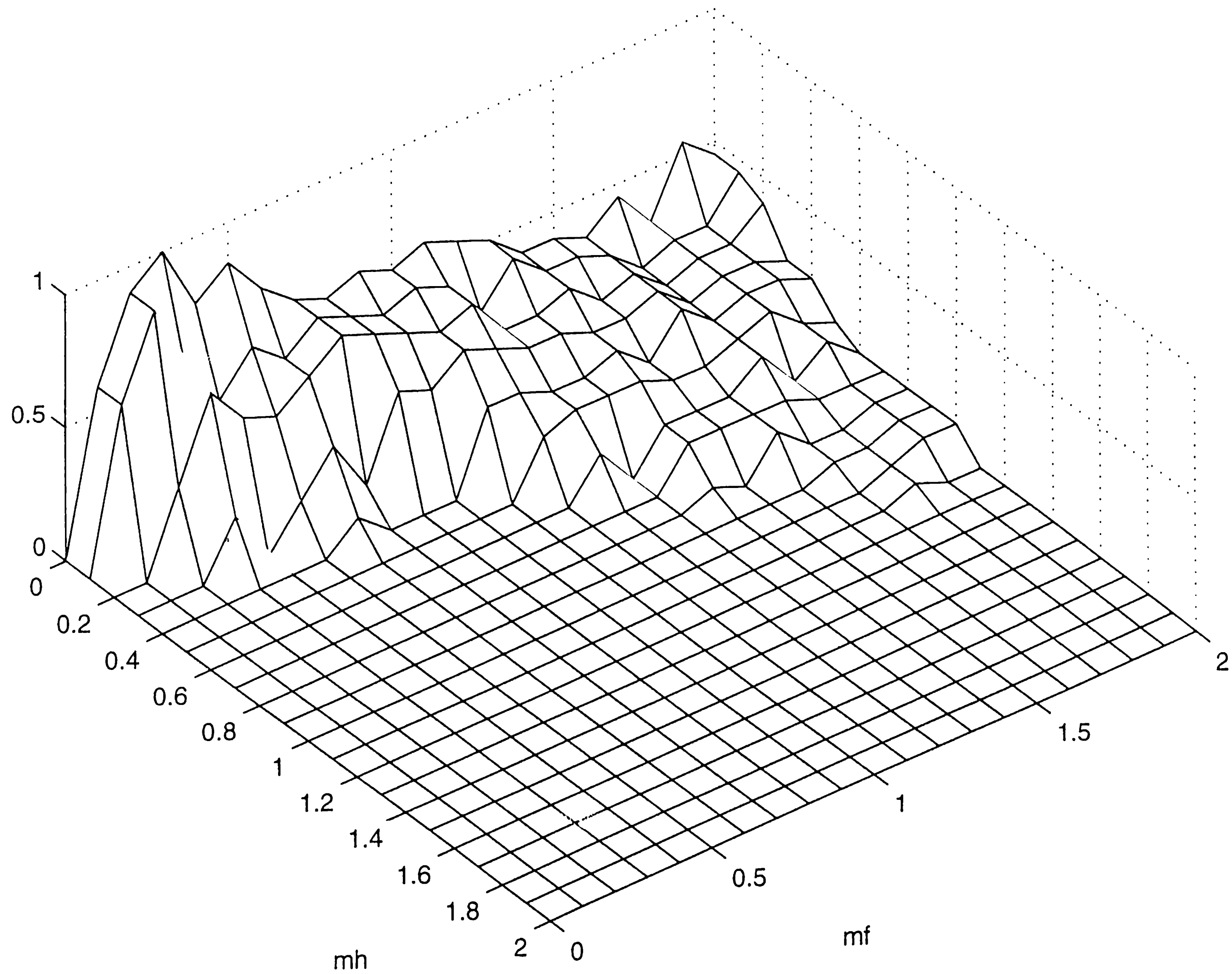


Figure 4a – FOREIGN REAL BALANCES FUNCTION at LOW INFLATION (d=0.15)

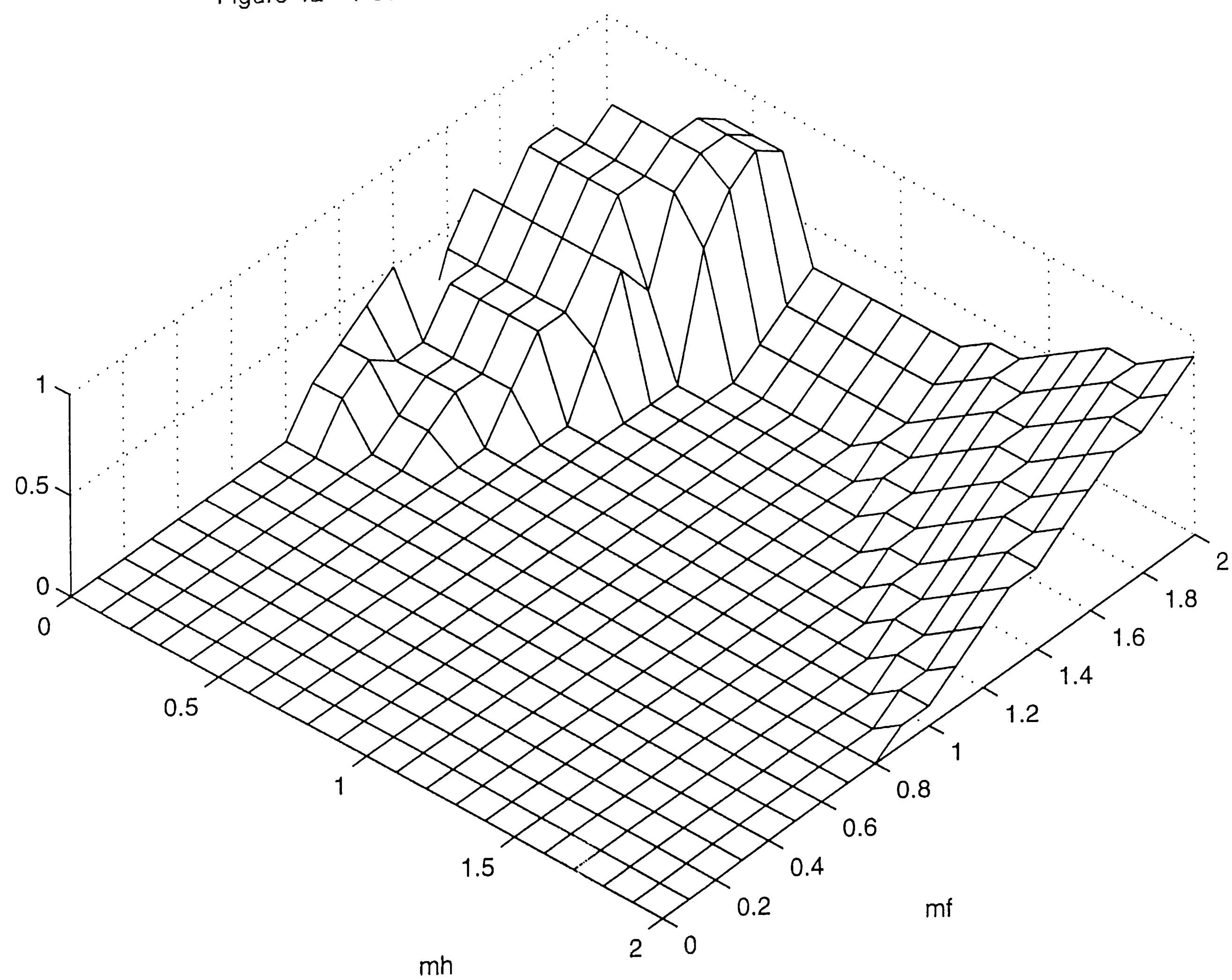


Figure 4b – FOREIGN REAL BALANCES FUNCTION at MID INFLATION (d=0.45)

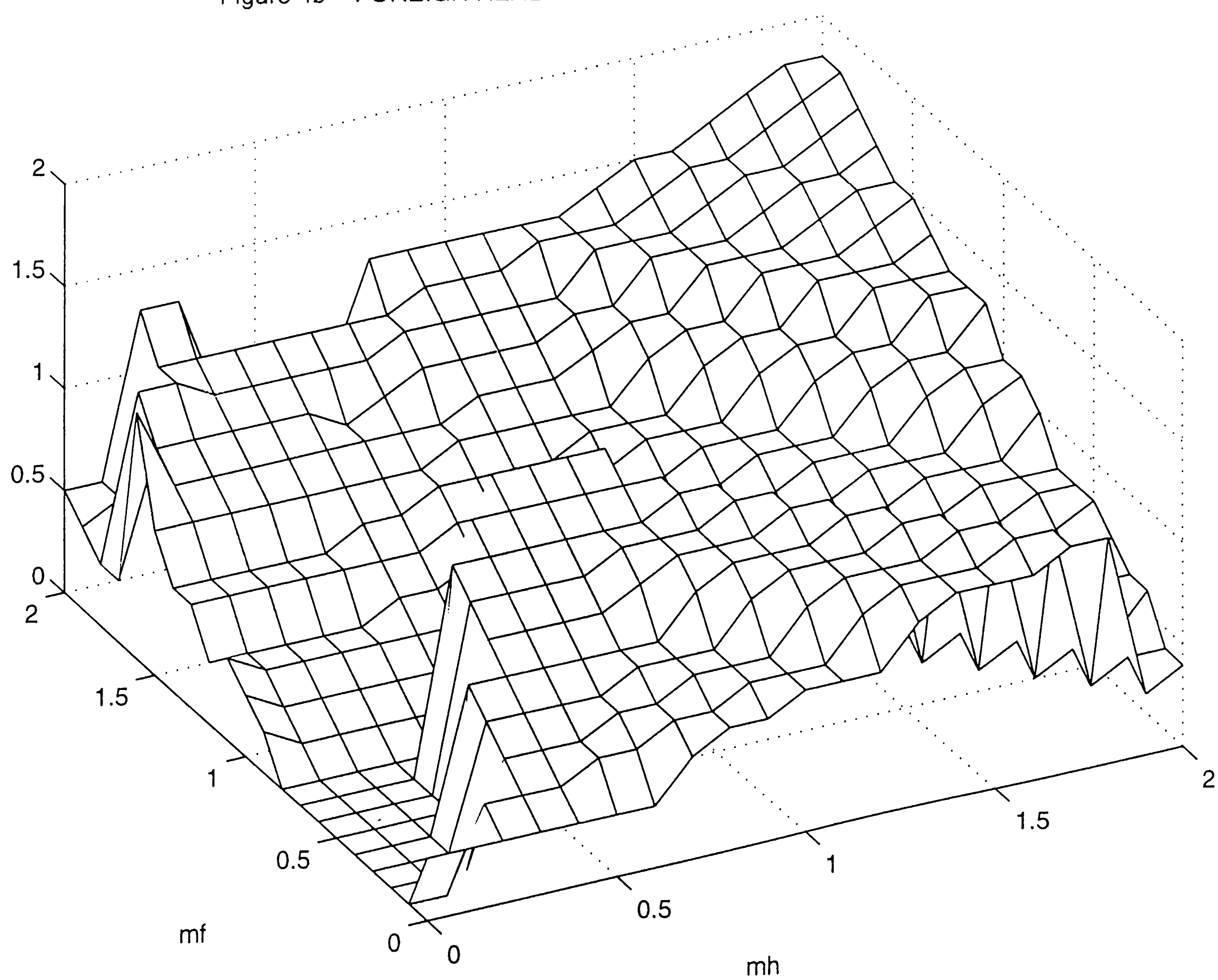


Figure 4c – FOREIGN REAL BALANCES FUNCTION at HIGH INFLATION (d= 0.15)

