DELEGATION IN A DUOPOLISTIC DIFFERENTIATED GOODS MARKET WITH COURNOT COMPETITION

A Thesis Submitted to the Department of Economics and the Institute of Economics and Social Sciences of Bilkent University in Partial Fulfillment of the Requirements for the Degree of

MASTER OF ARTS IN ECONOMICS

by
MUSTAFA UTKU ÜNVER
July, 1995

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I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Prof. Dr. Semih Koray

I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Prof. Dr. Asad Zaman

I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Prof. Dr. Farhad Husseinov

Approved by the Institute of Economics and Social Sciences

Director:

ABSTRACT

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MUSTAFA UTKU ÜNVER
MA in Economics
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We consider the impact of delegation in a Cournotic duopoly with differentiated goods upon the firms' profit maximization behavior. In an oligopoly, delegation in each firm can be modeled through a specific non-cooperative game. Delegation games in a differentiated goods market with affine demand are studied within the Cournot competition concept where redelegation is permitted in a symmetric duopoly. The following results are demonstrated: The maximand delegated by each primary delegator, i.e. owner of each firm, converges in monotonically decreasing fashion to the true profit function in the absence of delegation costs, and total industry output at the Cournot equilibrium converges in monotonically increasing fashion to some output level. Welfare changes due to redelegation are also considered.

KEYWORDS: Delegation, Efficiency, Theory of Firm, Regulation, Non-cooperative Games, Cournot Competition, Bertrand Competition, Duopoly, Differentiated Goods, Principal-Agent Games, Nash Equilibrium.

ÖZ

BİR DÜOPOLDEN OLUŞAN FARKLILAŞTIRILMIŞ MAL PİYASASINDA "COURNOT" REKABETÇİ DELEGASYON

MUSTAFA UTKU ÜNVER
Yüksek Lisans Tezi, İktisat Bölümü
Tez Yöneticisi: Prof. Dr. Semih Koray
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Farklılaştırılmış mallar üreten bir "Cournot" rekabetçi düopoldeki delegasyonun, firmaların kar maksimizasyonu davranışı üzerindeki etkisini incelemekteyiz. Bir oligopolde, firmalar içindeki delegasyon özel bir işbirliksiz oyunla modellenebilir. Simetrik bir düopol kapsamında, afin talepli bir farklılaştırılmış mal piyasasında, yönetici atamalarına izin verilmesi durumunda "Cournot" rekabetçi delegasyon oyunları incelenmektedir. Su sonuçlar sergilenmektedir: Her firmadaki firma sahibi gibi ilksel yetkilendirici tarafından belirlenen amaç fonksiyonu delegasyon giderleri yokluğunda monoton azalarak gerçek kar fonksiyonuna, endüstri boyutunda üretilen toplam "Cournot" denge miktarı monoton artarak bir üretim düzeyine yakınsar. Tekrarlanan delegasyon sonucunda ortaya çıkan toplumsal refah değişimleri de incelenmektedir.

ANAHTAR SÖZCÜKLER: Delegasyon, Etkinlik, Firma Teorisi, Regülasyon, İşbirliksiz Oyunlar, Cournot Rekabet Modeli, Bertrand Rekabet Modeli, Düopol, Farklılaştırılmış Mallar, İşveren-İşçi Oyunları, Nash Dengesi.

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CONTENTS

I. INTRODUCTION

In microeconomic theory, firms are generally treated as agents with sole objective of profit maximization. Many economists have argued this as simplistic and claimed that the relations between managerial processes within a firm should also be considered to determine the true behavior of a firm.

The objective of a large corporation may deviate from the maximization of profits. The complicated decision processes in the corporation and separation of ownership and management may be reasons for deviating from profit maximization. That can cause the firms to be interested in maximizing revenues, market share or another objective related to those rather than profits and to have market power as their ultimate goal.

In the case of entry deterrence in a single good industry, in which a monopoly is producing and selling its products through profit maximizing behavior and which is threatened by an outside firm with entry, the latter firm will not be deterred. However, if the monopoly hires managers who maximize revenues and the action of the managers are announced, the entrance of a profit maximizing firm will be deterred. Although the underlying non-cooperative games are different in both situations, the monopoly which has managers maximizing the market share gains more profit than the classical duopolistic firm (Vickers (1985)). Hence utility maximizers do not indeed directly maximize utility but may still attain highest utility possible.

Hence the consideration of separating ownership and management from each other may be a good starting point of stressing this type of behavior. Within large firms forming an oligopoly in a single good market, the separation of ownership and management gives the owner the opportunity not to directly maximize firm's profits. The behavior of the manager totally dependent on the owner's and his rival managers' in the other firms may then reflect a totally complicated action, i.e. profit maximization based on a fictitious marginal cost assigned by the owner.

Another complicated question arises when the number of managers vertically delegated increases: What will the behavior of the owners be? How far will they deviate from profit maximization? For the homogeneous good market such studies have been done through redelegation schemes by Koray - Sertel (1989a, 1989b, 1989c).

The main aim of this study can be summarized as demonstrating the behavior of owners and managers in a delegation game if redelegation of managers in each firm is permitted in a duopolistic differentiated goods market with a certain maximand type delegated to each manager in Cournot competition concept.

I.1. A SURVEY ON DELEGATION LITERATURE

Such a delegation scheme is literally adopted by Vickers (1985). He first stresses that in the case of entry deterrence, a non-profit maximizing firm may gain more profits than a profit maximizing firm under certain circumstances. The scenario Vickers uses is a two-stage game in a duopoly or n-firm oligopoly in which in the first stage owners simultaneously select the managers' incentives and in the second stage the managers endowed with the maximands delegated by the owners simultaneously choose outputs of firms accordingly, i.e. they maximize profit functions of firms with fictitious costs determined by the owner. The case where the good market has an affine inverse demand and the firms have the same constant marginal cost is analyzed. However, outcomes of the game are not incentive compatible. Furthermore, the results show that quantity competition under delegation ends with lower profits, higher outputs and lower price for each firm than competing in ordinary Cournot fashion with an objective of direct profit maximization. In this scenario the managers hired do not receive any share from the profits of the firms but they have fixed wages.

Later study by Fershtman and Judd (1987) analyzes incentive contracts that owners will choose for their managers in an oligopolistic concept, focusing on how competing owners may strategically manipulate these incentive contracts and the resulting impact on the oligopoly outcome, introducing both quantity competition in a similar homogeneous good market with Vickers' maximand and price competition in a differentiated goods market with the same maximand. In the analysis they follow, firms with different constant marginal costs compete in a market with random linear demand. The randomness in the parameters of the demand is handled by taking the expected value of those and does not essentially change the method of analysis so that the decisions of managers remain the same as the case where the demand function is deterministic.

Sklivas (1987) deals with similar principal-agent games and further shows that in a differentiated goods market, duopolistic firms gain more profits as a result of price competition version of the owner-manager game than the ordinary Bertrand competition. Firms behave as profit maximizers with marginal costs more than the true cost, with the result that prices are higher than in the ordinary Bertrand game. It is also stated and proven that in a quantity competition version of owner manager game in a homogeneous good market, what firms produce is greater than the ordinary Cournot output and less than the socially efficient amount. Here, firms behave as profit maximizers with marginal costs lower than the true cost. In price competition in the owner-manager game in a differentiated goods market firms produce total output less than ordinary Bertrand level but more than the joint profit maximizing output (collusive case).

Koray and Sertel (1989a) develop the idea of redelegation in such an industry. By vertically redelegating new managers in the firm working with similar objectives that are dependent upon the incentive assigned by the manager above in the hierarchical chain where redelegation is assumed to be costless and by using the idea of improvement in the welfare in one manager delegation in quantity competition, they show that a regulatory mechanism can be imposed to improve the welfare in an oligopoly where information about marginal cost is private to each firm. This regulation is based on a method called as "Pretend-But-Perform" mechanism (PPM) and one delegation of it has the same subgame perfect Nash equilibrium with the game between owner and manager adopted by preceding economists. Another result Koray and Sertel propose is that in a m-firm oligopoly, the total output produced in the industry converges to the socially efficient one with the rate m^2 which was m in the ordinary Cournot game. So this scheme of Pretend-But-Perform resulted with m^2-m fictitious firms in the industry. Viewed this way, the m firms end up with producing the output load of m^2 firms, each firm producing the load of m copies of itself and m-1 of these fictitious firms. This brings to our attention another point, namely the number of market participants m. Under ordinary free entry, Cournot equilibrium with uniform setup costs c_0, let the equilibrium size of the industry be m*. For the free entry equilibrium number m' ending up actually producing positive output in their declaration and PPM, we have $m'^2 \geq m^* \geq m'$ except the situation (m',m*)=(1,2), while output with firms m' regulated by PPM is at least as large as that produced m* firms under ordinary free entry Cournot equilibrium. Hence PPM enhances regulation in an oligopoly.

Koray and Sertel (1989b) show that the one-delegation equilibrium concept in Vickers, Fershtman-Judd and Sklivas is unstable, and each principal has an incentive to redelegate, increasing the length of the delegation chain if decreasing the manager number already delegated is not permitted. When we consider the scenario in which if one owner redelegates then the second owner is obligated to redelegate because of the decrease in his profits arising otherwise, delegating only once is not a stable equilibrium. No manager in one firm and two managers in the other is a stable equilibrium as proposed by Koray and Sertel. However, another possibility is to increase the length of manager chain without bound. In this case, the industry-wide output at the Cournot equilibrium under delegation in the homogeneous good market converges in monotonically increasing fashion to the socially efficient one and the maximand delegated by each owner converges in monotonically decreasing fashion to the true profit function, the fictitious cost parameter goes to the true marginal cost in the limit i.e. the primary incentive parameter converges to 1. This outcome is also incentive compatible contrary to Vickers' one manager delegation. Now though it may seem paradoxical that as the length of the delegation chain increases, the owners exaggerate their efficiency less, yet the output gets larger. The efficiency (incentive parameter) assigned to the last manager who chooses the firm's output is still greater than 1 and converges to 3/2 as the manager chain grows beyond bound. More formally it is equal to $3(n+1)/(2n+3)$, where n is the number of managers in the chain.

