IMPLEMENTATION OF JOHANSEN PROCEDURE IN THE ESTIMATION OF DEMAND FOR M1 AND M2 USING THE TURKISH DATA

A Thesis

Submitted to the Department of Economics
and the Institute of Economics and Social Sciences
of Bilkent University

in Partial Fulfillment of the Requirements
for the Degree of

MASTER OF ARTS IN ECONOMICS

By

Emre ÖZDENÖREN

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I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Prof. Dr. Sübidey Togan

I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Assist. Prof. Dr. Osman Zaim

I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

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Emre ÖZDENÖREN

MA in Economics

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This study aims at estimating the money demand function for Turkey using quarterly data. Estimation is done, for both M1 and M2, using Johansen procedure, which is a variate of the theory of cointegration.

The results of the Johansen procedure shows that real income is positively and expected loss is negatively related with demand for M1 and M2. Also, some linear restrictions are tested, by restricting the money demand coefficients. The results of these tests show that Tobin-Baumal model and unit elasticity of income are rejected for both M1 and M2.

Key Words : money demand, cointegration, level of integration, stationarity, Johansen procedure.

ÖZET

PARA TALEP FONKSİYONUNUN TÜRKIYE İÇİN JOHANSEN METODUYLA TAHMİN EDİLMESİ

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Bu çalışmada üç aylık veriler kullanılarak Türkiye için para talep fonksiyonu tahmin edilmiştir. Tahminlerin yapılmasında Johansen metodu kullanılmıştır ve tüm hesaplamalar M1 ve M2 için tekrarlanmıştır.

Johansen metodu kullanılarak yapılan tahminlerin sonucunda, hem M1 hem M2 için, reel gelirlerin katsayısı pozitif ve paranın beklenen kaybının katsayısı negatif olarak bulunmuştur. Ayrıca, bazı doğrusal kısıtlamalar test edilmiştir. Bu testlerin sonuçlarına göre Tobin-Baumal modeli ve birim gelir esnekliği hipotezleri, hem M1 hem de M2 için reddedilmiştir.

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1 Introduction

The determination of a relationship between money, income, interest rates and other related variables and the stability of such relationships have been an important topic in the literature.

In this thesis, the long run demand for money function in Turkey for the period 1977(1)-1989(4) has been investigated using a maximum likelihood method suggested by Johansen(1988), which is a variate of the theory of cointegration. This method also gives the opportunity to test some economically meaningful hypoteses.

Cointegration analysis states that economic series which are non-stationary may drift together as a group. If there is such a relationship between a set of variables, this analysis helps to discover it. If an economic theory is correctly specified then these variables will be related with each other with constant parameters, so these variables would not drift increasingly further as time goes on. But if the variables are not cointegrated then there should be doubts about the underlying economic theory, or at least the model.

This study is organized in the following way. Section 2 gives the necessary theoretical background. In 2.1 theory of money demand is summarized very briefly, In 2.2 econometric framework is discussed. Here the theory of unit roots and cointegration is overviewed, and the methodology suggested by Johansen is described.

In section 3 empirical results are reported. In this section also the results of various hypoteses tests are given, which compares the Turkish money demand function with the theory of money demand. Lastly, all these results are summarized and discussed in the conclusion.

2 Theoretical Background

2.1 Economic Framework

One of the earliest approaches to the demand for money is the quantity theory of money. Quantity theory starts with the equation of exchange which can be written as,

$$MV \equiv PT \tag{1}$$

where M is the quantity of money, V is the velocity of circulation, P is the price level, and T is the volume of transactions. In this equation M, P, and T are directly measurable, but V is implicitly defined by (1). If we assume that velocity, V, is determined by technological or institutional factors and therefore is relatively constant then we can view (1) as a demand function for money. Then from (1) we see that demand for real balances is proportinal to the volume of transactions.

Since Angell (1936), monetary economists express the quantity equation in terms of income transactions rather than gross transactions. Let y be the national income at constant prices. We can write the quantity equation in income form as,

$$MV \equiv Py \tag{2}$$

where M is the quantity of money as before, but V is now defined as the average number of times per unit time that the money stock is used in making income transactions.

Keynes (1936) modified this simple money demand function by introducing the speculative motive for holding money, together with the trancactions motive. In the speculative motive approach, money and bonds are seen as alternative assets. As bond holding depends on the rate of return on bonds, interest rate enters into the demand equation for money. Once the interest rate is introduced, one does not need to assume constant velocity anymore.

Transactions approach emphasizes the role of money as something that everybody will accept in exchange as 'general purchasing power'. But there must also be something which can serve as a temporary abode of purchasing power during the time that passes between sale and purchase. This aspect of money can be seen from the cash-balance equation.

$$M \equiv kPy \tag{3}$$

where k is the ratio of money stock to income. Apperantly k is the reciprocal of V, a simple mathematical transformation, but it stresses the role of money as an asset which people must hold as a part of their portfolio. So this formulation suggests treating money as part of capital or wealth theory.

Friedman (1956) restated the quantity theory of money. He treated money like any other asset yielding a flow of services. He distinguishes between ultimate wealth holders to whom money is an asset which they choose to hold their wealth, and enterprises to whom money is a producer's good like machinery or inventories.

For ultimate wealth holders demand for money may be expected to be a function of;

(i) Total wealth,

(ii) The division of wealth between human and non-human forms,

(iii) The expected rates of return on money and other assets,

(iv) Other variables determining the utility attached to services rendered by money relative to those rendered by other assets.

