A CONTINUOUS REVIEW INVENTORY SYSTEM IN A RANDOM ENVIRONMENT

A THESIS

Submitted To The Department Of Management
And The Graduate School Of Business Administration
Of Bilkent University
In Partial Fulfillment Of The Requirements
For The Degree Of
Master Of Science

By
Aslı BAYAR
Eylül 1998

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25 09 1998

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Master of Science.

Assist. Prof. Emre Berk (Supervisor)

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Abstract

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M. S. in Management
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25 09 1998

In this thesis, we develop a continuous review inventory model in a random environment where holding, ordering, and purchasing cost parameters are dependent on the state of the environment. We derive the exact expressions of the operating characteristics of the model and discuss some convexity properties of the expected cost rate. A numerical analysis is provided to examine the sensitivity of the optimal policy parameters with respect to various system parameters. We compare the instantaneous shock model with our model and illustrate that ignoring finite duration of environmental states results in considerable error. Moreover, we compare our model with a time-average EOQ and myopic model. The results illustrate that when ordering and purchasing cost parameters change, our model performs significantly better.

Keywords: Inventory, continuous review, random environment.

Özet

DEĞİŞEN BİR ORTAMDA SÜREKLİ GÖZDEN GEÇİRİLEN BİR STOK KONTROL SİSTEMİ

Aslı Bayar
İşletme Yüksek Lisans
Tez Yöneticisi: Assist. Prof. Emre Berk
1998

Bu tezde stokta tutma, sipariş ve satın alma maliyet parametrelerinin dış çevre durumuna bağlı olduğu değişken bir ortamda sürekli gözden geçirilen bir stok kontrol modeli geliştirdik. Modelin işletim özelliklerinin kesin ifadeleri ve beklenen maliyet oranının bazı konvekslik özellikleri saptandı. En iyi politika parametrelerinin çeşitli sistem parametrelerine karşı hassasiyetini incelemek için sayısal bir analiz yapıldı. Anlık şok modeli ile geliştirilen model karşılaştırıldı ve dış çevre durumlarının sürelerini gözardı etmenin dikkate değer bir hata yarattığı gözlemlendi. Ayrıca, geliştirilen model, uzun zaman ortalamalı ekonomik sipariş miktari modeli ve miyopik bir modelle karşılaştırıldı. Sayısal çalişma sipariş ve satın alma maliyeti parametrelerinin değiştiği ortamlarda geliştirilen modelin diğer politikalara kıyasla daha iyi sonuç verdiğini ortaya çıkardı.

Anahtar sözcükler: Stok kontrolü, sürekli gözden geçirme, değişken ortam

To my family

Acknowledgement

I want to thank to my family first, for their continuous support during the preperation of thesis and throughout my life.

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Chapter 1

Introduction

In the real world, big economic crises can be observed, like the ones in 1929 in the USA, in 1994 in Turkey, and Mexico, in 1997 in Asia, and in 1998 Russia. In such crises, environmental conditions may change very frequently in a random manner, and big volatilities can be observed in some economic variables, such as GNP, interest rate, exchange rates, and demand rates.

In an economic crisis, inventory management is one of the areas, which are affected from the changes in the environmental state. In inventory management, all of the cost parameters depend on the state of the environment at a point in time. Depending on the changes in the environmental state, purchasing price of the item, fixed ordering cost, holding cost, demand rate, and supply conditions may change separately or together.

Weather conditions, shortages, or political crisis can create randomness in the supply conditions and they may lead to random lead time. Moreover, the demand for many products responds in part to changes in certain basic economic variables, such as GNP, or interest rate. For instance except for some cheap products such as bread, as the GNP increases the demand for the product increases. However, as interest rates increase, borrowing becomes more expensive, so the demand for the product declines. The demand for the product changes also depending on shifts in consumer tastes.

Changes in the interest rates in the financial environment affect the holding cost in inventory management. When the interest rates are high, if a firm holds large amount of inventory on hand, (since it can not earn interest from the inventory on hand), it gives up the interest it would earn if it made investment in financial instruments. So, depending on the changes in the interest rates, holding cost in the inventory management changes.

The price of the item is affected from the changes in inflation rate and the random deal offerings of the suppliers. For the imported items, changes in the exchange rate affect the price of items. When the exchange rate increases, the purchasing price of the item increases.

In the literature under periodic review assumption for a single product, there are many studies in which all the cost parameters change depending on the changes in the environmental conditions. However, under the continuous review, the focus of research has been on the case where there is a price increase/decrease at an announced time. Among the studies in this area, only Moinzadeh[15](1997), considers an inventory system in which the product can always be procured at the list price and at random points in time price discounts are offered to the system by the market or the supplier(s) at a reduced price. The study assumes that the price discounts have no duration (i.e., is instantaneous)

In this study, we consider a continuous review inventory system for a single product where the holding, ordering, and purchasing cost parameters change in a random manner. In the system, there are two environmental conditions called state 0, and state 1, and the frequency of occurrence of these states follows exponential distribution. However, unlike in Moinzadeh[15](1997), in our study state 1 is not instantaneous.

Under the assumptions of no backorder, and negligible lead time, using optimal three parameter (S_1,S_0,s) type inventory management policy, we derive the exact expressions of the key operating characteristics of the system for the deterministic demand.

Even though the main application area of this study is inventory management, it is also possible to use the considered model in cash management. In such an application, instead of determining the optimum amount of inventory on hand, the optimum amount of money an individual can keep in his pocket is determined.

This thesis covers the following chapters. In Chapter 2, we present the literature on a single product under both the periodic and continuous review inventory systems in a changing environment.

In Chapter 3, we explain the three parameter optimal policy, and derive the key operating characteristics of the model. Expected total costs per unit time both when the holding cost is constant and changing, are calculated. And under some assumptions convexity of the total cost per unit time in some key inventory parameters are illustrated.

In Chapter 4, we present our numerical results on a wide range of parameter settings. Since the total cost expression is very complex, it can not be possible to derive analytical optimum results for the system.. We make sensitivity analysis, for the expected total cost per unit time with respect to changes in the values of holding cost, ordering cost, purchasing cost, frequency of occurrence of state 0, and frequency of occurrence of state 1.

In chapter 5, we conclude the thesis by summarising our findings, and possible future researches.

Chapter 2

Literature Review

In this chapter, we present the previous work on inventory systems in random environments. The models developed for the random environment can be classified into two groups, as periodic review inventory models, and the continuous review inventory models.

The earliest study on random environment inventory system is Hunter, and Kaminsky[10](1968). In their work, an inventory control model is developed for an environment in which random opportunities for reduced cost replenishments occur. Both demand, and the opportunities for special reduced-cost replenishment come from Poisson processes. Order lead time is negligible. A three-parameter (S_1,L_2,S_2) ordering policy is proposed. Under this policy , an order of size S_1 is placed at the regular price as the inventory is depleted. When a special opportunity for reduced unit cost occurs, given that inventory level is less than L_2, an order is made to raise the inventory level to S_2. In their analysis, they consider two separate cases, $L_2 \geq S_1$, and $S_1 \geq L_2$. As a solution they suggest a three dimensional search on the three control parameters, S_1, L_2, and S_2.

Kalymon[11](1971) studies a multi-period inventory model in which future prices of the purchased items are determined by a Markov stochastic process. It is assumed that inventory decisions are made at regular intervals. The inventory level is known at the beginning of each period, and the current price is known before purchases are made. Inventory levels are depleted by demand which in each period is a random variable whose probability distribution may depend on the current price. The paper proves the optimality of (s_i,S_i) policy in the finite horizon case.

In Golabi[6](1985), a single item inventory control model is developed in an environment in which ordering price is randomly distributed according to a known distribution function. Demand is constant. Backlogging is not allowed. At the beginning of each period, a random ordering price is received according to a known distribution function. The objective of the study is identified as the calculation of the optimal order quantities in each period which satisfying all demands, minimizes the total expected cost. The main result of Golabi[6](1985) implies that at the beginning of any period, the firm must order an amount to satisfy the demand.

Song and Zipkin[23](1991) derive an inventory model, in which the demand rate varies with an underlying state of the world variable. The world is modeled as a continuous time Markov chain. In the model, when the world is in the state k, demand follows a Poisson process with rate λ_k. There is an order lead time, either fixed or stochastic. The main results of the paper are that if the order cost is linear in the quantity ordered, base-stock policy is optimal. On the other hand if there is also a fixed cost to place an order, a state-dependent (r,S) policy is optimal, in which r is the reorder point, S is the order-up-to level.

Silver, Robb, and Rahnama[21] (1993) consider the same problem with Hunter and Kaminsky[10] (1968). However, they make the assumption that $L_2 \geq S_1$. They then develop an approximate solution method which is much easier to use. They do not consider the condition that $S_1 \geq L_2$, because occurrence of a fairly large fixed cost for each special replenishment is very unlikely. there is a small difference in the unit cost of the material between regular and special replenishments, and very frequent opportunity for special replenishments do not occur in the real world.

Özekici, and Parlar[17](1997) provide an infinite-horizon periodic-review inventory model in a random environment where the supply, demand, and cost parameters may change instantaneously. The state of the environment is described by a Markov chain. There is complete backlogging of the demand. In the study, they show that an environment dependent order-up-to level policy is optimal when the order cost is linear in order quantity. When there is also a fixed cost of ordering, it is shown that a two-parameter environment dependent (s_i, S_i) policy is optimal under some specific conditions.

Finally, in the periodic review literature, we should mention Hall[9](1992) who compares the cost rate model, and the present value model in an environment in which from time to time, the unit-purchasing price of the good may change in discrete steps. In both models, there is no order lead time and demand is constant. In the study, comparison of these two approaches reveals that when the price reduction is more than 40 percent, present value approach is more preferred, since in the present value approach the cost of holding inventory is not undervalued.

Under the continuous review, the main focus of research has been on the case where there is a price increase/decrease at an announced time.

Among the continuous review inventory management models, Taylor, and Bradley[27] (1985), develop optimal ordering policies for an environment where the announced price increase becomes effective at any future specified time rather than at the end of economic order quantity cycle.

Ardalan[1](1988) analyses the effects of a special order on inventory cost when a supplier reduces the price of a product temporarily, and develops an EOQ-type optimal policy in response to a sale. The paper assumes that the sale period is short relative to the regular inventory cycle, and it is not required that reduced price is in effect at the replenishment time. In the model, the total holding and ordering costs may also be affected by size, and time of special order.

Lev and Weiss[12](1990), relaxing the basic assumptions of the classical EOQ inventory model (infinite time horizon, and static costs), develop models for finite and infinite-time horizon optimal inventory policies in a random environment in which all or any costs change. In the model, there is no order lead time, holding cost is fixed, and at the beginning the time of the price change is announced. They show that all orders placed before the time of the last opportunity to purchase at low price are of the same size, and all orders placed after the time of the last opportunity are of the same size.

Recently, Moinzadeh[15](1997) analized an inventory system where price discounts are offered at random points in time, which are exponentially distributed. In the model, demand is assumed constant, order lead time is assumed to be negligible and no backorders are allowed. The discount (deal) itself have no duration (i.e. is instantaneous). In the model a three parameter (S_1, S_0, s) control policy is employed, and its convexity properties which are imitates of EOQ-type approximations to some policy parameters are also provided.

We should also mention the work on supply conditions. Parlar and Berkin[18](1991) deal with a continuous review inventory model where supply availability is a random variable. It is assumed that during a "wet" period supply is available in any amount that is desired, whereas during a "dry" period, it is impossible to obtain any supply. The demand that is not satisfied during a dry period is lost. Under an EOQ type ordering policy, the system is analyzed.

Berk, and Arreola-Risa[2](1994), build their study on Parlar and Berkin[18] (1991) and remove an implicit assumption in their work.

Parlar and Perry[19](1995) consider a stochastic inventory model where supply conditions (lead times) are random due to some factors such as strikes. In the study, the supplier's availability process is represented as a two-state continuous time Markov Chain where one state corresponds to availability (ON), and the other state corresponds to unavailability of the supplier (OFF). They assume that if the order arrives, the state is identified as ON, and in this state lead time is zero. The duration of both the ON state and the OFF state are exponentially distributed. In the model, to make an order the firm does not have to wait until the inventory level reaches zero, reorder point is defined as one of the decision variables. They use (S, s) type inventory policy. The paper identifying the objective function as the long-run average cost, determines the optimal values for the reorder point, the order quantity when the system is in ON state, and how long to wait before the next order if the system is OFF state.

Gürler and Parlar[8](1997) develop a continuous review inventory model in a random environment in which supply availibility is subject to random fluctuations. In the system, there are two suppliers, and their availability may be individually either ON or OFF. They assume that the duration of the ON periods for the two suppliers are distributed as Erlang random variables, and the OFF periods for each supplier have a general distribution. They use (S, s) type policy, and define their objective as minimization of long-run average cost.

In our study, we develop a continuous review inventory model in a random environment in which holding, ordering, and purchasing cost parameters change depending on changes in the environment. Among the previous studies, Moinzadeh[15](1997) is the closest to our model. As in Moinzadeh[15](1997), we assume that the demand is constant, order lead times are negligible, no backorders are allowed, and the occurrence of state 1, and 0 are poisson process. However, unlike in Moinzadeh[15](1997), in our study state 1 has positive duration, (i.e. is not instantaneous), and thereby holding cost also changes depending on environmental conditions.

Chapter 3

Model

In this chapter, a continuous review inventory model is developed under a three-parameter control policy operating in an environment in which state changes in a random manner. We begin our analysis by stating the assumptions of the model. We then introduce the control policy employed in the model. We derive the expressions for the operating characteristics of the system, and the expected total cost rate. Lastly, we discuss some convexity properties of the cost rate.

3.1 Derivation of the Model

We consider an inventory system under the following assumptions:

1. Demand rate is constant over time, D.

2. The environment in which the system operates can be found in two states, 0 and 1.

3. The time between changes in the state of the environment is random, and is exponentially distributed. The change from state 0 to 1 occurs with rate λ_1, and the change from state 1 to 0 occurs with rate λ_0 .

4. Order lead times are negligible.

5. No backorders are allowed.

6. Holding cost is incurred at h_i per unit held in stock per unit of time in state i (i=0,1).

7. Each order placed in state i (i=0,1) incurs a fixed ordering cost k_i.

8. Purchasing cost is incurred at c_i per unit purchased in state i (i=0,1).

In chapter 5, we discuss how each of the assumptions may be relaxed and what impact they would have on the analysis.

We know from Song and Zipkin[23](1991). and Özekici and Parlar[17](1997) that the optimal control policy class is of a state-dependent (S_i, s_i) type for periodic review inventory systems. In Song and Zipkin[23](1991) (see Theorem 3 in page 358), in the case where there is a fixed cost to place an order, it is shown that a world-dependent (r,S) policy is optimal. That is under case of two states, optimal control policy is of four parameter (S_1, s_1, S_0, s_0). In continuous review inventory systems, three parameter (S_1, S_0, s) inventory control policies would be optimal when the order lead time is negligible. However, when there are order lead times, inclusion of another non-zero reorder point is necessary resulting in (S_i, s_i) control policies. Under continuous review, Moinzadeh[15](1997) proposes a three-parameter (S_1, S_0, s) policy without discussion of optimality in the case of two environmental states when there are no order lead times.

In this study, since we assume that there is no order lead time, we consider a state-dependent three-parameter inventory control policy.

The following control policy is employed:

POLICY:i) If the inventory level is at or below s when the system is in state 1, the inventory level is raised to S_1;

ii) If the system is in state 0, the inventory level is raised to S_0 when inventory level drops to zero.

This policy will be referred to as the (S_1, S_0, s) policy. Under this policy, if the inventory level is greater than s, no order is made. An order may also be placed anywhere between s and zero if the environment is found in state 0 at s, and a change in the state from 0 to 1 occurs afterwards. S_1 and S_0 are the order-up-to levels in state 1 and 0 respectively.

To define a cycle and a regeneration point, we need to know the state of the environment, and the inventory level at the regeneration point, and both of them (the state of the environment and the inventory level) should have the same values at each time the inventory level reaches the regeneration point. In our model, we define a cycle as the time between two consecutive replenishments when the state is 1 (in other words when the inventory level is raised to S_1). It is also possible to identify the instance when state 0 replenishments occur as a regeneration point. However, it is not possible to identify instance at which inventory level hits s, since at that instance the state may be 0 or 1.

For ease of exposure, we define Q_1=S_1-s and Q_0=S_0-s.

Note that there may be many changes in state from 1 to 0, and 0 to 1 in a cycle. However, only the ones that occur when inventory on hand is less than or equal to s affect the ordering decision. Depending on the definition of a cycle, and the possible values, S_1, (Q_1+s), S_0, (Q_0+s), and s may take, there are three possible cases. In case I, S_1 is greater than or equal to S_0, and S_0 is greater than or equal to s, ($S_1 \geq S_0 \geq$ s). In case II, again S_1 is greater than or equal to S_0, but S_0 is less than or equal to s, ($S_1 \geq$ s $\geq S_0$). In case III, S_0 is greater than or equal to S_1, and S_1 is greater or equal to than s. In the model, $S_0 \geq s \geq S_1$ case is not a realistic case. According to the policy when the inventory level is less than or equal to s level, if state 1 occurs, an order is made to rise the inventory level to S_1. However, in that case, since S_1 is less than or equal to s, it is not possible to make an order when the state becomes 1 and the inventory level on hand is between s and S_1.

For the sake of clarity, it may be assumed that state 1 refers to a discount state, i.e. low cost, and state 0 is a high cost state. When state is 1, if we analyze the system in terms of purchasing cost since the unit purchasing cost is low in state 1, one would want to buy as much as it can at low cost, so S_1 would be greater than S_0. Also due to high unit purchasing cost S_0 would be chosen as small as possible; in some cases, S_0 might be chosen even smaller than s. Therefore, it is necessary that we consider both case I and case II in the analysis.

If we analyze the system in terms of holding cost, when state is 1, since unit holding cost is low, opportunity cost of holding inventory is not high, so to keep large amount of inventory does not cost much. Hence we would consider only case I in which S_1 is greater than or equal to S_0.

However, if we analyze the system in terms of ordering cost, we see that the optimal policy might happen in both case I, where Q_1 is greater than or equal to Q_0, and case III where Q_1 is less than or equal to Q_0. Both of the cases are considered because as Q_1 increases, number of units purchased in state 1 increases, and the number of replenishment declines. On the other hand, as optimal Q_0 values increases, fixed ordering cost per unit of order declines.

3.1.1 Case I

As can be seen from Figure 3.1, in case I, there are four possible cycle types. The first cycle type starts with inventory level at S_1, and environment is in state 1 and until inventory level declines to s, some changes may occur in the environment from state 0 to state 1. But given that state is 1 at the inventory level s, Q_1 units of inventory is ordered when the inventory level is equal to s. In other words, cycle ends when inventory level hits s, and the state is 1.

The second cycle type starts with an inventory level at S_1 and environment is again in state 1 and until inventory level declines to s, environment changes from state 0 to state 1. However, the state is 0 at the inventory level s, and at the inventory level I , which is between s and 0, state changes from 0 to 1. As soon as this change occurs, (S_1-I) units of inventory is ordered. In short, cycle ends when the state becomes 1 at inventory level I.

