

**BUYBACK AND TARGET REBATE  
CONTRACTS WHEN THE  
MANUFACTURER OPERATES UNDER  
CARBON CAP AND TRADE MECHANISM**

A THESIS

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FOR THE DEGREE OF  
MASTER OF SCIENCE

By  
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July, 2014

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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## ABSTRACT

# BUYBACK AND TARGET REBATE CONTRACTS WHEN THE MANUFACTURER OPERATES UNDER CARBON CAP AND TRADE MECHANISM

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In this study, the coordination of a manufacturer and a retailer in a supply chain is considered, in a single period environment where the manufacturer has carbon emission restriction with trade option. The customer demand over a period is assumed to be a random variable with an arbitrary distribution. Two types of contracts, namely the buyback and the target rebate contracts are considered. For each type, the contract parameters which achieve channel coordination have been studied. The models show that for both contract types, under specific parameter settings coordination is achievable. In particular, we show that under a buyback contract with a carbon trader manufacturer, coordination can be achieved, even if no returns are allowed, contrary to the findings of Pasternack, who first studied a buyback contract in a setting where carbon emissions are not taken into consideration. The results are illustrated by numerical examples.

*Keywords:* Contracts, Newsvendor, Emission Restriction, Cap and Trade, Buyback, Target Rebate.

## ÖZET

# KARBON TİCARETİ MEKANİZMASI ALTINDA ÇALIŞAN BİR ÜRETİCİ İLE GERİ ALMA VE HEDEF SATIŞ İNDİRİMİ KONTRATLARIN ANALİZİ

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Bu çalışmada üreticinin karbon emisyon kısıtı ve karbon ticareti opsiyonu olduğu bir ticaret zinciri üreticisi ve perakendeci arasındaki tek dönemlik koordinasyon ele alınmıştır. Bir dönem içindeki talebin tesadüfi değişken olduğu varsayılmıştır. Geri alma ve hedef satış olmak üzere iki kontrat tipi incelenmiştir. Her kontrat tipi için koordinasyon sağlayan sistem parametreleri araştırılmıştır ve belli koşullar altında koordinasyonun sağlandığı gösterilmiştir. Geri alma kontratı altında, Pasternack'ın bulgularının tersine, geri almaya izin vermeyen özel durumlarda koordinasyon olduğu gösterilmiştir. Sonuçlara ilişkin numerik örnekler sunulmuştur.

*Anahtar sözcükler:* Kontratlar, Emisyon Kısıtı, Kota ve Ticaret, Geri Alım, Hedef Satış.

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# Chapter 1

## Introduction

### 1.1 Supply Chain Coordination

A supply chain is a complicated network that consists of a number of suppliers, distributors, retailers and the end customers, that operate under several uncertainties arising from customer demand, replenishment transportation lead times etc. The overall efficiency and performance of the system can be improved if the members of the supply chain collaborate and share information. The actions and strategies of each member depend on those of the others in a complex way. Supply chain coordination can be defined as recognizing the interdependent activities between the supply chain members and devise mechanisms to manage those interdependencies. Different profitability problems occurs in supply chains where coordination is ignored. Double marginalization and Bullwhip effect are two major problems encountered in uncoordinated supply chains. Double marginalization occurs when the retailer makes arbitrary decisions without considering the profit margin of supplier. In bullwhip effect, the supply chain members make ordering decision ignoring the others and as a result, the variability of demand is increased moving upstream.



timely, and accurate information available for decision makers. Information processing in organizations includes the gathering of data, the transformation of data into information, and the communication and storage of information [1].

Logistics synchronization refers to organizing the supply chain according to the market: to mediate customer demand and to adjust inventory management, production, and transportation to meet the demand. The synchronizing logistics processes is also called physical flow coordination. [2].

Among coordination mechanisms, contracts are valuable tools used in both theory and practice to coordinate various supply chains. According to Tsay [3], the supply chain contract is a coordination mechanism that provides incentives to all of its members so that the decentralized supply chain behaves nearly or exactly the same as the centralized one. By specifying contract parameters such as quantity, price and lead times, contracts are designed to improve supplier and buyer relationship. These contracts, based on their type, serve to the different objectives which include maximizing the total supply chain profit, minimizing the inventory related costs (overstock and understock) and fair risk sharing between the members.

Examples of contract types that can be used to achieve coordination, are returns (buyback), rebate, quantity discounts, quantity flexibility and the use of subsidies or penalties.

In the next chapter we will discuss each contract type with examples in literature. In this thesis, we will consider two specific contract types, namely the buyback contract and target rebate contract in a two echelon supply chain consisting of a manufacturer and a retailer when the manufacturer has carbon emission restrictions with trade option. Our research has been motivated by the fact that

1. global warming is becoming a real threat to the world, for which the greenhouse gases and especially  $CO_2$  are the most significant contributors.
2. There is a global attempt to reduce carbon emissions via several international regulations which lead to carbon trading.