Baumal (1952) and Tobin (1956) applied inventory-theoretic considerations to the transactions demand for money. Their studies led to the well-known square-root law, where average money holdings are given by,

$$M = (2bT/r)^{1/2} \tag{4}$$

Here, r is the interest rate on bonds and b is the brokerage charge.Dividing both sides of equation (4) by price level makes the demand for money depend on interest rate, real brokerage charges and the level of real transactions. Miller and Orr (1966) extended this analysis to allow for uncertainty in cash flows. Their analysis showed that a firm's average money holdings depends on the variance of its cash flow.

Some other studies have tried to reformulate Keynes' speculative motive in terms of portfolio theory[1]. But there is a serious problem with this approach. If there is a riskless asset, such as a savings deposit, which is paying a higher rate of return on money, than money is a dominated asset and will not be held. So one must combine the portfolio approach with transaction costs in order to find an asset demand for money.

In order to estimate a money demand function empirically, one needs explicit variables which measure money and its determinants. The first step is to decide what can be accepted as money. In general, theories based on the transactions motive lead to a narrow definition of money, which includes currency and demand deposits.

If one moves away from the transactions view, and assumes that money

[1]Tobin (1958)

yields some unspecified services, then the definition of money is even less clear because there may be other assets which yield the same services. In this study two definitions of money are considered. The first one is M1, consisting of currency plus demand deposits, and the second one is M2, consisting of M1 plus time deposits. M1 can be seen as a variable which reflects the transactions view of the world and M2 can be seen as a variable which also reflects the asset use of money.

The level of transactions is typically measured by the level of income or gross national product, which is also the case in this study.

Another measurement issue is the oportunity cost of holding money. Here, the own rate of return on money and the rate of return on assets alternative to money must be considered. For the latter under transactions view, in general, some interest rate on a savings deposit or a combination of such interest rates is used. But it must be noted that consumption of goods is an alternative to holding money and inflation is the rate of return on consumption goods. In this study consumption is taken as the only alternative to money. This is an assumption mostly utilized for the financially repressed economies.

The own rate of return on money is defined as follows for M1 and M2:

$r_{M1} = \frac{r_{DD}DD}{M1}(1 - T)$

$r_{M2} = \frac{r_{DD}DD + r_{TD}TD}{M2}(1 - T)$

where

DD: Demand Deposits

TD: Time Deposits

T: Tax Rate on Interest Income

r_{DD}: Interest on Demand Deposits

r_{TD}: Interest on Time Deposits

The equation estimated in this study is given as,

$$\ln M = a \ln y + bEL \quad (5)$$

where EL, or expected loss is defined as expected inflation minus interest on money. This is in fact negative of the real rate of return on money. The equation above can be obtained from the square-law by taking the logarithm, and replacing the amount of transactions with income, and real interest rate by expected loss. Expected signs are positive for income and negative for EL. Inflationary expectations are assumed to be static in the sense that expected inflation in any period is the inflation rate which occurred in the previous period.

2.2 Econometric Framework

2.2.1 Stationarity, Unit Roots and Orders of Integration

A time series (x_t is stationary if its mean, $E(x_t)$, is independent of t, and its variance, $E[x_t - E(x_t)]^2$ is bounded by some finite number and does not vary systematically with time. A stationary series tends to return to its mean and fluctuate around it within a more or less constant range whereas a non-stationary series would have a different mean at different points in time.[2]

If a series must be differenced d times before it becomes stationary, then it is said to be integrated of order d, denoted by $I(d)$.Alternatively, we can say that a series is $I(d)$ if it has a stable, invertable, non-deterministic ARMA representation after differencing d times.

We can write an $I(d)$ series as

$$(1 - L)^d \phi(L) x_t = \theta(L) e_t \tag{6}$$

where L is the lag operator, $\phi(L)$ and $\theta(L)$ are polynomials in the lag operator and e_t is a stationary process. The roots of the polynomial, $(1 - L)^d \phi(z) = 0$, are called *unit roots*. As there are d roots of $z = 1$ testing for the order of

[2]For more information on the topics of this section see Engle and Granger (1987).

integration of a series is also called testing for the unit roots.

In general, if we take two series integrated of different orders, any linear combination will be integrated at the highest of the two orders of integration. An exception to this rule is where the low-frequency components of two series exactly offset each other and give a stationary linear combination. This is the case of a set of cointegrating variables. If a set of series are cointegrated, in the long run, they move closely together, even though they are individually trended.

The components of a vector X_t are said to be cointegrated of order d, b, denoted by $x_t \sim CI(d,b)$, if:

(i) all components of X_t are $I(d)$ and,

(ii) there exists a vector $\alpha(\neq 0)$ such that $Z_t = \alpha' X_t \sim I(d-b), b > 0$.

2.2.2 Testing for the Level of Integration

Consider the following autoregressive representation of a variable x_t:

$$x_t = \lambda_0 + \lambda_1 x_{t-1} + \lambda_2 x_{t-2} + \dots + \lambda_{n+1} x_{t-n-1} + u_t \qquad (7)$$

where u_t is a white noise stationary term.

