MACHINING CONDITIONS OPTIMIZATION, TOOL ALLOCATION, AND TOOL MAGAZINE ARRANGEMENT ON A CNC TURNING CENTER

A THESIS

SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL ENGINEERING

AND THE INSTITUTE OF ENGINEERING AND SCIENCES

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

By

Selçuk Avcı

August, 1993

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Selim Aktürk

Assist. Prof. M. Selim Aktürk (Advisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

MACHINING CONDITIONS OPTIMIZATION, TOOL ALLOCATION, AND TOOL MAGAZINE ARRANGEMENT ON A CNC TURNING CENTER

Selçuk Avcı
M.S. in Industrial Engineering
Supervisor: Assist. Prof. M. Selim Aktürk
August, 1993

In the view of the high investment and tooling cost of a CNC machining center, the cutting and idle times should be optimized by considering the tool consumption and the non-machining time components for an effective utilization. Therefore, it is necessary to develop a new module as a part of the overall computer-aided process planning system, which will improve both the system effectiveness and provide consistent process plans.

In this thesis, it is proposed to build a detailed mathematical model for the operation of a CNC lathe which will include the system characterization, the cutting conditions and tool life relationship, and related constraints. Then an algorithm is presented to find tool–operation assignments, machining conditions, appropriate tool magazine organization, and an operations sequence which will result in a minimum production cost.

Key words: Machining Economics, Computer Aided Process Planning (CAPP), Geometric Programming

ÖZET

BİLGİSAYAR NUMERİK KONTROL TORNA TEZGAHINDA İŞLEME ŞARTLARI ENİYİLEME, KESİCİ ALET ATAMA VE KESİCİ ALET MAGAZİNİ DÜZENLEME

Selçuk Avcı
Endüstri Mühendisliği Bölümü Yüksek Lisans
Tez Yöneticisi: Yrd. Doç. Dr. M. Selim Aktürk
Ağustos, 1993

Bilgisayar Numerik Kontrol (BNK) tezgahların yatırım ve kesici alet maliyetinin yüksek olmasından dolayı, etkin bir yararlanma için, kesici alet sarfiyatı ve işleme dışı zamanlar göz önüne alınarak işleme ve işleme dişi zamanlar eniyilenmelidir. Bu nedenle genel Bilgisayar Destekli İşlem Planlama (BDİP) sisteminin bir parçası olarak, sistemin etkinliğini arttıracak ve tutarlı işlem planları sağlayacak yeni bir modülün geliştirilmesi gereklidir.

Bu çalısmada, bir BNK Torna tezgahı için, sistem özellikleri, işleme şartları, kesici alet ömrü ilişkisi ve ilgili kısıtlarını içeren ayrıntılı bir matematiksel model önerilmektedir. Ardından en az maliyetle sonuçlanan kesici alet–işlem atamaları, işlem şartları, uygun kesici alet magazini düzenlemesi ve işlem sırasını bulan bir algoritma sunulmaktadır.

Anahtar sözcükler: İşleme Ekonomisi, Bilgisayar Destekli İşlem Planlama (BDİP), Geometrik İzlenceleme

To my parents

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Chapter 1

INTRODUCTION

Numerical Control (NC) machines were introduced in the early 1950s and have since then remained in wide usage. With the advent of better and faster computational power Computer Numerical Control (CNC) and Direct Numerical Control (DNC) have emerged. By the introduction of these new machine tools, there has been a trend towards more flexible and automated manufacturing to meet the changing market needs. Flexible Manufacturing Systems (FMSs) are the result of this trend in automation and flexibility. The objective of these systems is to achieve the efficiency and utilization rates of mass production while retaining the versatility of the traditional job shop.

Process planning is the function within a manufacturing organization that determines how a raw material is to be transformed from its initial state into its final state. It can be viewed as the preparation of the detailed work instructions necessary to produce the desired part. In the modern factory, it is a major determinant of manufacturing cost and it contributes to the success of the manufacturing industry by providing the necessary link between the functions of Computer-Aided Design (CAD) and Computer-Aided Manufacturing (CAM). While CAD systems and CNC machine tools commonly used in industry, process planning is still carried out manually. However, manual method of process planning is time consuming, inconsistent and requires scarce resources, like an experienced process planner. Automation of the process planning function is required to eliminate the disadvantages of the manual method and to bridge the gap between CAD and CAM. Computer-Aided Process Planning (CAPP) systems have been proposed to achieve this automation of planning. The variant and generative process planning systems are aimed at automating and speeding up the process planning.

The outputs obtained from CAPP systems are essential for job scheduling, NC programming, shop floor control, and other manufacturing tasks. A sub-domain of process planning is operation planning. This downstream activity entails determination of operations, selection of tools, selection of cutting conditions and determination of cutter location data, etc., and those informations are embedded in a part program together with the cutting tool path and operations sequence.

The productivity of an FMS, hence CNC Machine Tool, is not only dependent upon cutting time, but more importantly non-cutting time. The latest generation of CNC machines support automatic tool changing and be programmed in real time. Furthermore, very little setup is required between batches. On the other hand, an emphasis should be placed on the part programming in order to reduce the total non-machining time in a part cycle. Part programming systems that generate automatically the control instructions for CNC machine tools are considered as an economic and efficient means of reducing the lead times in process planning and decreasing the cost of manufacturing.

In this study, we present a hierarchical approach which solves the machining conditions optimization, tool allocation, tool magazine arrangement, and operations sequencing problems arising in part programming and operation planning of a CNC turning machine. There exist three stages in this decision hierarchy. In the first level, tool allocation problem is solved and the tool–operation assignments are fixed by their governing machining conditions. In the second level, tool magazine arrangement is determined by considering the tool sharing possibilities and the duplicate tools. Finally, the sequencing of operations is made in the last level. This model can be considered as a module of a fully automated part programming system.

In the next chapter, a literature review on the related subjects is presented. In Chapter 3, a problem definition is given to define the scope of this study, and mathematical formulation of the model is presented. Consequently in Chapter 4, the proposed heuristic approach is introduced and this approach is applied on an example problem for illustration purpose, in Chapter 5. Finally in Chapter 6, the conclusion of this study is presented with future research recommendations.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

In the literature there are few studies directly related to our research problem, but the following sub-problems have been addressed in the context of Flexible Manufacturing Systems (FMSs), Computer Numerical Control (CNC) machine tools, and tool management literature with different point of views, for example in FMSs literature the following problems are usually addressed at the system level :

- Machining Conditions Optimization
- Tool Magazine Organization
- Operations Sequencing

Furthermore, in the literature there are many other studies under different topics mentioning to the motivations for this study and the aspect of our problem. To give the related literature in an organized manner, we start with the Flexible Manufacturing Systems in the following section. Then we will introduce the Computer Numerical Control (CNC) machine tools in Section

2.3. After identifying and mentioning to related problems in both system and the equipment level, we will continue with the tool management considerations in Section 2.4. In Sections 2.5, 2.6 and 2.7, the literature on the above sub-problems will be presented, respectively. We will mention the existing literature on computer aided process planning and part programming, and its relations with our research topic in Section 2.8. Finally, in the last section we will summarize our findings from these literature review and present the concluding remarks.

2.2 Flexible Manufacturing Systems (FMSs)

A flexible manufacturing system is designed to achieve the efficiency of both automated high-volume mass production and flexibility of low-volume job shop production. FMSs typically consist of Numerically Controlled (NC) machine tools capable of performing multiple functions to process parts, automated Material Handling System (MHS) to move parts and tools between machines, Automated Storage and Retrieval System (AS/RS), and on-line computer systems to control and manage all operations, such as the machining operation, part and material movement, tool interchanges, etc. So, it is a complex system containing many limited resources like tools, pallets and fixtures.

Due to this complex nature of FMSs, the related production management problems are also more complex than any other manufacturing system. Therefore the efficient operation of a FMS is very difficult task, and in many implementations the available capacity has been under utilized. In the view of high initial cost of the FMSs, however, it is very important to operate these systems efficiently as much as possible in order to get expected benefits of flexibility and economy. In the FMSs literature operational problems at the equipment level concerning operations sequencing, machining conditions optimization, and tool management have been rarely addressed. On the other hand, there are several studies on the system level problems that are underlying some important considerations of our problem like tool management, operations sequencing, machining conditions optimization, etc., in an aggregate manner. In the literature, a series of problems have been addressed and those are mainly related with the design and operation of the FMSs. Among the others these include; system selection and justification, part family selection, system component selection, system loading and part allocation, and operational control. In our study we are particularly interested in the last two problems, since they are closely related issues for the CNC machining centers as they are being the fundamental components of FMSs. In the FMSs literature, system loading, part allocation, and operational control problems are mentioned by many authors (see [2] [22] [23] [26] [27] [29] [30] [34]).

In most of these studies, the operational characteristics of the system components and tool management concept have not been considered during the system modelling phase, since they make the model rather complex that it is almost impossible to handle such a complicated problem. However, especially for the operational problems, these factors should be taken into account for a reliable modelling of FMSs, otherwise the absence of such crucial constraints may lead to infeasible results.

A study on the decision support requirements in Flexible Manufacturing Systems was presented by Suri [34]. In this paper, the FMSs have been installed around the world qualified as being underutilized in the view of their high investment and operating costs. Suri relates this ineffiency to the complexity of the system and to the resulting interactions in decision making which is also rather complex than the other production systems, and many times not easy to predict. Suri provided a Decision Support Structure (DSS) to aid in FMS operational decision making, and showed how to implement this structure by using appropriate software and hardware components. He recommends to consider such a DSS as an integral part of the FMS. This structure has three-level of operational decision making stages related with the corresponding time span. The third level, which is related with the short term decision requirements, involves the following items:

1. Work order scheduling and dispatching

2. Movement of work piece and material handling system

3. Tool management

4. System monitoring and diagnostics

5. Reacting to disruptions

These third level decisions are required for real-time operation of the FMS. An immediate decision making is necessary by monitoring the state of system for successful operation of the FMS. So, the enormous capability of information gathering and evaluating of the computer system should be used to support these short term decisions.

Kouvelis [22] worked on the optimal tool selection for FMSs as a preliminary design issue or long term planning consideration. In this paper a two-level hierarchical decision scheme for the problem is described. At the first level, a long-term operations assignment policy to machines is specified. The optimal tooling decision is made at the second level. Kouvelis also mentioned to the cutting tool utilization as being a key to entire system performance especially for the metal cutting industries. In this study, the importance of tooling for a FMS underlined, and cost of tooling has reported as 25-30% percent of the fixed and variable costs of production in these automated manufacturing environments. The reason for such a high contribution of the tooling to the total manufacturing cost is related to the high material removal rate in metal cutting processes, and the consequent increased tool consumption rates and tool replacement frequencies. From industrial data, he also reports that usually metal parts processed in FMSs require 10 to 30 different tools for their operation, with 200-300 tools needed to produce the different part types in any given week.

Sarin and Chen [26] presented a study on the machine loading and tool allocation problem for FMSs. In this problem, for a given fixed number of parts whose operations are to be processed through the machines limited tool magazine capacity, the routing of parts and the allocation of tools of limited tool life to the machines are determined in order to minimize the total cost of the operations. In the modeling of the system they assumed that the machine loading and tool allocation problem particularly applied to the lower level of the production planning problem where the time period may be a week, a day or a shift. Once the related decisions are made by this approach, it is assumed that the tools stay with the machines for the planning period, and the parts are routed through the machines where the needed tooling and NC programs are already loaded.

In the FMS loading literature another approach was presented by DeWerra and Widmer [8]. They emphasize the importance of the Tool Management concept in the modeling of such production planning problems. They are strongly recommending to consider tooling in the modeling phase, especially when the setup times are important with respect to the processing times. They are indicating the fact that the tools constitute a basic component in FMSs with the presence of automatic tool interchangers and the tool magazines, so FMS loading problem which tries to have appropriate tool in the right tool magazine at the moment is needed. Furthermore, at the planning level as well as at the scheduling level (Stecke [29]), one should take into account that the tools are a kind of resource for the production process since they are available in limited numbers. By following these facts they are also emphasizing the importance of tool considerations in the long term and as well as in the short term decisions in FMSs problems.

Another important observation on the operating of FMSs was reported by Tang and Denardo [35][36], that is most of the studies on the FMS loading problem assumes negligible time for the interchange of the tools. However according to their industrial experience, the situation is somewhat different and they noticed that most of the CNC machining centers require a significant time amount for fine tuning during the tool changes relative to the job processing times. This implies that time available for machining will be wasted unless the total number of tool changes minimized. They presented several models for two objectives, namely minimizing the number of tool switches [35] and the number of tool switching instants [36] on a single machine. In both studies, they are considering N jobs are processed on a machine that has positions for C tools, where no jobs requires more than C tools.

A similar problem of minimizing tool switches on a flexible machine was also defined by Bard [3]. In this study, he addresses the problem of scheduling N jobs on a single machine equipped with an automatic tool interchange mechanism, where the total number of tools required to process all N jobs could be greater than the capacity of the tool magazine. It is assumed that the processing times and the switching times are independent.

2.3 CNC and NC Machining Centers

Numerically Controlled (NC) machining centers and Computer Numerical Control (CNC) machining centers, which are also known as the Single Stage Multifunctional Systems (SSMS), are the essential parts of the both Flexible Manufacturing Systems and batch type production systems, due to their reduced set-up times, precise machining ability even for the complex surfaces, and high productivity and flexibility. The influence of CNC on manufacturing has resulted in the growth of advanced production system and technologies (Agapiou [1]). First, it changes the way a machining or manufacturing process is designed and planned. For example, a process plan for traditional machining provides information regarding machines, tools, fixtures and time rates, the sequence of processes involved, and the necessary machining specifications, such as feed rates and cutting speeds, however the operation of each individual process, such as turning and milling, is specified by the operator, usually on a trial and error basis. With the CNC and FMSs, which involve minimum intervention by human beings, a process plan and all the processes involved should be planned and defined in every detail so that they can be designed as a part program to be executed by a computer and followed exactly by the CNC machining center. Second, CNC machining centers change the way that an operation is performed. Operations are no longer controlled by operators but by the CNC part program so the skill of the operator is replaced by the information processing. Third, CNC changes the way that a process or operation is controlled. Manual control over an operation or process is replaced by CNC controller or a computer. Although this new technology introduces several control capabilities on the manufacturing process, other important decisions such as the sequence for operation processing, cutting conditions, and tool magazine arrangement are still defined manually by the operator or process planner at the development of part program. As a result, the productivity of CNC machining centers hence FMSs, highly depends on the skill and experience of the operator. Therefore the effective usage of CNC machine tools can only be realized by paying attention on the effective programming of these parts.

2.4 Tool Management

As it was mentioned before, the tooling of a CNC machining Center is a key factor for the overall system performance due to its impact on both cost figures and the operational considerations. The studies in the literature ignoring the tooling issues of such systems are found unrealistic, and the importance of tool management in the design and the operation of FMSs is mentioned by the most of the researchers (see [1], [2], [3], [8], [22], [26], [34], [35] and [36]). Among the others the following papers particularly criticize the current tool management systems and discuss related issues for FMSs (see [2], [20], [32], [35] and [36]).

Suri [34] provides three functions concerning operational area of tool management in FMSs, which are as follows:

1. Collecting and updating data regarding the tools on each machine
2. Keeping track of tool wear and replacing tools
3. Reacting to tool breakage

The problem of tool allocation and tool scheduling for a flexible manufacturing system is addressed by Amoako-Gyampah et al. [2]. In their study, the tool allocation and tool scheduling problem is defined as the grouping and movement of tools so that the proper tools are assigned to the right machines at the desired times for the processing of scheduled parts. They have specified four different tool allocation rules:

- **Bulk Change :** In this strategy, each time a part assigned to a machine, the number of tools that the part require is allocated to that machine and the tool slots on the magazine are correspondingly decremented. In other words, the remaining tool slots on the tool magazine are gradually decreased for each subsequent part assigned to the machine. This process continues until no more tools can be fitted on the tool magazine and therefore no more parts can be assigned to that machine for the given production time.
- **Tool Migration :** This process is similar to the bulk exchange policy in terms of part routing. The tools however do not stay at the machines for the entire planning period. Instead, after processing the tools used for the completed parts, they are removed from the machine to create empty tool slots on the tool magazine to permit the processing of another part.
- **Resident Tooling :** This strategy is based on the group technology principles. The group technology procedure is aimed at forming clusters of different combinations of tools at the various machines and keeping these tools at the machines permanently. Tool changing occurs when a particular tool reaches to the end of its scheduled useful life.
- **Tool Sharing :** In this study, it is a hybrid system between bulk exchange and resident tooling. Using the tool clusters, groups of parts are identified that largely use each of the tool clusters. Tool commonality is then recognized between the parts within the planning period. Then the planner adjusts the tool requirements for the latest part based on the quantity of tool it shares with other parts already scheduled for station.

It should be noted that for the bulk exchange and migration strategies, the assignment of parts to machines are done randomly while for the resident and tool sharing policies, specific parts can only be assigned to specific machines based on which machines have proper tools.

2.5 Machining Conditions Optimization

This topic is a well studied one in the literature and there exists many variations of this problem in terms of the modeling approaches, objectives, constraints, and the solution methodologies. However, mainly we can divide them into two classes in the scope of our study, these are namely single operation and multi-operation economics problems. Further, in these classes we can find both probabilistic or deterministic approaches to either constrained or unconstrained problems for different machining operations with several objectives.

In a machining operation, cutting speed, depth of cut and feed rate are the major decision variables, and they are closely related to the factors like workpiece and tool material, workpiece geometry, machine tool capacity, cutting fluid and some other conditions. The performance of machining operation is measured in terms of some physical measures such as surface roughness, surface integrity and surface error. The objective of machining is to obtain prescribed quality specifications given in the blueprint in terms of these performance measures. Since this primary objective can be satisfied for a wide range of decision variables, an evaluation criterion is needed for optimizing the cutting operation over this range.

The most popular criteria in machining economics are either minimum cost or maximum production rate. In order to apply these criteria to the machining economics problem, detailed functions of both the total cost and the total machining time are constructed in terms of decision variables. These two criteria also define a region of acceptable machining conditions. As a third criterion, many author studied on the maximum profit which usually lies within the region specified by the above two criteria.

These three evaluation criteria for determining optimum cutting conditions were related to production aspects by Hitomi [18], as follows:

- **The maximum production rate criterion** : This maximizes the amount of production in unit time interval, or minimizes the cycle time. It is the criterion to be adapted when the increase in productivity is aimed, by neglecting the production cost and/or profit.
- **The minimum production cost criterion** : This produces the unit product most cheaply and coincides with the maximum profit criterion if the unit revenue is constant.
- **The maximum profit rate criterion** : This maximizes the profit in a given time interval. It should be adapted when there are too many production orders to be fulfilled in a specified limited time interval.

2.5.1 Single Operation Optimization

An early study on the machining economics was presented by Field et al. for Turning [12], Milling, Drilling, Reaming, and Tapping operations [13]. In these articles, for each operation, the detailed cost and machining time equations were given for different types of cutting tools. In these expressions, the depth of cut was assumed to be constant and simplified Taylor's tool life formula has been used in order to express the tool life as a function of cutting speed, this type of Taylor's expression is known as the simplified version. In this way, for each operation and cutting tool pair, minimum cost cutting speed and corresponding production time, maximum production rate, and total production time formulas were derived by simple calculus. Therefore, these papers mainly concern with the calculation of cost and production time items. They compare the actual machinability data with their results for the possible projections out of the experimental data set and the goodness of fit for Taylor expression with coefficients calculated from the data sets. Furthermore, there exists an additional study about the acceptability of the defined operation [12]. For this purpose, number of pieces that can be machined before a tool failure are given in a diagram for a data set and an acceptability limit is indicated.

Another approach to the machining economics problem for the milling, drilling, reaming and tapping operations has been offered by Ermer and Shah [11]. They noted that optimization of any machining operation is a part of the problem of optimizing overall manufacturing system. So, optimizing an individual operation can be considered as an local optimization in the global problem. This local optimization can be made according to two criteria mentioned before, which are maximum production rate and minimum production cost. However, due to complex nature of the overall problem, the calculated value of local optimum may vary from the global optimum solution of system. In this case, the solution usually lies between the solutions corresponding two former criteria. A typical example to this case is using the criterion of maximum profit condition. In this paper an analytical method for sensitivity study in determination of the optimum machining conditions for Milling, Drilling, Reaming, and Tapping operations was presented, assuming that the optimizing criterion is either minimum cost or maximum production rate. By using these criteria, a range of optimum machining conditions were studied in which the criterion function does not depart significantly from its optimum value. Authors of this paper noted that such an application of the concept of sensitivity can be very useful for approaching full optimization of overall system. In this study, the detailed cost model of Field, et al. [12][13] was used with again the simplified version of the Taylor's tool life expression.

Another study on the maximum profit criterion was given by the Wu and Ermer [39]. They present a cost model including machining, tool and tool change, and handling costs for a rough turning operation. In a usual manner, the production cost and time functions were combined with the simplified Taylor expression and from these the cutting speed and tool life for the minimum cost and maximum production criteria were obtained, respectively. These values are the minimum and maximum values of the tool life and the cutting speed that define a range of optimal cutting conditions. Additionally, in this study a model was constructed for the selling price, volume, total revenue and the resulting profit. With these models, the effects of the demand function, selling price, and the other cost and time parameters on the optimal cutting speeds were analyzed by simply plotting the related functions on the graphs. There is no computational procedure except the determination of the optimum ranges of the cutting speed, and the tool life dictated by the minimum cost and maximum production criteria. They investigated maximum profit cutting conditions on these graphs and carried some sensitivity analysis around the optimal conditions for the effects of several parameters by giving some numerical examples. An important contribution of this paper is the inclusion of the feed rate as a decision variable in study of optimal cutting conditions. For this purpose, extended Taylor's tool life equation has been utilized and by using the common production cost and time model, the range of optimum cutting speeds was determined as a function of feed rate, the same analysis was applied.

Boothroyd and Rusek [7] have also studied the maximum rate of profit criterion in machining economics. In their paper, again production cost and time models were built and in a usual fashion the optimal tool lives for the minimum cost and maximum production rate criteria were obtained by using simplified Taylor expression, so this is almost same with the previous studies where generally the optimal speed cost was studied. Same analysis was applied for the maximum profit rate criterion and corresponding optimal tool life was found as a function of selling price. One important observation was made from a graph which shows the rate of profit for varying selling price and including all these three criteria, and it is found that the optimal cutting conditions lie close to the minimum cost condition. Additional considerations on the effects of worker incentive schemes and batch production were also analyzed on the machining conditions for maximum profit rate objective. The equations developed for maximum profit rate condition were found suitable to use when long production runs exist. However, they claimed that in small batch production working at maximum profit rate for a particular job may not result in a maximum profit rate for the particular machine tool over a period which is longer than the production time of a job. Then, they have assumed that the mean profit rate for the machine tool over a long period of time can be estimated and the total profit over a long period can be modeled in terms of mean profit rate, batch size, production time and unknown profit rate of the particular job. This model also gives an interesting result that the tool life for maximum profit rate in a batch production can be obtained from the minimum cost criterion by adding the estimated mean profit rate to the total machine rate in this expression.

By applying the fundamental economics principle that maximum profit occurs when the marginal revenue equals the marginal cost, Wu and Ermer [39] determined the optimum cutting speed for a turning operation (without any constraint) to maximize the profit. Later, Ermer [9] derived the optimum cutting conditions of a constrained machining economics problem for a single turning operation by applying geometric programming method.

It should be noted that introduction of constraints on the machining economics problem is resulting in some computational difficulties since all these constraints are nonlinear functions of the decision variables. In this case classical nonlinear programming techniques may not be appropriate since they require a rather complex analysis. A much simpler approach by exploiting the special structure of the constrained machining economics problem is using the geometric programming method. In the literature, there are several studies presenting geometric programming approaches to constrained machining economics problem (see [9], [10], [15], [21] and [33]).

Ermer [9] built a general cost function as a sum of operating and tool cost which may vary with the different costs, times and other production parameters. Several constraints have been imposed to this cost function and the overall problem was solved by geometric programming. In the geometric programming, number of constraints dictates the difficulty of problem for a fixed number of decision variables. As the number of constraints increases the difficulty also increases due the increased size of dual formulation. In this study the following three types of problems were studied:

1. **Zero degree of difficulty** : Minimization of the total unit cost with a constraint of maximum feed rate.

2. **One degree of difficulty :** Minimization of the total cost with maximum allowable feed and machine horsepower constraints.

3. **Two degrees of difficulty :** Minimization of the total cost with maximum allowable feed, machine horsepower and the surface roughness constraints.

An interesting property of the geometric programming approach is that it creates a set of equations in terms of dual variables corresponding to the positive terms of the cost function and the constraints. The number of difficulty is determined from the nature of this equation set, for example in the first case the resulting system has the same number of equations and variables so corresponds to the zero degrees of difficulty. However for the other cases the number of variables exceeds the number of equations so that the degrees of difficulty increases and there is no unique solution. For such cases the solution is found by expressing the solution set in terms of the dual variables corresponding to the coefficients of constraints, and maximizing with respect to the dual variables. An interesting feature of this optimization scheme is that it first finds the optimal way to distribute the total cost among the cost terms with their dual weights instead of first seeking the optimal values of variables as in the case of Lagrange's method. After the allocations are found the optimal cost can be calculated and then the values of the decision variables corresponding to this optimum cost can be obtained.

A more recent study on machining conditions selection for turning operation with constraints was presented by Gopalakrishnan and Al-Khayyal [15]. In this paper they built a common cost model with an expanded Taylor expression to obtain optimal values for cutting speed and feed subject to the surface finish and machine power constraints for a given depth of cut (one degree of difficulty). For the solution of this problem, an analytical approach based on geometric programming was implemented. This approach uses the complementary slackness conditions between dual variables and primal constraints in addition to constraints of both primal and dual formulation. The quality of the solution has been illustrated on several examples and compared to solutions obtained by some optimization methods proposed in the literature including Ermer [9], and Ermer and Kromodihardjo [10].

Another problem of machining economics is the number of passes to obtain prescribed removal of material with an optimum result. A single pass turning operation is found optimum if the operation is only restricted by the highest allowable feed rate. However, if there exists some other practical constraints like desired surface finish, minimum tool life, etc., then multiple pass operations may result in optimal solutions. Ermer and Kromodihardjo [10] presented a new approach to machining optimization problem with a multiple pass. In this paper, the optimal multi-pass turning conditions were determined for different combinations of horsepower, surface finish, tool life and feed constraints, using minimum cost criterion as the objective function. The optimization scheme proposed in this study was aimed to be capable of handling as many as constraints that may exist in a real life situation. However, this results in a more complex optimization problem with a high degree of difficulty. The authors have suggested an approach of geometric programming combined with the linear programming that can handle such a complex problem. They built cost models for single pass and double pass operations. For the single pass part, extended Taylor expression was used in which the speed, depth of cut and feed rate are the decision variables. In the double pass part, namely the roughing and finishing passes, the related cost terms and the Taylor expressions were used separately. The authors noted that this optimization scheme is equally applicable if the maximum production is the optimizing criterion, however it is not applicable for the maximum profit rate criterion because geometric programming is restricted to an objective function which is the sum of only positive terms.

