DETERMINACY OF EQUILIBRIUM IN A CASH-IN-ADVANCE MODEL

A Master's Thesis

by

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

DETERMINACY OF EQUILIBRIUM IN A CASH-IN-ADVANCE MODEL

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July 2000

This thesis studies on the determinacy of equilibrium in a cash-in-advance production economy. The model consists of two types of infinitely lived agents, producer and labor types. Producer types face financial constraints in their labor demands. There are two markets operating in a sequence, first labor market and then goods market open in each period. In this setup, the possibility of selffulfilling inflations, deflations and fluctuations are investigated. The first result of the thesis rules out these possibilities. As a second result, we observe that the initial distribution of money across types is irrelevant regarding equilibrium consumption and welfare levels.

Keywords: Determinacy of equilibrium, cash-in-advance, selffulfilling, welfare.

ÖZET

ÜRETİMDE MALİ KISITLARIN BULUNDUĞU BİR EKONOMİDE DENGENİN BELİRLENEBİLİRLİĞİ

Tekatlı, Necati

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Bu tez üretimde mali kısıtların olduğu bir ekonomide dengenin belirlenebilirliği üzerine bir çalışmadır. Modelimizde uzun ömürlü işveren ve işçi tiplerinden oluşan bir ekonomiyi incelemekteyiz. İşverenlerin işgücü taleplerini karşılayacak nakitlerinin bulunması gerekmektedir. Her dönem önce işgücü piyasası sonra mal piyasası açılmaktadır. Böyle bir ekonomide, salt beklentilerden ortaya çıkan enflasyon, deflasyon ve dalgalanmaların varolup olmayacağı incelenmektedir. İlk sonucumuz bu tür dengelerin mümkün olmadığıdır. Çalışmanın ikinci önemli sonucu, başlangıçtaki paranın işveren ve işçiler arasındaki dağılımının denge tüketim ve refah seviyesilerini etkilemediğidir.

Anahtar Kelimeler: Dengenin belirlenebilirliği, mali kısıt, salt beklentiler, refah seviyesi.

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TABLE OF CONTENTS

1 Introduction

We study the determinacy of equilibria in a cash-in-advance production economy. The economy consists of two types of infinitely lived agents. There are two markets operating in a sequence, first labor market and then goods market open in each period. We first establish the uniqueness of monetary competitive equilibrium (MCE) and then prove a policy irrelevance result.

Determinacy of equilibria in macroeconomic models has been considered an important issue in the literature. Examples of indeterminacy include selffulfilling inflations and deflations (Sargent and Wallace, 1973, Bruno and Fischer, 1990), indeterminacy of exchange rates (Karaken and Wallace, 1981), asset price bubbles (Santos and Woodford, 1997), selffulfilling prophecies about business activity (Diamond , 1982, Farmer and Guo, 1994) and sunspots (Azariadis, 1993), endogeneuos cycles (Grandmont, 1985). Indeterminacy is typically observed in models with rational expectations and microfoundations.

Indeterminacy is clearly a problem in policy analysis. There can be many possible equilibrium paths consistent with the same policy. As a method of selection out of these equilibria, either learning dynamics have been introduced (e.g. Marcet and Sargent, 1989) or governments role in coordinating expectations around the better equilibria have been proposed (Azariadis, 1993).

Models of money that have been considered as suitable for policy analysis are the overlapping generations (OLG) model (Wallace, 1980) and the cash-in-advance model (Lucas and Stokey, 1983). Indeterminacy of equilibrium in OLG models is very common (e.g. Santos and Bona, 1989, Marimon and Sunder, 1993). In cash-in-advance models, indeterminacy issue has been studied by Sims (1994) and Woodford (1994). They show that, under a regime of fixing the money growth rate, indeterminacy may occur under low levels of money growth. Sims and Woodford work in a setup where the cash-in-advance constraints are imposed on consumption spending. In a setup where financial constraints are imposed on the factor demands of the firms (e.g. Fuerst, 1992, Başçı and Sağlam, 1998) , the indeterminacy issue has not been addressed.

In this thesis, we study the uniqueness of equilibria in a cash-in-advance model with two types of agents where the producer types face financial constraints. The existence of stationary equilibrium in this setup has been demonstrated by Başçı and Sağlam (1998). Here we ask the question whether selffulfilling deflations or inflations or fluctuations can be consistent with competitive equilibrium in the same model. After establishing the uniqueness of equilibrium, we also study a monetary tax-transfer experiment.

The thesis is organized as follows. The next chapter introduces the model. The third chapter proves the determinacy of equilibria. The fourth chapter shows welfare neutrality of money transfer to the worker types. Concluding remarks are presented in the last chapter.

2 The model

2.1 The Basic Structure

The economy consists of two types of agents (indexed by i=1,2) which have population sizes of N_1 and N_2, respectively. [1] Without loss of generality, we take $N_1 = N_2$ for simplicity. At each period t, there are two commodities, labor and a nonstorable consumption good. The lifetime utility of each type is

$$\sum_{t=0}^{\infty} \beta_i^t U_i(c_{it}) \quad (1)$$

where $\beta_i \in (0,1)$ is the discount factor of type i, c_{it} is the period-t consumption of i and $U_i(.)$ is the instantaneous utility function of a representative agent of type i and $U_i(c) = log(c)$.