Unlike in the first, and the second cycle types, in the third, and fourth cycle types there are replenishments when state is 0. These cycle types occur when state 0 goes on for a long duration. The main difference between the third, and fourth cycle types is that unlike in the third cycle type in which state changes from 0 to 1 when the inventory level is greater than s, in the fourth cycle type the state changes from 0 to 1 when the inventory level is below the s.

To calculate the operating characteristics, some regions are defined, and the cycle types are divided into the identified regions called R_{1a}, R_{1b}, R_2, R_{3a}, R_{3b} and R_4 as illustrated in Figure 3.1.

R_{1a} region covers the area between inventory levels S_1 and s. In R_{1a} region, at the inventory level S_1, environment is in state 1. between inventory levels S_1 and s state can be 1 or 0, however, it must be 1 when the inventory level falls to s. R_{1a} region exists in cycle type (i) of case I.

Like R_{1a} region. R_{1b} region covers the area between inventory levels S_1 and s. However, in R_{1b} region. at the inventory level S_1. state is 1. between inventory levels S_1 and s state can be 1 or 0, but it must be 0. when the inventory level is equal to s. R_{1b} region occurs in cycle type (ii), cycle type (iii), and cycle type (iv) of case I.

R_2 region is observed when state 0 goes on for a long duration, and the number of replenishments when state is 0 is greater than zero. R_2 region covers the area between inventory levels s and 0. In that region, the state is always 0. Depending on how long state 0 goes on, it is possible to observe occurrence of R_2 region in a cycle type more than once. If state becomes 1 between inventory levels s and 0, a replenishment is made, cycle ends, and R_2 region can not occur in that cycle type any more. R_2 region takes place in the third and fourth cycle types.

Like R_2 region, R_{3a} region is observed when the number of replenishments when the state is 0 is greater than zero. R_{3a} region covers the area between inventory levels S_0 and s. In R_{3a} region, given that at S_0 inventory level the state is 0, between inventory levels S_0 and s there may be changes from 0 to 1 or 1 to 0 states, and. at s inventory level the state is 0. Since inventory level is above s level, unlike in R_2 region, in R_{3a} region such changes in the state of environment do not affect the system. R_{3a} region takes place both in the third and fourth cycle types.

Like R_{3a} region. R_{3b} region covers the area between inventory levels S_0 and s and it is observed when the number of replenishments when the state is 0 is greater than zero. In R_{3b} region, at inventory level S_0 the state is 0, it will be 1 at the s inventory level. Unlike R_{3a} region, R_{3b} region occurs only once in cycle type (iv). It does not occur in cycle type (iii).

The region between inventory levels s and I is called as R_4 region. In that region I is the inventory level at that the state changes from 0 to 1; I can take any value between s and 0. In R_4 region, until inventory level I is reached, state 0 is observed. At the inventory level I, conditions change from 0 to 1 and at that point (S_1-I) units of inventory is ordered. R_4 region takes place only in the second and fourth cycle types. In each cycle type, R_4 region does not occur more than once, and when R_4 region occurs, the cycle ends. The main difference between R_4 region and the other regions is that R_4 region is a random region. The length of the R_4 region may change depending on the possible values I may take between inventory levels s and 0.

3.1.2 Case II

In case II, given that S_1 is greater than or equal to s, S_0 is less than or equal to s , ($S_1 \geq$ s $\geq S_0$). As can be seen from Figure 3.2. , unlike in case I, in case II there are three different cycle types. First, and second cycle types are the same with the first and second cycle types in case I. Like in the first cycle type of case I, in the first cycle type of case II, only R_{1a} region occurs. Moreover, like in second cycle type of case I, in the second cycle type of case II, both R_{1b}, and R_4 regions take place. However, cycle type (iii) is different from cycle type (iii) in case I. In cycle type (iii) of case II, there are replenishments when the state is 0, and since S_0 value is less than or equal to s value, the replenishment may occur when the inventory level is greater than S_0.

Like in case I, to make calculation of the total cost, some regions are defined, and the cycle types are divided into the identified regions called, R_{1a}, R_{1b}, R_2, R_4, R_5, and R_6.

R_{1a}, and R_{1b} regions are the same with the R_{1a}, and R_{1b} regions identified in case I.

R_2 region covers the area between inventory levels s and 0. In that region, the state does not change from 0 to 1. Except for the probability (expected number) of occurrence, R_2 region is the same with the R_2 region in case I. In case II, R_2 region is observed only in the third cycle type.

R_4 region covers the area between inventory levels s and I. I, which can take value between s and 0 inventory levels, is again defined as the inventory level at which state changes from 0 to 1. Unlike in case I. in case II, R_4 region occurs only in the second cycle type. Thus except for the probability (expected number) of occurrence, R_4 region is the same with the R_4 region in case I. Like in case I, in case II, the length of the R_4 region changes depending on the possible values I can take between s and 0 inventory levels.

R_5 region is observed when replenishments in state 0 occurs. In other words, it is observed only in the third cycle type. R_5 region covers the area between inventory levels S_0 and 0. In that interval, the state is always 0; no state 1 occurs. When state 0 remains for a long duration, R_5 region occurs more than once.

The region between inventory levels S_0 and I is called as R_6 region. Like R_5 region, R_6 region is observed only in the third cycle type. In that region I is the inventory level at that environmental conditions changes from 0 to 1. I can take any value between S_0 and 0. In R_6 region, until inventory level I is reached state 0 is observed. At inventory level I the state changes from 0 to 1 and at that point (S_1-I) units of inventory are ordered. R_6 region is also a random region. The length of the R_6 region depends on the possible values I can take between S_0 and 0 inventory levels.

3.1.3 Case III

As can be seen from Figure 3.3. case III is almost the same with case I. Like in case I, in case III, there are four different cycle types, and in those cycles R_{1a}, R_{1b}, R_2, R_{3a}, R_{3b}, and R_4 regions are observed. The main big difference between case I, and case III is that unlike in case I, in case III S_0 is greater than or equal to S_1, ($S_0 \geq S_1 \geq$ s). Except for restriction that $S_0 \geq S_1$, case III is the same as case I. Therefore we do not enumerate cycle types, and the regions in this case.

3.2 Derivation of the Cost Function

In calculation of the expected total cost function, conditional probabilities identifying the state of the environment are needed. However, our problem is similar to the M/M/1/1 queuing system in which derivation of the transient probabilities that at an arbitrary time t, there are n customers in a single channel system with Poisson input, exponential service, and no waiting room is a straightforward procedure. In our model, state 0 refers to the condition that there is no customer in the M/M/1/1 queuing system. And state 1 refers to the condition that there is one customer in the M/M/1/1 queuing system.

Using M/M/1/1 queuing system, as explained in the appendix A the following conditional probabilities are calculated:

$P_{10}(t)$: The probability that the state will 0 at time t given that it is 1 at time zero.

$P_{00}(t)$: The probability that the state will 0 at time t given that it is 0 at time zero.

$P_{11}(t)$: The probability that the state will 1 at time t given that it is 1 at time zero.

$P_{01}(t)$: The probability that the state will 1 at time t given that it is 0 at time zero.

$$P_{10}(t) = \frac{\lambda_0(1-e^{-(\lambda_1+\lambda_0)t})}{(\lambda_1+\lambda_0)} \tag{3.1}$$

$$P_{00}(t) = \frac{\lambda_0+\lambda_1 e^{(-(\lambda_1+\lambda_0)t)}}{(\lambda_1+\lambda_0)} \tag{3.2}$$

$$P_{11}(t) = \frac{\lambda_1+\lambda_0 e^{-(\lambda_1+\lambda_0)t}}{(\lambda_1+\lambda_0)} \tag{3.3}$$

$$P_{01}(t) = \frac{\lambda_1(1-e^{-(\lambda_1+\lambda_0)t})}{(\lambda_1+\lambda_0)} \tag{3.4}$$

P_1 is the probability that the state will be 0 at the inventory level s given that at inventory level S_1 it is 1.

$P_1 = P_{10}(Q_1/\mathrm{D})$

$$P_1 = \frac{\lambda_0(1 - e^{-(\lambda_1+\lambda_0)(\frac{Q_1}{D})})}{(\lambda_1+\lambda_0)} \tag{3.5}$$

P_2 is the probability that between inventory levels s and 0 state 1 does not occur. In other words, in the 0- s/D time interval, state 1 is not observed. only state 0 is observed.

$$P_2 = e^{-\lambda_1(\frac{s}{D})} \tag{3.6}$$

P_3 is used only in case I. It refers the probability that given that at inventory level S_0 the state is 0, again it will be 0 when the inventory level is equal to s.

$P_3 = P_{00}(Q_0/\mathrm{D})$

$$P_3 = \frac{\lambda_0 + \lambda_1 e^{(-(\lambda_1+\lambda_0)(\frac{Q_0}{D}))}}{(\lambda_1+\lambda_0)} \tag{3.7}$$

P_4 is the probability that between inventory levels S_0 and 0 state 1 does not occur. In other words, in the 0- S_0/D time interval, only state 0 is observed. This probability is used only in case II.

$$P_4 = e^{-\lambda_1(\frac{S_0}{D})} \tag{3.8}$$

In the model, these probabilities are used in the calculation of the expected costs of the cases, and the regions.

3.2.1 Cost Functions for Case I

Expected Holding Cost Per Cycle

We use identified regions R_{1a} through R_6 to calculate the expected holding cost within a cycle. Expected holding cost for some of the identified regions is different from the holding cost in the standard EOQ model. In the standard EOQ model since the holding cost does not change from one state to another, holding cost is equal graphically to the area under the inventory level curve multiplied by the holding cost per item. However, since in our model the environment is random.

conditional probabilities of occurrences of states 0 and 1 are also taken into account in calculating the expected holding cost.

Expected holding cost for case I can be written as,

$$HC_1 = \quad HC_{11} + HC_{12} + HC_{13} + HC_{14} \tag{3.9}$$

where HC_{ij} is the expected holding cost of region j in case i, (i=1,2,3 and j=1,2,...,6)

HC_{11} consists of two possible realizations depending on whether region R_{1a} or R_{1b} occurs. Let HC_{11a} or/and HC_{11b} be the holding cost for each of these regions.

$$HC_{11a} = \quad h_0 \int_0^{Q_1/D}(S_1 - Dt)dt - (h_0 - h_1)\int_0^{Q_1/D}(S_1 - Dt)P_{111}(t|(Q_1/D))dt \tag{3.10}$$

where P_{111}(t$|$(Q_1/D)) is the conditional probability that state will be 1 at time t given that it is 0 at times zero, and (Q_1/D), for (Q_1/D)$\geq$t$\geq$0. From Bayes' Law we have

$$P_{111}(t|(Q_1/D)) = \quad \frac{P_{11}(t)P_{11}(\frac{Q_1}{D}-t)}{P_{11}(\frac{Q_1}{D})} \tag{3.11}$$

$$HC_{11b} = \quad h_0 \int_0^{Q_1/D}(S_1 - Dt)dt - (h_0 - h_1)\int_0^{Q_1/D}(S_1 - Dt)P_{110}(t|(Q_1/D))dt \tag{3.12}$$

where P_{110}(t$|$(Q_1/D)) is the conditional probability that state will be 1 at time t given that it is 1, and 0 at time zero and (Q_1/D) respectively.

$$P_{110}(t|(Q_1/D)) = \quad \frac{P_{11}(t)P_{10}(\frac{Q_1}{D}-t)}{P_{10}(\frac{Q_1}{D})} \tag{3.13}$$

$$HC_{11} = \quad P_{11}HC_{11a} + P_{10}HC_{11b} \tag{3.14}$$

In Equation 3.14, P_{11}(Q_1/D) is the probability of occurrence of R_{1a} region, and P_{10}(Q_1/D) is the probability of occurrence of R_{1b} region.

$$HC_{11} = \begin{aligned} & h_0 \int_0^{Q_1/D}(S_1 - Dt)dt - (h_0 - h_1) \int_0^{Q_1/D}(S_1 - Dt) \\ & [P_{11}(Q_1/D)P_{111}(t|(Q_1/D)) + P_{10}(Q_1/D)P_{110}(t|(Q_1/D))]dt \end{aligned} \tag{3.15}$$

Note that

$$P_{11}(t) = P_{11}(Q_1/D)P_{111}(t|(Q_1/D)) + P_{10}(Q_0/D)P_{110}(t|(Q_1/D)) \tag{3.16}$$

$$HC_{11} = h_0 \int_0^{Q_1/D}(S_1 - Dt)dt - (h_0 - h_1) \int_0^{Q_1/D}(S_1 - Dt)P_{11}(t)dt \tag{3.17}$$

(S_1- Dt) is the inventory level at time t. In the equation above, the "saving" made when the state becomes 1, ($\int_0^{Q_1/D}$ (S_1- Dt) (h_0- h_1) P_{11}(t)dt) is subtracted from holding cost when the state is 0. ($\int_0^{Q_1/D}$ h_0(S_1- Dt) dt). P_{11}(t) refers to the probability that the state will be 1 at time t in the (S_1-s) inventory levels interval given that at time zero, the state is 1.

$$
\begin{aligned}
HC_{11} = {} & h_0 \int_0^{Q_1/D} (S_1 - Dt)dt - (h_0 - h_1) \int_0^{Q_1/D} (S_1 - Dt) \\
& \frac{(\lambda_1 + \lambda_0 e^{-(\lambda_1+\lambda_0)t})}{(\lambda_1+\lambda_0)} dt \\
= {} & \frac{h_0 S_1 Q_1}{D} - \frac{h_0 D Q_1^2}{2D^2} - \int_0^{Q_1/D} (S_1 h_0 - S_1 h_1 - Dt h_0 + h_1 Dt) \\
& \frac{(\lambda_1 + \lambda_0 e^{-(\lambda_1+\lambda_0)t})}{(\lambda_1+\lambda_0)} dt \\
= {} & \frac{h_0 2(Q_1+s)Q_1 - h_0 Q_1^2}{2D} - \frac{1}{(\lambda_1+\lambda_0)} \int_0^{Q_1/D} (S_1 h_0 \lambda_1 - S_1 h_1 \lambda_1 \\
& - Dt h_0 \lambda_1 + h_1 Dt \lambda_1 + \lambda_0 e^{-(\lambda_1+\lambda_0)t} S_1 h_0 - S_1 h_1 \lambda_0 e^{-(\lambda_1+\lambda_0)t} \\
& - Dt h_0 \lambda_0 e^{-(\lambda_1+\lambda_0)t} + h_1 Dt \lambda_0 e^{-(\lambda_1+\lambda_0)t}) dt \\
= {} & \frac{h_0 Q_1^2 + 2Q_1 s h_0}{2D} - \frac{1}{(\lambda_1+\lambda_0)} [\frac{S_1 h_0 \lambda_1 Q_1}{D} - \frac{S_1 h_1 \lambda_1 Q_1}{D} \\
& - \frac{D h_0 Q_1^2 \lambda_1}{2D^2} + \frac{h_1 D Q_1^2 \lambda_1}{2D^2} \\
& + (\frac{\lambda_0 e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}} S_1 h_0}{-(\lambda_1+\lambda_0)} + \frac{\lambda_0 S_1 h_0}{(\lambda_1+\lambda_0)}) \\
& - (\frac{S_1 h_1 e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}} \lambda_0}{-(\lambda_1+\lambda_0)} + \frac{S_1 h_1 \lambda_0}{(\lambda_1+\lambda_0)}) \\
& - (\frac{D e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}} h_0 \lambda_0}{(\lambda_1+\lambda_0)^2} (\frac{-(\lambda_1+\lambda_0)Q_1}{D} - 1) + \frac{D h_0 \lambda H}{(\lambda_1+\lambda_0)^2}) \\
& + (\frac{D h_1 \lambda_0 e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}}}{(\lambda_1+\lambda_0)^2} (\frac{-(\lambda_1+\lambda_0)Q_1}{D} - 1) + \frac{D h_1 \lambda_0}{(\lambda_1+\lambda_0)^2})] \\
= {} & \frac{h_0 Q_1^2 + 2Q_1 s h_0}{2D} - \frac{1}{(\lambda_1+\lambda_0)} [\frac{Q_1^2 h_0 \lambda_1 + Q_1 s h_0 \lambda_1}{D} - \frac{Q_1^2 h_1 \lambda_1}{D} \\
& - \frac{Q_1 s h_1 \lambda_1}{D} + \frac{Q_1^2 \lambda_1 (h_1 - h_0)}{2D} - \frac{\lambda_0 e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}} Q_1 h_0}{(\lambda_1+\lambda_0)} \\
& - \frac{\lambda_0 e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}} s h_0}{(\lambda_1+\lambda_0)} + \frac{\lambda_0 Q_1 h_0}{(\lambda_1+\lambda_0)} + \frac{\lambda_0 s h_0}{(\lambda_1+\lambda_0)} + \frac{\lambda_0 e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}} Q_1 h_1}{(\lambda_1+\lambda_0)} \\
& + \frac{\lambda_0 e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}} s h_1}{(\lambda_1+\lambda_0)} - \frac{\lambda_0 Q_1 h_1}{(\lambda_1+\lambda_0)} - \frac{\lambda_0 s h_1}{(\lambda_1+\lambda_0)} + \frac{\lambda_0 e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}} Q_1 h_0}{(\lambda_1+\lambda_0)} \\
& + \frac{\lambda_0 e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}} D h_0}{(\lambda_1+\lambda_0)^2} - \frac{D h_0 \lambda_0}{(\lambda_1+\lambda_0)^2} - \frac{\lambda_0 e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}} Q_1 h_1}{(\lambda_1+\lambda_0)} \\
& - \frac{\lambda_0 e^{(\lambda_1+\lambda_0)\frac{Q_1}{D}} D h_1}{(\lambda_1+\lambda_0)^2} + \frac{D h_1 \lambda_0}{(\lambda_1+\lambda_0)^2}] \\
= {} & \frac{h_0 Q_1^2 + 2Q_1 s h_0}{2D} - \frac{1}{(\lambda_1+\lambda_0)} [\frac{Q_1^2 h_0 \lambda_1}{2D} - \frac{Q_1^2 h_1 \lambda_1}{2D} + \frac{Q_1 s \lambda_1 (h_0 - h_1)}{D} \\
& + \frac{Q_1 \lambda_0 (h_0 - h_1)}{(\lambda_1+\lambda_0)} + \frac{s \lambda_0 (h_0 - h_1)}{(\lambda_1+\lambda_0)} + \frac{s \lambda_0 (h_1 - h_0) e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}}}{(\lambda_1+\lambda_0)} \\
& + \frac{D \lambda_0 (h_0 - h_1) e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}}}{(\lambda_1+\lambda_0)^2} + \frac{D \lambda_0 (h_1 - h_0)}{(\lambda_1+\lambda_0)^2}] \\
= {} & \frac{h_0 Q_1^2 + 2Q_1 s h_0}{2D} - \frac{Q_1^2 \lambda_1 (h_0 - h_1)}{(\lambda_1+\lambda_0) 2D} - \frac{Q_1 s \lambda_1 (h_0 - h_1)}{(\lambda_1+\lambda_0) D} \\
& - \frac{Q_1 \lambda_0 (h_0 - h_1)}{(\lambda_1+\lambda_0)^2} - \frac{s \lambda_0 (h_0 - h_1)}{(\lambda_1+\lambda_0)^2} - \frac{s \lambda_0 (h_1 - h_0) e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}}}{(\lambda_1+\lambda_0)^2} \\
& - \frac{D \lambda_0 (h_0 - h_1) e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}}}{(\lambda_1+\lambda_0)^3} - \frac{D \lambda_0 (h_1 - h_0)}{(\lambda_1+\lambda_0)^3} \\
= {} & \frac{\lambda_1 h_0 Q_1^2 + \lambda_0 h_0 Q_1^2 + \lambda_1 h_0 2Q_1 s + \lambda_0 h_0 2Q_1 s - \lambda_1 h_0 Q_1^2}{(\lambda_1+\lambda_0) 2D} \\
& + \frac{\lambda_1 h_1 Q_1^2 - \lambda_1 h_0 2Q_1 s + \lambda_1 h_1 2Q_1 s}{(\lambda_1+\lambda_0) 2D} \\
& - \frac{Q_1 \lambda_0 (h_0 - h_1)}{(\lambda_1+\lambda_0)^2} - \frac{s \lambda_0 (h_0 - h_1)}{(\lambda_1+\lambda_0)^2} - \frac{D \lambda_0 (h_1 - h_0)}{(\lambda_1+\lambda_0)^3} \\
& - e^{-(\lambda_1+\lambda_0)\frac{Q_1}{D}} (\frac{s \lambda_0 (h_1 - h_0)}{(\lambda_1+\lambda_0)^2} + \frac{D \lambda_0 (h_0 - h_1)}{(\lambda_1+\lambda_0)^3})
\end{aligned}
\tag{3.18}
$$

$$
\begin{aligned}
HC_{11} = \; & \frac{Q_1^2(\lambda_0 h_0+\lambda_1 h_1)}{2D(\lambda_1+\lambda_0)} + Q_1\left(\frac{(\lambda_0 h_0+\lambda_1 h_1)s}{D(\lambda_1+\lambda_0)} + \frac{\lambda_0(h_1-h_0)}{(\lambda_1+\lambda_0)^2}\right) \\
& + \frac{s\lambda_0(h_1-h_0)}{(\lambda_1+\lambda_0)^2} + \frac{\lambda_0 D(h_0-h_1)}{(\lambda_1+\lambda_0)^3} \\
& + \epsilon^{\frac{-(\lambda_1+\lambda_0)Q_1}{D}}\left(\frac{s\lambda_0(h_0-h_1)}{(\lambda_1+\lambda_0)^2} - \frac{D\lambda_0(h_0-h_1)}{(\lambda_1+\lambda_0)^3}\right)
\end{aligned} \tag{3.19}
$$

Since there is at least one occurrence of R_2 region in the third and the fourth cycle types, expected number of occurrence of R_2 region is included in the calculation of the expected holding cost for this region. In the equation. P_1 refers to the probability of occurrence of R_{1b} region in the third and the fourth cycle types. P_2^N refers to the probability that R_2 region is observed N times. P_3^{N-1} in the third cycle type is the probability of occurrence of R_3 region N-1 times. In the third cycle type, (1-P_3) is the probability that when the inventory level reaches the s level, the state is 1. In other words, as soon as inventory level reaches s (S_1-s) units of inventory are ordered and the cycle ends.

