DELEGATION IN A DUOPOLISTIC DIFFERENTIATED GOODS MARKET WITH BERTRAND COMPETITION

A Thesis Submitted to the Department of Economics and the Institute of Economics and Social Sciences of Bilkent University in Partial Fulfillment of the Requirements for the Degree of

MASTER OF ARTS IN ECONOMICS

by

Hüseyin Yıldırım

July, 1995

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I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Semih Koray

Prof. Dr. Semih Koray

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I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

DELEGATION IN A DUOPOLISTIC DIFFERENTIATED GOOD MARKET WITH BERTRAND COMPETITION

Hüseyin Yıldırım

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The impact of delegation in a firm has been observed by many modern authors. Vickers(1985), Fershtman and Judd(1987), Sklivas(1987) considered the problem as part of positive economic theory whereas Koray and Sertel(1989) treated it as a regulation problem. We examine a similar problem for a duopolistic differentiated good market with Bertrand competition and lengthen the delegation chain to 5 managers. Our findings show that the firms' profits are monotonically increasing, i.e. there is a positive incentive to redelegate for each firm. Our natural conjecture is that, in the limit, firms reach collusion non-cooperatively.

KEYWORDS: Delegation - Regulation - Non-cooperative games - Bertrand competition - Cournot competition - Duopoly - Product differentiation - Principal-Agent games - Efficiency

ÖZET

BİR DÜOPOL BİÇİMDE FARKLILAŞTIRILMIŞ ÜRÜN PIYASASINDA "BERTRAND" REKABETÇİ DELEGASYON

Hüseyin Yıldırım

Yüksek Lisans Tezi, İktisat Bölümü

Tez Yöneticisi: Prof.Dr.Semih Koray

31 sayfa

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Delegasyonun firmalar üzerindeki etkisi birçok modern iktisatçı tarafından incelenmiştir. Vickers(1985), Fershtman,Judd(1987) ve Sklivas(1987) problemi pozitif iktisat teorisi açısından ele alırken, Koray ve Sertel(1989) problemi regülasyon olarak düşünmüşlerdir. Bu çalışmada farklılaştırılmış ürünlere sahip bir düopol piyasasinda Bertrand rekabeti altında benzer bir problem ele alınmaktadır. Delegasyon zinciri 5 işletmeciye kadar uzatıldığında firmaların elde edecekleri kârlarda görülen artış, iki firmanın da yeniden delegasyon yapmak için nedenleri olduğunu göstermektedir. Buradan çıkan doğal bir kestirim, firmaların limitte işbirlikçi bir sonuca işbirliksiz olarak gidecekleri yönündedir.

ANAHTAR KELİMELER: Delegasyon - Regülasyon - İşbirliksiz oyun - Bertrand rekabeti - Cournot rekabeti - Düopol - Ürün farklılaştırılması - İşçi-işveren oyunu - Verimlilik

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Contents

1 INTRODUCTION

"If control of my decisions is in the hands of an agent whose preferences are different from my own, I may nevertheless prefer the results to those that would come about if I took my own decisions." noted John Vickers in his 1985 paper.

Actually, economists have directed their attentions to the objective functions of large corporations. Some have suggested that large firms are more concerned with maximizing revenues or market shares rather than profits. Although there may be many reasons behind this intuitions and suggestions, the complexity of managerial decision processes and management has been shown as one of the main reasons.

A number of modern authors, Koray and Sertel(1989), Fershtman and Judd(1987), Sklivas(1987), Vickers(1985), analyzed the problem for the symmetric duopoly with constant marginal cost and with one owner-one manager in each firm such that owners simultaneously choose their managers' incentives and then each manager chooses the firm's price or quantity and owners receive the resulting profits, and each manager is rewarded according to the incentives chosen by his owner. The results are very interesting in the sense that firms are not maximizing profits directly, and when managers compete in quantities, the result more closely resembles perfect competition than does Cournot behavior; conversely, when they compete in prices, the result more closely resembles collusion than does Bertrand behavior. Vickers(1985) showed also that if there are n firms competing in quantity where all but one maximize profit, then the firm which is not maximizing directly its profit earns more no matter what the number of firms in the market is

It is natural to expect that in an oligopoly, where each firm has one owner-one manager and is competing in quantity, as the number of firms goes to infinity, the market will converge to the purely competitive one. On the other hand, Koray and Sertel showed that the convergence in any $m-$ firm symmetric oligopoly with delegation is much more rapid than under naked Cournot competition, and actually it is as if $m^2 - m$ ghost copies of a typical firm in the symmetric oligopoly have been activated in competition. This really needs further attention for in this way the industry produces $m^2 - m$ fictitious firms without any further fixed costs.

At this point, an interesting question is whether in a Cournotic symmetric duopoly with constant marginal cost, there is any incentive for delegation to more than one manager? This question was first posed by Koray and Sertel(1989) where they found the following results:

In absence of extraneous delegation costs,

1) each owner has an incentive to redelegate, increasing the length of his delegation chain.

2) as the length of the delegation chain grows beyond bound,

i) total output at the (Cournot) equilibrium on the industry floor converges in monotonically increasing fashion to the socially efficient one, and

ii) the maximand delegated by each primal delegator converges in monotonically decreasing fashion to the (true) profit function.

As a consequence, it is suggested that, in a linear duopoly context, socially efficient and truthful outcomes can be arbitrarily closely approximated by the use of a Pretend-but-Perform Mechanism of sufficiently large order.

The above result is very important in the sense that to get the socially efficient result, it is not needed to have many firms in the industry; it can also be attained by lengthening the delegation chains, with only two firms.

There is a problematic point in Fershtman and Judd(1987), Sklivas(1987)(FJS) propose this meta-cournotic equilibrium (Cournotic on the industry floor and, with this institutionalized, also a la Cournot-Nash, in the owners' club) as a positive economic theory. Koray and Sertel(1989) first to criticize this deficiency in the literature and discussed three main reasons for not accepting FJS' approach as a positive economic theory. They pointed out the following reasons;

1) There is no natural reason why FJS managers would come to a Cournot equilibrium under assigned maximands. Not only are the managers not the recepients of these maximands, but the owners have no reason for instructing them to behave according to the Cournot-Nash solution concept.

2) There is no reason why the owners should limit the maximands they specify for their managers to the class given in FJS; to the contrary, they have incentives not to do so.

3) If redelegation is permitted, then there is incentive to redelegate. This is discussed in Koray and Sertel(1989b) in detail for the Cournotic duopoly.

Thus, Koray and Sertel concluded if the FJS approach to the problem is from the view point of regulation, then it can be accepted. i.e. for them all artificial restrictions are admissible. Otherwise, the solutions cannot be imposed as a contribution to positive economic theory.

Now almost all ingredients for our motivation towards the present work are ready. In a duopoly with Bertrand competition, the results more closely resemble collusion than ordinary Bertrand Competition. Although this conclusion is reached in many papers like Sklivas(1987), Fershtman and Judd(1987), why delegation is stopped at one chain is not discussed. We owe this redelegation idea to Koray and Sertel(1989). Thus our aim is to show the following: In the context of a symmetric linear Bertrand duopoly where redelegation is permitted and in the absence of extraneous delegation costs:

1) each principal has an incentive to redelegate, increasing the length of his delegation chain.

2) as the length of the delegation chain grows beyond bound,

i) total output at the (Bertrand) equilibrium on the industry floor converges in monotonically decreasing fashion to the collusive one, and

ii) the maximand delegated by each primal delegator converges in monotonically increasing fashion to the (true) profit function.