The dimension that the above studies introduce leads to the following question: To what extent can the VFJS (Vickers, Fershtman - Judd, Sklivas) theory be considered as a work of Positive Economic Theory? Three problems arise with the theory. First, we can not restrict the managers to compete in quantity or in price in positive theoretic sense. They will want to çompete in most profitable fashion. Second, the maximand type can not be restricted to a certain one i.e. Vickers' maximand, owners can find a more profitable maximand. Indeed Koray and Sertel (1989c) find such maximands. Finally one delegation equilibrium is unstable and for stability firms should redelegate.

I.2. CRITIQUES TO VFJS THEORY

Vickers, Fershtman - Judd, Sklivas theory on delegation equilibria can not be considered as positive theory in many respects. If the managers of each firm are free to choose their competition type, i.e. in quantity or price, why will they choose to compete in Cournot fashion or why should the owners of the firms instruct them to do so? For example, in the differentiated goods market competing in price is more profitable as stated by Sklivas (1987). So it is wise for both owners to let their managers compete in Bertrand way. Unless a regulatory or dictatorial scheme imposes them to compete in Cournot fashion, price competition is better for each firm in economic sense. If any other type of competition resulting with more profits is found, this will be even better for each firm.

Another aspect that VFJS fails to explain is why the managers are restricted to the given maximand type, i.e. a linear combination of revenues and profits. Any other maximand yielding non-zero profit can be chosen as the objective. Especially, if the profits of firms are higher under the new maximand than the maximand given by VFJS, it will be wise for the owners to hand it down to their managers. Hence not limiting the maximand space to contain only the maximands specified by VFJS will be right in positive theoretic sense. The space of possible outcome quantities can allow many types of quantity competition with different maximands. Throughout our analysis, we take the maximand types restricted to Vickers' maximands (a linear combination of profit and revenue) and approach to the problem as one in economic design in an oligopoly instead of positive theoretic approach. Two further examples of other maximand spaces are given by Koray - Sertel (1989c).

The third problem regarding the delegation equilibrium concept is why to limit the delegation chain to one link. Koray - Sertel (1989b) show that this is unstable, since delegating to one more manager is more profitable for the owner than keeping the number of managers the same as his rival. We face with infinite delegation as one of the stable Nash equilibrium possibilities. If there are redelegation costs, then the chain will end up with a finite number of managers. However, the space of possible stable chain lengths also includes finite chain lengths in the absence of redelegation costs: One owner does not hire any managers and the other exactly hires two.

So approaching the problem as an economic design problem connected with some central planning mechanism is more realistic. Our study is an extended work on redelegation game in a duopolistic differentiated product market with quantity competition using the recursive delegation equilibrium concept (Koray - Sertel (1989b)). We have the theoretical set-up of VFJS and Koray - Sertel. Section II presents some illustrative examples and the basic model to analyze the differentiated goods market with symmetric firms. Section III presents the whole analysis of considered game forms with the ordinary Cournot competition, one-step delegation game in quantity and redelegation game of infinite length manager chain. We use a mathematics software package in analyzing that part in order to predict the behavior of the subgame perfect Nash equilibrium outcome in several-step delegation and give the computer program in the appendix. That section demonstrates our main result, i.e. the maximand delegated by the owner to the top manager approaches to the true profit function in monotonically decreasing fashion. In the last part of Section III we demonstrate changes in social welfare due to redelegation, and try to explain to what extent the design can be a regulatory mechanism.

II. SOME ILLUSTRATIVE EXAMPLES

As a starting point to the concept of delegation, the entry deterrence in a monopolistic market can be considered in Vickers' (1985) fashion. Firm A is deciding whether or not to enter a market currently monopolized by firm B. If A enters, B's managers must decide whether to respond aggressively or in an accommodating fashion. Entry is profitable for A if and only if B does not fight. In case of entry, B will profit more if it accommodates and not fights, and B's profits are greater if there is no entry still.

Two types of behaviors of the managers can be seen:

1) B's managers are concerned to maximize profits

2) B's managers are principally concerned to maintain their dominance over the market. Profits are in the second place.

In the first case entry will occur and it will be accommodated, since B will not be able to fight due to its managers' behavior. In second case entry will be deterred since A's managers will know B will fight since market domination is B's managers' primary concern, so knowing this A will not enter. Hence profit maximizing behavior will lead to less profits for firm B than the mixed strategy behavior of managers of B which has market dominance in mind in the first place.

Let the inverse demand be given by $P=a-x$. Assume entry involves payment of a sunk cost F^2. Suppose firm B is already in the market and managers maximize $\pi_B+\theta_B x_B$; a combination of profits and sales volume. Let c be the common constant marginal cost in each firm. Entry is profitable for firm A only if $\theta_B<a-c-3F$. Entry is deterred if this does not hold. Firm B's profits are higher than non-deterring profits if $((a-c)^2-\theta_B^2)/4>(a-c)^2/9$. So choosing θ_B in that way results with higher profits for firm B. Therefore entry deterrence by non-profit maximizers can be profitable if

$a-c>3F>(a-c)(1-\sqrt{5}/3)$.

Since our model is partially based on Sklivas' (1987) work, we can refer to his analysis as an insightful example. He examines a duopoly in which the firms, each having one owner and one manager, play a two-stage principal-agent game. In the first stage owners simultaneously write and publicly announce the contracts with their managers. In the second stage the managers simultaneously choose their firms' output. Owners receive the resulting profits and managers are rewarded according to their contracts but do not receive any share from the profit. Application of the Nash equilibrium concept to both stages results in a subgame-perfect Nash equilibrium. The performance of the manager of firm i is measured by a linear combination of the firm's profits π_i and its revenue R_i. So the duty of the manager is to try to maximize his performance. So he chooses his firm's output x_i to maximize g_i.

$$g_i = \lambda_i \pi_i(x_1,x_2) + (1 - \lambda_i)R_i(x_1,x_2)$$
$$= R_i(x_1,x_2) - \lambda_i C(x_i) \quad i=1,2$$

The owner of firm i simply chooses the incentive parameter λ_i to determine the manager's maximand in firm i.

Definition 1: (x_1^*, x_2^*) is Nash equilibrium output pair in managers' (agents') subgame if and only if $x_i^* = \textit{argmax}\ g_i(x_i, x_j^*)$ for $\{i,j\}=\{1,2\}$.

Definition 2: $(\lambda_1^*, \lambda_2^*)$ is Nash equilibrium outcome in owners' (principals') subgame if and only if $\lambda_i^* = \textit{argmax}\ \pi_i(x_1^*(\lambda_i, \lambda_j^*), x_2^*(\lambda_i, \lambda_j^*))$ for $\{i,j\}=\{1,2\}$.

The subgame perfect Nash equilibrium outcome is the market outcome $(x_1^*(\lambda_1^*,\lambda_2^*),x_2^*(\lambda_1^*,\lambda_2^*))$, the Nash equilibrium quantities in the managers' subgame that result from the Nash equilibrium incentives of the owners' subgame.

Sklivas considers both quantity competition (Cournot) in a homogeneous good market and price competition (Bertrand) in a differentiated goods market versions of the same game.

Quantity Competition: Let the inverse demand function be $P = a - b\,x$ and total output be given with $x = x_1 + x_2$, the sum of the firms' outputs in the market, and let the marginal cost for each firm be the same and constant. For the sake of simplicity let this cost satisfy $c = 1$. Let $a > c = 1$ for positive output production.

In the managers' subgame, manager i's best response function $\phi_i(x_j,\lambda_i)$ is found by maximizing g_i over x_i. And best response function of the owner i is found by maximizing π_i over λ_i given that x_i^* is determined by his manager in terms of λ_i. So it is found that

$$x_i = (a-\lambda_i-bx_j) / (2b) = \phi_i(x_j, \lambda_i),$$
$$x_i^* = (a-2\lambda_i+\lambda_j) / (3b),$$

and this gives the Nash equilibrium quantities of the managers' game as a function of (λ_1, λ_2). As owner i makes his manager more aggressive by decreasing λ_i, his own firm's output increases while his rival's decreases if the latter does not follow the former in doing so. The profit functions are given by

$\pi_i(\lambda_i,\lambda_j) = (M+\lambda_i(6-a-\lambda_j)-2\lambda_i^2) / 9b$, where M is given by

$M=a^2-3a-3\lambda_j+2a\lambda_j+\lambda_j^2$.

So the owners' best response functions are found as

$\lambda_i=(6-a-\lambda_j) / 4$.