Now, reparametrise (7):

$$\Delta x_t = \lambda_0 + (\sum_{i=1}^{n+1} \lambda_i - 1)x_{t-1} - \sum_{z=1}^{n+1}[(\sum_{i=z}^{n+1} \lambda_i)\Delta x_{t-z}] + u_t \tag{8}$$

Consider the regression

$$\Delta x_t = \beta_0 + \beta_1 x_{t-1} + \sum_{i=1}^{n} \alpha_i \Delta x_{t-i} + u_t \tag{9}$$

Now, comparing (7), (8) and (9), we can conclude that stationarity requires $\beta_1 < 0$, while if x_t is non-stationary, than β_1 would be equal to zero. The latter will also mean that the sum of the autoregressive parameters λ_i in (7) would be unity, implying that the series would have a unit root.

Then, one way of testing for stationarity would be to estimate a regression of the form (9), and test the hyphoteses that $\beta_1 = 0$. This can be done using the ratio of $\hat{\beta}$, to its estimated standart error. This ratio is the augmented Dickey-Fuller Statistic(ADF).

The distribution of ADF is not Student's t so Fuller(1976) has tabulated critical values for this statistic by Monte Carlo methods. The number of lags of Δx_t is normally chosen to ensure that the regression residual is white noise. In this study four lags are used because data used is quarterly.

If no lags of Δx_t are used, then the ratio is called Dickey-Fuller (DF) statistic. The critical values for DF and ADF statistics are the same for one variable case.

In order to test for second order integration, we have to run the following regression:

$$\Delta^2 x_t = \gamma_0 + \gamma_1 \Delta x_{t-1} + \sum_{i=1}^{n-1} \Psi_i \Delta^2 x_{t-i} + u_t \tag{10}$$

In this case, similarly, we test the null hypoteses that Δx_t is stationary, $\gamma_1 = 0$, against the hypoteses that $\gamma_1 < 0$.

2.2.3 A Maximum Likelihood Approach

One problem here is to know the number of cointegrating combinations which may exist between a set of variables. If one consider two variables each integrated of order one, $X_t \sim I(1)$ and $Y_t \sim I(1)$, we can show that there is a unique parameter α such that

$$u_t = X_t - \alpha Y_t \sim I(0) \tag{11}$$

To see this assume there is another cointegrating parameter β:

$$w_t = X_t - \beta Y_t \sim I(0) \tag{12}$$

Adding and subtracting βY_t in (11) we can obtain:

$$u_t = w_t - (\alpha - \beta)Y_t \tag{13}$$

According to our assumption u_t and w_t are both $I(0)$ while Y_t is $I(1)$. This can hold only if $\alpha = \beta$. So, α is unique. But when we consider more than two variables, it is not possible to guarantee the uniqueness of the cointegrating vector.

The original approach was to assume a unique cointegrating vector. This approch was developed in the influencial work of Engle and Granger (1987). Johansen(1988) suggests a method for estimating all the cointegrating vectors and for constructing some statistical tests. He proposes the following data generation process of a vector of N variables X:

$$X_t = \Pi_1 X_{t-1} + ... + \Pi_k X_{t-k} + e_t \tag{14}$$

where each Π_i is an (N x N) matrix of parameters. This equation can be reparametrised in the error correction form as:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + ... + \Gamma_{k-1} \Delta X_{t-k+1} + \Gamma_k X_{t-k} + e_t \tag{15}$$

where

$\Gamma_i = -I + \Pi_1 + ...\Pi_i; i = 1...k.$

So Γ_k is a long run solution to (14).

Now, if X_t is a vector of $I(1)$ variables, then the left hand side of (15) is $I(0)$. At the right hand side all terms, other than the final term, are also $I(0)$. This implies that the last term must also be $I(0)$, $\Gamma_k X_{t-k} \sim I(0)$. This means that either X contains a number of cointegrating vectors or Γ_k is a matrix of zeros. Now, we define two Nxr matrices α and β such that

$-\Gamma_k = \alpha\beta'$.

It is easy to see from here that the columns of β are cointegrating parameter vectors for X_t, if X consists of variables integrated of order one then r must be at most $N-1$, so $r \leq N-1$.

In his paper Johansen gives the following theorem.

Theorem: The maximum likelihood estimate of the space spanned by β is the space spanned by the r canonical variates corresponding to the r largest squared canonical correlations between the residuals of X_{t-k} and ΔX_t corrected for the effect of the lagged differences of the X process. The likelihood ratio test statistic for the hyphoteses that there are at most r cointegrating vectors is

$$- 2 \ln Q = -T \sum_{i=r+1}^{N} \ln(1 - \hat{\lambda}_i) \tag{16}$$

where $\hat{\lambda}_{r+1} ... \hat{\lambda}_N$ are the $(N - r)$ smallest squared canonical correlations. After this Johansen shows that the likelihood ratio test has an asymptotic distribution which is a function of an $N - r$ dimensional Brownian motion and he tabulates a set of critical values which will be correct for all the models. He also demonstrates that the space spanned by β is consistently estimated by the space spanned by $\hat{\beta}$.

According to the Johansen's procedure we first regress ΔX_t on the lagged differences of ΔX_t which gives a set of residuals R_{0t}. We then regress X_{t-k} on the lagged differences of ΔX_{t-j} which gives another set of residuals R_{kt}. The likelihood function is then proportional to

$$L(\alpha, \beta, \Omega) = |\Omega|^{-T/2} exp[-1/2 \sum_{t=1}^{T} (R_{0t} + \alpha\beta' R_{kt})' \Omega^{-1} (R_{0t} + \alpha\beta' R_{kt})] \tag{17}$$

where T is the number of observations and Ω is the covariance matrix of e_t.