3. As a result of the above two points new paradigms have been introduced to the classical problems in the IE/OR context.

To further motivate our problem we provide below some basic information regarding the international environmental activities to reduce carbon emissions.

Greenhouse gases from human activities are the most significant driver of climate change since the mid-20<sup>th</sup> century. As greenhouse gas emissions increase, they build up in the atmosphere and warm the climate, leading to many other changes around the world. The Kyoto Protocol to the United Nations Framework Convention on Climate Change (UNFCCC), agreed in 1997, is an amendment to the international treaty on climate change, assigning mandatory targets for the reduction of greenhouse gas. The UNFCCC's Kyoto Protocol authorizes three cooperative implementation mechanisms: emissions trading, joint implementation and the clean development mechanism (CDM).

The provision on emissions trading, allows trading of the assigned amounts of emission. Three distinct trading possibilities emerge from this authorization: trading among countries with domestic emissions trading systems, trading among countries without domestic trading systems, and trading among countries with and without domestic emission trading systems.

The mechanism known as "joint implementation," allows a country with an emission reduction or limitation commitment under the Kyoto Protocol to earn emission reduction units (ERUs) from an emission-reduction or emission removal project, each equivalent to one tonne of  $CO_2$ , which can be counted towards meeting its Kyoto target.

The clean development mechanism (CDM) allows emission-reduction projects in developing countries to earn certified emission reduction (CER) credits, each equivalent to one tonne of  $CO_2$ . These CERs can be traded and sold, and used by industrialized countries to meet a part of their emission reduction targets under the Kyoto Protocol.

EU Emissions Trading System (EU ETS), is the largest multi-country, multi-sector greenhouse gas emissions trading system in the world. The EU ETS scheme started in 2005 in order to help the EU meet its targets under the Kyoto Protocol (8% reduction in greenhouse gas emissions from 1990 levels). The EU ETS works on the 'cap and trade' principle. It puts a cap on the carbon dioxide ( $CO_2$ ) emitted by business and creates a market and price for carbon allowances. It covers 45% of EU emissions, including energy intensive sectors and approximately 12,000 installations. The 'Cap' was converted into allowances, known as EUAs (1 tonne of Carbon Dioxide = 1 EUA). The carbon price signifies the amount participants in the EU ETS are willing to pay per EU allowance (1 allowance (EUA) equals 1 tonne of  $CO_2$  or its equivalent) given demand and supply. In addition, EU Aviation allowances (EUAs) have been created to be used for compliance by airline operators.

The trading price differs with programs and the countries which apply the emission reduction program. Gangadharan [4] studies the Regional Clean Air Incentives Market (RECLAIM), which is a program implemented in the Los Angeles to use emissions trading to reduce pollutants in the Los Angeles basin. The trading price in this program is ranging between \$160 and \$2220 per ton. In 2008, the European Commission released legislative proposals to cut emissions of greenhouse gases by 20% and increase the share of renewable energy to 20% of final energy consumption.

With progression of new developments in the strategies and regulations of emission reduction, the industries and production companies will also capture the effects of greening policies in their production and distribution policies and will be forced into a more competing environment under emission restrictions. Consumer awareness together with legislation dictated by the government has also pushed the environmental considerations of a supply chain into spotlight of the firms, making it mandatory for the organizations to have greening plans. With regard to the rising global awareness of environmental protection, operations management researchers started to deal with this problem within a new stream of research, which is called green supply chain management.

Green supply chain management can be defined as the integration of environmental consideration into supply chain management, including material sourcing, supplier selection, product design, manufacturing processes, packaging, transshipment and delivery to the end consumers [5]. The OR researchers have considered green supply chain management from different perspectives. In the next chapter, we will present examples of literature work related to the inventory management problems considering greening policies.

In compliance with the emission reduction considerations, we have build our model based on a supply chain with carbon restricted manufacturer. The model is constructed under two contract types, namely buyback(return) and target rebate contracts. For the return contract, in which a proportion of unsold items is allowed to be returned, we found that even if the manufacturer allows no returns, under special conditions the channel may be coordinated. For the case of no returns, we derived the optimal wholesale price and the conditions for return rate and price which are required for coordination in each of the order range. We showed that when full return is allowed, if the return price is lower than the wholesale price, there are pricing schemes which leads to overall channel benefit and coordination in particular.

For the rebate contract, in which the retailer gets a certain credit for the sales beyond a target, we have derived the optimal state of each contract parameter. We have performed this analysis for each case corresponding to the assigned emission cap. The results show that a properly designed target rebate achieves coordination and a win-win outcome.

