EXISTENCE OF COMPETITIVE EQUILIBRIUM UNDER FINANCIAL CONSTRAINTS AND INCREASING RETURNS

A THESIS
SUBMITTED TO THE DEPARTMENT OF ECONOMICS
AND THE INSTITUTE OF ECONOMICS AND SOCIAL SCIENCES
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF ECONOMICS

By
H. Nur Ata
August, 2000

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Economics.

Assist. Prof. Dr. Erdem Başçı(Principal Advisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Economics.

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ABSTRACT

EXISTENCE OF COMPETITIVE EQUILIBRIUM UNDER FINANCIAL CONSTRAINTS AND INCREASING RETURNS

H. Nur Ata
M. A. in Economics
Advisor: Assist. Prof. Dr. Erdem Başçı
August, 2000

In this work we analyze the existence of equilibrium under increasing returns in a limited participation model. There are two types of agents. Producer type has an increasing returns to scale (IRS) technology with no labor endowment while worker type has only labor endowment. Economy consists of three periods. At each period, due to cash-in-advance constraints imposed on factor purchases, goods market opens after the labor market closes. Total money stock is assumed to be constant. With this setup we were able to establish the existence and uniqueness of competitive equlibrium with increasing returns for the special case that the agent's preferences are being represented by logarithmic utility.

Keywords and Phrases: Inreasing Returns, Limited Participation, Fiat money.

ÖZET

FİNANSAL KISITLAR VE ÖLÇEĞE GÖRE ARTAN GETİRİ ALTINDA REKABETÇİ DENGENİN VARLIĞI İLE İLGİLİ BİR ARAŞTIRMA

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Ağustos, 2000

Bu çalışmada sonlu periyotlu bir modelde, üretimde finansal kısıtlar ve ölçeğe göre artan getiri altında rekabetçi dengenin varlığı araştırıldı. Ekonomide iki tip ajanımız var. Üreticinin, ölçeğe göre artan getiri veren üretim teknolojisine sahip olduğunu ve isgücü arz etmediğini, İşçinin ise sadece isgücüne sahip olduğ unu varsayıyoruz. Her periyotta faktör piyasasındaki ön ödeme kısıtı nedeniyle, mal piyasası emek piyasası kapandıktan sonra açılabiliyor ve toplam para stoğu değişmiyor. Bu biçimde tanımlanmış bir ekonomide genel dengenin varlığı ve tekliği, ajanların fayda fonksiyonlarının logaritmik olduğu özel durum için ispatlanmıştır.

Anahtar Kelimeler ve İfadeler:Ölçeğe Göre Artan Getiri, Ön Ödeme Kısıtı, Kâğıt Para.

to Özgür

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Chapter 1

Introduction

It is well known that the competitive mechanism fails in the presence of increasing returns. In general, competitive equilibria do not exist. This fact can be observed even in the case of one consumer-one producer economy. Because if firm type agent has increasing returns to scale (IRS) technology, not even local profit maximization can be guaranteed at the prices that could support optimal allocation which maximizes consumer type's welfare. Moreover the presence of non-convexities prevents the pricing system from supporting the Pareto optimal allocation as a profit maximizing choice. This failure motivated the search for alternative mechanisms and resulted in the theory of marginal cost pricing (MCP).[1]In this new theory the second welfare theorem is reformulated as follows:

If an allocation is Pareto optimal, then there exists prices and wealth levels such that (i) firms follow the special pricing rule which requires that the price of output equals its marginal cost (ii) consumers maximize their utility (iii) markets clear. As it is obvious by (ii), MCP neglects the second order marginal conditions but satisfaction of first order conditions itself does not ensure that the allocation is Pareto optimal. Moreover condition (ii) means that in the

[1]MCP idea has been first introduced by Harold Hotelling (1938)

MCP equilibrium firms do not neccessarily maximize profits.

Until the end of 1970's this statement of the second welfare theorem was formulated and generalized.(Guesnerie (1975), Khan and Vohra (1987), Bonnisseau and Cournet (1988)). There are also some papers which are concerned with the existence of equilibrium (Dierker, Guesnerie, Neuefeind (1985), Vohra (1988)). The whole literature is vast and will not be surveyed here. For a typical example which deals with the existence of MCP equilibrium, one may one to look at the Paulina Beato's 1982 paper. Donald and Heal's 1983 paper would also be helpful.

There are now many results on the existence of marginal cost pricing equilibrium:

"It is recently that increasing returns have been rigorously incorporated into Arrow-Debreu general equilibrium model. The literature has focused on optimality issues as well as the existence of equilibrium. With an appropriate generalization of the notion of "marginal cost prices" it has become possible to derive a generalized second welfare theorem, asserting that corresponding to every Pareto optimal allocation there exists a vector of marginal cost prices for the firms such that, evaluated at these prices, every consumer's expenditures are minimized, subject to the given utility levels. This represents simultaneously a generalization of the second welfare theorem of Arrow and Debreu to economies with nonconvex production sets and a generalization of the Hotelling's result on the necessity of marginal cost pricing to the nonsmooth context. This result can, of course, be interpreted to say every Pareto optimal allocation can be sustained as a marginal cost pricing equilibrium with a suitable redistribution of income. Interestingly, it turns out that if the income distribution is fixed, then none of the marginal cost pricing equilibria may be Pareto optimal. Thus important normative issues concerning optimal regulation of increasing returns firms remain open. Nevertheless, irrespestive of which pricing rule is proposed,

on normative or on positive grounds, the equlibrium existence issue will remain an important part of the theory." (R. Vohra, 1992, pp 859-60)

Efficiency considerations in the context of second welfare theorem is out of the scope of this thesis and will not be pursued here. We analyze the problem of the existence of competitive equilibrium with increasing returns but in a limited participation model. Concerning this property of the model that we used, this is a new contribution to the existing literature.

Limited participation models have been used in macroeconomic framework after the paper of Fuerst (1992). Fuerst uses a representative family framework, where credit markets were operative. Başçı and Sağlam (1999) studies a version with heterogenous agents and without the credit markets. With the assumption that the labor market opens before the goods market (with agents having CRS production technology) they observe that the presence of cash-in-advance requirement in the labor market limits the demand for labor. Therefore an equilibrium with real wage below the marginal product of labor can be sustained. We use a version of this model with three period and with IRS technology in poduction.

We introduce heterogeneity by allowing two types of representative agents, a worker and a producer. Market organization is such that at each period, labor market opens before the goods market and producer type faces a cash constraint in his factor payments. He is restricted in his labor purchases with the amount of money he holds at beginning of each period. Worker type has only labor endowment which he can supply in return for money to purchase his consumption good. Total money stock in the economy is assumed to be constant.

In this setup we succesfully give a solution to the problem of maximizing a non-concave function on a non-convex constraint set. Moreover we establish the existence and uniqueness of the competitive equilibrium in the presence of a representitive firm type having IRS technology and a representitive worker type with labor endowment and when both agents have logarithmic instantenous utility functions which represents their preferences over the economy's single consumption good.

The thesis is organized as follows: In Chapter 2 general model is presented as well as the assumptions. The solution to the maximization problem of the producer type agent is given in Chapter 3. Chapter 4 gives the existence result and proof. Chapter 5 concludes.

Chapter 2

The Model

In our hypothetical finite-horizon economy, at each time t, [1] we have two different types of agents, differing in their access to production technology; "workers" and "producers". There are two types of commodities: a factor of production, labor L_t and a nonstorable consumption good, apple q_t. Agent 1 (worker) has only labor endowment $\overline{L} > 0$ and has no access to production technology while agent 2 (producer) has an IRS technology $f_2(L) = L^2$ to convert labor into apples. One can have apples only through these production possibilities i.e initially there are no endowment of apples.

Agents are indexed by $i = 1, 2$. Preferences of the agents over the consumption good, apple, (and only the consumption good because we assume that neither one values leisure) are represented by the same instantenous utility function U. Thus the preferences over the lifetime consumption for both types of agents are given by an additively separable form $\Sigma_{t=0}^{T}\beta^t U(C_{i,t})$ where $\beta \in (0,1)$ is the common discount factor, and $C_{i,t}$ is the consumption of agent i at time t. We assume that U is twice continuously differentiable $U(.)' > 0$ and $U''(.) < 0$.