Assuming β as fixed we can maximize over α and Ω by regressing R_{0t} on $-\beta' R_{kt}$. This will give us

$$\hat{\alpha}(\beta) = -S_{0k}\beta(\beta' S_{kk}\beta)^{-1}$$

$$\hat{\Omega}(\beta) = S_{00} - S_{0k}\beta(\beta' S_{kk}\beta)^{-1}\beta' S_{k0}$$

where

$S_{ij} = T^{-1} \sum_{t=1}^{T} R_{it} R'_{jt}. \quad i, f = 0, k$

After substituting these into the likelihood function, resulting function will be proportional to $|\hat{\Omega}(\beta)|^{-T/2}$. So maximizing the likelihood function may be reduced to minimising

$$|S_{00} - S_{0k}\beta(\beta' S_{kk}\beta)^{-1}\beta' S_{k0}| \tag{18}$$

with respect to β.

This can be done by solving an eigenvalue problem. The matrix $\hat{\beta}$ is obtained as a set of eigenvectors with a corresponding vector of eigenvalues $\hat{\lambda}$. The columns of β are significant if the corresponding eigenvalue is significantly different from zero. Let the elements of $\hat{\lambda}_i$ be ordered as:

$\hat{\lambda}_1 > \hat{\lambda}_2 > ... > \hat{\lambda}_{N-1}$ and let the columns of β be ordered accordingly.

Then the eigenvalues are defined such that the maximum likelihood estimate of Ω is given by:

$$\hat{\Omega}(\beta) = |S_{00}| \prod_{i=1}^{N} (1 - \lambda_i). \tag{19}$$

Now, if we want to test the following null hyphoteses:

$H_0 : \lambda_i = 0, i = r + 1, ..., N - 1,$

we have to restrict the estimate of Ω as:

$$\hat{\Omega}(\beta) = |S_{00}| \prod_{i=1}^{r} (1 - \lambda_i). \tag{20}$$

Then we can form a likelihood ratio statistic for the null hyphoteses of at most r cointegrating vectors as

$$LR(N - r) = -2\ln(Q) = -T \sum_{i=r+1}^{N} \ln(1 - \hat{\lambda}_i) \tag{21}$$

where $Q = \frac{restricted\ \ maximised\ \ likelihood}{unrestricted\ \ maximised\ \ likelihood}$

Johansen (1989) gives the critical values for this statistic.

2.2.4 Testing Linear Restrictions

Johansen(1988) also demonstrates how linear restrictions can be tested on the parameters of the cointegrating vector. He considers linear restrictions on β which reduce the number of independent cointegrating parameters from N to S where $S \leq N$. In general the restrictions will be written in the following form:

$$H_0 : \beta = H\phi \tag{22}$$

where H is an (NxS) matrix of full rank equal to S and ϕ is an (Sxr) matrix of unknown parameters. Since H is known, we will replace β with $H\phi$ to obtain an estimate ϕ^*. The restricted estimate of β will then be given by $\beta^* = H\phi^*$.

The restricted estimation will produce a set of eigenvalues, $\lambda_1^* > \lambda_2^* > ... > \lambda_r^*$. This will give us a test based on the first r cointegrating vectors:

$$LR^*[r(N-S)] = -2\ln(Q) = T\sum_{i=1}^{r}\ln(1-\lambda_i^*)/(1-\hat{\lambda}_i) \quad (23)$$

which has an asymptotic chi-square distribution with r(N-S) degrees of freedom.

2.2.5 Weak Exogeneity

Exogeneity is a basic feature of emprical modelling and it is studied in Richard (1980), Hendry and Richard (1982, 1983) and Engle, Hendry and Richard (1983). Suppose, x_t is a vector of observations on all variables in period t, and $X_{t-1} = (x_{t-1}...X_1)'$. Then the joint probability of the sample x_t may be written as

$$\prod D(x_t|X_{t-1};\Theta) \quad (24)$$

where Θ is a vector of unknown parameters.

In order to simplfy this very general formulation we have to marginalize the data generating process (DGP). This DGP contains more variables than we can deal with in practice, so we choose a subset of variables. Secondly, given this choice of variables of interest, we must select a subset of these variables to be the endogenous variables (Y_t). These are then determined by the remaining variables (Z_t) of interest.

We can represent these two assumptions by the following factorisation:

$$D(x_t|X_{t-1};\Theta) = A(W_t|X_t : \alpha)B(Y_t|Y_{t-1}, Z_t : \beta)C(Z_t|Y_{t-1}, Z_{t-1} : \gamma) \qquad (25)$$

A specifies the determination of W, the variables of no interest, as a function of all the variables X_t. B gives the endogenous variables of interest Y_t as a function of lagged Y and the exogenous variables Z_t. C gives the determination of the exogenous variables Z_t as a function of the lagged endogenous and exogenous variables.

The conditioning assumptions require that the Z_t variables are at least weakly exogenous. This means that Z_t is independent of Y_t, which is assumed in term C.

3 Empirical Study on Money Demand Functions for M1 and M2

In this study quarterly data for the period 1977(1)-1989(4) is used. The data consists of M1, M2, real income, price index, interest on demand deposits, interest on one year time deposits, quantity of demand deposits, quantity of time deposits, and tax on interest income[3].