This thesis is organized as follows: In chapter 2, we presents an overview of the literature related to this research. In Chapter 3, the two models are discussed and the preliminaries are presented. In Chapter 4, we deal with the model, in which we focus on the coordination of a manufacturer and a retailer under return contract. In Chapter 5, the model with rebate contract is developed. Finally, in chapter 6, we summarize the findings, draw conclusions, and point out avenues for future research.



# Chapter 2

## Related Literature

### 2.1 Supply Chain Coordination and Contracts

In this chapter we classified the current literature relevant to the core concepts in this work and, based on the gap in the literature, our motivation behind this study is explained.

An example of a coordinating contract for the newsvendor setting is the buy-back or markdown contract. In this contract, the manufacturer reduces the risk of overage for the retailer by offering to buy back a part of the stock that is not sold at a certain price . In this way, the buyer's full order quantity is partially protected against not selling. The cost per item due to overage decreases and the retailer orders more.

Pasternack [6], in a newsvendor structure described in paragraph one of chapter three , for a commodity with short shelf or demand life develops a hierarchical model for deciding the optimal pricing policy for the manufacturer. In his model, manufacturer has the control of channel and is free to set the pricing policy. Goodwill cost is incurred by both the retailer and the manufacturer. He shows that the pricing policy with full credit for all unsold items as well as the policy with no returns could not achieve coordination. He concludes that under a properly chosen wholesale price, full credit for partial returns in a single retailer

and partial credit for unlimited returns in a multi retailer environment is channel coordinated. However, in the case of multi retailer environment all retailers are not affected equally.

Lariviere and Porteus [7] discuss a return policy in a single manufacturer and retailer channel using the newsvendor-type demand. Results show that the unrestricted return quantity policy is independent of the demand distribution. They do not consider supply chain coordination in their model.

Wu [8] studies the impact of buyback policy on retail price, order quantity and wholesale price in a duopoly of two manufacturer-retailer supply chains using newsvendor model. He assumes a competing supply chain in which the manufacturer maximizes its profit under its own optimal choice of a wholesale price. Given the wholesale price, the retailer makes the procurement determination and obtain the purchasing items from the manufacturer and resells them at the retail level with a self-determined price. He examines both buyback and non-buyback policies for two competing supply chains using vertical integration (VI) and manufacturer Stackelberg (MS) game methods. The results show that in the VI case with buyback policy, the manufacturer and retailer act as the same agent to set up a return policy and decide on ordering quantity and retail price.

Xiao et.al. [9] Consider coordination of a supply chain consisting of one manufacturer and one retailer facing consumer return. They integrate consumer returns policy and manufacturer buyback policy within a modeling framework. They design a buyback/markdown money contract to coordinate the supply chain under partial refund policy. They show that the refund amount plays an important role in the decisions and profitability of the players. In the coordinated setting with given buyback price, the refund amount first increases the players' expected profits/quantity, and then decreases them. When the risk of the consumer's return increases, the manufacturer may increase the unit wholesale price to achieve a higher unit profit. They show that the supply chain is better off using full refund policy if the risk is very small; otherwise, the supply chain prefers no returns policy.

In the case of a sales-rebate contract, the manufacturer provides an incentive

for each item that is sold above a certain threshold.

Taylor [10] considers a contract with time restricted rebates in which manufacturer specifies wholesale price and a rebate value and then, the retailer chooses the order quantity. In his model wholesale price is linear in order quantity. Selling price, manufacturing cost and salvage value are exogenous and wholesale price, rebate value and sale threshold are endogenous. In the attempt to explore the role of rebates in a one period model, two different structure is analyzed. The model in which retailer deals only with order quantity issue and the model with order quantity plus sales effort effect. His findings show that when demand is not influenced by sales effort, properly designed target rebate achieves channel coordination. However if sales effort affects retailers demand, then properly designed target rebate plus returns contract achieves channel coordination.

Wong et al. [11] develop a model in the context of a two-echelon supply chain with a single supplier serving multiple retailers in vendor-managed inventory (VMI) partnership. In their model retailers are considered in two scenarios: independent retailers with a demand function sensitive only to their own price and competing retailers with a demand function depending on all retailers' prices. Their proposed model demonstrates that the supplier gains more profit with competing retailers than without as competition among the retailers lowers the prices and thus stimulates demand.

Another, closely related agreement is the revenue sharing contract. In this case, the retailer pays the manufacturer a wholesale price per unit plus a percentage of the revenue the retailer generates. In the video cassette rental industry, such contracts are commonly used.

Cachon and Lariviere [12] showed that under a revenue-sharing contract, the manufacturer will set a wholesale price below production cost if manufacturer and retailer agree to share the total revenue of the supply chain. They also showed that buyback contracts and revenue-sharing contracts are equivalent in the fixed-price newsvendor model; for any buyback contract, there exists a revenue-sharing contract that generates the same cash flows for any realization of demand. However, this is not the case when demand is price-dependent. They study the impact

of revenue sharing on the performance of a supply chain and discusses the order quantity and retail price properties. They suggest that in cases of cheaper wholesale price contract, revenue sharing provides negligible improvements. In contrast, in case of a supply chain with demand that depends on costly retail effort, revenue sharing fails to achieve channel coordination. He concludes that no matter the demand property (deterministic or stochastic), revenue sharing coordinates supply chain with multiple non competing retailers even if retailers have different revenue functions.

