CONSISTENCY/INCONSISTENCY BETWEEN ECONOMIC AND POLITICAL PLANES

by

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CONSISTENCY/INCONSISTENCY

BETWEEN ECONOMIC AND POLITICAL PLANES

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By

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ABSTRACT

CONSISTENCY/INCONSISTENCY

BETWEEN ECONOMIC AND POLITICAL PLANES

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M.A. in Economics

Supervisor: Prof. Dr. Semih Koray

August 1998

In this study, we introduce a different mechanism with a hybrid ownership definition lying in between public and private ownership. Agents have claims over the endowments and the total production of the economy instead of having absolute ownership rights. We define social desirability as the following: an alternative x is socially preferred to an alternative y if the majority of the agents prefer x to y. In this context, we investigate whether the competitive equilibrium outcome is socially the most desirable outcome and whether there are other efficient outcomes socially preferred to the competitive equilibrium outcome. We use a voting scheme where agents vote on the production alternatives of the economy. We investigate if there is a voting rule that leads to the competitive equilibrium outcome and what kind of a rule this latter is. The central finding of the study is that, for a class of production and utility functions, there is a voting rule that leads to the competitive equilibrium outcome. Moreover, this is a weighted voting rule where agents' votes are their initial claims. A second important contribution is the analysis of the process of candidate nomination, which is most of the time, neglected by social choice problems. Finally, we consider the transfer problem where agents make transfers to other agents to make them vote on specific alternatives.

Keywords: Social choice, political economy, voting behaviour, information, models of political processes.

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Bu calismada, kamu ve ozel mulkiyet kavramlarinin ortasinda kalan bir mulkiyet kavramini icinde bulunduran yeni bir mekanizma tanimliyoruz. Insanlarin elinde mutlak mulkiyet haklari yerine, ekonominin tum varliklari ve uretimi uzerinden aldiklari paylar var. Toplumsal tercihi soyle tanimliyoruz: Eger toplumun buyuk bir kesimi x alternatifini y alternatifine tercih ediyorsa, x alternatifi toplumsal olarak daha cok isteniyor demektir. Bu baglamda, rekabetci dengenin toplumsal olarak en cok istenen secenek olup olmadigini ve baska verimli olan ve rekabetci dengeye toplumsal olarak tercih edilen secenekler olup olmadigini arastiriyoruz. Bunu yaparken, insanlarin uretim alternatifleri uzerine oy verdikleri bir oylama sistemi kullaniyoruz. Rekabetci piyasa dengesine goturen bir oylama kurali var midir ve eger varsa bu ne tur bir oylama kuralidir sorularina yanit ariyoruz. Calismanin en onemli bulgusu, verili bir uretim ve haz fonksiyonlari sinifi icin, boyle bir oylama kuralinin varoldugudur. Ayrica da, bu kural, insanlara toplam uretim ve varliklardan aldiklari pay oranlarinin agirlik olarak verildigi bir agirlikli oylama kuralidir. Ikinci onemli katkimiz ise, sosyal secim kurami tarafindan cogu zaman es gecilen ve onemli oldugunu dusundugumuz aday gosterme surecinin analizidir. Son olarak da, insanlarin diger grup insanlara belirli bir uretim alternatifini oylatmak icin transfer yaptiklari bir modelin analizini sunuyoruz.

Anahtar Sozcukler: Sosyal secim kurami, siyasal iktisat, oylama davranislari, bilgi, siyasi surec modelleri.

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I want to say, "You know!" to the one who knows the meaning of the phrase and who has been to me, a reason for living.

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Chapter 1

Introduction

The debate between the defenders of private ownership and public ownership has a long history and a deep impact on political and economic theory. Several authors have taken positions on the public side. Rawls (1971), Roemer (1986), Cohen (1986) are the ones that one can come across in the literature. The common point they focus on is that natural resources should remain in public ownership to prevent anyone from extracting excessive benefits from the private ownership of producing some commodity.

The crucial argument that makes neo-classical allocation mechanisms under private ownership the mainstream one is the fact that they lead to "efficiency". Agents making decisions selfishly, to maximise their own utilities, leads to social optimum. The concept of competitive equilibrium is the outcome of this paradigm, which argues that agents are totally free in making their decisions and these freely made decisions pave the way to the efficient outcome. Now, the questions to be asked are: "Is this competitive equilibrium allocation socially the most desirable one?". "Are not there other efficient points at least as desirable as the competitive equilibrium point?".

What determines whether an outcome is socially more desirable than the other? It is apparent that there are an infinite number of other "efficient" points on the production possibilities frontier of an economy.