In the literature there exist some Multiple Criteria Decision Making (MCDM) approaches to the machining economics problem [14][24]. Ghiassi et al. [14] have presented such an approach by assuming the absence of economic evaluation criteria like profit or cost. They tried to optimize the individual physical measures like surface roughness, integrity, etc., in terms of speed, feed and depth of cut. For this purpose, they obtained predicting equations of physical measures in terms of the controllable variables from an actual data set via least square regression analysis. Malakooti [24] presented interactive on-line multi-objective optimization approach for turning operations.

Koulamas [21] presented another approach to constrained single machining operation optimization problem to determine the tool replacement policies and the optimum machining conditions simultaneously. In this study, a model is built assuming a probabilistic distribution for the tool life.

2.5.2 Multi-Operation Optimization

Since a product requiring only a single machining operation is seldom found in practice, a more realistic formulation for determining the optimum cutting conditions should take care of all the operations that are to be performed on the product (see [17] and [25]).

A detailed study about the determination of optimum machining conditions for a job requiring multiple operations has presented by Hati and Rao [17] with both deterministic and probabilistic approaches. This study includes three criteria as the objective functions and two models are build including several constraints. In the deterministic model, the constraints are as follows; cutting speed, feed rate and depth of cut bounds, cutting speed restriction, available power restriction, bounds on tool life, temperature constraints and limitation on the depth of cut. In the probabilistic model, it is assumed that the constraints may vary about their mean value therefore these are taken as the independent random variables following the normal distribution. In this paper, it was assumed that the bounds can be assigned by the experience for the controllable cutting variables of speed, feed, and depth of cut in order to avoid infeasible cuts due to formation of built-up edge, high surface roughness, etc. For the same reasons, some feasibility limits are required on the tool life, consumed power, temperature, and the cutting force. An important contribution of this paper is that it includes some expressions for the resulting cutting force, power, temperature in terms of the controllable variables. The model allows to have multiple pass cut with the same depth of cut by the equality constraint. Production cost, time and profit equations are similar to the others, but the machining time was given in a way that allows to apply the model to every operation like the one given by Hitomi [18]. Besides, in the profit expression, the selling price was given as a decreasing function with the increasing production which has been proposed by Wu and Ermer [39]. For the tool life, extended Taylor expression was utilized with cutting speed, depth of cut and the feed rate as the decision variables. The method of sequence of unconstrained minimization technique (SUMT) has been applied to solve this constraint minimization problem. However this technique cannot take care of the equality constraint presented for the single pass depth of cut. This problem was handled by defining the series of feasible depths at the beginning, than the problem has been solved for each depth, and the best result was selected from this series of results with the corresponding depth of cut. At the end of this paper some numerical examples were given for both approaches and a simple sensitivity analysis has done by changing the cost coefficients.

In a later publication, Rao and Hati [25] proposed a deterministic model for the multi-operation problem. In this study, they claimed that in addition to the usual constraints arising due to the machine tool capabilities such as bounds on the cutting force, horse power, temperature rise, desired surface finish and rigidity, the relative times taken for the various operations and operation sequencing will play an important role on the total cost of production per piece, the production rate, and the profit. In this work, the problem of determining the optimal cutting conditions for a job requiring multiple operations was formulated as a constrained mathematical programming problem. For illustration, the machining of a gear blank was considered. The problem was solved for two different cases by minimizing the cost of production. In the first case, no restriction has been placed on the times taken for the individual operations and the objective was taken as the minimization of the cost of turning, drilling and milling. In the second case, some restrictions were introduced on the relative times taken for the different operations and the idle costs were included along with the three machining costs in the objective function. The sequence operation was taken as turning, drilling and milling. The constrained mathematical programming problem has been solved by using nonlinear programming techniques. By using the interior penalty function technique, the constrained optimization problem has been converted to a sequence of unconstrained optimization problems. The Davidon-Fletcher-Powell method, coupled with the cubic interpolation method of one dimensional minimization, has been used to solve these sequence of unconstrained optimization problems.

Hitomi [18] proposed a mathematical model for the flow-type multistage machining system in order to determine optimal conditions to be set on production stages. This production model contains N production stages sequenced in the order of operations sequence. The most important assumption in this model is that no in-process inventory was permitted, hence, the work material remained at the same stage even after the machining has been completed until all the operations at all production stages in the machining system are finished. So, the cycle time of the system is governed by maximum production time among those N production stages. In the basic mathematical model, it is assumed that the total production time per unit piece produced at one stage comprises the following three components; preparation time, machining time, and tool changing time that counts for the portion of total tool replacing time per unit piece by the ratio of machining time to total tool life at that stage. The cycle is given by maximum value among the N total production times of these N stages. Since it is assumed that in-process inventory between stages is not permitted, a waiting time occurs for each work station except the bottleneck one, as the difference between the cycle time of system and the total production time of that stage. The production cost per unit piece for each stage includes six items; preparation cost, machining cost, tool changing cost, tool cost, waiting cost and overhead cost. The total production cost is presented by the sum of raw material cost and sum of production cost through N stages. The profit is defined as the difference between the unit revenue and the total production cost per unit product. Then, a profit rate term is defined as the ratio of profit to cycle time. In order to investigate the optimal cutting conditions, the cutting speed has chosen as the controllable variable for the sake of simplicity. The model is expressed as a function of the cutting speed by using Taylor tool life equation in the usual manner.

A probabilistic approach to multi-tool machining operations presented by Sheikh et al. [28] to find the optimal tool replacement intervals and the machining conditions for three different tool changing policies, namely preventive planned tool change, scheduled tool change and failure replacement.

2.6 Tool Magazine Organization

There are many aspects of tool magazine organization due to the related considerations on tool management, part loading and operation allocation, tool magazine capacity, etc. However in the literature these issues are mentioned independently, and no one addressed the interrelations among these aspects. (see [1], [2], and [32])

2.7 Operations Sequencing

This topic is probably the most important one due to limited number of studies existing in the literature and difficulties arising when building a model for such a geometry dependent problem. In study of Agapiou [1], this problem was combined with the machining economics problem. The optimum sequence of operations was obtained using a network that involves the transformation of a sequential multi-functional decision process into a series of single operation processes. Hence, the multi-functional machining problem becomes a sequence of single operation problems for which each operation is optimized independently. In this network presentation, first the operations are classified according to possible precedence relations or the type of the operation. For example, rapid tool motions are presented by an arc from one node to the other, for the operations including metal removal from one point to the other (e.g., milling) are presented again with an arc between these two nodes (one node for the starting point, and another one for the end point). Point operations like drilling, reaming, etc., are presented by a single node. This network presentation is utilized for the optimization of the operations sequence by using a travelling salesman problem approach which seeks the shortest Hamiltonian path or cycle passing through each node exactly once. In this paper, three types of constraints for the operations were pointed, which are precedence, order and pairwise constraints. However, the other considerations for the operations, like multi-pass cutting, or tool path optimization, or tool selection problem were not studied. Furthermore, it is not too clear how the given mathematical model is formulated and the solution schemes are proposed, but it just gives an idea about the basics and possible extensions of the part programming problem.

Another approach has been presented by Bard and Feo [4] [5] to the operations sequencing problem. They have considered this problem as a part of the Computer Aided Process Planning (CAPP) module. The aim of CAPP is to fill the gap between CAD and CAM, and to overcome the inefficiency of the manual preparation of manufacturing process plan, and avoiding human judgment and errors. In this study, a surface and volume representation has been proposed for the development of tool path and cutting tool data management. According to this scheme, the total volume removal is decomposed into meaningful primitive volumes, then data about the volumes and their corresponding available cutting tools or processes are derived as the initial data for the rest of study. Their mathematical model involves a non-dominated paths matrix and their cutting tools for the removal. They have claimed that the problems for such an automated system first appear while transforming or representing the information on the blue-print to a manageable information structure, since there is no fully automated system for this task. In their paper, some basics for the design of automatic system have been underlined. Another problem is the generation of the set or matrix of all possible non-dominated paths, again, for this problem there is no available automated system. As a result, they have proposed a mathematical model for the optimization of operation sequence and tool selections, but they have failed to find a way of automatic machinable volume and candidate path generation and they assumed all these data generated by the programmer beforehand.

2.8 Computer Aided Process Planning

A review about CAPP has been presented by Steudel [31]. In this study, Manufacturing Process Planning is explained as follows: " Manufacturing Process Planning is a common task in small-batch, discrete parts metal working industries. The task consist of translating part design specifications from engineering drawing into the manufacturing operation instructions required to convert a part from a rough state to a final state. First the geometric features, dimensional sizes, tolerances, and material specifications of the part must be evaluated in order to select an appropriate sequence of processing operations and specific machines/workstations. Operation detail such as cut planning, speeds, feeds, assembly steps, tooling and so forth are then determined, and standard times and costs are calculated. The resulting process plans then documented as either a cost estimate, a job routing (or operation) sheet, or as coded instructions for numerically controlled (N/C) equipment".

Process planning represents the link between engineering design and shop floor manufacturing. It is the major determinant of manufacturing costs and profitability. It is important to note that manufacturing process planning involves the part programming task, and part programming is just a coding of the instructions given in the process plan depending on the specifications of particular machine tool and the coding language.

There are three approaches to accomplish the task of process planning: the traditional manual approach, the computer-assisted variant approach, and the computerized generative approach.

Manual Approach : The traditional manual approach involves examining a part drawing, and developing manufacturing process plans and instructions based upon the knowledge of process and machine capabilities, tooling, materials, related costs and shop practices. It might be a good method for small companies with a few process plans to generate. However, this activity is highly subjective in terms of the experience of the manufacturing analyst, also, it is labour intensive and time consuming task. All this observations are also valid for the part programming task. As we have mentioned before there exists a need for the automation of such decision activities in order to minimize the subjectivity, and to achieve a fast and cost effective control on the system.

Variant Approach : This approach is essentially a computer assisted extension of the manual approach that uses a powerful data-base of process plans of parts that have been processed before. It has the advantage of the data management, retrieval, and text editing efficiencies of the computer that greatly reduces time. It is particularly suitable in a Group Technology environment due to availability of standardized part coding method which is a key for data management. However, the disadvantage is that an experienced planner is required to construct, maintain, modify, and consistently edit the standard process plan. The knowledge and experience of the analyst are still the key factors in determining the quality of the resulting plans.

Generative Approach : The generative approach to process planning utilizes an automatic computerized system consisting of decision logic, formulae, algorithms, and geometry based data to uniquely determine the many process decisions for converting a part from a rough state to a finished state. The increased degree of sophistication for such a automatic system naturally includes the automatic generation of the coded instructions necessary to control the tool paths and functions of the machine. Because of these aspects of the generative process planning approach, a special care has been given to the literature on the design of such systems in order to obtain some useful ideas for our problem.

Several aspects of the computer aided process planning and the part programming have been discussed by Yeo et al. [37]. In their study they are underlying the fact that the output of a CAPP is needed for CNC or NC part programming, and the existing computer aided part programming systems still requiring inputs like cutting tools, tool type, and an operations sequence from a skilled operator. They also emphasize the importance of totally integrated systems which are expected to link the CAD and CAPP for the growth of unmanned manufacturing environments in a near future. They proposed integrated system by employing the expert system techniques which automates operation planning, machinability data selection and part program code generation all of which sharing a common knowledge pool.

In the literature there are similar knowledge based systems for CAPP and CAD integration, and design of these systems (Joseph and David [19], Yeo et al. [37] [38]). Knowledge based systems require elicitation of the knowledge from the experts and construction of some decision trees. However, in these studies they have mentioned to some important problems of process planning like machining conditions, tool selection, cutting path generation, etc., in a general context, these problems were not addressed and solved in an integrated manner. In other words the studies for the integration of several decision making activities are still requiring the elicitation and construction knowledge from the human beings and there is no totally integrated system using computational procedures instead of a rule based system.

Sundaram and Cheng [33] presented another approach to CAPP, however their study is just a computerized calculation of machining conditions for several operations instead of a CAPP system, since in their system the process planner still plays a vital role and is able to decide on choosing machine tools and tooling for the operation depending on the status of machine shop. Furthermore, the other crucial considerations of the process planning like operations sequencing and tool path design were not mentioned in this study. In the machining conditions calculation they are also advising the geometric programming approach as a fast tool and the most suitable approach for the microcomputer applications.

2.9 Conclusion

By this literature survey, we found that the problems stated in the introduction part have been mentioned under different titles and with different aspects. There is no study that combines machining conditions optimization, tool magazine organization and operations sequencing problems into a single body, and investigates the interactions among them. However, the importance of these three concepts have been mentioned in both system and the equipment level. Furthermore, we identified the need for the such a problem formulation especially for computer aided process and part programming since the available literature underlying the need for the standardization of these processes and they are mostly proposing knowledge based systems for this purpose. However, such systems are still human dependent since they require an extensive knowledge elicitation from the expert process planners and part programming, and their success is limited by this process. On the other hand, the proposed conceptual framework can be addressed as a module of a fully integrated system, and it will be supported by other modules dealing with cutting tool path optimization, cutting tool selection, and machinable volume and operation identification.

Finally, in the literature there exists some studies on the other modules and in our research we aimed to propose a module to handle operations sequencing, machining conditions selection and the tool magazine arrangement tasks of the overall problem.

Chapter 3

PROBLEM STATEMENT and MODELING

3.1 Introduction

In the view of the high investment and tooling cost of a CNC machining center, the cutting and idle times should be optimized by considering the tool consumption and the other operating costs for an effective utilization. This can be realized during the part programming stage by considering the factors like characteristics of the system, cutting conditions and tool lives, tool management, and tool magazine organization. However, in practice, all these parameters are determined by the machine operator. Consequently the production rate and cost depend on the skill of the operator and his past experiences. However, this is conflicting with the tendency of reducing the human being and machinery interventions. Therefore an automated planning system is necessary for the preparation of part program that will minimize the subjective decision making.

In this study, it is proposed to build a detailed mathematical model for the operation of a CNC machining center which will include the system characterization, the cutting conditions and tool life relationship, and related constraints.

Then an algorithm will be proposed to find appropriate tool magazine organization, cutting conditions, and sequence of cutting operations those will result in a minimum production cost.

3.2 Problem Definition

The aim of this research is to determine optimum machining conditions, operations sequence, and tool magazine arrangement to manufacture a batch of identical parts by a CNC Turning Machine on a minimum cost basis. In the real life, it is possible to identify a series of similar problems differing in some factor like the system characteristics and capabilities, operating policies and manufacturing environments. In the following section, we, first, define the scope of our study by stating related assumptions about these factors.

3.3 Assumptions

In this study, a CNC Turning Machine equipped with a limited capacity tool magazine is considered. The limits of problem is defined by stating operating policy and characteristics of the system. The following assumptions are made to clarify these considerations:

- Tool magazine capacity restricts the number of tools that can be mounted on a single machine for a production period, hence the number of different tool types can be used in the processing of a part is also limited due to this capacity constraint.
- In the tool magazine arrangement, all tools weigh about the same and each takes only a single tool slot in the magazine. Consequently, the tool magazine weight balancing problem is not concerned in this study.
- In a part cycle, total processing time of the machining operations, which are all assigned to a single tool, cannot exceed the available tool life of

that tool.

- The tool switching is only allowed during the part changing and only a single tool can be changed at a time. This assumption implies that tool changing time occurred in a particular part loading/unloading period is additive. So tool changing times of different tools can be summed to find the total tool changing time occurred in this period.

- Tool interchanging occurs when a tool is removed from the tool holder to the tool magazine and a different tool is mounted to the holder. For the tool interchange events, there is a fixed point in the reach of automatic changer, so tool holder first moves to this particular point for interchanging operation.

- In our model, tool interchanging is not necessarily required after every operation, for example, if we have more than one operation requiring the same tool and the precedence relations among them are feasible for a sequential removal of these machinable volumes, then it might be preferred to continue with the same tool without changing to reduce total tool interchanging time.

- For the machining operations, the cutting speed and the feed rate will be taken as the decision variables, and the depth of cut is assumed to be given as an input by the machinable volume presentation. This assumption particularly limits our attention to single pass machining, so if a material removal requires more than one pass, those should be prespecified as different volumes with their depths.

- All parts of the batch are identical in both geometry and machining features. Each machining operation of a part should be completed by a single tool type throughout the manufacturing of whole batch. Therefore a machining operation cannot be assigned to different tool types for machining of same volume over a part of the batch.

- After the completion of the batch, remaining tool lives are *not* taken into the tooling cost calculations.

3.4 Model Building

In this section, a mathematical model is developed to describe all the aspects of the problem, in terms of operation sequencing, tool magazine arrangement and machining conditions optimization. We are planning to determine optimum machining conditions, operation sequences, and tool magazine arrangement to manufacture a whole batch of identical parts on a minimum total cost basis.

The machining conditions optimization for a single operation is a well known problem and several methods have been developed as discussed in Section 2.5.1. However, these methods only consider the contribution of machining time and tooling cost to the total cost of the operation, where the decision variables are the cutting feed, depth of cut and feed rate. However, in our study of multi-operation case, non-cutting time components resulting from different sources, like the tool switchings between successive operations, tool setting, tool locating, etc., have also significant contribution to the total cost of production via operating cost. This fact was also mentioned in the literature by several authors. Consequently, our model should also include some other additional decisions effecting the non-machining time items, like the sequence of operation, and selection of the tool and operation pairs.

As an introduction, now, we are going to define some possible time components that should be included in the objective function of total cost for the manufacturing of a batch. There exist two distinct categories for the time components, namely machining time (cutting time) and the non-machining time (non-cutting, or idle) times. These items are explained and the related formulations are given in Sections 3.4.1 and 3.4.4, respectively. After mentioning the machining time, Taylor's tool life expression will be introduced in Section 3.4.2 and tool usage rate will be defined in Section 3.4.3 as a function of the controllable variables of machining operation, those will be helpful to including tooling cost into total cost function in terms of machining conditions.

3.4.1 Machining Time

It is the time required to complete a metal cutting operation. For example, the *cutting time* for a *turning operation* is given by the following expression:

$$t_{m_{ij}} = \frac{\pi . D_i . L_i}{1000 . v_{ij} . f_{ij}} \tag{3.1}$$

where,

D_i : Diameter of the generated surface, (mm)

L_i : Length of the generated surface, (mm)

v_{ij} : Cutting speed for machining of volume i with tool j, (m/min)

f_{ij} : Feed rate for machining of volume i with tool j, $(mm/rev.)$

Similar expressions for a wide variety of machining operations are available in the literature. However, for the machining economics studies the above expression has been preferred to study on since it is a common expression to all researchers and easy to extend to some other operations. Therefore we will also work with this expression.

3.4.2 Taylor's Tool Life Expression

The relationship between the terms tool life and machining time can be expressed as a function of the machining conditions by using an extended form of *Taylor's tool life* equation, as follows:

$$T_{ij} = \frac{C_j}{v_{ij}^{\alpha_j} . f_{ij}^{\beta_j} . d_i^{\gamma_j}} \tag{3.2}$$

where,

T_{ij} : Tool life of tool j for operation i, (min)

C_j : Taylor's tool life constant for tool j

d_i : Depth of cut for operation i, (mm)

α_j, β_j, γ_j : Speed, feed, depth of cut exponents for tool j

The above expression is frequently used in the machining economics literature especially in the cases where there exist more than one machining conditions studied as the controllable variable.

3.4.3 Usage Rate Expression

For the turning operation, by combining Equation 3.1 and 3.2, the following expression can be derived for the machining time to tool life ratio:

$$U_{ij} = \frac{t_{m_{ij}}}{T_{ij}} = \frac{\pi \, . \, D_i \, . \, L_i \, . \, d_i^{\gamma_j}}{1000 \, . \, C_j \, . \, v_{ij}^{(1-\alpha_j)} \, . \, f_{ij}^{(1-\beta_j)}} \tag{3.3}$$

It is possible to derive similar expressions for the other operations. In the above expression, the subscript j is added to the machining time, since both machining time and the tool life are depending on the particular choice of the tool and operation pair. This ratio is called as the *usage rate of tool j in the operation i*, which is denoted by U_{ij}.

3.4.4 Non-Machining Time

All time consuming events except the actual cutting operation are called as the non-machining time components. These non-machining items are essentially related with the operation of the system. Furthermore, the minimization of these time components is also a part of this study due to their significant contribution in the operating cost. Basic set-up, tool interchanging, tool switching, rapid travel motion, workpiece loading/unloading, tool tuning, tool approach and stabilization, etc., are the typical examples of non-machining events. In these non-machining time components, machining conditions selection and the operations sequencing are the determining factors. The following gives these components and indicates their dependencies to the our decision variables, and the assumptions made in developing their expressions.

- **Tool Switching Time :** It is determined by the tool usage rate, hence the number of necessary tool switchings. Each tool will have different switching time depending on whether the tool utilizes some special accessory or not, and this information should be given at the beginning for each tool. Therefore, the *total tool switching time* of a particular tool j for a batch can be expressed as follows:

$$T_{s_j} = t_{s_j} \, . \, \sum_i \bar{n}_{s_j} \text{ , for every } j$$

where,

T_{s_j} : Total tool switching time for tool j in a batch, (sec)

$\bar{n}_{s_j}$: The number of tool switching operations for tool j

t_{s_j} : Tool switching time for tool j, (sec)

- **Rapid Travel Motion Time :** It is the time needed to relocate the tool from one point to another, e.g., from tool magazine to starting point of the cutting operation. This time component can be expressed as a function of length of the path being followed. For such a presentation, we must be able to identify the starting and ending points of the motion, for example, in the case of rapid travel between two operations, we should know the ending point of first operation and the starting point of the following machining operation. Now, the *rapid travel motion time* can be expressed as follows:

$$t_{r_{xy}} = \begin{cases} 2 \, . \sqrt{\dfrac{\Delta_{xy}}{\alpha_s}} + (t_a + t_{st}) & \text{, if } \Delta_{xy} \leq \dfrac{V_s}{\alpha_s} \\ \dfrac{\Delta_{xy}}{V_s} - \dfrac{V_s}{\alpha_s} + \sqrt{\dfrac{\Delta_{xy}}{\alpha_s}} + (t_a + t_{st}) & \text{, if } \Delta_{xy} > \dfrac{V_s}{\alpha_s} \end{cases}$$

where,

$t_{r_{xy}}$: Rapid travel motion time from point x to point y, (sec)

V_s : Speed of machine slides, (m/min)

α_s : Acceleration of slides, (m/sec^2)

t_a : Tool approach time, (sec)

t_{st} : Settling or stabilization time for the slide, (sec)

Δ_{xy}: Relative distance between the points x and y, according to the relative distances (mm) in X, Y, and Z axes, which can be written as,

$$\Delta_{xy} = \sqrt{X_{xy}^2 + Y_{xy}^2 + Z_{xy}^2}$$

- **Tool Interchanging Time :** It counts for the time necessary to move tool from tool holder to tool magazine and replace it back, or vice versa. It is assumed that we are indifferent about the location of the tool in the tool magazine. This means that we are not spending extra time for the tool magazine indexing, etc., and we are only interested in the time spent for tool interchanging event. In this operation, first the tool is moved to a fixed location in which the Automatic Tool Changer (ATC) can handle the tool, and replace to the tool magazine back. This operation sequence involves only two time components, namely, the rapid travel from the ending point of the last operation to the fixed changing point, and the time for replacing to tool back to the magazine. It is presented as follows:

$$t_{x_{ij}} = t_{r_{if}} + t_{c_j}$$

where,

$t_{x_{ij}}$: Tool interchange time for tool j after finishing the operation i, (sec)

$t_{r_{if}}$: Rapid motion time for moving the tool from the ending point of operation i to the fixed changing point, (sec)

t_{c_j} : Tool changing time for tool j, (sec)

The same logic applies for taking of a tool from the tool magazine, for example, in this case, we have a rapid travel motion from the fixed changing point to the starting point of the next operation, and the time for withdrawing the tool from magazine is assumed as the same of replacing back. In this case, the following expression can be written:

$$t_{x_{\bar{i}j}} = t_{r_{f\bar{i}}} + t_{c_j}$$

Here in this notation subscript $\bar{i}$ stands for the starting point of the operation i, whereas $\underline{i}$ stands for the ending point of the operation i.

- **Basic Set-Up Time :** It is a component of the total non-cutting time due to the setup time counting for tool magazine preparation, and the loading of tools and part program for this specific batch. However, even there might be more than one alternative part programs their installation time will be almost constant, so it will also be excluded from the problem definition. Since we are assuming constant times depending on the type of the tool only, the loading time can be found as follows:

$$T_B = \sum_{j=1}^{n} t_{l_j}$$

where,

T_B : Tool Magazine loading time of tool set J for a batch, (sec)

t_{l_j} : Tool Magazine loading time for a single tool j, (sec)

Even there might be many other distinct non-machining time components counting for spindle acceleration/deceleration, workpiece loading/unloading, tool tuning, etc., we are only interested in the ones that might have a significant effect on the optimization problem. The non-machining time components that can be expressed as a function of our decision variables like cutting speed, feed rate, etc., or those can vary between the different alternative tool and operation combinations are only considered in the problem formulation, e.g., the tool interchange or switching times may vary for different possible tools for the same operation. On the other hand, spindle acceleration/deceleration time ignored since it is relatively small with respect to the other non-machining time components. Besides, loading/unloading time is not included either since it is related with the weight of the part, which is assumed as constant.

3.4.5 Total Cost Function

The cost of production for a particular batch can be expressed as the sum of production (machining or tooling cost) and the non-production cost (operating cost of the machine tool), by excluding the constant cost items, e.g., the material cost. For a single machining operation economics problem, the operating cost of the machine tool can be ignored since it is a linear function of the machining time. However, in our case, it will be included due to the existence of independent non-production time items. Therefore, the total cost should be expressed in terms of both machining time and non-machining time components, which can be written as follows:

Total Production Cost = Total Operating Cost + Total Tooling Cost

$$C_{tm} = C_o \, . \, t_{tm} \; + \; \sum_{j=1}^{n} C_{t_j} \, . \, \bar{n}_{t_j} \tag{3.4}$$

where,

C_{tm} : Total manufacturing cost for a particular batch, (\$)

C_o : Operating cost of the machine tool, (\$/*min*)

t_{tm} : Total manufacturing time of the batch, (*min*)

$\bar{n}_{t_j}$: Total number of *tool* j needed for complete machining of the batch, where, $j \in J=\{\ 1,\ 2,\ \ldots,\ n\ \}$

C_{t_j} : Cost of the *tool* j, (\$/*per tool*)

Total manufacturing time of a batch consists of both machining and non-machining time, for example, it is the time period starting with the basic set-up of loading the tool magazine and the part program, ending with the unloading of the last processed workpiece of the batch. By assuming machining time for a particular operation remains same throughout the batch, the *total* *manufacturing time* can be expressed as follows:

$$t_{tm} = N_B \, . \sum_{i=1}^{m} t_{m_i} \; + \; \sum_{k=1}^{o} n_{n_k} \, . \, t_{n_k} \tag{3.5}$$

where,

N_B : Batch size

t_{m_i} : Machining time for a single *machining operation* i, (min) where, $i \in I = \{\ 1,\ 2,\ \ldots,\ m\ \}$

n_{n_k} : Number of occurrences for *non-machining operation* k in a batch, where, $k \in K = \{\ 1,\ 2,\ \ldots,\ o\ \}$

t_{n_k} : Total time spent for the *non-machining operation* k, (min)

By substituting Equation 3.5 into 3.4, total cost expression can be written in terms of all time consuming events, as follows:

$$C_{tm} = C_o \, . \, N_B \, . \sum_{i=1}^{m} t_{m_i} \; + \; C_o \, . \sum_{k=1}^{o} n_{n_k} \, . \, t_{n_k} \; + \; \sum_{j=1}^{n} C_{t_j} \, . \, \bar{n}_{t_j}$$

In these expressions, the index sets I, J, and K correspond to the *set of machining operations, set of tools and set of non-production operations*, respectively, for a particular alternative part program (or process plan) of the batch. The sets of machining operations I and J should cover the operations necessary to manufacture a single part, where the set K is assumed to cover overall processing of the batch resulting from machining conditions selection, operation and tool assignments, and operation sequencing. Furthermore, for machining of a single machinable volume, a set of candidate tools will be specified as a subset of this initial tool set J.