The production technology of each type is

$$f_i(L) = \gamma_i L \quad (2)$$

where L is the labor input, γ_i is the marginal product of labor in agent i's pro-

[1]The basic structure of the model considered here is almost the same as Başçı and Sağlam (1998) .

duction plant and we take γ_1=0 for simplicity and γ_1=γ. Type two agent has a technology with higher productivity. Each type of agent is endowed with labor $\bar{L}_i$, a production function $f_i(.)$ and an initial money balance of $M_{i,0}$, but no endowment of consumption goods. We assume that $M_{20} > 0$. Also, it is obviously seen that agent 1(worker type) has no production possibility in its own plant since $\gamma = 0$ by consumption.

Notations we use are as follows:

Choice variables of a type i agent in period t are consumption, labor demand ((+) demand, (-) supply), goods demand ((+) demand, (-) supply) and money carried over to period $t+1$ and are denoted c_{it}, L_{it}, q_{it} and $M_{i,t+1}$, respectively.

Prices in period t are nominal wage rate and nominal good price, denoted by w_t and p_t, respectively.

At time zero, agents of the same type start with the same amount of money, $M_{i,0}$. Let M denote the total quantity of money in the economy. We assume that there is no further government intervention to the economy, so that total money stock does not change over time. For obvious reasons, we require that M is strictly positive, and

$$M_{1,0} + M_{2,0} = M \tag{3}$$

2.2 The Structure of Trading

Each period t consists of two subperiods. In the first sub-period, labor market and in the second sub-period, goods market open.

In the first sub-period, each type of agent starts the period with a money balance of $M_{i,t}$, i=1,2. In the labor market, agents, using either their money $M_{i,t}$ or their labor endowment $\bar{L}_{it}$, buy or sell labor at the nominal wage w_t. Since there is no loan market, wages must be paid before the second sub-period so that the agents can use this income in the goods market for trade. After the labor market, production of non-storable consumption good takes place with the purchased labor.

In the second sub-period, the goods market opens and the non-storable good is traded at the nominal price p_t with the money held after the labor market operations. These transactions determine the money balance in the next period.

2.3 The Maximization Problem

Agent's problem is to choose how much to consume and how much to supply or hire labor subject to the sequence of budget constraints, given the endowments and the sequence of strictly positive prices $\{w_t, p_t\}_{t=0}^{\infty}$:

$$\max \sum_{t=0}^{\infty} \beta_i^t U_i(c_{it}) \tag{4}$$

subject to, for all t

$$c_{it} = f_i(\bar{L}_i + L_{it}) + q_{it},$$

$$-\bar{L}_i \leq L_{it} \leq \frac{M_{i,t}}{w_t},$$

$$-f_i(\bar{L}_i + L_{it}) \leq q_{it} \leq \frac{M_{i,t} - w_t L_{it}}{p_t},$$

$$M_{i,t+1} = M_{i,t} - w_t L_{it} - p_t q_{it},$$

$$M_{i,0} \geq 0 \quad \text{is given.}$$

The choice variables and constraints are same for all agents. The first constraint says that agent consumes what he produces and what he buys. First inequality constraints the labor supply and demand. Labor can be supplied at most the amount endowed and there is a finance constraint, $\frac{M_{i,t}}{w_t}$, for labor demand. Third constraint gives the upper and lower bounds for quantity demanded. Since labor market opens first, in the goods market agent can use the money left after he has bought labor. Second equation of the costraints gives the money balance in period t+1 which is the money remained after the labor and goods market operations. Since all the agents are financially constrained in each period and sub-period, we call it a *financially constrained economy (FCE)*. Since in the model initial money endowments are taken as exogeneous, the levels of M_{10} and M_{20} are included in definition of a *FCE*.

$\{w_t, p_t, L_{it}, q_{it}, c_{it}, M_{it+1} \mid i = 1,2\}_{t=0}^{\infty}$ is called a *monetary competitive equilibrium (MCE)* of the financially constrained economy *FCE*, if $w_t, p_t > 0$ for all t, and

- for all i, $\{L_{it}, q_{it}, c_{it}, M_{it+1}\}_{t=0}^{\infty}$ solves the problem (4) under $\{w_t, p_t\}_{t=0}^{\infty}$,
  $N_1 L_{1t} + N_2 L_{2t} = 0$ for all t,
  $N_1 q_{1t} + N_2 q_{2t} = 0$ for all t,
  and $N_1 M_{1,t} + N_2 M_{2,t} = M$ for all t.

If the prices, wages, consumptions, demands, supplies and money holdings are all constant over time, then the monetary competitive equilibrium *(MCE)* of the financially constrained economy *(FCE)* is called a *stationay monetary competitive equilibrium(SMCE).* [2]

One should also note that, by Walras' Law, if two of the three markets -goods market, labor market and money market- clear, the remaining market clears as well.