In this market what Sklivas finds is that the Nash equilibrium incentives are determined through $\lambda_i^* = (6-a) / 5 <1$, i.e. the owners make their managers fight (which means to behave aggressively) in the market. The subgame perfect Nash equilibrium output is found as

$x_i^*(\lambda_1^*, \lambda_2^*) = 2(a-1) / (5b)$, and this is greater than

$x_i^*(1,1) = (a-1) / (3b)$

which is the ordinary Cournot output per firm. The price $P(x_1^*(\lambda_1^*,\lambda_2^*),x_2^*(\lambda_1^*,\lambda_2^*))$ is lower than the naked Cournot price. Finally the profit per firm is lower than the ordinary Cournot profit. The reason why owners hire managers is simple. If one firm hires a manager and the other does not, the profit of the manager hiring firm will exceed its Cournot profits and that of the non-deviating firm will decrease below the Cournot level. The reason why owners play such a game is the same as the simple Prisoners' Dilemma: There are two prisoners and they are threatened to confess their guilt. If one cheats and says that the other is the criminal, he will not be punished and , the one claimed to be guilty will be punished strictly. But if both cheat, they will both be punished. So both prisoners cheat at Nash equilibrium. Cheating corresponds here to hiring a manager and not cheating corresponds to competing by direct profit maximization without any delegation.

Price Competition: In the case of symmetric product differentiation where the market has an affine demand function and firms have constant marginal cost c, Sklivas considers the price competition version of the owner-manager game. And this will be the set-up of the model that we will consider later.

Demand function for firm i's output is given by an affine function

$x_i = \alpha - p_i + \beta p_j$, where $0 < \beta < 1$, $\{i,j\}=\{1,2\}$

and $0 < c < \alpha / (1-\beta)$ for positive quantity production, where p_i is the firm i's , p_j is firm j's price, x_i is the output produced by firm i.

Manager i's best response function $\psi_i(p_j, \lambda_j)$ is found by maximizing g_i over p_i. By increasing λ_i owner i makes his manager less aggressive, i.e. he responds with a greater p_i for any p_j:

$$\psi_i(p_j,\lambda_i) = (\alpha + \lambda_i c + p_j\beta) / 2 = p_i$$

The Nash equilibrium prices as a function of (λ_1,λ_2) are

$$p_i^* = (2\alpha + 2\lambda_i c + \alpha\beta + \beta\lambda_j c) / (4-\beta^2)$$

When λ_i varies, both prices move in the same direction. This yields the profit function

$$\pi_i(\lambda_i,\lambda_j) = K + \lambda_i (2\alpha\beta^2 c+\alpha\beta^3 c+\beta^3 c^2\lambda_j-6\beta^2 c^2+ \beta^4 c^2+8c^2+\lambda i^2(2\beta^2 c^2-4c^2)) / (4-\beta^2)^2$$

with $K = (2\alpha + \alpha\beta + \lambda_j\beta c) (2\alpha + \alpha\beta + \beta\lambda_j c-4c+\beta^2 c)$

So Nash equilibrium incentives are given by

$$\lambda_i=(2\alpha\beta^2+\alpha\beta^3+\beta^3\lambda_j c-6\beta^2 c+\beta^4 c+8c) / c(8-4\beta^2),$$

$$\lambda_i^*=(2\alpha\beta^2+\alpha\beta^3-6\beta^2 c+\beta^4 c+8c) / c(8-4\beta^2-\beta^3).$$

So in the owner-manager game, firms that compete in prices behave less aggressively than profit maximizers, i.e. $\lambda_i^*>1$. This results with higher prices than the Bertrand model i.e. $p_i^*(\lambda_1^*,\lambda_2^*) > p_i^*(1,1)$. The consequences of the separation of ownership and management reverse under price competition; firms act as profit maximizers with fictitious marginal costs greater than true ones, resulting in higher prices in this model. Firms also receive higher profits than the Bertrand model case: Firm 1's unilateral deviation from profit maximization raises both firms' profits, commitment has a cooperative effect on the price competing duopoly, so firms earn higher profits but still less than the perfectly collusive case. The next section presents the delegation model adopted by this study using Vickers' maximands.

III. THE MODEL

Our model considers a duopoly with symmetric firms in a differentiated goods market with affine demand for each firm. For the sake of compatibility, we assume that the form of the demand is the same as Sklivas'. If we state it once more for notation convenience:

$x_i = \alpha - p_i + \beta p_j$,where $0 < \beta < 1$ for $\{i,j\}=\{1,2\}$,

and $\alpha / (1-\beta) > c > 0$ for positive amount of output production, p_i is the price of output of i'th firm, p_j is the price of the output of j'th firm, x_i is the output of i'th firm, and c is the constant marginal cost of both firms. We examine the Cournot competition concept in this market to see whether Sklivas' (1987) results for the behavior of owners in a homogeneous good market hold for our case.

III.1. COURNOT COMPETITION

Here we use the ordinary Cournot model for the duopoly. We assume that the two firms in the market are competing for quantity in direct profit maximizing fashion. Here there are no managers in the firms buf owners decide the quantity in each firm by themselves. Inverse demand function for each firm is given by

$p_i = (\alpha(1+\beta)-x_i-\beta x_j) / (1-\beta^2)$ for $\{i,j\}=\{1,2\}$.

Definition 3: The quantity pair (x_1^*,x_2^*) is Nash equilibrium output of the Cournot competition if and only if $x_i^*=\mathit{argmax}\ \pi_i(x_i,x_j^*)$ for $\{i,j\}=\{1,2\}$.

Given this definition best response function of i'th firm is

$\phi_i(x_j) = [\alpha(1+\beta)-\beta x_j-c(1-\beta^2)] / 2 = x_i$.

So the Nash equilibrium output of the ordinary Cournot competition is

$x_i^* = [\alpha(1+\beta)-c(1-\beta^2)] / (2+\beta)$, resulting with the price

$p_i^* = [\alpha+c(1-\beta^2)] / [(2+\beta)(1-\beta)]$ for $i=1,2$.

III.2. DELEGATION GAME

Using the same market framework, we can wonder what will happen if the owners delegate only one manager. They will play a two stage non-cooperative game called the owner-manager game. In the first stage owners simultaneously write managers' contracts and using these contracts managers maximize their maximands and choose the decision variable of the game, output of the firm in which they work.

Here information is assumed to be perfect however, the marginal cost of the firm is private to the owner and the manager of it. Managers are assumed to work with a fixed wage, and do not receive any share from the profit of the firm. The profits of firm i can be written as:

$\pi_i = p_i\, x_i - c\, x_i$ and revenue:

$R_i = p_i\, x_i$.

We restrict the maximand of managers to $g_i = \lambda_i\, \pi_i - (1 - \lambda_i)\, R_i = p_i\, x_i - \lambda_i\, c\, x_i$

Using definition (1) and definition (2), best response function of manager i can be written as

$$\phi_{1,i}(x_j, \lambda_i) = [\alpha(1+\beta)-\beta x_j-\lambda_i c(1-\beta^2)] / 2 = x_i \text{ for } \{i, j\}=\{1,2\},$$

and Nash equilibrium output of managers' subgame is determined as

$$x_i^*(\lambda_i,\lambda_j) = [\alpha(2-\beta)(1+\beta)-2\lambda_i c(1-\beta^2)+\lambda_j c\beta(1-\beta^2)] / (4-\beta^2) \text{ for } \{i,j\}=\{1,2\}.$$

The higher the value of λ_i, the less output is produced in firm i, and more is produced in the rival firm j. Note that $x_i^*(1,1)$ is nothing but the naked Cournot output.

Owners take this value and maximize their objectives over their incentive on their managers. Best response function of the owners can be written as

$$\phi_{0,i}(\lambda_j) = [3(1-\beta^2)\beta c\lambda_j + N] / 4c(\beta^2-2)(1-\beta^2) = \lambda_i \text{ ,where N denotes}$$

$$N = -2(1+\beta)(4-\beta^2) - 2\beta\alpha(2-\beta)(1+\beta) - 2c(1-\beta^2)(4-\beta^2) - \alpha(4-\beta^2)(2-\beta)(1+\beta) \text{ for } \{i,j\}=\{1,2\}.$$

The subgame perfect Nash equilibrium incentive is found as

$$\lambda_i^* = - [\alpha\beta^2+2c(\beta-1)(\beta+2)] / [c(\beta-1)(\beta^2-2\beta-4)],$$

with output $x_i^* = - 2[(1+\beta)\{(\beta-1)c+\alpha\}] / (\beta^2-2\beta-4)$,

and price $p_i^* = - [2c\beta^2 + \alpha\beta^2 - 2c - 2\alpha] / [(\beta-1)(\beta^2-2\beta-4)]$ for $\{i,j\}=\{1,2\}$.

Statement 1: In the owner-manager game managers behave more aggressively than profit maximizers i.e. $\lambda_i^* < 1$, i=1,2. This results with higher outputs, lower prices than the ordinary Cournot model.

Proof: The fact $\lambda_1^*=\lambda_2^*<1$ follows from $\alpha / (1-\beta) > c > 0$, and $\lambda_1^*=\lambda_2^* < 1$ implies $x_i^*(\lambda_1^*,\lambda_2^*) > x_i^*(1,1)$, since the effect of a higher λ_i dominates the effect of higher λ_j so that the output which is a decreasing function in λ_i decreases with respect to previous level, for i=1,2. Then prices are directly lower than the ordinary Cournot model.

When profits are calculated, it is seen that these are lower than the usual Cournot level. Note that this is just the converse of the result found by Sklivas(1987) for price competition version of the game in the same market, and it supports his result for the quantity competition version for a homogeneous good market.

The intuition for the seemingly paradoxical result of profit maximizing owners' committing their managers to non-profit maximizing behavior lies in the fact that delegating one manager against the sole owner in the rival firm results with higher profits than without delegating a manager but solely competing a'la Cournot.