Unlike in the third cycle type, in the fourth cycle type R_{3a} region is observed N times, and the state changes from 0 to 1 when the inventory level is less than s. In the fourth cycle types, (1-P_2) refers to the probability that state changes from 0 to 1 when the inventory level is less than s.

Both in the third and the fourth cycle types, R_2 region is observed N times. Depending on how long state 0 goes on, N can take values between one and infinity.

N: Number of replenishments when the state is 0.

N_{ij}: Expected number of occurrence of region j in case i, j=1,2,...,6. i=1.2,3

$$N_{12} = \sum_{N=1}^{\infty} P_1 P_2^N P_3^{N-1} N(1-P_3) + \sum_{N=1}^{\infty} P_1 P_2^N P_3^N N(1-P_2) \tag{3.20}$$

$$N_{12} = \frac{P_1(1-P_3)P_2}{(1-P_2P_3)^2} + \frac{P_1P_3(1-P_2)P_2}{(1-P_2P_3)^2} \tag{3.21}$$

$$N_{12} = \frac{P_1P_2 - P_1P_2P_3 + P_1P_2P_3 - P_1P_2^2P_3}{(1-P_2P_3)^2} \tag{3.22}$$

$$N_{12} = \frac{P_1P_2}{(1-P_2P_3)} \tag{3.23}$$

$$HC_{12} = N_{12}[h_0 \int_0^{s/D} (s - Dt)dt] \quad (3.24)$$

$$HC_{12} = \frac{P_1 P_2}{(1-P_2 P_3)}(\frac{h_0 s^2}{D} - \frac{h_0 D s^2}{2D^2}) \quad (3.25)$$

$$HC_{12} = \frac{P_1 P_2 h_0 s^2}{(1-P_2 P_3)2D} \quad (3.26)$$

Similar to the calculation of holding cost in R_{1a} and R_{1b} regions, in the calculation of holding cost in R_{3a} and R_{3b} regions, instead of writing holding costs for R_{3a} and R_{3b} regions separately it is possible to write summation of holding costs of these regions as a holding cost of the R_3 region. HC_{13} consists of two possible realizations depending on whether region R_{3a} or R_{3b} occurs. Let HC_{13a} or/and HC_{13b} be the holding cost for each of these regions.

$$HC_{13a} = h_0 \int_0^{Q_0/D} (S_0 - Dt)dt - (h_0 - h_1) \int_0^{Q_1/D} (S_0 - Dt) P_{010}(t|(Q_0/D))dt \quad (3.27)$$

$$HC_{13b} = h_0 \int_0^{Q_0/D} (S_0 - Dt)dt - (h_0 - h_1) \int_0^{Q_0/D} (S_0 - Dt) P_{011}(t|(Q_0/D))dt \quad (3.28)$$

where P_{011}(t|(Q_0/D)) is the conditional probability that state will be 1 at time t given that it is 0. and 1 at time zero and (Q_0/D) for (Q_0/D)≥t≥0 respectively.

$$P_{011}(t|(Q_0/D)) = \frac{P_{01}(t)P_{11}(\frac{Q_0}{D}-t)}{P_{01}(\frac{Q_0}{D})} \quad (3.29)$$

P_{010}(t|(Q_0/D)) is the conditional probability that state will be 1 at time t given that it is 0, at times zero and (Q_0/D), for Q_0/D≥t≥0.

$$P_{010}(t|(Q_0/D)) = \frac{P_{01}(t)P_{10}(\frac{Q_0}{D}-t)}{P_{00}(\frac{Q_0}{D})} \quad (3.30)$$

As in the calculation of expected holding cost of region R_2, in the calculation of expected holding cost of the region R_{3a}, expected numbers of occurrence of region R_{3a} both in cycle type (iii), and cycle type (iv) are calculated. R_{3b} region is observed only in cycle type (iv), and the number of occurrence of the region is equal to one.

Expected Number of Occurrence of R_{3a} Region:

$$N_{13a} = \sum_{N=1}^{\infty} P_1 P_2^N P_3^{N-1}(N-1)(1-P_3) + \sum_{N=1}^{\infty} P_1 P_2^N P_3^N N(1-P_2) \tag{3.31}$$

$$N_{13a} = \frac{P_1(1-P_3)P_2}{(1-P_2P_3)^2} - \frac{P_1(1-P_3)P_2}{(1-P_2P_3)} + \frac{P_1P_3(1-P_2)P_2}{(1-P_2P_3)^2} \tag{3.32}$$

$$N_{13a} = \frac{P_1P_2P_3}{(1-P_2P_3)} \tag{3.33}$$

The probability of occurrence of R_{3b} region,

$$N_{13b} = \sum_{N=1}^{\infty} P_1 P_2^N P_3^{N-1}(1-P_3) \tag{3.34}$$

$$N_{13b} = \frac{P_1P_2(1-P_3)}{(1-P_2P_3)} \tag{3.35}$$

$$HC_{13} = N_{13a}HC_{13a} + N_{13b}HC_{13b} \tag{3.36}$$

$$HC_{13} = \frac{P_1P_2P_3}{(1-P_2P_3)}HC_{13a} + \frac{P_1P_2(1-P_3)}{(1-P_2P_3)}HC_{13b} \tag{3.37}$$

$$
\begin{aligned}
HC_{13} = & \frac{P_1P_2}{(1-P_2P_3)}(\int_0^{Q_0/D} h_0(S_0 - Dt)dt - \int_0^{Q_0/D}(h_0 - h_1) \\
& (S_0 - Dt)\frac{\lambda_1(1-e^{-(\lambda_1+\lambda_0)t})}{(\lambda_1+\lambda_0)}dt) \\
= & \frac{P_1P_2}{(1-P_2P_3)}(\int_0^{Q_0/D}(h_0S_0 - Dth_0)dt - \int_0^{Q_0/D}(h_0S_0 \\
& -h_1S_0 - h_0Dt + h_1Dt)\frac{\lambda_1(1-e^{-(\lambda_1+\lambda_0)t})}{(\lambda_1+\lambda_0)}dt) \\
= & \frac{P_1P_2}{(1-P_2P_3)}[\frac{h_0S_0Q_0}{D} - \frac{Dh_0Q_0^2}{2D^2} \\
& -\frac{1}{(\lambda_1+\lambda_0)}(\int_0^{Q_0/D}(\lambda_1h_0S_0 - \lambda_1h_1S_0 - \lambda_1h_0Dt + \lambda_1h_1Dt \\
& -\lambda_1e^{-(\lambda_1+\lambda_0)t}h_0S_0 + \lambda_1e^{-(\lambda_1+\lambda_0)t}h_1S_0 + \lambda_1e^{-(\lambda_1+\lambda_0)t}h_0Dt \\
& -\lambda_1e^{-(\lambda_1+\lambda_0)t}h_1Dt)dt)] \\
= & \frac{P_1P_2}{(1-P_2P_3)}[\frac{Q_0^2h_0+2h_0Q_0s}{2D} - \frac{1}{(\lambda_1+\lambda_0)} \\
& (\frac{\lambda_1h_0S_0Q_0}{D} - \frac{\lambda_1h_1S_0Q_0}{D} - \frac{\lambda_1h_0Q_0^2D}{2D^2} \\
& +\frac{\lambda_1h_1Q_0^2D}{2D^2} - (\frac{\lambda_1e^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}h_0S_0}{-(\lambda_1+\lambda_0)} + (\frac{\lambda_1h_0S_0}{(\lambda_1+\lambda_0)})) \\
& +(\frac{\lambda_1e^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}h_1S_0}{-(\lambda_1+\lambda_0)} + (\frac{\lambda_1h_1S_0}{(\lambda_1+\lambda_0)})) + \lambda_1h_0D(\frac{e^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}}{(\lambda_1+\lambda_0)^2} \\
& (\frac{-(\lambda_1+\lambda_0)Q_0}{D} - 1) + \frac{1}{(\lambda_1+\lambda_0)^2}) - h_1D\lambda_1(\frac{e^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}}{(\lambda_1+\lambda_0)^2} \\
& (\frac{-(\lambda_1+\lambda_0)Q_0}{D} - 1) + \frac{1}{(\lambda_1+\lambda_0)^2})] \\
= & \frac{P_1P_2}{(1-P_2P_3)}(\frac{Q_0^2h_0+2h_0Q_0s}{2D} - \frac{1}{(\lambda_1+\lambda_0)} \\
& (\frac{\lambda_1h_0Q_0^2+Q_0s\lambda_1h_0-\lambda_1h_1Q_0^2-\lambda_1h_1Q_0s}{D} + \frac{Q_0^2\lambda_1(h_1-h_0)}{2D} \\
& +\frac{\lambda_1Q_0e^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}h_0}{(\lambda_1+\lambda_0)} + \frac{\lambda_1se^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}h_0}{(\lambda_1+\lambda_0)} \\
& -\frac{\lambda_1h_0Q_0}{(\lambda_1+\lambda_0)} - \frac{\lambda_1h_0s}{(\lambda_1+\lambda_0)} - \frac{\lambda_1Q_0e^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}h_1}{(\lambda_1+\lambda_0)} \\
& -\frac{\lambda_1se^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}h_1}{(\lambda_1+\lambda_0)} + \frac{\lambda_1h_1Q_0}{(\lambda_1+\lambda_0)} + \frac{\lambda_1h_1s}{(\lambda_1+\lambda_0)} \\
& -\frac{\lambda_1Q_0e^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}h_0}{(\lambda_1+\lambda_0)} - \frac{\lambda_1De^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}h_0}{(\lambda_1+\lambda_0)^2} + \frac{\lambda_1h_0D}{(\lambda_1+\lambda_0)^2} \\
& +\frac{\lambda_1Q_0e^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}h_1}{(\lambda_1+\lambda_0)} + \frac{\lambda_1De^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}h_1}{(\lambda_1+\lambda_0)^2} - \frac{\lambda_1h_1D}{(\lambda_1+\lambda_0)^2})) \\
= & \frac{P_1P_2}{(1-P_2P_3)}(\frac{Q_0^2h_0+2h_0Q_0s}{2D} - \frac{1}{(\lambda_1+\lambda_0)} \\
& (\frac{\lambda_1 2h_0Q_0^2+2Q_0\lambda_1sh_0-2\lambda_1h_1Q_0^2-2\lambda_1h_1Q_0s}{2D} \\
& +\frac{\lambda_1h_1Q_0^2-Q_0^2\lambda_1h_0}{2D} + \frac{\lambda_1se^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}(h_0-h_1)}{(\lambda_1+\lambda_0)} \\
& +\frac{\lambda_1(h_1-h_0)Q_0}{(\lambda_1+\lambda_0)} + \frac{\lambda_1(h_1-h_0)s}{(\lambda_1+\lambda_0)} \\
& +\frac{\lambda_1De^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}(h_1-h_0)}{(\lambda_1+\lambda_0)^2} + \frac{\lambda_1D(h_0-h_1)}{(\lambda_1+\lambda_0)^2})) \\
= & \frac{P_1P_2}{(1-P_2P_3)}(\frac{Q_0^2h_0\lambda_1+\lambda_0Q_0^2h_0+2h_0Q_0s\lambda_1}{2D(\lambda_1+\lambda_0)} \\
& +\frac{2\lambda_0Q_0sh_0-2\lambda_1Q_0^2h_0-2\lambda_1Q_0sh_0+2\lambda_1Q_0^2h_1}{2D(\lambda_1+\lambda_0)} \\
& +\frac{2\lambda_1Q_0sh_1-\lambda_1Q_0^2h_1+\lambda_1Q_0^2h_0}{2D(\lambda_1+\lambda_0)} - \frac{s\lambda_1e^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}(h_0-h_1)}{(\lambda_1+\lambda_0)^2} \\
& -\frac{Q_0\lambda_1(h_1-h_0)}{(\lambda_1+\lambda_0)^2} - \frac{s\lambda_1(h_1-h_0)}{(\lambda_1+\lambda_0)^2} - \frac{D\lambda_1e^{-(\lambda_1+\lambda_0)\frac{Q_0}{D}}(h_1-h_0)}{(\lambda_1+\lambda_0)^3} \\
& -\frac{D\lambda_1(h_0-h_1)}{(\lambda_1+\lambda_0)^3})
\end{aligned}
\tag{3.38}
$$

$$
\begin{aligned}
HC_{13} = \; & \frac{P_1P_2}{(1-P_2P_3)}\Big[\frac{Q_0^2(\lambda_0h_0+\lambda_1h_1)}{2D(\lambda_1+\lambda_0)} \\
& +Q_0\Big(\frac{s(\lambda_0h_0+\lambda_1h_1)}{D(\lambda_1+\lambda_0)}+\frac{(h_0-h_1)\lambda_1}{(\lambda_1+\lambda_0)^2}\Big) \\
& +\frac{(h_0-h_1)\lambda_1 s}{(\lambda_1+\lambda_0)^2}+\frac{(h_1-h_0)\lambda_1 D}{(\lambda_1+\lambda_0)^3} \\
& +e^{-(\lambda_1+\lambda_0)Q_0/D}\Big(\frac{\lambda_1(h_1-h_0)s}{(\lambda_1+\lambda_0)^2}+\frac{\lambda_1D(h_0-h_1)}{(\lambda_1+\lambda_0)^3}\Big)\Big]
\end{aligned}
\tag{3.39}
$$

R_4 region is observed in the second and fourth cycle types. To calculate the expected holding cost for R_4 region, the probability of occurrence of R_4 region is included in the equation. $\frac{P_1}{(1-P_2P_3)}$ refers to the probability of occurrence of the other regions before R_4 region in case I, and $\lambda_1 e^{\frac{-\lambda_1(s-x)}{D}}$ refers to the probability that at inventory level I the state will become 1.

$$
HC_{14} \doteq \frac{P_1}{(1-P_2P_3)}\Big(h_0\int_0^s\int_0^{\frac{s-x}{D}}(s-Dt)e^{\frac{-\lambda_1(s-x)}{D}}\lambda_1 dt\frac{dx}{D}\Big) \tag{3.40}
$$

$$
\begin{aligned}
HC_{14} = \; & \frac{P_1}{(1-P_2P_3)}\frac{h_0\lambda_1}{D}e^{-\lambda_1 s/D}\int_0^s e^{\lambda_1 x/D}\Big(\frac{s(s-x)}{D}-\frac{(s-x)^2}{2D}\Big)dx \\
= \; & \frac{P_1}{(1-P_2P_3)}\frac{h_0\lambda_1}{D}e^{-\lambda_1 s/D}\int_0^s e^{\lambda_1 x/D}\Big(\frac{2s^2-2sx-s^2+2sx-x^2}{2D}\Big)dx \\
= \; & \frac{P_1}{(1-P_2P_3)}\frac{h_0\lambda_1}{D}e^{-\lambda_1 s/D}\int_0^s e^{\lambda_1 x/D}\Big(\frac{s^2-x^2}{2D}\Big)dx \\
= \; & \frac{P_1}{(1-P_2P_3)}\frac{h_0\lambda_1}{D}e^{-\lambda_1 s/D}\Big(\frac{e^{\lambda_1 s/D}s^2}{2\lambda_1}-\frac{s^2}{2\lambda_1} \\
& -\int_0^s\frac{x^2e^{\lambda_1 x/D}}{2D}dx\Big) \\
= \; & \frac{P_1}{(1-P_2P_3)}\frac{h_0\lambda_1}{D}e^{-\lambda_1 s/D}\Big(\frac{e^{\lambda_1 s/D}s^2-s^2}{2\lambda_1}-\frac{1}{2D} \\
& \Big(e^{\lambda_1 s/D}\sum_{r=0}^2\frac{(-1)^r 2!s^{2-r}}{(2-r)!(\frac{\lambda_1}{D})^{r+1}}-\sum_{r=0}^2\frac{(-1)^r 2!0^{2-r}}{(2-r)!(\frac{\lambda_1}{D})^{r+1}}\Big)\Big) \\
= \; & \frac{P_1}{(1-P_2P_3)}\frac{h_0\lambda_1}{D}e^{-\lambda_1 s/D}\Big(\frac{e^{\lambda_1 s/D}s^2-s^2}{2\lambda_1}-\frac{1}{2D} \\
& \Big(e^{\lambda_1 s/D}\Big(\frac{s^2D}{\lambda_1}-\frac{2sD^2}{\lambda_1^2}+\frac{2D^3}{\lambda_1^3}\Big)-\frac{2!D^3}{\lambda_1^3}\Big) \\
= \; & \frac{P_1}{(1-P_2P_3)}\Big(-\frac{s^2h_0e^{-\lambda_1 s/D}}{2D}+\frac{h_0 s}{\lambda_1}-\frac{Dh_0}{\lambda_1^2}+\frac{Dh_0e^{-\lambda_1 s/D}}{\lambda_1^2}\Big)
\end{aligned}
\tag{3.41}
$$

$$
HC_{14} = \frac{P_1}{(1-P_2P_3)}\Big[\frac{sh_0}{\lambda_1}-\frac{Dh_0}{\lambda_1^2}+e^{\frac{-\lambda_1 s}{D}}\Big(\frac{Dh_0}{\lambda_1^2}-\frac{s^2h_0}{2D}\Big)\Big] \tag{3.42}
$$