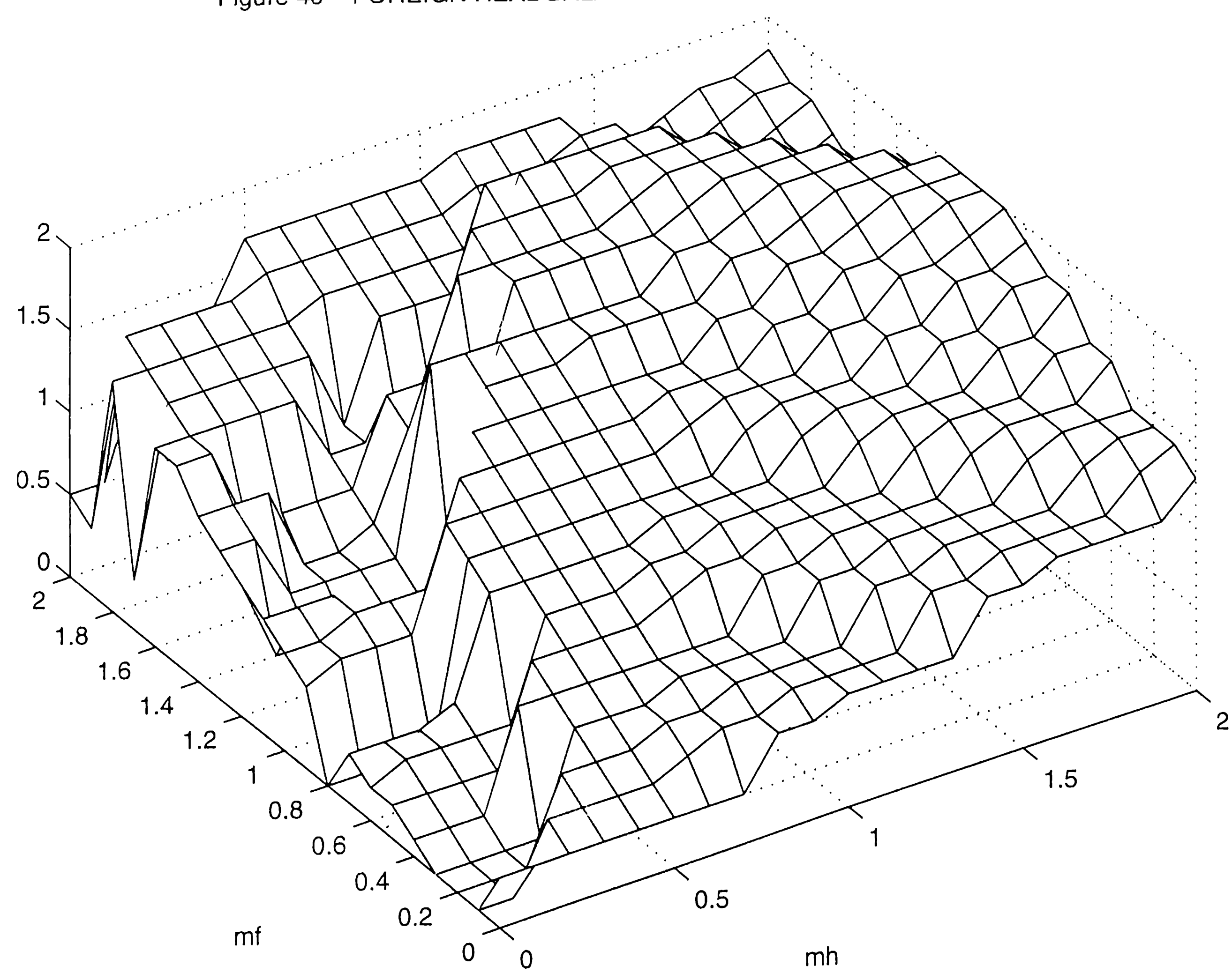


Figure 5a – DOMESTIC REAL BALANCES FUNCTION at LOW INFLATION (d=0.15)

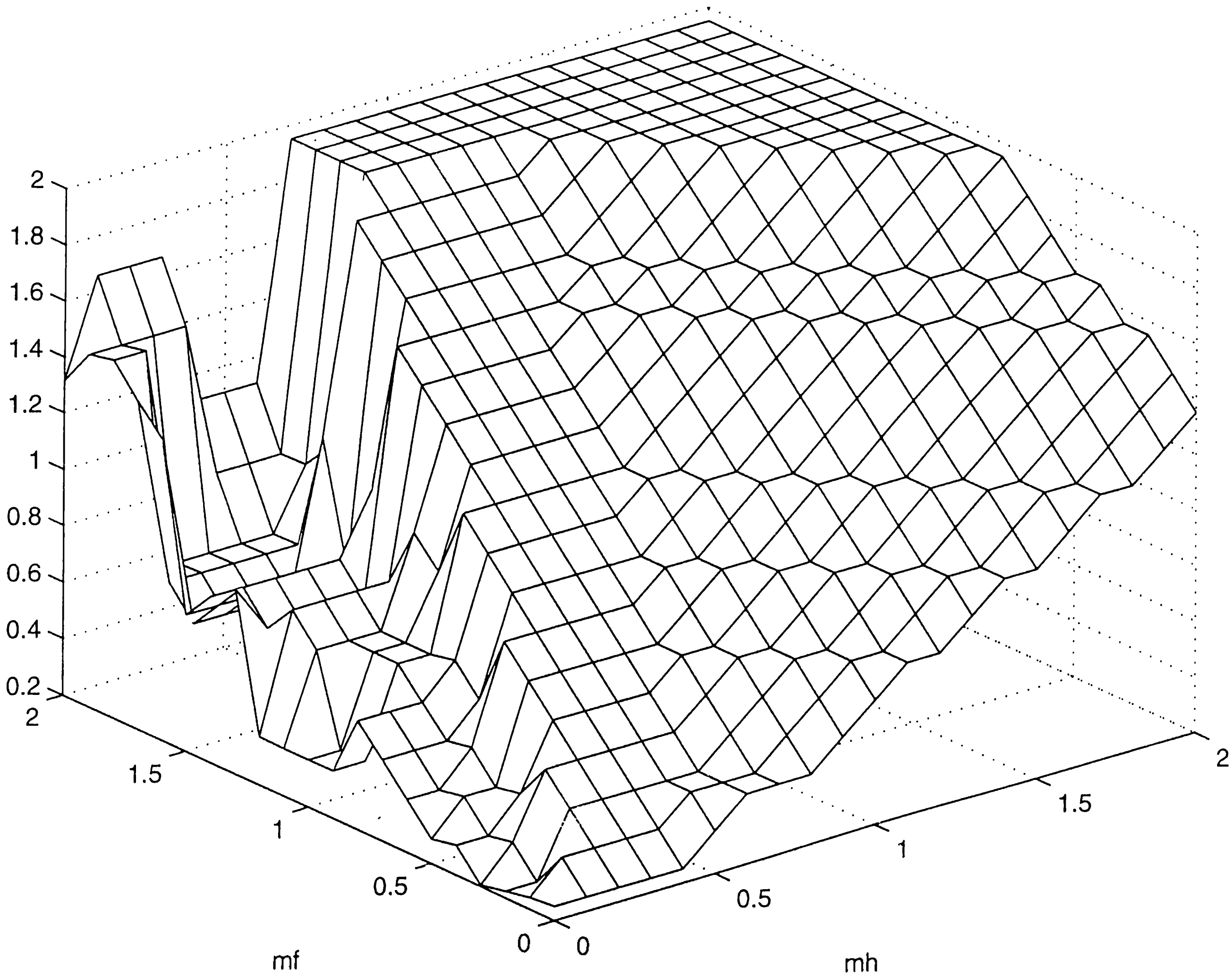


Figure 5b – DOMESTIC REAL BALANCES FUNCTION at MID INFLATION (d = 0.15)

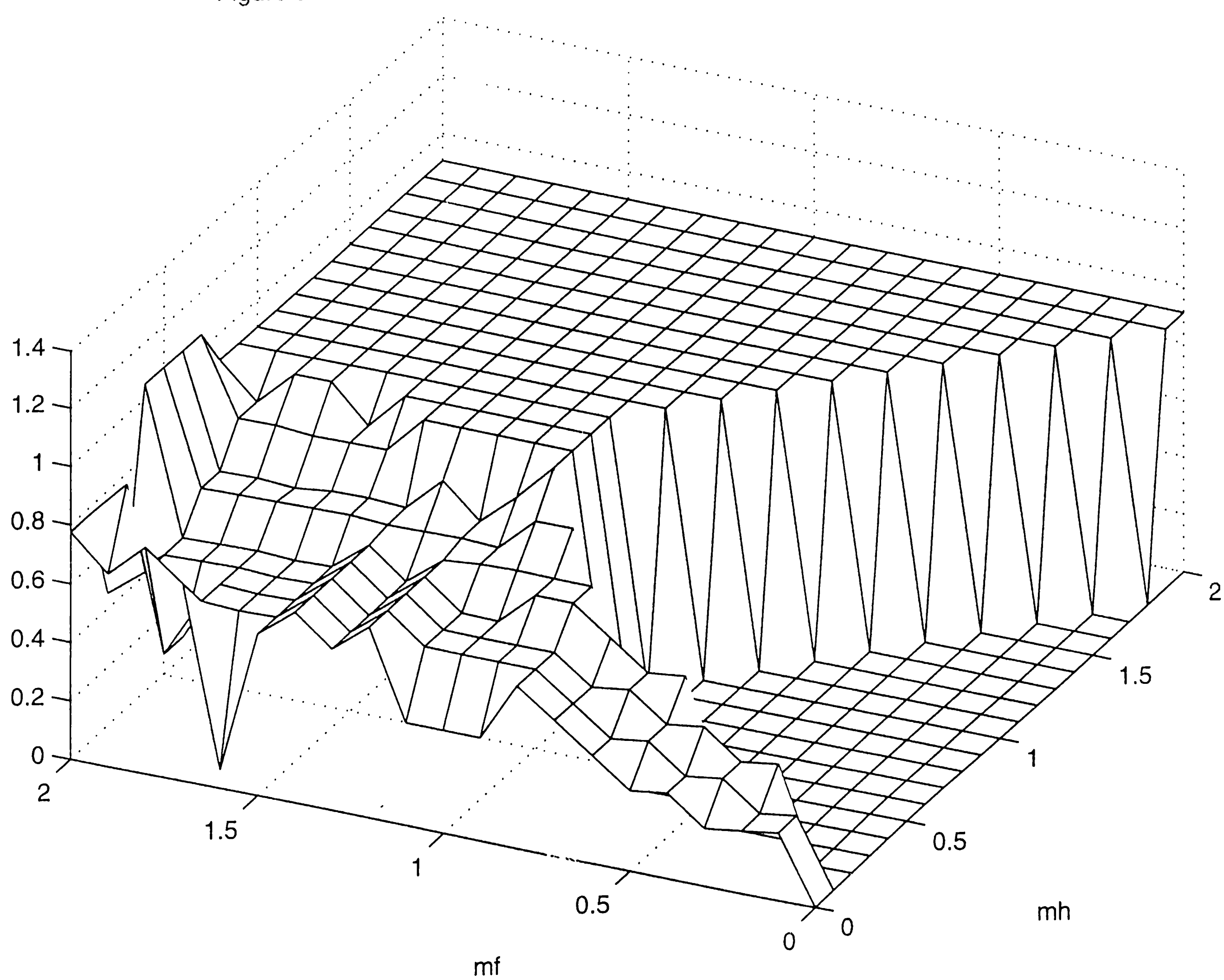


Figure 5c – DOMESTIC REAL BALANCES FUNCTION at HIGH INFLATION (d=0.15)