3.1 Testing for Unit Roots

Estimation procedure begins with the determination of the level of integration for the relevant variables. This is important because if any of the variables are stationary than we can not talk about cointegration between that variable and the other variables.

The level of integration is tested using DF and ADF tests for logarithm of M1 (LM1/P), logarithm of M2 (LM2/P), logarithm of real income (LREALY), expected loss for M1 (EL(M1)) and expected loss for M2 (EL(M2)).

The null hyphoteses in each case is that the variable in question is I(1). Naturally, if first difference of a variable is I(1), then the variable itself is I(2). The 5 % rejection region for both Dickey-Fuller and Augmented Dickey-Fuller

[3]See the data sources.

Table 1: Unit Root Tests for Money, Income and Expected Loss

	DF	ADF
LM1/P	-0.83	-0.35
ΔLM1/P	-5.95	-2.84
LM2/P	0.75	0.74
ΔLM2/P	-4.92	-2.42
LREALY	0.21	2.64
ΔLREALY	-7.57	-1.47
EL(M1)	-2.86	-1.73
ΔEL(M1)	-9.10	-3.64
EL(M2)	-0.89	-0.76
ΔEL(M2)	-7.87	-2.94

statistics are the same, $DForADF < -2.93$[4].

Looking at Table 1, we can conclude that Dickey-Fuller test rejects the null hypoteses for the first differences of the variables, but both of the tests support the hypoteses that all of the variables in levels are I(1). This means that we can use the Johansen procedure described previously.

[4] Fuller (1976)

Table 2: Likelihood Ratio Statistic for M1 and M2 for the number of Cointegrating Vectors

# cointegrating vectors r	LRS for M1	LRS for M2	5 % critical value
$r \leq 2$	2.62	6.26	9.09
$r \leq 1$	9.56	24.71	20.16
$r = 0$	47.53	48.74	35.06

3.2 Testing for the Number of Cointegrating Vectors

As all the variables of interest are I(1), it is possible to implement the procedure described in the previous chapter. In order to implement the procedure, it seems plausible to use fourth order lags for the vector autoregression, because the data used is quarterly.

From Table 2 we see that for M1 the hyphoteses that $r = 0$ is rejected at 95 % significance level, while the hyphoteses of one or more cointegrating vectors is not. So we can conclude that there is a single statistically significant cointegrating vector.

For M2 the hyphoteses that $r = 0$ and $r \leq 1$ are rejected at 95 % significance level, while the hyphoteses of two cointegrating vectors is not. So here we can conclude that there is definitely one, but possibly there are two statistically significant cointegrating vectors.

3.3 Results of the Johansen Procedure

Now, in the light of section (2.2.3), we will set up a VAR model which allows for fourth order lags of each variable, a constant and a trend. This means each equation will consist of 14 variables. The result are reported in Appendix A for M1, and M2.

The eigenvectors presented in Appendix A are normalized by real M1 for the first, by real income for the second, and by expected loss for the third. From the first row of β', one cointegrating combination represents a real money demand relationship as, (1, -3.23, 0.61).

So this gives us the following relationship for the long-run solution for real M1 balances.

$$LM1/P = 3.23LREALY - 0.61EL(M1) \tag{26}$$

The eigenvectors presented in Appendix A are normalized by real M2 for the first, by real income for the second, and by expected loss for the third. From the first row of β', one cointegrating combination represents a real money demand relationship as, (1, -6.75, 1.52).

So this gives us the following relationship for the long-run solution for real M2 balances.

$$LM2/P = 6.75LREALY - 1.52EL(M2) \tag{27}$$

From the previous section we know that there is a second possible cointegrating vector for M2. This can be solved by eliminating LM2/P from the second row using the first row. The solution of this process will give us the folowing relationship between the real income and the expexted loss on M2.

$$LREALY = -0.17EL(M2) \tag{28}$$

3.4 Testing Linear Restrictions on β

Firstly, we restrict β such that the long-run income elasticity of income is unity. In other words the coefficients of real money and income are equal with opposite sign. The hypoteses is formulated as $\beta = H\phi$ where H is the restriction matrix of dimension (p x s) and ϕ is a (s x r) matrix of unknown parameters.

Eigenvalues and eigenvectors under this restriction is given in Appendix

Table 3: H matrix for the unit income elasticity restriction

variable	column 1	column2
LM1/P	1	0
REALY	-1	0
EL	0	1

B for both M1 and M2. Using this data we can calculate the test statistic for M1 as:

$-2lnQ = T\sum_{i=1}^{r}[ln(1-\mu_i^*)-ln(1-\mu_i)] = 51[ln(1-0.18)-ln(1-0.54)] = 30.09$ which will be compared with $\chi^2_{.95r(p-s)} = \chi^2_{.95,1} = 3.84$ where p-s indicates the number of restrictions on β and r is the number of cointegrating vectors. Therefore the hypoteses of unit income elasticity for M1 demand is rejected at 95 % significance level. Also, calculating the same statistic for M2 will yield: $-2lnQ = T\sum_{i=1}^{r}[ln(1-\mu_i^*)-ln(1-\mu_i)] = 51[ln(1-0.31)-ln(1-0.39)] = 5.86$ Comparing this value with 3.84, we also reject this hyphoteses for M2 at 95 % significance level.