The economy operates with money under the cash-in-advance constraints in

[1]Time is indexed by t and period t is the time interval between t and $t+1$.

both labor and apple markets. Money is perfectly storable and $M_{i,t}$ denotes the money holding of agent i at time t. We assume that initially all the currency in the economy, M_0, is owned by agent 2, that is, $M_{1,0} = 0$ and $M_{2,0} = M_0$. We assume that the total money stock does not change over time. (The paper money is backed by the government with a promised price of $(\frac{1}{p_2})$ in the last period).

2.1 Markets

We will consider a three period (t=0,1,2) market organization in which labor market opens before the goods market. In period 0, agent 2 (since he has initially all the currency in the economy) purchases labor and we assume that he does so with all of his money i.e $L_{2,t} = \frac{M_{2,t}}{w_t}$. Then he produces apples with the IRS technology. After the production of apples is complete agent 1 has money, agent 2 has apples and goods market opens. Agent 2 sells his apples to agent 1 in return for money and now both agent 1 and agent 2 has apples to consume and money to be used for the next period. In the last period, money held by agents is backed by the government by selling apples to them.

With the endowment structure described above and given the strictly positive prices w_t, p_t for each period t, finite horizon utility maximization problem of the two agents can be written as

Agent 1 (Worker)

$$
\begin{aligned}
\text{(P1)} \qquad & max \sum_{t=0}^{T=2} \beta^t U(C_{1,t}) \\
& \text{subject to for all } t \\
& C_{1,t} = q_t^d \\
& L_t^s \leq \overline{L} \\
& M_{1,t+1} = M_{1,t} + w_t L_t^s - p_t q_t^d
\end{aligned}
$$

where $M_{1,t}, C_{1,t}, q_t^d, L_t^s \geq 0$ and $M_{1,0} = 0$ is given.

Agent 2 (Producer)

$$
\text{(P2)} \qquad max \sum_{t=0}^{t=2} \beta^t U(C_{2,t})
$$

subject to for all t

$$
C_{2,t} = \begin{cases} f(L_t^d) - q_t^s & \text{for } t \neq 2 \\ f(L_t^d) & \text{for } t = 2 \end{cases}
$$

$$
w_t L_t^d \leq M_{2,t}
$$

$$
M_{2,t+1} = M_{2,t} - w_t L_t^d + p_t q_t^s
$$

where $M_{2,t}, C_{2,t}, q_t^s, L_t^d \geq 0$ and $q_2^s = M_{2,3} = 0, M_{2,0} = M_0$ is given.

Assume that $L_t^d = \frac{M_{2,t}}{w_t}$, that is, assume that agent 2 uses all of his money to purchase labor, [2] then problem (P2) becomes

$$
\text{(P2)}' \qquad max \sum_{t=0}^{t=2} \beta^t U(C_{2,t})
$$

subject to for all t

$$
C_{2,t} = \begin{cases} f(\frac{M_{2,t}}{w_t}) - q_t^s & \text{for } t \neq 2 \\ f(\frac{M_{2,t}}{w_t}) & \text{for } t = 2 \end{cases} \tag{2.1}
$$

$$
M_{2,t} \leq M_{2,t} \quad \text{(trivially)} \tag{2.2}
$$

$$
q_t^s = \frac{M_{2,t+1}}{p_t} \tag{2.3}
$$

where $M_{2,t}, C_{2,t}, q_t^s, L_t^d \geq 0$ and $q_2^s = M_{2,3} = 0, M_{2,0} = M_0$ is given.

Substituting (2.3) into (2.1) and then (2.1) into the objective function problem becomes

[2] The rationale behind the assumption $L_{i,t} = \frac{M_{i,t}}{w_t}$ will be more understandable when we impose some kind of "profitability condition" later on in chapter 4.

$$(\mathrm{P2})'' \qquad max\sum_{t=0}^{t=2}\beta^t U(f(\frac{M_{2,t}}{w_t}) - \frac{M_{2,t+1}}{p_t})$$

subject to for all $t = 0, \cdots, 2$

$$f(\frac{M_{2,t}}{w_t}) - \frac{M_{2,t+1}}{p_t} \geq 0$$

where $M_{2,t} \geq 0, M_{2,3} = 0, M_{2,0} = M_0 > 0$ and $\beta \in (0,1)$.

An *equilibrium* in this economy consists of a finite sequence of apple prices, money wages, labor demands, labor supplies, apple demands, apple supplies and money holdings by the two agents such that at each date, demands, supplies and money holdings are optimal under the given wage and price sequences, demand equals supply in both labor and apple markets and money holdings sum up to the total money supply at each time.

Formally we say that $< p_t, w_t, L_t^d, L_t^s >_{t=0}^{t=2}$ and $< q_t^d, q_t^s, M_{1,t+1}, M_{2,t+1} >_{t=0}^{1}$ is an *equilibrium* if

(i) $\quad < L_t^s >_{t=0}^{2}, < q_t^d, M_{1,t+1} >_{t=0}^{1}$ solves $(P1)$ and

$< L_t^d >_{t=0}^{t=2}, < q_t^s, M_{2,t+1} >_{t=0}^{1}$ solves $(P2)$ under $< w_t, p_t >_{t=0}^{2}$

(ii) $\quad L_t^d = L_t^s \ \forall t$

(iii) $\quad q_t^d = q_t^s \ for \ t = 0, 1$

(iii) $\quad M_{1,t+1} + M_{2,t+1} = \overline{M} \ for \ t = 0, 1$

With the afforementioned assumption that for both types of agents we have

$$L_{i,t} = \frac{M_{i,t}}{w_t} \ \forall i = 1, 2$$

it is somewhat simpler to deduce the "optimal" behaviour of agent 1, worker. For this reason we will postpone dealing with the problem of agent 1 until the chapter 4 which we will give the existence results of the competitive equilibrium of the hyphothetical economy in question.

Most of the following analysis will be an attempt to find a solution to agent 2's optimization problem. Therefore we will drop the index i in variables of interest and the term "optimization problem" will refer to agent 2's optimization problem until chapter 4.

2.2 Producer's Optimization Problem

Let $V : \mathbb{R}^2_+ \to \mathbb{R}$ and $h_i : \mathbb{R}^2_+ \to \mathbb{R}_+ \; for \; i = 1, \cdots, 5$ be defined as

$$V(M_1, M_2) = U(f(\frac{M_0}{w_0}) - \frac{M_1}{p_0}) + \beta U(f(\frac{M_1}{w_1}) - \frac{M_2}{p_1}) + \beta^2 U(f(\frac{M_2}{w_2}))$$
$$h_1 = M_1 \geq 0$$
$$h_2 = M_2 \geq 0$$
$$h_3 = C_0 = f(\frac{M_0}{w_0}) - \frac{M_1}{p_0} \geq 0$$
$$h_4 = C_1 = f(\frac{M_1}{w_1}) - \frac{M_2}{p_1} \geq 0$$
$$h_5 = C_2 = f(\frac{M_2}{w_2}) \geq 0$$

where U is the twice continuously differentiable, instantenous utility function satisfying $U'(.) > 0, U''(.) < 0$ and $\lim_{c \to 0} U'(C) = \infty$. f denotes the IRS production function and satisfies $f'(L) \geq 0, f''(L) > 0$. All the parameters $\{w_t, p_t, M_0, \beta\}_{t=0}^{T=2}$ are assumed to be strictly positive, β is the discount factor, $\beta \in (0, 1)$.[3]

Three period utility maximization problem of the agent 2 can then be reformulated as

$$max V(M_1, M_2) \qquad \text{over the constraint set}$$
$$\Gamma = \{(M_1, M_2) \in \mathbb{R}^2_+ | h_i(M_1, M_2) \geq 0 \;\; i = 1, \cdots, 5\}$$

[3]Note that the objective function V is bounded above.