The quantity discount is a way to minimize the system-wide cost of operation. The manufacturer can offer quantity discount such that the retailer finds it optimal to order a larger quantity that minimizes the total channel's operating cost. Jeuland and Shugan [13] focused on channel coordination in the context of a single manufacturer and a single retailer structure. They find that coordination between a manufacturer and a retailer using a quantity discount schedule could lead to higher profit for channel members.

Li et al. [14] develop a model for illustrating how to use quantity discount policy to achieve supply chain coordination. A supplier-buyer system selling one type of product with multi-period and probabilistic customer demand is considered. They show that if both the buyer and supplier can find a coordination mechanism to make joint decisions, the joint profit in this situation is more than the sum of their profits in the decentralized decision situation. Their results illustrate that there is a bound of quantity discount in which both sides can accept and the increased profit due to joint decision can be measured using this bound. They design a method to divide it between the buyer and supplier, and the optimal quantity discount policy is obtained by using this profit sharing method.

One other contract is quantity flexibility contract, under which the manufacturer fully protects a proportion of the order against underselling. A retailer can receive an extra amount of products for which payment is due only when the product is actually sold.

## 2.2 Green Supply Chains and Coordination

Although most channel literature deal with pricing strategies and some with non-pricing strategies like advertising effort, very few studies have dealt with channel issues arising out in green supply chain initiatives. [15]

Song et al. [16] analyze a single period problem under different carbon emission reduction policies. These policies include mandatory cap, cap and trade and carbon taxing. For a newsvendor model under these policies they find the optimal production quantity and corresponding expected profit. They conclude that in order to reduce carbon emissions by a certain percentage, the tax rate imposed on the high-margin should be less than that on the low-margin for the high-profit perishable products, whereas the high-margin should absorb a high tax than the low-margin for the low-profit products. They also found that, from the perspective of the policy-maker, the emissions capacity should be set to a level such that the marginal profit of the firm is less than the carbon credit purchasing price, because, otherwise, the firm would produce more than the emissions capacity. They derive the specific conditions under which, as a result of implementing the cap-and-trade policy, the firm's expected profit is increased and carbon emissions are reduced.

Liang et al. [17] consider a supply chain which contains one manufacturer and one retailer in which the manufacturer has sufficient channel power over the retailer to act as the Stackelberg leader. In their model the demand is sensitive to the level of green innovation. They investigate the emissions reduction cost-sharing contract through green innovation under carbon emission constraints. Their objective is to establish an analysis model to obtain the optimal cost-sharing proportion established by the manufacturer and the retail price established by the retailer. Their results show the impact of the cost of achieving green innovation and the level of green innovation on the retailers cost-sharing proportion, the retail price and the wholesale price.

In another study Liang et al. [18] study the effects of customer and regulatory environmental pressures on the optimal price and carbon emission. They show

that the optimal wholesale price and retail price are positively related to regulatory environmental pressure and negatively related to customer environmental pressure, and the manufacturer's profit-maximizing level of emissions is determined by the customer environmental sensitivity and initial level of emissions, and the environmental performance of the manufacturer and the retailer's profit are affected by customer and regulatory environmental pressures and market size and costs for environmental investments.

Ma et al. [19] construct a supply chain decision model under carbon tax. They build the demand distribution function including consumer characters which is more in line with the reality of the actual retailer operations. They find that carbon emissions allocation coefficient does not affect the optimal profits of suppliers and retailer under wholesale price contract; Supply chain enterprise could gain more profit through the carbon emissions collaborative, choosing reasonable carbon emissions levels.

Benjaafar et al. [20] study the effect of business practices and operational policies on carbon emission. They consider three models: A single firm with strict carbon caps , a single firm with carbon tax, carbon cap-and-trade, or carbon offsets and multiple firms with and without collaboration. Using the framework of the economic order quantity (EOQ) They discuss three systems of emission regulation. For each system their observations are summarized as follows: For the systems with strict emission caps they argue that it is possible to significantly reduce emissions without significantly increasing cost. Moreover emission caps could be met more cost-effectively by adjusting operational decisions than by investing in costly more energy-efficient technology. They show that tighter caps on emissions can paradoxically lead to higher total emissions. For the systems with carbon offsets, carbon tax, and cap-and-trade they argue that carbon offsets enable tighter emission caps by mitigating the impact of lowering emission caps on cost. However under cap and trade when the price is fixed, emission levels are not affected by emission caps and are affected only by the price for carbon. In addition to the previous cases, under cap-and-trade a higher carbon price can lead to lower total cost. Finally for the systems with multiple firms with and without collaboration they argue that imposing supply chain-wide emission caps

achieves lower emissions at lower costs; it also increases the value of collaboration. They conclude that it is possible (through operational adjustments only) to significantly reduce emissions without significantly increasing cost; and although different regulatory policies can achieve the same reduction in emission, the corresponding costs can be significantly different. In addition, the cost of reducing emissions across the supply chain can be significantly lower if firms within the same supply chain collaborate (however, the cost and emission of individual firms may go up because of the collaboration).