After eliminating c_{it} and q_{it} in each agent's problem, the reduced form of the maximization problem (4) is:

$$\max \sum_{t=0}^{\infty} \beta_i^t U_i \left(f_i(\bar{L}_i + L_{it}) - \frac{w_t}{p_t} L_{it} + \frac{M_{i,t} - M_{i,t+1}}{p_t} \right) \tag{5}$$

[2] *SMCE* is demonstrated by Başçı and Sağlam (1998)

subject to, for all t

$$-\bar{L}_i \leq L_{it} \leq \frac{M_{i,t}}{w_t},$$

$$0 \leq M_{i,t+1} \leq M_{i,t} - w_t L_{it} + p_t f_i(\bar{L}_i + L_{it}),$$

$$M_{i,0} \geq 0 \quad \text{is given.}$$

Lemma 1 *Labor Demand Function* is given by

$$L_{it}(w/p) = \begin{cases} -\bar{L}_i & if \;\; w/p > \gamma_i, \\ L_{it} \in [-\bar{L}_i, M_{i,t}/w] & if \;\; w/p = \gamma_i, \\ M_{i,t}/w & if \;\; w/p < \gamma_i. \end{cases} \tag{6}$$

for any given path of money holdings $\{M_{i,t}\}_{t=0}^{\infty}$. [3]

Lemma 2 In a monetary competitive equilibrium of this economy, equilibrium real wage should satisfy $w_t/p_t \in [0, \gamma]$ for all t. [4]

[3]The proof is given in Başçı and Sağlam (1998).

[4]The proof is given in Başçı and Sağlam (1998)

3 Determinacy of Equilibrium

In this chapter, we will study the determinacy of the equilibrium in this financially constrained economy *(FCE)*. For given price series $\{p_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$ with $\frac{w_t}{p_t} \in [0, \gamma]$ the model has the following feature:

From labor demand function $L_{1t} = -\bar{L_1}$ and $L_{2t} = M_{2t}/w_t$. [5] From labor market clearing, $L_{2t} = \bar{L_1}$. Thus, $L_{2t} = M_{2t}/w_t = \bar{L_1}$ and so $M_{2t} = w_t \bar{L_1}$ which says that agent 2 buys labor with all his money. Using the fourth constraint of problem (4) and labor demand of agent 2, money flow becomes $M_{2,t+1} = -p_t q_{2,t}$.

Since what is produced is all consumed, $c_{1t} + c_{2t} = f_2(\bar{L}_1)$. From goods market clearing, $q_{1t} = -q_{2t}$. Since agent 1 does not produce, he consumes only what he buys from agent 2, ie., $c_{1t} = q_{1t}$.

Agent 2 supplies the amount, $q_{2t} = M_{2,t+1}/p_t$, of the good he produced and consumes the amount $c_{2t} = f_2(\bar{L}_1) + q_{2t}$.

By money flow of agent 1, $q_{1t} = -\frac{M_{1,t+1} - M_{1,t}}{p_t} + \frac{w_t}{p_t}\bar{L}_1$.

Euler conditions and transversality conditions are :

Euler Conditions:

[5]In case $\frac{w_t}{p_t} = \gamma$, the firms will be indifferent with regard to the fraction of their working capital to spend in the labor market. We take that fraction as 1.

Euler.1 For the problem of agent 1,

$U_1'(c_{1,t}) \geq \beta_1 \frac{p_t}{p_{t+1}} U_1'(c_{1,t+1})$

since agent 1 holds either positive or zero amount of money in period t+1.

Euler.2 For the problem of agent 2,

$U_2'(c_{2,t}) = \frac{p_t}{w_{t+1}} \gamma \beta_2 U_2'(c_{2,t+1})$

since agent 2 always holds positive amount of money. [6]his is due to the fact that log(c) exhibits an infinitely large marginal utility of consumption as c goes to 0

Transversality Conditions:

TVC.1 For agent 1,

$\lim_{t\to\infty} \beta_1^t (-\frac{1}{p_t}) U_1'(c_{1,t}) M_{1,t+1} = 0$

TVC.2 For agent 2,

$\lim_{t\to\infty} \beta_2^t (-\frac{1}{p_t}) U_2'(c_{2,t}) M_{2,t+1} = 0$

Note that the Euler and transversality conditions are necessary and sufficient

[6]T

for optimization problems.

The following results will be useful in proving the propositions in the next sections.

Lemma 3 In a *MCE* of the financially constrained economy, $M_{1t} > 0$ and $M_{1t+1} = 0$ for some t≥1 is not possible.

Proof:

Let us take three successive periods of the economy such that agent 1holds positive amount of money in the second period and but holds no money in the third period, whatever he does in the first of these three periods. Let us denote these periods by t, t+1and t+2.