III.3. REDELEGATION EQUILIBRIUM

Now consider the scenario in which duopolists have permission to redelegate, i.e. to hire other managers vertically in any chain length. At the same manager number already hired for both firms, Koray - Sertel (1989b) show that in a homogeneous good market principals have an incentive to redelegate. This means that unilateral deviation for firm i causes it to profit more and the rival to profit less than the previous level. For the case where firms have no managers, we have demonstrated that principals have an incentive to delegate. Similarly at any level of managers, it can be proven that principals have the incentive to redelegate one more, if decreasing manager chain length is not permitted. Unilateral deviation by delegating one more manager than the rival increases the firm's own profits again.

This meta-Cournotic game has n+1 stages, where n is the number of vertically hired managers. In the first stage, the owners, as usual, simultaneously write down and announce the incentive parameters $\lambda_{i,0}$ for i=1,2. In the second stage, the managers at the first link of the chain simultaneously maximize the maximands of their own determined according to the incentive parameters assigned by the owners and determine the incentive parameters $\lambda_{i,1}$ for i=1,2, which is assigned to the managers at the second link of the chain. So in the k'th stage where 1<k<n+1, the managers at the (k-1)'th link of the chain simultaneously maximize their maximands and determine the incentive parameters $\lambda_{i,(k-1)}$ for i=1,2 according to the incentive parameters $\lambda_{i,(k-2)}$ for i=1,2 assigned by the managers above. Finally in the (n+1)'th stage, the managers at the n'th link of the chain simultaneously determine the outputs of their firms according to the incentive parameters $\lambda_{i,(n-1)}$ for i=1,2 assigned by the managers at the preceding link of the chain. The objective of the owner of i'th firm for i=1,2 is profit maximization:

$g_{i,0} = \pi_i = (p_i - c)\, x_i$.

The maximand of the k'th manager in i'th firm for i=1,2 is given by

$g_{i,k} = (p_i - \lambda_{i,(k-1)}\, c)\, x_i$ for each $k \in \{1, \ldots ,n\}$.

Definition 4: (x_1^*,x_2^*) is Nash equilibrium output pair of (n+1)'th stage in the redelegation game if and only if $x_i^*=\mathit{argmax}\ g_{i,n}(x_i,x_j^*)$ for $\{i,j\}=\{1,2\}$.

Definition 5: $(\lambda_{1,k}^*,\lambda_{2,k}^*)$ is Nash equilibrium incentive parameter pair of (k+1)'th stage in the redelegation game if and only if

$\lambda_{i,k}^* = \mathit{argmax}\ g_{i,k}(x_i^*(\lambda_{i,k},\lambda_{j,k}^*),x_j^*(\lambda_{i,k},\lambda_{j,k}^*))$

for each $k \in \{0, \ldots ,n-1\}$ and for $\{i,j\}=\{1,2\}$.

Hence $(x_1^*(\lambda_{1,0}^*,\lambda_{2,0}^*),x_2^*(\lambda_{1,0}^*,\lambda_{2,0}^*))$ is the subgame perfect Nash equilibrium output pair of the game. Note that delegation is assumed to be costless throughout the analysis.

For n=1,2,3,4 the subgame perfect Nash equilibrium incentive parameter delegated by owner i, namely $\lambda_{i,0}$; for n=0,1,2,3,4 output x_i, price p_i and profit π_i are demonstrated through a mathematics software package XMAPLE (See appendix for computer program at list 1 and for graphical demonstrations at graphs 1 to 5). Note that the case n=0 corresponds to the naked Cournot competition. In these graphs the specific case for $\alpha=2$ and c=1 are considered. We see that the output per firm increases (graph 2), price of each firm's product decreases (graph 3), profit per firm decreases (graph 4), and primary incentive parameter increases (graph 1), but still stays below 1 for each $\beta \in (0,1)$, as the length of manager chain increases with equal number in each firm. The rate of decrease or increase for each item decreases in each redelegation. This fact creates the idea that in the limit they will converge to some specific value. Now we can make some comments on these graphs. β is the parameter of market differentiation. At $\beta=1$, the market characterizes a constant demand for each price level. The effect of both firms' pricing cancel each other. Since the firms have the objective of attaining highest utility possible, they assign the price $+\infty$. At $\beta=0$, the market splits into two markets with completely differentiated goods, i.e. different goods. Hence each firm behaves as if it were the monopoly in its industry. Hence delegating managers have no effect on the profits, prices and output. The owners already assign $\lambda_{i,0}=1$ for i=1,2 at $\beta=0$ for any manager length. The form of $\lambda_{i,0}$ for i=1,2 at a manager length n seems to be dependent upon β such that as β gets closer to

1, the owners make the managers fight so aggressively that they behave as if the marginal cost were close to $-\infty$. However, as n increases, the market should have a higher β than the previous case to have extreme levels of aggressive behavior occur. Following deduction has been stated but has not been proven.

Statement 2: $(\lambda_{1,0}^*, \lambda_{2,0}^*)$, the Nash equilibrium incentive parameters of the first stage of the meta-Cournotic game of (n+1) stages where n denotes the number of managers in each firm can explicitly be written with following functional form:

$$\lambda_{i,0}(n) = \lambda_{i,0}^* = \begin{cases} -\dfrac{\{\alpha\beta^{2n} + c(\beta-1)[\beta^{2n} - g(n)]\}}{c(\beta-1)g(n)} & \text{if } n \text{ is even} \\ -\dfrac{\{\alpha\beta^{2n} + c(\beta-1)g(n)\}}{c(\beta-1)\{\beta^{2n} - g(n)\}} & \text{if } n \text{ is odd} \end{cases} \qquad i=1,2$$

where polynomial g(n) is defined by

$$g(n) = \sum_{j=0}^{2n-1} (-1)^{[[j/2]]+n+1} 2^{2n-j} h_{j,n} \beta^j$$

and with coefficients $h_{j,n}$ satisfying for each $j \in \{0,1, \dots ,2n\}$,

$$h_{j,n} = \begin{cases} 1 & \text{if } j = 0 \\ 1 & \text{if } j = 1 \\ h_{j,n-1} + h_{j-1,n-1} + h_{j-2,n-1} & \text{if } j \text{ is even and } 2n > j > 0 \\ h_{j-1,n-1} + h_{j-2,n-1} & \text{if } j \text{ is odd and } 2n > j > 1 \\ 1 & \text{if } n \text{ is even and } j = 2n \\ 0 & \text{if } n \text{ is odd and } j = 2n \end{cases}$$

Note that, the polynomial g(n) satisfies the inequality $g(n)>\beta^{2n}$ if n is odd and $g(n)<0$ if n is even.This statement still requires proof and it is only a deduction yield from the forms of the function in hand for manager chain lengths of n=0,1,2,3,4. We have not proven it, but it seems that induction on n can be a correct way of proof. Proofs in the following statements are done assuming that above statement is true.

Statement 3: The owners make the top-manager behave less aggressively but still aggressively as the length of the manager chain increases, and in the limit, the primary incentive parameter converges to 1, i.e. the Nash equilibrium pair of incentive parameters $(\lambda_{1,0}^*, \lambda_{2,0}^*)$ of n+1 stage meta-Cournotic game satisfies

(i) $\lambda_{i,0}^* = \lambda_{i,0}(n) < 1$

(ii) $\lambda_{i,0}(n) > \lambda_{i,0}(n-1)$

(iii) $\lim_{n\to\infty} \lambda_{i,0}(n) = 1$, for i=1,2.

Proof: $\lambda_{1,0}(n)=\lambda_{2,0}(n) < 1$ directly follows from $g(n) > \beta^{2n}$ if n is odd and $g(n)<0$ if n is even, and $\alpha / (1-\beta) > c$. $\lambda_{i,0}(n) > \lambda_{i,0}(n-1)$,and as $n\to \infty$, $\lambda_{i,0}(n)\to 1$ are direct consequences of statement (2) and, since g(n) does not become zero in its domain and does not make $\lambda_{i,0}$ $-\infty$ for i=1,2. Note that, although the primary incentive parameter converges to 1, the last incentive parameter delegated on the industry floor does not converge to 1, hence output level does not converge to the ordinary Cournot quantity.

Statement 4: The maximand delegated by the owner to the top-manager converges in monotonically decreasing fashion to the true profit function.

Proof: This fact follows from $\lim_{n\to\infty} \lambda_{i,0}(n) = 1$. So $\lim_{n\to\infty} g_{i,1}(n) = \lim_{n\to\infty} (p_i x_i - \lambda_{i,0}(n)\, c\, x_i) = \pi_i$ for i=1, 2. Now $\lambda_{i,0}(n)$ converges in monotonically increasing fashion to 1. So $g_{i,1}(n)$ converges in monotonically decreasing fashion since it is a decreasing function in $\lambda_{i,0}(n)$.

At this point, it will be wise to consult to the study done on the same market framework for Bertrand competition case by Yildirim (1995). He works on the same redelegation game within the Bertrand competition framework. The functional form of incentive parameters delegated by the primary delegator to the secondary delegator is studied. What Yildirim finds is that the incentive parameter is just symmetric of the one we find above, around $\lambda_{i,0}=1$ for i=1,2 i.e. the primary incentive parameter in the Cournot model approaches from below to 1 and the primary incentive parameter in the Bertrand model approaches from above to 1 as the length of chain increases without bound.