As an extension to the work of Benjaafar, Chen [21] finds analytical support for the idea that by making adjustments in the order quantity, it is possible to achieve more carbon emission reduction without significantly increasing cost. For his model the opportunity to reduce carbon emission via operational adjustments exists whenever the operational drivers of emissions are different from the operational drivers of costs. He concludes that there is an opportunity to reduce emissions by modifying the order quantity.

Working on the problem of minimizing the inventory related and greenhouse gas emissions cost of a supply chain with penalties for exceeding emissions limit (considering EU-ETS), Jaber et al. [22] assume a coordinated two level supply chain with vendor and buyer in which emission is a function of production rate, exceeding the emission limit is penalized and by purchasing new certificates the emission limit can be increased. In their model emissions generated by facility is a convex function of its output rate. Using mathematical programming for minimizing supply chain cost their results show that a policy that considers a combination of carbon tax and emissions penalty is the most effective one as the optimal solution generated was associated with low emissions. They argue that supply chain coordination minimizes the total system cost but the reduction was in the inventory-related cost with no change in the sum of emissions and penalty cost.

Du et al. [23] consider an emission dependent supply chain with a permit supplier and an emission dependent manufacturer. The manufacturer may sell or buy the permit if necessary. The manufacturer faces stochastic demand for

a single product. No return, salvage and inventory holding costs are assumed. They game theoretically analyze the optimal decisions in a cap and trade system. They argue that in the modeling of Stackelberg game there is a Nash equilibrium where both parties achieve the maximum profit. The manufacturer's profit as well as the system-wide profit increase as the cap increase but the permit supplier's profit decrease. In their model under special conditions there will be room for the emission dependent manufacturer and permit supplier to coordinate to get more profit per production. They also employed Bernoulli-Nash social welfare function to analyze the optimal cap. Based on their findings the system wide and the manufacturer's profit increase with the emission cap while the permit supplier's decrease.

Ghosh and Shah [15] examine an apparel serial supply chain whose players initiate product greening. They consider situations in which the players cooperate or act individually. Their study is based on the clothing industry with short shelf/demand life. Using game theoretic models they build a two-part tariff contract to coordinate the green channel made of one manufacturer and one retailer. The demand is a linear function of retail price and the level of green innovation. They argue that cooperation between players does lead to higher greening levels as seen in market structures, manufacturer Stackelberg cooperative policy (MSCP) and vertical Nash cooperative policy (VNCP), however greening leads to higher retail prices of the apparel. The findings necessitate support from governments and policy making bodies which need to provide suitable incentives to companies going green in order to lower the prices of green apparels. Further, contrary to expectations, although cooperation through bargaining is advantageous to the overall supply chain and the retailer in certain market structures, the manufacturer does not benefit through cooperation and there is a need for the retailer to provide suitable incentives to the manufacturer for him to participate in the bargaining process.

Hua et al. [24] studied how firms manage carbon footprints in inventory management under the cap-and-trade mechanism. Their research focuses on the carbon emissions caused by logistics and warehousing activities. They assume that the carbon emissions from logistics per order is linear in the order quantity and the



carbon emissions from warehouse is linear in the inventory and the carbon price is not affected by the carbon cap allocated to a single retailer. They compared the optimal decision under cap and trade with that for the classical EOQ model, and examined the impacts of carbon cap and carbon price on order size, carbon emissions, and total cost. They argue that the optimal order size is between the optimal EOQ order size and the order size that minimizes carbon emissions. They show that compared with the classical EOQ model, the cap-and-trade mechanism induces the retailer to reduce carbon emissions, which may result in an increase in the total cost. However, the retailer may reduce carbon footprints and total cost simultaneously under some conditions. Carbon cap and carbon price have a great impact on the retailers order decisions, carbon footprints, and total cost. In their model whether the retailer should buy carbon credit depends on the carbon cap as follows: when the cap is less (higher) than a threshold, the retailer should buy (sell) carbon credit, whereas when the cap equals the threshold, he should neither buy nor sell. With increasing carbon price, the retailer may order more or fewer products, which depends on the cost and carbon emissions from logistics and warehouse. With increasing carbon price, the total cost may increase or decrease, depending on the carbon cap. When the cap is lower than one threshold, the total cost will increase; when the cap is higher than another higher threshold, the total cost will decrease; and when the cap is between the two thresholds, the cost will initially increase and then decrease.