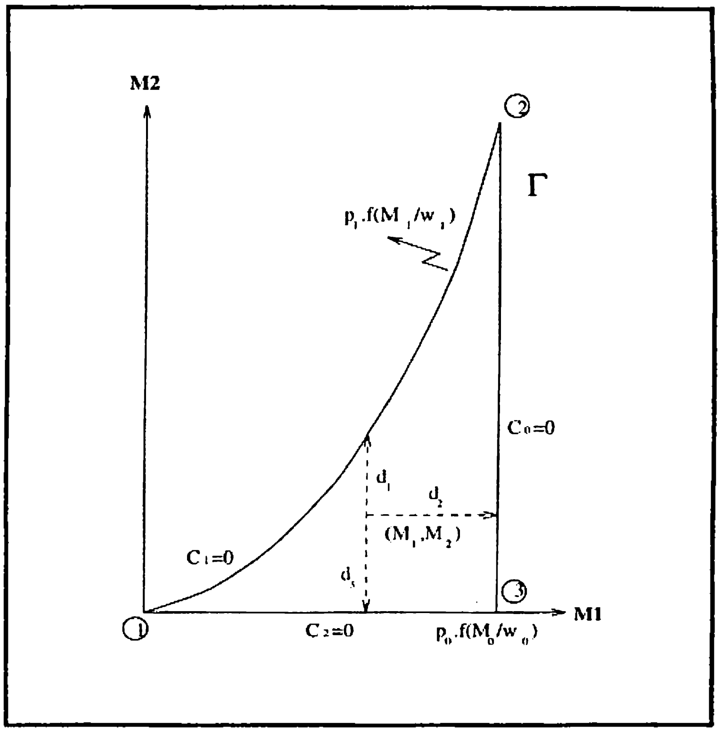

Figure 2.1: Constraint Set

2.3 Properties of the Constraint Set

Figure 2.1 shows the constraint set Γ on the (M_1, M_2) plane where $M_1, M_2 \in \mathbb{R}^2_+$.

It is easy to see that the set $\Gamma \subset \mathbb{R}^2$ is compact (closed and bounded) and non-convex. On the nonlinear section $M_2 = p_1 f(\frac{M_1}{w_1})$ we have $C_1 = 0$, on the vertical line $M_1 = p_0 f(\frac{M_0}{w_0})$ we have $C_0 = 0$ and the horizontal line $M_2 = 0$ is the set of points (M_1, M_2) where $C_2 = 0$. At the corners which are numbered by 1,2,3 we have $C_1 = C_2 = 0, C_1 = C_0 = 0, C_2 = C_0 = 0$ respectively.

At point $(M_1, M_2) \in int\Gamma$, the distance $d_1 = p_1 f(\frac{M_1}{w_1}) - M_2 = p_1 C_1$ measures the first period consumption and $d_2 = p_0 f(\frac{M_0}{w_0}) - M_1 = p_0 C_0$ measures the second period consumption. Distance from the point $(M_1, M_2) = \mathbf{x}$ to the horizontal line (M_1 axis) is a monotone transformation $g(C_2)$ of the third period consumption C_2. That is $d_3 = M_2 = w_2 f^{-1}(C_2) = g(C_2)$ [4]

[4]The production function f is a continuous, strictly increasing function (of L) hence it has an inverse and $f^{-1} = g$ is a monotone transformation.

Chapter 3

Solution

3.1 Existence of Solution to Producer's Optimization Problem

If the constraint set Γ were convex and the objective function V were concave and continuous on Γ then the Kuhn-Tucker sufficient conditions would be applicable to our problem. Clearly Γ is not convex and V is not concave, moreover V is not continuous on the boundary, $\partial\Gamma$, for certain types of utility functions like logarithmic ones. Proposition 3.2 states that even if this is the case, imposing the following condition on V guarantees the existence of a global maximum $\mathbf{x}^*$ of V which is in the interior of the constraint set Γ where

$$\Gamma = \{(M_1, M_2) \in \mathbb{R}^2_+ | h_i(M_1, M_2) \geq 0 \;\; i = 1, \cdots, 5\}$$

The following definition is needed before stating the related condition:

Definition 3.1 *Let $\Psi : [0,1] \to \Gamma$ be a curve and let $\mathbf{A}$ be its tangent vector field. Define the derivative* [1]

$$D_{\mathbf{A}} V(\Psi(0)) = \lim_{\epsilon \to 0} \frac{1}{\epsilon}\{V(\Psi(\epsilon)) - V(\Psi(0))\}$$

[1] *Indeed this derivative is known as the Lie derivative but we used here a special form of it adapted to scalar functions. (Abraham, Marsden, Ratiu (1983))*

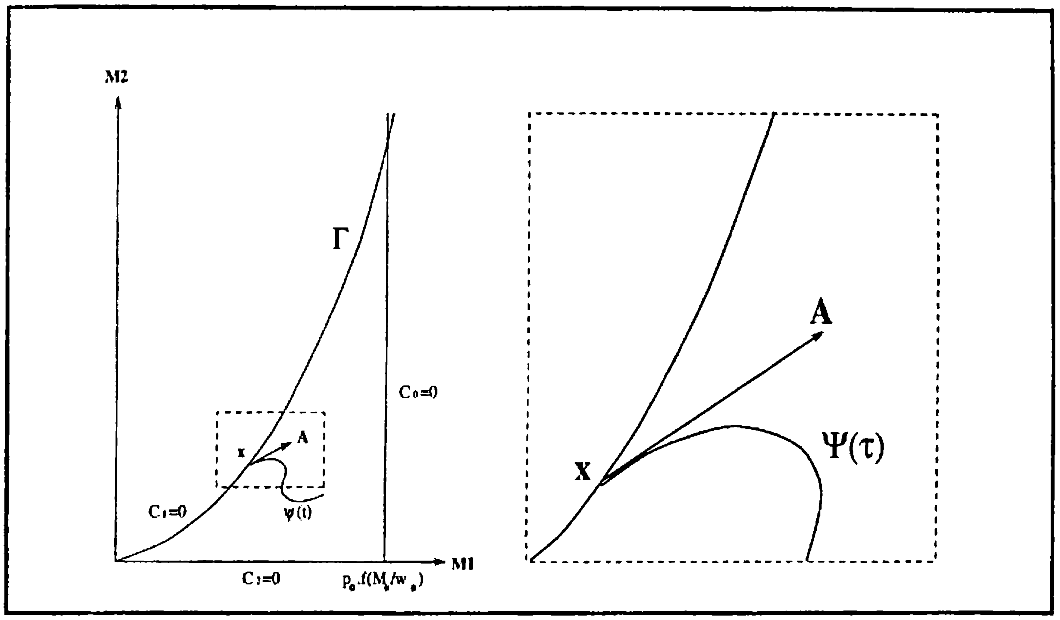

Figure 3.1: Illustration of **Condition***

We will impose the following condition on V:

Condition*: For all $\mathbf{x} \in \partial\Gamma$ there exists $\Psi_{\mathbf{x}} : [0,1] \to \Gamma$ with $\Psi_{\mathbf{x}}(0) = \mathbf{x}$ such that $D_{\mathbf{A}}V(\Psi_{\mathbf{x}}(0)) > 0$

Proposition 3.2 *Assume that the function V is continuous on $int\Gamma$ and that for any given parameter set, V satisfies condition*. Then a global maximum $\mathbf{x}^* = (M_1^*, M_2^*) \in int\Gamma$ exists to the inequality constraint problem $maxV(M_1, M_2)$ over the constraint set Γ. Therefore the point $\mathbf{x}^*$ satisfies the Kuhn-Tucker first order (necessary) conditions for a maximum, that is there exists λ^* such that the following conditions are met.*

$$[KT-1] \qquad \lambda_i^* \geq 0 \ \ and \ \ \lambda_i^* h_i(\mathbf{x}^*) = 0 \ \ for \ i = 1, \cdots, 5$$

$$[KT-2] \qquad DV(\mathbf{x}^*) + \sum_{i=1}^{5} \lambda_i^* Dh_i(\mathbf{x}^*) = 0$$

Proof: Take the point $\overline{\mathbf{x}} \in \partial\Gamma$ such that $V(\overline{\mathbf{x}}) \geq V(\mathbf{x}) \ \forall \mathbf{x} \in \partial\Gamma$. Such an $\overline{\mathbf{x}}$ exists because we know that the function V is bounded above. Define the set $K_{\mathbf{x},\epsilon} = \{\mathbf{x}' | \ ||\mathbf{x} - \mathbf{x}'|| < \epsilon\} \cap \Gamma$. Since V satisfies *condition** $\exists$ at least one

$\mathbf{x_0} \in int K_{\overline{\mathbf{x}},\epsilon}$ such that $V(\mathbf{x_0}) > V(\overline{\mathbf{x}})$. Note that $\Psi(0) = \overline{\mathbf{x}}$ implies $\Psi \not\subset \partial\Gamma$ since $\overline{\mathbf{x}}$ is the maximum on $\partial\Gamma$. Fix $\mathbf{x_0}$ and $\epsilon' > 0$ such that $\mathbf{x_0} \in \partial\Gamma_{\epsilon'}$ where $\Gamma_{\epsilon'} = \Gamma \backslash \bigcup_{\mathbf{x}\in\partial\Gamma} K_{\mathbf{x},\epsilon'}$. (see figure 3.2). Note that $\Gamma_{\epsilon'} \subset \Gamma$ is a compact set and V is continuous on $\Gamma_{\epsilon'}$. Then by Weierstrass Theorem $\exists \mathbf{x}^* \in argmax\{V(\mathbf{x})|\mathbf{x} \in \Gamma_{\epsilon'}\}$ which implies $V(\mathbf{x}^*) \geq V(\mathbf{x_0}) > V(\overline{\mathbf{x}})$. Since V is continuous on the interior of Γ this argument is valid for all $\epsilon > 0$. Hence $\mathbf{x}^*$ is the maximum of V on Γ as well. Moreover $\mathbf{x}^* \in int\Gamma$.$\square$

Remark 3.3 *Note that this proof is valid regardless of the dimension and geometry of* Γ*, provided that* Γ *is compact. This means that we have an existence result for the n-period economy. However there are serious technical difficulties in solving* $(M_1, \cdots, M_n)$ *explicitly. Therefore we prefered to give the existence reult for* $n = 2$.