We know that $c_{1t} = q_{1t} = \frac{M_{2t+1}}{p_t}$. That $M_{1t+1} > 0$ makes Euler 1 equality, ie., $U_1'(c_{1t}) = \beta_1 \frac{p_t}{p_{t+1}} U_1'(c_{1t+1})$. After substituting $c_{1t} = \frac{M_{2t+1}}{p_t}$ and logarithmic utility function into the Euler equation, simplifications results in $M_{2t+1} = \frac{1}{\beta_1} M_{2t+2}$ or equivalently $M_{2t+1} = \frac{1}{\beta_1} M$. Thus, because money holding of agent 2 at period t+1 exceeds the total amount of money in the market, Euler 1 leads to a contradiction.■

Lemma 4 In a *MCE* of the financially constrained economy, $M_{1t} > 0$ and $M_{2t} > 0$ for all t>T for some T≥0 is not possible.

Proof:

Assume that $M_{1t} > 0$ and $M_{2t} > 0$ for all t>T for some T≥0 . All of the following are regarding t>T. So, Euler 1 becomes equality. Substituting $c_{1t} = \frac{M_{2t+1}}{p_t}$ into Euler 1 leads to $\frac{M_{2t+2}}{p_{t+1}} = \beta_1 \frac{p_t}{p_{t+1}} \frac{M_{2t+1}}{p_t}$ which is equivalent to $M_{2t+2} = \beta_1 M_{2t+1}$. That is, M_{2t} is decreasing. So, by money market clearing M_{1t} is increasing. Again by Euler 1, $c_{1t+1} = \beta_1 \frac{p_t}{p_{t+1}} c_{1t}$. The solution is $c_{1t} = \beta_1^t \frac{p_0}{p_t} c_{10}$. Combining this with transversality of the worker type yields $\lim_{t \to \infty} (p_0 c_{10})^{-1} (-M_{1t+1})$ which is not zero since $M_{1t} > 0$ and it is increasing for all t. So, TVC 1 is not satisfied.■

3.1 Stationary Prices

When prices are stationary, all variables of the economy follow a stationary path. But it is considerable that they follow a unique path and this is the *SMCE*.

Proposition 1 *In a MCE of the financially constrained economy, for stationary prices, allocations (the consumptions, labor demands, goods demands and money demands per representative agent) must be stationary. Moreover, these stationary prices and allocations are the same as the ones in SMCE.*

Proof:

Let prices and wages be constant for all t, and set $p_t = p$ and $w_t = w$ for all t. By labor demand function, $L_{1t} = -\bar{L}_1$. From labor market clearing $L_{2t} = \bar{L}_1$.

Using the labor demand function, $L_{2t} = M_{2t}/w = \bar{L}_1$. So $M_{2t} = w\bar{L}_1$, ie., agent 2 holds a constant (positive) money in each period. Set $M_{2t} = M_2$ for all t, for some $M_2 > 0$. From the money flow equation, $q_{2t} = -\frac{w}{p}L_{2t} = -\frac{w}{p}\bar{L}_1$. Agent 2 has a constant production, $f_2(\bar{L}_1)$, and a constant supply, q_{2t}, therefore he consumes same amount, $c_{2t} = f_2(\bar{L}_1) + q_{2t} = \gamma\bar{L}_1 - \frac{w}{p}\bar{L}_1$, in all periods. Goods market should clear and we have $q_{1t} = -q_{2t} = \frac{w}{p}L_{2t} = \frac{w}{p}\bar{L}_1$ that implies a constant demand by agent 1. Since consumption of agent 2,c_{2t}, and good produced by agent 2, $f_2(\bar{L}_1)$, are constant, consumption of agent 1,c_{2t}, is also same in each period and $c_{1t} = \frac{w}{p}\bar{L}_1$.

Euler equation of the agent 2 is in the following form $U_2'(c_{2t}) = \frac{p}{w}\gamma\beta_2U_2'(c_{2,t+1})$.

But since agent 2 has a constant consumption in each period, Euler Equation implies that $p = \frac{w}{\beta_2\gamma}$.

Since M_{2t} is constant for all t, by money market clearing M_{1t} is constant, too. Then we set $M_1 = M_{1t}$.

Now suppose $M_1 > 0$.Then Euler 1 holds as an equality. But since c_{1t} and p_t are constant in each period, we get $\beta_1 = 1$ from Euler 1. But we are given that $\beta_1 < 1$. So our assumption is not correct and $M_1 = 0$. Thus $M_2 = M$.

Hence, proof is completed and allocations are stationary. ■

This proposition states that we need only the prices to be stationary in order to reach the stationary equilibrium (which is unique and to be shown in section

3.5) when it is profitable, $w_t/p_t < \gamma$, for the producer type.

3.2 Selffulfilling Inflations

In our model, agents have perfect foresight of all future prices and wages. Since money stock does not grow, a "quantity theory" prediction would be zero inflation, which is the case in a stationary equilibrium *(SMCE)*. Selffulfilling inflations and deflations are two potential examples of indeterminacy. These are the cases where prices go up and down simply because agents expect them to do so. Typically with a selffulfilling inflation (deflation), a reduction (an increase) in real money stock over time is observed (Woodford, 1994).