Zhang et al. [25] study a supply chain with a permission dependent manufacturer and a retailer. The manufacturer has an emission quota predetermined by the government. Beside the permission for carbon trading under cap and trade regulation, they consider single and multi time purification in their model. Within such a structure they aim to maximize the expected profit with appropriate production scale. In other words, for each purification case they find the optimal production quantity to balance the trade-off between purchase and purification.

Zanoni et al. [26] developed a joint economic lot sizing model for coordinated inventory replenishment decisions for price and environmentally sensitive demand. In their model two parameters are associated with the environmental performance and sensitivity of the demand to the environmental considerations. Using  $(Q,R)$

model they deal with the vendor's production lot size, selling price and the capital invested to improve product quality.

Toptal et al. [27] study a retailer's joint decisions on inventory replenishment and carbon emission reduction investment under three carbon emission regulation policies. They extend the economic order quantity model to consider carbon emissions reduction investment availability under carbon cap, tax and cap-and-trade policies. They analytically show that carbon emission reduction investment opportunities, additional to reducing emissions as per regulations, further reduce carbon emissions while reducing costs. They provide an analytical comparison between various investment opportunities and compare different carbon emission regulation policies in terms of costs and emissions.

Swami et al. [28] discusses the problem of coordination of a manufacturer and a retailer in a vertical supply chain, who consider greening actions in their operations. They address some pertinent questions in this regard such as extent of effort in greening of operations by manufacturer or retailer, level of cooperation between the two parties, and how to coordinate their operations in a supply chain. The greening efforts by the manufacturer and retailer result in demand expansion at the retail end. The decision variables of the manufacturer are wholesale price and greening effort, while those of the retailer are retail price and its greening effort. They show that the ratio of the optimal greening efforts put in by the manufacturer and retailer is equal to the ratio of their green sensitivity ratios and greening cost ratios. Further, profits and efforts are higher in the integrated channel as compared to the case of the decentralized channel. Finally, they define a two-part tariff contract to produce channel coordination.

From the above review we observe that there is a gap in the literature regarding different contracts under carbon restrictions and trading. We therefore attempt to fill in this gap by considering two alternative contracts. In particular, we obtain the parameter settings under which the supply chain coordinates both with buy-back and target rebate contracts.

# Chapter 3

## Problem Setting and Preliminaries

One of the classical problems in the inventory management literature is the newsvendor problem. Consider a retailer who places an order for a single product at the beginning of each period. The quantity procured is used solely to satisfy demand during the current period. No inventory is kept from one period to the next. The demand for this product during the current period is not known in advance. Instead, it is represented by a non-negative random variable  $X$ . The probability density function is  $f(X)$  and the cumulative distribution function of  $X$  is  $F$ , i.e.,  $F(X) = P(X \leq x)$ . The retailer must determine the order quantity  $Q$  which minimizes the expected cost at the end of the period. There are two cost components associated with the newsvendor problem, overstock cost  $C_o$  and understock cost  $C_u$ . The objective of the newsvendor problem is to find the optimal order quantity which minimizes the expected total cost. The newsvendor problem is suitable for the inventory management systems of perishable goods which can not be carried from one period to another. Newspaper, grocery items like milk, food and any other item with short shelf or demand life that can be purchased only once in a selling season are examples of perishable goods. Whitin [29] first presented the newsvendor problem. Since then, there has been a growing

interest in the model with its various versions in the literature of inventory management. Many extension has been proposed to the newsvendor problem. Lau and Lau [30], Khouja [31] and Shore [32] have considered the newsvendor problem from different perspectives and extended the results to different settings of supply chain.

In this research we have considered two problems, referring to two contracts under the newsvendor setting. The decentralized objective functions of the manufacturer and retailer are expected profits. The aim of the study is to find the contract parameters to harmonize their manufacturing and ordering quantities and maximizing their expected profit. Such parameters are said to coordinate the channel. For each contract the optimal ordering quantities of the manufacturer and the retailer is found, then the contract parameters are set such that these quantities are equal. After coordinating the orders, we look for the special set of parameters which result in a win-win outcome in terms of expected profit for both retailer and manufacturer.