The total expected holding cost is equal to:

$$
\begin{aligned}
HC_1 = \; & \frac{Q_1^2(\lambda_0 h_0+\lambda_1 h_1)}{2D(\lambda_1+\lambda_0)} + Q_1\left(\frac{(\lambda_0 h_0+\lambda_1 h_1)s}{D(\lambda_1+\lambda_0)} + \frac{\lambda_0(h_1-h_0)}{(\lambda_1+\lambda_0)^2}\right) \\
& + \frac{s\lambda_0(h_1-h_0)}{(\lambda_1+\lambda_0)^2} + \frac{\lambda_0 D(h_0-h_1)}{(\lambda_1+\lambda_0)^3} \\
& + e^{\frac{-(\lambda_1+\lambda_0)Q_1}{D}}\left(\frac{s\lambda_0 h_0-\lambda_0 h_1 s}{(\lambda_1+\lambda_0)^2} + \frac{D\lambda_0 h_1 - D h_0 \lambda_0}{(\lambda_1+\lambda_0)^3}\right) \\
& + \frac{P_1 P_2}{(1-P_2 P_3)}\left[\frac{Q_0^2(\lambda_0 h_0+\lambda_1 h_1)}{2D(\lambda_1+\lambda_0)}\right. \\
& + Q_0\left(\frac{s(\lambda_0 h_0+\lambda_1 h_1)}{D(\lambda_1+\lambda_0)} + \frac{(h_0-h_1)\lambda_1}{(\lambda_1+\lambda_0)^2}\right) \\
& + \frac{(h_0-h_1)\lambda_1 s}{(\lambda_1+\lambda_0)^2} + \frac{(h_1-h_0)\lambda_1 D}{(\lambda_1+\lambda_0)^3} \\
& \left. + e^{-(\lambda_1+\lambda_0)Q_0/D}\left(\frac{\lambda_1(h_1-h_0)s}{(\lambda_1+\lambda_0)^2} + \frac{\lambda_1 D(h_0-h_1)}{(\lambda_1+\lambda_0)^3}\right)\right] \\
& + \frac{P_1}{(1-P_2 P_3)}\left[\frac{sh_0}{\lambda_1} - \frac{Dh_0}{\lambda_1^2} + e^{\frac{-\lambda_1 s}{D}}\left(\frac{Dh_0}{\lambda_1^2}\right)\right]
\end{aligned} \tag{3.43}
$$

Expected Purchasing Cost Per Cycle

Depending on the number of replenishments, ordering and purchasing costs are calculated for all of the four cycle types. We can express the expected purchasing cost as

$$PC_1 = \; PC_{11} + PC_{12} + PC_{13} + PC_{14} \tag{3.44}$$

where PC_{ik} is expected purchasing cost of cycle type k in case i,(i=1,2,3, and k=1,2,3,4)

In cycle type (i), which covers only R_{1a} region (when the inventory level is equal to s, the state is 1), only one order is made, and the order occurs as soon as inventory level reaches s. In the equation (1-P_1) is the probability of the occurrence of the cycle type, and Q_1 is the order amount.

$$PC_{11} = \; (1-P_1)c_1 Q_1 \tag{3.45}$$

In cycle type (ii), which covers both R_{1b} region and R_4 region, again only one order is made. An order occurs when the inventory level is equal to I which can take value between zero and s. In this cycle type, (S_1-I) units of order is made.

$$
\begin{aligned}
PC_{12} = & \; c_1 P_1 \int_0^s \lambda_1 e^{-\lambda_1 \frac{s-x}{D}} (S_1 - x) \frac{dx}{D} \\
= & \; \frac{P_1 \lambda_1 e^{\frac{-\lambda_1 s}{D}}}{D} \Big(\frac{S_1 c_1 D (e^{\frac{\lambda_1 s}{D}} - 1)}{\lambda_1} \\
& - c_1 \Big(\frac{e^{\frac{\lambda_1 s}{D}} D^2}{\lambda_1^2} \Big(\frac{\lambda_1 s}{D} - 1 \Big) + \frac{D^2}{\lambda_1^2} \Big) \Big) \\
= & \; P_1 S_1 c_1 - e^{\frac{-\lambda_1 s}{D}} S_1 c_1 P_1 \\
& - c_1 s P_1 + \frac{D c_1 P_1}{\lambda_1} - \frac{D c_1 P_1 e^{\frac{-\lambda_1 s}{D}}}{\lambda_1} \\
= & \; P_1 S_1 c_1 (1 - P_2) - c_1 s P_1 + \frac{c_1 D P_1 (1 - P_2)}{\lambda_1}
\end{aligned}
\tag{3.46}
$$

In cycle type (iii). R_{1b}, R_2, R_{3a}, and R_{3b} regions occur. So, in this cycle type number of orders is greater than one, and unlike in cycle type (i), and cycle type (ii), in cycle type (iii). some of the orders are made when the state is 0. In the equation, N refers to the number of state 0 replenishments in which S_0 units of inventory is ordered. In this cycle type in the last replenishment which occurs when the state is 1, Q_1 units of inventory is ordered.

$$
\begin{aligned}
PC_{13} = & \; c_0 S_0 \sum_{N=1}^{\infty} N (P_1 P_2^N P_3^{N-1} (1 - P_3)) \\
& + c_1 (S_1 - s) \sum_{N=1}^{\infty} P_1 P_2^N P_3^{N-1} (1 - P_3) \\
= & \; \frac{S_0 c_0 P_1 (1 - P_3) P_2}{(1 - P_2 P_3)^2} + \frac{Q_1 c_1 P_1 (1 - P_3) P_2}{(1 - P_2 P_3)} \\
= & \; \frac{P_1 P_2 (1 - P_3)}{(1 - P_2 P_3)} \Big(S_0 c_0 + \frac{Q_1 c_1}{(1 - P_2 P_3)} \Big)
\end{aligned}
\tag{3.47}
$$

Cycle type (iv) covers R_{1b}, R_2, R_{3a}, and R_4 regions. Like in cycle type (iii). in cycle type (iv), replenishments when the state is 0 occur. However, in cycle type (iv) amount of the last order, (S_1-I), is greater than amount of the last order in cycle type (iii), (S_1-s). Amount of order when the state is 0 is again S_0.

$$
\begin{aligned}
PC_{14} = & \; c_0 S_0 \sum_{N=1}^{\infty} P_1 P_2^N P_3^N (1 - P_2) N \\
& + c_1 \int_0^s \sum_{N=1}^{\infty} P_1 P_2^N P_3^N \lambda_1 e^{-\lambda_1 (\frac{s-x}{D})} (S_1 - x) \frac{dx}{D}
\end{aligned}
\tag{3.48}
$$

$$
\begin{aligned}
PC_{14} = &\ (\tfrac{P_1P_2P_3(1-P_2)}{(1-P_2P_3)^2})c_0S_0 + \tfrac{P_1P_2P_3}{(1-P_2P_3)}\tfrac{\lambda_1 e^{-\lambda_1 \frac{s}{D}}}{D} \\
&(\int_0^s e^{\lambda_1 \frac{x}{D}}(c_1S_1 - c_1x)dx) \\
= &\ (\tfrac{P_1P_2P_3(1-P_2)}{(1-P_2P_3)^2})(c_0S_0) + \tfrac{P_1P_2P_3}{(1-P_2P_3)}\tfrac{\lambda_1 e^{-\lambda_1 \frac{s}{D}}}{D} \\
&(c_1S_1(\tfrac{De^{\lambda_1 \frac{s}{D}}}{\lambda_1} - \tfrac{D}{\lambda_1}) - c_1(\tfrac{D^2e^{\lambda_1 \frac{s}{D}}}{\lambda_1^2}(\tfrac{\lambda_1 s}{D} - 1) + \tfrac{D^2}{\lambda_1^2})) \\
= &\ (\tfrac{P_1P_2P_3(1-P_2)}{(1-P_2P_3)^2})(c_0S_0) + \tfrac{P_1P_2P_3}{(1-P_2P_3)}(c_1S_1 - c_1S_1e^{-\lambda_1 \frac{s}{D}} \\
&-c_1s + \tfrac{c_1D}{\lambda_1} - \tfrac{c_1De^{-\lambda_1 \frac{s}{D}}}{\lambda_1}) \\
= &\ (\tfrac{P_1P_2P_3(1-P_2)}{(1-P_2P_3)^2})(c_0S_0) + \tfrac{P_1P_2(1-P_2)P_3c_1S_1}{(1-P_2P_3)} \\
&-\tfrac{P_1P_2P_3c_1s}{(1-P_2P_3)} + \tfrac{P_1P_2(1-P_2)P_3c_1D}{(1-P_2P_3)\lambda_1}
\end{aligned} \tag{3.49}
$$

$$
\begin{aligned}
PC_{14} = &\ \tfrac{P_1P_2P_3(1-P_2)}{(1-P_2P_3)^2}(Q_0 + s)c_0 \\
&+\tfrac{P_1P_2P_3S_1c_1(1-P_2)}{(1-P_2P_3)} - \tfrac{P_1P_2P_3c_1s}{(1-P_2P_3)} \\
&+\tfrac{P_1P_2P_3}{\lambda_1(1-P_2P_3)}Dc_1(1 - P_2)
\end{aligned} \tag{3.50}
$$

The total expected purchasing cost is equal to

$$PC_1 = c_1Q_1 + \frac{P_1(1-P_2)}{(1-P_3P_2)}\frac{c_1D}{\lambda_1} + \frac{P_1P_2}{(1-P_3P_2)}s(c_0 - c_1) + \frac{P_1P_2}{(1-P_3P_2)}Q_0c_0 \tag{3.51}$$

Expected Ordering Cost Per Cycle

The calculation of the ordering cost is very similar to the calculation of the purchasing cost. Like purchasing cost, we can express ordering cost as a summation of the expected ordering cost of each cycle types.

$$OC_1 = OC_{11} + OC_{12} + OC_{13} + OC_{14} \tag{3.52}$$

where OC_{ik} is expected ordering cost of cycle type k in case i, (i=1,2,3, and k=1,2,3,4.)

In the first cycle type, there is only one replenishment, and the replenishment occurs when the state is 1. Again (1- P_1) is the probability of occurrence of cycle type (i).

$$OC_{11} = (1 - P_1)k_1 \tag{3.53}$$

Like in the first cycle type, in the second cycle type there is only one replenishment and it occurs when the state is 1. However, in this cycle type, the probability of occurrence of the cycle is different.

$$\begin{aligned} OC_{12} = &\; k_1 P_1 \int_0^s \lambda_1 e^{-\lambda_1 \frac{s-x}{D}} \frac{dx}{D} \\ = &\; \frac{P_1 \lambda_1 e^{\frac{-\lambda_1 s}{D}}}{D} \frac{k_1 D (e^{\frac{\lambda_1 s}{D}} - 1)}{\lambda_1} \\ = &\; k_1 P_1 - e^{\frac{-\lambda_1 s}{D}} k_1 P_1 \\ OC_{12} = &\; P_1 k_1 (1 - P_2) \end{aligned} \tag{3.54}$$

In cycle type (iii), since state 0 goes on for a long duration, there are N replenishments when the state is 0. And, at the end of the cycle type there is only one replenishment when the state is 1.

$$\begin{aligned} OC_{13} = &\; \textstyle\sum_{N=1}^{\infty} (N k_0)(P_1 P_2^N P_3^{N-1}(1 - P_3)) \\ &+ \textstyle\sum_{N=1}^{\infty} k_1 (P_1 P_2^N P_3^{N-1}(1 - P_3)) \\ OC_{13} = &\; \frac{P_1(1-P_3)P_2}{(1-P_2P_3)} \left(\frac{k_0}{(1-P_2P_3)} + k_1\right) \end{aligned} \tag{3.55}$$

Like in cycle type (iii), in cycle type (iv), there are N replenishments when the state is 0, and at the end, only one replenishment occurs when the state is 1. However, the probability of the occurrence of cycle type (iv), is different from the probability of occurrence of cycle type (iii).

$$\begin{aligned} OC_{14} = &\; \textstyle\sum_{N=1}^{\infty} P_1 P_2^N P_3^N (1 - P_2) N k_0 \\ &+ \int_0^s \textstyle\sum_{N=1}^{\infty} P_1 P_2^N P_3^N \lambda_1 e^{-\lambda_1 (\frac{s-x}{D})} k_1 \frac{dx}{D} \end{aligned} \tag{3.56}$$

$$\begin{aligned} OC_{14} = &\; \left(\frac{P_1P_2P_3(1-P_2)}{(1-P_2P_3)^2}\right) k_0 + \frac{P_1P_2P_3}{(1-P_2P_3)} \frac{\lambda_1 e^{-\lambda_1 \frac{s}{D}}}{D} \\ &\left(\int_0^s e^{\lambda_1 \frac{x}{D}} k_1 dx\right) \\ = &\; \left(\frac{P_1P_2P_3(1-P_2)}{(1-P_2P_3)^2}\right) k_0 + k_1 \frac{P_1P_2P_3}{(1-P_2P_3)} \frac{\lambda_1 e^{-\lambda_1 \frac{s}{D}}}{D} \\ &\left(\frac{D e^{\lambda_1 \frac{s}{D}}}{\lambda_1} - \frac{D}{\lambda_1}\right) \\ = &\; \left(\frac{P_1P_2P_3(1-P_2)}{(1-P_2P_3)^2}\right) k_0 + \frac{P_1P_2P_3}{(1-P_2P_3)} (k_1 - k_1 e^{-\lambda_1 \frac{s}{D}}) \\ = &\; \left(\frac{P_1P_2P_3(1-P_2)}{(1-P_2P_3)^2}\right) k_0 + \frac{P_1P_2(1-P_2)P_3 k_1}{(1-P_2P_3)} \end{aligned} \tag{3.57}$$

$$OC_{14} = \frac{P_1P_2P_3(1-P_2)}{(1-P_2P_3)} \left(\frac{k_0}{(1-P_2P_3)} + k_1\right) \tag{3.58}$$

Total expected purchasing cost is equal to,

$$OC_1 = k_1 + \frac{P_1 P_2}{(1-P_2P_3)} k_0 \tag{3.59}$$

As can be seen from the equation in total $P_1P_2/(1\text{-}P_2P_3)$ times replenishments occur when the state is 0, and only one replenishment occurs when the state is 1.

Expected Cycle Time

Expected cycle time is defined as the summation of the expected cycle time for all of the regions. The length of R_{1a} region and R_{1b} region are equal to (Q_1/D) and the total probability of occurrence of these regions is equal to one. In the equation second term is the expected number of occurrences of R_2 and R_3 regions, times the length of these regions, (s/D), and (Q_0/D) respectively. And finally, the remaining terms are the probability of occurrence of R_4 region and the length of the R_4 region, (s-I)/D.

$$\begin{aligned}
CT_1 = {} & \frac{Q_1}{D} + \frac{P_1P_2}{(1-P_2P_3)}\frac{(s+Q_0)}{D} + \frac{P_1}{(1-P_2P_3)}\Big(\frac{\lambda_1 e^{\frac{-\lambda_1 s}{D}}}{D} \\
& \int_0^s \Big(\frac{se^{\frac{\lambda_1 x}{D}} - xe^{\frac{\lambda_1 x}{D}}}{D}\Big)dx\Big) \\
= {} & \frac{Q_1}{D} + \frac{P_1P_2}{(1-P_2P_3)}\frac{(s+Q_0)}{D} + \frac{P_1}{(1-P_2P_3)}\frac{\lambda_1 e^{\frac{-\lambda_1 s}{D}}}{D^2} \\
& \Big(\frac{Dse^{\frac{\lambda_1 s}{D}}}{\lambda_1} - \frac{Ds}{\lambda_1}\Big) - \frac{P_1}{(1-P_2P_3)}\frac{\lambda_1 e^{\frac{-\lambda_1 s}{D}}}{D^2} \\
& \Big(\frac{D^2 e^{\frac{\lambda_1 s}{D}}}{\lambda_1^2}\Big(\frac{\lambda_1 s}{D} - 1\Big) + \frac{D^2}{\lambda_1^2}\Big) \\
= {} & \frac{Q_1}{D} + \frac{P_1P_2}{(1-P_2P_3)}\frac{(s+Q_0)}{D} + \frac{P_1 s}{(1-P_2P_3)D} \\
& - \frac{P_1}{(1-P_2P_3)}\Big(\frac{se^{\frac{-\lambda_1 s}{D}}}{D}\Big) - \frac{P_1 s}{(1-P_2P_3)D} + \frac{P_1}{(1-P_2P_3)\lambda_1} \\
& - \frac{P_1}{(1-P_2P_3)}\frac{e^{\frac{-\lambda_1 s}{D}}}{\lambda_1} \\
= {} & \frac{Q_1}{D} + \frac{P_1P_2}{(1-P_2P_3)}\frac{(s+Q_0)}{D} - \frac{P_1P_2 s}{(1-P_2P_3)D} \\
& + \frac{P_1}{(1-P_2P_3)\lambda_1} - \frac{P_1P_2}{(1-P_2P_3)\lambda_1} \\
CT_1 = {} & \frac{Q_1}{D} + \frac{P_1P_2}{(1-P_2P_3)}\frac{Q_0}{D} + \frac{P_1(1-P_2)}{(1-P_2P_3)}\frac{1}{\lambda_1}
\end{aligned} \tag{3.60}$$

3.2.2 Cost Functions for Case II

In this section, expected holding, ordering, purchasing costs, and expected cycle time are calculated for case II.