As a consequence, it is suggested that in a linear duopoly context collusive and truthful outcomes can be arbitrarily closely approximated by the use of redelegation of sufficiently large order. But we wish to emphasize that the results presented here are proved for particular cases where the delegation chain lenght is $0, 1, .., 5$, whereas the general formulas regarding the economic variables for arbitrary chain lenght are still in the status of conjectures, though there are auspicious clues leading to these.

2 INTUITIVE ILLUSTRATION

Before introducing our model formally, we wish to consider some simple examples to illustrate that under some institutions "non-profit maximizers" actually can surpass profit maximizers.

1) John Vickers gave the following example in his paper (1985)

Suppose firm A is deciding whether or not to enter a market currently monopolized by firm B.

Entry of A is profitable if and only if B does not fight. Faced with potential entry , it is more profitable for B to accommodate than to fight, but B's profits are greater still if there is no entry.

Consider how the game would unfold in each of the following circumstances (which are assumed to be common knowledge) :

I) B's managers are always concerned to maximize profits

II)B's managers are principally concerned to maintain their dominance over the market : considerations of profit are secondary.

In case I, it is clear that entry will take place and will be accommodated. If B's managers were to fight entry, they would be failing to maximize profits. Relying on this fact, A will enter the market.

In case II, however, entry will be deterred, because A knows that B's managers would fight entry. The detail of this illustration is discussed in the paper.

2) Here, I give a simple example.

Assume inverse demand is given by $P = a - b(x_A + x_B)$ with $a > 3c$, where x_A, x_B are the outputs of firm A and firm B, respectively. Assume also that the duopolistic industry is Cournotic and each firm has a constant marginal cost c.

Case i) Firm A and B both maximize their profits, and this is common knowledge. Then we will get the following results:

$x_A = x_B = (a - c)/3b$

$P = (a + 2c)/3$

$\Pi_A = \Pi_B = (a - c)^2/9b$

Case ii) Firm A maximizes its profit again but firm B maximizes its sales. Then we will obtain:

$x_A = (a - 2c)/3b, x_B = (a + c)/3b$

$P = (a + c)/3$

$\Pi_A = (a - c)^2/9b$, $\Pi_B = (a - 2c)(a + c)/9b$

If we compare the two cases, it can be concluded that $\Pi_A^{II} < \Pi_A^{I}$ and $\Pi_B^{II} > \Pi_B^{I}$. Hence, firm B earns more profit while maximizing sales. Roughly speaking, for a firm it is not needed to directly maximize profit to get maximum profit. John Vickers noted this idea by saying :" it is not nonsense to say that u-maximizers do not necessarily maximize u.".

To come closer to the problem mentioned in the introduction, let us first look at Vickers' example about one owner-one manager case for a symmetric oligopoly.

We assume that there are n firms in the industry and the objective of managers of firm i is to maximize

$$M_i = \Pi_i + \theta_i q_i \tag{1}$$

where

$$\Pi_i = p(Q)q_i - cq_i \tag{2}$$

and $Q = \sum q_i$. Combining (1) and (2), we have

$M_i = p(Q)q_i - (c - \theta_i)q_i$

It can be seen immediately that M_i is the same as the objective function of a profit-maximizing firm with unit cost of $c - \theta_i$. We assume that the game is solved in the Nash-Cournot fashion. Suppose that $p(Q) = A - Q$. Then in equilibrium , we have

$q_i^* = (p^* - c + \theta_i)$

$p^* = (A + nc - \sum \theta_j)/(n+1)$

$M_i^* = q_i^{*2}$

(It is assumed that $p^* > c - \theta_i$ for all i)

Since $\Pi_i = M_i - \theta_i q_i$, it follows that

$\Pi_i = (A - c - \sum \theta_j)[A - c + (n+1)\theta_i - \sum \theta_j]/(n+1)^2$.

The level of θ_i which maximizes Π_i given θ_j for $j \neq 1$ is

$\hat{\theta}_i = (n-1)(A - c - \sum_{j \neq i} \theta_j)/2n$

The Nash equilibrium of θ-setting game is symmetric with

$\hat{\theta} = (n-1)(A-c)/(n^2+1) > 0$

Correspondingly, we have

$\hat{\theta} = n(A-c)/(n^2+1)$

$\hat{P} = (A + n^2 c)/(n^2+1)$

$\hat{\Pi} = n(A-c)^2/(n^2+1)^2$

Compared with the case in which all firms are managed by profit-maximizers, output per firm is higher, price is lower and profits are lower.

Note that for $n > 1$, $\hat{\theta}$ is decreasing in n and goes to zero in the limit.

So, in this example, the extent of deviation from profit-maximization at the symmetric equilibrium vanishes as competition grows.

Now, briefly consider the case in which $\theta_j = 0$ for $j = 2..n$. That is, all firms but one are profit-maximizers. It can be seen that then $\theta_1 = (n-1)(A-c)/2n$ in which case

$\Pi_1 = (A-c)^2/4n = n\Pi_j$, $j = 2..n$

This shows rather vividly the extent to which non-profit maximizers can surpass profit-maximizers in terms of profits. Indeed, here the non-profit-maximizer earns greater profits than those of his rivals added together, no matter how many rivals there are.

3 DELEGATING WITH ONE MANAGER

Now, we will discuss the Sklivas (1987) model in more detail since our work will also follow the similar model. Also Koray and Sertel(1989) observed the same conclusions for asymmetric costs and only for the Cournot case.

In Sklivas' model, there is a duopoly in which firms, each having one owner and one manager , play a two-stage game. In the first stage the owners simultaneously write and publicly announce contracts with their managers that specify how they will be rewarded. In the second stage, the managers simultaneously choose their firms' output. Owners receive the resulting profits and managers are rewarded according to their contracts. Actually Fershtman and Judd (1985) independently and simultaneously obtain results similar to those in Sklivas'. By applying Nash equilibrium to both stages of the game, we obtain a subgame-perfect equilibrium as our solution.

Owner i measures his manager's performance according to some function of his firm's profits(Π_i) and revenues (R_i). We call this measure g_i, $i = 1, 2$. The higher g_i , the higher is manager $i's$ bonus or the lower is the likelihood that he will be fired. Because firm $i's$ output (x_i) does not enter manager $i's$ utility directly, he chooses x_i to maximize g_i. g_i is measured to be a linear combination of profits and revenues :

$$g_i = \lambda_i \Pi_i(x_1, x_2) + (1 - \lambda_i) R_i(x_1, x_2) = R_i(x_1, x_2) - \lambda_i C_i(x_i) \ , \quad i = 1, 2$$

Owner i simply chooses the parameter λ_i to determine his manager's incentives.

DEFINITION 1:

(x_1^*,x_2^*) is a Nash equilibrium in the managers' subgame if and only if $x_i^* = argmax \ g_i(x_i, x_j^*)$, $\{i, j\} = \{1, 2\}$

It is assumed that the owner knows demand and costs.