In our model, under the newsvendor setting, the manufacturer produces an item with short life cycle for sale to the retailer. Each unit of production emits  $\beta$  units of carbon dioxide. The manufacturer has an emission allowance (cap)  $\kappa$  in each production period. However the carbon trading regulation enables the manufacturer to compensate deficiency and sufficiency of carbon through the carbon market. The  $\kappa$  value can be considered as a cap imposed by the government or the environmental organizations. Selling and buying prices of the carbon in the carbon market are assumed to be  $p_s$  and  $p_b$  respectively. The conventional relation between selling and buying prices is such that, the buying price is costly than the selling price to the carbon market. Let  $Q$  represent the production quantity of the manufacturer. We assumed that unit item production emits  $\beta$  units of carbon dioxide. As a result  $Q\beta$  units of carbon will be emitted if  $Q$  items are produced. If the specified  $\kappa$  by the government is greater than  $Q\beta$ , then selling of the remaining  $(\kappa - Q\beta)$  with  $p_s$  to the carbon market will add  $(\kappa - Q\beta)p_s$  to the channel profit. In return, a  $\kappa$  value less than  $Q\beta$  pushes the manufacturer to compensate the shortcoming of  $(Q\beta - \kappa)$  from the carbon market with a price of  $p_b$ . This will reduce the channel profit by  $(Q\beta - \kappa)p_b$ .

The manufacturer has to decide on the wholesale price to the retailer. The wholesale price to charge for the product depends on the cost function and market bear in terms of price. Once the price is set, retailer decides on the amount of order quantity. Retailer places the orders once in a selling period and sells during the selling period. The lead time is assumed to be negligible.

Under two contract types between manufacturer and retailer, namely return and rebate we set the contract parameters in such a way that the manufacturer and retailer have the same production and order quantity.

### 3.1 Buyback Contract

For the contract with returns of the unsold items, the scenario is as follows: The retailer places an order of size  $Q$  at the beginning of a selling season. The manufacturer sells the finished item to the retailer with wholesale price  $c_1$  and allows the retailer to return some or all of the unsold items at the end of selling season. Before the procurement of the order quantity, the manufacturer announces the price  $c_2$  which he pays to the retailer for the unsold items. Considering his expected profit margin, the retailer decides on the end-customer selling price  $p$ . Beside the order amount one other important decision to make is whether to let the retailer to return some or all of the remained items at the end of a selling season. That is, the retailer in return of a certain credit returns a proportion  $0 \leq R \leq 1$  of inventory on hand to the manufacturer for the disposal with salvage value  $c_3$  and the target is achieving coordination by letting such a return policy.

The following relations are assumed to hold: A1)  $c_3 < c < c_1 < p$  and A2)  $c_3 < c_2 \leq c_1 < p$ .

The goodwill cost associated with the end customer's unsatisfied demand is partially incurred by retailer  $g$  and partially by the manufacturer  $g_1$ . The total goodwill cost is  $g_2 = g + g_1$ . The order quantity affects the total profit of the

manufacturer. The contract parameters which need to be optimized in order to set coordination are:

- i) The price charged to the retailer per unit product, i.e.,  $c_1$ .
- ii) The per unit credit for returned items paid by manufacturer to retailer, i.e.,  $c_2$ .
- iii) The percentage of unsold allowed to be returned, i.e.,  $R$ .

Pasternack [6] was the first researcher who considered coordination with buy-back contract. In his model, when the manufacturer permits no return, the channel turns out to be uncoordinated. This is true for the case of full returns with full credit as well. However, for a single retailer when full return is allowed and the return price is smaller than the wholesale price, then under a specific relation between wholesale and return price, coordination is achievable.

## 3.2 Target Rebate

In addition to return contracts, channel rebates are also common mechanisms for manufacturers to entice retailers to increase their order quantities and sales ultimately. Rebate is different from discount. Rebate is the money paid by manufacturer to the retailer for the units sold. This could be linear rebate or target rebate. In linear rebate manufacturer pays retailer a rebate for each unit sold. Target rebate is the money paid by manufacturer to the retailer for the sales beyond a target  $T$ . Linear rebate is a converse contract type of revenue sharing contract in the sense that in revenue sharing retailer pays manufacturer a portion of retail price for each unit sold to the customer.

Manufacturer sells the items to the retailer with wholesale price  $c_1$ . Based on the target rebate contract, the manufacturer pays the rebate amount  $u$  to the retailer for the sales beyond a specific target  $T$ . These three values are the system or contract parameters which we studied to find the optimal values which lead to coordination.

Selling, manufacturing and salvage prices are assumed to be exogenous and

wholesale price;  $c_1$ , rebate value;  $u$  and target sale;  $T$  are assumed to be endogenous. No lump sum side payment is allowed. The order quantity is chosen by the retailer after the manufacturer specifies the rebate amount. The followings are assumed: B1)  $0 < c < c_1 < p$  and B2)  $c_3 < c, u > 0, T > 0$ . Furthermore we assume that the goodwill cost corresponding to the lost demand is negligible,  $g = g_1 = 0$  as in Taylor.

According to the structure of the rebate contract no returns is allowed. That is, for the rebate contract  $R = 0$  and  $c_2 = 0$ . The contract parameters which need to be optimized in order to set coordination are:

- i) The price charged to the retailer per unit product, i.e.,  $c_1$ .
- ii) The target sales, i.e.,  $T$ .
- iii) The rebate for the sales beyond target, i.e.,  $u$ .