Expected Holding Cost Per Cycle

For the calculation of the holding cost, again some regions are identified called R_{14}, R_{15}, R_2, R_4, R_5, and R_6. We can express expected holding cost for case II as follows,

$$HC_2 = HC_{21} + HC_{22} + HC_{24} + HC_{25} + HC_{26} \tag{3.61}$$

Since R_1 region is the same with R_1 region identified in case I. The expected cost equation is not rewritten here. HC_{11}=HC_{21}

Except for the probability (expected number) of occurrence, R_2 region is the same with R_2 region in case I.

$$HC_{22} = P_1 P_2 h_0 \int_0^{s/D} (s - Dt)dt \tag{3.62}$$

$$HC_{22} = P_1 P_2 h_0 \frac{s^2}{2D} \tag{3.63}$$

In case II, R_4 region occurs only in the second cycle type. Like in case I it covers (s-I) region. Until the inventory level reaches I, state 0 goes on, and at I, state 1 occurs, and (S_1-I) units inventory are ordered.

$$HC_{24} = P_1 h_0 \int_0^s \int_0^{\frac{(s-x)}{D}} \lambda_1 e^{\frac{-\lambda_1 (s-x)}{D}} (s - Dt)dt \frac{dx}{D} \tag{3.64}$$

$$\begin{aligned}
HC_{24} =& \ P_1 \int_0^s ((\lambda_1 e^{-\lambda_1(\frac{s-x}{D})}) \frac{h_0}{D} \int_0^{\frac{(s-x)}{D}} (s - Dt)dt)dx \\
=& \ P_1 \lambda_1 e^{-\lambda_1(\frac{s}{D})} \frac{h_0}{D} \int_0^s e^{\lambda_1(\frac{x}{D})} (\frac{s(s-x)}{D} - \frac{(s-x)^2}{2D})dx \\
=& \ P_1 \lambda_1 e^{-\lambda_1(\frac{s}{D})} \frac{h_0}{D} \int_0^s e^{\lambda_1(\frac{x}{D})} \frac{(s^2-x^2)}{2D} dx \\
=& \ P_1 \lambda_1 e^{-\lambda_1(\frac{s}{D})} \frac{h_0}{D} (\frac{s^2 e^{\lambda_1(\frac{s}{D})}}{2\lambda_1} - \frac{s^2}{2\lambda_1} - \frac{1}{2D} \\
& (e^{\lambda_1(\frac{s}{D})} \sum_{r=0}^{2} \frac{(-1)^r 2! s^{2-r}}{(2-r)!(\frac{\lambda_1}{D})^{r+1}} - \sum_{r=0}^{2} \frac{(-1)^r 2! 0^{2-r}}{(2-r)!(\frac{\lambda_1}{D})^{r+1}})) \\
=& \ P_1 \lambda_1 e^{-\lambda_1(\frac{s}{D})} \frac{h_0}{D} (\frac{s^2 e^{\lambda_1(\frac{s}{D})} - s^2}{2\lambda_1} - \frac{1}{2D} \\
& (e^{\lambda_1(\frac{s}{D})} (\frac{s^2 D}{\lambda_1} - \frac{2D^2 s}{\lambda_1^2} + \frac{2D^3}{\lambda_1^3}) - \frac{2D^3}{\lambda_1^3})) \\
=& \ P_1 \lambda_1 e^{-\lambda_1(\frac{s}{D})} \frac{h_0}{D} (\frac{s^2 e^{\lambda_1(\frac{s}{D})} - s^2}{2\lambda_1} - \frac{1}{2D} \frac{e^{\lambda_1(\frac{s}{D})} s^2 D}{\lambda_1} \\
& + \frac{e^{\lambda_1(\frac{s}{D})} sD}{\lambda_1^2} - \frac{e^{\lambda_1(\frac{s}{D})} D^2}{\lambda_1^3} + \frac{D^2}{\lambda_1^3}) \\
=& \ P_1 h_0 (\frac{s}{\lambda_1} - \frac{D}{\lambda_1} + e^{-\lambda_1(\frac{s}{D})} (\frac{D}{\lambda_1^2} - \frac{s^2}{2D}))
\end{aligned} \tag{3.65}$$

$$HC_{24} = P_1 (\frac{sh_0}{\lambda_1} - \frac{Dh_0}{\lambda_1^2} + e^{-\lambda_1 s/D} (\frac{Dh_0}{\lambda_1^2} - \frac{s^2 h_0}{2D})) \tag{3.66}$$

Since R_5 region occurs more than once, expected number of occurrence of R_5 region is also included in the calculation of the expected holding cost. In cycle type (iii), R_5 region occurs N-1 times.

Expected Number of Occurrence of R_5:

$$\begin{aligned}
N_{25} =& \ P_1 P_2 \sum_{N=1}^{\infty} (N-1) P_4^{N-1} (1 - P_4) \\
=& \ P_1 P_2 (1 - P_4) (\frac{1}{(1-P_4)^2} - \frac{1}{(1-P_4)}) \\
=& \ \frac{P_1 P_2 P_4}{(1-P_4)}
\end{aligned} \tag{3.67}$$

$$HC_{25} = N_{25} h_0 \int_0^{S_0/D} (S_0 - Dt)dt \tag{3.68}$$

$$HC_{25} = \frac{P_1 P_2 P_4}{(1-P_4)} h_0 (\frac{S_0 S_0}{D} - \frac{S_0^2 D}{2D^2}) \tag{3.69}$$

$$HC_{25} = \frac{P_1 P_2 P_4}{(1-P_4)} \frac{h_0 S_0^2}{2D} \tag{3.70}$$

R_6 region occurs only in the third cycle type, and the number of occurrence of this region in this cycle type is equal to one. The region covers the area between inventory levels S_0 and I. I is the inventory level at which state 1 occurs, and it can take values between S_0 and 0. In the equation $\sum_{N=1}^{\infty} P_1 P_2 P_4^{N-1}$ refers to the probability of occurrence of the other regions in the third cycle type.

$$\begin{aligned}
HC_{26} = & \sum_{N=1}^{\infty} P_1 P_2 P_4^{N-1} \int_0^{S_0} \int_0^{\frac{S_0-x}{D}} \lambda_1 e^{\frac{-\lambda_1(S_0-x)}{D}} h_0(S_0 - Dt)dt\frac{dx}{D} \\
= & \frac{P_1P_2}{(1-P_4)} \int_0^{S_0} \lambda_1 e^{\frac{-\lambda_1(S_0-x)}{D}} \frac{h_0}{D}\left(\frac{S_0(S_0-x)}{D} - \frac{D(S_0-x)^2}{2D^2}\right)dx \\
= & \frac{P_1P_2}{(1-P_4)} \int_0^{S_0} \lambda_1 e^{\frac{-\lambda_1(S_0-x)}{D}} \frac{h_0}{D}\left(\frac{2S_0^2-2S_0x-S_0^2+2S_0x-x^2}{2D}\right)dx \\
= & \frac{P_1P_2}{(1-P_4)}\lambda_1 e^{\frac{-\lambda_1 S_0}{D}} \frac{h_0}{2D^2} \int_0^{S_0} e^{\frac{\lambda_1 x}{D}}(S_0^2 - x^2)dx \\
= & \frac{P_1P_2}{(1-P_4)}\lambda_1 e^{\frac{-\lambda_1 S_0}{D}} \frac{h_0}{2D^2}\left(\frac{S_0^2 De^{\frac{\lambda_1 S_0}{D}}}{\lambda_1} - \frac{S_0^2 D}{\lambda_1} - \int_0^{S_0} x^2 e^{\frac{\lambda_1 x}{D}}dx\right) \\
= & \frac{P_1P_2}{(1-P_4)}\lambda_1 e^{\frac{-\lambda_1 S_0}{D}} \frac{h_0}{2D^2}\left(\frac{S_0^2 De^{\frac{\lambda_1 S_0}{D}}}{\lambda_1} - \frac{S_0^2 D}{\lambda_1}\right. \\
& \left. -\left(e^{\frac{\lambda_1 S_0}{D}} \sum_{r=0}^{2} \frac{(-1)^r 2! S_0^{2-r}}{(2-r)!(\frac{\lambda_1}{D})^{r+1}} - \sum_{r=0}^{2} \frac{(-1)^r 2! 0^{2-r}}{(2-r)!(\frac{\lambda_1}{D})^{r+1}}\right)\right) \\
= & \frac{P_1P_2}{(1-P_4)}\lambda_1 e^{\frac{-\lambda_1 S_0}{D}} \frac{h_0}{2D^2}\left(\frac{S_0^2 De^{\frac{\lambda_1 S_0}{D}}}{\lambda_1} - \frac{S_0^2 D}{\lambda_1} - \left(e^{\frac{\lambda_1 S_0}{D}}\left(\frac{S_0^2 D}{\lambda_1}\right.\right.\right. \\
& \left.\left.\left. - \frac{2S_0D^2}{\lambda_1^2} + \frac{2D^3}{\lambda_1^3}\right) - \frac{2D^3}{\lambda_1^3}\right)\right) \\
= & \frac{P_1P_2}{(1-P_4)}\lambda_1 e^{\frac{-\lambda_1 S_0}{D}} \frac{h_0}{2D^2}\left(\frac{S_0^2 De^{\frac{\lambda_1 S_0}{D}}}{\lambda_1} - \frac{S_0^2 D}{\lambda_1} - \frac{S_0^2 De^{\frac{\lambda_1 S_0}{D}}}{\lambda_1}\right. \\
& \left. + \frac{2S_0D^2 e^{\frac{\lambda_1 S_0}{D}}}{\lambda_1^2} - \frac{2D^3 e^{\frac{\lambda_1 S_0}{D}}}{\lambda_1^3} + \frac{2D^3}{\lambda_1^3}\right) \\
= & \frac{P_1P_2}{(1-P_4)}h_0\left(\frac{S_0}{\lambda_1} - \frac{e^{\frac{-\lambda_1 S_0}{D}} S_0^2}{2D} + \frac{e^{\frac{-\lambda_1 S_0}{D}} D}{\lambda_1^2} - \frac{D}{\lambda_1^2}\right)
\end{aligned} \quad (3.71)$$

$$HC_{26} = \frac{P_1P_2}{(1-P_4)}h_0\left(\frac{S_0}{\lambda_1} - \frac{D}{\lambda_1^2} + e^{-\lambda_1 S_0/D}\left(\frac{D}{\lambda_1^2} - \frac{S_0^2}{2D}\right)\right) \quad (3.72)$$

The total expected holding cost is equal to,

$$\begin{aligned}
HC_2 = & \frac{Q_1^2(\lambda_0 h_0+\lambda_1 h_1)}{2D(\lambda_1+\lambda_0)} + Q_1\left(\frac{(\lambda_0 h_0+\lambda_1 h_1)s}{D(\lambda_1+\lambda_0)} + \frac{\lambda_0(h_1-h_0)}{(\lambda_1+\lambda_0)^2}\right) \\
& + \frac{s\lambda_0(h_1-h_0)}{(\lambda_1+\lambda_0)^2} + \frac{\lambda_0 D(h_0-h_1)}{(\lambda_1+\lambda_0)^3} \\
& + e^{\frac{-(\lambda_1+\lambda_0)Q_1}{D}}\left(\frac{s\lambda_0 h_0-\lambda_0 h_1 s}{(\lambda_1+\lambda_0)^2} + \frac{D\lambda_0 h_1 - Dh_0\lambda_0}{(\lambda_1+\lambda_0)^3}\right) \\
& + P_1\left(\frac{sh_0}{\lambda_1} - \frac{Dh_0}{\lambda_1^2} + e^{-\lambda_1 s/D}\left(\frac{Dh_0}{\lambda_1^2}\right)\right) \\
& + \frac{P_1P_2}{(1-P_4)}h_0\left(\frac{S_0}{\lambda_1} - \frac{D}{\lambda_1^2} + e^{-\lambda_1 S_0/D}\left(\frac{D}{\lambda_1^2}\right)\right)
\end{aligned} \quad (3.73)$$

Expected Purchasing Costs Per Cycle

Expected purchasing costs are calculated for all of the three cycle types separately.

$$PC_2 = PC_{21} + PC_{22} + PC_{23} \quad (3.74)$$

In the first cycle type which covers only R_{1a} region, only Q_1 units of replenishment is made, when the state is 1.

$$PC_{21} = (1 - P_1)c_1Q_1 \tag{3.75}$$

In the second cycle type, again there is only one replenishment, and the replenishment occurs when the state is 1. However, in this cycle type amount of order (S_1-I) is greater than the amount of order in the first cycle type, (S_1-s).

$$\begin{aligned} PC_{22} = & P_1c_1 \int_0^s \lambda_1 e^{-\lambda_1 \frac{s-x}{D}}(S_1 - x)\frac{dx}{D} \\ = & \frac{P_1\lambda_1 e^{\frac{-\lambda_1 s}{D}}}{D}(\frac{S_1c_1D(e^{\frac{\lambda_1 s}{D}}-1)}{\lambda_1} \\ & -c_1(\frac{e^{\frac{\lambda_1 s}{D}}D^2}{\lambda_1^2}(\frac{\lambda_1 s}{D} - 1) + \frac{D^2}{\lambda_1^2})) \\ = & P_1S_1c_1 - e^{\frac{-\lambda_1 s}{D}}S_1c_1P_1 \\ & -c_1sP_1 + \frac{Dc_1P_1}{\lambda_1} - \frac{Dc_1P_1e^{\frac{-\lambda_1 s}{D}}}{\lambda_1} \\ = & P_1S_1c_1(1 - P_2) - c_1sP_1 + \frac{c_1DP_1(1-P_2)}{\lambda_1} \end{aligned} \tag{3.76}$$

In the third cycle type, number of order is greater than one, and the number of replenishments when the state is 0 is also greater than zero. There are N times S_0 units of order in state 0, and one (S_1-I) units of order in state 1.

$$\begin{aligned} PC_{23} = & \textstyle\sum_{N=1}^{\infty} NP_1P_2P_4^{N-1}(1 - P_4)S_0c_0 + \sum_{N=1}^{\infty} P_1P_2P_4^{N-1} \\ & \int_0^{S_0} \lambda_1 e^{-\lambda_1(\frac{S_0-x}{D})}c_1(S_1 - x)\frac{dx}{D} \\ = & \frac{P_1P_2}{(1-P_4)}(S_0c_0) + \frac{P_1P_2}{(1-P_4)}[\frac{\lambda_1 e^{-\lambda_1(\frac{S_0}{D})}c_1}{D} \\ & (S_1(\frac{e^{\lambda_1(\frac{S_0}{D})}D}{\lambda_1} - \frac{D}{\lambda_1}) - (\frac{e^{\lambda_1(\frac{S_0}{D})}D^2}{\lambda_1^2}(\frac{\lambda_1 S_0}{D} - 1) + \frac{D^2}{\lambda_1^2}))] \\ = & \frac{P_1P_2}{(1-P_4)}(S_0c_0) + \frac{P_1P_2}{(1-P_4)}(S_1c_1) - \frac{P_1P_2S_1c_1e^{-\lambda_1(\frac{S_0}{D})}}{(1-P_4)} \\ & -\frac{P_1P_2S_0c_1}{(1-P_4)} + \frac{P_1P_2Dc_1}{\lambda_1(1-P_4)} - \frac{P_1P_2Dc_1e^{-\lambda_1(\frac{S_0}{D})}}{\lambda_1(1-P_4)} \\ = & \frac{P_1P_2}{(1-P_4)}(S_0(c_0 - c_1)) + P_1P_2c_1S_1 + \frac{P_1P_2c_1D}{\lambda_1} \end{aligned} \tag{3.77}$$

Total expected purchasing cost is equal to,

$$PC_2 = c_1Q_1 + \frac{P_1P_2}{(1-P_4)}[S_0(c_0 - c_1)] + \frac{P_1Dc_1}{\lambda_L} \tag{3.78}$$

Expected Ordering Cost Per Cycle

Ordering cost is also calculated for all of the cycle types separately. In the calculation only number of orders in each cycle types. and the probability of occurrence of the cycle types are taken into account.

$$OC_2 = OC_{21} + OC_{22} + OC_{23} \tag{3.79}$$

$$OC_{21} = (1 - P_1)k_1 \tag{3.80}$$

$$\begin{aligned} OC_{22} &= P_1 k_1 \int_0^s \lambda_1 e^{-\lambda_1 \frac{s-x}{D}} \frac{dx}{D} \\ &= \frac{P_1 \lambda_1 e^{\frac{-\lambda_1 s}{D}}}{D} \frac{k_1 D(e^{\frac{\lambda_1 s}{D}} - 1)}{\lambda_1} \\ &= k_1 P_1 - e^{\frac{-\lambda_1 s}{D}} k_1 P_1 \\ &= P_1 k_1 (1 - P_2) \end{aligned} \tag{3.81}$$

$$\begin{aligned} OC_{23} &= P_1 P_2 \textstyle\sum_{N=1}^{\infty} P_4^N (1 - P_4) N k_0 \\ &\quad + P_1 P_2 \int_0^{S_0} \textstyle\sum_{N=1}^{\infty} P_4^{N-1} (k_1) \lambda_1 e^{-\lambda_1 (\frac{S_0 - x}{D})} \frac{dx}{D} \\ &= \frac{P_1 P_2}{(1-P_4)} k_0 + \frac{P_1 P_2}{(1-P_4)} \frac{\lambda_1 e^{-\lambda_1 (\frac{S_0}{D})} k_1}{D} \int_0^{S_0} e^{\lambda_1 (\frac{x}{D})} dx \\ &= \frac{P_1 P_2}{(1-P_4)} k_0 + \frac{P_1 P_2}{(1-P_4)} \frac{\lambda_1 e^{-\lambda_1 (\frac{S_0}{D})} k_1}{D} \\ &\quad (\frac{e^{\lambda_1 (\frac{S_0}{D})} D}{\lambda_1} - \frac{D}{\lambda_1}) \\ &= \frac{P_1 P_2}{(1-P_4)} k_0 + \frac{P_1 P_2 k_1}{(1-P_4)} - \frac{P_1 P_2 e^{-\lambda_1 (\frac{S_0}{D})} k_1}{(1-P_4)} \\ &= \frac{P_1 P_2}{(1-P_4)} k_0 + P_1 P_2 k_1 \end{aligned} \tag{3.82}$$

The total expected ordering cost is equal to,

$$OC_2 = k_1 + \frac{P_1 P_2}{(1-P_4)} k_0 \tag{3.83}$$

As can be seen from the equation there is only one replenishment when the state is 1, but there are $P_1 P_2/(1\text{-}P_4)$ times replenishments when the state is 0. OC_2 can be identified as ordering cost in state 1 plus cost of order in state 0.

Expected Cycle Time

Expected cycle time is calculated as the summation of the expected length of the R_{1a}, and R_{1b} regions and the probability of occurrence of R_{1b} region times expected mean time until state 1 occurs.

$$CT_2 = \frac{Q_1}{D} + \frac{P_1}{\lambda_1} \tag{3.84}$$

3.3 Expected Constant Holding Cost

When it is assumed that the holding cost is constant, and the duration of the state is very short, our study will be similar to Moinzadeh[15](1997). However, in this part we assume only constant holding cost. State 1 still has some duration, it is not instantaneous. When the holding cost is constant, conditional probabilities P_{11}, and P_{01} used for occurrences of state 1 are not used.

3.3.1 Case I

If it is assumed that holding cost is constant and equal to H, holding cost for R_1 region is,

$$HC_{11} = \int_0^{Q_1/D} H(S_1 - Dt)dt \tag{3.85}$$

$$HC_{11} = \frac{Q_1^2 H + 2sQ_1 H}{2D} \tag{3.86}$$

Since in R_2 region state is always 0, when it is assumed that the holding cost is constant in R_2 region, there will be no change in the expected holding cost equation for R_2 region.

$$HC_{12} = \frac{P_1 P_2}{(1-P_2P_3)} \int_0^{s/D} H(s - Dt)dt \tag{3.87}$$

$$HC_{12} = \frac{P_1 P_2 s^2 H}{2D(1-P_2P_3)} \tag{3.88}$$

When the holding cost does not change in R_3 region, expected holding cost equation is,

$$HC_{13} = \frac{P_1P_2}{(1-P_2P_3)} \int_0^{Q_0/D} H(S_0 - Dt)dt \tag{3.89}$$

$$HC_{13} = \left(\frac{Q_0^2H + 2HsQ_0}{2D}\right)\frac{P_1P_2}{(1-P_2P_3)} \tag{3.90}$$

If the holding cost does not change in R_4 region, like in R_2 region there will not be major changes in the expected holding cost equation.

$$HC_{14} = \frac{P_1}{(1-P_2P_3)} \int_0^s \int_0^{\frac{s-x}{D}} (s - Dt)He^{\frac{-\lambda_1(s-x)}{D}} \lambda_1 dt \frac{dx}{D} \tag{3.91}$$

$$HC_{14} = \frac{P_1}{(1-P_2P_3)}\left(\frac{sH}{\lambda_1} - \frac{DH}{\lambda_1^2} + e^{\frac{-\lambda_1 s}{D}}\left(\frac{DH}{\lambda_1^2} - \frac{s^2H}{2D}\right)\right) \tag{3.92}$$

3.3.2 Case II

When it is assumed that holding cost is constant, again R_1 region in case II is the same with R_1 region in case I.