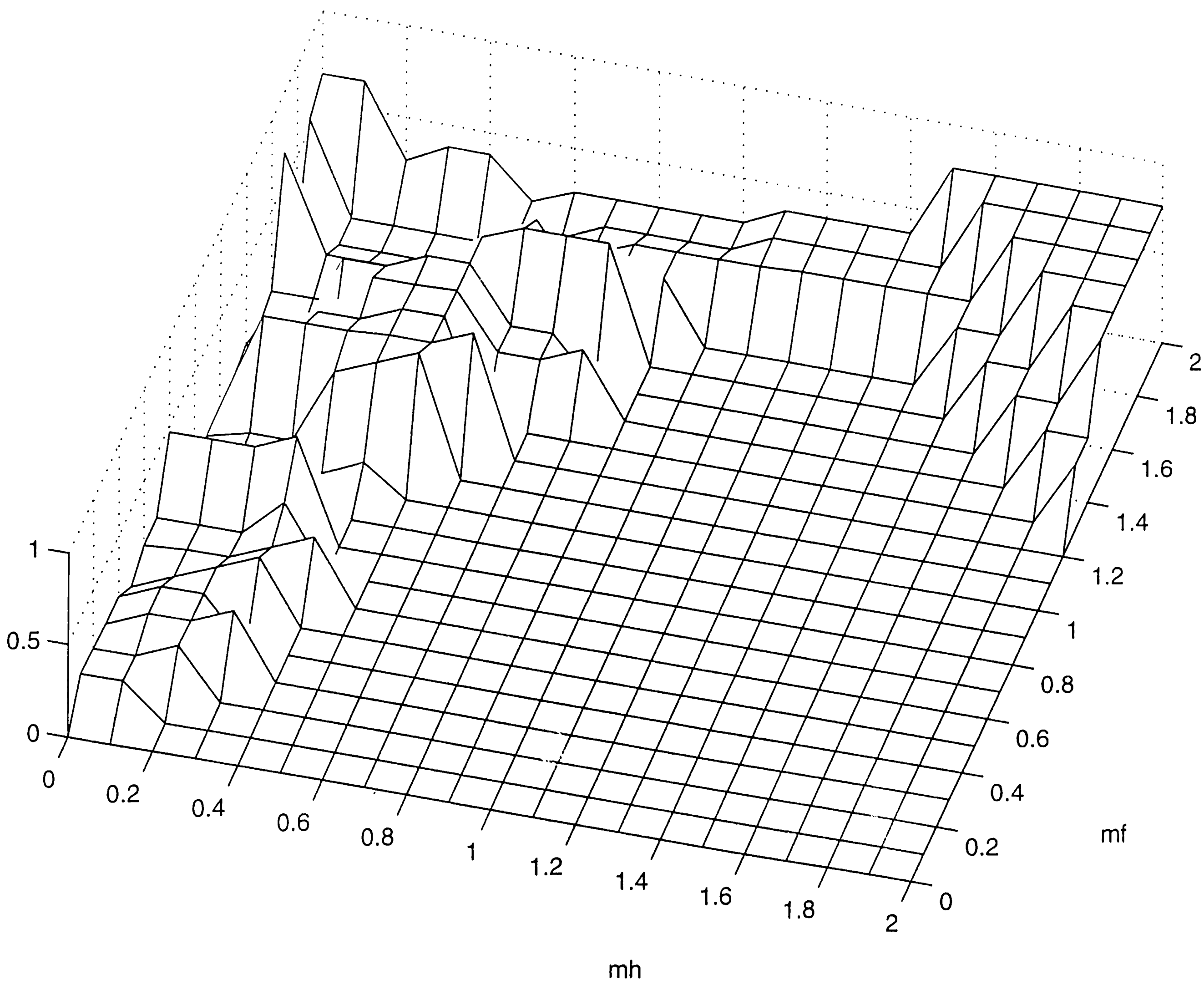


Figure 6a – CONSUMPTION FUNCTION at LOW INFLATION (d=0)

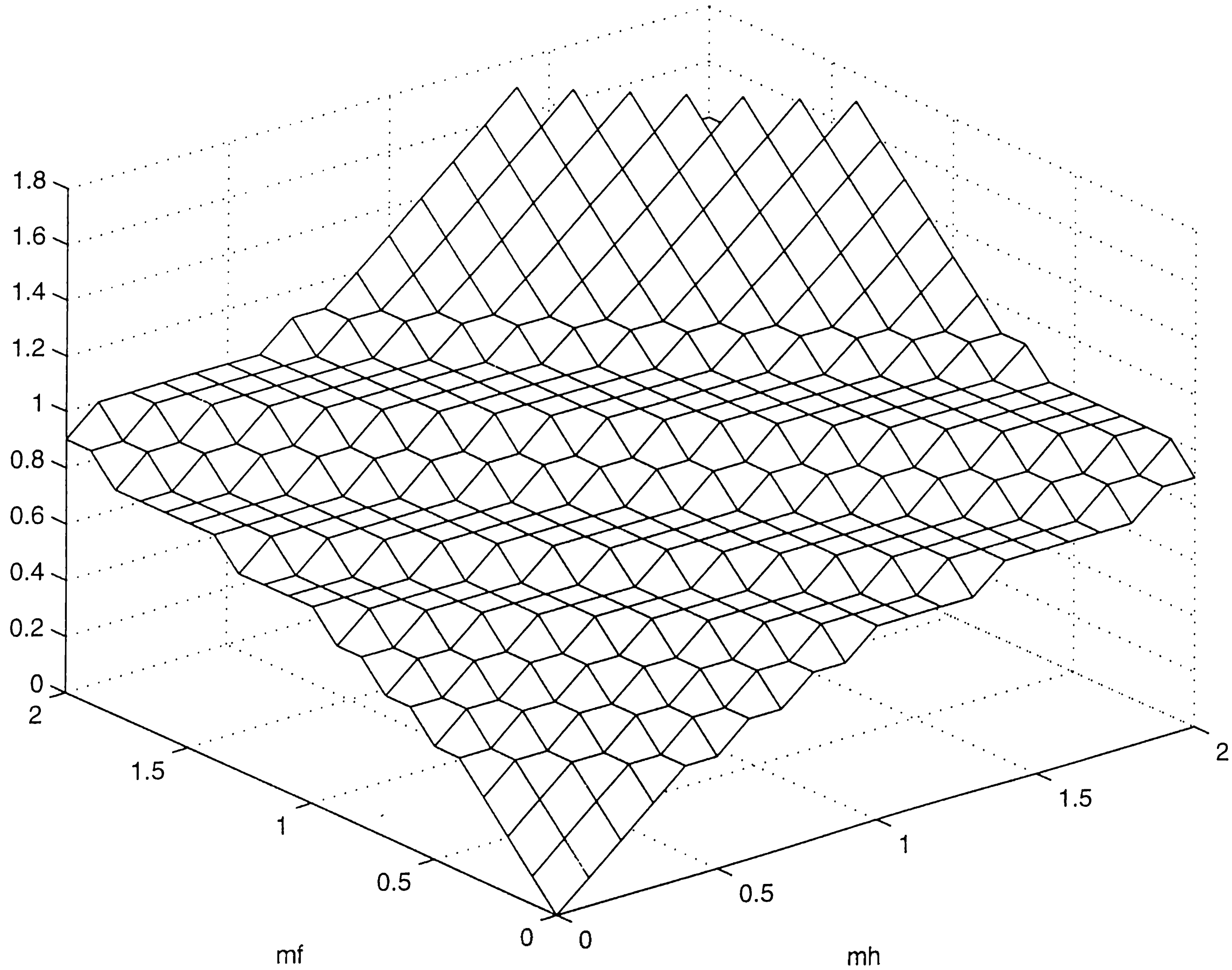


Figure 6b - CONSUMPTION FUNCTION at MID INFLATION (d=0)

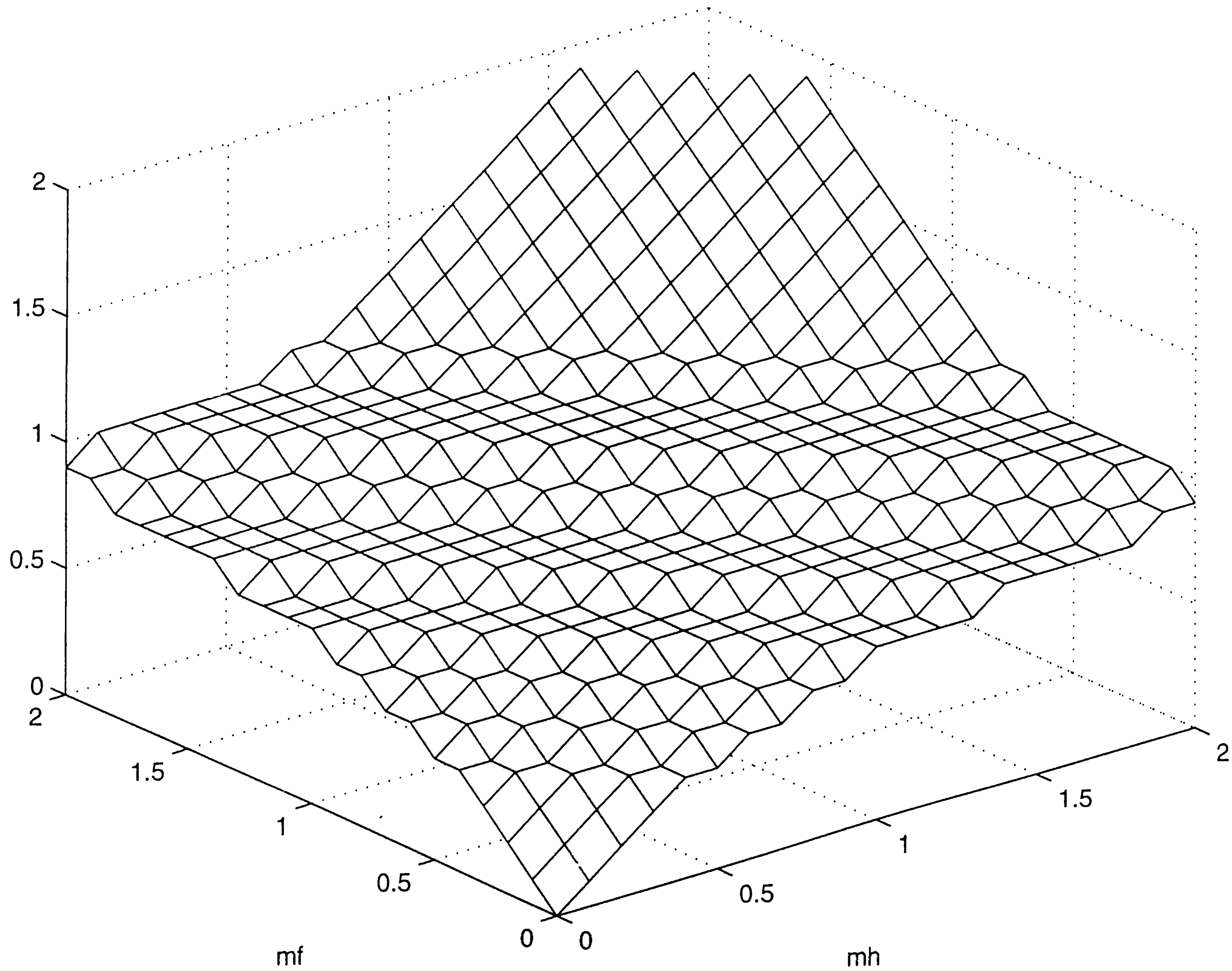


Figure 6c – CONSUMPTION FUNCTION at HIGH INFLATION (d=0)