Remark 3.4 *As noted earlier this proof is valid only when the objective function* V *is continuous on* $int\Gamma$*. A typical example is obtained for the case where U(C)=lnC. With a minor modification same lines of arguments used in the proof of proposition 3.2 can be used to prove the following corollary.*

Corollary 3.5 *Assume now that the objective function* V *is continuous everywhere on* Γ*. Proposition 3.2 is still valid.*

Proof: Take the point $\overline{\mathbf{x}} \in \partial\Gamma$ such that $V(\overline{\mathbf{x}}) \geq V(\mathbf{x})\ \forall \mathbf{x} \in \partial\Gamma$. Such an $\overline{\mathbf{x}}$ exists because we know that the function V is bounded above. By *condition** there exists $\mathbf{x_0} \in int\Gamma$ such that $V(\mathbf{x_0}) > V(\overline{\mathbf{x}})$. But since Γ is compact and V is continuous on Γ, by Weierstrass Theorem $\exists\ \mathbf{x}^* \in argmax\{V(\mathbf{x})|x \in \Gamma\}$.Hence $V(\mathbf{x}^*) \geq V(\mathbf{x_0}) > V(\mathbf{x})$. Thus $\mathbf{x}^* \in int\Gamma$. $\square$

For $\mathbf{x}^* \in int\Gamma$, we have $h_i(\mathbf{x}^*) > 0\ \forall i = 1, \cdots, 5$. Thus $[KT-1]$ is satisfied at $(\mathbf{x}^*, \lambda_i^*)$ with $\lambda_i^* = 0\ \forall i = 1, \cdots, 5$. Therefore $[KT-2]$ reduces to $DV(\mathbf{x}^*) = 0$.

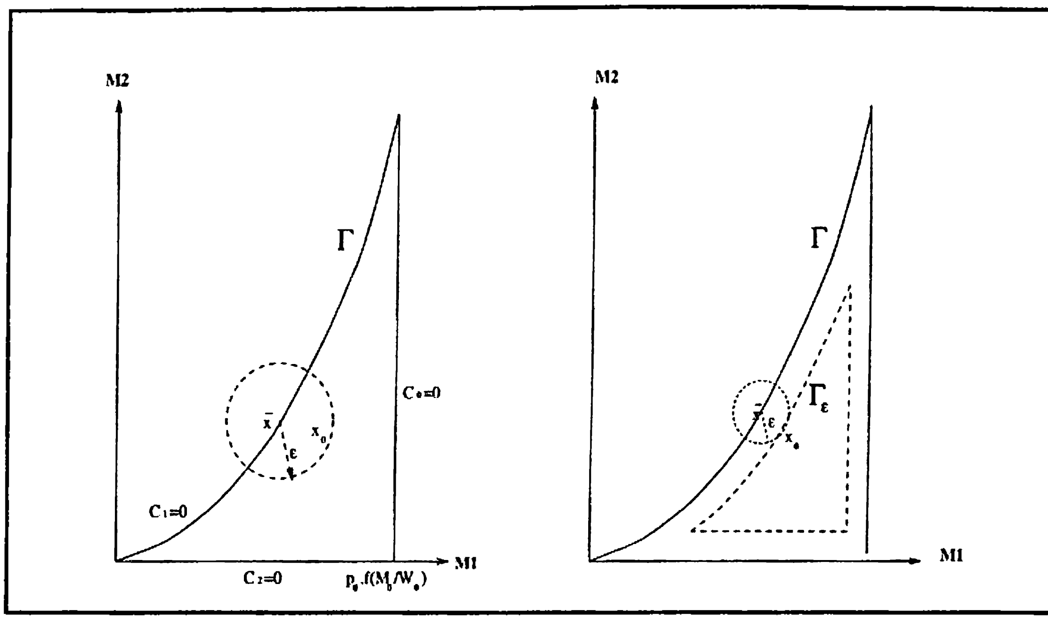

Figure 3.2: Illustraion of the proof of proposition 3.2

Constraint qualification (C.Q) is automatically satisfied at $\mathbf{x}^*$ because the set of effective constraints h_E is empty. In our problem, $[KT-2]$ can be written as

$$\frac{\partial \mathcal{V}}{\partial x_i} = 0 \qquad i = 1, \cdots, 2 \quad \text{or} \quad \frac{\partial \mathcal{V}}{\partial M_1} = 0 \quad \frac{\partial \mathcal{V}}{\partial M_2} = 0 \tag{3.1}$$

For three period (t=0,1,2) this is equivalent to

$$-\frac{1}{p_0}U'(f(\frac{M_0}{w_0}) - \frac{M_1}{p_0}) + \frac{\beta}{w_1}U'(f(\frac{M_1}{w_1}) - \frac{M_2}{p_1})f'(\frac{M_1}{w_1}) = 0 \tag{3.2}$$

$$\frac{\beta}{p_1}U'(f(\frac{M_1}{w_1}) - \frac{M_2}{p_1}) + \frac{\beta^2}{w_2}U'(f(\frac{M_2}{w_2}))f'(\frac{M_2}{w_2}) = 0 \tag{3.3}$$

or in terms of C_t variables

$$-\frac{1}{p_0}U'(C_0) + \frac{\beta}{w_1}U'(C_1)f'(\frac{M_1}{w_1}) = 0 \tag{3.4}$$

$$-\frac{\beta}{p_1}U'(C_1) + \frac{\beta^2}{w_2}U'(C_2)f'(\frac{M_2}{w_2}) = 0 \tag{3.5}$$

If we have a unique point $(\mathbf{x}^*, \lambda^*)$ which satisfies $[KT-2]$, it follows that this point also identifies the problem's global maximum $\mathbf{x}^*$.

Remark 3.6 *We at first thought that imposing the* **Inada Condition** , $\lim_{c\to 0} U'(C) = \infty$, *on the utility function* $U(C)$ *(instead of condition*) would be enough to ensure that the optimum will not occur at the boundary of (our constraint set)* Γ. *Natural examples would then be* $U(C) = lnC$ *and* $U(C) = \sqrt{C}$, *both of which satisfy the so called "Inada condition". But somewhat surprisingly* $U(C) = \sqrt{C}$ *appeared as a counterexample to this conjecture. If one tries to solve the equations (with* $U(C) = \sqrt{C}$*)*

$$\frac{\partial V}{\partial M_1} = 0, \ \frac{\partial V}{\partial M_2} = 0, \tag{3.6}$$

one will see that the existence of the solution depends on the parameter values, a fact which contradicts with the conclusion of the proposition 3.2 that there exists optima for any given parameter set. For example with all the parameters of interest $\{w_t, p_t, M_0\}_{t=0}^{T}$, *except* β, *set equal to one, equation 3.6 leads to the following set of equations to be solved :*

$$1) \qquad M_2 = M_1^2 - \frac{1}{4\beta^2} \tag{3.7}$$

$$2) \qquad M_2 = 4\beta^2 M_1^3 - (4\beta^2 - 1)M_1^2 \tag{3.8}$$

Equating 3.7 and 3.8 we get the cubic equation

$$4\beta^2 M_1^3 - 4\beta^2 M_1^2 + \frac{1}{4\beta^2} = 0 \tag{3.9}$$

which has double root when $M_1 = 0$ *or* $M_1 = \frac{2}{3}$ *with* $\beta_{critical} = (\frac{27}{64})^{\frac{1}{4}}$. *This means that for* $\beta < \beta_{critical}$ *equation 3.9 has no positive real solution at all.*