When the holding cost does not change in R_2 region, the equation is almost the same with the equation when the holding cost is random.

$$\begin{aligned} HC_{22} &= P_1P_2\int_0^{s/D} H(s - Dt)dt \\ HC_{22} &= \frac{P_1P_2Hs^2}{2D} \end{aligned} \tag{3.93}$$

If the holding cost is constant in R_4 region,

$$HC_{24} = P_1 \int_0^s \int_0^{\frac{(s-x)}{D}} \lambda_1 e^{\frac{-\lambda_1(s-x)}{D}} H(s - Dt)dt \frac{dx}{D} \tag{3.94}$$

$$HC_{24} = P_1\left(\frac{sH}{\lambda_1^2} - \frac{DH}{\lambda_1^2} + e^{-\lambda_1 s/D}\left(\frac{DH}{\lambda_1^2} - \frac{s^2H}{2D}\right)\right) \tag{3.95}$$

When the holding cost is constant in R_5 region, there will not be major change in the cost equation, since the state is always 0 in R_5 region.

$$HC_{25} = \frac{P_1P_2P_4}{(1-P_4)}\int_0^{S_0/D} H(S_0 - Dt)dt \tag{3.96}$$

$$HC_{25} = \frac{P_1P_2P_4}{(1-P_4)}\frac{HS_0^2}{2D} \tag{3.97}$$

When holding cost does not change in the R_6 region,

$$HC_{26} = \frac{P_1P_2}{(1-P_4)}\int_0^{S_0}\int_0^{\frac{S_0-x}{D}} \lambda_1 e^{\frac{-\lambda_1(S_0-x)}{D}} H(S_0 - Dt)dt\frac{dx}{D} \tag{3.98}$$

$$HC_{26} = \frac{P_1P_2}{(1-P_4)}H\left(\frac{S_0}{\lambda_1} - \frac{D}{\lambda_1^2} + e^{-\lambda_1 S_0/D}\left(\frac{D}{\lambda_1^2} - \frac{S_0^2}{2D}\right)\right) \tag{3.99}$$

3.4 Convexity of the Total Cost Rate

The expected total cost per unit time is identified as a function of expected holding cost, ordering cost, purchasing cost, and cycle length.

$$TC_i = \frac{HC_i+PC_i+OC_i}{CT_i} \tag{3.100}$$

where TC_i is expected total cost in case i per unit time (i=1,2,3).

In this section, convexity of total cost per unit time in Q_1, and Q_0, is proved. Since the total cost function is very complex, for simplicity it is assumed that Q_1 and Q_0 are very large in some cases.

Proposition1: a) For ($S_1 \geq S_0 \geq$ s) and ($S_0 \geq S_1 \geq$ s) total cost function per unit time is convex in Q_1 and Q_0 for large Q_1 and Q_0.

b) For ($S_1 \geq$ s $\geq S_0$), total cost function per unit time is convex in Q_1 for large Q_1.

c) For ($S_1 \geq$ s $\geq S_0$), total cost function per unit time is convex in S_0 when $c_0 \geq c_1$.

Proof of Proposition 1: *a)*

$$P_1 = \frac{\lambda_0(1-e^{\frac{-(\lambda_1+\lambda_0)Q_1}{D}})}{(\lambda_1+\lambda_0)} \tag{3.101}$$

$$P_3 = \frac{\lambda_0+\lambda_1 e^{(\frac{-(\lambda_1+\lambda_0)Q_0}{D})}}{(\lambda_1+\lambda_0)} \tag{3.102}$$

When Q_1 and Q_0 are high, $P_1 = P_3$

$$P_3 = \frac{\lambda_0}{(\lambda_0+\lambda_1)} \tag{3.103}$$

For convexity, if A is equal to expected total cost, and B is equal to expected cycle time,

The first derivative with respect to Q_1 gives

$$\frac{A'B-B'A}{B^2} = 0 \tag{3.104}$$

The second derivative with respect to Q_1 is

$$\frac{A''B-B''A}{B^4} \tag{3.105}$$

Since for Q_1 and Q_0 , second derivative of B with respect to Q_1 is equal to zero. For convexity, the condition that second derivative of A with respect to Q_1 is greater than zero is required.

$$A = \quad TC_1 = HC_1 + PC_1 + OC_1 \tag{3.106}$$

$$B \quad = \quad CT_1 \tag{3.107}$$

To test convexity of Q_1, we check the second derivative of A with respect to Q_1, when P_1 is replaced by P_3, since both are identical for large Q_1 and Q_0.

$$\begin{aligned} A' = \quad & 2Q_1\left(\frac{\lambda_0 h_0+\lambda_1 h_1}{2D(\lambda_1+\lambda_0)}\right) + \frac{s(\lambda_0 h_0+\lambda_1 h_1}{D(\lambda_1+\lambda_0)} + \frac{\lambda_0(h_1-h_0)}{(\lambda_1+\lambda_0)^2} \\ & + e^{\frac{-(\lambda_1+\lambda_0)Q_1}{D}}\left(\frac{s\lambda_0(h_1-h_0)}{D(\lambda_1+\lambda_0)} + \frac{\lambda_0(h_0-h_1)}{(\lambda_1+\lambda_0)^2}\right) \\ & + \frac{c_1}{(1-P_2P_3)} + \frac{c_1 P_3^2}{(1-P_2P_3)} - P_2P_3c_1 \end{aligned} \tag{3.108}$$

$$\begin{aligned} A'' = \quad & \frac{\lambda_0 h_0+\lambda_1 h_1}{D(\lambda_1+\lambda_0)} - \frac{(\lambda_1+\lambda_0)}{D} e^{\frac{-(\lambda_1+\lambda_0)Q_1}{D}} \left(\frac{s\lambda_0(h_1-h_0)}{D(\lambda_1+\lambda_0)} + \frac{\lambda_0(h_0-h_1)}{(\lambda_1+\lambda_0)^2}\right) \\ A'' = \quad & \frac{\lambda_0 h_0+\lambda_1 h_1}{D(\lambda_1+\lambda_0)} - e^{\frac{-(\lambda_1+\lambda_0)Q_1}{D}} \left(\frac{s\lambda_0(h_1-h_0)}{D^2} + \frac{\lambda_0(h_0-h_1)}{D(\lambda_1+\lambda_0)}\right. \end{aligned} \tag{3.109}$$

For large Q_1,

$$e^{-\frac{(\lambda_1+\lambda_0)Q_1}{D}} = 0 \tag{3.110}$$

Hence,

$$A'' = \frac{\lambda_0 h_0+\lambda_1 h_1}{D(\lambda_1+\lambda_0)} \tag{3.111}$$

Since second derivative of A is always positive, TC is convex in Q_1.

To test the convexity of Q_0, second derivative of A is taken with respect to Q_0.

$$\begin{aligned} A' = & \frac{P_2P_3}{(1-P_2P_3)}\Big(\frac{2Q_0(\lambda_0h_0+\lambda_1h_1)}{2D(\lambda_1+\lambda_0)} \\ & +\frac{s(\lambda_0h_0+\lambda_1h_1)}{D(\lambda_1+\lambda_0)}+\frac{\lambda_1(h_0-h_1)}{(\lambda_1+\lambda_0)^2} \\ & +\frac{(\lambda_1+\lambda_0)}{D}e^{\frac{-(\lambda_1+\lambda_0)Q_0}{D}}\Big(\frac{s\lambda_1(h_0-h_1)}{(\lambda_1+\lambda_0)^2}+\frac{\lambda_1D(h_1-h_0)}{(\lambda_1+\lambda_0)^3}\Big)\Big) \\ & +\frac{C_0P_2P_3(1-P_3)}{(1-P_2P_3)^2}+\frac{C_0P_2P_3(1-P_2)}{(1-P_2P_3)^2} \end{aligned} \tag{3.112}$$

$$\begin{aligned} A'' = & \frac{P_2P_3}{(1-P_2P_3)}\Big(\frac{(\lambda_0h_0+\lambda_1h_1)}{D(\lambda_1+\lambda_0)}\Big) \\ & -\frac{P_2P_3}{(1-P_2P_3)}e^{\frac{-(\lambda_1+\lambda_0)Q_0}{D}}\Big(\frac{s\lambda_1(h_0-h_1)}{D^2}+\frac{\lambda_1(h_1-h_0)}{D(\lambda_1+\lambda_0)}\Big) \end{aligned} \tag{3.113}$$

For large Q_0

$$e^{-\frac{(\lambda_1+\lambda_0)Q_0}{D}} = 0 \tag{3.114}$$

Hence,

$$A'' = \frac{P_2P_3}{(1-P_2P_3)}\Big(\frac{(\lambda_0h_0+\lambda_1h_1)}{D(\lambda_1+\lambda_0)}\Big) \tag{3.115}$$

Since A'' is always positive, TC is convex in Q_0.

b) The proof is very similar to the proof in case I, so only the result is illustrated

$$A'' = \frac{\lambda_0h_0+\lambda_1h_1}{D(\lambda_1+\lambda_0)} \tag{3.116}$$

It is obvious that A'' is greater than zero, so it can be said that Q_1 is convex.

c) Total cost function for case II,

$$A = \quad TC_2 = HC_2 + PC_2 + OC_2 \tag{3.117}$$

The expected cycle time for case II,

$$B = \quad CT_2 = \frac{Q_1}{D} + \frac{P_1}{\lambda_1} \tag{3.118}$$

To test convexity of S_0 second derivative of A with respect to S_0 is calculated. Let

$$\begin{aligned} K(Q_1, s) = \quad & \frac{Q_1^2(\lambda_0 h_0+\lambda_1 h_1)}{2D(\lambda_1+\lambda_0)} + Q_1\left(\frac{(\lambda_0 h_0+\lambda_1 h_1)s}{D(\lambda_1+\lambda_0)} + \frac{\lambda_0(h_1-h_0)}{(\lambda_1+\lambda_0)^2}\right) + \frac{s\lambda_0(h_1-h_0)}{(\lambda_1+\lambda_0)^2} \\ & + \frac{\lambda_0 D(h_0-h_1)}{\lambda_1+\lambda_0)^3} + e^{\frac{-(\lambda_1+\lambda_0)Q_1}{D}}\left(\frac{s\lambda_0 h_0-\lambda_0 h_1 s}{(\lambda_1+\lambda_0)^2} + \frac{D\lambda_0 h_1-Dh_0\lambda_0}{(\lambda_1+\lambda_0)^3}\right) \\ & + \frac{P_1P_2h_0s^2}{2D} + P_1\left(\frac{sh_0}{\lambda_1} - \frac{Dh_0}{\lambda_1^2} + e^{-\lambda_1 s/D}\left(\frac{Dh_0}{\lambda_1^2} - \frac{s^2h_0}{2D}\right)\right) \\ & + c_1Q_1 + k_1 + \frac{c_1DP_1}{\lambda_L} \end{aligned} \tag{3.119}$$

$$\begin{aligned} f(S_0) = \quad & h_0\left(\frac{S_0}{\lambda_1} - \frac{D}{\lambda_1^2} + e^{-\lambda_1 S_0/D}\left(\frac{D}{\lambda_1^2} - \frac{S_0^2}{2D}\right)\right) \\ & +(k_0 + S_0c_0 - S_0c_1) \end{aligned} \tag{3.120}$$

$$A = \quad \frac{P_4h_0S_0^2P_1P_2}{2D} + \frac{P_1P_2f(S_0)}{(1-P_4)} + K(Q_1, s) \tag{3.121}$$

$$\begin{aligned} A' = \quad & \frac{P_4'P_1P_2\frac{h_0S_0^2}{2D}+P_1P_2P_4\frac{h_0S_0}{D}+f(S_0)'P_1P_2}{(1-P_4)} \\ & +\frac{P_4'}{(1-P_4)}\left(\frac{\frac{P_4h_0S_0^2P_1P_2}{2D}+P_1P_2f(S_0)}{(1-P_4)}\right) \end{aligned} \tag{3.122}$$

$$A' = \quad \frac{\frac{P_4'h_0S_0^2P_1P_2}{2D}+\frac{P_1P_2P_4h_0S_0}{D}+f(S_0)'P_1P_2+P_4'(a-K(Q_1.s))}{(1-P_4)} \tag{3.123}$$

$$A' = \quad \frac{P_1P_2}{(1-P_4)}\left(\frac{P_4'h_0S_0^2}{2D} + \frac{P_4h_0S_0}{D} + f(S_0)' + \frac{P_4'(a-K(Q_1.s))}{P_1P_2}\right) \tag{3.124}$$

$$c = \quad \left(\frac{P_4'h_0S_0^2}{2D} + \frac{P_4h_0S_0}{D} + f(S_0)' + \frac{P_4'(a-K(Q_1.s))}{P_1P_2}\right) \tag{3.125}$$

$$A' = \quad \frac{P_1P_2c}{(1-P_4)} = 0 \tag{3.126}$$

$$A'' = \frac{P_1 P_2 c'}{(1-P_4)} + \frac{P_4' c P_1 P_2}{(1-P_4)^2} \tag{3.127}$$

$$\frac{P_4' c P_1 P_2}{(1-P_4)^2} = a' = 0 \tag{3.128}$$

$$A'' = \frac{P_1 P_2 c'}{(1-P_4)} \tag{3.129}$$

$$c' = \frac{-\lambda_1}{D}(c - f(S_0)') + \frac{2S_0 P_4' h_0}{2D} + \frac{P_4 h_0}{D} + f(S_0)'' + \frac{P_4' a'}{P_1 P_2} \tag{3.130}$$

$$c' = \frac{\lambda_1}{D} f(S_0)' + f(S_0)'' + \frac{P_4' h_0 S_0}{D} + \frac{P_4 h_0}{D} \tag{3.131}$$

$$\begin{aligned} f(S_0) = & \frac{h_0 S_0}{\lambda_1} - \frac{h_0 D}{\lambda_1^2} + \frac{P_4 D h_0}{\lambda_1^2} \\ & - \frac{P_4 S_0^2 h_0}{2D} + k_0 + S_0(C_0 - C_1) \end{aligned} \tag{3.132}$$

$$f(S_0)' = \frac{h_0}{\lambda_1} - \frac{P_4 h_0}{\lambda_1} - \left(\frac{\frac{-\lambda_1 P_4 S_0^2}{D} + 2S_0 P_4}{2D}\right) h_0 + (c_0 - c_1) \tag{3.133}$$

$$f(S_0)'' = \frac{P_4 h_0}{D} - \frac{\lambda_1^2 S_0^2 P_4 h_0}{2D^3} + \frac{\lambda_1 P_4 S_0 h_0}{D^2} + \frac{\lambda_1 P_4 S_0 h_0}{D^2} - \frac{P_4 h_0}{D} \tag{3.134}$$

$$\begin{aligned} c' = & \frac{\lambda_1}{D}\left(\frac{h_0}{\lambda_1} - \frac{P_4 h_0}{\lambda_1} + \frac{P_4 \lambda_1 h_0 S_0^2}{2D^2} - \frac{P_4 h_0 S_0}{D}\right. \\ & \left. + (C_0 - C_1)\right) + \frac{P_4 h_0}{D} - \frac{P_4 \lambda_1^2 h_0 S_0^2}{2D^3} + \frac{P_4 \lambda_1 h_0 S_0}{D^2} \end{aligned} \tag{3.135}$$

$$c' = \frac{1}{D}(h_0 + \lambda_1(c_0 - c_1)) \tag{3.136}$$

Since the first derivative of c is always positive for $c_0 \geq c_1$, the second derivative of A is always positive. Therefore TC function is convex in S_0