First, we will consider selffulfilling inflation. Selffulfilling sustained inflations prevail with increasing consumption of agent 2 and decreasing consumption of agent 1. As a pressure of inflation, agent 1 decreases his consumption day by day. But optimal consumption plan of agent 2 increases so high that it would violate the feasibility. By this context, following proposition states that MCE with selffulfilling sustained inflations does not exist in a cash-in-advance production economy.

Proposition 2 *In a MCE of the financially constrained economy, selffulfilling sustained inflations are not possible.*

Proof:

By lemma 3 and lemma 4, there is only one case which we should consider for selffulfilling sustained inflations and it is that $M_{1t} = 0$ and $M_{2t} = M$ for all t.

Let $M_{1t} = 0$. Thus $M_{2t} = M$ by money market clearing. So by equation $M_{2t} = w_t \bar{L}_1$, we have $M = w_t \bar{L}_1$ or $w_t = \frac{M}{L_1}$. So constant money leads to constant nominal wages. Set $w_t = w$.

Now, suppose that there is inflation. By $c_{1t} = q_{1t} = \frac{w}{p_t}$, c_{1t} is decreasing due to inflation. Thus, c_{2t} is increasing by market clearing (and so is q_{2t}). Then, Euler 2 with $c_{2t} < c_{2t+1}$ implies $\frac{p_t}{w_{t+1}}\gamma\beta_2 > 1$.

Since both type of agents have the logarithmic instantaneous utility function $U(c) = log(c)$, by Euler (2), $\frac{1}{c_{2t}} = \frac{p_t}{w}\gamma\beta_2\frac{1}{c_{2,t+1}}$. Solution of this difference equation is $c_{2t} = (\frac{\gamma\beta_2}{w})^t c_{20}\prod_{i=0}^{t-1} p_i$

Since price, p_t, is increasing, $p_t > p_0$ for all t>0. Therefore

$c_{2t} = (\frac{\gamma\beta_2}{w})^t c_{20}\prod_{i=0}^{t-1} p_i$

$> (\frac{\gamma\beta_2}{w})^t c_{20}\prod_{i=0}^{t-1} p_0$

$= (\frac{\gamma\beta_2 p_0}{w})^t c_{20}$.

But while time,t, goes to infinity, the last expression also goes to infinity since $\frac{\gamma\beta_2 p_0}{w} > 1$. Thus, consumption,$c_{2t}$, grows infinitely which is not feasible. ■

3.3 Selffulfilling Deflations

In this section, we ask if selffulfilling sustained deflation is possible.

Proposition 3 *In a MCE of the financially constrained economy, selffulfilling sustained deflations are not possible.*

Proof:

By lemma 3 and lemma 4, there is only one case which we should consider for selffulfilling sustained deflations and it is that $M_{1t} = 0$ and $M_{2t} = M$ for all t.

Assume that $M_{1t} = 0$ which implies $M_{2t} = M$ by money market clearing. Thus $M = w_t\bar{L}_1$ or $w_t = \frac{M}{\bar{L}_1}$. So constant money leads to constant wages. Set $w_t = w$.

Now, suppose that there is deflation. By $c_{1t} = q_{1t} = \frac{w}{p_t}$, c_{1t} is increasing due to deflation. Thus, c_{2t} is decreasing by market clearing (and so is q_{1t}). Then, Euler 2 with $c_{2t} > c_{2t+1}$ implies $\frac{p_t}{w_{t+1}}\gamma\beta_2 < 1$.

Since both type of agents have the logarithmic instantaneous utility function $U(c) = log(c)$, by Euler (2), $\frac{1}{c_{2t}} = \frac{p_t}{w_{t+1}}\gamma\beta_2\frac{1}{c_{2t+1}}$. Solution of this difference equation is $c_{2t} = (\frac{\gamma\beta_2}{w})^t c_{20}\prod_{i=0}^{t-1} p_i$.

TVC.1 is satisfied since $M_{1t} = 0$ for all t and hence $\lim_{t\to\infty}\beta_1^t(\frac{1}{p_t})U_1'(c_{1t})M_{1t+1} = 0$

Claim: TVC.2 is not satisfied, that is,

$\lim_{t\to\infty} \beta_2^t(-\frac{1}{p_t})U_2'(c_{2t})M_{2t+1} \neq 0.$

Proof of the claim:

First we will do substitutions into TVC.2 and do the simplifying cancellations. The limit, after substituting for consumption, becomes $\lim_{t\to\infty} \beta_2^t((\frac{\gamma\beta_2}{w})^t c_{20} \prod_{i=0}^{t-1} p_i)^{-1} q_{2t}$ and then becomes $\lim_{t\to\infty} \prod_{i=0}^{t-1}(\frac{w}{\gamma p_i})^t \frac{q_{2t}}{c_{20}}$.

c_{20} is a positive constant and q_{2t} is increasing in absolute value and bounded. So we should check if $\lim_{t\to\infty} \prod_{i=0}^{t-1}(\frac{w}{\gamma p_i})$ is zero. But $\lim_{t\to\infty} \prod_{i=0}^{t-1}(\frac{w}{\gamma p_i})$ is zero if and only if $\lim_{t\to\infty} \sum_{i=0}^{t-1} ln(\frac{w}{\gamma p_i})$ is minus infinity.