DEFINITION 2:

$(\lambda_1^*, \lambda_2^*)$ is a Nash equilibrium in the owner's subgame if and only if $\lambda_i^* = argmax\ \Pi_i(x_1^*(\lambda_i, \lambda_j^*), x_2^*(\lambda_i, \lambda_j^*) \quad \{i, j\} = \{1, 2\}$,

3.1 QUANTITY COMPETITION

Let there be a homogeneous product and let the marginal cost be constant. Without loss of generality let $c = 1$. We have $P = a - bx$, where P is the price, $a > 1$ and $x = x_1 + x_2$. We find manager i's best response function, $\phi_i(x_j, \lambda_i)$, by maximizing $g_i(.)$ over x_i. As λ_i is decreased, costs are weighted less, and $\phi_i(.)$ shifts out. Hence, decreasing λ_i commits manager i to behave more aggressively.

$x_i = (a - \lambda_i - bx_j)/2b = \phi_i(x_j, \lambda_i)$

The Nash equilibrium quantities as a function of (λ_1, λ_2) are

$x_i^* = (a - 2\lambda_i + \lambda_j)/3b \quad , i, j = 1, 2, \quad i \neq j$

Notice that as the owner i makes his manager more aggressive, by decreasing λ_i, his own firm's output increases, while his rival's decreases in equilibrium. We have the following profit function for the owners:

$\Pi(\lambda_i, \lambda_j) = [M + \lambda_i(6 - a - \lambda_j) - 2\lambda_i^2]/9b$

where, $M = a^2 - 3a - 3\lambda_i + 2a\lambda_j + \lambda_j^2$

The owner's best-response function and Nash equilibrium are given as:

$\lambda_i = (6 - a - \lambda_j)/4$

$\lambda_i^* = (6 - a)/5, \ i = 1, 2$

PROPOSITION 1:

In the owner-manager game managers behave more aggressively than profit maximizers, i.e. $\lambda_i^* < 1 \ i = 1,2$. This results in outputs that are higher than in Cournot model, yet still below the social optimum, i.e. $a/2b > x_i^*(\lambda_1^*, \lambda_2^*) > x_i(1,1) \quad , i = 1,2$

3.2 PRICE COMPETITION

In this section, we will look at Sklivas' price competition case in more detail.

We analyze this for the case of symmetric product differentiation, linear demand, and constant marginal cost c. We write linear demand as:

$x_i = \alpha - P_i + \beta P_j, \ 0 < \beta < 1 \ , i,j = 1,2 \ i \neq j \quad 0 < c < \frac{\alpha}{(1-\beta)}$

where P_i is firm i's price. The solution concept is the same as above one. Manager $i's$ best-response function is:

$P_i = (\alpha + \lambda_i c + \beta P_j)/2 = \phi_i(P_j, \lambda_i)$

The Nash equilibrium prices as a function of (λ_1, λ_2) are

$P_i^* = (2\alpha + 2\lambda_i c + \alpha\beta + \beta\lambda_j c)/(4 - \beta^2)$

Notice that as λ_i varies, both prices move in the same direction. This yields the following profit function for the owners, where $K = (2\alpha + \alpha\beta + \lambda_j\beta c)(2\alpha + \alpha\beta + \beta\lambda_j c - 4c + \beta^2 c)$ is a constant.

$\Pi_i(\lambda_i, \lambda_j) = [K + \lambda_i(2\alpha\beta^2 c + \alpha\beta^3 c + \beta^3 c^2 \lambda_j - 6\beta^2 c^2 + \beta^4 c^2 + 8c^2) + \lambda_i^2(2\beta^2 c^2 - 4c^2)]/(4 - \beta^2)^2 \quad i,j = 1,2 \ , \ i \neq j$ The owner's best-response functions and Nash equilibrium incentives are given as follows:

$\lambda_i = (2\alpha\beta^2 + \alpha\beta^3 + \beta^3\lambda_j c - 6\beta^2 c + \beta^4 c + 8c)/c(8 - 4\beta^2)$

$\lambda_i^* = (2\alpha\beta^2 + \alpha\beta^3 - 6\beta^2 c + \beta^4 c + 8c)/c(8 - 4\beta^2 - \beta^3) \quad , i = 1,2$

PROPOSITION 2:

In the owner-manager game firms that compete in prices behave less aggressively than profit maximizers, i.e. $\lambda_i^* > 1$. This results in higher prices than in Bertrand model i.e. $P_i(\lambda_1^*, \lambda_2^*) > P_i^*(1,1)$.

The consequences of the separation of ownership and management are reversed under price competition; firms act as profit maximizers with greater than true cost, resulting in higher prices.

Here, the reader may wonder what the wages of managers are. One possible explanation is as follows: Wages paid to managers are fixed and there are many of equal quality managers so that owners can find others if the present ones do not behave in accordance with the delegated maximands. We adopt the same explanation in our discussions.

4 EXTENDING THE DELEGATION CHAIN LENGTH

4.1 THE COURNOT CASE WITH HOMOGENEOUS PRODUCT

As mentioned in the introduction, one may ,moreover, wonder that what restrains owners redelegating further. Koray and Sertel (1989) discussed this problem in detail and first found that if there is no restriction on redelegating in a symmetric Cournotic duopoly, there is incentive to do so. Actually, if both owners have k chain below, then one lengthening one more will gain more profit than the other owner. It should be noted that the following results follow under the assumption that none of the owners can decrease the chain.

Let price be $P = a - (x_1 + x_2)$ where 1 and 2 are names of firms producing the same good. The equilibrium λ's of owners to be assigned to the below managers are:

$\lambda^*_{1,0} = \lambda^*_{2,0} = \frac{(k+1)(2k+1)}{k(2k+3)}\alpha \quad , \alpha = a - c.$

where k is the number of managers in a firm.

Note that

$\frac{\partial \lambda^*_{1,0}}{\partial k} < 0$ and $\lim_{k \to \infty} \lambda^*_{1,0} = \alpha$ which means that as chain grows, owner will exeggarate less his true efficiency and in the limit, he will tell the true one, $\alpha = a - c$.

Total output in the industry in equilibrium is:

$x^* = x^*_1 + x^*_2 = \frac{2(k+1)}{2k+3}\alpha$ and $\frac{\partial x^*}{\partial k} > 0$ and $\lim_{k \to \infty} x^* = \alpha$

Thus, output will increase and reach the socially efficient one. They noted that for any fixed $k \in N$, each owner exaggerates the efficiency of his firm when he sends down a maximand to his immediate subordinate is greater than his firm's true efficiency α (except for trivial case where $\alpha = 0$). Moreover, the efficiency is further exaggerated which h^{th} level delegate receives from $(h+1)^{th}$ level delegate the parameter $\frac{(h+1)(2h+1)}{h(2h+3)}\alpha^{h+1}$ which is greater than α^{h+1} whenever $h \in K - \{0\}$. So, total industrial output corresponds to that at the ordinary Cournot equilibrium of a symmetric linear duopoly with an exaggerated efficiency and is thus greater than total output at the ordinary Cournot equilibrium of the actually existing symmetric linear duopoly whose true efficiency is α.

Furthermore, they also noticed that the paradoxical thing as the length of the delegation chain gets larger, the owners exaggerate their efficiency less, yet total industrial output becomes greater. But it can be explained that at the industrial floor $efficiency = \frac{3(k+1)}{2k+3}\alpha$ which is monotonically increasing function of k. and as $k \to \infty$, efficiency of floor goes to $\frac{3}{2}\alpha$ which is consistent with the fact that output at the ordinary Cournot equilibrium of a symmetric linear duopoly with $efficiency$ $\frac{3}{2}\alpha$ is equal to efficient output where $efficiency = \alpha$.