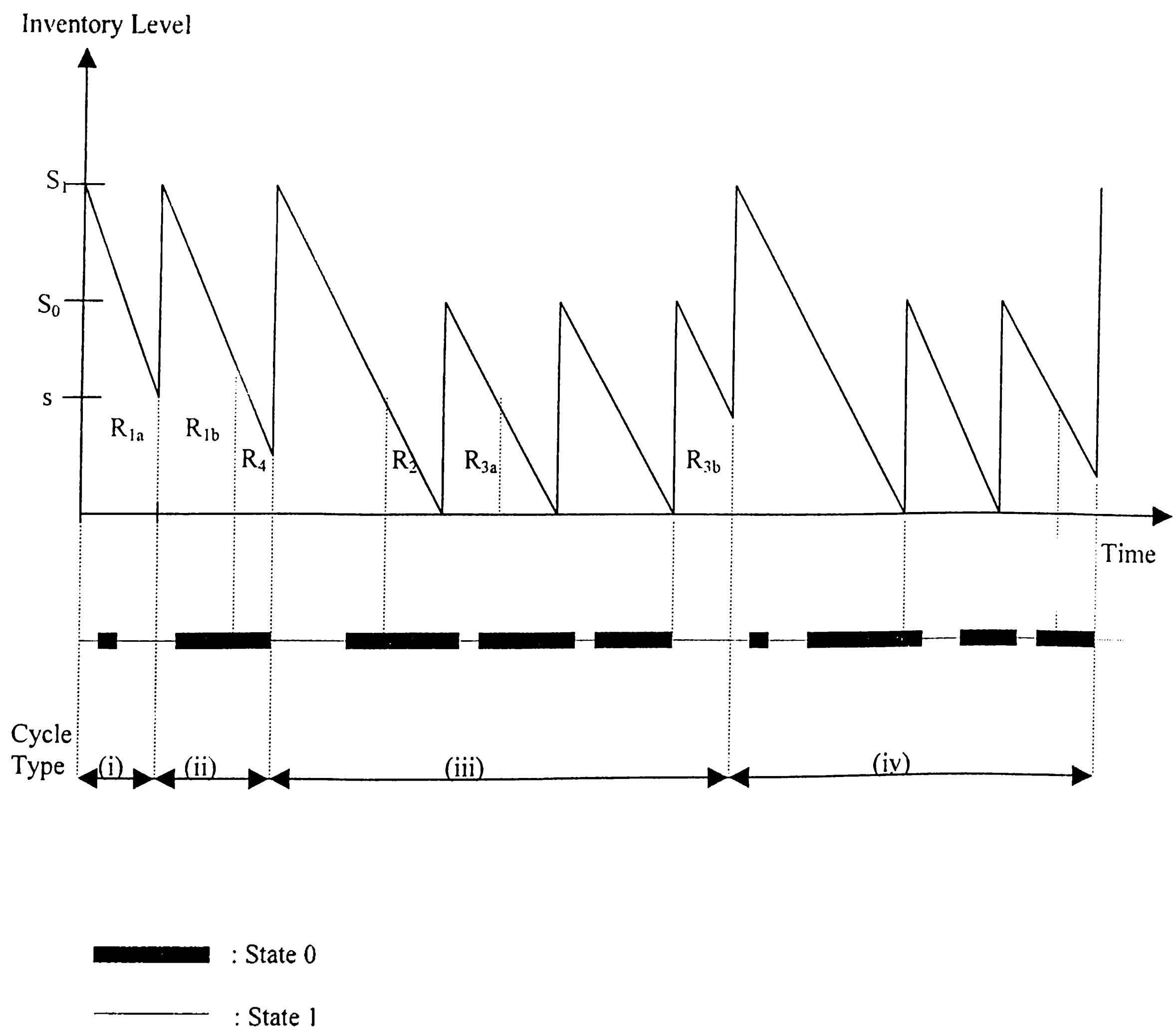

Figure 3.1. Case I, $S_1 \geq S_0 \geq s$

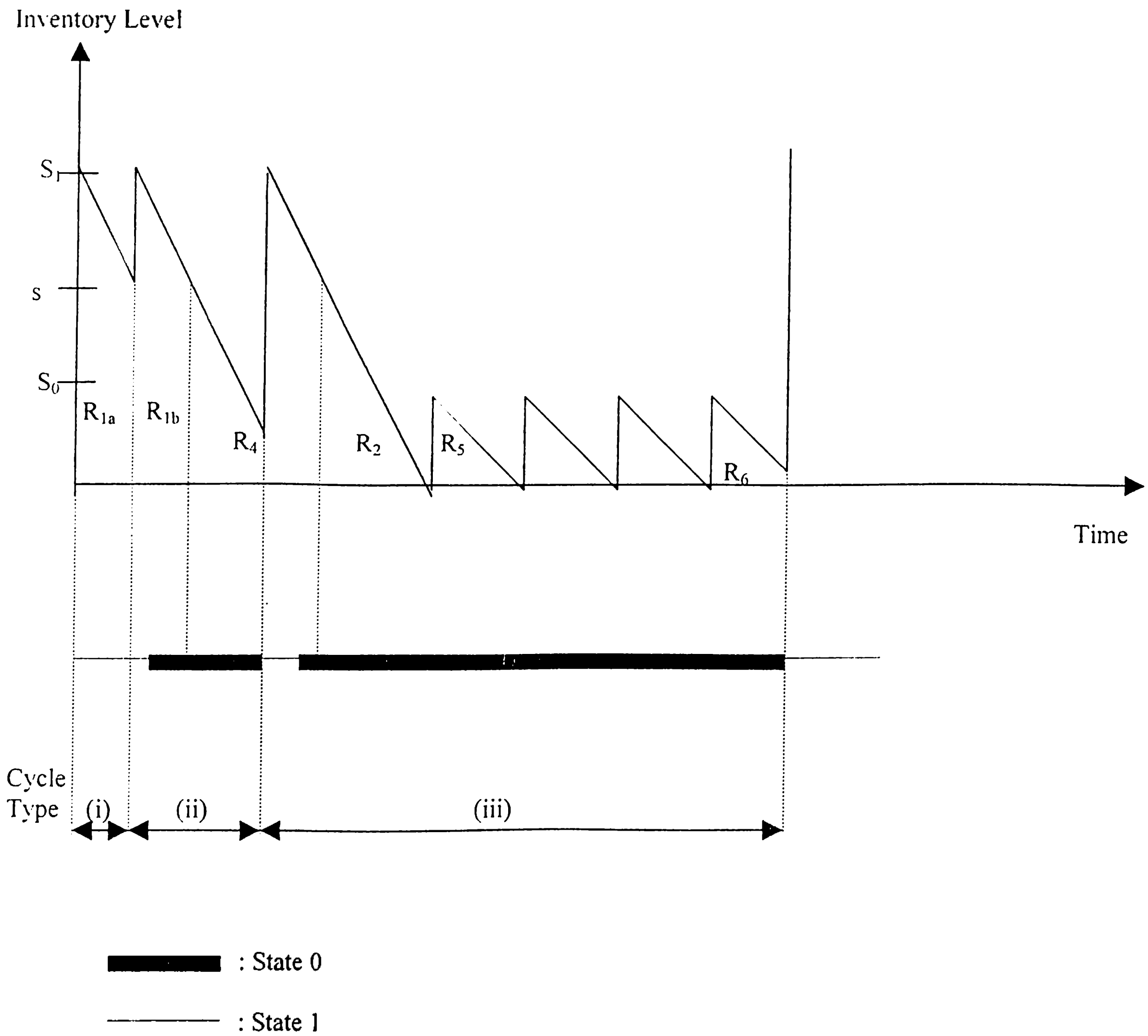

Figure 3.2. Case II, $S_1 \geq s \geq S_0$

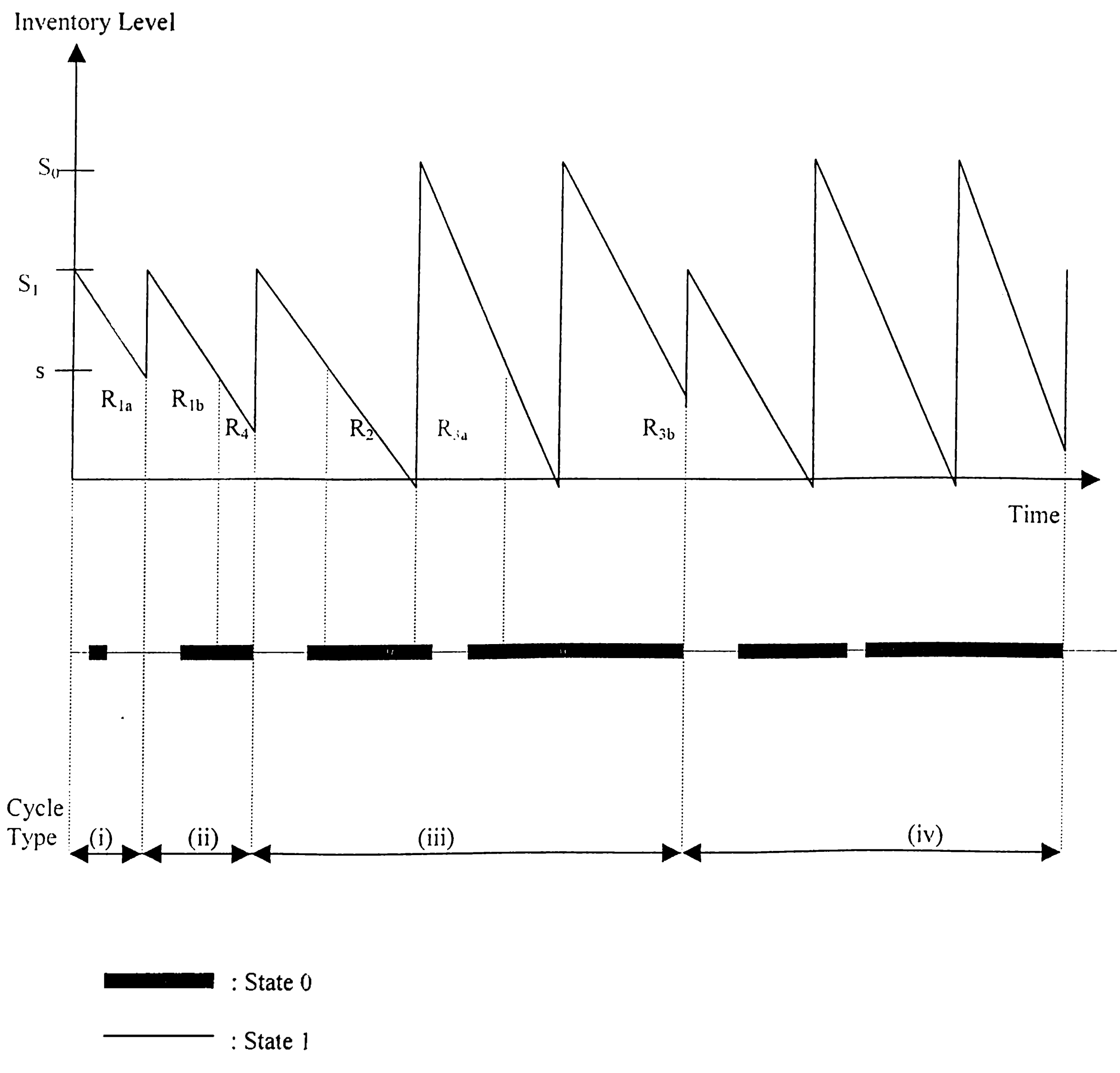

Figure 3.3. Case III, $S_0 \geq S_1 \geq s$

Chapter 4

Numerical Results

In this chapter, numerical results for the (S_1,S_0,s) ordering policy are presented. Based on a wide range of parameter settings, the sensitivity of the model is analyzed with respect to changes in the numerical values of various parameters of the inventory system under consideration. A comparision of the model herein is also made with the instantaneous shock model.

4.1 Parameter Set

For sensitivity analysis we set h_1 equal to 1, c_1 equal to 5, k_1 equal to 50, and the demand equal to 100, and vary the ratio of the parameters as shown in the Table 4.1.

h_0/h_1	1, 2, 5
k_0/k_1	1, 2, 5, 10
λ_1	0.1, 0.5, 1, 5, 10
λ_0/λ_1	0.1, 0.5, 1, 5, 10
c_0/c_1	1, 1.5, 2

Table 4.1: Parameters Set Used in the Sensitivity Analysis

In the sensitivity analysis, total 250 cases are considered. 75 cases (=5*5*3) for sensitivity analysis for h_0/h_1, 75 cases (=5*5*3) for sensitivity analysis for c_0/c_1, and 100 (=5*5*4) cases for sensitivity analysis for k_0/k_1 are considered.

4.2 Sensitivity Results

In this section, for different h_0/h_1, c_0/c_1, and k_0/k_1, λ_0/λ_1 ratios, numerical results are presented and the effect of those changes on the optimal S_1, S_0, s, and $TC(Q_1,Q_0,s)$ values are explained. Moreover, optimal results using our model are compared with the optimal results using instantaneous shock model of Moinzadeh[15](1997).

4.2.1 Sensitivity Results for h_0

When k_0=k_1=50, D=100, c_0=c_1=5, and h_1=1, for changing λ_0, λ_1, and h_0 values, the following results are obtained:

The numerical results illustrate that when fixed ordering costs, and purchasing costs in state 1 and 0 are equal to each other, even though ratio of holding cost in state 0 to holding cost in state 1 changes, s^* is always equal to zero. As s^* increases number of replenishments when the state is 1 increases, since when the state is 1, amount of order is higher, average inventory on hand increases, and to decrease the holding cost the policy wants to set s^* equal to zero.

Even though it is not illustrated in Table 4.2 it is observed that when (h_0/h_1) is equal to one, changes in the (λ_0/λ_1) ratio do not result in changes in the optimal parameters. In this case, since all of state 1 costs are equal to state 0 costs, in terms of cost there is no difference between state 1 and 0, as expected. So the optimal results are the same with the optimal results using EOQ type model.

When (h_0/h_1) is greater than one, as (λ_0/λ_1) ratio increases, optimal S_1, and optimal S_0 values decline, and optimal total cost per unit time increases. This is because, increases in the (λ_0/λ_1) ratio refer to the increases in the frequency of occurrence of state 0. When (h_0/h_1) ratio is greater than one, as the frequency of occurrence of state 0 increases, to decrease the amount of inventory on hand, and thereby the total cost of holding inventory, optimal S_1, and S_0 values decline. Moreover, given that λ_0 and λ_1 do not change as (h_0/h_1) ratio increases optimal S_1, and S_0 values decline due to the same reason.

4.2.2 Sensitivity Results for c_0

When $k_0=k_1=50$, D=100, $h_0=h_1=1$, and $c_1=5$, for changing λ_0, λ_1, and c_0 values, the following results are obtained:

As can be seen from Table 4.3 as (λ_0/λ_1) ratio increases, optimal S_1, and s values increase. As the frequency of occurrence of state 0 increases, to be able to make replenishments when the state is 1, optimal s value increases. Moreover, when (λ_0/λ_1) ratio increases, the system wants to purchase high amount of inventory when the state is 1, so it increases the optimal value of S_1, so rises units of items purchased in state 1.

Unlike in the sensitivity results for h_0, in the sensitivity analysis of c_0, we observed that there are some cases in which optimal S_0 is equal to optimal s. In such cases, optimal results can not be calculated in case I in which optimal S_0 is greater than or equal to optimal s. So, in these cases, we have to search for optimal values for S_1, S_0, and s in case II.

Moreover, given that all of the parameters are constant, as the purchasing cost in the state 0 increases, the system increases optimal S_1 and s values, but decreases optimal S_0 value. So, the number of units purchased in state 0 decreases, and the number of units purchased in state 1 increases.

4.2.3 Sensitivity Results for k_0

When $k_1=50$, D=100, $h_0=h_1=1$, and $c_1=c_0=5$, for changing λ_0, λ_1, and k_0 values, the following results are obtained:

Table 4.4 illustrates that as (λ_0/λ_1) ratio increases, optimal S_1, S_0, and s increase. When both λ_0 and λ_1 values are small, in other words frequencies of occurences of state 1 and state 0 are low, optimal s values is equal to zero. However, when frequency of occurence of state 0 increases, to be able to make replenishments when state is 1, control policy increases optimal value of s.

Unlike in the sensitivity results for h_0 and c_0 in the sensitivity results for k_0 it is required to use case III in which $Q_0 \geq Q_1$. In this case the reason for Q_0 being greater than or equal to Q_1 is that when the state is 0, increasing the value of

Q_0 system declines fixed ordering cost per unit of order.

As can be calculated from the table, as stochasticity in the system increases, in other words, λ_0 and λ_1 values are high, optimal Q_1 value approaches optimal Q_0 value. The policy wants to increase both optimal Q_1 and optimal Q_0 values. Because as Q_1 increases, number of units purchased in state 1 increases, and the number of replenishment declines. On the other hand, as optimal Q_0 values increases, fixed ordering cost per unit of order declines.

As a result, the general trends in the optimal values for changing cost parameters when λ_0/λ_1 is constant, are as illustared in Table 4.5.

When h_0=h_1=1, c_0=c_1=5, k_0=k_1=50, as expected optimal results provided using our model are the same as the optimal results using EOQ model. (Q_1^*=100, Q_0^*=100, s^*=0, TC^*=600)

4.3 Comparison With Instantaneous Shock Model

In this section optimal results of our model are compared with the instantaneous shock model studied in Moinzadeh[15](1997), and the percentage deviation in optimal cost rate is examined. As λ_0 goes to infinity optimal policy parameters $S_1^*(\infty)$,$S_0^*(\infty)$, and $s^*(\infty)$ are obtained; we then compute total cost rate from our model with the determined policy parameters from the shock model, $TC(\infty)$.

4.3.1 Comparison of the Models When h_0 Changes

In this section, when h_0, and frequency parameters change, percentage differences between the optimal total cost in our model, and the optimal total cost in the instantaneous shock model are compared.

Percentage difference between costs used in Table 4.6-Table 4.12 is expressed as follows,

$$\%\Delta = \frac{(TC(\infty)-TC^*)}{TC^*} * 100 \qquad (4.1)$$

Given that the frequency of state 1 is high, the frequency of the state 0 is low. and the (h_0/h_1) ratio is high, the percentage difference between the costs goes up to 5.058.

When the frequency of state 1 is constant, as the frequency of state 0 increases. the percentage difference declines. Moreover, Table 4.6 illustrates that for the same (h_0/h_1) ratio. as the frequencies of state 1 and state 0 increase at the same rate, the percentage difference declines. Therefore, instantaneous shock model becomes relatively suitable to the systems operating in highly volatile environments.

4.3.2 Comparison of the Models When c_0 Changes

In this section, when c_0, and frequency parameters change, percentage differences between the optimal total cost in our model and the total cost incurred when using the instantaneous inventory shock model policy parameters are presented.

As can be seen from Table 4.7, for the given parameter set, when (c_0/c_1) ratio is high, to use our model rather than the instantaneous shock model results in the percentage deviation as high as 53. For the given (c_0/c_1) ratio and frequency of occurance of state 1, as the frequency of occurence of state 0 increases, the percentage deviation from optimal cost declines. Because in the system state 0 is observed more as its frequency increases.

4.3.3 Comparison of the Models When k_0 Changes

In this section, when k_0, and frequency parameters change, the optimal total cost in the instantaneous shock model and the optimal total cost in our model are compared.

As can be seen from Table 4.8, for the give parameter set. when (k_0/k_1) ratio is high. using our model rather than instantaneous shock model percentage difference between the optimal total costs of the models increases up to 21.77 For the given (k_0/k_1) ratio and frequency of occurrence of state 1, as the frequency of occurrence of state 0 increases, the percentage difference between the costs declines, because state 0 is observed more as its frequency of occurrence increases.

4.4 Optimal Total Cost per Unit Time When s^*=0

In this section, we set optimal s equal to zero, and given that optimal s is equal to zero, we find optimal values for S_1, S_0, and TC(Q_1,Q_0,s). When k_0=k_1=50, D=100, h_0=h_1=1, and c_1=5, we calculate percentage difference between optimal total cost per unit time in our model and optimal total cost per unit time when optimal s is equal to zero. In other words we want to calculate how much total cost changes, if we follow myopic two parameter inventory control policy rather than optimal three parameter (S_1, S_0, s) inventory control policy used in our model. In the two parameter (S_1, S_0) inventory control policy order is made when the inventory level is equal to zero. When the inventory level is equal to zero, if the state is 1 inventory level is increased to S_1 level, however, if the state is 0, inventory level is increased up to S_0 level.

Table 4.9 illustrates that when c_0 changes, optimal total cost using our model gives 75.49 percent better results than the model in which s^*=0. On the other hand, Table 4.10 illustarets that when k_0 changes, optimal total cost using our model gives 25.98 percent better results than the other model. As λ_0, the frequency of occurrence of state 0, increases percentage deviation in the optimal costs increases. This is because, when s^*=0, number of replenishents in state 1 declines, and we can say that when s^*=0, number of replenishments in state 1 is less than the number of replenishments in state 1 when s^* is greater than zero.

In this section we did not make comparison of the models when h_0 changes. Because, as can be seen from Table 4.2 optimal s value is always equal to zero. In other words our model gives exactly the same results as the model when s^*=0.

4.5 Numerical Comparison With A Time-Average EOQ Model

In this section, for the given cost and frequency cost parameters, the percentage difference between optimal total cost per unit time in our model and optimal total cost per unit time in the time-average EOQ model is calculated.

Ordering, and holding cost parameters used in the EOQ model are calculated as a weighted time-average:

$$\tilde{k} = \frac{\lambda_0 k_0 + \lambda_1 k_1}{(\lambda_1 + \lambda_0)} \tag{4.2}$$

$$\tilde{h} = \frac{\lambda_0 h_0 + \lambda_1 h_1}{(\lambda_1 + \lambda_0)} \tag{4.3}$$

$$EOQ = \left(\frac{2D\tilde{k}}{\tilde{h}}\right)^{1/2} \tag{4.4}$$

In the time-average EOQ model, it is assumed that optimal s is equal to zero. and optimal S_1, and S_0 values are equal to EOQ in (4.4). In the numerical solutions, we set D=100, $h_1=h_0=1$, $c_1=5$, and $k_1=50$,

Table 4.11 illustrates that when c_0 changes, optimal total cost using our model gives 75.51 percent better results than the average EOQ model On the other hand, Table 4.12 illustrates that when k_0 changes, optimal total cost using our model gives 28.05 percent better results than the average EOQ model. Again as can be seen from the both tables as λ_0, the frequency of occurrence of state 0, increases percentage deviation in the optimal costs increases. Because, when $s^*=0$, number of replenishents in state 1 declines. Again we did not compare the models when h_0 changes, because the optimal S_1, S_0 and optimal s parameters are very close to the parameters in time-average EOQ model.