The simple reason for $U(C) = \sqrt{C}$ *be appearing as a counter example is that the behaviour of the value function* $V(M_1, M_2)$ *depends on the* β *values. Following observations can be made :*

Case(1): $0 < \beta < \frac{1}{2}$

With $\psi(0) = (0, 0) = \tilde{\mathbf{x}}$*, we have* $D_{\mathbf{A}}V < 0$ *and global maximum accurs at the point* $\tilde{\mathbf{x}}$ *on the boundary. (see figure 3.3).*

Case(2): $\frac{1}{2} < \beta < (\frac{27}{64})^{\frac{1}{4}}$

Let $\tilde{\mathbf{x}} = (0,0)$ *and* $\overline{\mathbf{x}} = (1,0)$. *Then with* $\psi(0) = \mathbf{x}$ *for all* $\mathbf{x} \in (\tilde{\mathbf{x}}, \overline{\mathbf{x}})$ *we have* $D_{\mathbf{A}}V < 0$ *and maximum occurs at* $(1 - \frac{1}{4\beta^2}, 0)$ *on the boundary. (see figure 3.4)*

Case(3): $\beta > (\frac{27}{64})^{\frac{1}{4}}$. *In this case maximum occurs at* $x^* \in int\Gamma$.*(see figure 3.5)*