We can define social desirability in the context of social choice theory and voting procedures, specifically. By voting procedures, we mean the rules that govern how votes in an election are aggregated and how a winner or winners are determined. All the human populations live under one type of a social contract or another. And the majority of these groups live under systems of which periodic elections are an important part. The crucial criteria which are highly valued by these systems are fairness, neutrality and anonymity. *Anonymity* says that each voter's opinion should be equally important (one man, one vote); *Neutrality* says that no candidate should be a priori discriminated against (Moulin (1994)). A very simple definition of social desirability could be the following: An alternative x is socially more desirable than an alternative y if the number of agents that prefer alternative x to alternative y is larger than the number of agents that prefer y to x.

We introduce a model economy with a "hybrid" ownership definition, that lies in-between public and private ownership. This is such an economy where agents have claims over the total production and the initial endowments but they are not the absolute owners of them. Redefining ownership this way has a nice use. Had we given the agents absolute ownership rights over the endowments, a public decision upon what to produce would be inappropriate since no one would have the right to compel another to allocate his endowment to a production alternative that he does not like. In our case, this type of a common decision is meaningful.

Then, we ask the following questions: "Is there a voting rule that sustains the competitive equilibrium outcome given the aggregated preferences of the agents? What kind of a rule is this?". "Is the outcome that this rule selects socially the most desirable one?". We provide answers to these questions with the help of our new model economy.

The literature on allocation of resources by voting follows two broad paths. One is the literature on the allocation of public goods. Deciding whether or not to undertake a public project and how to distribute the cost of it to the public is the classic problem of this genre. Most of the attention has been given to the design of optimal mechanisms under conditions of asymmetric information. Clarke (1971), Slutsky (1977), Groves and Ledyard (1977), Harris and Townsend (1981), Jackson and Moulin (1992) are the typical examples to cite.

The second type of work considers the corporate voting literature. This field of research has focused primarily on the following two issues: the assignment of votes in relation to income claims within a corporation, and the selection of the voting percentage required to transfer control of a corporation (Barzel and Sass(1990)). Classic and more recent works to cite in this area are DeAngelo (1981), Easterbrook and Fischel (1983), Nitzan and Procaccia (1986), Grossman and Hart (1980), and Harris and Raviv (1988).

The paper is organised as follows. In Chapter 2, we formulate the problem and introduce the model. We provide the intuition behind our definition of ownership and undertake a standard general equilibrium analysis. In Chapter 3, we deal with the issue of nominating the candidates, which we think is one of the most crucial steps in the selection of a socially desirable outcome, and is almost always neglected by Social Choice theorists. Chapter 4 brings with it a different approach to look at the market mechanism through the use of transfers. Chapter 5 summarises and concludes.

Chapter 2

The Model

We consider a polarised economy with production. There are 2 firms each producing one of the two goods of the economy, namely **good A** and **good B**. Both firms use the same and the only input of the economy. $\mathbf{K_1}$ and $\mathbf{K_2}$ denote the amounts of input allocated to firm1 and firm2, respectively, where $K_1+K_2=\mathbf{C}$, the total amount of input in the economy. $\mathbf{P_A}$, $\mathbf{P_B}$, and $\mathbf{w}$ are the prices of good A and good B and the input in that order. Firms' production functions are of the following form: $f_1(K_1)=\mathbf{aK_1}^\gamma$ for the first firm and $f_2(K_2)=\mathbf{bK_2}^\gamma$ for the second firm, where a,b>0 and $0<\gamma<1$ (We also consider the linear case where $\gamma=1$, separately). Firms maximise profit. They are owned publicly, meaning that the agents in the economy are the shareholders of the firms and they have different claims over the total profits. Shareholders constitute the two groups of consumers in the economy, namely **group1** (informally A-lovers) and **group2** (B-lovers). There are $\mathbf{n_1}$ (>0) and $\mathbf{n_2}$ (>0) identical agents in group1 and group2. The agents in both groups have Cobb-Douglas utility functions defined over the bundles of goods, meaning that they value both goods but they love one of the goods more than the other: $U_i\left(X_A^i, X_B^i\right)=\left(X_A^i\right)^{\alpha_i}\left(X_B^i\right)^{\beta_i}$ where α_i (>0) and β_i (>0) are the parameters of the utility functions of i^{th} group, i=1,2 and X_A^i and X_B^i are the i^{th} type agents' demands for good A and good B. $\alpha_1>\alpha_2$, meaning that agents in the first group like to consume good A more than the agents in the second group. $\alpha_i+\beta_i=1$ (this is not a serious restriction since utility functions represent the same preferences up to increasing transformations). Each agent in group i (i=1,2) is endowed with $\mathbf{s_i C}$ amount of input and receives $\mathbf{s_i}$ of the total profits generated by the firms, where $s_i>0$. We normalise the shares so that $\mathbf{n_1 s_1+n_2 s_2=1}$.