We will show that the expression on the right hand side of the "if and only if" condition is not minus infinity. For this purpose, now we will consider the Taylor expansion of p_t. But we should first derive the solution path for prices consistent with Euler conditions and market clearing. Since $c_{1t} = q_{1t}$, $c_{1t} = \frac{w}{p_t}\bar{L}_1$. Set $\Delta c_1 = c_{1t+1} - c_{1t} = \bar{L}_1 w(\frac{1}{p_{t+1}} - \frac{1}{p_t})$. Using Euler 2, we will do the same for c_2. Euler 2 with logarithmic utility is $\frac{1}{c_{2t}} = \frac{p_t}{w_{t+1}}\gamma\beta_2\frac{1}{c_{2t+1}}$, or equivalently $c_{2t+1} = \frac{p_t}{w}\gamma\beta_2 c_{2t}$ since $w_t = w$ for all t. Similarly, $\Delta c_2 = (\frac{p_t}{w}\gamma\beta_2 - 1)c_{2t}$. From equation $c_{1t} + c_{2t} = f_2(\bar{L}_1)$ and the preceding equations, one can easily show that $\Delta c_1 + \Delta c_2 = 0$ which is equivalent to $\bar{L}_1 w(\frac{1}{p_{t+1}} - \frac{1}{p_t}) + (\frac{p_t}{w}\gamma\beta_2 - 1)c_{2t} = 0$. After substitution of $c_{2t} = \gamma\bar{L}_1 - \frac{w}{p_t}\bar{L}_1$ into the last equation and necessary cancellations, the solution for p_{t+1} is $p_{t+1} = \frac{w^2}{w\gamma(\beta_2+1)-\gamma^2\beta_2 p_t}$ (see Figure 1). p_t converves to the point $\frac{w}{\gamma}$ since $\frac{w}{\gamma} < p_t < \frac{w}{\gamma\beta_2}$ and so does p_0. And while p_t goes to $\frac{w}{\gamma}$, $\frac{w}{\gamma p_i}$ goes to 1.

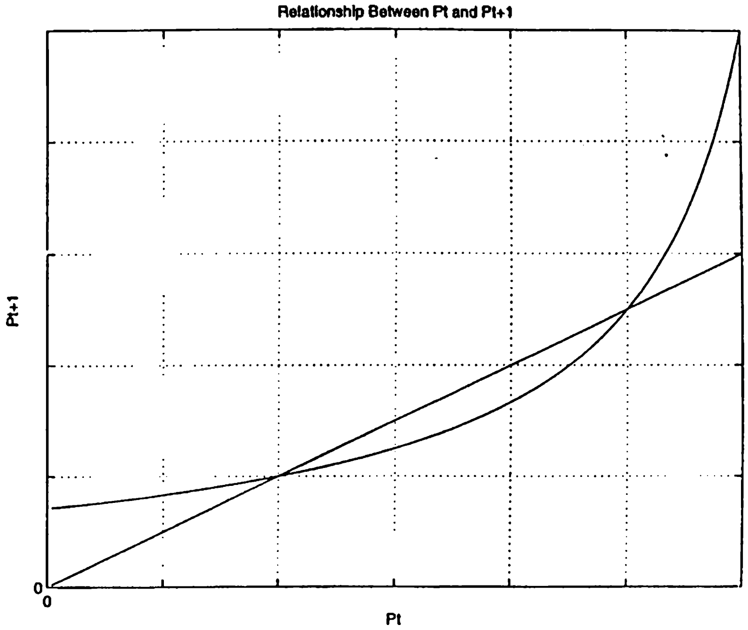

Figure 1: *The graph of* $p_{t+1} = \frac{w^2}{w\gamma(\beta_2+1)-\gamma^2\beta_2 p_t}$. *For sketching the graph, Euler and market clearing conditions are imposed but transversality conditions are not.*

Set $f(p_t) = p_{t+1}$. Taking Taylor Expansion of p_{t+1} around $p_t = \frac{w}{\gamma}$ yields $p_{t+1} = \frac{w}{\gamma} + (p_t - \frac{w}{\gamma})\beta_2$ which is equivalent to $f(p_t) = f(\frac{w}{\gamma}) + (p_t - \frac{w}{\gamma})f'(\frac{w}{\gamma})$. Arranging the Taylor Expansion, we get $p_{t+1} = \beta_2 p_t + (1-\beta_2)\frac{w}{\gamma}$. The solution is $p_t = \beta_2^t p_0 + (1-\beta_2)\frac{w}{\gamma}\sum_{i=0}^{t-1}\beta_2^i$. Thus $p_t = \beta_2^t p_0 + (1-\beta_2)\frac{w}{\gamma}\frac{1-\beta_2^t}{1-\beta_2}$. After cancellations, $p_t = \beta_2^t p_0 + \frac{w}{\gamma}(1-\beta_2^t)$.