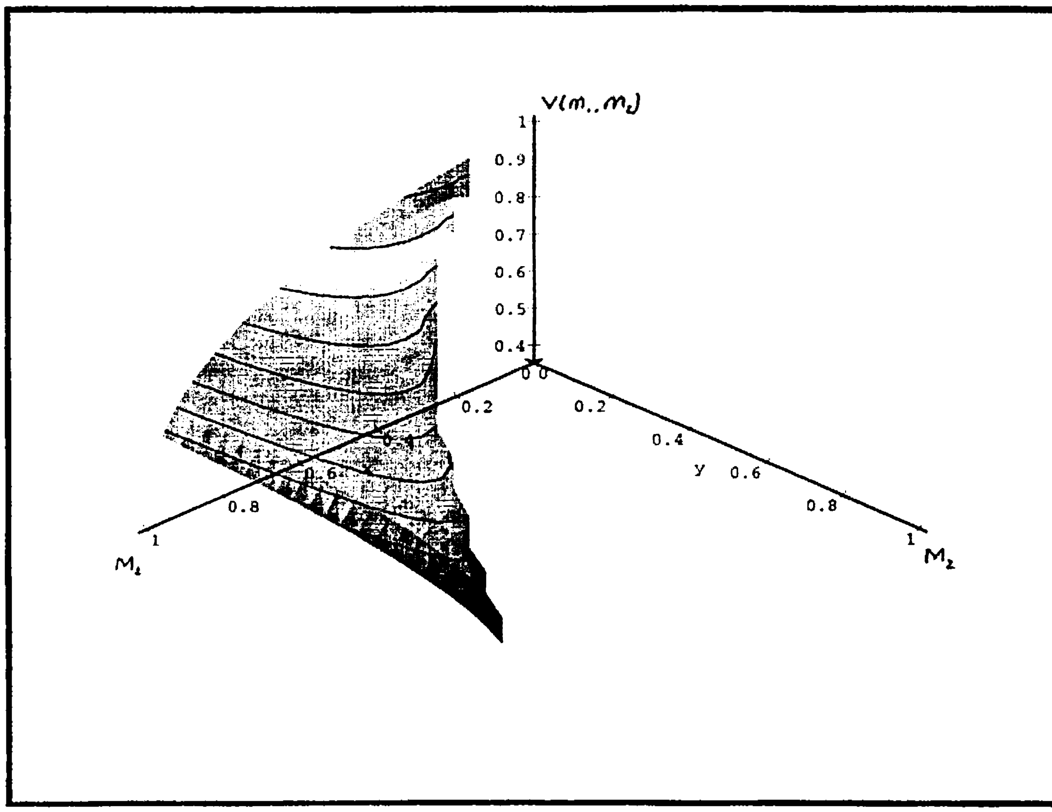

Figure 3.3: $\beta = 0.4$

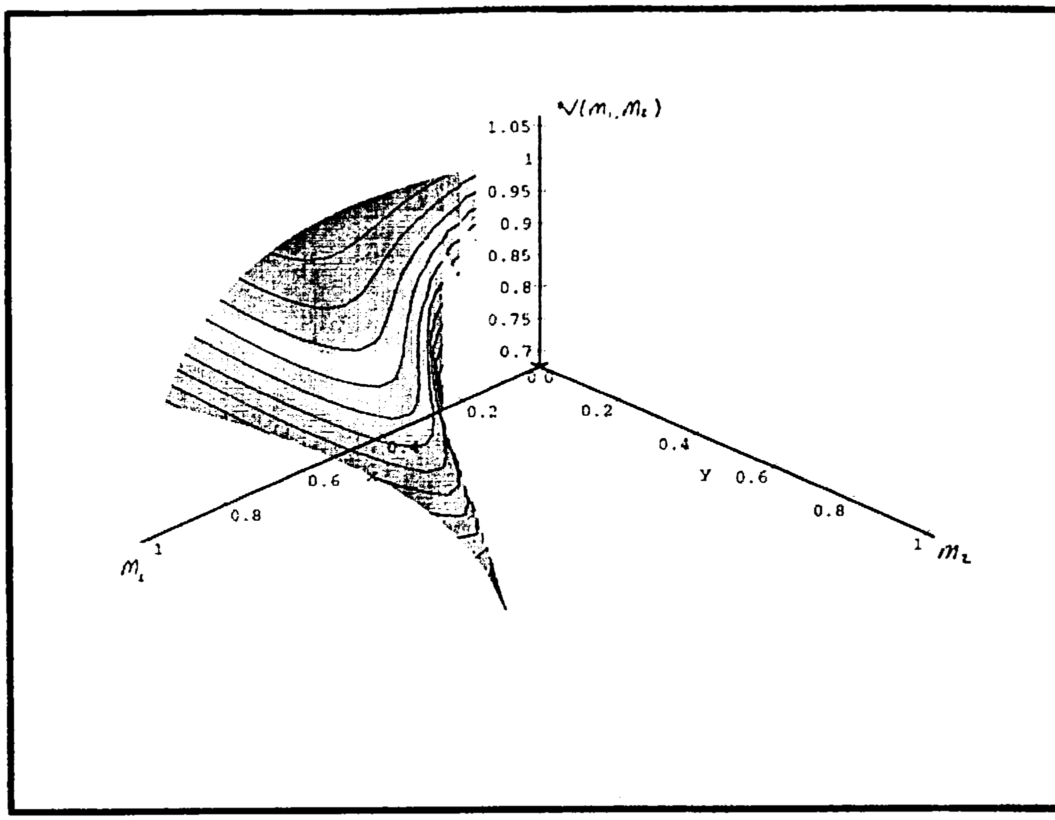

Figure 3.4: $\beta = 0.7$

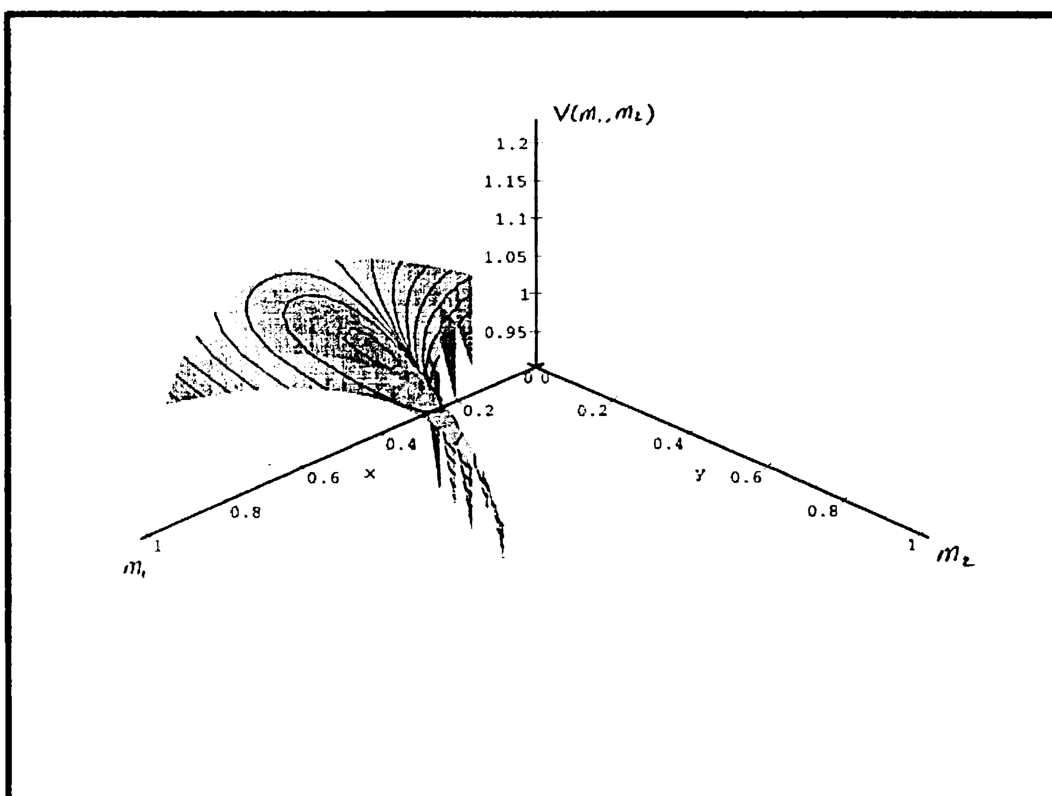

Figure 3.5: $\beta = 0.9$

Remark 3.7 *What we observe here is that when β is low ($\beta < \frac{1}{2}$), agent 2 chooses to consume all he/she has at period 0.that is, does not hold currency to be used for the next period. As β rises, agent 2 discounts future consumption less and we observe a consumption smoothing behaviour. For some $\beta > \beta_{critical}$ we have $C_0, C_1, C_2 > 0$. With this switching behaviour of the optimum, this problem looks like a "bang-bang" optimal control problem.(Bryson, Yu-Chi-Ho (1975)). The name "bang-bang" comes from the fact that the controls move suddenly from one point on the boundary of the feasible control region to another point on the boundary. It would be interesting to try to formulate our problem in this way and see its implications, which will not be pursued here.*

3.2 Unique Solution For Logarithmic Utility

It is straigtforward to check that the objective function V satisfies *condition** if we choose the intantenous utility function $U(C)$ from the logarithmic family. Then proposition 3.2 says that there exists an interior global maximum x^* of V. For the uniqueness of x^* we will use the following proposition proof of which can easily be done by using ordinary calculus.

Proposition 3.8 *Let $g_1, g_2 : [x_0, \infty) \to \mathbb{R}$, $g_1, g_2 \in C^2[x_0, \infty)$ be two functions satisfying the following conditions:*

$$(i)\ \ g_1(x_0) \leq g_2(x_0)$$
$$(ii)\ \ g_1'(x_0) < g_2'(x_0)$$
$$(iii)\ \ g_1''(x) > g_2''(x)\ \ \forall x > x_0$$
$$(iv)\ \ \exists \overline{x} > x_0\ \ \text{such that}\ g_1(\overline{x}) > g_2(\overline{x})$$

Then there exists a unique point $\tilde{x} \in (x_0, \overline{x})$ such that $g_1(\tilde{x}) = g_2(\tilde{x})$.

Proof: Consider the difference function $f(x) = g_1(x) - g_2(x)$. Clearly $f(x)$ satisfies the following conditions:

$$(i)\ f(x_0) \leq 0$$
$$(ii)\ f'(x_0) < 0$$
$$(iii)\ f''(x) > 0\ \forall x > x_0$$
$$(iv)\ \exists \overline{x} > x_0 \text{ such that } f(\overline{x}) > 0$$

Using condition (ii) we can say that $\exists \epsilon > 0$ such that $f(x_0 + \epsilon) < 0$. Also from condition (iv) we know that $\exists \overline{x} > x_o$ such that $f(\overline{x}) > 0$. But then "Mean Value Theorem of calculus says that $\exists \tilde{x} \in (x_0, \overline{x})$ such that $f(\tilde{x}) = 0$. This proves the existence of $\tilde{x}$.

For the uniqueness of $\tilde{x}$, assume contrary. That is assume that $f(\tilde{x}) = f(\check{x}) = 0$ for some $\check{x}$ where $x_0 < \tilde{x} < \check{x} < \overline{x}$. By Rolle's Theorem $\exists x_1, x_2$ such that $x_0 < x_1 < \tilde{x} < x_2 < \check{x} < \overline{x}$, at which $f'(x_1) = f'(x_2) = 0$. Again applying Rolle's Theorem to f', we can say that $\exists x_3 > x_0$ such that $f''(x_3) = 0$ which contradicts with condition (iii). This proves the uniqueness of $\tilde{x}$. □

If we assume that $U(C) = lnC$ conditions 3.4 and 3.5 becomes

$$(-\frac{1}{p_0})\frac{1}{(\frac{M_0^2}{w_0^2} - \frac{M_1}{p_0})} + (\frac{\beta}{w_1})\frac{1}{(\frac{M_1^2}{w_1^2} - \frac{M_2}{p_1})}(\frac{2M_1}{w_1}) = 0 \tag{3.10}$$

$$(-\frac{\beta}{p_1})\frac{1}{(\frac{M_1^2}{w_1^2} - \frac{M_2}{p_1})} + (\frac{\beta^2}{w_2})\frac{1}{(\frac{M_2^2}{w_2^2})}(\frac{2M_2}{w_2}) = 0 \tag{3.11}$$

Let $M_1 \equiv x$ and solve for M_2 in terms of M_1 in 3.10 and 3.11 to get

$$g_1(x) = \frac{p_1(1+2\beta)}{w_1^2}x^2 - \frac{2\beta p_0 p_1 M_0^2}{w_0^2 w_1^2}x$$
$$g_2(x) = \frac{2\beta p_1}{(1+2\beta)w_1^2}x^2$$

It is easy to check that with $x_0 = 0$ and $\overline{x} = p_0 f(\frac{M_0}{w_o}) = p_0 \frac{M_0^2}{w_0^2}$, conditions $(i - iv)$ of proposition 3.8 are satisfied. This means that equations 3.10 and

3.11 can be solved to find the unique solution $x^* = (M_1^*, M_2^*)$ which will be the unique global maximum of our optimization problem with $U(C) = lnC$, if we refer to propositions 3.2.

From 3.10 we have

$$(1+2\beta)M_1^2 - \frac{w_1^2}{p_1}M_2 - 2\beta M_1 p_0 \frac{M_0^2}{w_0^2} = 0 \tag{3.12}$$

and from 3.11 we have

$$M_2 = \frac{2\beta}{(1+2\beta)} p_1 \frac{M_1^2}{w_1^2} \tag{3.13}$$

Substitute (3.13) into (3.12) to get

$$(1+2\beta)M_1^2 - \frac{2\beta}{(1+2\beta)}M_1^2 - 2\beta p_0 \frac{M_0^2}{w_0^2} M_1 = 0$$

or, equivalently

$$M_1\{\frac{(1+2\beta)^2 - 2\beta}{(1+2\beta)}M_1 - 2\beta \frac{p_0 M_0^2}{w_0^2}\} = 0$$

which has two solutions

$$M_1^* = 0 \quad or \quad M_1^* = \frac{2\beta(1+2\beta)}{(1+2\beta+4\beta^2)} p_0 \frac{M_0^2}{w_0^2}$$

$M_1^* \neq 0$ because $(M_1^*, M_2^*) \in int\Gamma$ by proposition 3.2. Thus we have

$$M_2^* = \frac{(2\beta)^3(1+2\beta)}{(1+2\beta+4\beta^2)^2} \frac{p_0^2 p_1}{w_1^2 w_0^4} M_0^4$$

(M_1^*, M_2^*) is the unique solution pair that we are searching for.

Chapter 4

General equlilibrium

We are now ready to state our existence result on general competitive equilibrium.

4.1 Existence of Competitive Equilibrium

Let $M_0 \equiv \overline{M} > 0$ be the initial money agent 2 has and $\overline{L} > 0$ be the labor endowent of agent 1.