2.1 The General Case with a Strictly Concave Technology

A standard general equilibrium analysis is conducted. The competitive equilibrium levels of production and individual demand functions are calculated.

Firms' problem

Firms maximise profits and the profits are distributed to the agents according to their shares. Firms solve the following problem:

$$\max P_a a K_1^{\gamma} - wK_1$$

$$\text{s.t. } 0 \le K_1 \quad (1)$$

and

$$\max P_b b K_2^{\gamma} - wK_2$$

$$\text{s.t. } 0 \le K_2$$

given P_A, P_B, w >0

After imposing the market clearing condition $K_1+K_2=C$. The equilibrium input bundles to be allocated to firm1 and firm 2 are the following:

$$K_1 = \frac{C(P_A a)^{\delta}}{(P_A a)^{\delta} + (P_B b)^{\delta}} \tag{2}$$

$$K_2 = \frac{C(P_B b)^{\delta}}{(P_A a)^{\delta} + (P_B b)^{\delta}} \tag{3}$$

where $\delta = \frac{1}{1-\gamma}$

By determining (K_1, K_2), we have also determined the point on the production possibilities frontier the firms want to produce at, given the prices. Here is the price of the input determined by the first-order conditions:

$$w = \frac{\gamma C^{\gamma - 1}}{\left[(P_A a)^{\delta} + (P_B b)^{\delta}\right]^{\gamma - 1}} \tag{4}$$

We denote by Π_1 and Π_2 the profits generated by firm1 and firm2 respectively:

$$\Pi_1 = \frac{(1-\gamma) C^{\gamma} (P_A a)^{\delta}}{\left[(P_A a)^{\delta} + (P_B b)^{\delta}\right]^{\gamma - 1}} \tag{5}$$

$$\Pi_2 = \frac{(1-\gamma)C^\gamma (P_B b)^\delta}{\left[(P_A a)^\delta + (P_B b)^\delta\right]^{\gamma-1}} \tag{6}$$

Since the income stream generated by profits and input price is common to each individual's demand function, we compute it here:

$$wC + \Pi_1 + \Pi_2 = \frac{\gamma C^\gamma + (1-\gamma)C^\gamma\left[(P_A a)^\delta + (P_B b)^\delta\right]}{\left[(P_A a)^\delta + (P_B b)^\delta\right]^{\gamma-1}} \tag{7}$$

Having worked out the firms' demand of inputs, we move onto the consumers' side.

Consumers' problem

Consumers solve the following problem:

$$\max\left(X_A^1\right)^{\alpha_1}\left(X_B^1\right)^{\beta_1}$$

$$\text{s.t.} \quad \begin{aligned} & P_A X_A^1 + P_B X_B^1 \leq s_1\left[wC + \Pi_1 + \Pi_2\right] \\ & X_A^1, X_B^1 \geq 0 \end{aligned}$$

given P_A, P_B, $w>0$

(8)

The optimal demand functions are the following:

$$
\begin{aligned}
X_A^1 &= \frac{\alpha_1 s_1}{P_A}\left[\frac{\gamma C^\gamma + (1-\gamma)C^\gamma((P_A a)^\delta + (P_B b)^\delta)}{((P_A a)^\delta + (P_B b)^\delta)^{\gamma-1}}\right] \\
X_B^1 &= \frac{\beta_1 s_1}{P_B}\left[\frac{\gamma C^\gamma + (1-\gamma)C^\gamma((P_A a)^\delta + (P_B b)^\delta)}{((P_A a)^\delta + (P_B b)^\delta)^{\gamma-1}}\right] \\
X_A^2 &= \frac{\alpha_2 s_2}{P_A}\left[\frac{\gamma C^\gamma + (1-\gamma)C^\gamma((P_A a)^\delta + (P_B b)^\delta)}{((P_A a)^\delta + (P_B b)^\delta)^{\gamma-1}}\right] \\
X_B^2 &= \frac{\beta_2 s_2}{P_B}\left[\frac{\gamma C^\gamma + (1-\gamma)C^\gamma((P_A a)^\delta + (P_B b)^\delta)}{((P_A a)^\delta + (P_B b)^\delta)^{\gamma-1}}\right]
\end{aligned}
\tag{9}
$$

Market Clearing

Here are the market clearing conditions:

$$
\begin{aligned}
n_1 X_A^1 + n_2 X_A^2 &= aK_1^\gamma \\
n_2 X_B^1 + n_2 X_B^2 &= bK_2^\gamma
\end{aligned}
\tag{10}
$$

We know that there exists a positive price vector that satisfies the above equations[1].