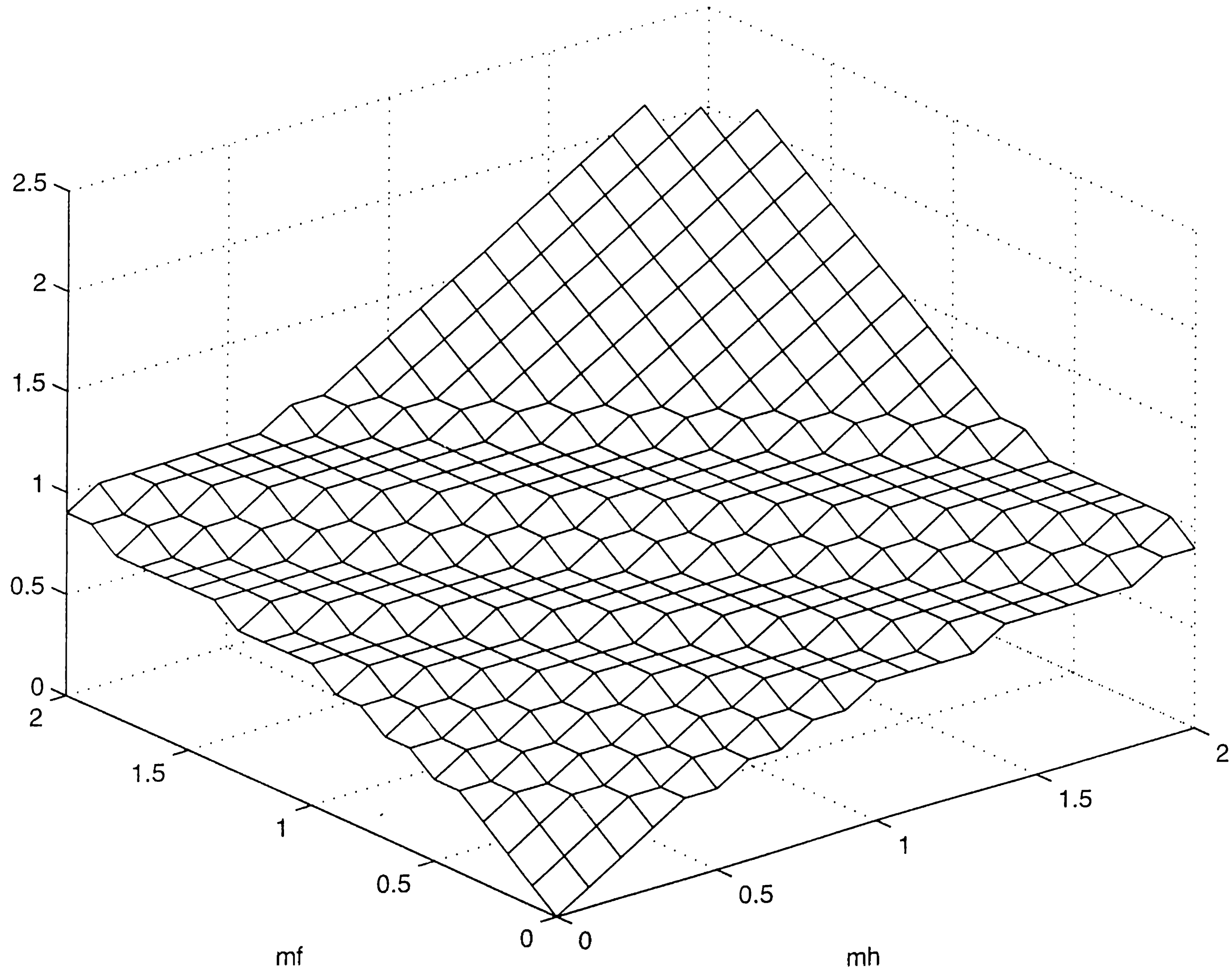


Figure 7a - FOREIGN REAL BALANCES FUNCTION at LOW INFLATION (d=0)

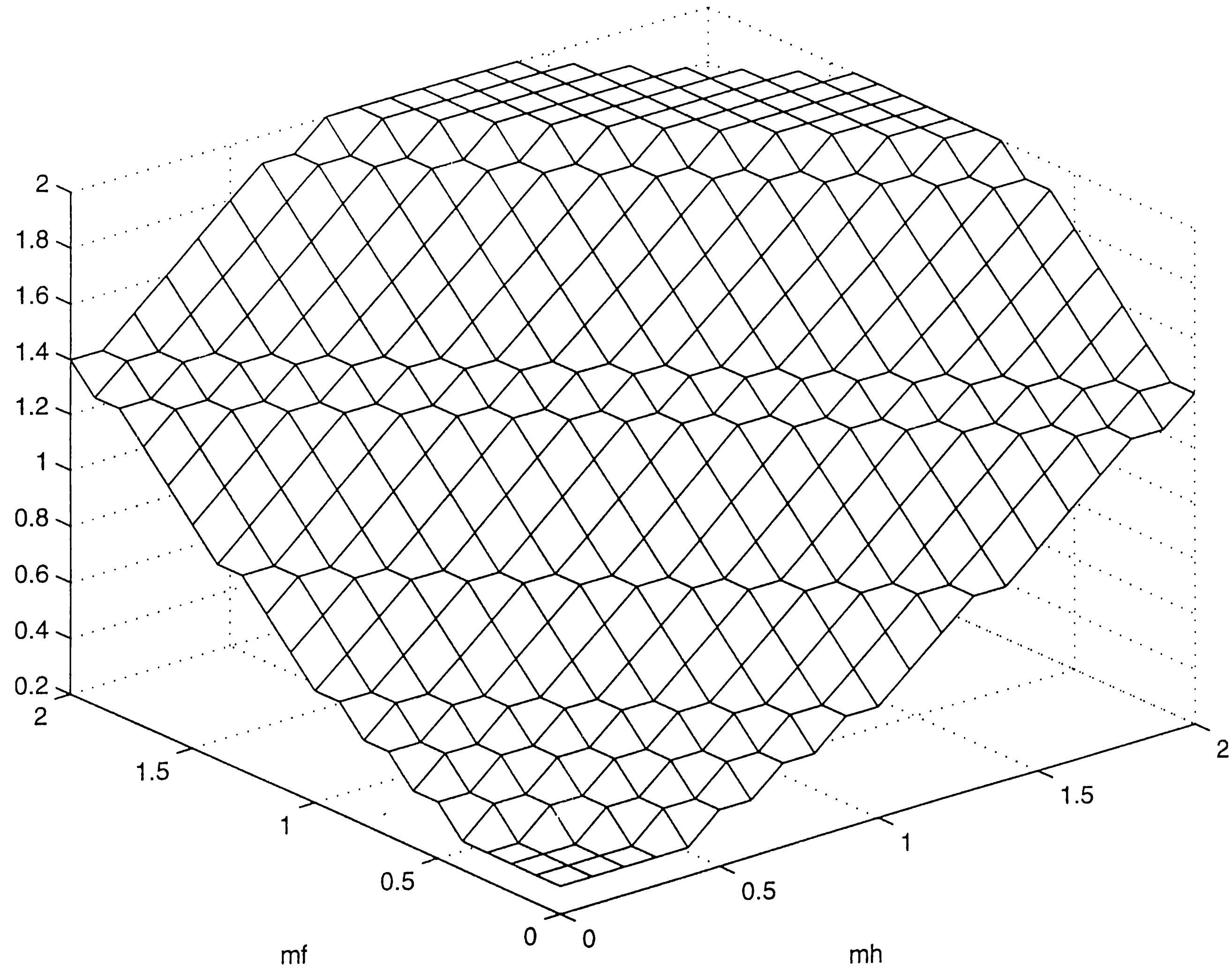


Figure 7b – FOREIGN REAL BALANCES FUNCTION at MID INFLATION (d=0)

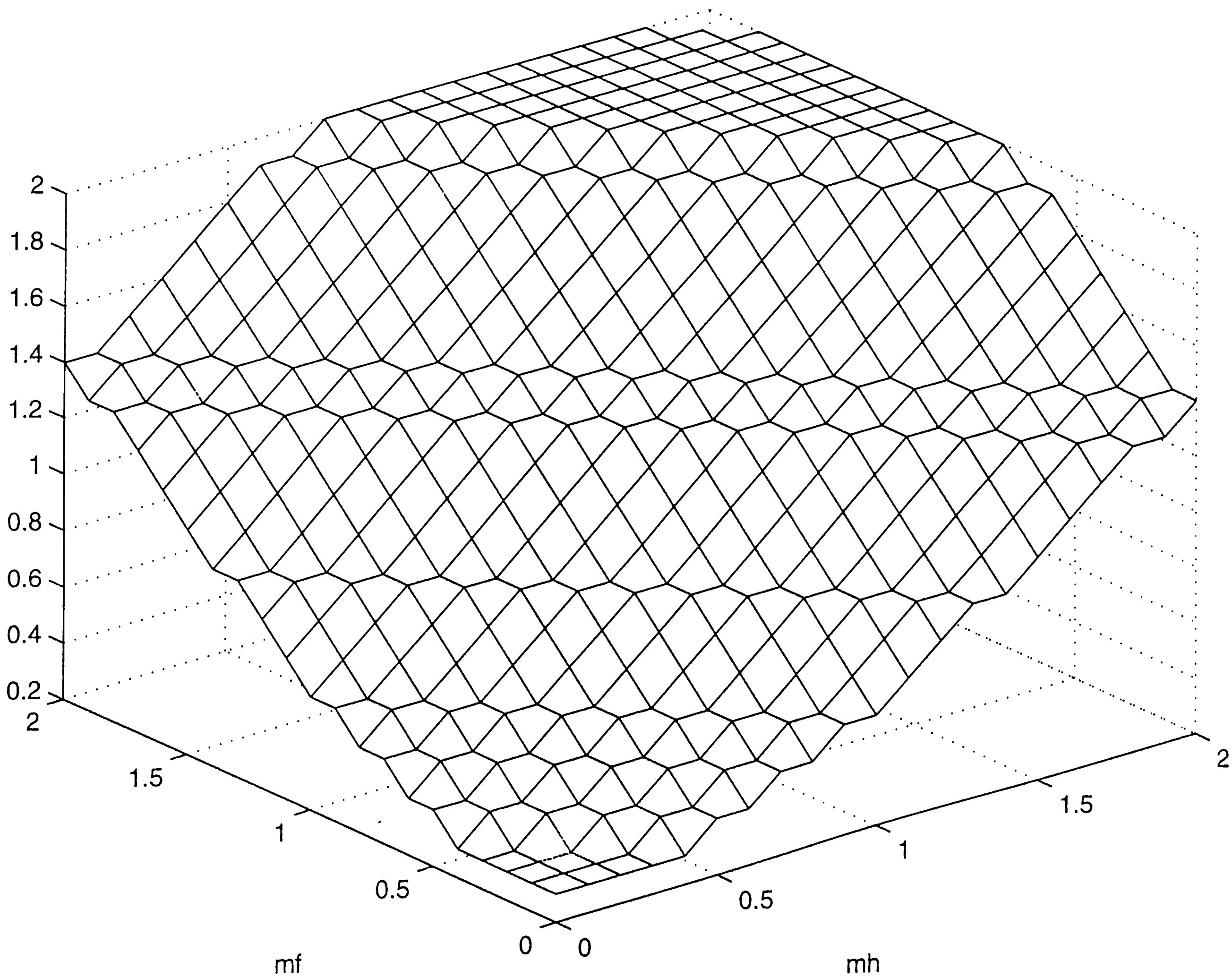


Figure 7c - FOREIGN REAL BALANCES FUNCTION at HIGH INFLATION (d=0)