Proposition 4.1 *An equilibrium exists in this economy with $U(C) = lnC$ and $f(L) = L^2$ and is given by*

$$w_t = \frac{\overline{M}}{\overline{L}} \ \forall t \tag{4.1}$$

$$p_0 = \frac{\overline{M}}{\overline{L}^2} \frac{(1 + 2\beta + 4\beta^2)}{2\beta(1 + 2\beta)} \tag{4.2}$$

$$p_1 = \frac{\overline{M}}{\overline{L}^2} \frac{(1 + 2\beta)}{2\beta} \tag{4.3}$$

$$p_2 = \overline{p_2} \in (\frac{\overline{M}}{f(\overline{L})}, \infty) \tag{4.4}$$

$$L_t^d = L_t^s = \overline{L} \ \forall t \tag{4.5}$$

$$q_t^d = q_t^s = \frac{\overline{M}}{p_t} \ for \ \ t = 0, 1 \tag{4.6}$$

$$M_{1,t+1} = 0 \;\; M_{2,t+1} = \overline{M} \;\; for \;\; t = 0,1 \tag{4.7}$$

$$C_{1,t} = q_t^d = \frac{\overline{M}}{p_t} \;\; \forall t \tag{4.8}$$

$$C_{2,t} = f(\overline{L}) - \frac{\overline{M}}{p_t} \;\; for \;\; t = 0,1 \tag{4.9}$$

$$C_{2,T} = f(\overline{L}) \;\; T = 2 \tag{4.10}$$

Proof: For money market clearing we must have $M_{2,t+1} = \overline{M}$ for $t = 0,1$ in equilibrium since $M_{1,t+1} = 0$ for $t = 0,1$. Note that such a money holding plan is feasible and optimal for agent 2 by proposition 3.2 It is feasible because $(M_1^*, M_2^*) \in int\Gamma$ hence budget constraints are satisfied at (M_1^*, M_2^*). It is optimal because (M_1^*, M_2^*) maximizes the discounted sum of utilities , V. Therefore from the money market clearing we have the following two equations:

$$\overline{M} = \frac{2\beta(1+2\beta)}{(1+2\beta+4\beta^2)} p_0 \frac{\overline{M}^2}{w_0^2} \tag{4.11}$$

$$\overline{M} = \frac{(2\beta)^3(1+2\beta)}{(1+2\beta+4\beta^2)^2} \frac{p_0^2 p_1}{w_1^2 w_0^4} \overline{M}^4 \tag{4.12}$$

Labor market clearing conditions $L_t^s = \overline{L} = L_t^d = \frac{\overline{M}}{w_t}$ $\forall t$ can be used to find the money wages w_t:

$$w_0 = w_1 = w_2 = \frac{\overline{M}}{\overline{L}} \tag{4.13}$$

Substituting (4.13) into (4.11) and (4.12) prices p_t can be solved:

$$p_0 = \frac{\overline{M}}{\overline{L}^2} \frac{(1+2\beta+4\beta^2)}{2\beta(1+2\beta)} \tag{4.14}$$

$$p_1 = \frac{\overline{M}}{\overline{L}^2} \frac{(1+2\beta)}{2\beta} \tag{4.15}$$

Optimality For Agent 1:

(i) Supplying $\overline{L} > 0$ for all t is always optimal for agent 1 because his utility is strictly increasing in L_t^s. To see this consider the agent 1's optimization problem:

$$max \sum_{t=0}^{T=2} \beta^t U(\frac{M_t - M_{t+1}}{p_t} + \frac{w_t}{p_t} L_t^s)$$

subject to for all t

$$(\frac{M_t - M_{t+1}}{p_t} + \frac{w_t}{p_t} L_t^s) = q_t^d$$

$$L_t^s \leq \overline{L}$$

$$M_{1,t+1} = M_{1,t} + w_t L_t^s - p_t q_t^d$$

where $M_t, L_t^s \geq 0$ and $M_{1,0} = 0$ is given. Since $\frac{w_t}{p_t} > 0$ and $U'() > 0, U(\frac{w_t}{p_t} L_t^s)$ increases if L_t^s increases. Therefore supplying $\overline{L}$ is optimal for agent 1.

(ii)Holding zero currency at each period is optimal for agent 1, when the following condition is satisfied at each period:

$$U'(C_t) > \frac{p_t}{p_{t+1}} \beta U'(C_{t+1}) \qquad (4.16)$$

At period 0, with $U(C) = lnC$ and $C_t = \frac{\overline{M}}{p_0},\ p_t = p_0,\ p_{t+1} = p_1,\ C_{t+1} = \frac{\overline{M}}{p_1}$ above condition becomes $\beta < 1$ therefore it is automatically satisfied.For the other periods same argument applies.

Optimality For Agent 2:

Last period deserves attention. Agent 2 has two choices:

(i)He does not produce apples and uses his money to purchase apples from the government at $\overline{p}_2$.

(ii)he hires labor $(\overline{L})$, produces $(f(\frac{\overline{M}}{w_2}) = f(\overline{L}))$ and consumes all.

We want to make agent 2 to hire labor and produce apples so $\overline{p}_2$ must be set to satify

$$U(\frac{\overline{M}}{\overline{p}_2}) < U(f(\frac{\overline{M}}{w_2})) \qquad (4.17)$$

Since $U'(.) > 0$ this means $\overline{p}_2 > \frac{\overline{M}}{f(\overline{L})}$. So with the last periods price p_2 is set at $\overline{p}_2$, agent 2 will hire labor $\overline{L}$, produce $f(\overline{L})$ and consumes all. Government sells apples to agent 1 at $\overline{p}_2$, thus agent1 consumes $c_{1,2} = \frac{\overline{M}}{\overline{p}_2}$ which is clearly decreasing in $\overline{p}_2$.

Remark 4.2 *Since we have $\overline{p}_2 \in (\frac{\overline{M}}{f(\overline{L})}, \infty)$ the optimal policy would be to set $\overline{p}_2 = \frac{\overline{M}}{f(\overline{L})} + \epsilon$ for arbitrarily small $\epsilon > 0$.*

Indeed the condition $\overline{p}_2 > \frac{\overline{M}}{f(\overline{L})}$ can be tought of as a **profitability condition** and should hold at each period. But when we look at the equilibrium prices p_0, p_1 we see that this condition is automatically satisfied for the other periods.

It is now clear that agent 1 and agent 2 are maximized at the described equilibrium. This completes the proof.□

4.2 Uniqueness of Competitive Equilibrium

In chapter 2 we showed that optimal allocations (M_1^*, M_2^*) are unique for $U(C) = lnC$ and $f(L) = L^2$. Money and labor market clearing was used to find equlibrium prices p_0, p_1.

Proposition 4.3 *Prices p_0 and p_1 are the unique prices that support the competitive equilibrium which is given by proposition 4.1.*

Proof: At the equlibrium we have $w_t = \frac{\overline{M}}{\overline{L}}$ with $M_1, M_2 = \overline{M}$. Thus F.O.C becomes

$$\frac{\beta}{p_1} U'(f(\overline{L}) - \frac{\overline{M}}{p_1}) + \frac{\beta^2 \overline{L}}{\overline{M}} U'(f(\overline{L})) f'(\overline{L}) = 0 \tag{4.18}$$

$$-\frac{1}{p_0}U'(f(\overline{L}) - \frac{\overline{M}}{p_0}) + \frac{\beta\overline{L}}{\overline{M}}U'(f(\overline{L}) - \frac{\overline{M}}{p_1})f'(\overline{L}) = 0 \quad (4.19)$$

Taking the total differentials we get

$$\{\frac{\beta}{p_1^2}U'(C_1) + \frac{\overline{M}}{p_1^2}U''(C_1)(\frac{-\beta}{p_1})\}\mathbf{dp_1} = 0 \quad (4.20)$$

$$\{\frac{1}{p_0^2}U'(C_0) - \frac{\overline{M}}{p_0^3}U''(C_0)\}\mathbf{dp_0} + \frac{\beta\overline{L}}{p_1^2}f'(\overline{L})U''(C_1)\mathbf{dp_1} = 0 \quad (4.21)$$

From (4.20) we have

$$dp_1 = 0 \ \ or \ \ p_1 = \frac{\overline{M}U''(C_1)}{U'(C_1)} < 0 \quad (4.22)$$

and from (4.21) we have:

$$\frac{dp_0}{dp_1} = -\frac{\frac{\beta\overline{L}}{p_1^2}f'(\overline{L})U''(C_1)}{\frac{1}{p_0^2}U'(C_0) - \frac{\overline{M}}{p_0^3}U''(C_0)} \quad (4.23)$$

Looking at 4.23 we see that $\frac{dp_0}{dp_1} < 0$ iff $p_0 < \frac{\overline{M}U''(C_0)}{U'(C_0)} < 0$ which is impossible by the assumption of strictly positive prices. Again by 4.23 we have $\frac{dp_0}{dp_1} > 0$ iff $p_0 > \frac{\overline{M}U''(C_0)}{U'(C_0)}$ which is trivial. For the same reason 4.22 implies $dp_1 = 0$, that is, p_1 is constant. Therefore equations (4.20) and (4.21) can be solved to find the *unique* solution (p_0, p_1) which is given by proposition 4.1 at the beginning of this chapter. This completes the proof.□

4.3 Comparative Statics

In this section we conduct comparative statics to see how changes in β_2 [1] affect prices p_0, p_1. We see that equilibrium prices p_0 and p_1 decrease with β_2 so with impatient firm type (low β) p_0 and p_1 will be higher at the equilibrium reducing the real value of apples.