Statement 5: $(x_1^*(\lambda_{1,0}^*,\lambda_{2,0}^*),x_2^*(\lambda_{1,0}^*,\lambda_{2,0}^*))$, the subgame perfect Nash equilibrium output pair of (n+1) stage meta-Cournotic game, where n denotes the number of managers is given for i =1,2 by

$$x_i(n) = x_i^* = \{[f(n)(1+\beta)]\,[\alpha - (1-\beta)c]\}/\,[f(n+1) + \beta f(n)]$$

where $f(n) = \sum_{j=0}^{n} (-1)^{[[j/2]]}\, k_{j,n}\, 2^{n-j}\, \beta^{j}$

and with the coefficients $k_{j,n}$ satisfying for each $j \in \{0,1,2, \ldots\}$,

$$k_{j,n} = \begin{cases} 0 & \text{if } j \text{ is odd or } j > n \\ 1 & \text{if } \{j = 0\} \text{ or } \{j \text{ is even and } n = j\} \\ k_{j,n-1} + k_{j-2,n-2} & \text{if } j \text{ is even and } j > 0 \end{cases}$$

This is also deduced by the help of the computer outputs and still requires proof. We have the conjecture that this value is convergent as number of managers increase.

III.4. COMPARISONS REGARDING SOCIAL WELFARE

In this subsection we demonstrate the impact of delegation on social welfare. First, we should give the definitions of consumers' and producers' surplus in a differentiated goods industry to define the social welfare in our context.

Definition 6: Consumers' surplus in the differentiated goods industry at an output level (x_1,x_2) is defined by

$$CS(x_1,x_2)= \sum_{\substack{i=1 \\ j=1,2 \text{ and } j\neq i}}^{2} \int_0^{x_i} (p_i(x_i',x_j)-p_i(x_i,x_j))dx_i' = \sum_{i=1}^{2} x_i^2 / (2(1-\beta^2)),$$

producers' surplus at output level (x_1,x_2) is defined by sum of profits in the market,

$$PS(x_1,x_2) = \sum_{i=1}^{2} \pi_i(x_1,x_2) = \sum_{i=1 \text{ and } j\neq i,\, j=1,2}^{2} \{\alpha(1+\beta)x_i-x_i^2-\beta x_j x_i-c(1-\beta^2)x_i\}/(1-\beta^2).$$

Welfare in the society can be some increasing function of both producers' and consumers' surplus, since it indicates aggregate utility function of the whole society. Many ways of defining welfare are possible. We further assume that welfare of the society at output level (x_1,x_2) is defined by sum of the producers' and the consumers' surpluses:

$$W(x_1,x_2) = PS(x_1,x_2) + CS(x_1,x_2).$$

We assume that the society as a whole have decided on it, but not only the central planner or authority who deals with welfare maximization.

Definition 7: The quantity pair (x_1^*,x_2^*) is the socially efficient output level if and only if $x_i^* = \mathit{argmax}\, W(x_i,x_j^*)$ for $\{i,j\}=\{1,2\}$.

Socially efficient outcome per firm is determined as

$x_i = \{\alpha-c(1-\beta)\}(1+\beta) / (1+2\beta)$, for $i=1,2$ in our market.

For welfare, the graph for $\alpha=2$, $c=1$ are demonstrated in appendix as welfare differentials in each delegation vs. β (graph 5) and social welfare for each delegation (graph 6). If we consider that graph, we see that the welfare does not uniformly increase or decrease on each delegation, but it is dependent on the degree of market differentiation β. It can be proposed that there exists some $\beta^+(n) \in (0,1)$ such that the difference in welfare between each delegation

$$\Delta W(n) = (W(x_1(n),x_2(n))-W(x_1(n-1),x_2(n-1))$$

is positive for all $\beta \in (0,\beta^+(n))$ and negative for all $\beta \in (\beta^+(n),1)$. Certainly the value of $\beta^+(n)$ is dependent on the value of equilibrium outputs for the delegation length n. However, for large manager lengths the welfare differential stays around zero for small values of β. Furthermore, for each manager length the output produced gets the same value with the socially efficient one for some β. Hence, the welfare attained by the concerned manager length is the highest one for that β. (For special case n=4, see graph 7). The analytic expression of this β as a function of delegation length can also explicitly be demonstrated in a further study. Hence welfare is equal to the socially efficient one at this β. So we can propose that, there exist certain market differentiation rates such that by delegating the enough number of managers, we can attain the highest welfare possible. So it can characterize a suboptimal regulatory mechanism. Such a regulation mechanism can include following steps:

1) Using the subgame perfect Nash equilibrium concept in non-cooperative games, determine the number of managers that provide the welfare closest to the efficient one,

2) Impose the owners to compete in that manager length.

Statement 6: As number of managers increases without bound in the meta-Cournotic game, the industry-wide output level is convergent and converges in monotonically increasing fashion to some output level. The limit output is different from the socially efficient one.

This statement again needs proof and it is only a conjecture on the behavior of the output forms in the game. Hence if this result is true, infinite delegation game can not be used as an optimal regulatory mechanism for each β for improving welfare with respect to the ordinary Cournot competition in a duopoly. This may be source for a further study on this concept. Note that the proposition stated by Koray-Sertel (1989b) for a homogeneous good market implies that the output level converges in monotonically increasing fashion to the socially efficient one. Hence this does not hold for a differentiated goods market. However our earlier statement saying that the primary incentive parameter converges to 1 is also valid for the homogeneous good market as stated by Koray-Sertel (1989b).

IV. CONCLUDING REMARKS

The main contribution of this study is to gain some further understanding about delegation in a market and to display the behavior of delegators. We demonstrate the behavior of owners and managers in a duopolistic market in the presence of differentiated goods. The meta-Cournotic competition concept in a differentiated goods market is rather different than the naked Cournot competition. However the redelegation game has not a pure Cournotic equilibrium for only the last managers are competing in quantity, but not the intermediate delegators. So the type of the game is determined by only the last manager on the industry floor. Hence the preferences and the award contract of the last manager is important in deciding whether the firm should compete in quantity or price. In our study, we take the competition type to be limited to quantity and the preference of each delegator to a particular maximand type. The contracts of the managers are such that they do not receive any share from profit of the firm i.e. the labor market for managers is highly competitive and there exist many managers to do the job perfectly with some fixed wage. It is also important that the marginal cost of each firm is constant. If it were not, the meaning of the incentive parameter would not be exactly the same with our case where we take it as a fictitious cost coefficient. Also if the firms were not symmetric i.e. they had different costs or technologies in producing the good, the existence of equilibrium would arise as an important question.

First, results of the ordinary Cournot competition in the differentiated goods market are demonstrated. Second, one-manager delegation game is modeled with the concept of Cournot competition and results for homogeneous products are shown to be compatible with results of this study: The owners make the managers behave aggressively. Later, the recursive delegation is studied through the limiting behavior of owners. It is important that when the manager chain grows beyond bound, the maximands delegated by the owners of firms converge to the true profit function, hence the mechanism converges to an incentive compatible one. Price of good and profits of each firm monotonically decreases, output per firm monotonically increases in each delegation, and both are convergent in the limit. Regarding welfare, there exists only one β for each value of α and c, such that social welfare converges to the socially efficient one. So the infinite delegation enhances regulation for those markets only. However, in contrast to the homogeneous good case, the infinite delegation equilibrium concept may not be used as a regulatory scheme for all values of the market differentiation rate β. Since the social welfare concept as defined in our study is not a monotonic function in β, the total industry output does not converge to the socially efficient one. However, there exists some $\beta \in (0,1)$ for each delegation length such that the equilibrium pair of outputs gives the highest attainable welfare in the society. These finite delegation lengths can be used in designing a suboptimal regulatory mechanism.

The framework presented throughout the study can be a basic theoretic setup to design regulation mechanisms for certain oligopolistic structures i.e. the varying marginal cost or different marginal cost for each firm, or markets with more complicated demand behavior. A further study may also try to prove the statements stated throughout the preceding sections and to determine β where the socially efficient welfare value becomes the same with the welfare attained by the particular finite delegation length.

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APPENDIX

Graph 1. Incentive parameter $\lambda_{i,0}(n)$ vs. β for delegation lengths 1,2,3,4 for $\alpha=2$, $c=1$.

Graph 2. Quantity produced in each firm $x_i(n)$ vs. β for delegation lengths of 0,1,2,3,4, for $\alpha=2$, $c=1$.

Graph 3. Price of each firm $p_i(n)$ vs. β for delegation lengths 0,1,2,3,4, for $\alpha=2$, $c=1$.