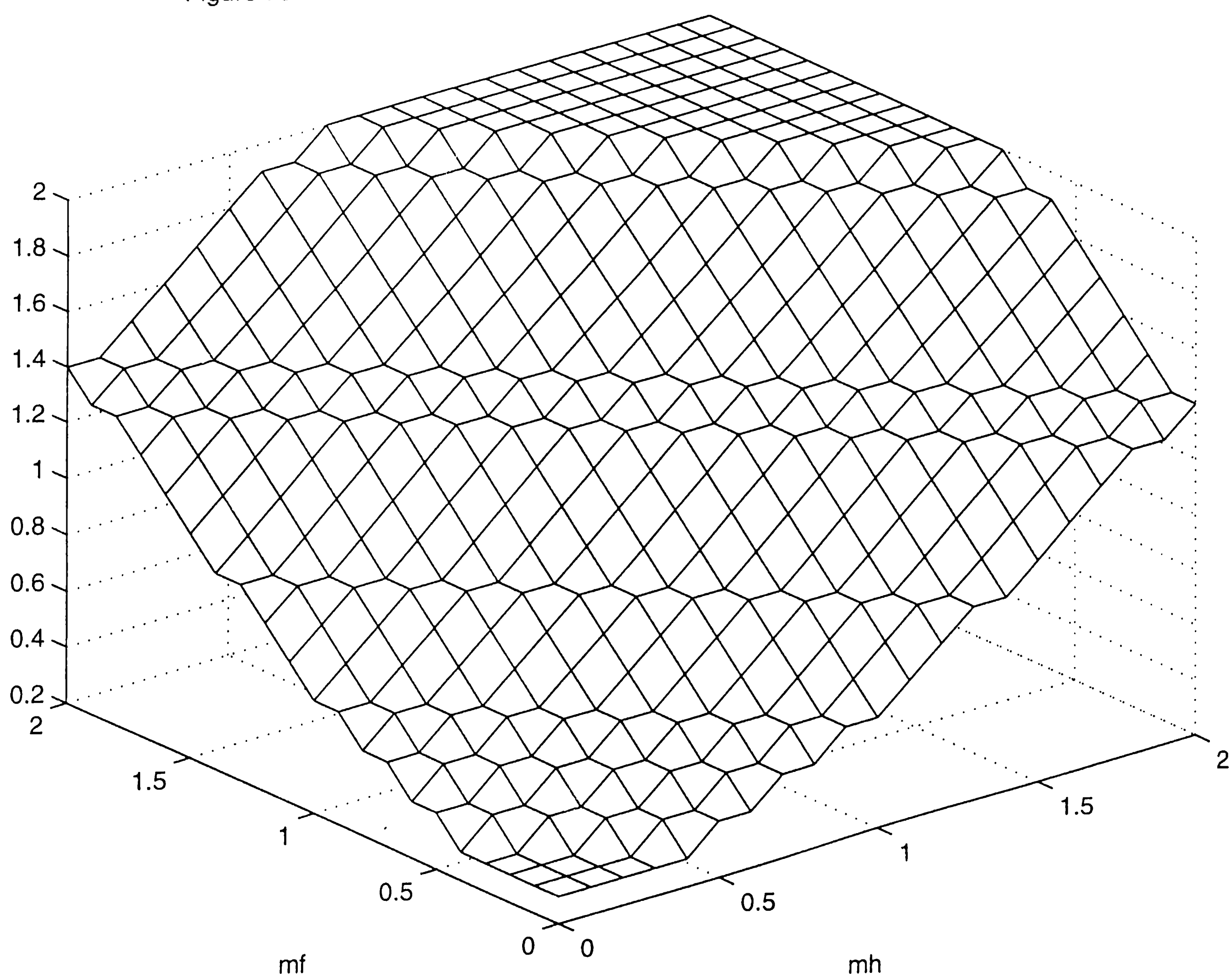


Figure 8a - CONSUMPTION FUNCTION at LOW INFLATION (d=0.5)

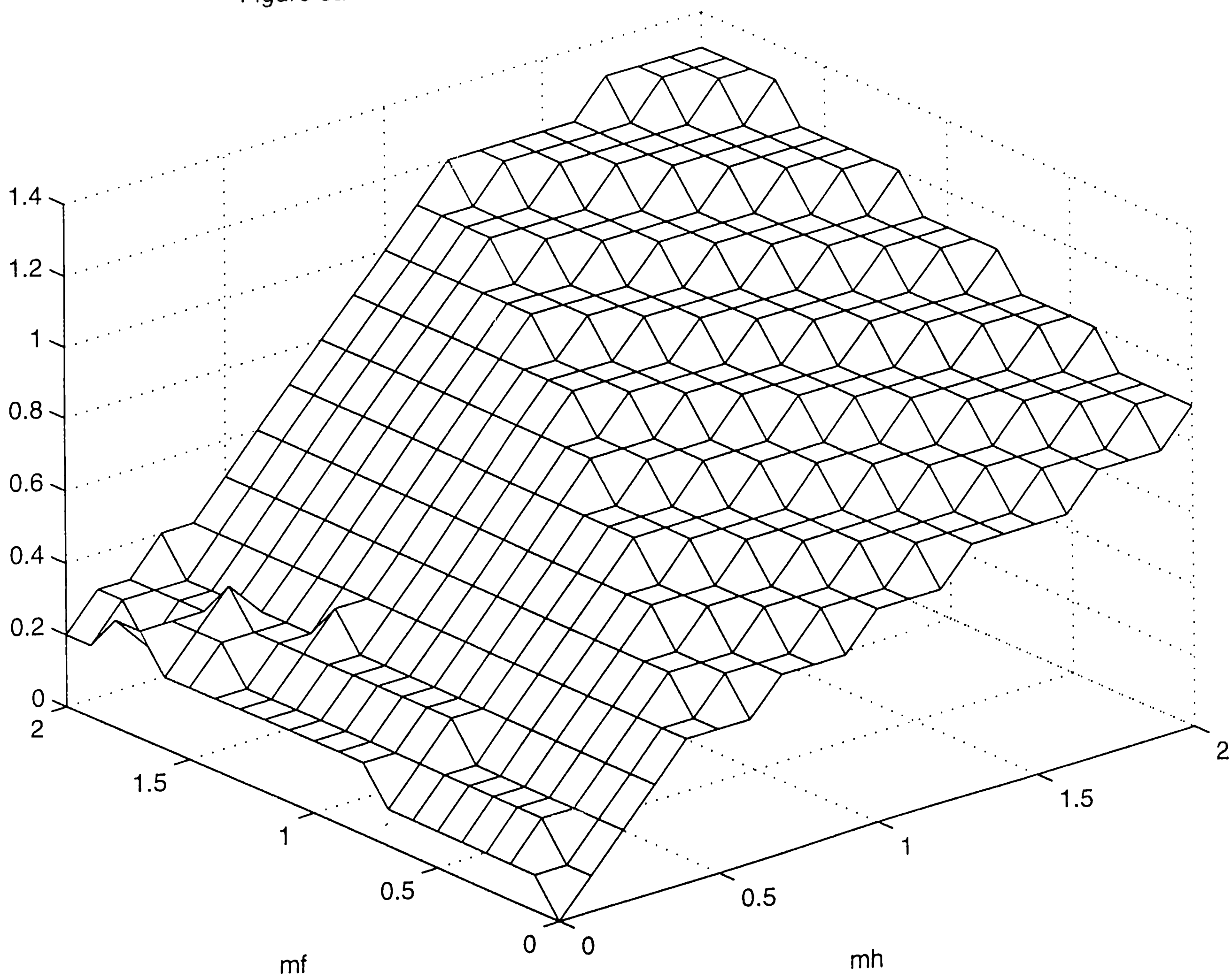


Figure 8b – CONSUMPTION FUNCTION at MID INFLATION (d=0.5)

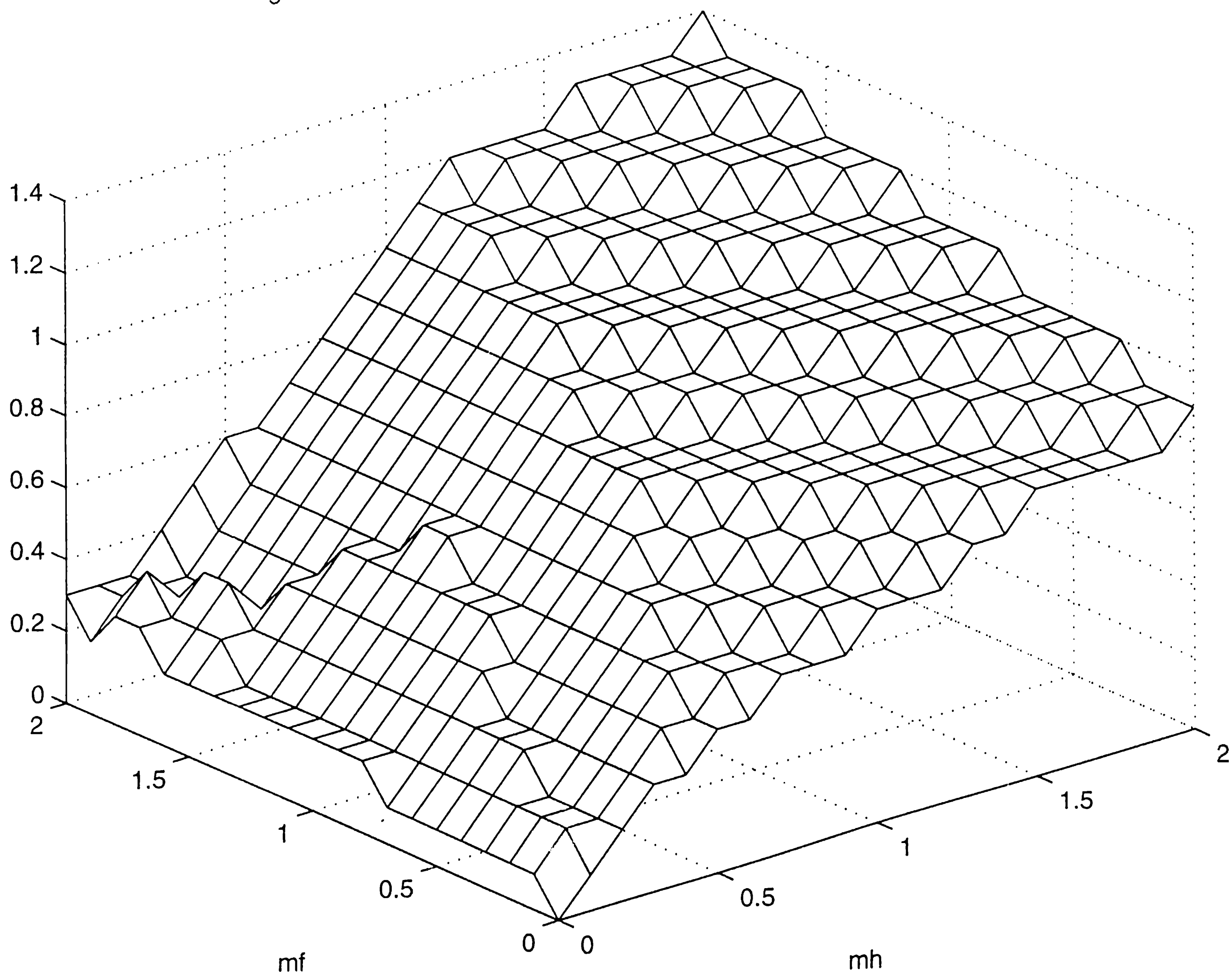


Figure 8c – CONSUMPTION FUNCTION at HIGH INFLATION (d=0.5)