Most wanted points

We have two groups with different preferences over the bundles of the two goods of the economy. Due to this, the points on the production possibilities frontier that the agents in two groups want to be at the most are different. Both groups solve the following problem to select these points:

$$
\max\left(\frac{aK_1^\gamma}{n_i}\right)^{\alpha_i}\left(\frac{bK_2^\gamma}{n_i}\right)^{\beta_i}
$$

[1] See Appendix for the proof of the existence.

$$\text{s.t.} \quad \begin{matrix} K_1 + K_2 \le C \\ K_1, K_2 \ge 0 \end{matrix} \tag{11}$$

The optimal solutions for the resource allocation problem they considered are summarised below Figure 1, where A* and B* are the most-preferred points of the first and second type agents.

Finally, we have a rough figure of the production possibilities frontier (PPF), the competitive equilibrium point and the most-wanted points of both groups.

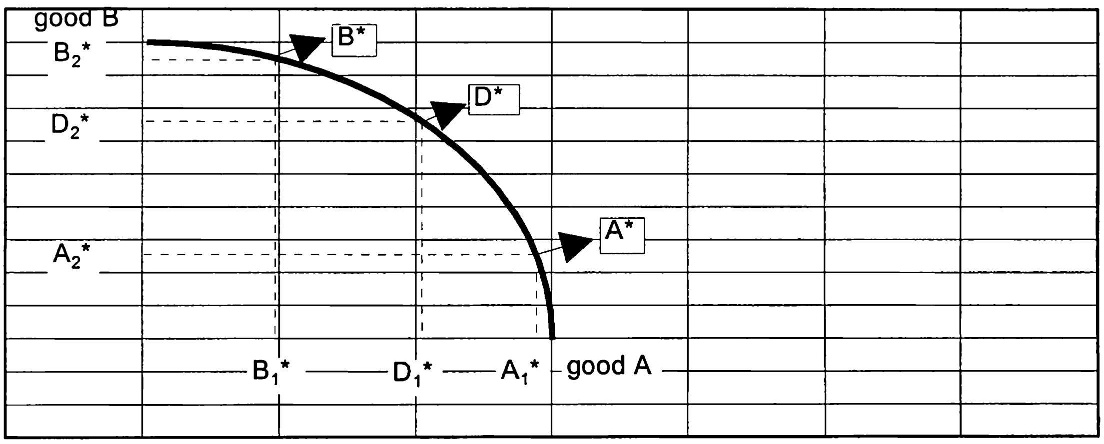

Figure 1: The PPF of the strictly concave case.

A*: most-wanted point for A-lovers, $K1=\alpha_1\mathbf{C}$, $K2=\beta_1\mathbf{C}$.

B*: most-wanted point for B-lovers, $K1=\alpha_2\mathbf{C}$, $K2=\beta_2\mathbf{C}$.

D*: competitive equilibrium point, $K1= \mathbf{n_1 s_1}\alpha_1+\mathbf{n_2 s_2}\alpha_2$, $K2= \mathbf{n_1 s_1}\beta_1+\mathbf{n_2 s_2}\beta_2$.

- $\alpha_1=\alpha_1(n_1s_1+n_2s_2)= \alpha_1n_1s_1+\alpha_1n_2s_2>\alpha_1n_1s_1+\alpha_2n_2s_2$ $(\alpha_1>\alpha_2) \Rightarrow A_1^*>D_1^*$
- $\beta_1=\beta_1(n_1s_1+n_2s_2)= \beta_1n_1s_1+\beta_1n_2s_2<\beta_1n_1s_1+\beta_2n_2s_2$ $(\beta_1<\beta_2) \Rightarrow A_2^*<D_2^*$

It is easy to show by a similar comparison that $B1^* < D1^*$ and $B2^* > D2^*$. So, we have $A1^* > D1^* > B1^*$ and $A2^* < D2^* < B2^*$. The analysis concludes that the competitive equilibrium is always between the two groups' most preferred points.

When the preferences of both types are the same, the above-mentioned points coincide at the competitive equilibrium. More interesting case is where the preferences are different and the competitive equilibrium point is between the most-wanted points.

It is finally the right time to ask the most important question of the paper. Does there exist a voting rule that selects the competitive equilibrium outcome given the aggregated preferences of the society? Our answer to this question is positive. The exciting result is that the quantities of inputs allocated to the production alternatives are, explicitly, linear combinations of the most-wanted points of both groups where the weights are simply $\mathbf{n}_i\mathbf{s}_i$.