3.4.6 Constraints

Before formulating the model, we will explain the constraints those should be imposed on the problem for the feasibility of solution and problem definition. These constraints can be grouped into three different classes, namely, operational constraints, tool related constraints and machining operation constraints. In the following sections these constraints are discussed and their governing expressions are given.

3.4.6.1 Operational Constraints

For the initial feasibility of the solution, operational requirements of the system should be satisfied that is the number of assigned tools must be less then or equal to the capacity of the tool magazine, and each operation must be assigned to a single tool. These conditions can be presented by the following expressions:

$$\sum_{j \in J} a_j \leq N_m$$

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ , for every } i \in I$$

where,

a_j : The number of tool slots required for tool type j

N_m : The capacity of the tool magazine

$$x_{ij} = \begin{cases} 1 & \text{, if } \textit{tool } j \text{ is used in } \textit{operation } i \\ 0 & \text{, otherwise} \end{cases}$$

Another important operational constraint is the precedence relationships among the operations. These precedence relations are given at the beginning as an input by the definition of machinable volumes. The following binary variable will be used to present this information:

$$r_{ii'} = \begin{cases} 1 & \text{, if operation } i \text{ precedes to operation } i'\text{, for every } i' \neq i \in I \\ 0 & \text{, otherwise} \end{cases}$$

In our model, the operations sequence will be presented by the vector w in which the i^{th} member gives the machinable volume number that will be processed as i^{th} operation. For example, $w_o = i$ means that o^{th} machining operation of the part program will be the removal of volume i, where $o \in \{1, 2 \ldots m\}$ and $i \in I$.

Now by using the above notation, the precedence constraint can be written in terms of the operations sequence decision and the precedence relationships, as follows:

$$\text{if } r_{ii'} = 1 \text{ for } i \neq i' \in I\text{, then } x < y \text{ such that } w_x = i \text{ and } w_y = i'$$

3.4.6.2 Tool Related Constraints

Another constraint for the initial feasibility of the solution is that the total machining time assigned to a tool for processing of a single part should not exceed the tool life of this particular tool. This restriction is called as the *tool life covering constraint* and it is expressed as follows:

$$\sum_{i=1}^{m} x_{ij} \,.\, U_{ij} \leq 1 \text{ , for every } j \in J$$

If above constraint cannot be satisfied by a tool type and the tool life of this tool is capable of finishing every single operation assigned to it, then by allowing to the presence of duplicate tools in the tool magazine and rearranging the assignments, the feasibility of the problem can be established. Furthermore, while achieving the feasibility of the problem by tool allocation, there exists several trade-offs regarding the tooling cost and non-machining times. These are the consideration leading to the "Tool Management" facet of our problem, perhaps, it is the key idea which makes our problem more complicated and different from the other multi-operation machining problems.

The last constraint of this class is the *tool availability* constraint which avoids exceeding the number of available tool quantity on hand for any tool type, that is t_j tools for type j, while determining the tool assignments and machining condition. This is as follows:

$$\sum_i \bar{n}_{t_{ij}} \leq t_j$$

where,

$\bar{n}_{t_{ij}}$: The number of tools required for the machining of volume i by tool j

3.4.6.3 Machining Operation Constraints

In the machining economics literature, there are many constraints imposed on the problem to prevent infeasible operations and to maintain specified quality of machining. A number of deterministic constraints exist such as allowable maximum cutting force, cutting temperature, available machine power, etc., and usually these are nonlinear expressions in terms of depth of cut, feed rate and cutting speed. Although there exists a wide variety of constraints in most of the studies, two constraints are usually concerned. These are the surface roughness and available machine power constraints. The surface roughness presents the quality requirement on the operation and the machine power constraint provides to operate machine tool without being subject to any damage. We will also use these two constraints in our study and they are expressed as follows (Gopalakrishnan and Al-Khayyal [14]):

$$C_s.v_{ij}^g.f_{ij}^h.d_i^i \leq SF_{max_i} \quad (3.6)$$

$$C_m.v_{ij}^b.f_{ij}^c.d_i^e \leq HP_{max} \quad (3.7)$$

where,

HP_{max} : Maximum allowable machine power for all operations

SF_{max_i} : Maximum allowable surface roughness for the volume i

C_m, b, c, e: Specific coefficient and exponents of the machine power constraint for a particular tool and volume pair

C_s, g, h, i : Specific coefficient and exponents of the surface roughness constraint for a particular tool and volume pair

3.5 Input Requirements

In our study, we are assuming that a part will be specified by the machinable volume presentation by giving the precedence relations among these volumes (see Bard and Thomas [4], [5]). For each machinable volume, a set of candidate tools will be specified for the machining of this volume. Furthermore, the geometrical information about the starting and ending point of an operation will also be supplied with this presentation. For every feasible machinable volume and tool pair, we will have a possible machining operation, so the parameters related for this pair should be specified to build corresponding Taylor's tool life expression. In order to construct the objective function and allocate non-machining time components, we need to know standard tool interchange, tool switching and initial loading times, and tool cost for each tool type. For the constraints of the problem, we need to know related parameters in surface roughness and machine power expressions for every operation, and the number of tools on hand should be specified for each tool type. Finally we need to know system related parameters which are number of slots in the tool magazine and operating cost of the system.

3.6 General Formulation

In Section 3.4, related mathematical expressions for the constraints of the problem and the total cost function have been given in the view of assumptions made in Section 3.3 . A general formulation of our mathematical model is given as follows:

$$\min_{m \in M = \{m | \mathbf{f}(v,f,w,\bar{J} \subset J)\}} \quad C_{tm}(m)$$

Subject to:

- Tool Magazine Constraint, (v, f, J):
  $\sum_{j \in J} a_j \leq N_m$
- Tool Assignment Constraint, (J):
  $\sum_{j=1}^{n} x_{ij} = 1$, for every $i \in I, j \in J$
- Tool Availability Constraint, (v, f, J):
  $\sum_{i} \bar{n}_{t_{ij}} \leq t_j$, for every $j \in J$
- Tool Life Covering Constraint, (v, f, J):
  $\sum_{i=1}^{m} x_{ij} \,.\, U_{ij} \leq 1$, for every $i \in I, j \in J$, and $x_{ij} = 1$
- Surface Roughness Constraint, (v, f, J):
  $C_s.v_{ij}^{g}.f_{ij}^{h}.d_i^{i} \leq SF_{max_i}$, for every $i \in I, j \in J$, and $x_{ij} = 1$
- Available Machine Power Constraint, (v, f, J):
  $C_m.v_{ij}^{b}.f_{ij}^{c}.d_i^{e} \leq HP_{max}$, for every $i \in I, j \in J$, and $x_{ij} = 1$

In the above formulation, the objective function is expressed as a function of machining conditions selection, operations sequencing, and tool assignment. In this model, M presents the set of alternatives generated by the decision variables of the overall problem, and $\bar{J}$ presents the set of allocated tools, which is a subset of available tool set, J. Since our problem is a multi-level problem having several interrelated decisions to be made at different levels, we could not develop a manageable closed form of the objective function. A more detailed presentation of the total cost function is given Section 3.4.5. Furthermore, tool magazine and tool assignment constraints are given as the operational constraints, in Section 3.4.6.1. In Section 3.4.6.2, the tool availability and tool life covering constraints are studied as tool related constraints. Finally the last two constraints of this general formulation are imposed to present constraints of machining economics problem which are also given in Section 3.4.6.3.

3.7 Conclusion

In this section we have built a mathematical model having a complicated cost function and several constraints to be imposed on it. However, our formulation is a nonlinear one having several integer and continuous variables in both objective function and the constraints, therefore it is impossible to solve this optimally by using a known mathematical programming method in a reasonable computation time. On the other hand, we can a develop heuristic method to solve this problem by using the properties of our problem. In this respect it is a better starting point to consider each machining operation as a single machining operation optimization problem, since this simplified formulation will include both tool cost and the operating cost due to the machining time in the objective function. These cost items are mentioned in the literature, as they are the most effective cost items in operating of a CNC Machining Center. Furthermore, we have a powerful tool to solve this single machining economics problem in a quite reasonable computation time, which is known as the Geometric Programming combined with the Analytical Method. This method uses properties of the machining economics problem and produces explicit form optimal solutions for the machining conditions in a reasonable computational expense, even we have a non-linear mathematical programming problem at the beginning. After optimizing each operation independently over a set of candidate tools then we can search for some possible trade-offs while maintaining the feasibility of the problem and the solution.

In following chapter we will present a three level hierarchical approach to solve our problem in a heuristic manner. This approach has the following levels:

- Single Machining Optimization and Tool Allocation
- Tool Magazine Arrangement
- Operations Sequencing

Chapter 4

PROPOSED HEURISTIC METHOD

4.1 Introduction

In the previous chapter, a mathematical formulation of the research problem has been presented and it was concluded that it is impossible to solve this problem in a reasonable computation time, since it involves a mixed integer non-linear optimization. On the other hand, this problem can be decomposed into smaller ones to build a heuristic procedure even they may remain highly interrelated. This chapter deals with the development of such a heuristic method to solve the problem. The following section introduces some underlying ideas and assumptions those will be involved in the development of our heuristics. In Section 4.4, single machining optimization problem will be presented and its extension to the multiple operations case will be studied in Section 4.5. In Sections 4.6, 4.7 and 4.8, the tool allocation, tool magazine arrangement and operations sequencing algorithms will be given respectively.

4.2 General Procedure

In Section 3.4.5, a cost function has been presented as the objective of mathematical model which includes mainly a tooling cost and operating cost. The operating cost is incurred due to both machining time and non-machining time components. In our study, the non-machining time components are more emphasized since they are likely to increase total cost in the case of multi-operation, whereas the machining time and tooling cost can be optimized together as in the case of a single operation optimization. Furthermore, our problem can also be considered as a feasibility problem since there exists several constraints those should be imposed on the objective function. These constraints are grouped into three classes; which are operational, machining and tool related.

The constraints and the decisions variables for machining conditions, tool and operation assignments, and operations sequence are closely interact with each other, therefore it is impossible to decompose the overall problem into independent smaller problems. Although, our problem is highly complicated due to these interrelations, it is still possible to develop a heuristic approach by using the single machining optimization as a key. In the single machining optimization, the objective function includes the tooling cost and operating cost due to the machining time, and it is possible to impose the machining optimization constraints on that problem together with a tool life covering constraint. Basically this formulation is just concerned with the machining conditions and it is independent of the other decisions required by the problem. However, by using the results of this approach, a procedure can be devised for selecting the set of tools which cover all the machinable volumes. In our study, we are calling this approach as tool allocation and this will be the first step of proposed hierarchal approach together with the single machining optimization. After fixing the operation-tool assignments, which means that reducing the candidate tool set of each volume to a single tool, the solution will be improved by considering other decision variables and the feasibility constraints. For this purpose, at the second level the tool magazine capacity constraint will be imposed on the tool allocation determined in the first level by considering possible tool sharing events. In the second level, first the minimum number of tool slots required for the tool allocation will be determined and if there exists any empty slot available then a further improvement on the current solution will be sought by a relaxation algorithm which allows tool sharing among the operations to minimize non-machining time components. In the third level, the operations sequencing decision will be made by considering the tool and operation assignments to minimize rapid travel motion and tool interchanging times. The flow chart given in Figure 4.1 shows the levels of our decision hierarchy and the decisions made at each level.

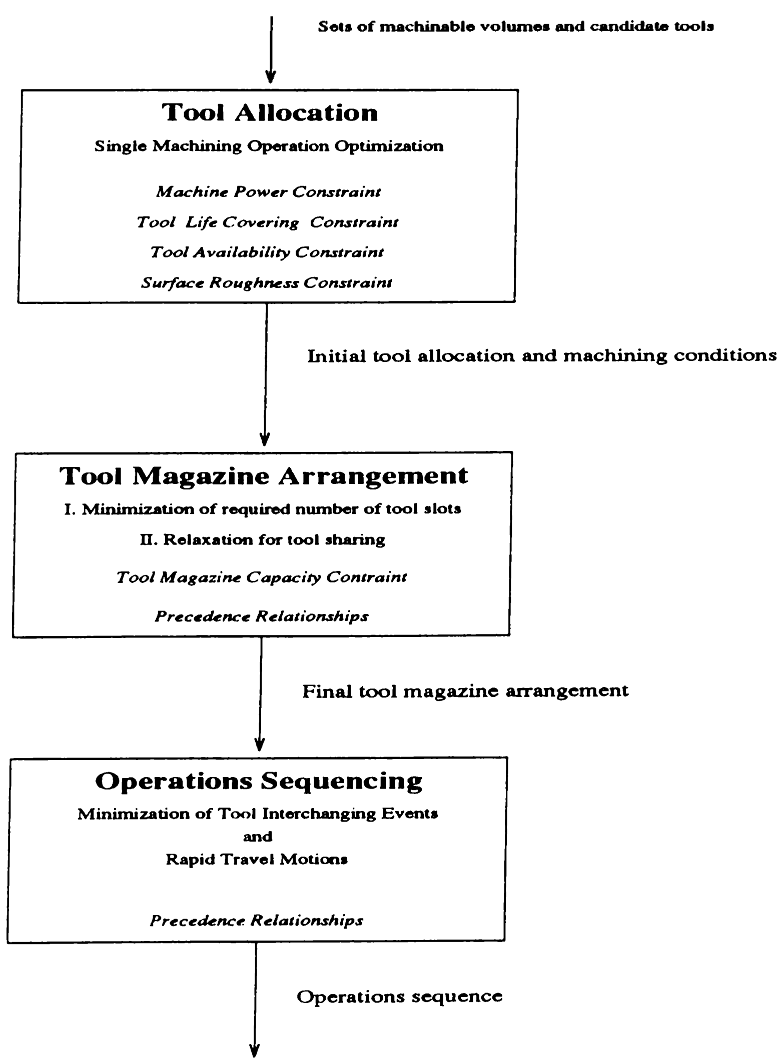

Figure 4.1: Flow Chart of the Proposed Hierarchy

As indicated in the flow chart, some of the constraints such as precedence and tool magazine capacity constraints are relaxed at the first level, then the feasibility of the solution is checked at the second level while searching for possible improvements due to tool sharing concept.

4.3 Notation

In our problem definition, there is a machinable volumes set I and each volume of this set should be removed by a machining operation for the processing of workpiece. Furthermore, every single machinable volume i has a governing candidate tools set J_i, which is a subset of the general tool set J. Here in this representation every pair (i,j), such that $(i,j) \in \{(i,j)|i \in I \wedge j \in J_i\}$, presents a single machining operation that can be solved for the optimal machining conditions on the minimum cost basis by imposing several constraints. Similarly, I_j presents the set of operations those can be processed by the tool j. A list of notations is given in Appendix C.

4.4 Single Machining Operation Optimization

In the literature, the tooling cost is found as the most effective part of total cost for operation of a CNC machining center. All non-machining terms except the operations sequence are related with the both tool allocation and machining conditions selection. Although the effect of the non-machining cost terms is considered as being significant in the total cost, their relation with the machining conditions can be taken into account in the development of the heuristic. Therefore, in this section, a single machining optimization problem will be introduced first, then its extensions to our problem will be studied.

The single machining optimization problem can be defined as follows:

Minimize Operating Cost + Tooling Cost

$$C_{m_{ij}} = C_o.t_{m_{ij}} + C_{t_j}.U_{ij}$$

Subject to:

- Tool Life Covering Constraint:
$$U_{ij} \leq \frac{1}{\bar{p}_{ij}}$$
- Surface Roughness Constraint:
$$C_s.v_{ij}^g.f_{ij}^h.d_i^i \leq SF_{max_i}$$
- Machine Power Constraint:
$$C_m.v_{ij}^b.f_{ij}^c.d_i^e \leq HP_{max}$$
- $v_{ij}, f_{ij} > 0$

In our study, an additional constraint is included which initially accounts for the tool life covering. Furthermore, this constraint will be used to present other tool related constraints by changing the parameter $\bar{p}_{ij}$ when the infeasibility occurs. Now, by substituting the Equations 3.1, 3.3, 3.6, and 3.7, and rearranging the terms the following mathematical programming formulation can be written as follows:

Minimize

$$C_{m_{ij}} = C_1.v_{ij}^{-1}.f_{ij}^{-1} + C_2.v_{ij}^{(\alpha_j-1)}.f_{ij}^{(\beta_j-1)}$$

Subject to:

- Tool Life Covering Constraint:
$$C_t'.v_{ij}^{(\alpha_j-1)}.f_{ij}^{(\beta_j-1)} \leq 1$$
- Surface Roughness Constraint:
$$C_s'.v_{ij}^g.f_{ij}^h \leq 1$$
- Machine Power Constraint:
$$C_m'.v_{ij}^b.f_{ij}^c \leq 1$$
- $v_{ij}, f_{ij} > 0$

where,

$$C_1 = \frac{\pi.D_i.L_i.C_o}{1000} \quad , C_2 = \frac{\pi.D_i.L_i.d_i^{\gamma_j}.C_{t_j}}{1000.C_j}$$

$$C_t' = \frac{\pi.D_i.L_i.d_i^{\gamma_j}.\bar{p}_{ij}}{1000.C_j} \quad , C_m' = \frac{C_m.d_i^e}{HP_{max}} \quad , \text{and } C_s' = \frac{C_s.d_i^i}{SF_{max_i}}$$

The above problem can be solved by using the Geometric Programming (GP)-Analytic Method in a reasonable computational burden. The details of Geometric Programming is given in the Appendix A. Furthermore, the notation used throughout this chapter is also given in Appendix C.

The associated geometric programming dual problem for the above single machining optimization formulation is given below:

Maximize $$Q^* = (\frac{C_1}{D_1})^{D_1}.(\frac{C_2}{D_2})^{D_2}.(C_t')^{D_3}.(C_m')^{D_4}.(C_s')^{D_5}$$

Subject to:
- Normality Condition:
  $D_1 + D_2 = 1$
- Orthogonality Conditions:
  $-D_1 + (\alpha - 1).D_2 + (\alpha - 1).D_3 + b.D_4 + g.D_5 = 0$
  $-D_1 + (\beta - 1).D_2 + (\beta - 1).D_3 + c.D_4 + h.D_5 = 0$
- $D_1, D_2, D_3, D_4, D_5 \geq 0$

The dual problem above has two degrees of difficulty among a class of geometric programming problems. Even the objective function for the dual problem is still non-linear one, the constraints of the dual formulation are well-defined linear equations. The dual problem is solved by Analytical Approach [15], that uses the complementary slackness conditions between dual variables and primal constraints in addition to constraints of both the primal and dual problems. These complementary slackness conditions are as follows:

$$\begin{aligned} &D_3.(C_t'.v_{ij}^{(\alpha_j-1)}.f_{ij}^{(\beta_j-1)} - 1) = 0 \\ &D_4.(C_m'.v_{ij}^{b}.f_{ij}^{c} - 1) = 0 \\ &D_5.(C_s'.v_{ij}^{g}.f_{ij}^{h} - 1) = 0 \end{aligned} \tag{4.1}$$

Each of the constraints of primal problem is either loose or tight at the optimality. Therefore, the principle to solve this dual problem is checking every possibility for the constraints of dual problem and solving the corresponding dual variables. If a dual feasible solution is found then the corresponding primal solution can be evaluated in terms of decision variables, and consequently the primal feasibility of the solution will be checked. At the optimality, the corresponding solution should be feasible in both dual and the primal problems, and the objective function value for both problems should be the same. Since we have three constraints in the primal problem, there exist eight different cases for the dual problem. Each case is analyzed as follows :

- **CASE 1:** Only Surface Roughness Constraint is tight

In this case, the tool life covering and the machine power constraints are loose, so the corresponding dual variables D_3 and D_4 are both equal to zero. For the surface roughness constraint, D_5 should be greater than or equal to zero because of the dual feasibility constraints. Therefore, the constraints of dual problem are reduced to the following system:

$$\begin{array}{rcccccl} & D_1 & + & D_2 & & & = 1 \\ - & D_1 & + & (\alpha_j - 1).D_2 & + & g.D_5 & = 0 \\ - & D_1 & + & (\beta_j - 1).D_2 & + & h.D_5 & = 0 \end{array} \tag{4.2}$$

By substituting the first equation of 4.2 into the others in terms of D_2, the following linear system of equations with two equations and two unknowns is formed:

$$\begin{array}{lcll} \alpha_j.D_2 & + & g.D_5 & = 1 \\ \beta_j.D_2 & + & h.D_5 & = 1 \end{array}$$

The solution for this system can be stated explicitly as follows:

$$D_2 = \frac{g - h}{g.\beta_j - h.\alpha_j} \text{ , and } D_5 = \frac{\alpha_j - \beta_j}{h.\alpha_j - g.\beta_j}$$

$$\text{where, } g.\beta_j - h.\alpha_j \neq 0$$

The following conditions should be satisfied to verify dual feasibility of the solution:

$$0 \leq D_2 \leq 1 \text{ , and } D_5 \geq 0$$

- **CASE 2:** Only Machine Power Constraint is tight

In this case, the tool life covering and the surface roughness constraints are loose, so the corresponding dual variables D_3 and D_5 are equal to zero, and for the machine power constraint, D_4 should be greater than or equal to zero. Therefore, the constraints of dual problem are reduced to the following system:

$$\begin{array}{rlllll} & D_1 & + & D_2 & & & = 1 \\ - & D_1 & + & (\alpha_j - 1).D_2 & + & b.D_4 & = 0 \\ - & D_1 & + & (\beta_j - 1).D_2 & + & h.D_4 & = 0 \end{array} \tag{4.3}$$

By substituting the first equation of 4.3 into the others in terms of D_2, the following linear system of equations with two equations and two unknowns is formed:

$$\begin{array}{llll} \alpha_j.D_2 & + & b.D_4 & = 1 \\ \beta_j.D_2 & + & c.D_4 & = 1 \end{array}$$

The solution for this system can be stated explicitly as follows:

$$D_2 = \frac{b - c}{b.\beta_j - c.\alpha_j} \text{ , and } D_4 = \frac{\alpha_j - \beta_j}{c.\alpha_j - c.\beta_j}$$
$$\text{where, } b.\beta_j - c.\alpha_j \neq 0$$

The following conditions should be satisfied to verify dual feasibility of the solution:

$$0 \leq D_2 \leq 1 \text{ , and } D_4 \geq 0$$

- **CASE 3:** Only Tool Life Covering Constraint is tight

In this case, the machine power and the surface roughness constraints are loose, so the corresponding dual variables D_4 and D_5 are equal to zero, and for the surface roughness constraint, D_3 should be greater than or equal to zero. Therefore, the constraints of dual problem are reduced to the following system:

$$\begin{array}{rlllll} & D_1 & + & D_2 & & & = 1 \\ - & D_1 & + & (\alpha_j - 1).D_2 & + & (\alpha_j - 1).D_3 & = 0 \\ - & D_1 & + & (\beta_j - 1).D_2 & + & (\beta_j - 1).D_3 & = 0 \end{array} \tag{4.4}$$

By rearranging the terms of equation 4.4 :

$$D_1 + D_2 = 1$$
$$(\alpha_j - 1).(D_2 + D_3) = D_1$$
$$(\beta_j - 1).(D_2 + D_3) = D_1$$

We know that the following inequality always holds for extended Taylor's tool life expression [16] :

$$\alpha_j > \beta_j, \gamma_j > 1 \text{ , for } T_{ij} = \frac{C_j}{v_{ij}^{\alpha_j}.f_{ij}^{\beta_j}.d_i^{\gamma_j}}$$

Since $\alpha_j \neq \beta_j$, the solution for this case is :

$$D_1 = 0 \text{ , } D_2 = 1, \text{ and } D_3 = -1$$

Therefore, this case is *infeasible* since $D_3 < 0$. As a conclusion, the tool life covering constraint *cannot* be tight just itself.

- **CASE 4:** Both Surface Roughness and Tool Life Covering Constraints are tight

In this case, the machine power constraint is loose, so the corresponding dual variable D_4 is equal to zero, and for the surface roughness and tool life covering constraints the corresponding dual variables D_3 and D_5 should be greater than or equal to zero. Therefore, the following system can be written by using the complementary slackness conditions (Equation 4.1) :

$$\begin{array}{lllll} C_t' & . & v_{ij}^{\alpha-1} & . & f_{ij}^{\beta-1} & = 1 \\ C_s' & . & v_{ij}^{g} & . & f_{ij}^{h} & = 1 \end{array} \tag{4.5}$$

By taking logarithmic transform, above system turns to a linear system of equations with two equations and two unknowns. This system is solved for v_{ij} and f_{ij}. After finding v_{ij} and f_{ij} and calculating the primal objective function value $C_{m_{ij}}$, dual variables D_1 and D_2 can be calculated as they show the weights of terms in the objective function:

$$D_1 = \frac{C_1.v_{ij}^{-1}.f_{ij}^{-1}}{C_{m_{ij}}} \text{ , and } D_2 = 1 - D_1$$

If the solution satisfies the condition for D_1, that is $0 \leq D_1 \leq 1$, then the following system is solved for D_3 and D_5 :

$$\begin{array}{rcrcrcr} (\alpha_j - 1).D_3 & + & g.D_5 & = & D_1 & - & (\alpha - 1).D_2 \\ (\beta_j - 1).D_3 & + & h.D_5 & = & D_1 & - & (\beta - 1).D_2 \end{array}$$

The overall solution is dual feasible if $D_3, D_5 \geq 0$ condition is also satisfied.