λ_1	h_0	λ_0	S_1^*	S_0^*	s^*	TC^*
0.1	2	0.01	99.9	70.8	0	603.784
0.1	2	0.05	99.5	70.8	0	613.874
0.1	2	0.1	98.9	70.8	0	620.810
0.1	2	0.5	95.2	70.8	0	634.667
0.1	2	1	91.9	70.8	0	637.801
0.1	2	∞	70.7	70.7	0	641.392
0.1	5	0.01	99.3	44.9	0	611.402
0.1	5	0.05	97.1	44.8	0	641.783
0.1	5	0.1	94.3	44.7	0	662.632
0.1	5	0.5	82.7	44.7	0	704.057
0.1	5	1	75.5	44.7	0	713.296
0.1	5	∞	44.7	44.7	0	723.541
10	2	1	96.5	88.4	0	604.414
10	2	5	88.3	83.2	0	615.427
10	2	10	83.3	80	0	622.449
10	2	50	74.7	73.7	0	635.399
10	2	100	73	72.4	0	638.169
10	2	∞	70.7	70.7	0	641.412
10	5	1	87.7	61.4	0	616.309
10	5	5	69	57.7	0	652.235
10	5	10	61	54.3	0	672.916
10	5	50	49.3	47.8	0	708.151
10	5	100	47.2	46.4	0	715.319
10	5	∞	44.7	44.7	0	723.591

Table 4.2: Sensitivity Results for h_0

λ_1	c_0	λ_0	S_1^*	S_0^*	s^*	TC^*
0.1	7.5	0.01	101.3	89.6	0	622.655
0.1	7.5	0.05	106.7	89.6	0	682.967
0.1	7.5	0.1	114.7	89.6	0	724.240
0.1	7.5	0.5	222.1	89.1	84.7	803.088
0.1	7.5	1	284.8	88.3	105	813.295*
0.1	7.5	∞	317.1	87.9	216.2	815.963*
0.1	10	0.01	102.5	82	0	645.197
0.1	10	0.05	114.7	81.7	0	765.275
0.1	10	0.1	136.3	81.7	0	846.865
0.1	10	0.5	410.1	80.8	180.2	978.394*
0.1	10	1	481.015	80.7	278.2	988.208*
0.1	10	∞	498.3	80.4	391.3	990.734*
10	7.5	1	108.5	16.6	0	608.557
10	7.5	5	119.3	16.1	12.5	619.326
10	7.5	10	121.8	14.9	16.5	621.614*
10	7.5	50	123.4	14.8	22.5	623.332*
10	7.5	100	123.4	14.8	22.5	623.392*
10	7.5	∞	123.5	14.8	23.4	623.432*
10	10	1	112.3	12.5	2.6	612.131
10	10	5	122.6	11.4	15.7	622.592*
10	10	10	125	11.4	19.8	624.891*
10	10	50	126.6	11.4	24.9	626.565*
10	10	100	126.7	11.4	25.7	626.669*
10	10	∞	126.8	11.4	26.7	626.709*

Table 4.3: Sensitivity Results for c_0. (*) indicates that optimal solution is found in Case II.

λ_1	k_0	λ_0	S_1^*	S_0^*	s^*	TC^*
0.1	100	0.01	100.2	138.7	0	603.839**
0.1	100	0.05	101	138.7	0	614.069**
0.1	100	0.1	102.2	138.7	0	621.093**
0.1	100	0.5	110.9	139.2	0	635.025**
0.1	100	1.0	121.2	139.6	0	638.072**
0.1	100	∞	142.9	142.9	2.8	641.330**
0.1	250	0.01	100.7	211	0	611.891**
0.1	250	0.05	103.3	211.9	0	643.522**
0.1	250	0.1	107	212.3	0	665.143**
0.1	250	0.5	144	215.4	0	706.949**
0.1	250	1.0	186.6	217.3	0	715.178**
0.1	250	∞	247.5	247.5	66.4	721.300**
0.1	500	0.01	101.3	288.5	0	621.644**
0.1	500	0.05	106.4	289.7	0	679.045**
0.1	500	0.1	113.8	291.4	0	717.997**
0.1	500	0.5	209.7	299.8	0	790.309**
0.1	500	1.0	280.7	303	91	800.785**
0.1	500	∞	321.6	321.6	129.8	805.814**
10	100	1	104.5	104.7	0	604.447**
10	100	5	113.3	113.3.6	6.3	613.227**
10	100	10	115.4	115.5	10.5	615.564**
10	100	50	117.4	117.4	15.6	617.265**
10	100	100	117.4	117.5	16.4	617.370**
10	100	∞	117.5	117.7	17.5	617.412**
10	250	1	115.2	115.3	5.5	615.098**
10	250	5	125.6	125.9	18.6	625.542**
10	250	10	127.9	128	22.7	627.853**
10	250	50	129.6	130	28	629.536**
10	250	100	129.6	130	28.8	629.639**
10	250	∞	129.8	130	29.6	629.680**
10	500	1	122.9	122.9	13.4	622.891**
10	500	5	133.3	133.8	26.5	633.310**
10	500	10	135.7	135.8	30.5	635.614**
10	500	50	137.3	137.3	35.7	637.292**
10	500	100	137.4	137.5	36.4	637.396**
10	500	∞	137.5	137.7	37.5	637.436**

Table 4.4: Sensitivity Results for k_0.(**) indicates that optimal solution is found in Case III.

Changes	S_1^*	S_0^*	s^*	TC^*
$h_0/h_1 \uparrow$	$\downarrow$	$\downarrow$	=0	$\uparrow$
$c_0/c_1 \uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$k_0/k_1 \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$

Table 4.5: Behavior of Optimal Values with Respect to Tested Cost Parameters

λ_1	h_0	λ_0	tc(∞)	%$\triangle$
0.1	2	0.01	609.283	0.910
0.1	2	0.05	617.848	0.647
0.1	2	0.1	623.736	0.471
0.1	2	0.5	635.513	0.133
0.1	2	1	638.189	0.006
0.1	5	0.01	642.330	5.058
0.1	5	0.05	663.990	3.460
0.1	5	0.1	678.881	2.452
0.1	5	0.5	708.665	0.655
0.1	5	1	715.434	0.300
10	2	1	609.285	0.806
10	2	5	617.855	0.394
10	2	10	623.746	0.208
10	2	50	635.530	0.002
10	2	100	638.207	0
10	5	1	642.334	4.222
10	5	5	664.007	1.805
10	5	10	678.907	0.890
10	5	50	708.706	0.008
10	5	100	715.479	0.002

Table 4.6: Comparison of the Models When h_0 Changes

λ_1	c_0	λ_0	TC(∞)	%Δ
0.1	7.5	0.01	816.179	31.081
0.1	7.5	0.05	816.119	19.497
0.1	7.5	0.1	816.077	12.681
0.1	7.5	0.5	815.995	1.607
0.1	7.5	1	815.997	0.330
0.1	10	0.01	991.462	53.668
0.1	10	0.05	991.257	29.530
0.1	10	0.1	991.117	17.034
0.1	10	0.5	990.838	1.272
0.1	10	1	990.778	0.260
10	7.5	1	623.457	2.448
10	7.5	5	623.467	0.669
10	7.5	10	623.474	0.300
10	7.5	50	623.488	0.003
10	7.5	100	623.491	0.001
10	10	1	627.288	2.476
10	10	5	627.288	0.754
10	10	10	627.288	0.374
10	10	50	627.289	0.006
10	10	100	627.289	0.002

Table 4.7: Comparison of the Models When c_0 Changes

λ_1	k_0	λ_0	TC(∞)	%Δ
0.1	100	0.01	611.524	1.273
0.1	100	0.05	619.485	0.882
0.1	100	0.1	624.959	0.622
0.1	100	0.5	635.902	0.138
0.1	100	1	638.385	0.005
0.1	250	0.01	687.978	12.435
0.1	250	0.05	643.522	8.324
0.1	250	0.1	703.350	5.744
0.1	250	0.5	715.788	1.250
0.1	250	1	718.517	0.647
0.1	500	0.01	756.865	21.775
0.1	500	0.05	770.447	13.460
0.1	500	0.1	779.769	8.603
0.1	500	0.5	798.214	1.000
0.1	500	1	802.165	0.172
10	100	1	617.491	2.158
10	100	5	617.469	0.692
10	100	10	617.454	0.307
10	100	50	617.425	0.003
10	100	100	617.419	0
10	250	1	629.608	2.359
10	250	5	629.628	0.653
10	250	10	629.642	0.285
10	250	50	629.667	0.002
10	250	100	629.673	0
10	500	1	637.494	2.344
10	500	5	637.477	0.658
10	500	10	637.466	0.291
10	500	50	637.446	0.002
10	500	100	637.441	0
10	500	∞	637.436	0

Table 4.8: Comparison of the Models When k_0 Changes

λ_1	c_0	λ_0	$TC^*(s^*=0)$	$\%\Delta$
10	7.5	1	608.558	0
10	7.5	5	632.944	2.20
10	7.5	10	653.887	5.19
10	7.5	50	746.814	19.81
10	7.5	100	796.799	27.82
10	7.5	∞	849.924	36.33
10	10	1	612.485	0.01
10	10	5	645.147	3.62
10	10	10	670.786	7.34
10	10	50	773.165	23.40
10	10	100	850.631	35.74
10	10	∞	1099.78	75.49

Table 4.9: Comparison of the Results When c_0 Changes and $s^*=0$

λ_1	k_0	λ_0	$TC^*(s^*=0)$	$\%\Delta$
10	100	1	604.447	0
10	100	5	615.470	0.37
10	100	10	622.474	1.12
10	100	50	635.400	2.94
10	100	100	638.169	3.37
10	100	∞	641.558	3.91
10	250	1	616.775	0.27
10	250	5	652.752	4.35
10	250	10	673.205	7.22
10	250	50	708.166	12.49
10	250	100	715.321	13.61
10	250	∞	723.703	14.93
10	500	1	634.840	1.92
10	500	5	699.999	10.53
10	500	10	734.520	15.56
10	500	50	791.546	24.20
10	500	100	803.013	25.98
10	500	∞	816.013	28.01

Table 4.10: Comparison of the Results When k_0 Changes and $s^*=0$

λ_1	c_0	λ_0	TC^*(EOQ)	%Δ
10	7.5	1	622.727	2.33
10	7.5	5	683.332	10.33
10	7.5	10	724.999	16.63
10	7.5	50	808.331	29.68
10	7.5	100	827.270	32.70
10	7.5	∞	849.962	36.34
10	10	1	645.454	5.44
10	10	5	766.665	23.14
10	10	10	849.997	36.02
10	10	50	1016.66	62.26
10	10	100	1054.54	68.28
10	10	∞	1099.93	75.51

Table 4.11: Comparison with Time-Average EOQ When c_0 Changes

λ_1	k_0	λ_0	TC^*(EOQ)	%Δ
10	100	1	604.448	0
10	100	5	615.471	0.37
10	100	10	622.474	1.12
10	100	50	635.401	2.94
10	100	100	638.170	3.37
10	100	∞	641.413	3.89
10	250	1	616.775	0.27
10	250	5	652.753	4.35
10	250	10	673.205	7.22
10	250	50	708.166	12.49
10	250	100	715.321	13.61
10	250	∞	723.591	14.91
10	500	1	634.841	1.92
10	500	5	699.999	10.53
10	500	10	734.520	15.56
10	500	50	791.546	24.20
10	500	100	803.013	25.98
10	500	∞	816.205	28.05

Table 4.12: Comparison with Time-Average EOQ When k_0 Changes

Chapter 5

Conclusion

In this study, we consider an optimum three parameter inventory control policy for a continuous review inventory system where except for the demand rate all of the system parameters change in a random environment. We derive the exact expressions of the key operating characteristics of the model and obtain asymptotic convexity results. A numerical analysis is provided to illustrate how the optimum inventory parameters, and the optimal total cost are affected from the changes in the cost parameters. The results illustrate that given that λ_0 and λ_1 do not change, as (h_0/h_1) ratio increases S_1^*, and S_0^* values decline to be able to keep small amount of inventory on hand. Moreover, for changing (h_0/h_1) s^* is always equal to zero due to the same reason.

As the purchasing cost in state 0 increases, the system increases S_1^* and s^* values, but decreases S_0^* value. Thus, the number of units purchased in state 0 decreases, and the number of units purchased in state 1 increases. And by increasing s^*, the opportunity to make a replenishment when state is 1, is increased.

However, as k_0 increases, S_1^*, S_0^*, and s^* increase. s^* increases to be able to make replenishment when state is 1. As Q_0^* values increases, fixed ordering cost per unit of order declines. Finally, Q_1^* increases, because, number of units purchased in the state 1 increases, and the number of replenishment declines.

Moreover, optimal results in our model are compared with the optimal results in instantaneous shock model studied in Moinzadeh[15](1997). The comparison done for different cost parameter ratios illustrate that when the frequency of occurrence of states are low and the (c_0/c_1) ratio is high, to use our model rather than the instantaneous shock model results in the highest percentage deviation between the costs. In all of the comparisons, for the given frequency of occurrence of state 1, as the frequency of occurrence of state 0 increases, the percentage deviation from optimal cost declines.

Moreover, we compare our model with average EOQ model and the model in which reorder point is equal to zero. The results illustrate that when c_0, and/or k_0 change our model preforms better than the other models.

In this study we assume that volatility in the demand rate is so small that demand rate is constant over time. There is no duration between order time and delivery time. And, time between changes in the state of the environment is random, and exponentially distributed. However, in the future studies these assumptions may be relaxed. If we assume that the demand rate is also random, we would identify it as D_i, demand rate in state i=0,1. In that case, the conditional probabilities which are expressed in terms of time, may be expressed in terms of demand occurred over a random time period. For example the conditional probability $P_{10}(Q_1/\mathrm{D})$ would be written as $P_{10}(Q_1)$.

If we assume that there is a positive order lead time, then the three-parameter (S_1, S_0, s) policy considered would not be optimal any more. Instead of (S_1, S_0, s) policy, we would use a four parameter policy (i.e. (S_1,s_1, S_0, s_0)). However, in such a case our cycle types would change and the number of cycle types would increase.

In this study, since no backorders are allowed, minimum inventory level is always non-negative. If backorders are allowed with finite penalty cost, inventory level would drop below zero, but except for that, analysis would be essentially the same.

Even though it is not too complicated to relax the assumptions about demand rate, order lead time, and backordering, it is very complicated however to relax the assumption that the shocks are exponentially distributed.

Appendix

A.1 Appendix A

A.1.1 Derivation of Conditional Probabilities

Let P_1(t) denotes the probability that environmental conditions are 1 at time t.

$$P_0(t+h) = \ (1-\lambda_1 h)P_0(t) + \lambda_0 h P_1(t) + O(h) \tag{A.1.1}$$

In the M/M/1/1 queuing system, P_0(t+h) is the probability that at time (t+h), there is no customer in the system. In the equation, (1-λ_0h) is the conditional probability that given that at time t there is no customers in the system, at time (t+h) zero arrivals will occur. P_0(t) is the probability that at time t there is no customer in the system. P_1(t) is the probability that at time t there is one customer in the system. In our model, P_0(t+h) is the probability that at time (t+h), state is 0. And, in the equation, (1-λ_1h)P_0(t) refers to the condition that given that at time t the state is 0, there will be no change in state at time (t+h). P_0(t) is the probability that at time t, state is 0, and P_1(t) is the probability that at time t, state is 1. In the equation above λ_0hP_1(t) refers to the condition that given that at time t state is 1, it will be 0 at time (t+h).

$$P_0(t+h) = \ P_0 - \lambda_1 h P_0(t) + \lambda_0 h P_1(t) + O(h) \tag{A.1.2}$$

$$\frac{(P_0(t+h) - P_0(t))}{h} = \ \frac{-\lambda_1 P_0 h}{h} + \frac{\lambda_0 h P_1}{h} + \frac{O(h)}{h} \tag{A.1.3}$$

$$lim_{h-\infty}(\frac{P_0(t+h)-P_0(t)}{h}) = \quad -\lambda_1 P_0(t) + \lambda_0 P_1 = P_0(t)' \tag{A.1.4}$$

Similarly,

$$P_1(t)' = \quad -\lambda_0 P_1(t) + \lambda_1(1 - P_1(t)) \tag{A.1.5}$$

$$P_1(t)' = \quad -\lambda_0 P_1(t) + \lambda_1 - \lambda_1 P_1(t) \tag{A.1.6}$$

$$\lambda_0 = \quad P_1(t)' + (\lambda_0 + \lambda_1)P_1(t) \tag{A.1.7}$$

$$P_1(t) = \quad Ce^{-(\lambda_1+\lambda_0)t} + \frac{\lambda_1}{(\lambda_1+\lambda_0)} \tag{A.1.8}$$

$$P_1(t) = \quad Ce^{-(\lambda_1+\lambda_0)t} + \frac{\lambda_1}{(\lambda_1+\lambda_0)} \tag{A.1.9}$$

Boundary value of P_1(t) at t=0 that is $P_1(0)$

$$C = \quad P_1(0) - \frac{\lambda_1}{(\lambda_1+\lambda_0)} \tag{A.1.10}$$

$$P_1(t) = \quad \frac{\lambda_1(1-e^{-(\lambda_1+\lambda_0)t})}{(\lambda_1+\lambda_0)} + P_L(0)e^{-(\lambda_1+\lambda_0)t} \tag{A.1.11}$$

Also,

$$P_1(t) = \quad 1 - P_0(t) \tag{A.1.12}$$

$$P_0(t) = \quad \frac{\lambda_0(1-e^{-(\lambda_1+\lambda_0)t})}{(\lambda_1+\lambda_0)} + P_H(0)e^{-(\lambda_1+\lambda_0)t} \tag{A.1.13}$$

Depending on the initial conditions at time zero, the probability function changes.

$P_1(0)$=0, $P_1(0)$=1

$P_0(0)$=0, $P_0(0)$=1

$P_{10}(t)$: The probability that the state will 0 at time t given that it is 1 at time zero.

$P_{00}(t)$: The probability that the state will 0 at time t given that it is 0 at time zero.

$P_{11}(t)$: The probability that the state will 1 at time t given that it is 1 at time zero.

$P_{01}(t)$: The probability that the state will 1 at time t given that it is 0 at time zero.

$$P_{10}(t) = \frac{\lambda_0(1-e^{-(\lambda_1+\lambda_0)t})}{(\lambda_1+\lambda_0)} \tag{A.1.14}$$

$$P_{00}(t) = \frac{\lambda_0+\lambda_1 e^{(-(\lambda_1+\lambda_0)t)}}{(\lambda_1+\lambda_0)} \tag{A.1.15}$$

$$P_{11}(t) = \frac{\lambda_1+\lambda_0 e^{-(\lambda_1+\lambda_0)t}}{(\lambda_1+\lambda_0)} \tag{A.1.16}$$

$$P_{01}(t) = \frac{\lambda_1(1-e^{-(\lambda_1+\lambda_0)t})}{(\lambda_1+\lambda_0)} \tag{A.1.17}$$

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