Secondly, we restrict β such that the cefficient of REALY is 1/2 and the coefficient of EL is -1/2, which may be looked as a test of the Tobin-Baumal

Table 4: II matrix restricting coefficient of REALY as 1/2 and coefficient of EL as -1/2

variable	column 1
LM1/P	1
REALY	-0.5
EL	0.5

money demand function.

Eigenvalues and eigenvectors under this restriction is given in Appendix C for both M1 and M2. Calculating the test statistic for M1 will give:

$-2lnQ = T\sum_{i=1}^{r}[ln(1-\mu_i^*)-ln(1-\mu_i)] = 51[ln(1-0.12)-ln(1-0.54)] = 34.17$

which will be compared with $\chi^2_{.95r(p-s)} = \chi^2_{.95,2} = 5.99$ where p-s indicates the number of restrictions on β and r is the number of cointegrating vectors. Therefore the hypoteses that coefficient of REALY being equal to 1/2 and coefficient of EL(M1) being equal to -1/2 is rejected at 95 % significance level.

Calculating the same statistic for M2 will yield:

$-2lnQ = T\sum_{i=1}^{r}[ln(1-\mu_i^*)-ln(1-\mu_i)] = 51[ln(1-0.78)-ln(1-0.39)] = 15.81$

The hypoteses is rejected also for M2.

3.5 Testing Linear Restrictions on α

Satisfactory modelling requires weak exogeneity of the regressors. Weak exogeneity can be tested by putting restrictions on α matrix.

One can test weak exogeneity, by restricting α of the form $\alpha = A\phi$ where A is a (p x m) matrix. Also define B which is (p x (p-m)) and othogonal to A, B'A=0. Therefore, $B'\alpha = 0$ indicating that some of the rows of α should be zero.

Here, we will consider tests for weak exogeneity of real income ($\alpha_2 = 0$) and expected loss ($\alpha_3 = 0$). Finally a joint hyphoteses of ($\alpha_2 = \alpha_3 = 0$) is considered. The eigenvalue and eigenvectors under this restrictions are reported in Appendix D.

For, the three cases the restriction matrices A and B are shown in tables 5,6 and 7 respectively.

Test statistic for $\alpha_2 = 0$ is:

$-2lnQ = 51[ln(1-0.21) - ln(1-0.54) = 27.2]$

The test statistic is asymptotically distributed as χ^2 with f=r(p-m) degrees of freedom. Comparing this value with $\chi^2_{0.95,1}$, one can see that real income is not

Table 5: A and B matrices for testing $\alpha_2 = 0$

A matrix			B matrix	
variable	LM1/P	LREALY	variable	LM1/P
Row 1	1	0	Row 1	0
Row 2	0	0	Row 2	1
Row 3	0	1	Row 3	0

Table 6: A and B matrices for testing $\alpha_3 = 0$

A matrix			B matrix	
variable	LM1/P	LREALY	variable	LM1/P
Row 1	1	0	Row 1	0
Row 2	0	1	Row 2	0
Row 3	0	0	Row 3	1

Table 7: A and B matrices for testing $\alpha_2 = \alpha_3 = 0$

A matrix		B matrix		
variable	LM1/P	variable	LM1/P	REALY
Row 1	1	Row 1	0	0
Row 2	0	Row 2	1	0
Row 3	0	Row 3	0	1

weakly exogenous for the long run demand for M1.

Test statistic for $\alpha_3 = 0$ is:

$-2lnQ = 51[ln(1 - 0.54) - ln(1 - 0.54)] = 0$

Comparing with $\chi^2_{0.95,1}$ shows that expected loss is weakly exogenous.

Test statistic for $\alpha_2 = \alpha_3 = 0$ is:

$-2lnQ = 51[ln(1 - 0.19) - ln(1 - 0.54)] = 28.52$

Comparing with $\chi^2_{0.95,2}$ shows that the joint hyphoteses of weak exogeneity is also rejected.

For M2 the same statistics can be reported as follows:

Test statistic for $\alpha_2 = 0$ is:

$-2lnQ = 51[ln(1-0.31) - ln(1-0.39) = 6.06]$

$\chi^2_{0.95,1}$ suggests that real income is not weakly exogenous for the long run demand for M1.

Test statistic for $\alpha_3 = 0$ is:

$-2lnQ = 51[ln(1-0.38) - ln(1-0.39)] = 0$

Comparing with $\chi^2_{0.95,1}$ shows that expected loss is weakly exogenous.

Test statistic for $\alpha_2 = \alpha_3 = 0$ is:

$-2lnQ = 51[ln(1-0.31) - ln(1-0.39)] = 6.06$

Comparing with $\chi^2_{0.95,2}$ shows that the joint hyphoteses of weak exogeneity is not rejected.

4 Conclusion

In this study, the main aim was to test the money demand function for the period 1977(1)-1989(4). This is done by using the Johansen procedure. This procedure is in essence a maximum likelihood approach to theory of cointegration. This method has some advantages against Engle and Granger two step procedure. Engle and Granger two step procedure is done by estimating a static regression and ignoring the dynamics in the first step. The complete omission of dynamics in the first step creates problems since dynamics are important in finite samples to reduce bias in both short-run and long-run coefficient estimates. In addition, a two-step estimating procedure does not have well defined limiting distributions, but the dynamic models often allow to use the standart Normal asymptotic theory.[5]

In money demand studies highly aggregated time-series data are used. Initially the data were annually in most studies but in recent studies the focus is on shorter periods. Following this path quarterly data is used in this study, which is the shortest period available in Turkey. The main reason for using shorter periods is that these are more useful for guiding monetary policy.