4.2 THE BERTRAND CASE WITH TWO DIFFERENTIATED PRODUCTS

Now we are ready to explain our contributions. Actually, we will follow the same model as Sklivas' one such that there is a symmetric duopoly with constant marginal cost ,c and firms perform the Bertrand competition at the floor level.

There are two differentiated products 1, 2 and demands are:

$x_i = \alpha - P_i + \beta P_j,\ i, j = 1, 2,\ i \neq j,\ \ 0 < \beta < 1.$

where $P_i's$ are prices of commodities. Actually, we will assume $0 < c < \frac{\alpha}{1-\beta}$ so that we eliminate the case of inaction in the equilibrium. The reasoning of $0 < \beta < 1$ is obvious. Moreover, we assume that there is no extraneous cost to redelegate and it is permissible. Our conjectures are almost evident that as opposed to Cournot case (Koray and Sertel(1989)) one can expect that as delegation chain grows, the equilibrium profits, prices and outputs will converge to that of collusion case, i.e. joint-profit maximization one.

First let us give the results of collusion case:

$Max_{P_1,P_2} \Pi_1 + \Pi_2$

where Π_i is the i^{th} firm's profit, $i = 1, 2$.

$Max_{P_1,P_2}(P_1 - c)(\alpha - P_1 + \beta P_2) + (P_2 - c)(\alpha - P_2 + \beta P_1) = f(P_1, P_2)$

F.O.C.

$\frac{\partial f}{\partial P_1} = 0$

$\frac{\partial f}{\partial P_2} = 0$

From here, one can easily find that at the optimal point :

$P_1^* = P_2^* = \frac{\alpha}{2(1-\beta)} + \frac{c}{2}$

$x_1^* = x_2^* = \frac{\alpha - c(1-\beta)}{2}$

$\Pi_1^* = \Pi_2^* = \frac{[\alpha - c(1-\beta)]^2}{4(1-\beta)}$

Let us explain how to find subgame perfect Nash equilibrium in our model. Although following backward or forward induction is not important, we will follow backward one. Now given a fixed number, n, of delegation, at the floor, level n, they will decide prices via making Bertrand competition.

$h_{n,j} = n + 1, \ j > 2n - 1$

and n is the number of managers in each firm.

Then $\lim_{n\to\infty} \lambda^*_{1,0} = 1$ and $\lambda^*_{1,0}(n+1) < \lambda^*_{1,0}(n)$

Although we have found $\lambda^*_{1,0}$ by looking at the results found by the program , one has to prove it. Also note that, for now, we disregard any indeterminacy in the limit via relying on our $g's$ regularity. These results, if true, enable us to make the interpretation that as the delegation chain gets larger, owner's delegation will approach to the true one, i.e. $\lambda^*_{1,0} = 1$ and this convergence will be monotonic. Notice that according to these results, Cournot case (Koray and Sertel(1989)) and Bertrand case give the same qualitative convergences. Moreover, Unver (1995) who tried the same problem by using Cournot competition ,i.e. symmetric linear Cournotic duopoly with differentiated products, has found graphically and intuitively that $\lim_{n\to\infty} \lambda^*_{1,0} = \lambda^*_{2,0} = 1$ and $\lambda^*_{1,0}(n+1) > \lambda^*_{1,0}(n)$. Interestingly, for a fixed $n \in N$, our $\lambda^*_{1,0}(n)$ and Unver's one are symmetric with respect to $\lambda_{1,0} = 1$.

STATEMENT 2:

In the above game, the equilibrium output of firms are:

$x^*_2 = x^*_1 = (-1)^{n+1} \frac{g(n)[\alpha - c(1-\beta)]}{f(n)}$

where

$g(n) = \sum_{j=0}^{n+1} (-1)^{\lfloor \frac{j+n+1}{2} \rfloor} \, h_{nj} 2^{n+1-j} \, \beta^j$

$$h_{nj} = \begin{cases} h_{n-1,j} + h_{n-2,j-2} & \text{if } j \textit{ even and } 1 \leq j < n \\ 0 & \text{if } j \textit{ odd and } 1 \leq j < n \end{cases}$$

$h_{n0} = 1$

$$h_{n,n+1} = \begin{cases} 1 & \text{if } n \textit{ odd} \\ 0 & \text{if } n \textit{ even} \end{cases}$$

$$h_{n,n} = \begin{cases} 0 & \text{if } n \textit{ odd} \\ \frac{(n+1)}{2} & \text{if } n \textit{ even} \end{cases}$$

and

$f(n) = \sum_{j=0}^{n+1} (-1)^{\lfloor \frac{j+1}{2} \rfloor} \, h_{nj} 2^{n+1-j} \, \beta^j$

$$h_{nj} = \begin{cases} h_{n-1,j} + h_{n-1,j-1} & \text{if } j \textit{ even } 2 \leq j < n+1 \\ h_{n,j-1} & \text{if } j \textit{ odd } 2 \leq j < n+1 \end{cases}$$

$h_{n0} = h_{n1} = h_{n,n+1} = 1$

Here, we can have one more conjecture that:

$\lim_{n \to \infty} \frac{(-1)^{n+1} g(n)}{f(n)} = \frac{1}{2}$

5 WELFARE COMPARISONS AND CONCLUDING REMARKS

DEFINITION:

Welfare is defined as the sum of producers' and consumers' surplus, i.e.

$W = PS + CS$

$W = (\Pi_1 + \Pi_2) + (\sum_{i=1}^{2} \int_0^{x_i^*} [P_i(x_i, x_j^*) - P_i(x_i^*, x_j^*)]dx_i$

Since, in our model, demands are affine, it is easy to find a compact form for the welfare function. The i^{th} commodity's producers' and consumers' surplus are found as follows:

$PS_i = x_i^*(P_i^* - c)$

$CS_i = (P_i^- - P_i^*)\frac{x_i^*}{2}, \quad i = 1, 2$

where x_i^*, P_i^* are the equilibrium values and P_i^- is the the price at which the demand curve intersects the price-axis.

Since we know that at equilibrium both firms have the same price and quantities, we can write the welfare formula as follows:

$W = W_1 + W_2 = 2(PS_1 + CS_2)$

$W = \frac{2\alpha(1+\beta)x_1^* - (1+2\beta)x_1^{*2} - 2c(1-\beta^2)x_1^*}{(1-\beta^2)}$

One should notice that we are dealing with calculating only equilibrium welfares. Actually, there may be many different kinds of welfare functions, but here we accepted the usual one which gives the same weight to both producer and consumer sides.

Using this formula, we got the graphs for pure Bertrand case, one owner-one manager,..., one owner-5 manager and collusion case and took $\alpha = 2$, $c = 1$.(see A6)

It is seen from the graphs that for sufficiently small $\beta's$ there is no significant difference among the welfares; however, one can easily conclude that all welfare graphs coincide at some $\beta - value$ and this has a very strong implication that there exists some market in which application of any two of different regulations mentioned above give the same welfare for the society under our welfare function. That is, none of the above cases has a uniform superiority according to welfare. Moreover, if we examine the marginal welfare graphs, i.e. the graphs showing the difference between one case and the other that has one more manager, we see that there are some $\beta's$ at which they are equal and for sufficiently large β, the marginal welfare is decreasing (see A7). On the other hand, collusion case compared with one owner-5 manager case is worse for most of the $\beta - values$. This is not surprising because as firms try to collude, consumers will lose more.