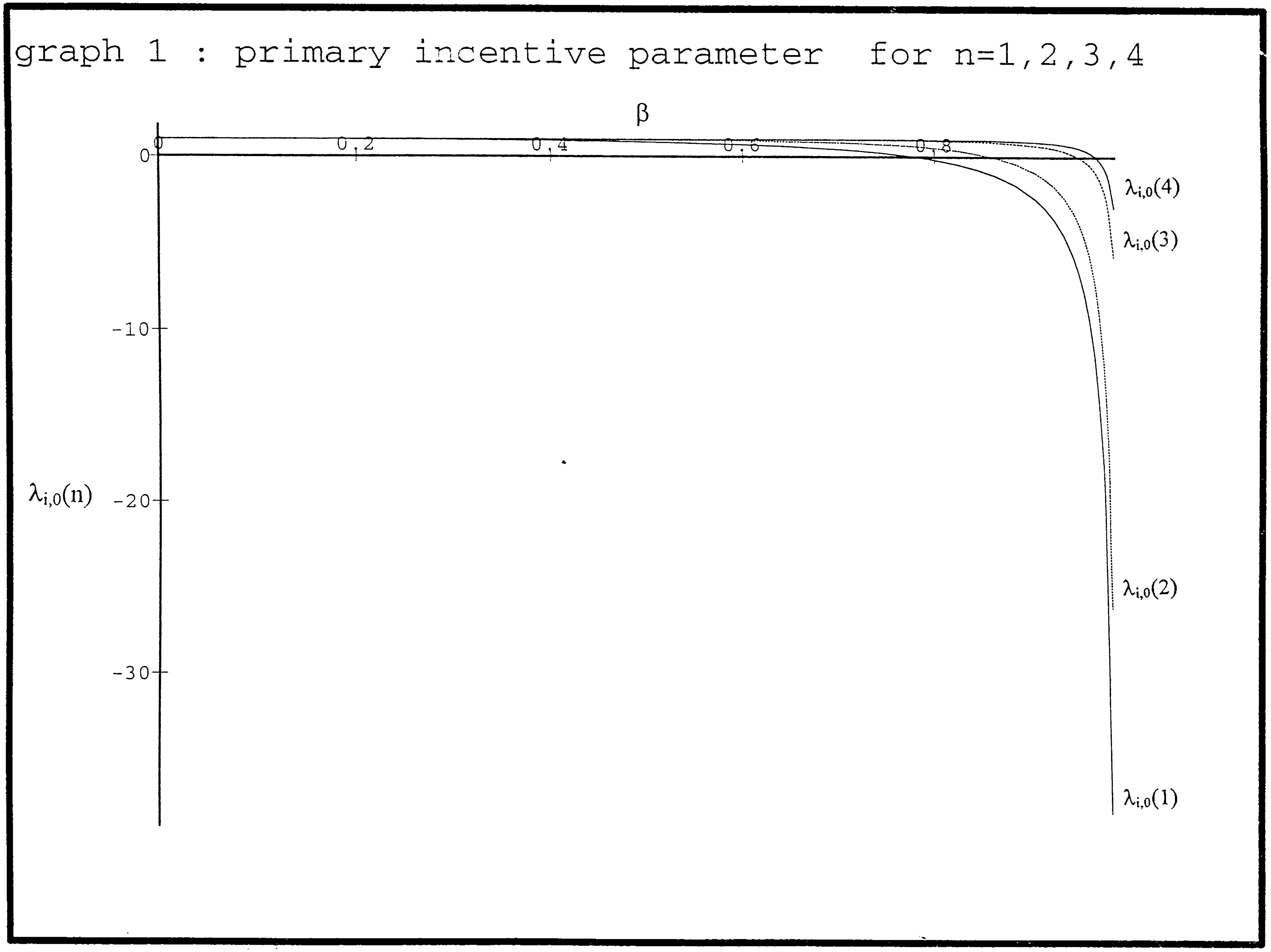
Graph 4. Profit of each firm $\pi_i(n)$ vs. β for delegation lengths 0,1,2,3,4 for $\alpha=2$, $c=1$.

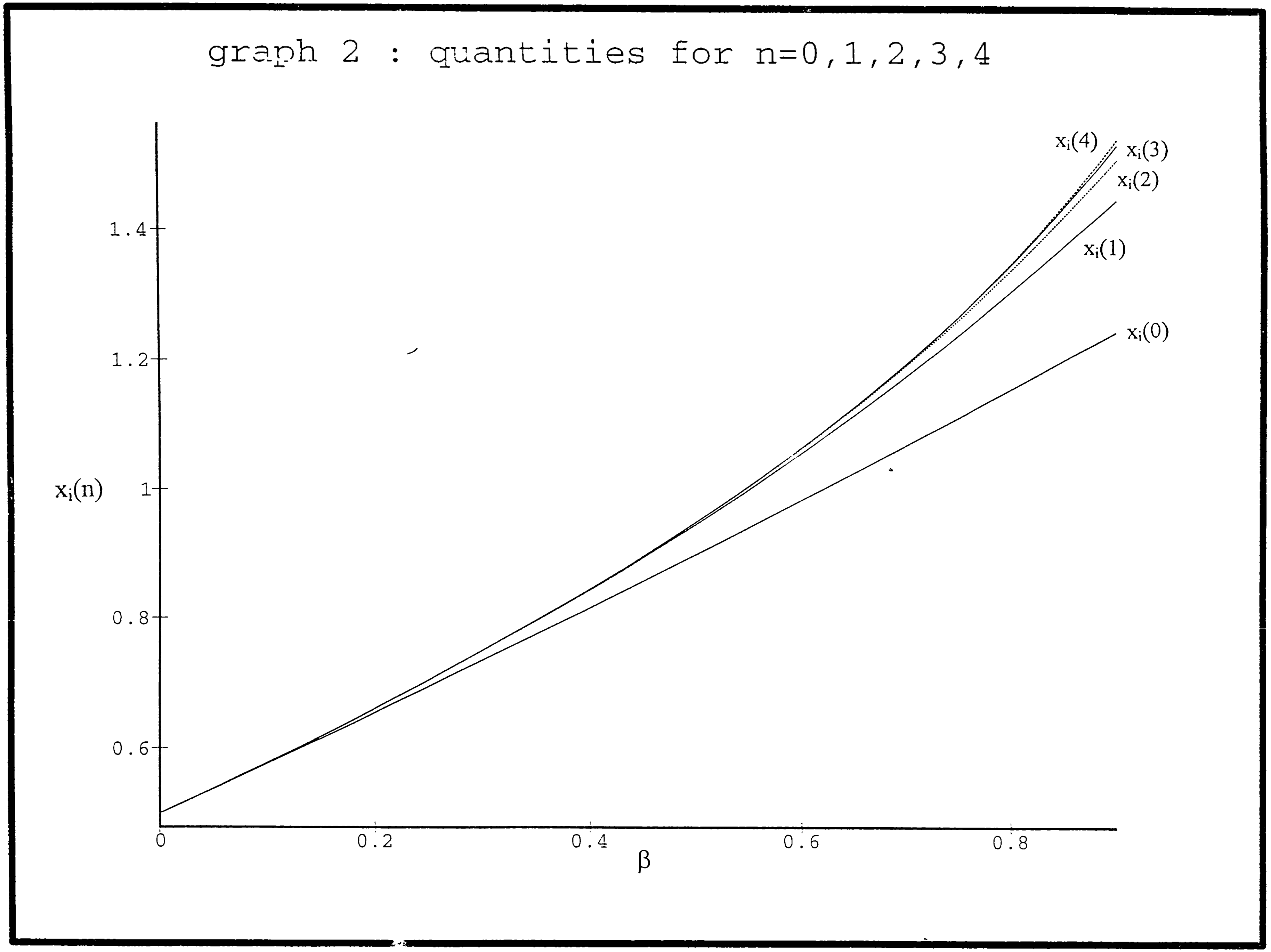
Graph 5. Difference in social welfare in each delegation $\Delta W(n)$ vs. β for delegation lengths 0,1,2,3,4 and efficient one for $\alpha=2$, $c=1$.

Graph 6. Social welfare in each delegation $W(n)$ vs. β for delegation lengths 0,1,2,3,4, and efficient one for $\alpha=2$, $c=1$.

Graph 7. Social welfare $W(n)$ vs. β for $n=4$ and efficient one.

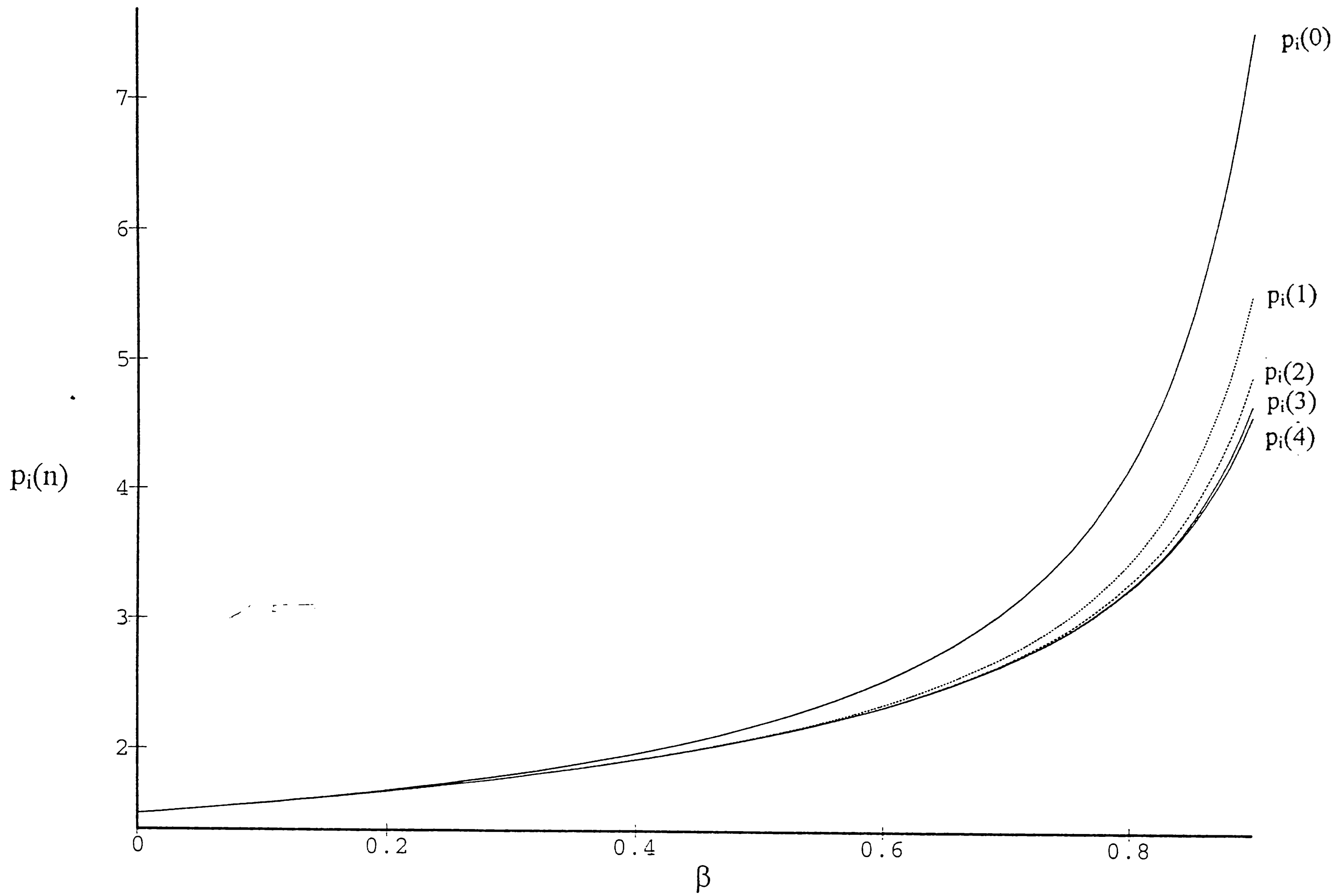
List 1. Computer Program.