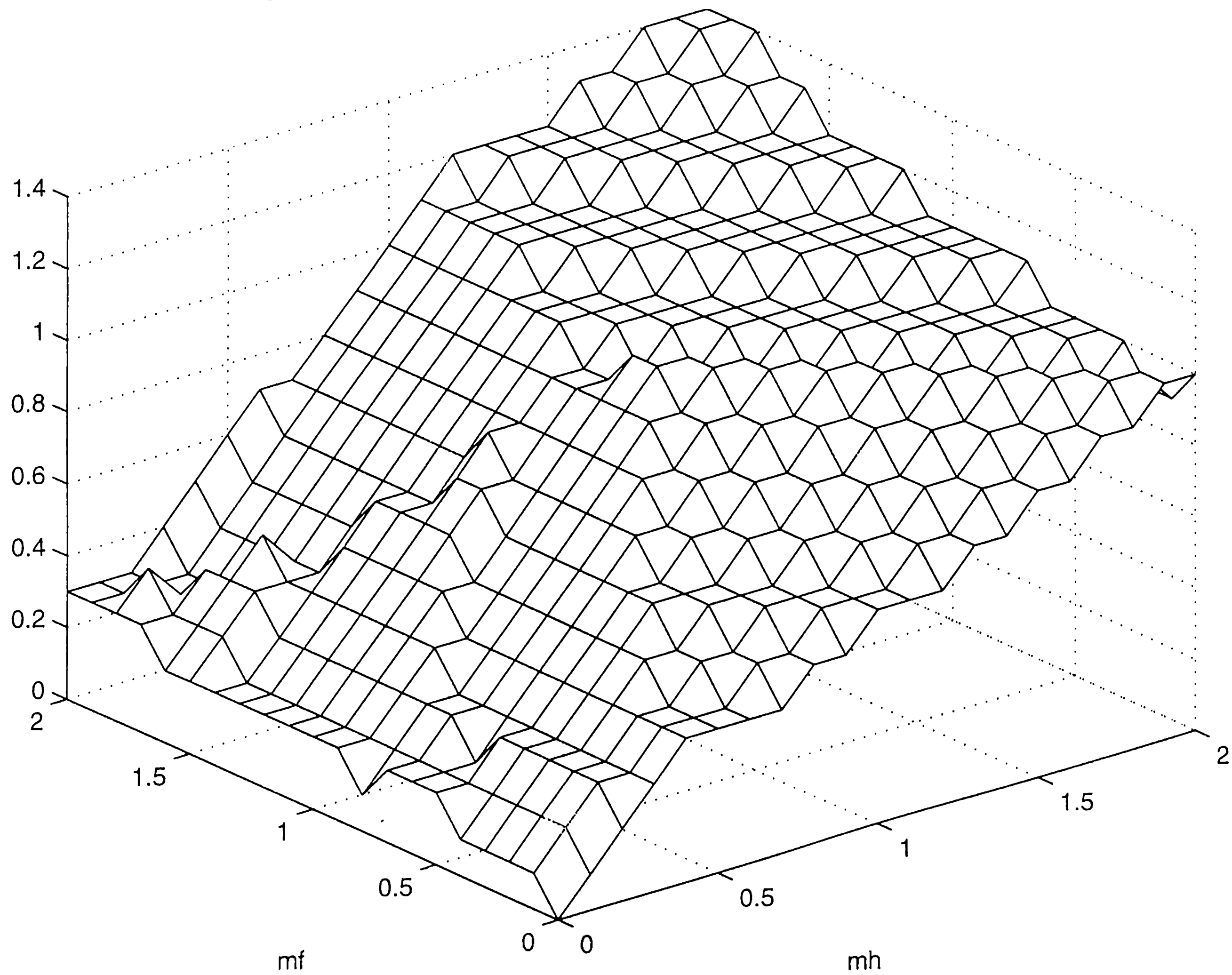


Figure 9a – FOREIGN REAL BALANCES FUNCTION at LOW INFLATION (d=0.5)

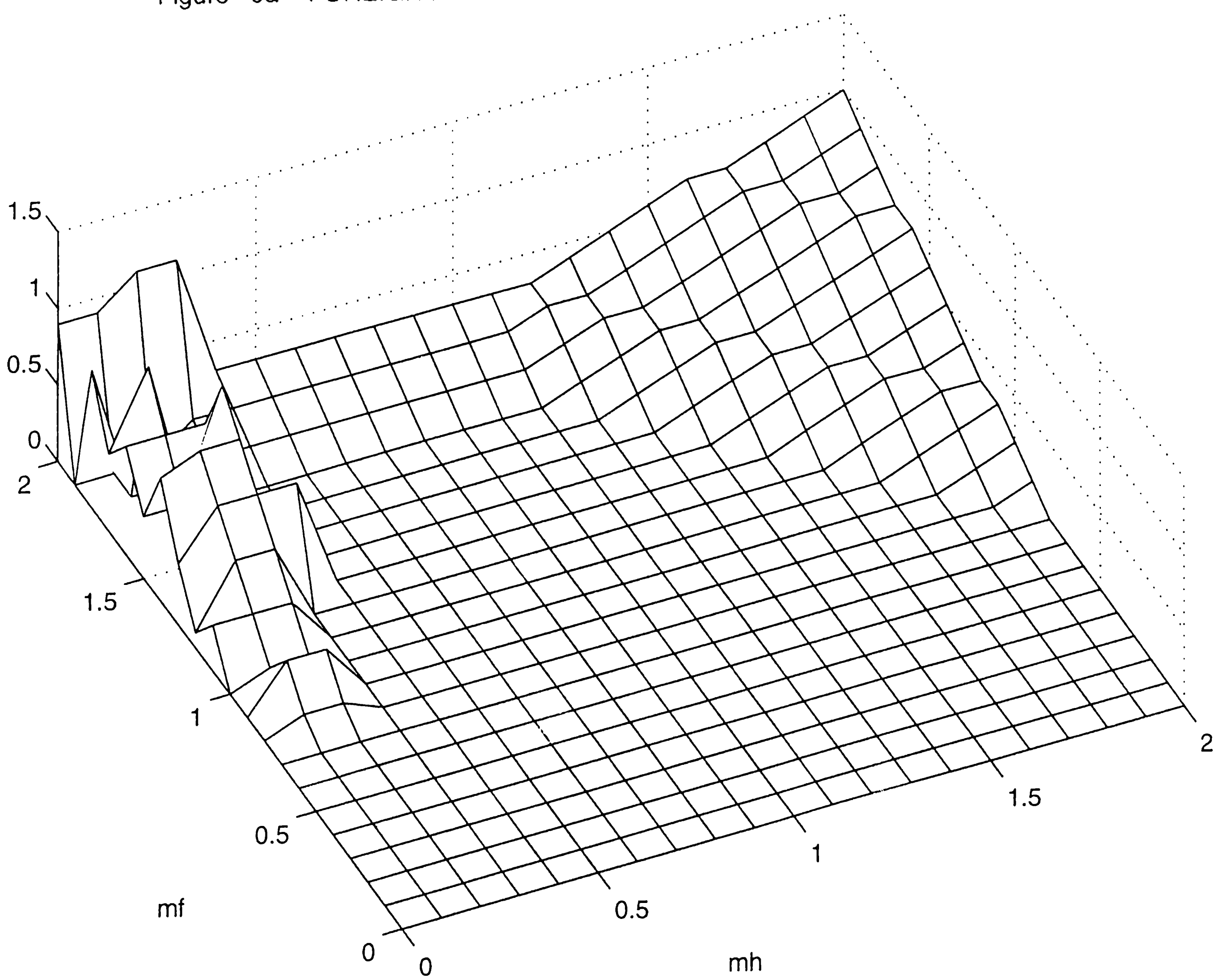


Figure 9b - FOREIGN REAL BALANCES FUNCTION at MID INFLATION (d=0.5)

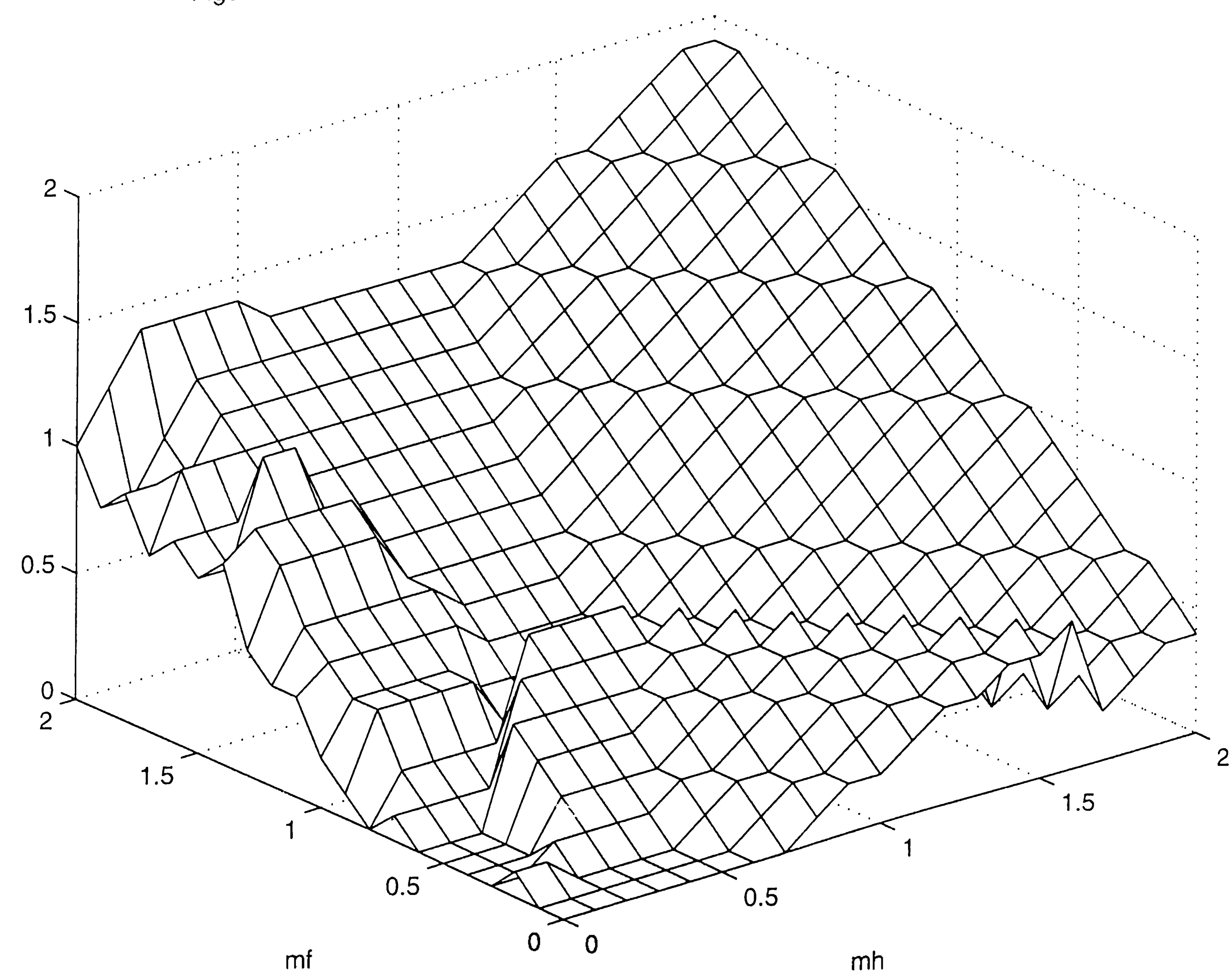
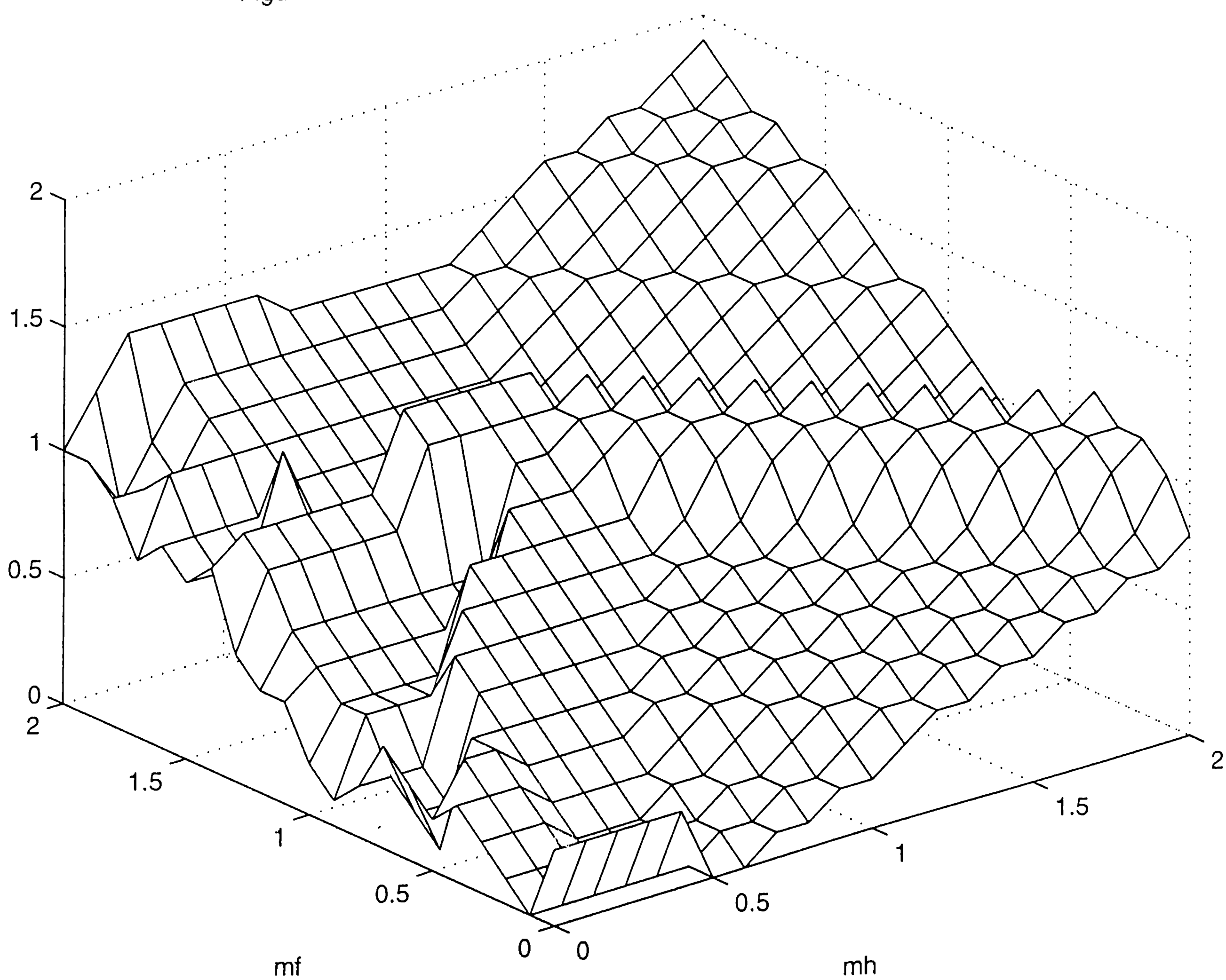