In the study, we basically combined the ideas in papers Koray and Sertel(1989) and Sklivas(1987) for a symmetric duopoly that compete in prices, where redelegation is permitted. In fact, Sklivas(1987) proved that under rather mild conditions, in a symmetric duopoly in which each firm has one owner-one manager and that compete in prices, firms will behave less aggressively i.e. equilibrium prices will be higher than that of the naked Bertrand Model. In addition, profits of firms will increase. Using the same model as Sklivas and having the motivation of redelegating from Koray and Sertel (1989), we have a very important clue that according to the profit graphs, firms will have positive incentive to redelegate and it seems that profit approaches the collusive one. Although we have a strong intuition for our conjecture that as the number of managers goes to infinity, firms will approach to collusion, the statements have not been proved yet. We believe, if true, this collusion result is very crucial in the sense two firms can collude non-cooperatively and this can be proposed as a regulation mechanism.

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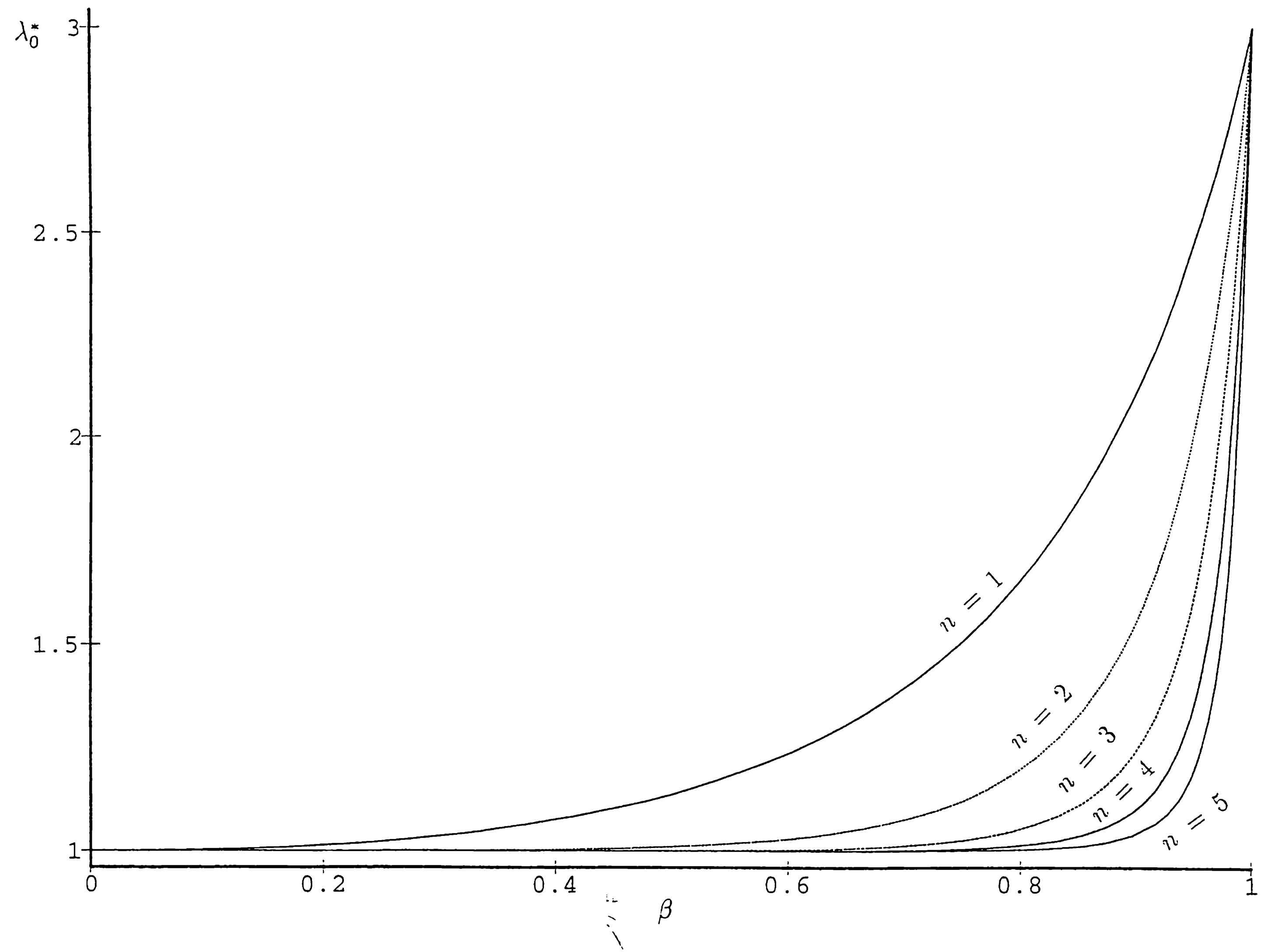
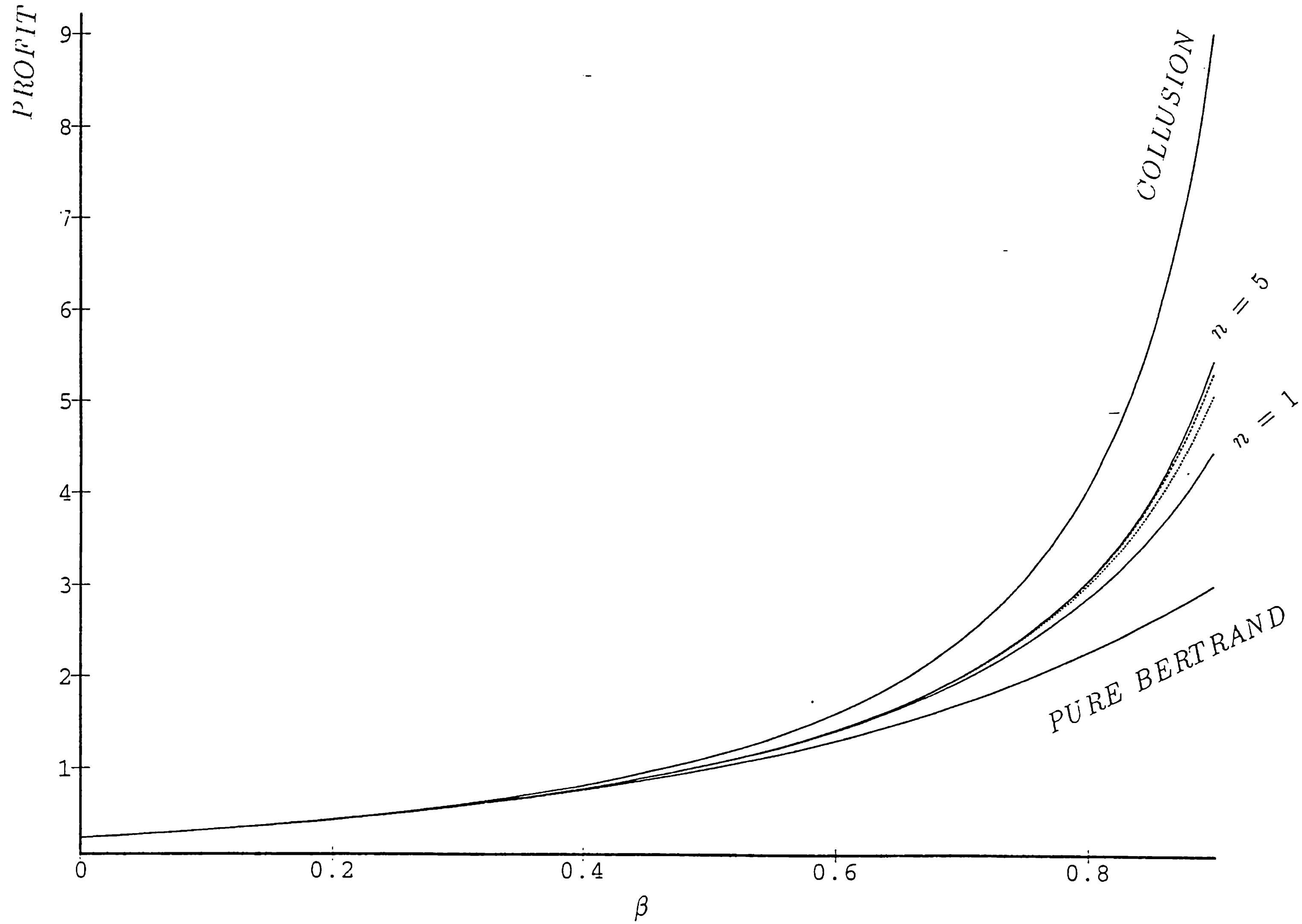
Appendix A