Substituting p_t into $\lim_{t\to\infty}\sum_{i=0}^{t-1} ln(\frac{w}{\gamma p_i})$ gives the following limit problems

$$\lim_{t\to\infty}\sum_{i=0}^{t-1} ln(\frac{w}{\gamma(\beta_2^i p_0 + \frac{w}{\gamma}(1-\beta_2^i))})$$
$$=\lim_{t\to\infty}\sum_{i=0}^{t-1} ln(\frac{w}{\gamma\beta_2^i p_0 - w\beta_2^i + w})$$
$$=-\lim_{t\to\infty}\sum_{i=0}^{t-1} ln(\frac{\gamma\beta_2^i p_0 - w\beta_2^i + w}{w})$$
$$=-\lim_{t\to\infty}\sum_{i=0}^{t-1} ln(\frac{\gamma\beta_2^i p_0}{w} - \beta_2^i + 1)$$

Now, we apply the integral test for series to the last expression. In other words, $\lim_{t\to\infty}\sum_{i=0}^{t-1} ln(\frac{\gamma\beta_2^i p_0}{w} - \beta_2^i + 1)$ converges if $\int_c^\infty ln(\frac{\gamma\beta_2^x p_0}{w} - \beta_2^x + 1)dx$ where c is a positive finite constant converges. Notice that $\frac{\gamma\beta_2^i p_0}{w} - \beta_2^i + 1 = \beta_2^i(\frac{\gamma p_0}{w} - 1) + 1 > 1$, since $\frac{\gamma p_0}{w} > 1$.

Note that Taylor Expansion of $ln(z)$ around 1 is $ln(z) = ln(1) + (z-1)\frac{1}{z_0}\frac{1}{1!} = z - 1$.

Now we apply integral test to the summation of Taylor Expansion of $ln(z)$ where $z = \frac{\gamma\beta_2^x p_0}{w} - \beta_2^x + 1$. Integration is as follows.

$\int_c^\infty [z - 1]dx$

$= \int_c^\infty [(\frac{\gamma\beta_2^x p_0}{w} - \beta_2^x + 1) - 1]dx$

$= \int_c^\infty [\beta_2^x(\frac{\gamma p_0}{w} - 1)]dx$

$= (\frac{\gamma p_0}{w} - 1)\int_c^\infty \beta_2^x dx$

$= (\frac{\gamma p_0}{w} - 1)(-\frac{\beta_2^{c+1}}{c+1})$

The last expreession is a finite constant other than zero because $\frac{\gamma p_0}{w} > 1$. So, the summation of Taylor Expansion of $ln(z)$ converges. Thus, $\lim_{t\to\infty}\sum_{i=0}^{t-1} ln(\frac{\gamma\beta_2^i p_0}{w} - \beta_2^i + 1)$ converges. And hence, $\lim_{t\to\infty}\sum_{t=0}^{t-1} ln(\frac{w}{\gamma p_i})$ converges, too and is not minus infinity. So, $\lim_{t\to\infty}\prod_{t=0}^{t-1}(\frac{w}{\gamma p_i})$ is not zero and TVC.2 is not satisfied. ■

3.4 Selffulfilling Fluctuations in Wages and Prices

In propositions 2 and 3, we show that selffulfiling sustained inflations and deflations can not occur. Now we investigate if selffulfilling fluctuations could happen.

Proposition 4 *In a MCE of the financially constrained economy, selffulfilling fluctuations in wages and prices are not possible.*

Proof:

By lemma 3 and lemma 4, the remaining possible cases are as follows.

$M_{1t} = 0$ for all t≥1 implies *SMCE* by same arguments used in proof of propositions 1, 2 and 3. Therefore fluctuations in prices cannot be consistent with $M_{1t} = 0$ for all t≥1.

Likewise $M_{1t} = 0$ for all t≥1 is ruled out by lemma 4. These arguments are valid regardless of the price sequence. Hence the only possibility is $M_{1t} = 0$ for all t≥1 and *SMCE* prices aand allocations.■

3.5 Uniqueness of Equilibrium

Now, we reach to the main point of this chapter. All propositions given above bring us to the following conclusive proposition.

Proposition 5 *There is a unique MCE of the financially constrained economy for the case $M_{10} = 0$, $M_{20} = M$ and it is the stationary one(SMCE).*

Proof:

By propositions 1 to 4. ■

As a result of this theorem, cash-in-advance economies follow a path of unique MCE when the firms gain positive profits. There are no optimal and feasible paths to which economy can deviate.

4 Welfare Neutrality of Money Transfers

Following proposition has a special interest, since it has an interesting policy conclusion. The way you distribute money to the people is not important with regard to the welfare allocation.

Proposition 6 *If type 1 agents are endowed with positive initial money balances, the economy instantly goes to $SMCE$ once and for all in the second period (period 1) when worker type enters the economy with positive amount of money. Moreover in the first period (period 0) only the wages change and are strictly below the ones in $SMCE$ and the other variables of the economy are all same. This is the only monetary competitive equilibrium with $M_{10} > 0$ and $M_{20} > 0$.*

Proof:

By lemma 4, $M_{1t} > 0$ and $M_{2t} > 0$ for all t>T for some T≥0 is not possible.