Figure 9c – FOREIGN REAL BALANCES at HIGH INFLATION (d=0.5)



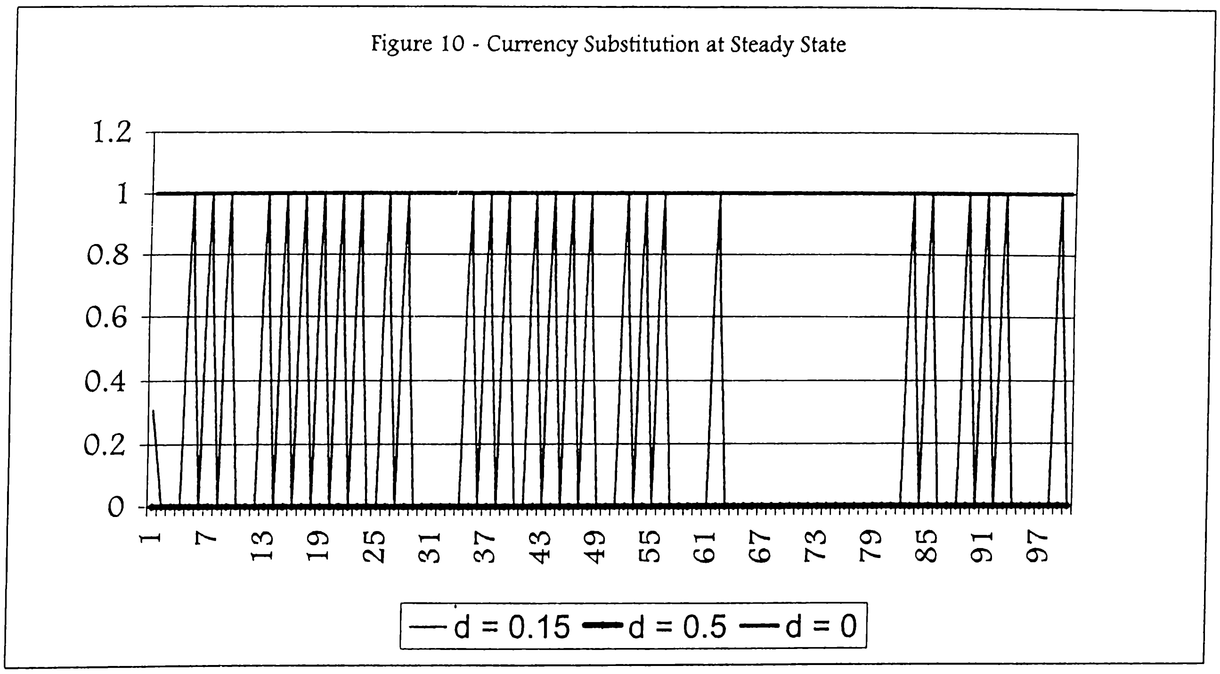


Figure 10 - Currency Substitution at Steady State
1.2
1
0.8
0.6
0.4
0.2
0
1
7
13
19
25
31
37
43
49
55
61
67
73
79
85
91
97
d = 0.15
d = 0.5
d = 0

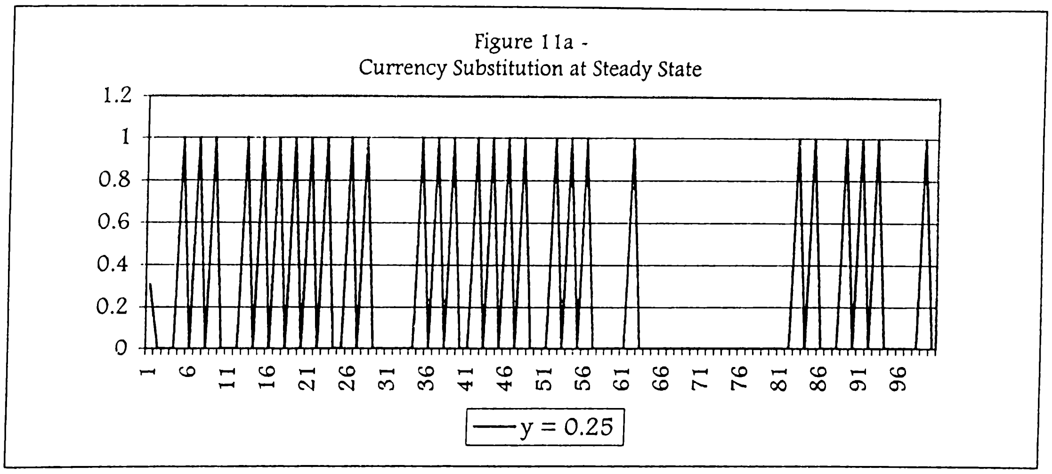


Figure 11a -
Currency Substitution at Steady State
1.2
1
0.8
0.6
0.4
0.2
0
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96
—y = 0.25

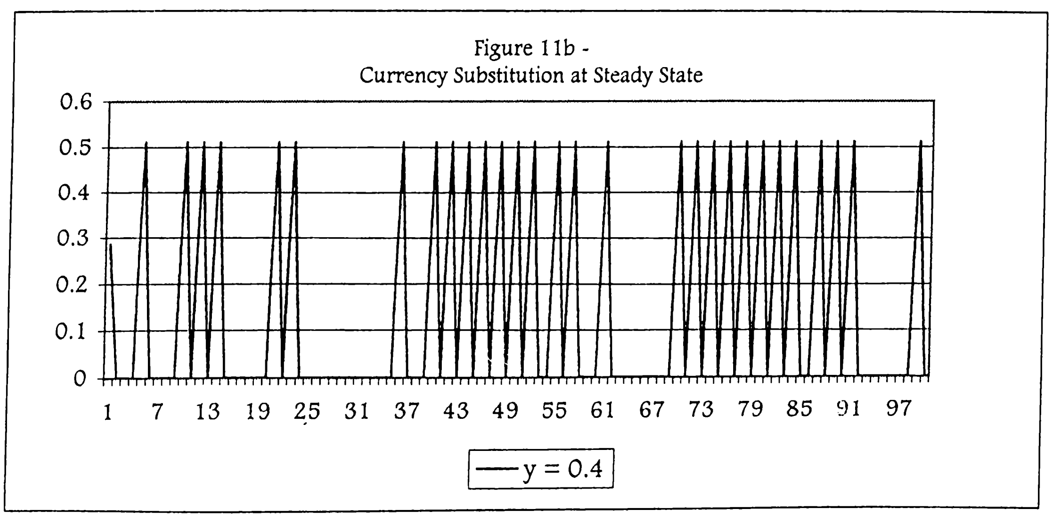


Figure 11b -
Currency Substitution at Steady State
0.6
0.5
0.4
0.3
0.2
0.1
0
1 7 13 19 25 31 37 43 49 55 61 67 73 79 85 91 97
—y = 0.4