XMAPLE PROGRAM

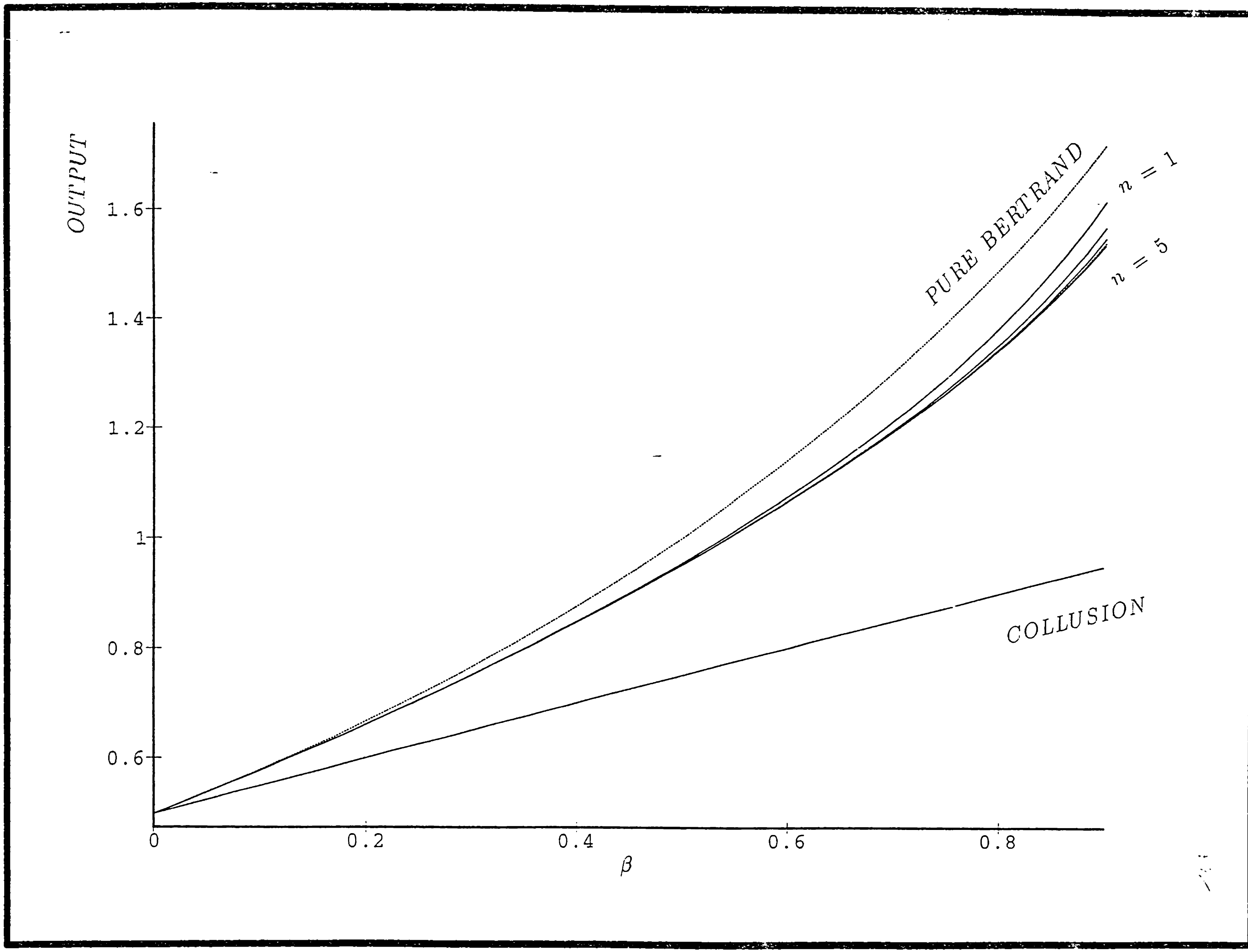
```
n:=1;                                  {number of managers}

x1:=a-p1+b*p2;                         {demand equation for good 1}

x2:=a-p2+b*p1;                         {demand equation for good 2}

pr1[n]:=(p1-l1[n-1]*c)*x1;             {firm 1 profit}

pr2[n]:=(p2-l2[n-1]*c)*x2;             {firm 2 profit}

assign(solve({diff(pr1[n],p1)=0,diff(pr2[n],p2)=0},{p1,p2}));{bertrand
competition at floor level}

for i from n-1 by -1 to 1 do

 pr1[i]:=(p1-l1[i-1]*c)*x1;
 pr2[i]:=(p2-l2[i-1]*c)*x2;

assign(solve({diff(pr1[i],l1[i])=0,diff(pr2[i],l2[i])=0},{l1[i],l2[i]}))
od;{solves all lamdas till the owner's

one}

factor(l1[1]);                         {factorizes lambda}
```