- **CASE 5:** Both Machine Power and Tool Life Covering Constraints are tight

In this case, the surface roughness constraint is loose, so the corresponding dual variable D_5 is equal to zero, and for the machine power and tool life covering constraints the corresponding dual variables D_3 and D_4 should be greater than or equal to zero. Therefore, the following system can be written by using the complementary slackness conditions (Equation 4.1):

$$\begin{array}{lllllll} C'_t & . & v_{ij}^{\alpha-1} & . & f_{ij}^{\beta-1} & = 1 \\ C'_m & . & v_{ij}^{b} & . & f_{ij}^{c} & = 1 \end{array}$$

By taking logarithmic transform, above system turns to a linear system of equations with two equations and two unknowns. This system is solved for v_{ij} and f_{ij}. After finding v_{ij} and f_{ij} and calculating the primal objective function value $C_{m_{ij}}$, dual variables D_1 and D_2 can be calculated as follows:

$$D_1 = \frac{C_1.v_{ij}^{-1}.f_{ij}^{-1}}{C_{m_{ij}}} \text{ , and } D_2 = 1 - D_1$$

If the solution satisfies the condition for D_1, that is $0 \leq D_1 \leq 1$, then the following system is solved for D_3 and D_4 :

$$\begin{array}{rcrcrcr} (\alpha_j - 1).D_3 & + & b.D_4 & = & D_1 & - & (\alpha - 1).D_2 \\ (\beta_j - 1).D_3 & + & c.D_4 & = & D_1 & - & (\beta - 1).D_2 \end{array}$$

The overall solution is dual feasible if D_3, $D_4 \geq 0$ condition is also satisfied.

- **CASE 6:** Both Surface Roughness and Machine Power Constraints are tight

In this case, the tool life covering constraint is loose, so the corresponding dual variable D_3 is equal to zero, and for the machine power and surface roughness constraints the corresponding dual variables D_4 and D_5 should be greater than or equal to zero. Therefore, the following system can be written by using the complementary slackness conditions (Equation 4.1):

$$\begin{array}{lllll} C'_m & . & v_{ij}^b & . & f_{ij}^c & = 1 \\ C'_s & . & v_{ij}^g & . & f_{ij}^h & = 1 \end{array}$$

By taking logarithmic transform, above system turns to a linear system of equations with two equations and two unknowns. This system is solved for v_{ij} and f_{ij}. After finding v_{ij} and f_{ij} and calculating the primal objective function value $C_{m_{ij}}$, dual variables D_1 and D_2 can be calculated as follows:

$$D_1 = \frac{C_1.v_{ij}^{-1}.f_{ij}^{-1}}{C_{m_{ij}}} \text{ , and } D_2 = 1 - D_1$$

If the solution satisfies the condition for D_1, that is $0 \leq D_1 \leq 1$, then the following system is solved for D_4 and D_5 :

$$\begin{array}{lllllll} b.D_4 & + & g.D_5 & = & D_1 & - & (\alpha - 1).D_2 \\ c.D_4 & + & h.D_5 & = & D_1 & - & (\beta - 1).D_2 \end{array}$$

The overall solution is dual feasible if D_3, $D_4 \geq 0$ condition is also satisfied.

- **CASE 7:** All the constraints are tight

In this case, all dual variables corresponding to constraints should be greater than or equal to zero. Therefore the following complementary slackness conditions should be satisfied:

$$\begin{array}{lllll} C'_t & . \; v_{ij}^{(\alpha_j-1)} & . \; f_{ij}^{(\beta_j-1)} & = 1 \\ C'_m & . \; v_{ij}^{b} & . \; f_{ij}^{c} & = 1 \\ C'_s & . \; v_{ij}^{g} & . \; f_{ij}^{h} & = 1 \end{array}$$

By rearranging the above system the following set of equations can be written:

$$\begin{array}{llll} \frac{C'_m}{C'_s} & . \; v_{ij}^{b-g} & . \; f_{ij}^{c-h} & = 1 \\ C'_t & . \; v_{ij}^{(\alpha_j-1)} & . \; f_{ij}^{(\beta_j-1)} & = 1 \end{array}$$

By taking logarithmic transform, above system turns to a linear system of equations with two equations and two unknowns. This system is solved for v_{ij} and f_{ij}. After finding v_{ij} and f_{ij} and calculating the primal objective function value $C_{m_{ij}}$, dual variables D_1 and D_2 can be calculated as follows:

$$D_1 = \frac{C_1.v_{ij}^{-1}.f_{ij}^{-1}}{C_{m_{ij}}} \text{ , and } D_2 = 1 - D_1$$

If the solution is satisfied for the condition for D_1, that is $0 \leq D_1 \leq 1$, then the following optimality condition can be given:

$$C_{m_{ij}} = Q^* = (\frac{C_1}{D_1})^{D_1}.(\frac{C_2}{D_2})^{D_2}.(C'_t)^{D_3}.(C'_m)^{D_4}.(C'_s)^{D_5}$$

By taking the logarithmic transform and rearranging the term:

$$\ln(C'_t).D_3 + \ln(C'_m).D_4 + \ln(C'_s).D_5 = \ln\left((\frac{D_1}{C_1})^{D_1}.(\frac{D_2}{C_2})^{D_2}.C_{m_{ij}}\right)$$

By using the above equality together with the complementary slackness conditions the following system of three equations and three unknowns is obtained which is solved for D_3, D_4 and D_5 :

$$\ln(C'_t).D_3 + \ln(C'_m).D_4 + \ln(C'_s).D_5 = \ln\left((\frac{D_1}{C_1})^{D_1}.(\frac{D_2}{C_2})^{D_2}.C_{m_{ij}}\right)$$
$$(\alpha - 1).D_3 + b.D_4 + g.D_5 = D_1 - (\alpha - 1).D_2$$
$$(\beta - 1).D_3 + c.D_4 + h.D_5 = D_1 - (\beta - 1).D_2$$

The overall solution is dual feasible if $D_3, D_4, D_5 \geq 0$ condition is also satisfied.

- **CASE 8:** All the constraints are loose

In this case all dual variables corresponding to the constraints are equal to zero. Therefore the constraint set of the dual problem turns to following system :

$$\begin{array}{rcrcl} & D_1 & + & D_2 & = 1 \\ - & D_1 & + & (\alpha_j - 1).D_2 & = 0 \\ - & D_1 & + & (\beta_j - 1).D_2 & = 0 \end{array}$$

This system is infeasible since α_j and β_j cannot be equal which makes the system of equality inconsistent. Therefore, the occurrence of such a case in constrained single machining operation optimization is impossible.

For all of the feasible cases, after finding a dual feasible solution, the primal feasibility is checked by the constraints of the primal problem. If the resulting solution is optimal, then both primal and dual problems should have the same objective function value. The following theorem is a result of the above dual feasibility study :

Theorem 4.1 : *In the Single Machining Operation Optimization, at least one of the Surface Roughness or Machining Power Constraints must be tight at the optimal solution.*

Proof: Case 3 and Case 8 are the only cases where both constraints are assumed to be loose at the optimality. Since both cases are shown to be infeasible and the remaining cases include only one of the mentioned constraints, above theorem has been proved □ .

4.5 Extension of SMOP to the Multi-Operations Case

After finding the optimal machining conditions and the machining cost for possible single operations, then the available tool capacity should be allocated among the machinable volumes to accomplish their removal by a single tool from their candidate set having enough tool life capacity. This problem is particularly difficult as a mathematical programming problem since we should find a optimal tool set that will result in the minimum machining cost while satisfying several kinds of feasibility constraints like the tool magazine capacity, tool availability, and tool life covering. Further, such an approach assumes that the optimality of single machining economics problem will be still valid for the development of the tool allocation model. However this is not the actual case since there is a limited tool quantity available on hand. There might exist several infeasibilities due to this tool capacity constraints and they can only be resolved by shifting from the optimal conditions dictated by the GP-Analytical method. As a result, the proposed procedure should be capable of identifying the infeasibilities and resolving the problem for such critical operations while it is allocating the available tool capacity and covering all the members of I set.

In order to develop such a heuristic procedure, first we should device a measure that will provide us to rank a set of alternative tools for a particular operation in terms of their desirability for this operation. Since our global objective is aimed to reduce the cost of manufacturing, naturally the cost can be preferred as a measure for this purpose and this cost measure may include the following items:

- Machining Cost of the Single Machining Operation (Tool cost and the operating cost due to the machining time)
- Tool Switching Cost
- Tool Loading Cost (Basic Set-up Cost)
- Waste Tool Life Cost

So the cost measure for an operation (i, j) can be written as follows:

$$\bar{C}_{ij} = N_B.C_{m_{ij}} + C_o.(\bar{n}_{s_j}.t_{s_j} + t_{l_j}) + C_{w_j}$$

In the above cost function, the first term represents the cost of single machining optimization which includes the operating cost due to actual cutting time and the tool cost. The second term accounts for operating cost resulting from non-machining time components. Here, all non-machining time components of our original mathematical model are included except the ones depending on the operations sequencing decision, such as the tool interchanging and rapid tool motion costs, which will be considered at the third level. The last term of this measure represents the waste tool life cost.

Our simplified model for the single machining operation optimization problem assumes the manufacturing of a single part instead of the whole batch, so it just counts for consumed tool life for a single part. However, in our problem, a batch of identical parts is considered and some tool life can be wasted since a single tool can produce an integer number of parts due to our assumptions on the operation of a machining center. This cost should be added as the waste tool cost. For the waste tool life cost, an explicit expression can be derived by the following assumption that the tool life remaining prior to the switching operation is a waste, since we are paying for it whether we are using the entire tool life or not. Note that this assumption is valid if more than one tool is needed for a set of operations during the manufacturing of whole batch and the last tool will have no waste due to our assumptions mentioned in Section 3.3, for the tool management strategy. Now, the waste tool cost, in general, can be expressed as follows:

$$C_{w_{ij}} = \begin{cases} C_{t_j}.[\bar{n}_{s_j} . (1 - \bar{p}_j \sum_i U_{ij})] & \text{, if } \bar{n}_{s_j} \geq 1 \\ 0 & \text{, otherwise} \end{cases}$$

where,

$\bar{p}_j$: Number of parts can be manufactured by using a single tool of type j, that is expressed as follows:

$$\bar{p}_j = \left\lfloor \frac{1}{\sum_i U_{ij}} \right\rfloor$$

$\bar{n}_{s_j}$: Number of tool switching operations necessary for tool type j, that can be expressed as follows:

$$\bar{n}_{s_j} = \left\lceil \frac{N_B}{\bar{p}_j} \right\rceil - 1$$

The optimum machining conditions (v, f) and the corresponding machining cost $(C_{m_{ij}})$ in the simplified model of single machining operation problem can be calculated by using GP-Analytical Method by relaxing the tool capacity constraints and requiring one-to-one assignments between the tools and the machinable volumes, which corresponds to no tool sharing case.

4.6 Tool Allocation

So far we have discussed the nature of a possible single machining operation in the content of our main problem having multiple operations. In order to solve tool allocation problem, we should search for the applicability of the results of single operation optimization to the multiple operation case by imposing the tool availability and the tool magazine constraints. For this purpose, a heuristic procedure can be developed by assuming that if there exists no tool availability limit then the best choice is most probably considering each operation as a single machining operation, and if the results of this assumption violates our capacity constraints then the feasibility will be maintained back by resolving the problem.

Since our intend is to develop a procedure that maintains the feasibility of problem while minimizing the total cost as much as possible, we should first deal with the possible sources of infeasibility. In this point of view, the most critical operations and tools those may cause the infeasibility should be identified. At the first glance, the most critical operations in our problem can be identified as the ones having only a single candidate tool for their processing. This type of operations must be covered and the required tool quantity should be allocated.

The following heuristic procedure is proposed to solve the tool allocation problem without tool sharing consideration. After finding the best tool allocation, we will continue with introducing the tool sharing case to improve the current solution. The aim of this algorithm is just to reduce initial candidate tool set to a single tool for every machinable volume and to determine the machining conditions, by considering the tool availability constraint. As it is discussed in the previous paragraphs, first the most critical operations will be handled in this algorithm to minimize the computational expense and to enhance the solution. For this purpose, in the first five steps of the algorithm, all possible operations are generated, and the candidate tools of of every machinable volume are ranked with the proposed cost measure. In Step 6, a set of primal tools are determined as they are the most economical tools to be allocated first, and in the next step, operation assignment sets are created for these tool types. In Step 8, the machinable volumes having only a single candidate tool for their processing are considered as the most critical operations and their allocation is handled by imposing the tool availability constraint on the SMOP. For the machinable volumes having more than one candidate, the algorithm first focuses on the most scarce tools as they are the bottle neck of the problem, and they are determined by using the required perturbation ratio for each tool type in Step 9. In Step 10, starting from the most critical tool, every possible reduction for the operations of a tool is generated as the perturbations of this tool type and the best perturbation set resulting in minimum total cost increment is computed by using a 0-1 IP model. This perturbation model is particularly useful to identify trade-offs between several possible assignments. In the last step of the algorithm, inclusion of a new tool type in the tool magazine is justified by adding a penalty cost due to additional tool interchange events required for this tool.

Tool Allocation Heuristic

- **Step 1 :** Create the candidate tool set, J_i, for every machinable volume and possible volume assignment set, I_j, for every tool, as follows:

$$J_i = \{j | x_{ij} = 1\}, \text{ for every } i \in I$$

$$I_j = \{i | x_{ij} = 1\}, \text{ for every } j \in J$$

- **Step 2 :** For every possible operation, $(i,j) \in O = \{(i,j) | x_{ij} = 1\}$, solve single machining operation optimization problem (SMOP) using the procedure defined in Section 4.4, initially $\bar{p}_{ij}$ values are equal to one.

- **Step 3 :** For every operation, calculate the number of parts that can be manufactured, $\bar{p}_{ij}$, and the number of tools required for this process, $\bar{n}_{t_{ij}}$, as follows:

$$\bar{p}_{ij} = \left\lfloor \frac{1}{U_{ij}} \right\rfloor \text{ and } \bar{n}_{t_{ij}} = \left\lceil \frac{N_B}{\bar{p}_{ij}} \right\rceil$$

- **Step 4 :** For every operation, update the $\bar{p}_{ij}$ by decreasing the tool requirement found in the previous step and resolve SMOP for the new value of $\bar{p}_{ij}$.

  - Calculate the following cost measure, which is defined in Section 4.5, for every operation $(i,j) \in O$ and $\bar{p}_{ij}$ by assuming no tool sharing among the operations as follows:

$$\bar{C}_{ij} = N_B . C_{m_{ij}} + C_o . (\bar{n}_{s_j} . t_{s_j} + t_{l_j}) + C_{w_{ij}}$$

- Pick the alternative having minimum cost measure.

- **Step 5 :** Rank the members of each J_i set in increasing order, with respect to cost measure of the operation (i, j), such that $j \in J_i$.

- **Step 6 :** Create an initial " Primal Tools " set J_F, by picking the first rank tool of the set J_i for every volume i

- **Step 7 :** Create the machinable volume set for every primal tool, $j \in J_F$:

$$I_{j_F} = \{i | x_{ij} = 1 \text{ and } i = \arg\min_{I_{j_F}} \bar{C}_{ij}\}$$

- **Step 8 :** For the machinable volumes having only a single tool in their candidate tool set, such that $s(J_i) = 1$, check the tool availability constraint

$$\bar{n}_{t_{ij}} \leq t_j$$

  - If the above condition is satisfied, then allocate this tool and its operation by adding them into sets $\bar{I}$, $\bar{J}$, and update the available number of tools, t_j. If the above condition is satisfied at equality, then remove this tool from the candidate sets of other operations.
  - Otherwise update $\bar{p}_{ij}$ and resolve SMOP, update $\bar{I}$, $\bar{J}$ and remove the tool j from every J_i, where $i \in I \backslash \bar{I}$.

- **Step 9 :** For the machinable volumes having more than one candidate, $s(J_i) > 1$, calculate the total tool requirement for every primal tool, as follows:

$$R_j = \sum_{i \in I_{j_F}} \bar{n}_{t_{ij}}, \text{ where } j \in J_F$$

  - If $R_j \leq t_j$, then update $\bar{I}$, $\bar{J}$, and t_j. Solution satisfies the tool availability constraint so check the next constraint.

- Otherwise, calculate the deficit tool amount as follows:

$$\delta_j = \sum_{i \in I_{j_F}} \bar{n}_{t_{ij}} - t_j, \text{ where } n_{t_{ij}} \in \mathcal{R},$$

Then calculate the required perturbation ratio as follows:

$$\rho_j = \frac{\delta_j}{R_j}$$

- **Step 10 :** Starting from the tile having highest ρ_j value continue with the following algorithm:

  Step 10.1 : Span the possible reductions in the tool requirement, π, for every operation of j, where $i \in I_{j_F}$ and $j \in I \backslash \bar{I}$:

$$\pi \in \{0, \ldots, \delta_j\}$$

  Step 10.2 : For every perturbation π update $\bar{p}_{ij}$ and resolve SMOP. Compare resulting cost measure, $\bar{C}^{\pi}_{ij}$, with the secondary candidate tool of this machinable volume, j', and calculate the corresponding cost increment for this perturbation, $\Delta\bar{C}^{\pi}_i$ as follows:

  Step 10.2.1 : For the secondary tool, j', check the tool availability constraint, if it is violated, then update $\bar{p}_{ij'}$ and resolve SMOP. If the resulting cost measure is larger than the other secondary tools than re-rank the candidate tool set and again check the tool availability constraint until finding a feasible secondary tool

  Step 10.2.2 : Compare the resulting cost measure of the secondary tool with the π^{th} perturbation of the primary tool and also with the other secondary tools:

  - If the π^{th} perturbation results in a lower cost measure with respect to the secondary tool, then the cost increment is:

$$\Delta\bar{C}^{\pi}_i = \bar{C}^{\pi}_{ij} - \bar{C}^{0}_{ij}$$

  where $\bar{C}^{0}_{ij}$ corresponds to the cost measure found at **Step 4**.

- Otherwise the cost increment for the machinable volume i with perturbation π is:

$$\Delta \bar{C}_i^{\pi} = \bar{C}_{ij'}^{\pi'} - \bar{C}_{ij}^{0}$$

Step 11 : Solve the following 0-1 IP to find the best perturbation amounts and the allocation set for every tool that the tool availability constraint is satisfied:

Minimize $\quad \Delta C_j = \sum_{i \in I_{j_F}} \sum_{\pi} y_i^{\pi} . \Delta \bar{C}_i^{\pi}$

Subject to:

$$\sum_{\pi} y_i^{\pi} = 1 \qquad \text{, for every } i \in I_{j_F}$$

$$\sum_{i \in I_{j_F}} \sum_{\pi} y_i^{\pi} . \bar{n}_{t_{ij}}^{\pi} = t_j$$

$$\sum_{i \in I_{j_F}} \sum_{\pi'} y_i^{\pi'} . \bar{n}_{t_{ij}}^{\pi'} \leq t_{j'} \qquad \text{, for every } j'$$

where,

$$y_i^{\pi} = \begin{cases} 1 & \text{, if } \pi^{th} \text{ perturbation is selected for } (i,j) \\ 0 & \text{, otherwise} \end{cases}$$

In the above model, the first constraint ensures that for every operation only a single perturbation will be selected, and the second constraint represents that the tool usage equals to available quantity. Third constraint also ensures that tool availability constraint for the secondary tools will be satisfied. The objective is to find the best combination that satisfies the tool availability constraint with a minimum total cost increment, ΔC_j.

Step 12 : In the solution of the above model, if one or more of the secondary tools are allocated for some machinable volumes then check whether they have been already assigned for some operation before or not:

- If $j' \in \bar{J}$ then stop.

- Otherwise, add a penalty cost for the introduction of this new tool type since it will increase the total non-machining time considerably due to the tool interchanging required in every part for this particular operation. Resolve the above problem with the new cost measure again until getting a feasible solution that is all tools in the set $\bar{J}$ or introduction of new tool is justified. The following additional cost will be included in cost measure for the tool interchange events by ignoring the rapid travel motions:

$$C_p = C_o.N_B.t_{c_j}$$

4.7 Tool Magazine Arrangement

After the tool allocation procedure, the tool magazine capacity constraint and tool sharing phenomena should be considered to avoid any infeasibility due to initial tool loading. Furthermore, non-machining times can be minimized by introducing the tool sharing.

At the beginning, a violation of tool magazine capacity constraint can be identified by assuming that it is possible to allocate only a single tool in the tool magazine for every tool type of final tool allocation. In other words, a total tool sharing is possible for every operation of a tool type during the manufacturing of a single part. Now, the following condition can be written for a preliminary justification of the tool allocation:

$$s(\bar{J}) \leq N_m$$

If our solution to tool allocation problem violates this constraint, then problem should be resolved by reducing the initial tool set in a proper manner. Further, even the above condition might be satisfied, duplicate tools in the magazine can be needed to recover the operation assignment sets of some tool types, and that may cause the violation of the tool magazine capacity constraint.

In the following algorithm, the minimum tool slot requirement for each tool is found in the first two steps by using a 0-1 MIP model which tries to minimize total slack tool life. In Step 3, tool magazine capacity constraint is checked with the minimum slot requirements. If this constraint is satisfied with a number of empty slots, then alternative tool arrangements are generated for each tool type in Step 4, by using a 0-1 IP model which maximizes the tool sharing events. In the next step every alternative measured by a cost measure and in Step 6, the best tool magazine arrangement is found by using the another 0-1 IP model.

Tool Magazine Capacity Checking and Tool Sharing Algorithm

- **Step 1 :** For the tool types having only a single operation assignment, a single slot should be allocated in the magazine, that is :

$$\text{For every } j \in \bar{J}, \text{ such that } s(\bar{I}_j) = 1, \text{ set } a_j = 1, \text{ and } j \in \check{J}$$

where,

$$\bar{I}_j = \{i | x_{ij} = 1 \text{ and } i \in \bar{I}\}$$

a_j : The number tool slots allocated for the tool type j

$\check{J}$: Set of the arranged tool types

- **Step 2 :** For the tools having more than one operation in their assignment set, that is:

$$\text{For every } j \in \bar{J}, \text{ such that } s(\bar{I}_j) \geq 2$$

Both feasibility of the tool magazine constraint and the tool availability constraint must be considered during the initial tool loading. The following steps will identify the alternative tool allocations for a particular tool type in terms of the number of duplicates and their operation assignment sets, and evaluate them to determine the final tool magazine arrangement.

Step 2.1 : In the following steps, the input data to feed the 0-1 LP model given in the **Step 2.2**, which is pre-processed a priori, in order to avoid some infeasible cases and reduce the problem size:

Step 2.1.1 : Determine the possible tool requirement levels, l_j, and the corresponding number of parts can be manufactured at that level of usage, p_j^l, for tool type j such that:

$$l_j \in L_j = \left\{ \min\{\bar{n}_{t_{ij}} | i \in \bar{I}_j \right\}, \cdots, t_j \} \text{ and } p_j^l = \left\lfloor \frac{N_B}{l_j} \right\rfloor$$

where, there is an one-to-one correspondence between l_j and p_j^l, such that:

$$\text{for every } l_j \text{ and } l_j' \in L_j,\ p_j^l \neq p_j^{l'}$$

Step 2.1.2 : For every tool requirement level, evaluate the maximum number of duplicates, k_{l_j}, that can be placed into tool magazine:

$$k_{l_j} = \left\lfloor \frac{t_j}{l_j} \right\rfloor$$

Step 2.1.3 : For every tool requirement level, calculate the maximum allowable tool usage rate, $\bar{U}_{l_j}$, that can be assigned for a set of operations:

$$\bar{U}_{l_j} = \frac{1}{p_j^l}$$

Furthermore, identify the operations which can be assigned on this level such that:

$$i \in V_{l_j} = \{i | i \in \bar{I}_j \text{ and } U_{ij} \leq \bar{U}_{l_j}\}$$

Step 2.2 : Create an alternative tool arrangement for every tool j by solving the following 0-1 MIP model:

$$\text{Minimize} \quad \sum_{l_j} \sum_{k_{l_j}} s_{lk}$$

Subject to:

$$\sum_{i \in \bar{I}_j} U_{ij}.z_{ilk} + s_{lk} = \bar{U}_l \quad , \text{for every } l_j,\ k_{l_j} \text{ and } i \in V_{l_j}$$

$$z_{ilk} \leq m_{lk} \quad , \text{for every } l_j,\ k_{l_j} \text{ and } i \in V_{l_j}$$

$$\sum_{l_j} \sum_{k_{l_j}} z_{ilk} = 1 \quad , \text{for every } i \in \bar{I}_j$$

$$\sum_{l_j} \sum_{k_{l_j}} l_j.m_{lk} \leq t_j$$

$$\sum_{l_j} \sum_{k_{l_j}} m_{lk} \leq s(\bar{I}_j)$$

where,

$$z_{ilk} = \begin{cases} 1 & \text{, if the } i^{th} \text{ operation is allocated to } k^{th} \text{ duplicate of} \\ & \quad \text{tool requirement level } l \\ 0 & \text{, otherwise} \end{cases}$$

$$m_{lk} = \begin{cases} 1 & \text{, if the } k^{th} \text{ duplicate of tool } l \text{ is located into tool magazine} \\ 0 & \text{, otherwise} \end{cases}$$

In the above model, the first constraint ensures that maximum allowable tool usage rate for every tool will not be exceeded. Second constraint ensures that if an operation is allocated to a particular tool requirement level, then this tool will be allocated in the tool magazine. Third constraint ensures that every operation of the $\bar{I}_j$ assigned to a single tool. Fourth constraint avoids exceeding the number of available tools. Finally, the last constraint represents the worst case bound on the number of tool slot requirements, which is initially equal to the number of machinable volumes in the global assignment set by assuming no tool sharing. This constraint is updated in the following steps to find the tool arrangement resulting in the minimum tool slot requirement. Our objective is to minimize the tool waste cost.

Step 2.3 : Update the bound on the total slot requirements, $s(\bar{I}_j)$, and solve the problem again using the dual–simplex method to determine the minimum tool slot number, S_j.

- **Step 3** : Check the tool magazine capacity constraint, as-follows:

$$\sum_{j \in J \setminus \bar{J}} S_j \leq N_m - \sum_{j \in \bar{J}} a_j$$

  - If the above constraint is violated, then the problem is infeasible. This means that for the given available tools set it is impossible to solve this problem economically. The feasibility of the problem can be maintained back either increasing the tool availability or decreasing the usage rates of the available tools. However the second choice is undesirable for any tool type since it increases machining cost very rapidly.
  - Otherwise, continue with the following steps to find the best tool magazine arrangement.

- **Step 4** : For every tool $j \in \bar{J}$, build a precedence relationship graph to illustrate possible ordering of the operations assigned on a tool type j.