[5] For details see Banerjee and others (1986), Stock (1987), and Stock and West (1988).

The results showed that M1, M2, real income and expected loss terms are all I(1). For both M1 and M2 the resuls of the analysis suggested that money is cointegrated with real income and expected loss. This is strong evidence supporting quantity theory of money which is suggesting a relationship between these variables. Existence of a long-run relationship for money demand is confirmed by the tests concerning the number of cointegrating vectors. There is at least one cointegrating vector for both M1 and M2 for sure.

We have argued before that quantity theory in the form formulated by Friedman requires the money demand to be a function of income, expected interest rate on money and expected interest rates on alternative assets. In our formulation the alternative interest rate is taken as inflation. The rationale behind this is in a financially repressed economy the only alternative to money is consumption. Under this assumption expected loss is defined as inflation minus nominal interest rate on money. This is in fact negative of the real interest rate on money.

We expect the coefficient of income to be positive. In Friedman's formulation coefficient of return on money is expected to be positive and coefficients of alternative rates are expected to be negative. In our formulation the coefficients of inflation and interest rate on money are restricted to have the same magnitude with opposite signs. Under this restriction we know that if real interest rate on money increases then demand for money increases. Thus, we expect the sign of expected loss to be negative.

The results of the Johannsen procedure supports the arguments stated above. The signs of the variables are as expected. Sign of real income is positive and sign of expected loss is negative, although the magnitude of income term is larger than expected. The results also gave some evidence of a second cointegrating vector for M2. This vector suggests a long run negative relationship between income and expected loss.

Tests of linear restrictions on β is used to restrict the money demand coefficients. Tobin-Baumal model is rejected for both M1 and M2. This test is performed by restricting the coefficients of real income and expected loss to 1/2 and -1/2 respectively. Unit elasticity of income is rejected for M1, it is also rejected for M2. This test is performed by restricting the coefficient of real income to one.

Tests of weak exogeneity are done by imposing restrictions on α. It is worth noting that income seems not to be weakly exogenous, but expected loss is weakly exogenous for both M1 and M2. Also a joint test is performed. For M1 income and expected loss are not weakly exogenous jointly. For M2, income and expected loss are weakly exogenous jointly. This may be taken as evidence for the fact that our model for M2 is valid. And, using M2 gives better and econmically more interpretable results for money demand estimation.

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Data Sources

Quarterly Income : Calculated by Ercan UYGUR and Fatih ÖZATAY from the Central Bank of Turkey.

Interest Rates : From various issues of *Central Bank Quarterly Bulletins* between 1977-1989.

M1, M2, Demand and Time Deposits : From various issues of *Central Bank Quarterly Bulletins* between 1977-1989.

Price Index : From SIS *Statistical Indicators 1923-1990.*

Appendix A

Table 8: Results of multicointegration analysis for M1

Eigenvalues μ_i	$-Tlog(1-\mu_i)$	$-T\sum\log(1-\mu_i)$
0.53203	2.624177	2.624117
0.13456	6.937189	9.561366
0.54667	37.97514	47.536507

Standardized β' eigenvectors			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	1.00000	-3.23274	-0.61594
LREALY	0.78513	1.00000	-2.36515
EL(M1)	0.31677	-0.29470	1.00000

Standardized α coefficients			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	0.00606	-0.08432	-0.28045
LREALY	0.66458	-0.04824	-0.12108
EL(M1)	0.05446	0.05439	-0.16109

Table 9: Results of multicointegration analysis for M2

Eigenvalues μ_i	$-Tlog(1-\mu_i)$	$-T\sum \log(1-\mu_i)$
0.12241	6.267791	6.267791
0.31910	18.448940	24.716732
0.393841	24.029423	48.746155

Standardized β' eigenvectors			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	1.00000	-6.75859	1.52937
LREALY	-0.86222	1.00000	-2.16909
EL(M1)	-0.44532	0.25959	1.00000

Standardized α coefficients			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	0.06602	0.36532	-0.00639
LREALY	0.21942	-0.06418	-0.05019
EL(M1)	-0.02237	0.00169	-0.14420

Appendix B

Table 10: Results of multicointegration analysis for M1 under the unit income elaticity restriction

Eigenvalues μ_i	$-Tlog(1-\mu_i)$	$-T\sum\log(1-\mu_i)$
0.05335	2.632074	2.632074
0.184422	9.785205	12.417279

Standardized β' eigenvectors			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	1	-1	-1.07090
LREALY	-1	1	-3.19955
EL(M1)	-1.07489	1.07489	1

Standardized α coefficients			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	0.12089	-0.08480	-0.0001527
LREALY	-0.09373	-0.02932	0.0001151
EL(M1)	-0.09004	-0.05326	0.0001096

Table 11: Results of multicointegration analysis for M2 under the unit income elaticity restriction

Eigenvalues μ_i	$-Tlog(1-\mu_i)$	$-T\sum\log(1-\mu_i)$
0.124269	6.369445	6.369445
0.319566	18.481167	24.850612