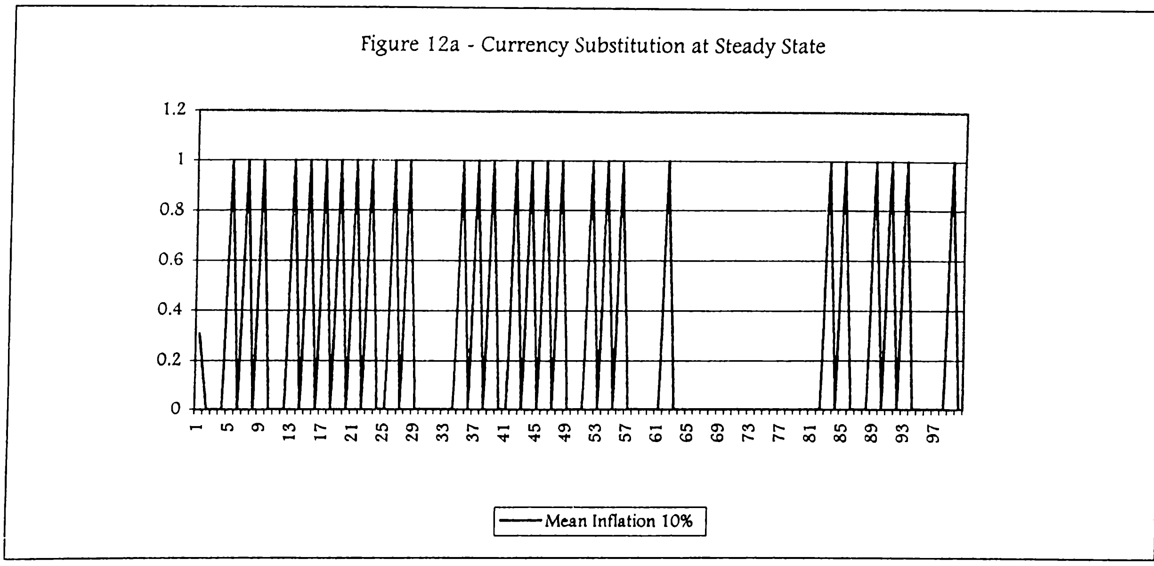


Figure 12a - Currency Substitution at Steady State
Mean Inflation 10%

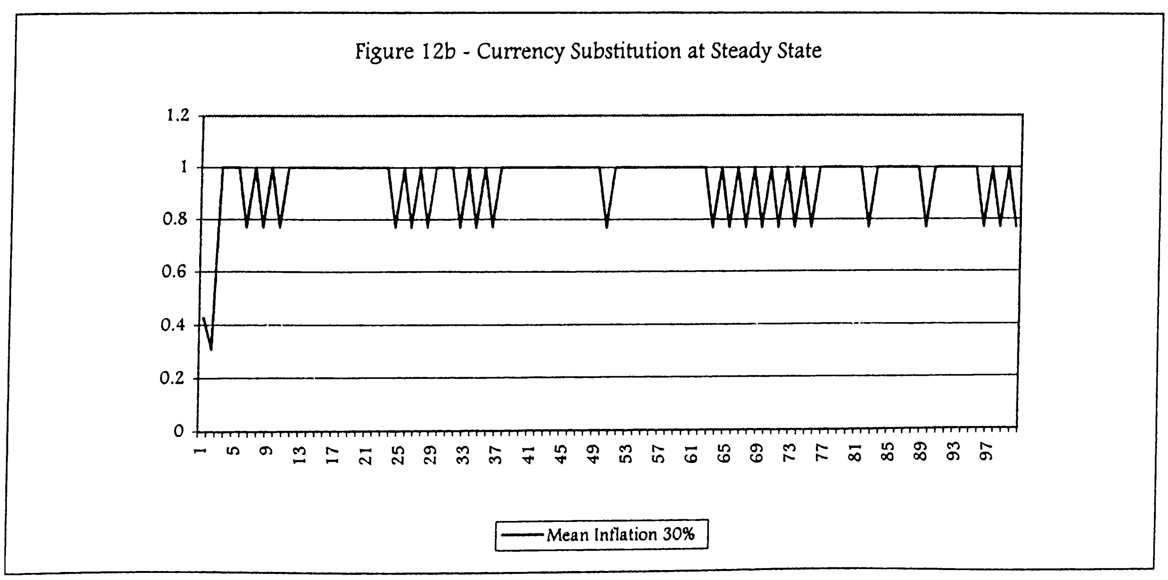


Figure 12b - Currency Substitution at Steady State
Mean Inflation 30%

APPENDIX B

THE COMPUTER PROGRAM

```
import java.lang.Math;
class program {
public static void main (String args[]) {
int i,j,k,l,n,o,p,z;
int ind=0;

double y= {0.25, 0.4};   income, deterministic
double d= {0, 0.15, 0.5};   transaction costs
double [] f = new double [21]; the proportion of foreign currency converted for consumption
double uplimmfp; upperlimit of foreign real balances savings
double [] inf = new double [3]; inflation rate
double [] mf = new double [21]; foreign real balances
double [] mh = new double [21]; domestic real balances
double uplimcons=0;
double u; utility
double [] mhp = new double [inf.length]; domestic real balances savings
int [] index = new int [inf.length];
double [] [] [] polf = new double [mh.length] [mf.length] [inf.length];

double [] [] [] polc = new double [mh.length] [mf.length] [inf.length];
double [] [] [] polmf = new double [mh.length] [mf.length] [inf.length];
double [] [] [] polmh = new double [mh.length] [mf.length] [inf.length];

double [] [] [] finalv = new double [mh.length] [mf.length] [inf.length];
double [] [] [] value = new double [mf.length] [mh.length] [inf.length];
double [] [] [] newvalue = new double [mf.length] [mh.length] [inf.length];
```

```
double challenge;
double maxvalue;

int cind=0;
int [] policy = new int [3];
double [] con= new double [41]; consumption
double diff=1.0;
double beta=0.9;
double sigma =0.5;
double iter1=0.1;
double iter2=0.05;

inf[0] = 1; inf[1]=1.05; inf[2]=1.25; gross inflation rate is a three state Markov process
double [] [] prob=new double [inf.length] [inf.length];
prob [0] [0] = 0.6; prob [0] [1] = 0.4; prob [0] [2] = 0;
prob [1] [0] = 0.2; prob [1] [1] = 0.6; prob [1] [2] = 0.2;
prob [2] [0] = 0; prob [2] [1] = 0.4; prob [2] [2] = 0.6;

Creating the domain for the variables f, mf, mh, and con
for (i=0; i<mf.length; i++) {

        mf[i] = i*iter1;
        mh[i] = i*iter1;

}
for (i=0; i<con.length; i++) {

     con[i] = i*iter1;

}


for(i=0;i<f.length;i++){
        f[i] = i*iter2;
```

```
}
System.out.println("sigma= "+sigma);
System.out.println("beta= "+beta);
System.out.println("transactions cost= "+d);
System.out.println("income= "+y);

int counter=0;
while (diff>0.035) {

Value function is by default initialized with zero
Step 2:
for (i=0; i<mh.length; i++) {                 1st loop for mh
   for (j=0; j<mf.length; j++) {              2nd loop for mf
      for (k=0; k<inf.length; k++) {          3rd loop for inflation
           maxvalue=-1000;
         for (l=0; l<f.length; l++) {           4th loop for f

         uplimcons = mh[i]+( (1-d)*f[l]*mf[j] );
                  for (z=0; z<con.length; z++){
                  if (Math.abs((10*uplimcons)-(10*con[z]))<0.51)
                  {
                  cind = z;
                  }else if(z==con.length-1 && uplimcons>con[z]) {
                  cind = z;
                  }else if(z==0 && uplimcons<con[z]){
                  cind = 0;
                  }
      }
                  for (int h=0; h<cind+1; h++){                5th loop for consumption

                  u = Math.pow(con[h],sigma)/(sigma);
                  uplimmfp =(uplimcons-con[h])+y+(1-f[l])*mf[j];
                           for (p=0; p<mf.length; p++) {
                           if (Math.abs((10*uplimmfp)-(10*mf[p]))<0.51)
```

```
{
ind = p;
}else if(p==mf.length-1 && uplimmfp>mf[p]) {
ind = p;
}else if(p==0 && uplimmfp<mf[p]){
ind = 0;
}
}
        for (n=0; n<ind+1; n++) {       6th loop for mfp
                for (o=0; o<inf.length; o++) {
        mhp[o] = ((uplimcons-con[h])+y+((1-f[l])*mf[j])-mf[n])/inf[o];
                for (p=0; p<mh.length; p++) {
                if (p==0 && mhp[o]<mh[p])
                { index[o] = 0;
                }else if(Math.abs((10*mhp[o])-(10*mh[p]))<0.51){
                index[o] = p;
                }else if(p==mh.length-1 && mhp[o]>mh[p]) {
                index[o] = p;
                }
                }
        }

double exp=0;
for (o=0; o<inf.length; o++) {
 exp = exp+(prob[k][o]*value[index[o]][n][o]);
}

challenge = u+(beta*exp);
if(maxvalue<challenge)
{
maxvalue=challenge;
policy [0] = l;
policy [1] = n;
policy [2] = h;
```

```
            }
                                }  6th loop ends
                        }  5th loop ends
            }   4th loop ends

                  if (diff<0.05)
            {
             polf [i] [j] [k] = f[policy[0]];
             polc [i] [j] [k] = con[policy[2]];
             polmf [i] [j] [k] = mf[policy[1]];
             if ( (mh[i]+((1-d)*f[policy[0]]*mf[j])-con[policy[2]]+y+((1-f[policy[0]])*mf[j])-
            mf[policy[1]])/inf[k]<0 )
            {
             polmh [i] [j] [k] = 0; }
            else polmh [i] [j] [k] = (mh[i]+((1-d)*f[policy[0]]*mf[j])-con[policy[2]]+y+((1-
            f[policy[0]])*mf[j])-mf[policy[1]])/inf[k];
                    }
            newvalue[i][j][k] = maxvalue;

        }  3rd loop ends
      }    2nd loop ends
    }      1st loop ends
```

Step 3 & 4 :

```
diff=0;
for (i=0; i<mh.length; i++) {
  for (j=0; j<mf.length; j++) {
    for (k=0; k<inf.length; k++) {
      if(diff<Math.abs(newvalue[i][j][k]-value[i][j][k]))
      diff=Math.abs(newvalue[i][j][k]-value[i][j][k]);
    }
  }
}
if (diff<0.05){
```

```
  for (i=0; i<mh.length; i++) {
     for (j=0; j<mf.length; j++) {
       for (k=0; k<inf.length; k++) {
           finalv[i][j][k]=newvalue[i][j][k];
        }
     }
   }
 }
 for (i=0; i<mh.length; i++) {
    for (j=0; j<mf.length; j++) {
        for (k=0; k<inf.length; k++) {
 value[i][j][k] = newvalue [i][j][k];
     }
   }
 }
 counter++;
 }

 System.out.println("convergence completion after"+counter+"iteration");
 for (k=0; k<inf.length; k++) {
 for (i=0; i<mh.length; i++) {
   for (j=0; j<mf.length; j++) {
        System.out.println("value("+mh[i]+","+mf[j]+","+inf[k]+"):  "+finalv[i][j][k]);
  }
 }
}
System.out.println("policy function f");
for (k=0; k<inf.length; k++){
System.out.println("when inflation rate is"+inf[k]);
  for (i=0; i<mh.length; i++) {
    System.out.println("domestic real balance   "+mh[i]);
       for (j=0; j<mf.length; j++) {


           System.out.println("                    "+polf [i] [j] [k]);
```

```
      }
    }
  }
  System.out.println("policy function consumption");
    for (k=0; k<inf.length; k++){
        System.out.println("when inflation rate is"+inf[k]);
            for (i=0; i<mh.length; i++) {
               System.out.println("domestic real balance   "+mh[i]);
                 for (j=0; j<mf.length; j++) {

   System.out.println("                    "+polc [i] [j] [k]);
                 }
             }
    }

  System.out.println("policy function foreign real balance");
      for (k=0; k<inf.length; k++){
        System.out.println("when inflation rate is"+inf[k]);
            for (i=0; i<mh.length; i++) {
               System.out.println("domestic real balance   "+mh[i]);
                    for (j=0; j<mf.length; j++) {
                           System.out.println("                    "+polmf [i] [j] [k]);
                    }
            }
        }

System.out.println("policy function domestic real balance");
        for (k=0; k<inf.length; k++){
            System.out.println("when inflation rate is"+inf[k]);
              for (i=0; i<mh.length; i++) {
                  System.out.println("domestic real balance   "+mh[i]);
                    for (j=0; j<mf.length; j++) {
                     System.out.println("                    "+polmh [i] [j] [k]);
                    }
```

```
}
}

Steady state values are found
double die=0;
int a=0;
int b=0;
int c=(int) (Math.random()*2);
i=10;
j=10;

for (n=0; n<100; n++){
  die = Math.random();
if (c==0) {
if (die>0 && die<=0.6) {
 c=0; }
else if (die>0.6 && die<=1){
 c=1; }
}
if (c==1) {
if (die>0 && die<=0.6) {
 c=1; }
else if (die>0.6 && die<=0.8){
 c=0; }
else if (die>0.8 && die<=1){
 c=2; }
}
if (c==2) {
if (die>0 && die<=0.6) {
 c=2; }
else if (die>0.6 && die<=1){
 c=1; }
}
System.out.println(n+" "+polmh[i][j][c]+" "+polmf[i][j][c]);
```

```
for (p=0; p<mf.length; p++) {
  if (polmf[i][j][c]==mf[p])
        b = p;
}
 for (p=0; p<mh.length; p++) {
    if (p==0 && polmh[i][j][c]<mh[p])
 {
        a = 0;
     }else if(Math.abs((10*polmh[i][j][c])-(10*mh[p]))<0.51) {
        a = p;
     }else if(p==mh.length-1 && polmh[i][j][c]>mh[p]){
        a = p;
     }
    }
i=a;
j=b;

}
}
}
```