  Step 4.1 : For every graph, determine the set of adjacent operation pairs, which is defined as follows:

$$I_{A_j} = \{(i, i') | r_{ii'} = 1 \text{ or } r_{i'i} = 1\} \text{ , where } j \in \bar{J}$$

  Step 4.2 : For every operation pair, determine the candidate tool requirement levels to avoid some infeasible assignments which violates tool life covering constraint, as follows:

$$L_{(i,i')} = \{l_j | U_{ij} + U_{i'j} \leq \bar{U}_{l_j}\}$$

  Step 4.3 : Solve the following 0-1 IP model for every possible tool slot requirement, SR_j, such that:

$$a_j \in SR_j = \left\{ S_j, \cdots, \min\{s(\bar{I}_j) - 1, N_m - \sum_{j \in \bar{J}} a_j\} \right\}$$

The aim of this formulation is to find an alternative tool magazine arrangement for a tool type, $A_j^{a_j}$, resulting in a minimum number of tool interchanging events, which gives the best tool sharing combination, for a given number of available tool slots, a_j :

$$\text{Maximize} \quad \sum_{(i,i')\in I_{A_j}} \sum_{l_j \in L_{(i,i')}} \sum_{k_{l_j}} (z_{ilk} + z_{i'lk} - m_{lk})$$

Subject to :

$$\sum_{i\in \bar{I}_j} U_{ij}.z_{ilk} \leq \bar{U}_{l_j} \quad \text{, for every } l_j,\ k_{l_j} \text{ and } i \in V_{l_j}$$

$$z_{ilk} \leq m_{lk} \quad \text{, for every } l_j,\ k_{l_j} \text{ and } i \in V_{l_j}$$

$$\sum_{l_j} \sum_{k_{l_j}} z_{ilk} = 1 \quad \text{, for every } i \in \bar{I}_j$$

$$\sum_{l_j} \sum_{k_{l_j}} l_j.m_{lk} \leq t_j$$

$$\sum_{l_j} \sum_{k_{l_j}} m_{lk} = a_j$$

The above formulation is similar to the 0-1 MIP model presented in **Step 2.2**, which generates feasible tool arrangements. However, the above model tries to maximize the number of operations sharing the same tool for a fixed number of available tool slots, rather than minimizing the total slack tool life. So, the slacks are eliminated in the first constraint of this model and tool slot availability constraint, last constraint, turns to an equality constraint. Here note that the above formulation may have a smaller size with respect to the previous model, if the some of the tool requirement levels are eliminated by **Step 4.2**. In such a case the following condition holds :

$$L_j \ \setminus \Big(\bigcup_{(i,i')\in I_{A_j}} L_{(i,i')} \Big) \neq \emptyset$$

- **Step 5** : Evaluate the following cost measure, $C_{A_j^n}$, for all alternatives of tool type j :

$$
\begin{aligned}
C_{A_j^n} = \; & \sum_{\{l_j | z_{ilk}=1\}} C_0.N_B \Big(\sum_{\{r_{ii'}=1 | l_j \in L_{(i,i')}\}} (t_{r_{f\bar{i}}} + t_{r_{\underline{i}\bar{i}'}} + t_{r_{\underline{i}'f}} + 2.t_{c_j}) \\
& + \sum_{\{r_{ii'}=0 | l_j \in L_{(i,i')}\}} (t_{r_{f,\bar{i}}} + t_{r_{\underline{i}f}} + 2.t_{c_j} + t_{r_{f\bar{i}'}} + t_{r_{\underline{i}'f}} + 2.t_{c_j}) \Big) \\
& + C_o.\Big((\sum_{d=1}^{a_j} l^d - a_j).t_{s_j} \Big) \\
& + C_o.a_j.t_{l_j} \\
& + C_{t_j}.\Big(\sum_{d=1}^{a_j} (l^d - 1).s_{l^d} \Big)
\end{aligned}
$$

where,

l^d : Tool requirement level at the d^{th} tool slot

s_{l^d} : Slack tool life at the d^{th} tool slot

In the above cost measure, we are excluding the machining cost since they have been fixed by the tool allocation algorithm and they remain the same. In the first two terms, rapid travel motion times and the tool interchanging times are found by considering the tool sharing event. The other terms represents the tool switching, loading and waste costs, respectively. This cost measure includes all the non-machining time components and the waste tool cost since they are closely related the tool magazine arrangement and the tool sharing. Further, for the operations sharing the same tool, the operations sequence is determined by ranking their starting and ending points according to the relative distance of those points from a datum point laying in the vertical plane of workpiece axis. In this sequencing procedure, we start from the farthest starting point, then, determine the subsequent operations by minimizing the travelling distance and applying the precedence relationships. This above procedure is proposed to minimize the total rapid travel motion between the operations of a shared tool.

- **Step 6 :** Determine the best tool arrangement of every tool type in $\bar{J} \backslash \check{J}$ by solving the following 0-1 LP :

$$
\begin{aligned}
&\text{Minimize} \quad \sum_{j \in J} \sum_{n=1}^{n_j} d_{A_j^n} . C_{A_j^n} \\
&\text{Subject to:} \\
&\qquad \sum_{n=1}^{n_j} d_{A_j^n} = 1 \qquad\qquad , \text{for every } j \in \bar{J} \\
&\qquad \sum_{j \in \bar{J}} \sum_{n=1}^{n_j} d_{A_j^n} . a_j^n \leq N_m
\end{aligned}
$$

where,

$$
d_{A_j^n} = \begin{cases} 1 & , \text{if the } n^{th} \text{ alternative arrangement of tool } j \text{ is selected} \\ 0 & , \text{otherwise} \end{cases}
$$

a_j^n : Number of tool slots required by the alternative A_j^n

n_j : Number of different arrangements for tool type j

In this formulation, first constraint requires that only one of the alternatives will be selected and the second constraint ensures that total tool slot requirement of the entire tool magazine arrangement will not exceed the tool magazine capacity. The objective function aims to minimize total cost of final tool magazine arrangement for the tools have not been allocated yet.

4.8 Operations Sequencing

After fixing the operation and tool assignments, and tool magazine arrangement, only the operation sequencing decision remains to be made. Tool interchanging and rapid travel motion times are the only variables to be concerned at this stage. These non-machining time components should be minimized by avoiding unnecessary tool interchanging events and keeping rapid travel motion requirements at a minimum level.

This sequencing decision is transformed into a network model in order to illustrate the problem. In this presentation, nodes correspond to the several phases of a workpiece, for example, initially it is a blank material in state s, and every cutting operation changes the state of workpiece. At the end, the final state having m operations is denoted by the node f. Cutting operations are presented by the arcs and every arc will have a cost value corresponding to the non-machining operations due to state transitions. While building this network flow presentation, the precedence relations will be imposed on the problem by placing the arcs between the volumes satisfying precedence relations only. Further, as it is shown in Figure 4.2, there exist $m+2$ stages in manufacturing of the part. In every cutting stage, there exist at most m nodes representing the machinable volumes of set I.

In the previous algorithm, some of the machinable volumes are assigned to the same tool in order to get potential benefits of tool sharing and for those operations, an operations sequence has been also found. These operations can be aggregated into a single machinable volume by preserving their predetermined operations sequence in order to simplify the operations sequencing problem. Therefore, in this stage of the overall decision hierarchy, a reduced machinable volume set is used with other related data updates.

Our problem is to find a path from state s to state f which has the minimum cost. This path should include each state only once, in other words all of the required cutting operations must be performed during the manufacturing of a part. The following algorithm is proposed to make a full enumeration by spanning all feasible alternatives.

Operation Sequencing Algorithm

- **Step 1 :** Define new machinable volumes set if any tool sharing has been defined at the tool magazine arrangement, as follows:

  Step 1.1 : For the tools having a sequence of adjacent operations, defined a new machinable volume by taking the starting point of the

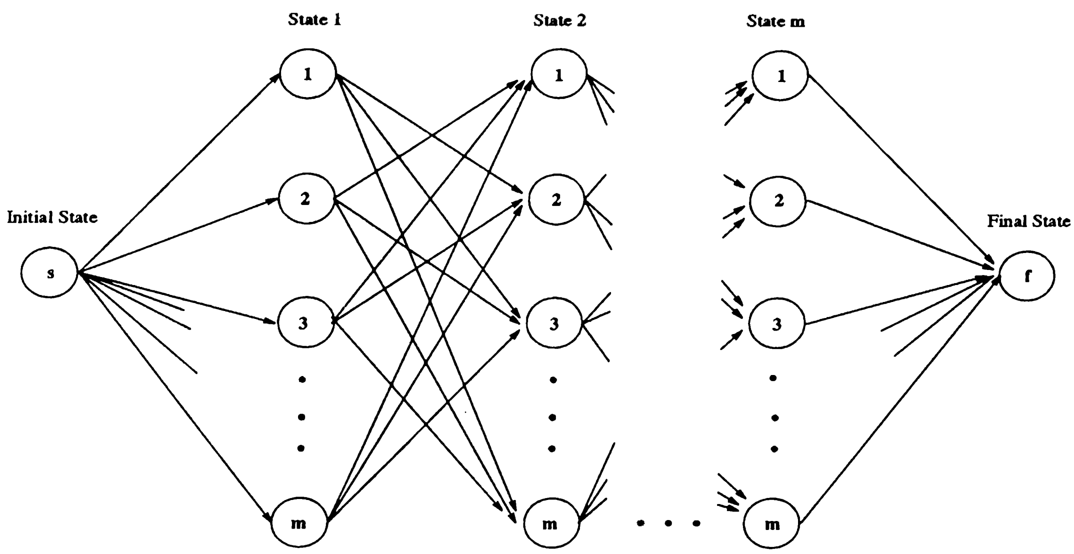

Figure 4.2: Network presentation for operation sequencing problem

first operation of this chain as the starting point, and ending point of the last operation as the ending point of aggregated volume,

Step 1.2 : Update the precedence relations, R, by deleting the intermediate operations of every chain

- **Step 2 :** Create machinable volume set for State 1, such that:

$$S_1 = \{i | r_{i'i} = 0 \text{ , for every } i' \neq i \in I\}$$

Step 3 : Calculate the cost of arcs from state s to state 1 as follows:

$$c_{s,i} = t_{c_j} + t_{r_{fi}} \text{ , where } i \in S_1 \text{ and } x_{ij} = 1$$

- **Step 4 :** For every intermediate State n, where $n \in \{1, \ldots, m-1\}$, create the sub level of succeeding operation, S_{n+1}, by the following:

$$S_{n+1} = \{i | i \in I - T, \text{ if } r_{i'i} = 1 \text{ then } i' \in T\}$$

$$\text{where, } T = \bigcup_{t<n} S_t$$

- **Step 5 :** Calculate cost of arc directed from State S_n to S_{n+1}, where $i' \in S_n$ and $i \in S_{n+1}$

$$c_{i',i} = \begin{cases} t_{r_{\underline{i'f}}} + t_{c_{j'}} + t_{c_j} + t_{r_{f,i}} & \text{, if } j' \neq j \\ t_{r_{\underline{i'i}}} & \text{, if } j' = j \end{cases}$$

- **Step 6 :** Calculate the cost of arc from State m to State f by the following expression:

$$c_{i,f} = t_{r_{if}} + t_{c_j} \text{ , where } i \in S_m$$

- **Step 7 :** Calculate total cost for every path from root node s to leaf node f and pick the path with minimum cost for the operations sequence.

In the above heuristic, there exist three types of operations (arc) to represent the starting, intermediate and ending operations of the part cycle. For the starting and ending operations, the cost of arc will include only rapid travel motion time whereas the intermediate operations may also include a tool interchange time additionally if the successive operations require different tools. Further, this heuristic checks the precedence relations while creating a consecutive level in order to avoid infeasible operations sequences.

An example tree generated by the above heuristic is given in Figure 4.3 where the number of machinable volumes is taken as three and the only precedence relationship is given between volumes 1 and 3.

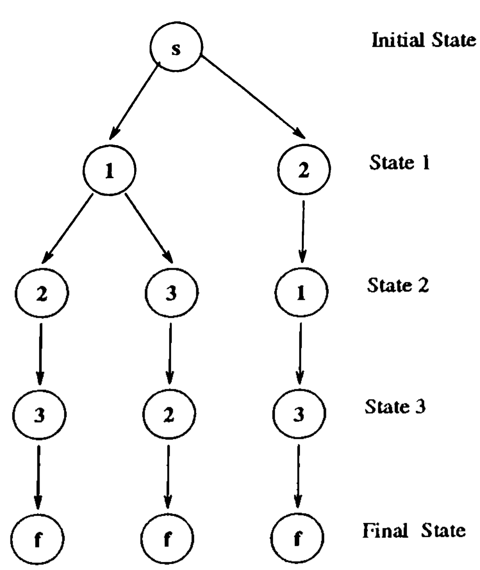

Figure 4.3: Enumeration tree for operations sequencing

4.9 Conclusion

In this chapter, a heuristic approach has been proposed to solve the problem defined in Chapter 3. The decisions to be made for this problem have close interaction between them and several constraints are imposed on the problem. As a solution procedure, a hierarchical approach is proposed to capture this complex nature of the problem. In this hierarchy, several decisions are made by relaxing some of the constraints and the feasibility of the solution is checked while searching for possible improvements by the proposed algorithms.

There exist three levels in this hierarchical approach. Briefly, at the first level, the tool allocation problem is solved by relaxing the tool magazine capacity constraint. The tool and operation assignments are fixed and the machining conditions are determined by assuming no tool sharing event among the operations. This level particularly utilizes the SMOP as a tool due to the fast solving method developed for that problem. Moreover, the results of SMOP are extended to a cost measure in order to handle the case of multiple operations by considering the related non-machining time components and tool waste. In the second level, tool magazine capacity constraint and precedence conditions are imposed on the problem and tool sharing is considered for possible improvements. The final composition of the tool magazine is fixed by considering the arrangement of each tool type. Furthermore, at this level an operations sequence is found for the operations which are sharing the same tool. Finally, the operations sequencing decision is made at the third level to minimize the rapid travel motions and tool interchange times which uses the information on final tool magazine arrangement and the operation assignments.

Next chapter will present an example problem to illustrate a possible application of the proposed approach.

Chapter 5

AN ILLUSTRATIVE EXAMPLE

5.1 Introduction

In this chapter, an example problem is studied to illustrate a possible application of the heuristic approach proposed in the previous chapter. The input data will be given in the next section. In sections 5.3.1, 5.3.2, and 5.3.3, the Tool Allocation, Tool Magazine Arrangement and the Operations Sequencing Algorithms will be applied on the example problem, respectively. In section 5.4, a discussion on the results of the proposed algorithm and the concluding remarks will be presented.

5.2 Input Data

In this chapter, turning of a rotational part on a CNC Turning Center, whose machinable volume presentation is illustrated in Figure 5.1, is studied. Our example part has twelve prespecified machinable volumes those should be removed by one of the tools in their candidate set. Geometrical data and the required surface qualities for these machinable volumes are presented in Table 5.1.

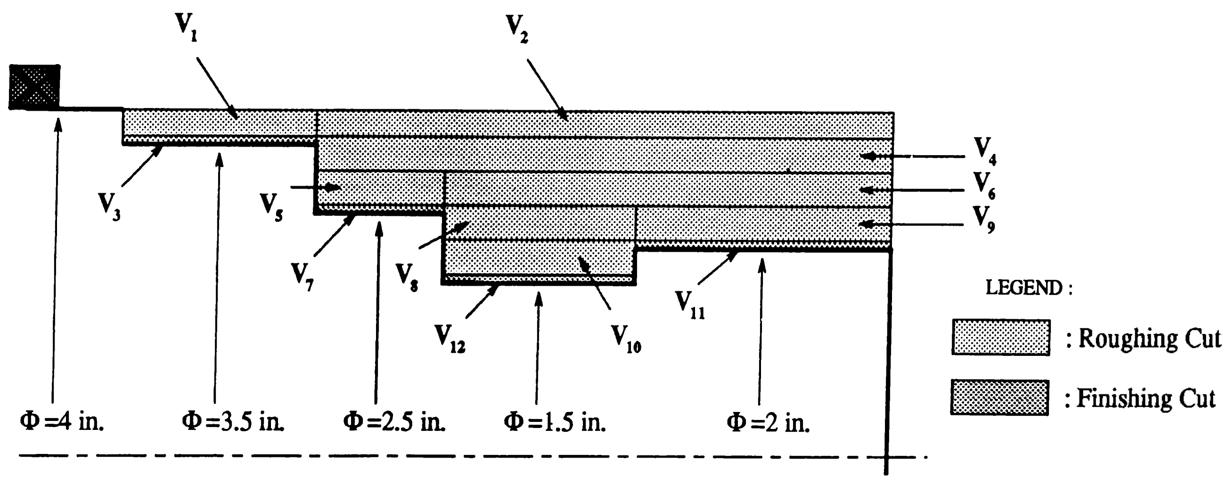

Figure 5.1: Machinable Volume Presentation

Volume	D_i,(in.)	L_i,(in.)	d_i,(in.)	SF_{max_i},(μin.)	Start Point	End Point
V_1	4	3	0.2	300	(4,2,0)	(1,2,0)
V_2	4	9	0.2	400	(13,2,0)	(4,2,0)
V_3	3.6	3	0.05	75	(4,1.8,0)	(1,1.8,0)
V_4	3.6	9	0.25	400	(13,1.8,0)	(4,1.8,0)
V_5	3.1	2	0.25	300	(6,1.55,0)	(4,1.55,0)
V_6	3.1	7	0.25	400	(13,1.55,0)	(6,1.55,0)
V_7	2.6	2	0.05	50	(6,1.3,0)	(4,1.3,0)
V_8	2.6	3	0.25	400	(9,1.3,0)	(6,1.3,0)
V_9	2.6	4	0.25	300	(13,1.3,0)	(9,1.3,0)
V_{10}	2.1	3	0.25	300	(9,1.05,0)	(6,1.05,0)
V_{11}	2.1	4	0.05	40	(13,1.05,0)	(9,1.05,0)
V_{12}	1.6	3	0.05	30	(9,0.8,0)	(6,0.8,0)

Table 5.1: Machinable Volume Data

The precedence relations between the machining of the machinable volumes are given in a 0-1 matrix, R, which indicates that the removal of volume i must be preceded by the machining of volume j, not necessarily directly. The possible machinable volume – cutting tool assignments are presented by the 0-1 matrix, X, which shows whether it is possible to remove a machinable volume by a particular tool type.

Precedence Relationship Matrix:

$$R = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Machinable Volume and Tool Pairs Matrix:

$$X = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

There are ten different tool types and their technological parameters are given in the Table 5.2. Tool switching, loading, changing times, the number of tools available on hand and the tool cost for each type are presented in Table 5.3.

Tool	α	β	γ	C_j	b	c	e	C_m	g	h	i	C_s
T_1	4.0	1.40	1.16	40960000	0.91	0.78	0.75	2.394	-1.52	1.004	0.25	204620000
T_2	4.3	1.60	1.20	37015056	0.96	0.70	0.71	1.637	-1.60	1.005	0.30	259500000
T_3	3.7	1.30	1.10	13767340	0.90	0.75	0.72	2.315	-1.45	1.015	0.25	202010000
T_4	3.7	1.28	1.05	11001020	0.80	0.75	0.70	2.415	-1.63	1.052	0.30	205740000
T_5	4.1	1.26	1.05	48724925	0.80	0.77	0.69	2.545	-1.69	1.005	0.40	204500000
T_6	4.1	1.30	1.10	57225273	0.87	0.77	0.69	2.213	-1.55	1.005	0.25	202220000
T_7	3.7	1.30	1.05	13767340	0.83	0.75	0.73	2.321	-1.63	1.015	0.30	203500000
T_8	3.8	1.20	1.05	23451637	0.88	0.83	0.72	2.321	-1.55	1.016	0.18	213570000
T_9	4.2	1.65	1.20	56158018	0.90	0.78	0.65	1.706	-1.54	1.104	0.32	211825000
T_{10}	3.8	1.20	1.05	23451637	0.81	0.75	0.72	2.298	-1.55	1.016	0.18	203500000

Table 5.2: Technological Exponents and Coefficients of the Available Tools

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
t_{s_j} (min.)	0.75	0.75	0.75	0.75	0.75	0.75	1	1	1	0.75
t_{l_j} (min.)	1	1	1	0.75	1	1	1	1.5	1	0.75
t_{c_j} (sec.)	5	5	5	5	5	5	5	5	5	5
t_j	4	3	5	20	10	12	4	11	2	15
C_{t_j}	0.50	0.70	0.90	0.70	0.50	0.60	0.75	0.50	0.75	0.60

Table 5.3: Tool Switching, Loading, Changing Times, Available Quantity on Hand and Cost for the each Tool Type

In this problem, manufacturing of a batch of 30 identical parts is concerned on a CNC Turning Center having the following system parameters:

- Operating Cost : \$0.5/min
- Maximum Allowable Machine Power : 3.5 hp
- Velocity of the Slides : 5 in./sec.
- Acceleration of the Slides : 5 in./sec.^{2}
- Approach and Stabilization Time : 5 sec.
- Tool Magazine Capacity : 10 slots

- Batch Size : 30 workpieces
- Coordinates of the Tool Changing Point (f) : (0,0,20)

5.3 Application of the Proposed Algorithm

In this section, the Tool Allocation, Tool Magazine Arrangement and the Operations Sequencing Algorithms are applied on the given problem by referring the Sections 4.6, 4.7, and 4.8, respectively. The algorithms are applied with the same step numbers presented in the corresponding sections.

5.3.1 Tool Allocation Algorithm

Step 1 : The following sets are created by the data given in tool and operations assignment matrix:

Candidate tool sets of the machinable volumes:

$$
\begin{array}{ll}
J_1 = \{4,5,6,7,8,10\} & J_7 = \{1,2,3,4,7,9\} \\
J_2 = \{4,5,6,7,8,10\} & J_8 = \{4,5,6,7,8,10\} \\
J_3 = \{1,2,3,4,7\} & J_9 = \{4,5,6,7,8,10\} \\
J_4 = \{4,5,6,7,8,10\} & J_{10} = \{4,5,6,7,8,10\} \\
J_5 = \{4,5,6,7,8,10\} & J_{11} = \{1,2,3,9\} \\
J_6 = \{4,5,6,7,8,10\} & J_{12} = \{1,2,3,9\}
\end{array}
$$

Operation assignment sets of the available tools:

$$
\begin{array}{ll}
I_1 = \{3,7,11,12\} & I_6 = \{1,2,4,5,6,8,9,10\} \\
I_2 = \{3,7,11,12\} & I_7 = \{1,2,3,4,5,6,7,8,9,10\} \\
I_3 = \{3,7,11,12\} & I_8 = \{1,2,4,5,6,8,9,10\} \\
I_4 = \{1,2,3,4,5,6,7,8,9,10\} & I_9 = \{7,11,12\} \\
I_5 = \{1,2,4,5,6,8,9,10\} & I_{10} = \{1,4,5,6,8,9,10\}
\end{array}
$$

Step 2 : For every possible operation, SMOP has been solved by taking $\bar{p}_{ij} = 1$ to impose the tool life covering constraint on the machining problem and the results are given in Appendix B, Tables B.1-B.12. It has observed that all the possible operations satisfy this constraint in a considerably large slack tool life and this result is consistent with the given problem since the machining operations are small in size and the tools are durable high quality tools.

Step 3 : The required number of tools and the corresponding $\bar{p}_{ij}$ value for every operation is also presented in the Tables B.1-B.12 of Appendix B.

Step 4 : Selection of the machining conditions having the minimum cost measure has been explained in the forth step of Tool Allocation algorithm in Section 4.6. An example trace of this procedure is given in Table B.13 for the machining of Volume-11 with Tool-9, that is Operation(11,9).

Step 5 : For every machinable volume, resulting minimum cost measures after ranking are presented in Tables B.14-B.25 of Appendix B, respectively.

Step 6 : There exist only three *Primal Tools* which are as follows:

$$J_F = \{4, 7, 9\}$$

This result can be explained by an observation on the assignment sets of these particular tools and the given surface roughness specifications for their machinable volumes. As a result, Tool-4 has been found as the best roughing tool where as the Tool-9 is the best finishing one and Tool-7 is particularly suitable for medium surface roughness specifications.

Step 7 : The machinable volume sets for the primal tools are created by picking the first rank tools from Tables B.14-B.25:

$$I_{4_F} = \{1, 2, 4, 5, 6, 8, 9, 10\}$$
$$I_{7_F} = \{3\}$$
$$I_{9_F} = \{7, 11, 12\}$$

Step 8 : In this problem there is no machinable volume having only a single tool for its manufacturing so this step of the algorithm is skipped. If such a case would occur than this particular machinable volume-tool pair would be a critical operation in the overall problem.

Step 9 : In this step, total tool requirements for the primal tools are calculated first by using the Tables B.14-B.25, and then the most deficit tool is found by comparing the available quantity on hand, which is defined as the bottleneck resource of problem. These calculations are as follows:

$$\begin{aligned}
R_4 = &\ \bar{n}_{t_{1,4}} + \bar{n}_{t_{2,4}} + \bar{n}_{t_{4,4}} + \bar{n}_{t_{5,4}} + \bar{n}_{t_{6,4}} + \bar{n}_{t_{8,4}} + \bar{n}_{t_{9,4}} + \bar{n}_{t_{10,4}} \\
= &\ 3 + 6 + 6 + 2 + 4 + 2 + 3 + 2 \\
R_4 = &\ 28 > t_4 = 24
\end{aligned}$$

$$R_7 = \bar{n}_{t_{3,7}} = 2 < t_7 = 4$$

$$\begin{aligned}
R_9 = &\ \bar{n}_{t_{7,9}} + \bar{n}_{t_{11,9}} + \bar{n}_{t_{12,9}} \\
= &\ 1 + 2 + 1 \\
R_9 = &\ 4 > t_9 = 2
\end{aligned}$$

Only Tool-7 has an excess amount of two tools, for the others the deficit ratios are as follows:

$$\rho_4 = \frac{28 - 20}{28} \simeq 0.2857 \text{ and } \rho_9 = \frac{4 - 2}{4} = 0.5$$

From the above values, Tool-9 is found as the most scarce resource, therefore in the next step first the allocation of this tool is completed, then we will continue with the Tool-4. For the Tool-7, there exists an excess amount of 2 tools, so this tool and and its corresponding volume are appended in the following reservation sets and the available quantity on hand is updated:

$$\bar{I} = \{3\}\ ,\ \bar{J} = \{7\}\ ,\ t_7 = 2$$

Step 10 : As it has mentioned before, this algorithm first focuses on the most scarce resources and tries to allocate them as efficiently as possible. In this respect first Tool-9 is handled in this part, and then continued with the Tool-4:

Allocation of Tool-9

Step 10.1 : First, the theoretically possible reductions are determined in terms of the number of required tools for the machining operation. In this convention, $\pi = 0$ means that do not perturb this operation and accept the conditions corresponding to minimum cost measure, and $\pi \geq \bar{n}_{t_{ij}}$ means that use a secondary tool instead of the primary one for keeping $\bar{n}_{t_{ij}}$ tools available for the other operations. On the other hand, for some $\pi \leq \bar{n}_{t_{ij}}$, the secondary tool can be preferred if it gives a lower cost measure.

Step 10.2 : In this step, the resulting increase in the cost measure is compared with the cost of secondary tool to make a selection among them. For this purpose, first the availability of the secondary tool is checked and then, if it is necessary a modification on the $\bar{p}_{ij}$ is done and the entire tool list is re-ordered. After finding a feasible secondary tool, the primary tool is revised for π^{th} perturbation and finally the resulting cost increment for manufacturing of a volume is determined as mentioned in Section 4.6. If the secondary tool exists and gives a smaller cost figure with respect to π^{th} perturbation of the primal tool, then, it means that this tool might be a better substitute and the saving in tool requirement of the primal tool for this perturbation will be the whole amount required for this operation, such that, $\bar{n}^{\pi}_{t_{ij}} = \bar{n}_{t_{ij}}$, so we will stop perturbating this operation. Further, after finding the regular perturbations, if any one of the secondaries appears in the set $J_F \cup \bar{J}$, this tool is also taken as a possible substitute reducing the usage of primal tool at all.