Now, we design a voting scheme that leads to the competitive equilibrium outcome. We let the agents vote on the three possible points on the PPF. It is apparent that each agent votes for the point he wants to be at the most. Contrary would be meaningless since the rule is a weighted average of these points and there is no incentive for an agent in one of those groups to vote for one of the other two points. At this point, we do not know whether the vote weights that agents have are equal. Later, we show that they must be different in order for the voting rule to select the competitive equilibrium outcome.

Let $\tilde{s}_i$ denote the vote that agents in the i[th] group have. Agents in the first group vote on A* and the agents in the second group vote on B*. There are n_1+n_2 votes, n_1 for A* and n_2 for B*. Then, the ratio of K_1 to K_2 is the following:

$$\frac{K_1}{K_2}=\frac{n_1\tilde{s}_1\alpha_1+n_2\tilde{s}_2\alpha_2}{n_1\tilde{s}_1\beta_1+n_2\tilde{s}_2\beta_2} \tag{12}$$

We also know from (2) and (3) that:

$$\frac{K_1}{K_2}=\frac{(P_A a)^\delta}{(P_B b)^\delta}=\frac{a^\delta}{(P_B b)^\delta} \tag{13}$$

Now, we use the market clearing conditions in (10):

$$\begin{aligned}(n_1s_1\alpha_1+n_2s_2\alpha_2)\big(\gamma+(1-\gamma)\big((a^\delta)+(P_Bb)^\delta\big)\big(a^\delta+(P_Bb)^\delta\big)&=a^\delta\\(n_1s_1\beta_1+n_2s_2\beta_2)\big(\gamma+(1-\gamma)\big((a^\delta)+(P_Bb)^\delta\big)\big(a^\delta+(P_Bb)^\delta\big)&=(P_Bb)^\delta\end{aligned} \tag{14}$$

Dividing the first equation by the second gives us:

$$\frac{(n_1s_1\alpha_1+n_2s_2\alpha_2)}{(n_1s_1\beta_1+n_2s_2\beta_2)}=\frac{a^\delta}{(P_Bb)^\delta}=\frac{K_1}{K_2}=\frac{(n_1\tilde{s}_1\alpha_1+n_2\tilde{s}_2\alpha_2)}{(n_1\tilde{s}_1\beta_1+n_2\tilde{s}_2\beta_2)} \tag{15}$$

$\Rightarrow \tilde{s}_1=s_1$ and $\tilde{s}_2=s_2$ is a solution to the equation.

Thus, a voting rule where each agent is given a vote equal to his claim over the total produce, leads us to the competitive equilibrium. This very fact shades light on a number of questions in our mind. The crucial one is whether the projection of the problem of resource allocation on the economic plane to the political one satisfies certain criteria. The criteria we have in mind are anonymity and neutrality. We saw that the answer is not positive. Suppose each agent had been given equal votes and that a majoritarian voting rule had been applied. The outcome would have been the point on the PPF which is the most-wanted point of the more populated group. What if we use a mixture of both procedures, namely each one having an equal vote but a weighted voting rule is applied. Here comes "social desirability". The social outcome would not be one of the most-wanted points but it would be closer to the point of the group which is more populated. It would exhibit a better representation of the needs and preferences of the society.

2.2 Linear Technology

This one is a special case of the model in 2.1, with the only change that the production functions of the firms are linear which pave the way to an affine PPF. As expected, everything works more smoothly with an affine PPF. A similar general equilibrium analysis is conducted and all the demand functions and the production levels are calculated.

Firms' side

Production functions are of the form $f_1(K_1)=\mathbf{aK_1}$ and $f_2(K_2)=\mathbf{bK_2}$ for firm1 and firm2. Here are the firms' demand for inputs.

$$\begin{matrix} K_1 \in (0,C) \\ K_2 \in (0,C) \end{matrix} \quad \text{s.t.} \quad \frac{P_A}{P_B} = \frac{b}{a} \tag{16}$$

Consumers' side

$$
\begin{aligned}
X_A^1 &= \alpha_1 s_1 a C \\
X_B^1 &= \beta_1 s_1 b C \\
X_A^2 &= \alpha_2 s_2 a C \\
X_B^2 &= \beta_2 s_2 b C
\end{aligned}
\qquad (17)
$$

Market Clearing

$$
\begin{aligned}
n_1 X_A^1 + n_2 X_A^2 &= aK_1 \\
n_1 X_B^1 + n_2 X_B^2 &= bK_2
\end{aligned}
\qquad (18)
$$

$$
\Rightarrow \begin{aligned}
K_1 &= C(n_1 s_1 \alpha_1 + n_2 s_2 \alpha_2) \\
K_2 &= C(n_1 s_1 \beta_1 + n_2 s_2 \beta_2)
\end{aligned}
\qquad (19)
$$

We have computed the competitive equilibrium production and allocation levels.