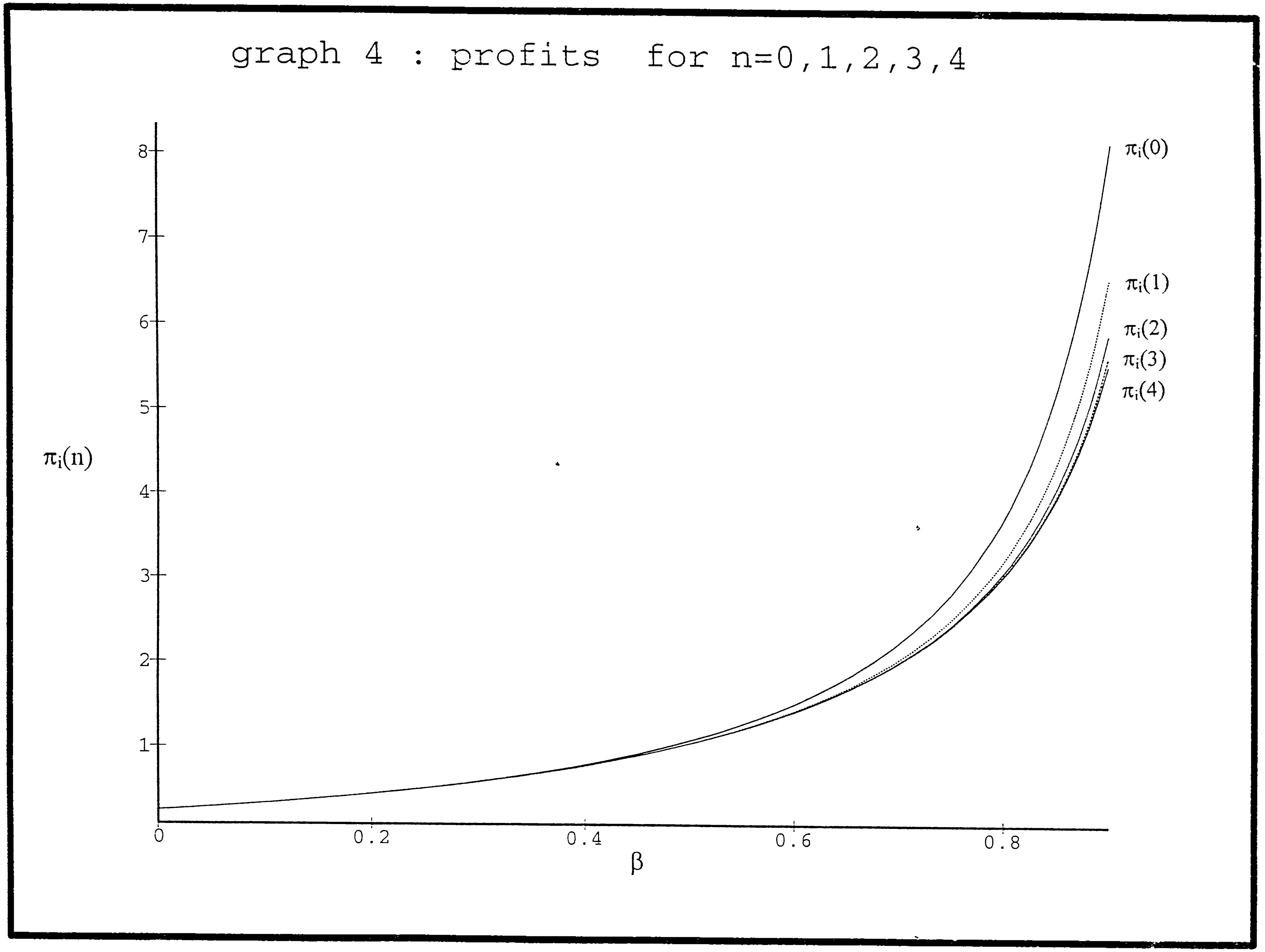

graph 1 : primary incentive parameter for n=1,2,3,4
β
0
0,2
0,4
0,6
0,8
0
-10
-20
-30
λi,0(n)
λi,0(4)
λi,0(3)
λi,0(2)
λi,0(1)



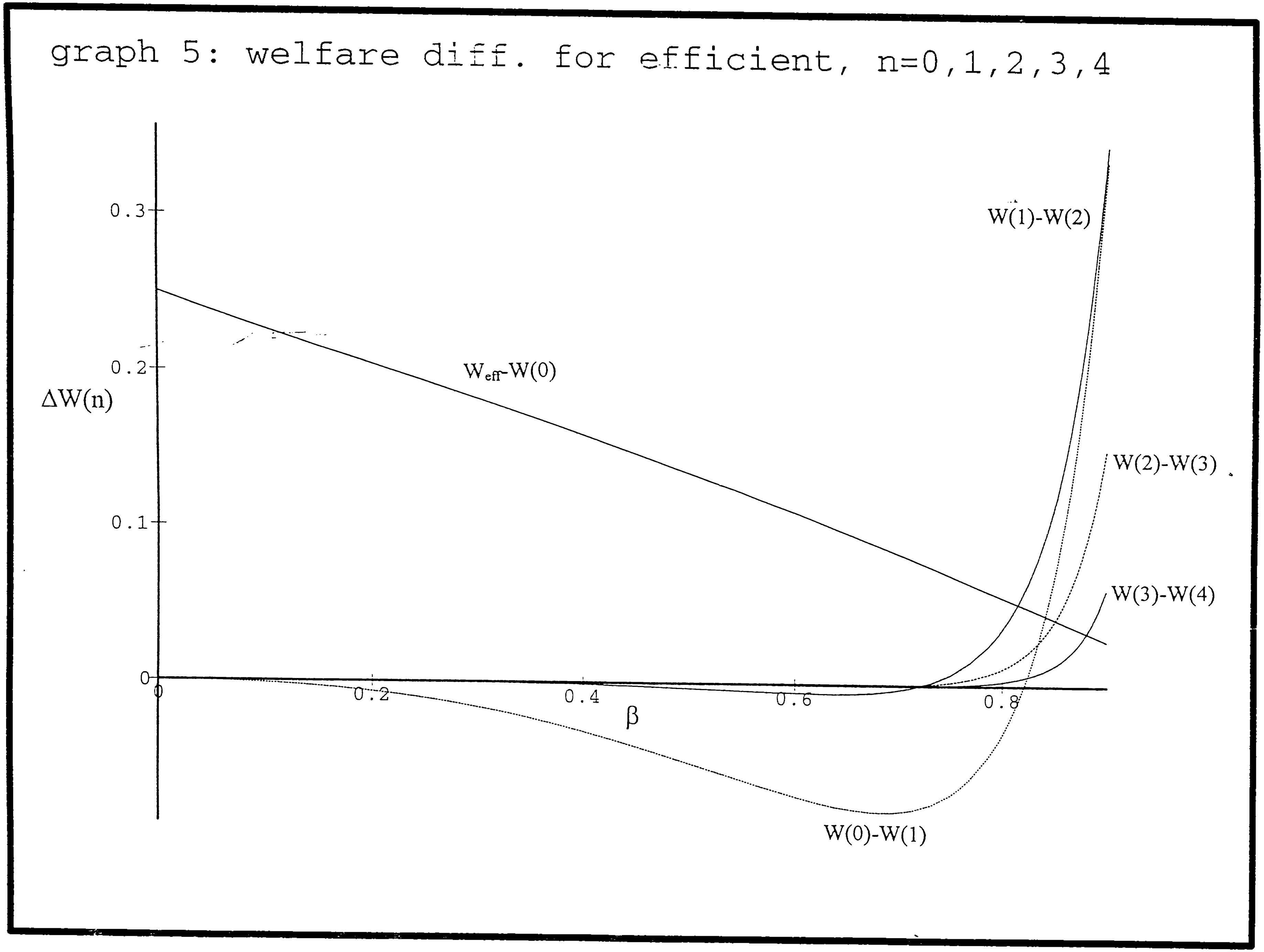
graph 2 : quantities for n=0,1,2,3,4
$x_i(n)$
1.4
1.2
1
0.8
0.6
0
0.2
0.4
0.6
0.8
β
$x_i(4)$
$x_i(3)$
$x_i(2)$
$x_i(1)$
$x_i(0)$