We know from lemma 3 that if we take the sucessive three periods of the economy, it is not possible that agent 2 holds some money in the second period but holds no money in the third period, whatever he does in the first period of these three periods. So, if in period 1 M_{1t} does not become zero, afterwards it has no chance to be zero. But as shown in lemma 4, if M_{1t} keeps positive over time, it violates TVC.1.

What remains is the case where both agents start with positive money holdings and agent 2 holds all the money supply in period 1 and afterwards.

We proved in propositions 2, 3 and 4 that sustained inflations, deflations and fluctuations are not consistent with equilibrium for initial $M_1 = 0$. Therefore, stationary prices should prevail after period 0. Wages are stationary, too, since $w_t = \frac{M}{L_1}$. By proposition.1, all allocations are also stationary. So, in the second period(period 1) and afterwards economy is in the stationary($SMCE$) position.

Now let us consider period 0 and period 1. The transition between these periods is as follows. In period 1, the variables are as follows: $w_1 = \frac{M}{L_1}, p_1 = \frac{M}{\beta_2 \gamma L_1}, M_{11} = 0, M_{21} = M, q_{11} = c_{11} = \beta_2 \gamma \bar{L}_1, c_{21} = (1-\beta_2)\gamma \bar{L}_1, q_{21} = -q_{11}$. In period 0, $w_0 = \frac{M_{20}}{L_1}$, $q_{10} = \frac{M_{10}}{p_0} + \frac{w_0}{p_0}\bar{L}_1 = \frac{M_{10}+M_{20}}{p_0} = \frac{M}{p_0}$, hence $c_{10} = \frac{M}{p_0}$. By $c_{10} + c_{20} = f_2(\bar{L}_1) = \gamma \bar{L}_1$, we get $c_{20} = \gamma \bar{L}_1 - \frac{M}{p_0}$. Euler 2 for logarithmic utility is $\frac{1}{c_{20}} = \frac{p_0}{w_1}\gamma\beta_2\frac{1}{c_{21}}$. Substituting c_{20}, c_{21} and w_1 into Euler 2 and solving for p_0 yields $p_0 = \frac{M}{\beta_2 \gamma L_1}$ which is exactly the same price in $SMCE$. Thus, c_{10}, c_{20}, q_{10} and q_{20} are also the stationary ones. Transversality conditions are also satisfied, since economy follows the path of $SMCE$ except period 0. Euler 2 is satisfied. Euler 1 can be verified easily. Tat is, by substituting c_{10} and c_{11} into $\frac{1}{c_{10}} \geq \beta_1 \frac{p_0}{p_1}\frac{1}{c_{11}}$, one can show that $1 \geq \beta_1$. Hence, the proof is completed.■

Note that in period 0 only the wages, $w_0 = \frac{M_{20}}{L_1}$, change and are less than the ones in $SMCE$, the other variables except money are all same. Summing the transferred money and his wage, his total money holding does not change. Thus he consumes same amount since the prices are the same as the ones in $SMCE$. The consumption of type two stay the same level, too. Therefore, initial distribution of money does not matter regarding welfare.

We can apply this result of initial money allocations to the case where at some period t government unexpectedly collects tax from the producers and transfer this money to the worker. Then, we have a remarkable result that unanticipated tax-transfer policies has no effect on welfare.

5 Concluding Remarks

There are two main contributions of this study. Firstly, we study the determinacy of equilibria in a cash-in-advance producion economy in the setup of Başçı and Sağlam (1998). The determinacy of equilibria occur when the financial constraints are imposed on the factor spendings of the firm. Secodly, we have got a policy result. We observe that unanticipated government interventions by means of taxes and transfers do not affect the welfare.

There are papers (Sims, 1994 and Woodford, 1993) on indeterminacy in cash-in-advance models. They have a setup in which cash-in-advance constraints are imposed on consumption. In this kind of setup indeterminacy may exist under low levels of money growth. Also there are studies (e.g. Fuerst, 1992, Başçı and Sağlam, 1998) using a setup in which financial constraints are imposed on the factor demands. But in these studies, authors do not work on the indeterminacy issue. In our setup, we impose financial constraints on the factor purchases of the firms. Eventually, we reach to an important result that determinacy of equilibria exists in such a cash-in-advance production economy. This reduces uncertainty in government's policy making.

The result on the irrelevance of unanticipated tax-transfer policies is quite interesting. Since we study a model of market clearing, the mechanism that eliminates the benefits of a monetary transfer to worker types operates through a reduction in nominal (and real wages) within the same period . The reduction in wages arises due to a decline in the labor demand of the firm types, resulting from the lump-sum monetary tax that they have to pay. If we were to allow for the sticky wages in our model, the corresponding effect would be observed as unemployment. Such an analysis is the subject of future work.

In some of our results, we use the logarithmic utility function assumption. Relaxing this assumption towards a more general concave utility function does not seem to be straightforward and is left as an open research question.

Other natural extensions would be the analysis of determinacy and tax-transfer policies under monetary expansions and contractions. Credit markets can also be introduced and effects of government borrowing can be incorporated in the context of the Ricardian equivalence question.

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