Most-wanted points

A similar analysis to the one made in 2.1 is conducted and the following most-wanted points are computed for both groups:

A-lovers: $K1=\alpha_1 C$, $K2=\beta_1 C$.

B-lovers: $K1=\alpha_2 C$, $K2=\beta_2 C$.

We again have the figure of the PPF with the competitive equilibrium and the most-wanted points on it.

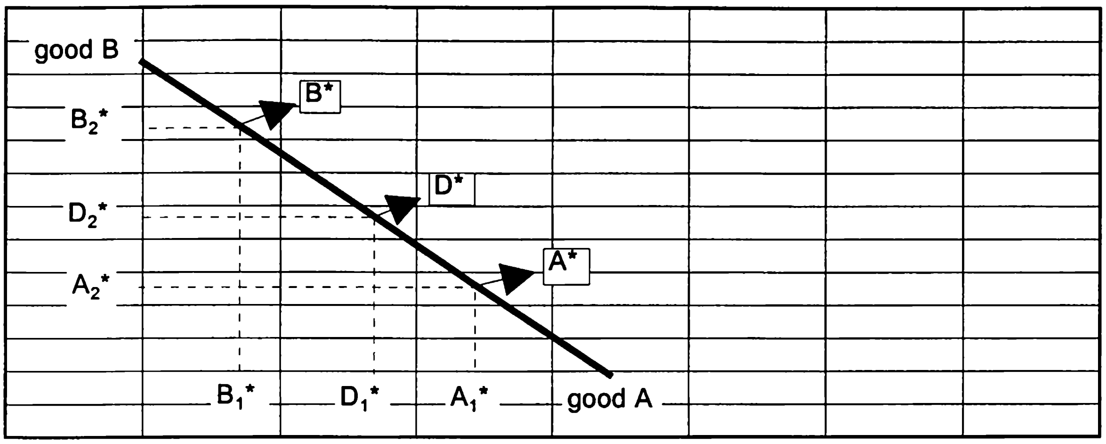

Figure 2: The PPF of the affine case.

Chapter 3

Candidate Nomination

The problems in social choice theory are always constructed given an alternative set to choose from. The process of determining these alternatives is not considered. We think that this process is important since constraining the set of alternatives can lead to different outcomes.

In our model, we let the agents declare the points on the PPF that they want to be at the most, without assuming knowledge of their preferred points. Then, the outcome is determined by the weighted voting rule that we defined above. Now, this process is not strategy-proof, since declaring a point which is different than the "most-wanted" point can lead to the selection of a better outcome for both type agents.

Let $1 \geq \xi \geq 0$ and $1 \geq \eta \geq 0$. Now, ξ denotes the point chosen by the first type agents meaning that $K_1=\xi C$, $K_2=(1-\xi)C$ and η denotes the point chosen by the second type agents meaning that $K_1=\eta C$, $K_2=(1-\eta)C$. We have the following lemma:

Lemma: Assume that (ξ,η) is a NE. If $\xi\neq 1$, then the equilibrium outcome is the most preferred point of A-lovers. If $\eta\neq 0$, then the equilibrium outcome is the most preferred point of B-lovers.

Proof: Suppose that $\xi\neq 1$ and that the equilibrium outcome is not the most preferred point of A-lovers. The equilibrium outcome is $K_1=(n_1s_1\xi+n_2s_2\eta)C$, $K_2=(1-(n_1s_1\xi+n_2s_2\eta))C$. It is clear that the outcome is feasible for any value of ξ and η within the given range. Let $K_1^*=\xi^* C$ denote the most preferred point for A-lovers. If $K_1> K_1^*$ then declaring a smaller ξ will make A-lovers better-off. If $K_1< K_1^*$ then declaring a larger ξ will make A-lovers better-off. In both cases, A-lovers can do better by deviating, a contradiction to (ξ,η) being a NE. Hence, the equilibrium outcome is the most preferred point of A-lovers. Similarly for the case where $\eta\neq0$. QED.

From the above Lemma, we know that the case where $\xi\neq1$ and $\eta\neq0$ is not possible. So $1 \geq \xi \geq \xi^*$ and $\eta^* \geq \eta \geq 0$. One can also compute the conditions under which (1,0) is a NE.

Chapter 4

Transfer Problem

Here, we provide another approach to analyse the problem of resource allocation. We use the same model described in Section 2. The difference is that instead of the weighted voting rule we used before, we utilise the majoritarian voting rule. This means that the alternative that receives more than half of the votes is selected. All the agents have equal votes. They vote on their most-wanted points. Without transfer, the outcome of this process is the most-wanted point of the more populated group.