[1] β_2 stands for β of agent 2 (producer).

Chapter 5

Conclusion

Concerning the structure of the model used here, our existence results are not directly comparable with the ones in the existing literature. Almost all of them use standard assumptions of the classical Arrow-Debreu model except for convexity of the production set. Existence issue is analyzed in this framework and results are obtained when firms follow special pricing rules without necessarily maximizing profits. Moreover, important part of the theory is devoted to the efficiency considerations (in the context of the second welfare theorem) which is not studied here. Nevertheless, our findings are interesting. We, in a competitive setup, showed the existence of equilibrium under increasing returns with firm type agent making positive profits. There is one possible explanation for this nonstandard result:

We assume that factor payments must be paid in cash and producer can not use the money earned from selling output in the goods market at period t, to pay for period t factor services. This limits the demand for labor. Therefore producer does not face unbounded increasing returns in the sense that there is an upper bound on the labor input used in production. This makes us think that the limited participation assumption as well as the finite time is responsible for the existence result.

If one looks at the equilibrium prices, one will see that the last period's price $\overline{p}_2$ can be set arbitrarily large without distorting the equilibrium. This means that we have an equilibrium ,in a finite horizon economy, with valued fiat money. There are many examples in the literature on the existence of equilibria (even without the cash-in-advance constraints) with valued fiat money (Benveniste, Cass (1986), Kiyotaki, Wright (1988)). But it is well known that finite horizon makes the value of money unstable because agents do not want to hold money near horizon (McCabe (1989)). For this reason it is interesting to see the possibility that even if the horizon is finite, individuals may want to hold money. Nevertheless, this result is a peculinarity of the logarithmic utility function. Moreover the motivation of this attribute, the role of the government, is a real weakness in the model.

It would be a natural extension to search for the competitive equilibrium with infinitely lived agents. Unfortunately, non-concavity of objective function causes problems in the application of dynamic optimization techniques. It is not impossible to overcome this technical difficulty but one should not expect to get the the existence result easily. Indeed Sotomayor, in his 1987 paper, claims that, under certain restrictions, the value function for the dynamic optimization problem (resulting from a discrete time one-good model of optimal accumulation) is concave and the optimal stationary policy exhibits properties similar to that obtained in the model where the technology is assumed to be convex. However later on Roy (1993) showes that the conditions on the utility and production function functions imposed in Sotomayor's paper are insufficient to ensure the results claimed about the concavity of the value function and other classical properties. These findings suggests that existence issue still deserves further investigation and it may very well be the case to have indeterminacy with infinite horizon. Nevertheless concerning the structure of the model, the solution technique introduced and results obtained, our work is a new contribution to the literature when horizon is finite.

There are some papers dealing with existence of equilibrium under increasing returns but they are different in one important aspect; in the assumption on the type of increasing returns. They allow either an initial face of inreasing returns or an aggregate increasing returns with individual firms having CRS technology (external economies of scale). For example Majumdar an Mitra (1993) have some existence results for a dynamic optimization example with a non-convex technology in the case of a linear objective function but the convexity is such that production function exhibits an initial phase of increasing returns.

Jang-Ting Guo (1998) analyses indeterminacy with external economies of scale in a monetary economy with limited participation and he finds that the region of indeterminacy depends crucially on the (i)coefficient of relative risk aversion (ii)labor supply elasticity and (iii) the degree of increasing returns to scale. Therefore in this paper an existence result is not given which would make it comparable to our findings.

As it is mentioned in chapter 2 we have an existence result for $n - period$ economy. However there are technical difficulties in solving the variables $M_1, \cdots, M_n$ explicitly. A potential future research, in spite of the technical difficulties faced, would be to generalize our findings to cover infinite horizon or at least to $n - period$ case and see its implications.

References

Abraham M., Marsden J.E. Ratiu T., *Manifolds, Tensor Analysis and Applications*, Applied Mathematical Sciences; V. 75, 1983, Springer-Verlag, New York Inc.

Başçı E., Sağlam I., *Does Money Matter? A Deterministic Model with Cash-in Advance Constraints in Factor Markets.*, in Current Trends in Economics: Theory and Applocations (edited by: Alkan A., Aliprantis C.D, Yannelis N.C.), 1999, Springer-Verlag, 107-118.

Beato, Paulina, *The Existence of Marginal Cost Pricing Equilibria with Increasing Returns*, The Quarterly Journal of Economics, November 1982, 389, 669-688.

Benveniste, Lawrence M., Cass, David, *On the Existence of Optimal Stationary Equilibria with a Fixed Supply of Fiat Money:I. The Case of a Single Consumer*, Journal of Political Economy: 94(2), April 1986, pages 402-417

Bonnisseau, J. M., And B. Cornet, *Existence of Equilibria when Firms Follow Bounded Losses Pricing Rules*,Journal of Mathematical Economics,17, 1988, 119-147.

Brown J. , Heal M. , Geoffry, *Marginal vs. Avarage Cost Pricing in the Presence of a Public Monopoly*, The American Economic Review, Volume 73, Issue 2, Papers and Proceedings of the Ninety-Fifth Annual Meeting of the American Economic Association, (May, 1983), 189-193.

Bryson, E. Arthur,Yu-Chi-Ho , *Applied Optimal Control*, Taylor and Francis, 1975.

Dierker, E., R. Guesnerie, And W.Neuefeind, *General Equlibrium when Some Firms Follow Special Pricing Rules*, Econometrica, 53, 1985, 1369-1393.

Fuerst T. , *Liquidity, Loanable Funds and Real Activity*, Journal of Monetary Economics, Volume 29(1), (Feb., 1992), 3-24.

Guesnerie R.,,*Pareto Optimality in Non-Convex Economies*, Econometrica, 45, 1975

Guo Jan-Ting *Indeterminacy and Sunspots in a Monetary Economy with Limited Participation*, Economics Letters, 59, 1998 337-345.

Hotelling,Harold, *The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates* , Econometrica, July 1999,6,242-269.

Khan, A., And R. Vohra, *An Extension of the Second Welfare Theorem to Economies with Non-Convexities and Public Goods*, Quarterly Journal of Economics, 102, 1987, 223-241.

Kiyotaki, Naburo, Wright, Randall, *Fiat Money in Search Equilibrium*, University of Pensilvanya Center for Analytic Research in Economics and the Social Sciences Working Paper:88-23, November 1988, pages 49..

Majumdar, M., Mitra, T., *Dynamic Optimization with a Non-Convex Technology: The Case of a Linear Objective Function* , The Review of Economic Studies, v. 50, Issue 1, 1993, 143-151

McCabe, Kevin A. *Fiat Money as a Store of Value in an Experimental Market,* Journal of Economic Behaviour and Organization; 12(2), October 1989, pages 215-231.

Roy S. *A note On Income Fluctuations and Capital Gains with a Convex Production Function*, Journal of Economic Dynamics and Control, 18, 1993, 1199-1202.

Sotomayor, M.O., *On Income Fluctuations and Capital Gains with a Convex Production Function,* Journal of Economic Dynamics and Control, 11, 1987, 285-312.

Vohra Rajiv *Marginal Cost Pricing Under Bounded Marginal Returns,* Econometrica, Volume 60, Issue 4 (Jul., 1992), 859-876.

Vohra Rajiv, *On the Existence of Equlibria in Economies with Increasing Returns* , Journal of Mathematical Economics, 17, 1988, 179-192.