In the following part, these calculations are outlined for every operation of Tool-9:

- **Operation (7,9)** : For this operation, the secondary tool is Tool-7 and the required amount is $\bar{n}_{t_{7,7}} = 1$ and it is less than the available number $t_7 = 2$. Therefore there is no need to perturb the secondary tool for a reduction in its usage, so the Table B.20 remains same.

$\underline{\pi = 0:}$ It means that use same tool with the data given Table B.20, so the cost does not change and the required tool amount remains same, as follows:

$$\bar{C}^0_{7,9} = 3.18 \ , \ \bar{n}^0_{t_{7,9}} = 1 \ , \ \Delta C^0_7 = 0$$

$\underline{\pi = 1:}$ In this case the secondary tool, Tool-7, is used and the resulting increase in the cost is:

$$\bar{C}^0_{7,7} = 3.43 \ , \ \Delta C^1_7 = 3.43 - 3.18 = 0.25$$

$\underline{\pi = 2:}$ As the last perturbation, usage of the Tool-4 is considered since it is a primal tool and it is available since it has not been allocated:

$$\bar{C}^0_{7,4} = 3.44 \ , \ \Delta C^2_7 = 3.44 - 3.18 = 0.26$$

The above perturbations for the Operation (7,9) are summarized in Table B.26.

- **Operation (11,9)** : Now, the secondary tool is Tool-1 and $t_1 = 2$ (see Table B.24), however for this operation the required amount is $\bar{n}_{t_{11,1}} = 3$. Therefore, by taking $\bar{p}_{11,1} = 15$, new cost measure is found as $\bar{C}^1_{11,1} = 8.21$, and Tool-1 is still the secondary tool since there is no other tool with a lower cost.

  $\underline{\pi = 0:}$ Use Tool-9 with $\bar{C}^0_{11,9} = 5.57$ and $\bar{n}^0_{t_{11,9}} = 2$, so $\Delta C^0_{11} = 0$

  $\underline{\pi = 1:}$ Take $\bar{p}_{11,9} = 30$, then the resulting cost measure is $\bar{C}^1_{11,9} = 6.10$ with $\bar{n}^1_{11,9} = 1$. This perturbation results in a lower value than secondary tool, so the cost increment is:

$$\Delta C^1_{11} = \bar{C}^1_{11,9} - \bar{C}^0_{11,9} = 6.10 - 5.57 = 0.53 \text{ with } \bar{n}^1_{t_{11,9}} = 1$$

  $\underline{\pi = 2:}$ In this case, Tool-1 will be used instead of Tool-9:

$$\Delta C^2_{11} = \bar{C}^1_{11,1} - \bar{C}^0_{11,9} = 8.21 - 5.57 = 2.64 \text{ with } \bar{n}^2_{t_{11,9}} = 0$$

  The above perturbations for the Operation (11,9) are summarized in Table B.27.

- **Operation (12,9)** : Again, the secondary is Tool-1 (see Table B.25), but this time required tool amount is equal to the available on hand, $\bar{n}^0_{t_{12,1}} = t_1 = 2$, and this tool may be used instead of Tool-9 without any perturbation :

  $\underline{\pi = 0 :}$ Use Tool-9 in the primal set with the following values:

  $$\bar{C}^0_{12,9} = 3.54 \ , \ \Delta C^0_{12} = 0 \ , \ \bar{n}^0_{t_{12,9}} = 1$$

  $\underline{\pi = 1 :}$ Use the secondary tool and keep Tool-9 available for the other operations:

  $$\bar{C}^0_{12,1} = 4.98 \ , \ \Delta C^0_{12} = \bar{C}^0_{12,1} - \bar{C}^0_{12,9} = 4.89 - 3.54 = 1.35 \ , \ \bar{n}_{t_{12,1}} = 2$$

  The above perturbations for the Operation (12,9) are summarized in Table B.28.

Step 11 : The following 0-1 MIP is proposed to find the best combination as explained in Section 4.6. In this model, first three constraints ensure that only one of the several decisions (perturbations) is made for every machinable volume. The last four constraints require that the resulting perturbations should satisfy the tool availability constraint for secondary tools; Tool-1, Tool-4, Tool-7, and the primary, Tool-9, respectively. The objective function aims to minimize the resulting cost increase determined by the tool allocation.

Minimize $\quad \Delta C_9 = 0.25y^1_7 + 0.26y^2_7 + 0.53y^1_{11} + 2.64y^2_{11} + 1.35y^1_{12}$

Subject to:

$$\begin{aligned}
&y^0_7 + y^1_7 + y^2_7 = 1 \\
&y^0_{11} + y^1_{11} + y^2_{11} = 1 \\
&y^0_{12} + y^1_{12} = 1 \\
&2y^2_{11} + 2y^1_{12} \leq t_1 = 2 \\
&y^2_7 \leq t_4 = 20 \\
&y^1_7 \leq t_7 = 2 \\
&y^0_7 + 2y^0_{11} + y^1_{11} + y^0_{12} = t_9 = 2
\end{aligned}$$

The solution to the above problem is found by using LINDO, as follows:

$y^1_7 = y^1_{11} = y^0_{12} = 1$ and Total Cost Increment is, $\Delta C_9 = 0.78$

Step 12 : This solution suggests to use Tool-7 for the manufacturing of Volume-7, a reduction of a single Tool-9 in the processing of the Volume-11, and it leaves the original solution for the Volume-12 without any reduction in the usage of Tool-9. The current tool allocation set, $\bar{J} = \{7\}$, includes Tool-7 and the Tool-9 is a *Primal Tool.* Therefore this solution is acceptable since there is no tool j introduced where $j \in J \backslash (J_F \cup \bar{J})$. The updated values are given as follows:

$$\bar{I} = \{3, 7, 11, 12\} \text{ and } \bar{J} = \{7, 9\}$$

$$t_7 = 1 \ , \ t_9 = 0 \text{ so delete Tool-9 from } J$$

The remaining tools for the further allocations are:

$$J = \{1, 2, 3, 4, 5, 6, 7, 8, 10\}$$

Allocation of Tool-4

Step 10.1 & 10.2 : Possible perturbations for every operation, and the resulting increments with tool usages are summarized below:

- **Operation (1,4) :** In this particular machinable volume the secondary tool is Tool-7, however after some tool allocation the remaining quantity for this tool is $t_7 = 1$ so the corresponding cost measure found as $\bar{C}^1_{1,7} = 11.96$ and this cost measure carries it to fifth rank, now the secondary tool is Tool-5 (see Table B.29)

  $\underline{\pi = 0 :}$ Use Tool-4 without any perturbation

  $$\bar{n}^0_{t_{1,4}} = 3 \ , \ \Delta C^0_1 = 0$$

  $\underline{\pi = 1 :}$ Use Tool-4 by reducing tool requirement by one

  $$\bar{n}^1_{t_{1,4}} = 2 \ , \ \Delta C^1_1 = 9.17 - 8.79 = 0.38$$

  $\underline{\pi = 2 :}$ Use Tool-5 without any perturbation

  $$\bar{n}^2_{t_{1,5}} = 0 \ , \ \Delta C^2_1 = 10.60 - 8.79 = 1.81$$

$\underline{\pi = 3:}$ Use Tool-7 since it is in set $\bar{J}$ and $T_7 = 1$

$$\bar{n}^3_{t_{1,7}} = 0 \ , \ \Delta C^3_1 = 11.96 - 8.79 = 3.17$$

- **Operation (2,4)** : In this case again the secondary tool is Tool-5 (see Table B.30)

$\underline{\pi = 0:}$ Use Tool-4 without any perturbation

$$\bar{n}^0_{t_{2,4}} = 6 \ , \ \Delta C^0_2 = 0$$

$\underline{\pi = 1:}$ Use Tool-4 by reducing tool requirement by one

$$\bar{n}^1_{t_{2,4}} = 5 \ , \ \Delta C^1_2 = 24.94 - 23.81 = 1.13$$

$\underline{\pi = 2:}$ Use Tool-5 without any perturbation

$$\bar{n}^2_{t_{2,5}} = 0 \ , \ \Delta C^2_2 = 26.18 - 23.81 = 2.37$$

$\underline{\pi = 3:}$ Use Tool-4 by reducing tool requirement by two

$$\bar{n}^3_{t_{2,4}} = 4 \ , \ \Delta C^3_2 = 28.81 - 23.81 = 5$$

- **Operation (4,4)** : For this operation the secondary tool is Tool-5 (see Table B.31)

$\underline{\pi = 0:}$ Use the original solution with Tool-4 without any perturbation

$$\bar{n}^0_{t_{4,4}} = 6 \ , \ \Delta C^0_4 = 0$$

$\underline{\pi = 1:}$ Use Tool-4 by reducing tool requirement by one

$$\bar{n}^1_{t_{4,4}} = 5 \ , \ \Delta C^1_4 = 25.91 - 25.57 = 0.34$$

$\underline{\pi = 2:}$ Use Tool-5 without any perturbation

$$\bar{n}^2_{t_{4,5}} = 0 \ , \ \Delta C^2_4 = 29.53 - 25.57 = 3.96$$

$\underline{\pi = 3:}$ Use Tool-4 by reducing tool requirement by two

$$\bar{n}^3_{t_{4,4}} = 5 \ , \ \Delta C^3_4 = 30.04 - 25.57 = 4.47$$

- **Operation (5,4)** : For this operation the secondary tool is Tool-7 (see Table B.32)

  $\underline{\pi = 0:}$ Use Tool-4 without any perturbation

$$\bar{n}^0_{t_{5,4}} = 2 \ , \ \Delta C^0_5 = 0$$

  $\underline{\pi = 1:}$ Use Tool-7 with $\bar{n}_{t_{5,7}} = 1$

$$\bar{n}^1_{t_{5,7}} = 0 \ , \ \Delta C^1_7 = 5.60 - 5.50 = 0.10$$

  $\underline{\pi = 2:}$ Use Tool-4 by reducing tool requirement by one

$$\bar{n}^2_{t_{5,4}} = 1 \ , \ \Delta C^2_5 = 5.85 - 5.50 = 0.35$$

- **Operation (6,4)** : For this operation the secondary tool is Tool-5 (see Table B.33)

  $\underline{\pi = 0:}$ Use Tool-4 without any perturbation

$$\bar{n}^0_{t_{6,4}} = 4 \ , \ \Delta C^0_6 = 0$$

  $\underline{\pi = 1:}$ Use Tool-4 by reducing tool requirement by one

$$\bar{n}^1_{t_{6,4}} = 3 \ , \ \Delta C^1_6 = 18.30 - 17.00 = 1.30$$

  $\underline{\pi = 2:}$ Use Tool-5

$$\bar{n}^2_{t_{6,5}} = 0 \ , \ \Delta C^2_6 = 19.18 - 17.00 = 2.18$$

  $\underline{\pi = 3:}$ Use Tool-4 by reducing tool requirement by two

$$\bar{n}^3_{t_{6,4}} = 2 \ , \ \Delta C^3_6 = 23.06 - 17.00 = 6.06$$

- **Operation (8,4)** : For this operation the secondary tool is Tool-7 (see Table B.34)

  $\underline{\pi = 0:}$ Use Tool-4 without any perturbation

$$\bar{n}^0_{t_{8,4}} = 2 \ , \ \Delta C^0_8 = 0$$

$\underline{\pi = 1:}$ Use Tool-7 with $\bar{n}_{t_{5,7}} = 1$

$$\bar{n}^1_{t_{8,7}} = 0 \ , \ \Delta C^1_8 = 6.59 - 6.39 = 0.20$$

$\underline{\pi = 2:}$ Use Tool-4 by reducing tool requirement by one

$$\bar{n}^2_{t_{8,4}} = 1 \ , \ \Delta C^2_8 = 6.91 - 6.39 = 0.52$$

- **Operation (9,4)** : For this operation the secondary tool is Tool-5 (see Table B.35)

$\underline{\pi = 0:}$ Use Tool-4 without any perturbation

$$\bar{n}^0_{t_{9,4}} = 3 \ , \ \Delta C^0_9 = 0$$

$\underline{\pi = 1:}$ Use Tool-4 by reducing tool requirement by one

$$\bar{n}^1_{t_{9,4}} = 2 \ , \ \Delta C^1_9 = 9.04 - 9.02 = 0.02$$

$\underline{\pi = 2:}$ Use Tool-5

$$\bar{n}^2_{t_{9,5}} = 0 \ , \ \Delta C^2_9 = 10.36 - 9.02 = 1.34$$

$\underline{\pi = 3:}$ Use Tool-4 by reducing tool requirement by two

$$\bar{n}^3_{t_{9,4}} = 0 \ , \ \Delta C^3_9 = 13.09 - 9.02 = 4.07$$

- **Operation (10,4)** : For this operation the secondary tool is Tool-7 (see Table B.36)

$\underline{\pi = 0:}$ Use Tool-4 without any perturbation

$$\bar{n}^0_{t_{10,4}} = 2 \ , \ \Delta C^0_{10} = 0$$

$\underline{\pi = 1:}$ Use Tool-7 with $\bar{n}_{t_{5,7}} = 1$

$$\bar{n}^1_{t_{10,7}} = 0 \ , \ \Delta C^0_{10} = 5.68 - 5.56 = 0.12$$

$\underline{\pi = 2:}$ Use Tool-4 by reducing tool requirement by one

$$\bar{n}^1_{t_{10,4}} = 1 \ , \ \Delta C^0_{10} = 5.99 - 5.56 = 0.43$$

Step 11 : By using the above possible perturbations the following 0-1 MIP is constructed to find the best allocation of tools and their perturbations:

$$\begin{aligned}
\text{Minimize} \quad & \Delta C_4 = 0.38y_1^1 + 1.81y_1^2 + 3.17y_1^3 + \\
& 1.13y_2^1 + 2.37y_2^2 + 5y_2^3 + \\
& 0.34y_4^1 + 3.96y_4^2 + 4.47y_4^3 + \\
& 0.10y_5^1 + 0.35y_5^2 + \\
& 1.30y_6^1 + 2.18y_6^2 + 6.06y_6^3 \\
& 0.20y_8^1 + 0.52y_8^2 + \\
& 0.02y_9^1 + 1.34y_9^2 + 4.07y_9^3 + \\
& 0.12y_{10}^1 + 0.43y_{10}^2
\end{aligned}$$

Subject to:

$$\begin{aligned}
& y_1^0 + y_1^1 + y_1^2 + y_1^3 = 1 \\
& y_2^0 + y_2^1 + y_2^2 + y_2^3 = 1 \\
& y_4^0 + y_4^1 + y_4^2 + y_4^3 = 1 \\
& y_5^0 + y_5^1 + y_5^2 = 1 \\
& y_6^0 + y_6^1 + y_6^2 + y_6^3 = 1 \\
& y_8^0 + y_8^1 + y_8^2 = 1 \\
& y_9^0 + y_9^1 + y_9^2 + y_9^3 = 1 \\
& y_{10}^0 + y_{10}^1 + y_{10}^2 = 1
\end{aligned}$$

$$\begin{aligned}
& 3y_1^0 + 2y_1^1 + 6y_2^0 + 5y_2^1 + 4y_2^3 + \\
& 6y_4^0 + 5y_4^1 + 4y_4^3 + 2y_5^0 + y_5^2 + \\
& 4y_6^0 + 3y_6^1 + 2y_6^3 + 2y_8^0 + y_8^2 + \\
& 3y_9^0 + 2y_9^1 + y_9^3 + 2y_{10}^0 + y_{10}^2 = t_4 = 20
\end{aligned}$$

$$2y_1^2 + 10y_2^2 + 8y_4^2 + 6y_6^2 + 4y_9^2 \leq t_5 = 10$$

$$y_1^3 + y_5^1 + y_8^1 + y_{10}^1 \leq t_7 = 1$$

The solution of the above problem is:

$$y_1^0 = y_2^2 = y_4^0 = y_5^1 = y_6^0 = y_8^0 = y_9^0 = y_{10}^0 = 1$$

Step 12 : This solution suggests:

Volume-1 : Use Tool-4 with $\bar{n}^0_{t_{1,4}} = 3$, and $\Delta C^0_1 = 0$

Volume-2 : Use Tool-5 with $\bar{n}^2_{t_{2,5}} = 0$, and $\Delta C^2_2 = 2.37$

Volume-4 : Use Tool-4 with $\bar{n}^0_{t_{4,4}} = 6$, and $\Delta C^0_4 = 0$

Volume-5 : Use Tool-7 with $\bar{n}^1_{t_{5,7}} = 0$, and $\Delta C^1_7 = 0.10$

Volume-6 : Use Tool-4 with $\bar{n}^0_{t_{6,4}} = 4$, and $\Delta C^0_6 = 0$

Volume-8 : Use Tool-4 with $\bar{n}^0_{t_{8,4}} = 2$, and $\Delta C^0_8 = 0$

Volume-9 : Use Tool-4 with $\bar{n}^0_{t_{9,4}} = 3$, and $\Delta C^0_9 = 0$

Volume-10 : Use Tool-4 with $\bar{n}^0_{t_{10,4}} = 2$, and $\Delta C^0_{10} = 0$

The resulting increase in the total manufacturing cost is $\Delta C_4 = 2.47$ with the tool requirements of $t_4 = 20$, $t_5 = 10$, and $t_7 = 1$. However this solution requires the allocation of Tool-5 in the tool magazine, which has not been allocated before and it is neither a primal tool. Therefore, the resulting cost increment for this tool should be updated by taking the tool interchanging cost into consideration. For this purpose, the cost increment ΔC^2_2 has been revised as follows:

$$\begin{aligned} \Delta C^{2'}_2 &= \Delta C^2_2 + C_0.N_B.t_{c_j} \\ &= 2.37 + (0.5)(30)(\frac{5}{30}) = 3.62 \end{aligned}$$

After this revision the allocation problem has been re-solved by using the above penalized cost increment value of the Tool-5 and the results are as follows:

$$y^1_1 = y^0_2 = y^1_4 = y^2_5 = y^0_6 = y^0_8 = y^2_9 = y^1_{10} = 1$$

However, this solution also suggests to inclusion of Tool-5 in the tool magazine for the Operation (9,5) since $y^2_9 = 1$ (see Table B.35). In this case the total cost increment is $\Delta C_4 = 2.53$. As a result, the Tool-5 is allocated in the tool magazine as dictated by the first problem, since its introduction is justified by the second (penalized) problem. In other words, without including Tool-5 in the tool magazine, the alternative allocations will be much more expensive than the additional cost incurred due to the necessary tool interchanges for Tool-5.

The final tool allocation and the usages are, which satisfy the tool availability constraints, as follows:

$$\bar{I}_4 = \{1,4,6,8,9,10\}, \bar{I}_5 = \{2\}, \bar{I}_7 = \{3,5,7\}, \bar{I}_9 = \{11,12\}$$

$$\bar{J} = \{4,5,7,9\} \text{ and } t_4 = 20, t_5 = 10, t_7 = 4, t_9 = 2$$

The detailed results for the final tool allocation and the machining conditions are also given in the Table 5.4.

$V\#$	T#	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$\bar{n}_{t_{ij}}$	C_{ij}
1	4	6	285.01	0.02853	0.3862	4.6733	0.0826	3	8.79
2	5	6	226.92	0.03627	1.1445	3.7804	0.3027	10	27.98
3	7	4	528.39	0.02624	0.2038	3.0575	0.0667	2	5.81
4	4	6	242.92	0.02747	1.2706	7.0095	0.1813	6	25.57
5	7	6	270.56	0.02181	0.2749	8.5375	0.0322	1	5.60
6	4	6	242.92	0.02747	0.8510	7.0095	0.1214	4	17.00
7	7	4	555.22	0.01905	0.1286	3.8584	0.0333	1	3.43
8	4	4	242.63	0.02742	0.3068	7.0561	0.0435	2	6.39
9	4	6	269.68	0.02457	0.4107	5.4916	0.0748	3	9.02
10	4	6	269.68	0.02457	0.2488	5.4916	0.0453	2	5.56
11	9	4	535.20	0.01238	0.3318	9.9528	0.0333	1	6.10
12	9	4	639.16	0.01222	0.1608	4.8244	0.0333	1	3.54

Table 5.4: Final Tool Allocation and the Machining Conditions

5.3.2 Tool Magazine Capacity Checking and Tool Sharing Algorithm

For the details of this algorithm refer to the Section 4.7

Step 1 : Only Tool-5 has a single operation assignment, Volume-2, so a single tool slot is reserved for the Operation (2,5) in the set $\check{J}$.

Step 2 : Tool-9, Tool-7 and Tool-4 have more than one operation in their assignment sets:

$$\bar{I}_9 = \{11, 12\}, \bar{I}_7 = \{3, 5, 7\}, \text{ and } \bar{I}_4 = \{1, 4, 6, 8, 9, 10\}$$

Step 2.1 : Pre-processed data for the Tool-4, Tool-7, and Tool-9 are presented in Tables 5.5, 5.6 and 5.7, respectively.

Tool Req. (l_j)	Duplicates (k_{l_j})	# of Parts ($p_j^{l_j}$)	Usage Rate ($\bar{U}_{l_j}$)	Volumes (V_{l_j})
2	10	15	0.06667	$\{8, 10\}$
3	6	10	0.10000	$\{1, 8, 9, 10\}$
4	5	8	0.12500	$\{1, 6, 8, 9, 10\}$
5	4	6	0.16667	$\{1, 6, 8, 9, 10\}$
6	3	5	0.20000	$\{1, 4, 6, 8, 9, 10\}$
8	2	4	0.25000	$\{1, 4, 6, 8, 9, 10\}$
10	2	3	0.33334	$\{1, 4, 6, 8, 9, 10\}$
15	1	2	0.50000	$\{1, 4, 6, 8, 9, 10\}$

Table 5.5: Pre-processing of the data for Tool-4

Tool Req. (l_j)	Duplicates (k_{l_j})	# of Parts ($p_j^{l_j}$)	Usage Rate ($\bar{U}_{l_j}$)	Volumes (V_{l_j})
1	4	30	0.03334	$\{5, 7\}$
2	2	15	0.06667	$\{3, 5, 7\}$
3	1	10	0.10000	$\{3, 5, 7\}$
4	1	8	0.12500	$\{3, 5, 7\}$

Table 5.6: Pre-processing of the data for Tool-7

Tool Req. (l_j)	Duplicates (k_{l_j})	# of Parts ($p_j^{l_j}$)	Usage Rate ($\bar{U}_{l_j}$)	Volumes (V_{l_j})
1	2	30	0.03334	$\{11, 12\}$
2	1	15	0.06667	$\{11, 12\}$

Table 5.7: Pre-processing of the data for Tool-9

Step 2.2 : As it is mentioned in Section 4.6, alternative tool arrangements for each tool type are generated by solving a 0-1 MIP model which minimizes the total slack tool life. This model is presented only for Tool-7 in below for the sake of simplicity:

Maximize $s_{1,1} + s_{1,2} + s_{2,1} + s_{2,2} + s_{3,1} + s_{4,1}$

Subject to:

$$0.0322z_{5,1,1} + 0.0333z_{7,1,1} + s_{1,1} = 0.03334$$
$$0.0322z_{5,1,2} + 0.0333z_{7,1,2} + s_{1,2} = 0.03334$$
$$0.0667z_{3,2,1} + 0.0322z_{5,2,1} + 0.0333z_{7,2,1} + s_{2,1} = 0.06667$$
$$0.0667z_{3,2,2} + 0.0322z_{5,2,2} + 0.0333z_{7,2,2} + s_{2,2} = 0.06667$$
$$0.0667z_{3,3,1} + 0.0322z_{5,3,1} + 0.0333z_{7,3,1} + s_{3,1} = 0.1$$
$$0.0667z_{3,4,1} + 0.0322z_{5,4,1} + 0.0333z_{7,4,1} + s_{4,1} = 0.125$$
$$z_{5,1,1} \leq m_{1,1}$$
$$z_{7,1,1} \leq m_{1,1}$$
$$z_{5,1,2} \leq m_{1,2}$$
$$z_{7,1,2} \leq m_{1,2}$$
$$z_{3,2,1} \leq m_{2,1}$$
$$z_{5,2,1} \leq m_{2,1}$$
$$z_{7,2,1} \leq m_{2,1}$$
$$z_{3,2,2} \leq m_{2,2}$$
$$z_{5,2,2} \leq m_{2,2}$$
$$z_{7,2,2} \leq m_{2,2}$$
$$z_{3,3,1} \leq m_{3,1}$$
$$z_{5,3,1} \leq m_{3,1}$$
$$z_{7,3,1} \leq m_{3,1}$$
$$z_{3,4,1} \leq m_{4,1}$$
$$z_{5,4,1} \leq m_{4,1}$$
$$z_{7,4,1} \leq m_{4,1}$$
$$z_{3,2,1} + z_{3,2,2} + z_{3,3,1} + z_{3,4,1} = 1$$
$$z_{5,1,1} + z_{5,1,2} + z_{5,2,1} + z_{5,2,2} + z_{5,3,1} + z_{5,4,1} = 1$$
$$z_{7,1,1} + z_{7,1,2} + z_{7,2,1} + z_{7,2,2} + z_{7,3,1} + z_{7,4,1} = 1$$
$$m_{1,1} + m_{1,2} + 2m_{2,1} + 2m_{2,2} + 3m_{3,1} + 4m_{4,1} \leq 4$$
$$m_{1,1} + m_{1,2} + m_{2,1} + m_{2,2} + m_{3,1} + m_{4,1} \leq 3$$

The above model is solved for every tool type by taking the bound on the number of tool slots as the worst case arrangement, that allocates a separate tool for every operation without any tool sharing. These worst case allocations and the corresponding solutions of the above model are presented as follows:

⋆ For Tool-4 :

$$A_4^6: \; T_4^2 = \{8\}, \;\; T_4^2 = \{10\}$$
$$T_4^3 = \{1\}, \;\; T_4^3 = \{9\}$$
$$T_4^4 = \{6\}, \;\; T_4^6 = \{4\}$$

RHS=6

$$A_4^3: \;\; T_4^5 = \{8, 9, 10\}, \;\; T_4^6 = \{4\} \quad \text{, and } T_4^8 = \{1, 6\}$$

⋆ For Tool-7 :

$$A_7^3: \;\; T_7^1 = \{5\}, \;\; T_7^1 = \{7\}, \quad \text{and } T_7^2 = \{3\}$$

RHS=3

$$A_7^2: \;\; T_7^2 = \{7\} \quad \text{, and } T_7^3 = \{3, 5\}$$

⋆ For Tool-9 :

$$A_9^2: \;\; T_9^1 = \{11\}, \;\; T_9^1 = \{12\}$$

where,

A_j^n : An alternative arrangement for the tool type j requiring n tool slots

T_j^l : Operation assignment set for a duplicate of tool type j at the l^{th} tool requirement level

Step 2.3 Updating the right-hand-side of the Number of Tool Slots constraint and finding the minimum tool slot requirement:

⋆ For Tool-4 :
RHS=3

$$A_4^2: \;\; T_4^8 = \{4, 8\} \quad \text{, and } T_4^{10} = \{1, 6, 9, 10\}$$

Minimum number of tool slots required, $S_4 = 2$

- ⋆ <u>For Tool-7 :</u>

  <u>RHS=2</u>

  This results in the same allocation found in the previous step, so: minimum number of tool slots required, $S_7 = 2$

- ⋆ <u>For Tool-9 :</u>

  <u>RHS=2</u>

  $$A_9^1: \ T_9^1 = \{11, 12\}$$

  Minimum number of tool slots required, $S_9 = 1$

Step 3: Checking the tool magazine capacity constraint:

$$\bar{J} = \{4, 7, 9\}, \ \check{J} = \{5\} \text{ with } a_5 = 1$$

$$\text{Therefore } \sum_{J \setminus \check{J}} S_j = 5 \leq N_m - \sum_{\check{J}} a_j = 10 - 1 = 9$$

Since we have some slack tool slots at the minimal allocations we continue with the following steps.