In our setup, there is the following asymmetry between the shares and the population of the groups. The share that the agents in group2 receive from the total production is higher than that of the agents in group1; but the number of agents in group 2 is less than the number of agents in group 1. The problem that the agents in group 2 solve is the following:

$$\max\left(X_A^2(B^*)(1-T_A)\right)^{\alpha_2}\left(X_B^2(B^*)(1-T_B)\right)^{\beta_2}$$

s.t.

$$\left(X_A^1(B^*)(1+\frac{s_2 T_A n_2}{s_1 l})\right)^{\alpha_1}\left(X_B^1(B^*)(1+\frac{s_2 T_B n_2}{s_1 l})\right)^{\beta_1} \geq \left(X_A^1(A^*)\right)^{\alpha_1}\left(X_B^1(A^*)\right)^{\beta_1}$$

$$T_A, T_B \geq 0 \qquad (20)$$

$$T_{A,}, T_B \leq 1$$

What agents in group2 try to do is to have the necessary number of people in group2, namely *l*, vote on the most-wanted point of group2. To achieve this, agents in group 2 transfer that amount of good A and good B to those people in group 1 that will make them at least as good as they are at their most-wanted point. We provide here a very simple example to clarify it:

Let n_1=80, n_2=20 meaning that 20% of the total population are B-lovers. s_1=0.0025, s_2=0.04. This gives us n_1s_1=.20 and n_2s_2=.80 meaning that, although B-lovers constitute the minority group, they receive the 80% of the total production.α_1=.85 and α_2=.2. a=b=1 and γ=.5. After the above problem is solved, we have the following utility levels for B-lovers who transferred 7.5% of the amount of good A that they receive at B* to 31 A-lovers to make them vote on B*. These people are as good as at A*. B-lovers do not care about the remaining A-lovers since they convinced the necessary number of A-lovers to make them win the voting.

B-lovers after the transfer: 0.8

B-lovers would get at D*: 0.8

B-lovers would get at A*: 0.5

This finding shows that even after the transfers, B-lovers receive a utility level that they would get under the competitive equilibrium outcome.

The simulations run for some specific values of the parameters show that there is some "grain of truth" in it[2]. In most of the cases, the transfer is feasible in the sense that, the utility levels that the agents in group2 get are higher than the utility levels that they would get if the most-wanted point of the first group was selected. More interesting is that in some of the cases, the utility levels that the agents in group2 get are also very close to the utility levels that they would get under the competitive equilibrium allocation.

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[2] The simulation results can be found in the Appendix.

Chapter 5

Conclusion

In this study, we tried to open a new area of discussion. What if we use a voting model where agents vote on the production alternatives, instead of using the neo-classical resource allocation mechanism? This mechanism is new in the sense that we introduced a new definition of ownership. This new definition is a "hybrid" one that lies between public and private ownership. Instead of having absolute ownership rights on the initial endowments and the final production, agents possess initially given claims over the total income stream of the economy which is the price that is paid to the agents for their providing of the inputs plus the profits generated by the firms. Defining ownership this way has a nice use: It allows us to let people vote on the production alternatives of the economy, which would be impossible under the private ownership definition, without transfers.

Although there are examples of allocating the scarce resources of the economy by voting, they all consider the case of pure public goods where the cost of these goods have to be shared by the society. The tool we introduced and used throughout the study aimed to analyse whether the competitive equilibrium of the model economy, where the allocation and production of private goods is considered, is the "socially most desirable one". We defined a socially more desirable alternative as the one which is preferred to another alternative by the majority of the population. The answer to the above question turned out to be negative.

We know that the competitive equilibrium point on the PPF of an economy is not the only efficient one; moreover there are other points on the PPF which are efficient and socially preferred to the competitive equilibrium outcome. This fact encouraged us to ask the following question: If one can design a mechanism under which the majority of the society prefers the outcome, that comes to scene using this very mechanism, to the one generated by the neo-classical allocation mechanism, why not use it? One can bring the criticism that the "private ownership right" is the cornerstone of the liberalism and of the systems where agents are free to make their decisions at the personal level. One answer to this question could be that we live under one type of a social contract or another. We also know that these contracts are not divine. Hence, selecting the social contract, under which we want to live, is a right, too. If the majority of the society desires the economy to work under a mechanism that provides them with a better outcome, why not switching to that one.