graph 3 : prices for n=0,1,2,3,4

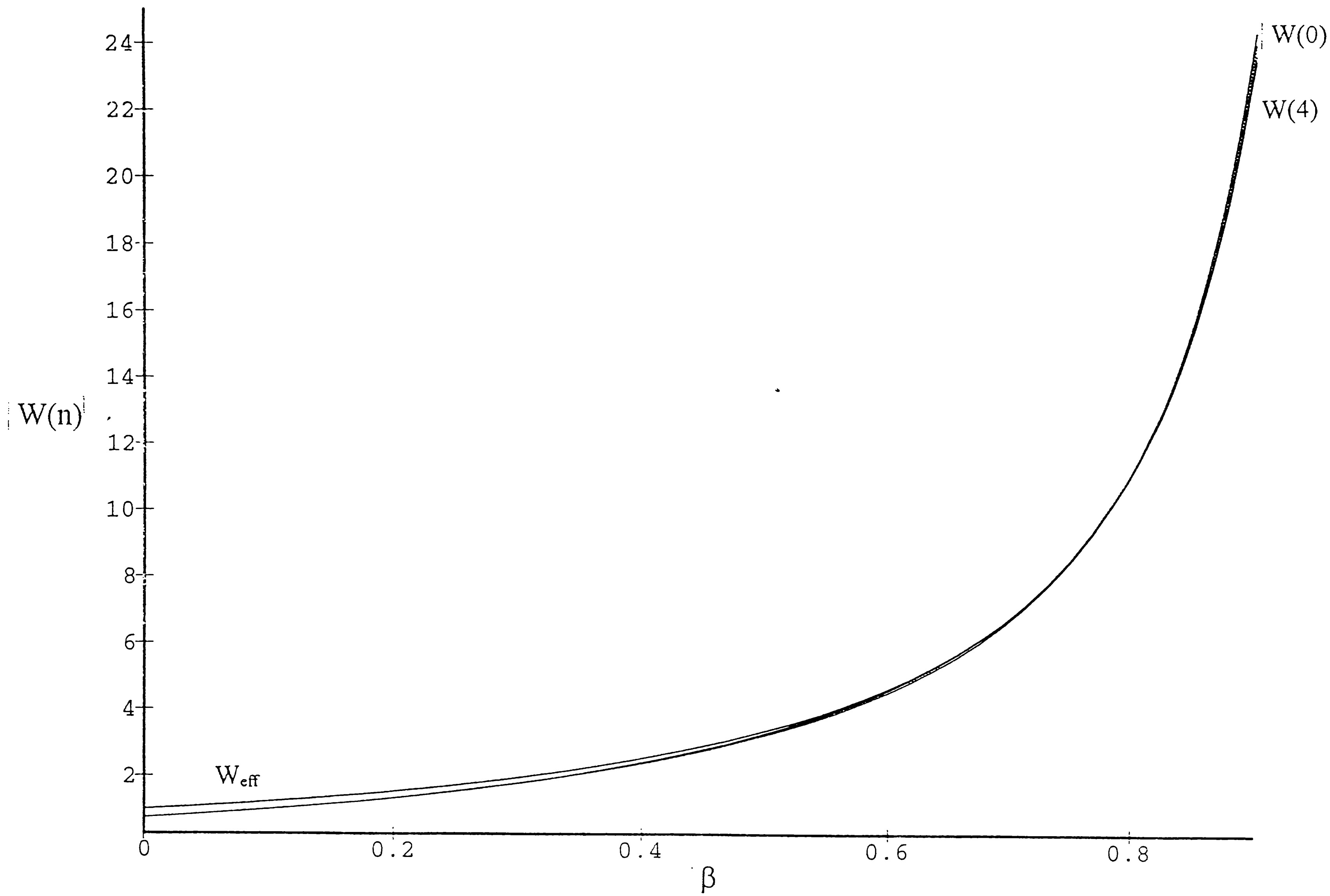




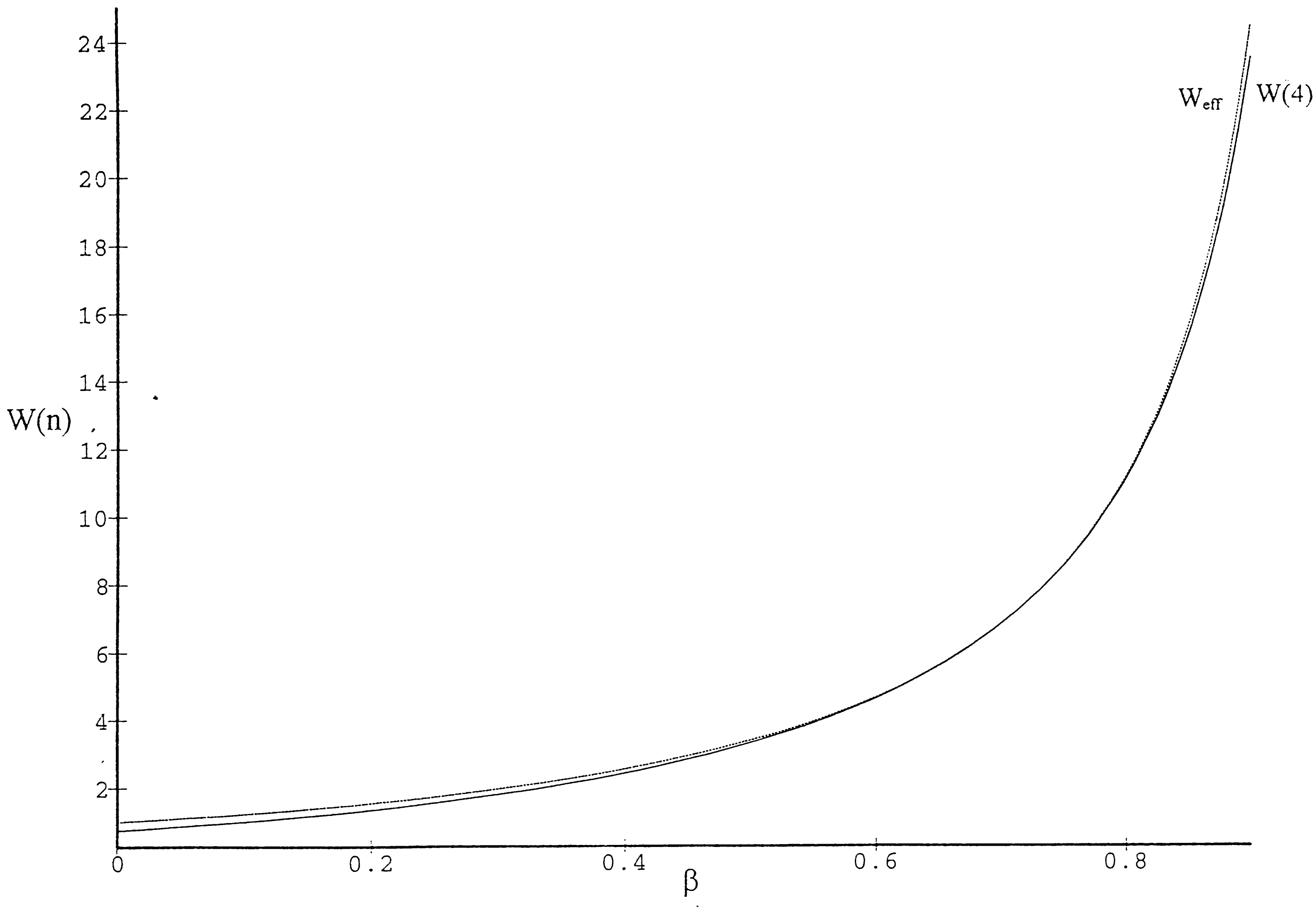
graph 4 : profits for n=0,1,2,3,4
$\pi_i(0)$
$\pi_i(1)$
$\pi_i(2)$
$\pi_i(3)$
$\pi_i(4)$
$\pi_i(n)$
8
7
6
5
4
3
2
1
0
0.2
0.4
0.6
0.8
β


graph 5: welfare diff. for efficient, n=0,1,2,3,4
0.3
0.2
0.1
0
ΔW(n)
0
0.2
0.4
0.6
0.8
β
W(1)-W(2)
W_{eff}-W(0)
W(2)-W(3)
W(3)-W(4)
W(0)-W(1)

graph 6: Social welfare for n=0,1,2,3,4, efficient



graph 7: Social welfare for n=4 and efficient



List 1: XMAPLE computer program

```
n:=4;{for delegation chain length 4}
p1:=(a*(1+b)-x1-b*x2)/(1-b^2);{inverse demand functions of each firm}
p2:=(a*(1+b)-x2-b*x1)/(1-b^2);
pr1[n]:=(p1-l1[n-1]*c)*x1;{maximands of the n+1'th stage}
pr2[n]:=(p2-l2[n-1]*c)*x2;
assign(solve({diff(pr1[n],x1)=0,diff(pr2[n],x2)=0},{x1,x2}));{solve for
Nash eq. of n+1'th stage}
for i from n-1 by -1 to 1 do  {solve for Nash eq. of stages 2 to n}
  pr1[i]:=(p1-l1[i-1]*c)*x1; {maximands of stage i}
  pr2[i]:=(p2-l2[i-1]*c)*x2;
  assign(solve({diff(pr1[i],l1[i])=0,diff(pr2[i],l2[i])=0},{l1[i],l2[i]})){solve
for Nash eq. of stage i}
  od;
pr1[0]:=(p1-c)*x1;{maximands of 1st stage}
pr2[0]:=(p2-c)*x2;
assign(solve({diff(pr1[0],l1[0])=0,diff(pr2[0],l2[0])=0},{l1[0],l2[0]}));
{solve for Nash eq. of 1st stage}
factor(l1[0]);{factor out primary incentive parameter, quantity, price, profit}
factor(x1);
factor(p1);
factor(pr1);
```