Step 4: The graphs presented in Figures 5.2, 5.3 and 5.4 are generated for the Tool-4, Tool-7 and Tool-9, respectively, by using the information given in precedence matrix R.

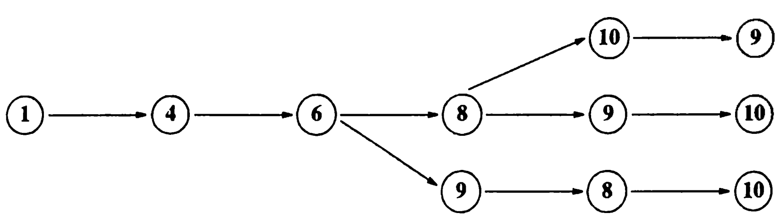

Figure 5.2: Network presentation for the operations of Tool-4

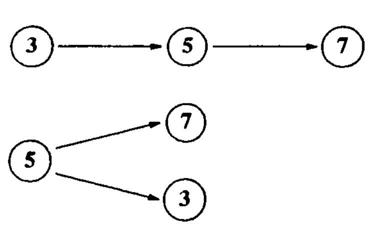

Figure 5.3: Network presentation for the operations of Tool-7

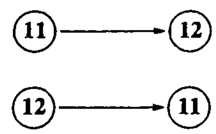

Figure 5.4: Network presentation for the operations of Tool-9

- **Step 4.1** : Set of adjacent operation pairs:

  ⋆ For Tool-4 :

  $I_{A_4} = \{(1,4),(4,6),(6,8),(6,9),(8,9),(8,10),(9,10)\}$

  ⋆ For Tool-7 :

  $I_{A_7} = \{(3,5),(5,7)\}$

  ⋆ For Tool-9 :

  $I_{A_9} = \{(11,12)\}$

- **Step 4.2** : Candidate tool requirement levels:

  ⋆ For Tool-4 :

  $L_{(1,4)} = \{10,15\}$ $\quad$ $L_{(8,9)} = \{4,5,6,8,10,15\}$
  $L_{(4,6)} = \{10,15\}$ $\quad$ $L_{(8,10)} = \{3,4,5,6,8,10,15\}$
  $L_{(6,8)} = \{5,6,8,10,15\}$ $\quad$ $L_{(9,10)} = \{4,5,6,8,10,15\}$
  $L_{(6,9)} = \{6,8,10,15\}$

  ⋆ For Tool-7 :

  $L_{(3,5)} = \{3,4\}$ $\quad$ $L_{(5,7)} = \{2,3,4\}$

  ⋆ For Tool-9 :

  $L_{(11,12)} = \{2\}$

- **Step 4.3** : Possible tool slot requirements and the resulting alternative arrangements:

  ⋆ For Tool-4 :

For this tool type, the possible slot requirements are as follows:

$$SR_4 = \{2, \cdots \min\{5,6\}\}$$

$$= \{2,3,4,5\}$$

For this tool type, the tool requirement level of two is excluded since:

$$L_j \backslash \bigcup_{(i,i')\in I_{A_4}} L_{(i,i')} = \{2\}$$

This means that at this level it is impossible to have a tool sharing between any two successive operations. The following alternatives are found by solving the 0-1 IP model mentioned in the **Step 4.3** of Tool Magazine Capacity Checking and Tool Sharing Algorithm for the members of SR_4 (formulation of this model is presented only for Tool-7 in the next part for the sake of simplicity):

- **Alternative A_4^2**
  $T_4^3 = \{1\}$ $T_4^{15} = \{4,6,8,9,10\}$
  Objective Function Value = 5
- **Alternative A_4^3**
  $T_4^3 = \{1\}$ $T_4^6 = \{8,9,10\}$ $T_4^{10} = \{4,6\}$
  Objective Function Value = 1
- **Alternative A_4^4**
  $T_4^3 = \{1\}$ $T_4^3 = \{10\}$ $T_4^4 = \{8,9\}$ $T_4^{10} = \{4,6\}$
  Objective Function Value = -2
- **Alternative A_4^5**
  $T_4^3 = \{1\}$ $T_4^3 = \{9\}$ $T_4^3 = \{10\}$ $T_4^5 = \{6,8\}$ $T_4^6 = \{4\}$
  Objective Function Value = -7

⋆ For Tool-7 :

Possible tool slot requirements:

$$SR_7 = \{2, \cdots \min\{2,6\}\} = \{2\}$$

O-1 IP Model for Tool Sharing:

$$\begin{aligned}
\text{Maximize} \quad & (z_{5,2,1} + z_{7,2,1} - m_{2,1}) + (z_{5,2,2} + z_{7,2,2} - m_{2,2}) \\
& +(z_{5,3,1} + z_{7,3,1} - m_{3,1}) + (z_{5,4,1} + z_{7,4,1} - m_{4,1}) \\
& +(z_{3,3,1} + z_{5,3,1} - m_{3,1}) + (z_{3,4,1} + z_{5,4,1} - m_{4,1})
\end{aligned}$$

Subject to:

$$\begin{aligned}
& 0.0667 z_{3,2,1} + 0.0322 z_{5,2,1} + 0.0333 z_{7,2,1} \leq 0.0667 \\
& 0.0667 z_{3,2,2} + 0.0322 z_{5,2,2} + 0.0333 z_{7,2,2} \leq 0.0667 \\
& 0.0667 z_{3,3,1} + 0.0322 z_{5,3,1} + 0.0333 z_{7,3,1} \leq 0.1 \\
& 0.0667 z_{3,4,1} + 0.0322 z_{5,4,1} + 0.0333 z_{7,4,1} \leq 0.125
\end{aligned}$$

$$\begin{aligned}
& z_{5,2,1} - m_{2,1} \leq 0 \\
& z_{7,2,1} - m_{2,1} \leq 0 \\
& z_{5,2,2} - m_{2,2} \leq 0 \\
& z_{7,2,2} - m_{2,2} \leq 0 \\
& z_{3,3,1} - m_{3,1} \leq 0 \\
& z_{5,3,1} - m_{3,1} \leq 0 \\
& z_{7,3,1} - m_{3,1} \leq 0 \\
& z_{3,4,1} - m_{4,1} \leq 0 \\
& z_{5,4,1} - m_{4,1} \leq 0 \\
& z_{7,4,1} - m_{4,1} \leq 0
\end{aligned}$$

$$\begin{aligned}
& z_{3,2,1} + z_{3,2,2} + z_{3,3,1} + z_{3,4,1} = 1 \\
& z_{5,2,1} + z_{5,2,2} + z_{5,3,1} + z_{5,4,1} = 1 \\
& z_{7,2,1} + z_{7,2,2} + z_{7,3,1} + z_{7,4,1} = 1
\end{aligned}$$

$$2m_{2,1} + 2m_{2,2} + 3m_{3,1} + 4m_{4,1} \leq 4$$

$$m_{2,1} + m_{2,2} + m_{3,1} + m_{4,1} = 2$$

The following alternative is generated by solving the above problem with LINDO for $a_7 = 2$:

- **Alternative** A_7^2
  $T_7^2 = \{3\} \quad T_7^2 = \{5, 7\}$

Objective Function Value = 0
This solution suggests to use two duplicates of Tool-7 at the requirement level of two, and operations are assigned as the above set. Therefore, this solution has identified a tool sharing for the volume 5 and 7.

$\star$ For Tool-9 :

Possible tool slot requirements:

$$SR_9 = \{1, \cdots \min\{1,5\}\} = \{1\}$$

In this case, there is no need to solve the 0-1 model since only a single tool will be allocated in the tool magazine and it will cover all the operations, so this tool will have a total tool sharing event. As a result, Tool-9 is excluded in **Step 6** and its arrangement is fixed in this step.

Step 5 : Evaluating alternatives by considering all non-machining times:

$\star$ For Tool-4 :

- **Alternative A_4^2**

Operations of T_4^{15} are all adjacent and to determine their order the following distances are used:

$$\Delta_{\overline{4}} = 13.12, \Delta_{\underline{4}} = 4.39, \Delta_{\overline{6}} = 13.09, \Delta_{\underline{6}} = 6.20, \Delta_{\overline{8}} = 9.09, \Delta_{\underline{8}} = 6.14$$

$$\Delta_{\overline{9}} = 13.04, \Delta_{\underline{9}} = 9.09, \Delta_{\overline{10}} = 9.06, \Delta_{\underline{10}} = 6.09$$

$$\Delta_{\underline{4}} < \Delta_{\underline{10}} < \Delta_{\underline{8}} < \Delta_{\underline{6}} < \Delta_{\overline{10}} < \Delta_{\underline{9}} = \Delta_{\overline{8}} < \Delta_{\overline{9}} < \Delta_{\overline{6}} < \Delta_{\overline{4}}$$

where,

$\Delta_{\overline{i}}$: Distance from the starting point of the volume i to the fixed point f

$\Delta_{\underline{i}}$: Distance from the ending point of the volume i to the fixed point f

We start with the Operation (4,4) since it has the farthest starting point and then we continue with the Operation (6,4) as the precedence relations dictates. As the third operation either Operation

(8,4) or (9,4) can be selected, at that point we choose Operation (9,4) since it has the farthest starting point, and continue with the Operations (8,4) and (10,4), respectively. In the view of this operation sequence the following cost is calculated for Tool-4:

$$\begin{aligned}
C_{A_4^2} = \; & C_o.N_B.(t_{r_{f,\bar{4}}} + t_{r_{\underline{4},\bar{6}}} + t_{r_{\underline{6},\bar{9}}} + t_{r_{\underline{9},\bar{8}}} + t_{r_{\underline{8},\overline{10}}} + t_{r_{\underline{10},f}} + 2.t_{c_4}) \\
& + C_o.(t_{r_{f,\bar{1}}} + t_{r_{\underline{1},f}} + 2.t_{c_4}) \\
& + C_o.\Big((3+15)-2\Big).t_{s_4} \\
& + C_o.2.t_{l_4} + C_{t_4}.\Big((3-1).(0.1-0.0826) \\
& + (15-1).(0.5-0.1813+0.1214+0.0435+0.0748+0.0453)\Big) \\
= \; & (0.5)\,(30)\,(\frac{1}{60})(19.931+10.682+9.443+5.1+6.966+18.062+2.5) \\
& + (0.5)\,(\frac{1}{60})(17.806+17.755+2.5) \\
& + (0.5)\,(16)\,(0.75) + (0.5)\,(2)\,(0.75) \\
& + (0.70)\Big(2.(0.0174)+14.(0.0337)\Big) \\
= \; & \$27.53
\end{aligned}$$

Similarly:

- **Alternative** A_4^3 : $C_{A_4^3} = \$48.20$
- **Alternative** A_4^4 : $C_{A_4^4} = \$58.78$
- **Alternative** A_4^5 : $C_{A_4^5} = \$65.91$

⋆ For Tool-7 and Tool-9 :

For these tool types, we do not need to evaluate their alternatives since there is only one alternative for each and their arrangement into tool magazine have been already determined in **Step 4**.

Step 6 : There exists only one single tool type, Tool-4, whose tool magazine arrangement decision is to be made at this step. Therefore the 0-1 IP model presented in the original algorithm is not needed to make a decision, rather by inspecting the resulting cost of every alternative for this particular tool type we pick the best alternative giving a minimum cost value. As it is shown in the previous step, minimum cost arrangement is found at the minimum tool slot requirement, which is Alternative A_4^2, and this implies that use this particular arrangement for Tool-4 and leave some empty slots in the magazine.

Final tool arrangement is as follows:

- For Tool-4 use two duplicates: $T_4^3 = \{1\}$, $T_4^{15} = \{4, 6, 8, 9, 10\}$
- For Tool-5 use a single tool: $T_5^{10} = \{2\}$
- For Tool-7 use two duplicates: $T_7^2 = \{3\}$, $T_7^2 = \{5, 7\}$
- For Tool-9 use a single tool: $T_9^2 = \{11, 12\}$

5.3.3 Operations Sequencing Algorithm

For the details of this algorithm refer to the Section 4.8.

Step 1 : The tools having *Tool Sharing* with their operation assignment sets:

$$T_4^{15} = \{4, 6, 8, 9, 10\},\ T_7^2 = \{5, 7\},\ \text{and } T_9^2 = \{11, 12\}$$

Step 1.1 : The operations sequence for each chain has been found in the previous algorithm. These chains and the corresponding aggregated volumes are presented with the related geometrical data (see Figure 5.1 and Table 5.1), as follows:

⋆ <u>For Tool-4 :</u>
Op. (4,4) → Op. (6,4) → Op. (9,4) → Op. (8,4) → Op. (10,4)
Defining aggregated machinable volume Volume-4^+ with:
Starting point : (13,1.8,0), by V_4; Ending point : (6,1.05,0), by V_{10}

⋆ <u>For Tool-7 :</u>
Op. (5,7) → Op. (7,7)
Defining aggregated machinable volume Volume-5^+ with:
Starting point : (6,1.55,0), by V_5; Ending point : (4,1.3,0), by V_7

* ⋆ For Tool-9 :

  Op. (11,9) → Op. (12,9)

  Defining aggregated machinable volume Volume-11^+ with:

  Starting point : (13,1.05,0), by V_{11}; Ending point : (6,0.8,0), by V_{12}

Step 1.2 : Updated precedence relationship matrix, R^+ for the recent machinable volume set $I^+ = \{1, 2, 3, 4^+, 5^+, 11^+\}$:

Precedence Relationship Matrix:

$$R^+ = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 2 : Machinable volumes set for State 1 is composed of the operations which have no predecessors, so they are the possible starting operations:

$$S_1 = \{1, 2\}$$

Step 3 : Cost of the arcs from State s to State 1:

$$\begin{aligned} c_{s,1} &= t_{c_4} + t_{r_{f,1}} \\ &= 5 + 7.1479 = 12.1279 \text{ sec.} \end{aligned} \qquad \begin{aligned} c_{s,2} &= t_{c_5} + t_{r_{f,2}} \\ &= 5 + 8.1635 = 13.1635 \text{ sec.} \end{aligned}$$

Step 4 : Intermediate States are presented as follows:

$$S_2 = \{1, 2, 3, 4^+\},\ S_3 = \{1, 2, 3, 4^+, 5^+, 11^+\},\ S_4 = \{1, 3, 4^+, 5^+, 11^+\}$$

$$S_5 = \{1, 3, 5^+, 11^+\}, \text{ and } S_6 = \{3, 5^+, 11^+\}$$

Steps 5,6 & 7: Cost of arcs between the intermediate states depends on whether these two successive operation uses the same tool or not. If they require different tools then tool magazine should be visited for a tool interchange, otherwise, either we may continue to removal of machining volumes of the same tool or we may stop using this tool by leaving its remaining operations and install another tool. In this example, all the tools are assigned to, either a single aggregated operation, or a single basic machinable volume.

In Table 5.8, the cost of arcs between every nodes are presented and an infinity value is given for the infeasible arcs.

	f	V_1	V_2	V_3	V_4^+	V_5^+	V_{11}^+
f	∞	12.1479	13.1635	∞	∞	∞	∞
V_1	∞	∞	25.2009	24.1797	25.1963	24.3177	25.1832
V_2	∞	24.2957	∞	24.2902	25.3068	∞	25.2938
V_3	12.0317	24.1796	25.1952	∞	25.1906	24.3120	25.1776
V_4^+	∞	24.4190	∞	24.4134	∞	24.5514	25.4170
V_5^+	12.1310	24.2789	∞	24.2733	∞	∞	25.2769
V_{11}^+	12.2678	24.4157	∞	24.4101	∞	24.5481	∞

Table 5.8: Total Tool Interchanging and Rapid Travel Motion Times between the Machinable Volumes

By using the above table, one of the alternative operations sequences and its time value is given as follows:

$$f \rightarrow V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_4^+ \rightarrow V_5^+ \rightarrow V_{11}^+ \rightarrow f$$

$$\begin{aligned} P_1 &= 12.1479 + 25.2009 + 24.2902 + 25.1906 + 24.5514 + 25.2769 + 12.2678 \\ &= 148.9257 \text{ sec.} \end{aligned}$$

5.4 Conclusion

In this chapter, the proposed approach to our problem has been applied on an example problem. At the first level, tool allocation decision was made by using the cost measure developed for multiple operations case and it has been observed that this measure performed better than the results of SMOP, since the resulting non-machining time and tool waste costs are ignored in the case of single operation. Further, tool availability constraint has been imposed on the SMOP and at that point our geometric programming model was particularly useful, since it allows to impose this constraint on the problem by a simple parameter revision and reasonable computational burden of this method made the tool allocation algorithm effective for searching all possibilities. At the second level, tool magazine arrangement has been fixed by imposing the tool magazine capacity and tool availability constraints on the problem, and maximizing the tool sharing events. The importance of tool sharing has been identified due to its reducing effect on every non-machining time component and it also provided a reduced problem for the operations sequencing level since the operations of every tool sharing event has been aggregated into a single machinable volume. Actually, at the third level, no sequencing decision has left to be made, due to the reduced machinable volume representation. Because, at the second level, every tool of the magazine has been assigned to a single aggregated machinable volume or a single basic machinable volume.

Chapter 6

CONCLUSION

In this study, a hierarchal approach was presented to solve machining conditions optimization, tool allocation, tool magazine arrangement and operations sequencing problems for the manufacturing of a batch of parts on a CNC lathe. The proposed decision hierarchy has three levels. At the first level, the tool–operation pairs have been determined by relaxing the tool magazine capacity constraint and assuming no tool sharing. At the second level, the magazine arrangement was fixed by imposing the tool magazine capacity constraint and precedence relationships on the problem, and tool sharing has been taken into account. Finally, at the third level, operations sequence was determined by using the output of the previous stages.

The proposed heuristic method was applied on an example problem. At the first level, we saw that our cost measure has given a lower cost measure with respect to result of SMOP. Furthermore, the effect of the tool availability constraint has been also observed at this level. At the second level, tool magazine arrangement was determined in terms of the number of duplicates and their operation assignment sets by considering the tool sharing. At this level, we concluded that an emphasis should be placed on the tool sharing due to its impact on the reduction of non-cutting time components and decreasing tool slot requirement. Besides, tool sharing has resulted in a simplification in the operations sequencing part of the hierarchy, since it provided some aggregations in the machinable volume presentation. Finally, at the last level, we arrived at a special case in which we were indifferent about the operations sequence because of the simplification of tool sharing via machinable volume aggregation.

In this study, we have combined three subproblems of the literature into a single body at the equipment level and investigated their interactions. This model includes tool life covering, tool magazine capacity, tool availability, and machining conditions constraints which have been usually ignored at the FMS tactical level planning models, which only consider the batching, loading and routing problems for the sake of simplicity. Furthermore, we also considered tool sharing concept at this level, again frequently excluded at the system level due to its non-linear nature.

Tool life covering constraint has been imposed on the well-known single machining operation optimization problem, as an additional constraint. This constraint is particularly important since it guarantees that the failure of the cutting tool during the machining operation will be avoided. Besides, we have shown that this constraint is particularly useful for multiple operations case.

For solving the SMOP, Geometric Programing combined with Analytical Approach was implemented as an exact method. In the analysis of this extended SMOP, we have proven that both surface roughness and machine power constraints cannot be loose at the optimality.

Furthermore, a cost measure was constructed for extending the results of SMOP to the multiple operations case by considering the possible non-machining terms. This cost measure is a tool to investigate the trade-offs among different alternatives resulting from the interactions of subproblems.

In the tool magazine arrangement algorithm, a 0-1 IP model was constructed for embedding tool sharing concept into our decision hierarchy.

For the future research, extension of this hierarchy to other CNC machine tools can be studied such as milling machine. As it was mentioned before, this model is considered as part of a fully automated process planning system or as a part of the Decision Support System (DSS) proposed by Suri [34]. Therefore, interfacing this model with the other modules is an important problem to be studied. In this study, we have assumed a machinable volume representation as an input, however creating this representation is also another problem to be attempted by extending the given solution procedure for SMOP to the case of multiple passes metal cutting.

Appendix A

THEORY of the GEOMETRIC PROGRAMMING

Geometric programming is technique developed for solving nonlinear programming problems subject to nonlinear constraints. This technique first suggested by Clarence Zener and the governing duality theory was developed by Richard Duffin. The original mathematical development of this new method used the arithmetic-geometric mean inequality relationship between the sums and products of positive numbers. It was this theoretical development that prompted Duffin to call this technique "Geometric Programming" [6]. This relation between the arithmetic-geometric is expressed as follows:

$$\prod_{t=1}^{T} c_t.p_t \geq \sum_{t=1}^{T} \left(\frac{c_t.p_t}{w_t}\right)^{w_t}$$

where c is a coefficient, p is a function and w is an associated weight, where $0 \leq w_t \leq 1$.

The generalized Geometric Programming Problem is stated as follows:

$$\text{Minimize} \quad y_o(\mathbf{x}) = \sum_{t=1}^{T_o} \left(\sigma_{ot}.c_{ot}. \prod_{n=1}^{N} x_n^{ato} \right)$$

Subject to:

$$y_m(\mathbf{x}) = \sum_{t=1}^{T_m} \sigma_{mt}.c_{mt}. \prod_{n=1}^{N} x_n^{a_{mtn}} \leq \sigma_m \ ,$$

$$m = 1, 2, \ldots, M$$

$$x_n > 0 \ , \ n = 1, 2, \ldots, N$$

where,

$$\sigma_{mt} = \pm 1 \ , \ \sigma_{ot} = \pm 1 \ , \text{ and } \sigma_m = \pm 1 \ ,$$

$$c_{mt} > 0 \ , \ c_{ot} > 0$$

$$T_m = \text{ number of terms in the } m^{th} \text{ constraint}$$

$$T_o = \text{ number of terms in the objective function}$$

In terms of engineering design formulations, the c_{mt} are economic coefficients, the x_n are design decision variables, the a_{mtn} are technological exponents of the decision variables, and the σ vector has as elements of binary variables (± 1), whose signs represent the sign of each term and inequality in the problem statement. If all the σ are positive, the problem is called a *posynomial* geometric programming problem, which corresponds to our case in this study; if one or more of the σ are negative, the problem is called a *signomial* geometric programming problem.