The central finding of the paper is that, a voting rule that sustains the competitive equilibrium as its outcome should be the one under which voters are allocated votes of different weights. We also explicitly formulated such voting rule and showed that it is the weighted average of the most preferred points of two groups of voters where the votes that the agents in both groups use are the initially given claims over the total income stream of the economy. This last finding helped us answer a couple of questions in our mind. One of them is: whether the voting rule that leads to the competitive equilibrium outcome, is anonymous. The answer is no. Under such a rule, one man one vote procedure leads to a different outcome which is socially more desirable. The only exception to the statement is where the shares that both groups receive from the total income stream are equal, i.e., $s_1=s_2$. It is not neutral either.

We also looked at the candidate nomination problem which is almost always neglected by the social choice theorists. We believe that determining the candidates to vote on is an important process, since restricting the set of alternatives can lead to different outcomes, We showed that this process is manipulable for our model. We also provided a Lemma stating that the points declared by the agents are either their most preferred ones or to the right of it for A-lovers and to the left of it for B-lovers.

In Section 4, we considered another approach to look at the process of resource allocation through the use of transfers. We found out that, the market mechanism can also be explained, to some extent, by the use of transfers. We used a one man one vote scheme where one of the groups have a number of agents in the other group vote on the formers' most preferred point. This is accomplished by making these agents exactly as good as at their own most preferred points by transferring them a basket of good A and good B. The interesting point to note is that in most of the cases, this was feasible. A more interesting and crucial one was that even after transfers are made, the agents that made the transfers received a utility level which is very close to the one they would get at the competitive equilibrium point. We concluded that there must be a "grain of truth" in it.

The final thing to note is that there are still a lot of things to do. One probable extension could be to look at the case where the number of groups are greater than 2. We believe that a voting rule which is very similar to the one we described will prevail. A second extension could be to look at the conditions under which declaration of (1,0) as the candidates of two groups is a NE. One can also consider the same problem for a wider class of production and utility functions. A fourth but maybe not the last extension to state is to investigate whether a strategy-proof voting scheme to be used for the allocation of resources can be designed.

As a last word, we believe that this new area of research is a very generous one. I f we can make people think about and work on it, this will be the most promising accomplishment for us.

Appendices

- We show here that there is a price vector that satisfies equations (10).

$$n_1 X_A^1 + n_2 X_A^2 = aK_1^\gamma$$
$$n_2 X_B^1 + n_2 X_B^2 = bK_2^\gamma$$

We substitute in the explicit forms of the demand functions and let P_A=1. After cancellations are made, we have the following:

$$(n_1 s_1 \alpha_1 + n_2 s_2 \alpha_2)\gamma + (n_1 s_1 \alpha_1 + n_2 s_2 \alpha_2)(1-\gamma)\left(a^\delta + (P_B b)^\delta\right) = \frac{a^\delta}{a^\delta + (P_B b)^\delta}$$

Let $z = a^\delta + (P_B b)^\delta$ and $u = n_1 s_1 \alpha_1 + n_2 s_2 \alpha_2$. Then we have the following equation:

$u\gamma z + (1-\gamma)uz^2 = a^\delta$. Cancellation gives:

$z^2 + \frac{\gamma}{1-\gamma}z - \frac{a^\delta}{(1-\gamma)u} = 0$. Finally, we have:

Now, since all the parameters are positive and the third term in the equation is negative, the determinant is positive. The equation has two roots, one positive and one negative. The positive root satisfies the market clearing conditions we mentioned.

- Here, we summarise the results of the simulations that we undertook. We considered an economy consisted of 100 agents where n_1 and n_2 in each case are the numbers of agents belonging to each group and n_1+n_2=100. For each such case, we have 5 subcases where the share that group1 receives from the total produce varies from 20%

to 80%. Moreover, for each of these cases, we considered a discrete space of Cobb-Douglas parameters for both groups where α_1 varies from 0.5 to 0.95 and α_2 varies from 0.05 to 0.5 with increments of 0.05. The numbers in the boxes corresponding to each case show, in what percentage of the cases considered, the problem has a feasible solution.

Linear Case:

				n_1s_1		
n_1	n_2	20%	40%	50%	60%	80%
50	50	100%	100%	100%	100%	98%
60	40	96%	95%	90%	82%	58%
70	30	96%	89%	82%	74%	44%
80	20	95%	88%	86%	54%	35%
90	10	95%	81%	73%	50%	31%

Concave Case:

				n_1s_1		
n_1	n_2	20%	40%	50%	60%	80%
50	50	100%	100%	100%	100%	100%
60	40	100%	100%	100%	100%	88%
70	30	100%	100%	100%	94%	75%
80	20	100%	100%	98%	83%	67%
90	10	100%	100%	96%	82%	59%

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