Standardized β' eigenvectors			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	1	-1	2.54889
LREALY	-1	1	1.90207
EL(M1)	-1.71548	1.71548	1

Standardized α coefficients			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	0.31817	0.0086	-0.00052
LREALY	-0.0063219	0.04716	0.00012
EL(M1)	-0.00301	0.06744	0.00018

Appendix C

Table 12: Results of multicointegration analysis for M1 under the hyphoteses that coefficient of real income is 1/2 and coefficient of expected loss is -1/2

Eigenvalues μ_i	$-Tlog(1-\mu_i)$	$-T\sum\log(1-\mu_i)$
0.120329	6.153948	6.153948

Standardized β' eigenvectors			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	1	-0.5	0.5
LREALY	-2	1	-1
EL(M1)	2	-1	1

Standardized α coefficients			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	0.17923	-0.0005321	0.0000866
LREALY	0.01940	-0.0000576	0.0000094
EL(M1)	-0.06084	0.0001806	-0.0000294

Table 13: Results of multicointegration analysis for M2 under the hyphoteses that coefficient of real income is 1/2 and coefficient of expected loss is -1/2

Eigenvalues μ_i	$-Tlog(1-\mu_i)$	$-T\sum\log(1-\mu_i)$
0.178967	9.465204	9.465204

Standardized β' eigenvectors			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	1	-0.5	0.5
LREALY	-2	1	-1
EL(M1)	2	-1	1

Standardized α coefficients			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	0.14396	-0.0005573	0.0001487
LREALY	0.02061	-0.0000798	0.0000213
EL(M1)	-0.11334	0.0004387	-0.0001170

Appendix D

Table 14: Results of multicointegration analysis for M1 under the hyphoteses that $\alpha_2 = 0$

Eigenvalues μ_i	$-Tlog(1-\mu_i)$	$-T\sum\log(1-\mu_i)$
0.075659	3.776361	3.776361
0.217166	11.752051	15.528412

Standardized β' eigenvectors			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	1	-2.27122	0.22121
LREALY	-0.35527	1	-1.17632
EL(M1)	-0.31402	5.32695	1

Standardized α coefficients			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	0.37124	-0.15768	-0.0015286
LREALY	0	0	0
EL(M1)	-0.12187	-0.24786	0.0024103

Table 15: Results of multicointegration analysis for M1 under the hyphoteses that $\alpha_3 = 0$

Eigenvalues μ_i	$-T log(1-\mu_i)$	$-T\sum \log(1-\mu_i)$
0.091236	4.592160	4.592160
0.540173	37.291481	41.883641

Standardized β' eigenvectors			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	1	-3.26059	0.69336
LREALY	-6.44721	1	-3.53183
EL(M1)	0.13822	-0.24843	1

Standardized α coefficients			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	-0.01446	-0.02529	-0.0041311
LREALY	-0.66519	-0.01234	0.00958
EL(M1)	0	0	0

Table 16: Results of multicointegration analysis for M1 under the hyphoteses that $\alpha_2 = \alpha_3 = 0$

Eigenvalues μ_i	$-Tlog(1-\mu_i)$	$-T\sum\log(1-\mu_i)$
0.191334	10.1937	10.1937

Standardized β' eigenvectors			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	1	-2.34674	0.65053
LREALY	-0.11384	1	-0.27720
EL(M1)	0.12072	-0.03261	1

Standardized α coefficients			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	0.42604	-0.0050732	-0.0065235
LREALY	0	0	0
EL(M1)	0	0	0

Table 17: Results of multicointegration analysis for M2 under the hyphoteses that $\alpha_2 = 0$

Eigenvalues μ_i	$-Tlog(1-\mu_i)$	$-T\sum \log(1-\mu_i)$
0.131831	6.785716	6.785716
0.319938	18.507416	25.293132

Standardized β' eigenvectors			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	1	-0.89862	2.55311
LREALY	0.26849	1	-1.59184
EL(M1)	1.03967	-4.63742	1

Standardized α coefficients			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	0.31399	0.0034527	0.004004
LREALY	0	0	0
EL(M1)	-0.0061214	-0.12486	0.0042168

Table 18: Results of multicointegration analysis for M2 under the hyphoteses that $\alpha_3 = 0$

Eigenvalues μ_i	$-Tlog(1-\mu_i)$	$-T\sum\log(1-\mu_i)$
0.319103	18.448554	18.448554
0.384789	23.317947	41.766501

Standardized β' eigenvectors			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	1	-6.24356	1.17835
LREALY	-0.85808	1	-2.15271
EL(M1)	-0.36294	-0.06535	1

Standardized α coefficients			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	-0.07113	-0.36735	0.0079036
LREALY	-0.23931	0.06695	0.0053095
EL(M1)	0	0	0

Table 19: Results of multicointegration analysis for M2 under the hyphoteses that $\alpha_2 = \alpha_3 = 0$

Eigenvalues μ_i	$-Tlog(1-\mu_i)$	$-T\sum \log(1-\mu_i)$
0.319848	18.50105	18.50105

Standardized β' eigenvectors			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	1	-0.9108	2.57545
LREALY	0.77619	1	-2.82770
EL(M1)	0.81667	-3.91049	1

Standardized α coefficients			
Variable	LM1/P	LREALY	EL(M1)
LM1/P	0.31284	-0.0020447	0.0051596
LREALY	0	0	0
EL(M1)	0	0	0