The following system is derived as a dual geometric program for posynomial primal problem:

$$\text{Minimize} \quad d_o(w) = \prod_{m=0}^{M} \prod_{t=1}^{T_m} \left(\frac{c_{mt}.w_{mo}}{w_{mt}} \right)^{w_{mt}}$$

Subject to:

Normality Condition:

$$\sum_{t=1}^{T_o} w_{ot} = 1$$

Orthogonality Conditions:

$$\sum_{m=1}^{M} \sum_{t=1}^{T_m} a_{mtn}.w_{mt} = 0 \ , \ n = 1, 2, \ldots, N$$

$$w_{mo} = \sum_{t=1}^{T_m} w_{mt} \ , \ m = 1, 2, \ldots, M$$

The difference between the number of variables and the number of independent linear equations determines the number of *degrees of freedom*. Duffin and Zener suggest calling this quantity the number of *degrees of difficulty*, since the larger this number, the harder the problem to solve. For the constrained posynomial problem this number is determined by the following expression:

$$\text{Degrees of Difficulty} = \left(\sum_{i=1}^{M} T_i + T_o \right) - (N + 1)$$

Appendix B

TABLES of the TOOL ALLOCATION ALGORITHM

T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
4	1	6	285.01	0.02853	0.3862	4.6733	0.0826	0.2509	0.0117	3	8.79
5	1	6	252.10	0.03252	0.3831	2.8184	0.1359	0.2867	0.1361	5	10.74
6	1	6	324.93	0.01770	0.5458	3.2034	0.1704	0.3752	0.4441	6	14.70
7	1	6	286.08	0.02548	0.4308	7.1731	0.0601	0.2604	0.0294	2	9.09
8	1	6	371.12	0.01921	0.4404	2.5098	0.1755	0.3079	0.3067	6	12.54
10	1	6	385.10	0.02132	0.3825	1.9249	0.1987	0.3105	0.0195	6	11.71

Table B.1: Volume 1, $\bar{p}_{ij} = 1$

T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
4	1	6	256.73	0.03189	1.1506	5.9650	0.1929	0.7103	0.1245	6	23.81
5	1	6	226.92	0.03627	1.1445	3.7804	0.3027	0.7842	0.5782	10	27.98
6	1	6	291.92	0.01998	1.6149	4.2464	0.3803	1.0356	2.0110	15	40.58
7	1	6	257.69	0.02860	1.2779	9.0849	0.1407	0.7445	0.0460	5	25.13
8	1	6	332.62	0.02158	1.3124	3.3102	0.3965	0.8545	1.4491	15	34.58
10	1	6	345.45	0.02397	1.1375	2.5269	0.4502	0.8388	0.8374	15	31.75

Table B.2: Volume 2, $\bar{p}_{ij} = 1$

T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
1	1	1	720.06	0.01736	0.2260	1.4342	0.1576	0.1918	0.1087	5	7.86
2	1	1	513.07	0.01572	0.3503	2.2983	0.1524	0.2819	0.2393	5	10.69
3	1	1	764.71	0.01272	0.2906	2.3188	0.1253	0.2581	0.4424	5	10.06
4	1	1	540.34	0.03055	0.1712	1.7209	0.0995	0.1552	0.0075	3	5.91
7	1	1	577.31	0.03025	0.1618	1.8315	0.0884	0.1472	0.0420	3	6.21

Table B.3: Volume 3, $\bar{p}_{ij} = 1$

T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
4	1	6	242.92	0.02747	1.2706	7.0095	0.1813	0.7622	0.3279	6	25.57
5	1	6	217.84	0.03099	1.2560	4.3124	0.2913	0.8319	0.7952	10	29.63
6	1	6	276.56	0.01739	1.7627	4.9669	0.3549	1.0943	2.4379	15	42.77
7	1	6	243.72	0.02449	1.4207	10.8130	0.1314	0.8089	0.2409	5	27.26
8	1	6	313.39	0.01894	1.4283	3.8403	0.3719	0.9001	1.7929	15	36.30
10	1	6	323.04	0.02080	1.2615	3.0576	0.4126	0.8783	1.4685	15	33.57

Table B.4: Volume 4, $\bar{p}_{ij} = 1$

T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
4	1	6	269.68	0.02457	0.2448	5.4916	0.0446	0.1536	0.0134	2	5.50
5	1	6	242.00	0.02778	0.2413	3.2150	0.0751	0.1732	0.0338	3	6.48
6	1	6	307.83	0.01541	0.3420	3.7469	0.0913	0.2258	0.1046	3	8.38
7	1	6	270.56	0.02181	0.2749	8.5375	0.0322	0.1616	0.0000	1	5.60
8	1	6	349.65	0.01686	0.2751	2.9118	0.0945	0.1848	0.0551	3	7.10
10	1	6	360.13	0.01850	0.2435	2.3292	0.1045	0.1845	0.1063	4	7.27

Table B.5: Volume 5, $\bar{p}_{ij} = 1$

T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
4	1	6	242.92	0.02747	0.8510	7.0095	0.1214	0.5105	0.0605	4	17.00
5	1	6	217.84	0.03099	0.8412	4.3124	0.1951	0.5572	0.0862	6	19.18
6	1	6	276.56	0.01739	1.1806	4.9669	0.2377	0.7329	0.2069	8	26.19
7	1	6	243.72	0.02449	0.9515	10.8130	0.0880	0.5418	0.0481	3	18.05
8	1	6	313.39	0.01894	0.9566	3.8403	0.2491	0.6029	0.0126	8	22.10
10	1	6	323.04	0.02080	0.8449	3.0576	0.2763	0.5883	0.9234	10	22.45

Table B.6: Volume 6, $\bar{p}_{ij} = 1$

T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
1	1	1	789.76	0.01333	0.1292	1.4342	0.0901	0.1096	0.0090	3	4.55
2	1	1	563.79	0.01220	0.1978	2.2983	0.0860	0.1591	0.0749	3	6.10
3	1	1	839.62	0.00975	0.1662	2.3188	0.0717	0.1476	0.1224	3	5.68
4	1	1	589.34	0.02377	0.0971	1.7209	0.0564	0.0881	0.0285	2	3.55
7	1	1	631.51	0.02343	0.0920	1.8315	0.0502	0.0836	0.0345	2	3.79
9	1	1	626.07	0.01886	0.1153	2.5721	0.0448	0.0912	0.0107	2	3.50

Table B.7: Volume 7, $\bar{p}_{ij} = 1$

T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
4	1	6	242.92	0.02747	0.3059	7.0095	0.0436	0.1835	0.0280	2	6.41
5	1	6	217.84	0.03099	0.3024	4.3124	0.0701	0.2003	0.0257	3	7.28
6	1	6	276.56	0.01739	0.4244	4.9669	0.0854	0.2634	0.0722	3	9.48
7	1	6	243.72	0.02449	0.3420	10.8130	0.0316	0.1947	0.0000	1	6.59
8	1	6	313.39	0.01894	0.3439	3.8403	0.0895	0.2167	0.0151	3	8.02
10	1	6	323.04	0.02080	0.3037	3.0576	0.0993	0.2114	0.0081	3	7.60

Table B.8: Volume 8, $\bar{p}_{ij} = 1$

T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	C_{ij}
4	1	6	269.68	0.02457	0.4107	5.4916	0.0748	0.2577	0.0389	3	9.02
5	1	6	242.00	0.02778	0.4048	3.2150	0.1259	0.2906	0.3320	5	11.05
6	1	6	307.83	0.01541	0.5737	3.7469	0.1531	0.3787	0.1951	5	14.06
7	1	6	270.56	0.02181	0.4611	8.5375	0.0540	0.2711	0.0208	2	9.40
8	1	6	349.65	0.01686	0.4615	2.9118	0.1585	0.3100	0.0980	5	11.90
10	1	6	360.13	0.01850	0.4085	2.3292	0.1754	0.3095	0.3694	6	12.03

Table B.9: Volume 9, $\bar{p}_{ij} = 1$

T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	C_{ij}
4	1	6	269.68	0.02457	0.2488	5.4916	0.0453	0.1561	0.0023	2	5.56
5	1	6	242.00	0.02778	0.2452	3.2150	0.0763	0.1760	0.0118	3	6.54
6	1	6	307.83	0.01541	0.3475	3.7469	0.0928	0.2294	0.0869	3	8.47
7	1	6	270.56	0.02181	0.2793	8.5375	0.0327	0.1642	0.0000	1	5.68
8	1	6	349.65	0.01686	0.2796	2.9118	0.0960	0.1878	0.0399	3	7.17
10	1	6	360.13	0.01850	0.2474	2.3292	0.1062	0.1875	0.0790	4	7.33

Table B.10: Volume 10, $\bar{p}_{ij} = 1$

T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	C_{ij}
1	1	1	830.95	0.01153	0.2294	1.4342	0.1599	0.1947	0.0807	5	7.92
2	1	1	593.81	0.01062	0.3487	2.2983	0.1517	0.2805	0.2512	5	10.67
3	1	1	883.93	0.00842	0.2953	2.3188	0.1273	0.2623	0.3909	5	10.13
9	1	1	659.02	0.01655	0.2015	2.5721	0.0784	0.1595	0.0896	3	6.00

Table B.11: Volume 11, $\bar{p}_{ij} = 1$

T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	C_{ij}
1	1	1	887.25	0.00956	0.1480	1.4342	0.1032	0.1256	0.1065	4	5.50
2	1	1	634.89	0.00887	0.2231	2.2983	0.0971	0.1795	0.0413	3	6.68
3	1	1	944.52	0.00697	0.1907	2.3188	0.0822	0.1694	0.0235	3	6.23
9	1	1	704.08	0.01399	0.1276	2.5721	0.0496	0.1010	0.0061	1	3.79

Table B.12: Volume 12, $\bar{p}_{ij} = 1$

$\bar{n}_{t_{ij}}$	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{C}_{ij}$
3	1	1	659.02	0.01655	0.2015	2.5721	0.0784	0.1595	0.0896	6.00
2	15	4	633.60	0.01567	0.2214	3.3217	0.0667	0.1607	0.0000	5.57
1	30	4	535.20	0.01238	0.3318	9.9528	0.0333	0.1909	0.0000	6.10

Table B.13: Finding the Minimum Cost Measure for Operation (11,9)

	T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
1	4	12	6	285.01	0.02853	0.3862	4.6733	0.0826	0.2509	0.0117	3	8.79
2	7	16	6	286.08	0.02548	0.4308	7.1731	0.0601	0.2604	0.0294	2	9.09
3	5	10	4	231.15	0.02810	0.4834	4.8338	0.1000	0.3117	0.0000	3	10.60
4	10	6	4	363.90	0.01955	0.4413	2.6476	0.1667	0.3206	0.0000	5	11.62
5	8	6	4	365.02	0.01873	0.4592	2.7551	0.1667	0.3129	0.0000	5	11.89
6	6	6	4	322.92	0.01754	0.5545	3.3271	0.1667	0.3773	0.0000	5	13.82

Table B.14: Alternative Tools of Volume 1

	T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
1	4	5	6	256.73	0.03189	1.1506	5.9650	0.1929	0.7103	0.1245	6	23.81
2	7	7	6	257.69	0.02860	1.2779	9.0849	0.1407	0.7445	0.0460	5	25.13
3	5	3	6	226.92	0.03627	1.1445	3.7804	0.3027	0.7842	0.5782	10	27.98
4	10	3	4	313.59	0.02068	1.4524	4.3572	0.3333	0.9262	0.0000	10	31.66
5	8	3	4	314.55	0.01982	1.5114	4.5341	0.3333	0.9223	0.0000	10	32.67
6	6	3	4	281.32	0.01887	1.7741	5.3224	0.3333	1.0871	0.0000	10	37.61

Table B.15: Alternative Tools of Volume 2

	T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
2	7	15	4	528.39	0.02624	0.2038	3.0575	0.0667	0.1519	0.0000	2	5.81
3	4	15	4	475.57	0.02507	0.2370	3.5554	0.0667	0.1652	0.0000	2	5.83
4	1	10	4	634.71	0.01434	0.3104	3.1042	0.1000	0.2052	0.0000	3	7.41
5	3	10	4	711.51	0.01147	0.3462	3.4616	0.1000	0.2631	0.0000	3	9.02
6	2	10	4	464.68	0.01343	0.4529	4.5287	0.1000	0.2964	0.0000	3	10.14

Table B.16: Alternative Tools of Volume 3

	T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
1	4	5	6	242.92	0.02747	1.2706	7.0095	0.1813	0.7622	0.3279	6	25.57
2	7	8	4	239.93	0.02388	1.4799	11.8389	0.1250	0.8337	0.0000	4	27.26
3	5	4	4	208.63	0.02882	1.4102	5.6409	0.2500	0.8801	0.0000	8	29.53
4	10	3	4	301.60	0.01873	1.5005	4.5016	0.3333	0.9503	0.0000	10	32.38
5	8	3	4	302.52	0.01795	1.5615	4.6844	0.3333	0.9474	0.0000	10	33.42
6	6	3	4	271.74	0.01693	1.8433	5.5299	0.3333	1.1216	0.0000	10	38.65

Table B.17: Alternative Tools of Volume 4

	T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
1	4	22	6	269.68	0.02457	0.2448	5.4916	0.0446	0.1536	0.0134	2	5.50
2	7	30	6	270.56	0.02181	0.2749	8.5375	0.0322	0.1616	0.0000	1	5.60
3	5	15	4	234.02	0.02625	0.2641	3.9608	0.0667	0.1787	0.0000	2	6.24
4	10	10	4	355.01	0.01810	0.2525	2.5248	0.1000	0.1862	0.0000	3	6.84
5	8	10	6	349.65	0.01686	0.2751	2.9118	0.0945	0.1848	0.0551	3	7.10
6	6	11	4	307.48	0.01538	0.3430	3.7734	0.0909	0.2261	0.0000	3	8.28

Table B.18: Alternative Tools of Volume 5

	T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
1	4	8	6	242.92	0.02747	0.8510	7.0095	0.1214	0.5105	0.0605	4	17.00
2	7	11	6	243.72	0.02449	0.9515	10.8130	0.0880	0.5418	0.0481	3	18.05
3	5	5	6	217.84	0.03099	0.8412	4.3124	0.1951	0.5572	0.0862	6	19.18
4	10	4	4	312.79	0.01980	0.9166	3.6665	0.2500	0.6083	0.0000	8	21.37
5	8	4	6	313.39	0.01894	0.9566	3.8403	0.2491	0.6029	0.0126	8	22.10
6	6	4	6	276.56	0.01739	1.1806	4.9669	0.2377	0.7329	0.2069	8	26.19

Table B.19: Alternative Tools of Volume 6

	T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
1	9	30	4	582.56	0.01705	0.1370	4.1087	0.0333	0.0935	0.0000	1	3.18
2	7	30	4	555.22	0.01905	0.1286	3.8584	0.0333	0.0893	0.0000	1	3.43
3	4	30	4	498.20	0.01833	0.1490	4.4712	0.0333	0.0979	0.0000	1	3.44
4	1	15	4	726.49	0.01175	0.1594	2.3908	0.0667	0.1130	0.0000	2	4.27
5	3	15	4	820.34	0.00943	0.1759	2.6383	0.0667	0.1479	0.0000	2	5.19
6	2	15	4	530.98	0.01109	0.2310	3.4652	0.0667	0.1622	0.0000	2	5.74

Table B.20: Alternative Tools of Volume 7

	T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
1	4	23	4	242.63	0.02742	0.3068	7.0561	0.0435	0.1838	0.0000	2	6.39
2	7	30	6	243.72	0.02449	0.3420	10.8130	0.0316	0.1947	0.0000	1	6.59
3	5	15	4	214.75	0.03025	0.3142	4.7125	0.0667	0.2038	0.0000	2	6.99
4	10	10	6	323.04	0.02080	0.3037	3.0576	0.0993	0.2114	0.0081	3	7.60
5	8	11	6	313.39	0.01894	0.3439	3.8403	0.0895	0.2167	0.0151	3	8.02
6	6	12	4	274.63	0.01720	0.4320	5.1836	0.0833	0.2660	0.0000	3	9.48

Table B.21: Alternative Tools of Volume 8

	T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
1	4	13	6	269.68	0.02457	0.4107	5.4916	0.0748	0.2577	0.0389	3	9.02
2	7	18	6	270.56	0.02181	0.4611	8.5375	0.0540	0.2711	0.0208	2	9.40
3	5	8	4	241.51	0.02768	0.4071	3.2565	0.1250	0.2910	0.0000	4	10.36
4	10	6	4	354.27	0.01804	0.4257	2.5544	0.1667	0.3129	0.0000	5	11.39
5	8	6	6	349.65	0.01686	0.4615	2.9118	0.1585	0.3100	0.0980	5	11.90
6	6	6	6	307.83	0.01541	0.5737	3.7469	0.1531	0.3787	0.1951	5	14.06

Table B.22: Alternative Tools of Volume 9

	T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	$\bar{C}_{ij}$
1	4	22	6	269.68	0.02457	0.2488	5.4916	0.0453	0.1561	0.0023	2	5.56
2	7	30	6	270.56	0.02181	0.2793	8.5375	0.0327	0.1642	0.0000	1	5.68
3	5	15	4	232.97	0.02606	0.2716	4.0738	0.0667	0.1825	0.0000	2	6.35
4	10	11	4	342.51	0.01714	0.2809	3.0895	0.0909	0.1950	0.0000	3	7.10
5	8	10	6	349.65	0.01686	0.2796	2.9118	0.0960	0.1878	0.0399	3	7.17
6	6	11	4	306.10	0.01527	0.3526	3.8783	0.0909	0.2308	0.0000	3	8.42

Table B.23: Alternative Tools of Volume 10

	T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	C_{ij}
1	9	15	4	633.60	0.01567	0.2214	3.3217	0.0667	0.1607	0.0000	2	5.57
2	1	10	4	729.47	0.00947	0.3183	3.1826	0.1000	0.2091	0.0000	3	7.52
3	3	10	4	818.20	0.00754	0.3562	3.5623	0.1000	0.2681	0.0000	3	9.17
4	2	10	4	538.40	0.00908	0.4495	4.4947	0.1000	0.2947	0.0000	3	10.09

Table B.24: Alternative Tools of Volume 11

	T#	$\bar{p}_{ij}$	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$C_{w_{ij}}$	$\bar{n}_{t_{ij}}$	C_{ij}
1	9	30	4	639.16	0.01222	0.1608	4.8244	0.0333	0.1054	0.0000	1	3.54
2	1	15	4	785.93	0.00796	0.2008	3.0122	0.0667	0.1337	0.0000	2	4.89
3	3	15	4	883.21	0.00634	0.2245	3.3670	0.0667	0.1722	0.0000	2	5.92
4	2	15	4	581.26	0.00771	0.2804	4.2059	0.0667	0.1869	0.0000	2	6.48

Table B.25: Alternative Tools of Volume 12

π	T#	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$\bar{n}_{t_{ij}}$	C_{ij}	$\bar{n}^{\pi}_{t_{ij}}$	ΔC^{π}_{i}
0	9	4	582.56	0.01705	0.1370	4.1087	0.0333	1	3.18	1	0
1	7	4	555.22	0.01905	0.1286	3.8584	0.0333	1	3.43	0	0.25
2	4	4	498.20	0.01833	0.1490	4.4712	0.0333	1	3.44	0	0.26

Table B.26: Perturbations of Operation(7,9)

π	T#	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$\bar{n}_{t_{ij}}$	C_{ij}	$\bar{n}^{\pi}_{t_{ij}}$	ΔC^{π}_{i}
0	9	4	633.60	0.01567	0.2214	3.3217	0.0667	2	5.57	2	0
1	9	4	535.20	0.01238	0.3318	9.9528	0.0333	1	6.10	1	0.53
2	1	4	651.89	0.00799	0.4222	6.3335	0.0667	2	8.21	0	2.64

Table B.27: Perturbations of Operation(11,9)

π	T#	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$\bar{n}_{t_{ij}}$	C_{ij}	$\bar{n}^{\pi}_{t_{ij}}$	ΔC^{π}_{i}
0	9	4	639.16	0.01222	0.1608	4.8244	0.0333	1	3.54	1	0
1	1	4	785.93	0.00796	0.2008	3.0122	0.0667	2	4.89	0	1.35

Table B.28: Perturbations of Operation(12,9)

π	T#	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$\bar{n}_{t_{ij}}$	C_{ij}	$\bar{n}^{\pi}_{t_{ij}}$	ΔC^{π}_{i}
0	4	6	285.01	0.02853	0.3862	4.6733	0.0826	3	8.79	3	0
1	4	4	266.13	0.02565	0.4599	6.8990	0.0667	2	9.17	2	0.38
2	5	4	231.15	0.02810	0.4834	4.8338	0.1000	2	10.60	0	1.81
3	7	4	237.76	0.01893	0.6976	20.9281	0.0333	1	11.96	0	3.17

Table B.29: Perturbations of Operation(1,4)

π	T#	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$\bar{n}_{t_{ij}}$	C_{ij}	$\bar{n}^{\pi}_{t_{ij}}$	ΔC^{π}_{i}
0	4	6	256.73	0.03189	1.1506	5.9650	0.1929	6	23.81	6	0
1	4	4	245.03	0.02967	1.2957	7.7745	0.1667	5	24.94	5	1.13
2	5	6	226.92	0.03627	1.1445	3.7804	0.3027	10	27.98	0	2.37
3	4	4	223.54	0.02574	1.6374	13.0994	0.1250	4	28.81	4	5

Table B.30: Perturbations of Operation(2,4)

π	T#	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$\bar{n}_{t_{ij}}$	C_{ij}	$\bar{n}^{\pi}_{t_{ij}}$	ΔC^{π}_{i}
0	4	6	242.92	0.02747	1.2706	7.0095	0.1813	6	25.57	6	0
1	4	4	236.50	0.02635	1.3604	8.1623	0.1667	5	25.91	5	0.34
2	5	4	208.63	0.02882	1.4102	5.6409	0.2500	8	29.53	0	3.96
3	4	4	215.75	0.02286	1.7191	13.7527	0.1250	4	30.04	4	4.47

Table B.31: Perturbations of Operation(4,4)

π	T#	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$\bar{n}_{t_{ij}}$	C_{ij}	$\bar{n}^{\pi}_{t_{ij}}$	ΔC^{π}_{i}
0	4	6	269.68	0.02457	0.2448	5.4916	0.0446	2	5.50	2	0
1	7	6	270.56	0.02181	0.2749	8.5375	0.0322	1	5.60	0	0.10
2	4	4	245.79	0.02128	0.3102	9.3053	0.0333	1	5.85	1	0.35

Table B.32: Perturbations of Operation(5,4)

π	T#	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$\bar{n}_{t_{ij}}$	C_{ij}	$\bar{n}^{\pi}_{t_{ij}}$	ΔC^{π}_{i}
0	4	6	242.92	0.02747	0.8510	7.0095	0.1214	4	17.00	4	0
1	4	4	228.34	0.02496	0.9964	9.9639	0.1000	3	18.30	3	1.30
2	5	6	217.84	0.03099	0.8412	4.3124	0.1951	6	19.18	0	2.18
3	4	4	200.63	0.02042	1.3857	20.7862	0.0667	2	23.06	2	6.06

Table B.33: Perturbations of Operation(6,4)

π	T#	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$\bar{n}_{t_{ij}}$	C_{ij}	$\bar{n}^{\pi}_{t_{ij}}$	ΔC^{π}_{i}
0	4	4	242.63	0.02742	0.3068	7.0561	0.0435	2	6.39	2	0
1	7	6	243.72	0.02449	0.3420	10.8130	0.0316	1	6.59	0	0.20
2	4	4	222.91	0.02404	0.3808	11.4244	0.0333	1	6.91	1	0.52

Table B.34: Perturbations of Operation(8,4)

π	T#	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$\bar{n}_{t_{ij}}$	C_{ij}	$\bar{n}^{\pi}_{t_{ij}}$	ΔC^{π}_{i}
0	4	6	269.68	0.02457	0.4107	5.4916	0.0748	3	9.02	3	0
1	4	4	259.98	0.02321	0.4509	6.7640	0.0667	2	9.04	2	0.02
2	5	4	241.51	0.02768	0.4071	3.2565	0.1250	4	10.36	0	1.34
3	4	4	208.39	0.01648	0.7925	23.7752	0.0333	1	13.09	1	4.07

Table B.35: Perturbations of Operation(9,4)

π	T#	Case	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$\bar{n}_{t_{ij}}$	C_{ij}	$\bar{n}^{\pi}_{t_{ij}}$	ΔC^{π}_{i}
0	4	6	269.68	0.02457	0.2488	5.4916	0.0453	2	5.56	2	0
1	7	6	270.56	0.02181	0.2793	8.5375	0.0327	1	5.68	0	0.12
2	4	4	244.53	0.02111	0.3193	9.5793	0.0333	1	5.99	1	0.43

Table B.36: Perturbations of Operation(10,4)

Appendix C

A LIST of NOTATIONS

α_j : Exponent of the cutting speed in Taylor's tool life expression for tool j

α_s : Acceleration of slides, (m/sec^2)

β_j : Exponent of the feed rate in Taylor's tool life expression for tool j

δ_j : Deficit tool amount for tool type j

Δ_{xy}: Relative distance between the points x and y

$\Delta\bar{C}_i^{\pi}$: Cost increment for the π^{th} perturbation of volume i

γ_j : Exponent of the depth of cut in Taylor's tool life expression for tool j

ρ_j : Required perturbation ratio for tool type j

a_j : Number of tool slots required for tool type j

a_j^n : Number of tool slots required for the alternative arrangement, A_j^n

b : Exponent of the cutting speed in machine power constraint

c : Exponent of the feed rate in machine power constraint

$c_{x,y}$: Cost of arc from node x to node y in operation sequencing graph

$C_{A_j^n}$: Cost of alternative tool arrangement, A_j^n, for tool type j

$\bar{C}_{ij}$: Cost measure proposed for Tool Allocation Algorithm

$\bar{C}_{ij}^{\pi}$: Resulting cost measure for the π^{th} perturbation of operation (i,j)

C_j : Taylor's tool life constant for tool j

C_m : Coefficient of the machine power constraint for a particular tool and volume pair

$C_{m_{ij}}$: Cost of machining found in SMOP, (\$)

C_o : Operating cost of the machine tool, ($\$/min$)

C_p : Penalty cost for additional tool interchanging events

C_s : Coefficient of the surface roughness constraint for a particular tool and volume pair

C_{t_j} : Cost of the tool j, ($\$/per\ tool$)

C_{tm} : Total manufacturing cost for a particular batch, ($\$$)

C_{w_j} : Waste tool cost for tool type j

$d_{A_j^n}$: A 0-1 binary variable which is equal to 1 if the n^{th} alternative arrangement of tool j is selected

d_i : Depth of cut for operation i, (mm)

D_i : Diameter of the generated surface, (mm)

D_n : Dual variables

e : Exponent of the depth of cut in machine power constraint

f_{ij} : Feed rate for operation i

g : Exponent of the cutting speed in surface roughness constraint

h : Exponent of the feed rate in surface roughness constraint

HP_{max} : Maximum allowable machine power for all operations

i : Exponent of the depth of cut in surface roughness constraint

I_j : Set of the possible operation assignments for the tool j

I_{A_j} : Set of adjacent operation pairs for the operation assignment set of tool j

J : Set of the available tools

J_i : Set of candidate tools for the volume i

J_F : Set of the primal tools

$\bar{J}$: Set of the allocated tools

$\check{J}$: Set of the arranged tools

k_{l_j} : Maximum number of duplicates that can be used at l_j tool requirement level

l^d : Tool requirement level at the d^{th} tool slot

L_i : Length of the generated surface, (mm)

l_j : Tool requirement level for tool type j

L_j : Set of possible tool requirement levels for tool type j

$L_{(i,i')}$: Set of possible tool candidate levels for the manufacturing of adjacent operation pair (i, i')

m_{lk} : A 0-1 binary variable which is equal to 1 if the k^{th} duplicate of tool requirement level l is located into tool magazine

n_j : Number of different arrangements of tool type j

N_B : Batch size

N_m : The capacity of the tool magazine

n_{n_k} : Number of occurrences for non-machining operation k in a batch

$\bar{n}_{s_j}$: Number of tool switching operations for tool j

$\bar{n}_{t_{ij}}$: Number of tools required for the machining of volume i by tool j

$\bar{n}^{\pi}_{t_{ij}}$: Number of tools required for the π^{th} perturbation of operation (i, j)

$\bar{n}_{t_j}$: Total number of tool j needed for complete machining of the batch

p^l_j : Number of parts can be manufactured at l_j tool requirement level

$\bar{p}_{ij}$: Number of parts can be manufactured by the removal of volume i by tool j

Q^* : Objective function of the dual problem

$r_{ii'}$: A 0-1 binary variable which is equal to 1 if operation i precedes to operation i', for every $i' \neq i \in I$

R_j : Total tool requirement for tool type j

s_{l^d} : Slack tool life at the d^{th} tool slot

S_j : Minimum number of tool slots required for tool type j

SF_{max_i} : Maximum allowable surface roughness for the volume i

SR_j : Set of the possible tool slot requirements for tool j

t_a : Tool approach time, (sec)

T_B : Tool Magazine loading time for a batch, (sec)

t_{c_j} : Tool changing time for tool j, (sec)

T_{ij} : Tool life of tool j for operation i, (min)

t_j : Number of available tools on hand for tool type j

t_{l_j} : Tool Magazine loading time for a single tool j, (sec)

t_{m_i} : Machining time for a single machining operation i, (min)

$t_{m_{ij}}$: Machining time for a single machining operation i with tool j, (min)

t_{n_k} : Total time spent for the non-machining operation k, (min)

$t_{r_{\bar{i}f}}$: Rapid motion time for moving the tool from the fixed point to the starting point of operation i, (sec)

$t_{r_{if}}$: Rapid motion time for moving the tool from the ending point of operation i to the fixed changing point, (sec)

$t_{r_{xy}}$: Rapid travel motion time from point x to point y, (sec)

T_{s_j} : Total tool switching time for tool j in a batch, (sec)

t_{s_j} : Tool switching time for tool j, (sec)

t_{st} : Settling or stabilization time for the slide, (sec)

t_{tm} : Total manufacturing time of the batch, (min)

U_{ij} : Usage rate of tool j in the operation i

$\bar{U}_{l_j}$: Maximum allowable tool usage rate at l_j tool requirement level

v_{ij} : Cutting speed for operation i, (m/min)

V_s : Speed of machine slides, (m/min)

w : Operations sequence vector

x_{ij} : A 0-1 binary variable,and it is equal to 1 if tool j is used in operation i

y_i^{π} : A 0-1 binary variable which is equal to 1 if π^{th} perturbation is selected for operation (i,j)

z_{ilk} : A 0-1 binary variable which is equal to 1 if the i^{th} operation is allocated to k^{th} duplicate of tool requirement level l

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