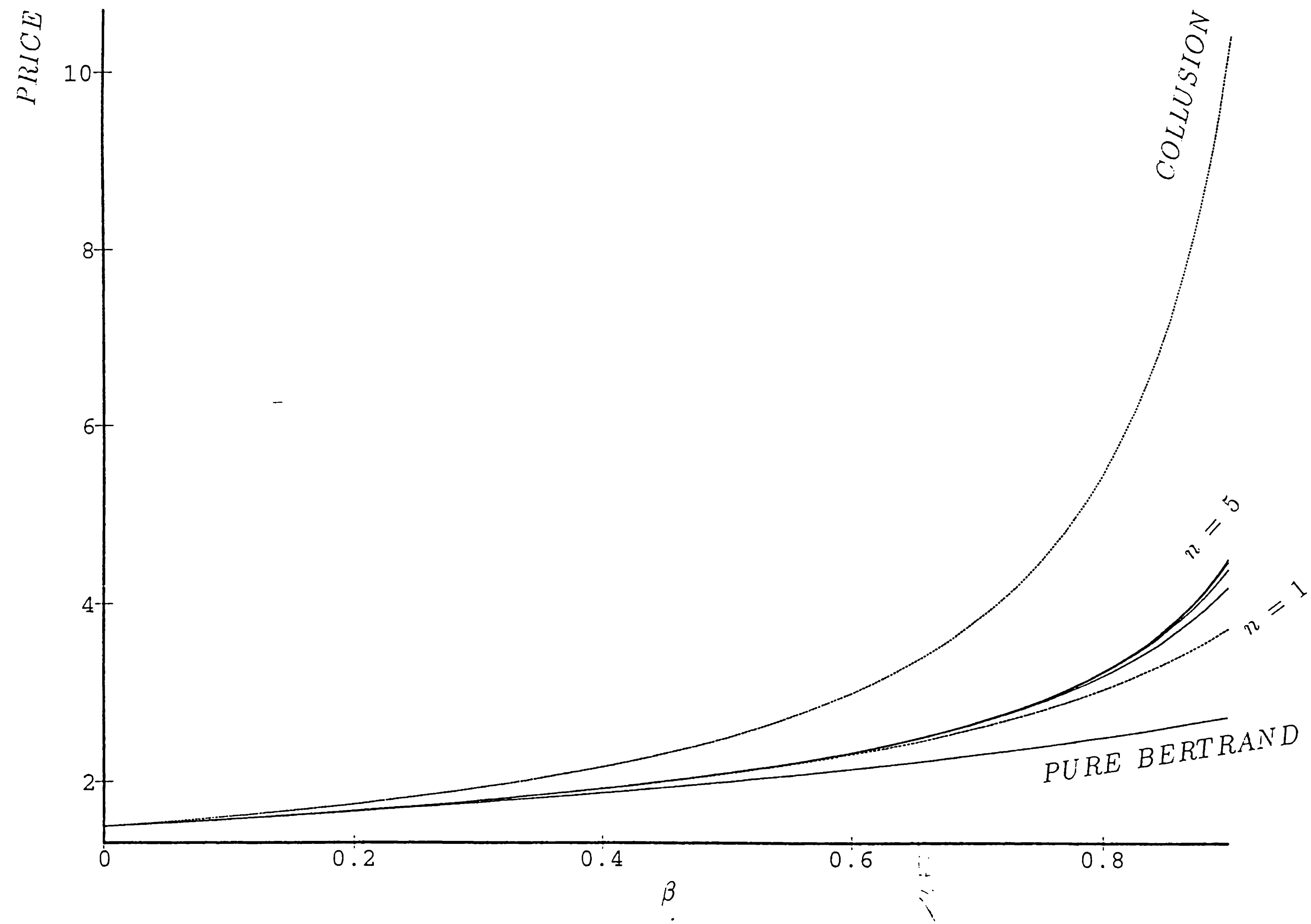


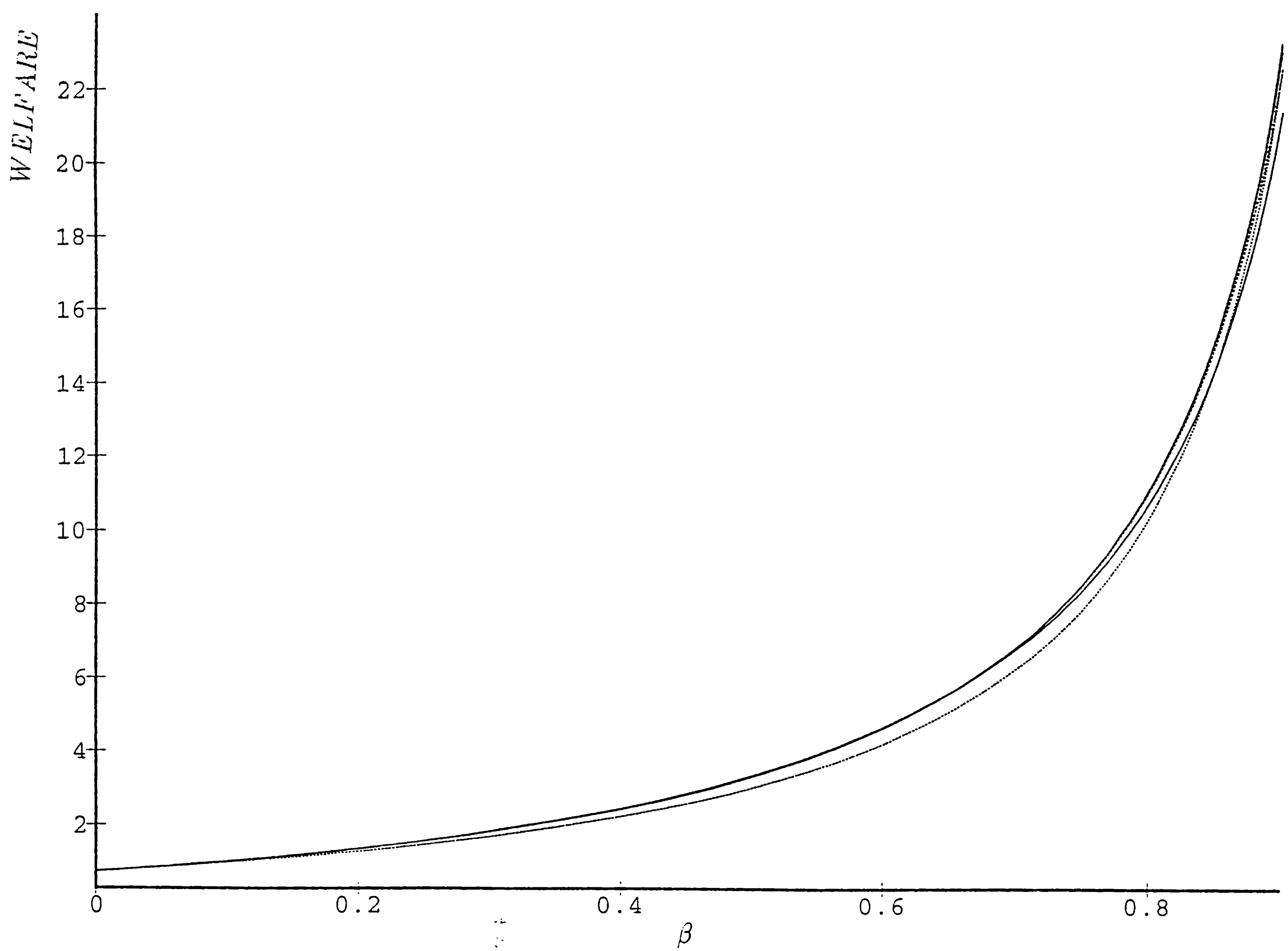
A2

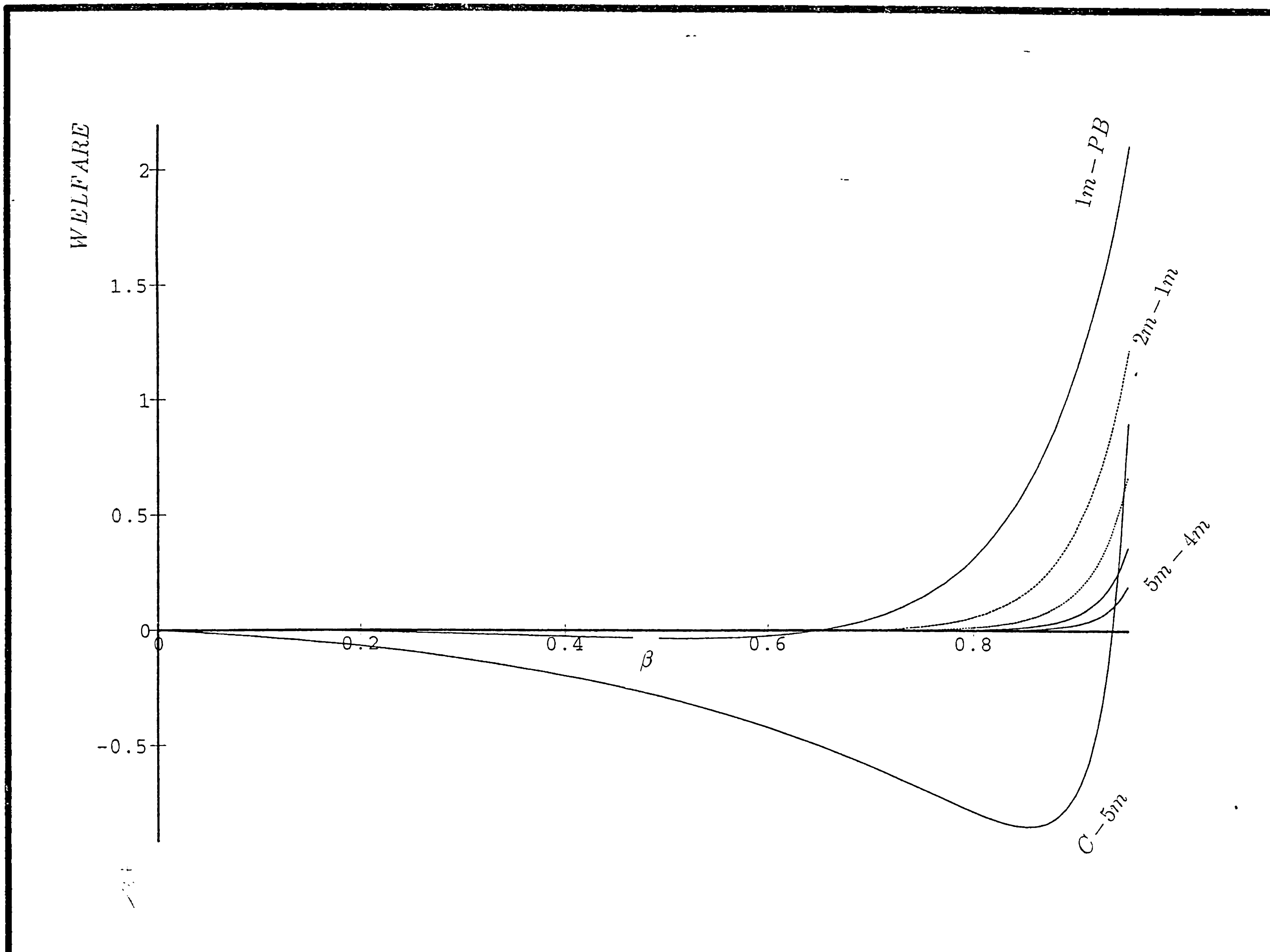


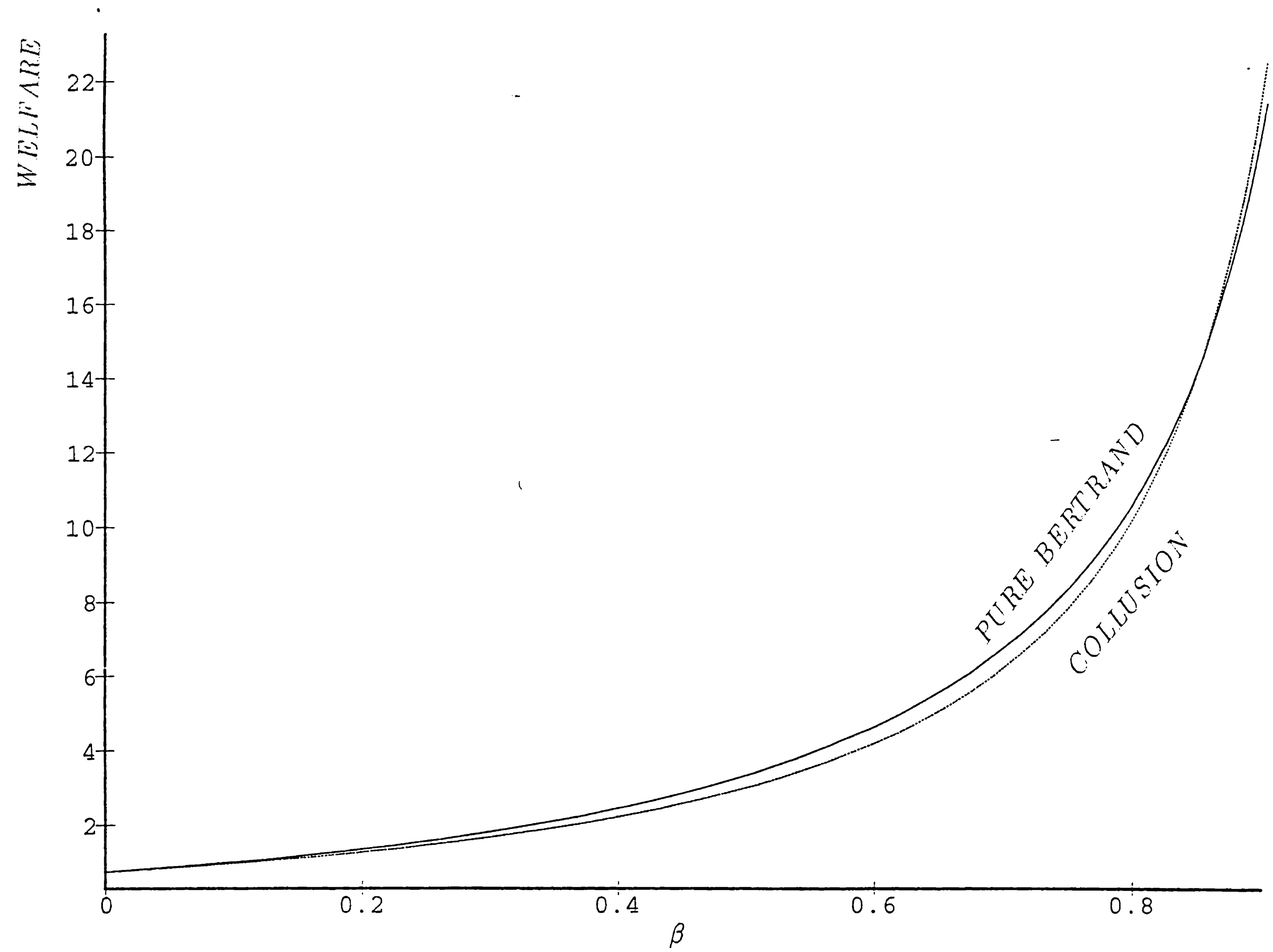
PROFIT
9
8
7
6
5
4
3
2
1
0
0.2
0.4
0.6
0.8
β
COLLUSION
n = 5
n = 1
PURE BERTRAND


OUTPUT
1.6
1.4
1.2
1
0.8
0.6
0
0.2
0.4
0.6
0.8
β
PURE BERTRAND
n = 1
n = 5
COLLUSION


PRICE
COLLUSION
$n = 5$
$n = 1$
PURE BERTRAND
10
8
6
4
2
0
0.2
0.4
0.6
0.8
β


WELFARE
22
20
18
16
14
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
β


WELFARE
2
1.5
1
0.5
0
-0.5
0.2
0.4
0.6
0.8
β
1m − PB
2m − 1m
5m − 4m
C − 5m



WELFARE
22
20
18
16
14
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
β
PURE BERTRAND
COLLUSION