All the notations used in the thesis is given in Appendix.

### 3.3 Integrated channel when manufacturer acts as his own retailer

When the manufacturer directly sells to the market without any retailer, then the expected channel's profit is given by:

$$\begin{aligned}
 EP_T(Q) = & -cQ + \int_0^Q [xp + (Q-x)c_3]f(x)dx + \int_Q^\infty [pQ - g_2(x-Q)]f(x)dx \\
 & + [m(\kappa - Q\beta)p_s] + [(1-m)(\kappa - Q\beta)p_b]
 \end{aligned} \tag{3.1}$$

where  $m = 1$  if  $\kappa > Q\beta$ , zero otherwise.

The newsvendor model with resource constraints have been studied earlier by Sözüer [33] and Korkmaz [34] in their thesis. However we provide below the

optimization results for the sake of completeness.

Regarding production policy of the manufacturer, we first derive the production quantities  $Q_s$  and  $Q_b$  as follows, which refer to the cases of selling and buying carbon emission respectively:

$$Q_s = F^{-1} \left( \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3} \right) \quad (3.2)$$

$$Q_b = F^{-1} \left( \frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3} \right) \quad (3.3)$$

### Optimal Ordering Policy

#### THEOREM 1.

(a) Suppose  $p_b > p_s$ , then the optimal policy of the manufacturer under integrated channel will be:

$$Q^* = \begin{cases} Q_b & \text{if } \kappa < Q_b\beta \\ \frac{\kappa}{\beta} & \text{if } Q_b < \kappa < Q_s \\ Q_s & \text{if } \kappa > Q_s\beta \end{cases} \quad (3.4)$$

(b) Suppose  $p_0 = p_b = p_s$ , then the optimal policy of the manufacturer under integrated channel will be:

$$Q^* = \frac{p + g_2 - c - \beta p_0}{p + g_2 - c_3}$$

Proof (a):

The first order condition (FOC) from (3.1) results in:

$$\begin{aligned} \frac{dEP_T(Q)}{dQ} &= -c + c_3F(Q) + p[1 - F(Q)] + g_2[1 - F(Q)] - m\beta p_s \\ &\quad - (1 - m)\beta p_b = 0 \\ &= -c + c_3F(Q) + p - pF(Q) + g_2 - g_2F(Q)c - m\beta p_s \\ &\quad - (1 - m)\beta p_b = 0 \\ &= F(Q)[c_3 - p - g_2] - c + p + g_2 - m\beta p_s - (1 - m)\beta p_b = 0 \end{aligned}$$



which implies:

$$F(Q) = \frac{c - p - g_2 + m\beta p_s + (1 - m)\beta p_b}{c_3 - p - g_2}$$

$Q_s$  is the optimal production quantity if the excess carbon is sold in the carbon market and  $Q_b$  is the optimal production quantity if additional carbon emission allowance is purchased from the carbon market. Then

$$F(Q_s) = \frac{c - p - g_2 + \beta p_s}{c_3 - p - g_2} = \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3}$$

$$F(Q_b) = \frac{c - p - g_2 + \beta p_b}{c_3 - p - g_2} = \frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3}$$

and we get

$$Q_s = F^{-1}\left(\frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3}\right)$$

$$Q_b = F^{-1}\left(\frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3}\right)$$

$$\frac{c - p - g_2 + \beta p_s}{c_3 - p - g_2} > \frac{c - p - g_2 + \beta p_b}{c_3 - p - g_2}$$

Assuming  $p_b > p_s$  we have

$$\frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3} > \frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3}$$

Since F is a one to one and increasing function, we have

$$F^{-1}\left(\frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3}\right) > F^{-1}\left(\frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3}\right)$$

which implies  $Q_s > Q_b$ .

The first and second order conditions of the expected profit function of the integrated channel for two cases of buying and selling will be:

$$\frac{dEP_T(Q)}{dQ} = \begin{cases} -c + c_3F(Q) + p[1 - F(Q)] + g_2[1 - F(Q)] - \beta p_s \\ -c + c_3F(Q) + p[1 - F(Q)] + g_2[1 - F(Q)] - \beta p_b \end{cases}$$

$$\frac{d^2 EP_T(Q)}{dQ^2} = \begin{cases} c_3 f(Q) - pf(Q) - g_2 f(Q) = -f(Q)(p + g_2 - c_3) < 0 \\ c_3 f(Q) - pf(Q) - g_2 f(Q) = -f(Q)(p + g_2 - c_3) < 0 \end{cases}$$

Obviously either  $\kappa > Q\beta$  or  $\kappa < Q\beta$ , the second derivative is negative. Consequently the  $Q_s$  and  $Q_b$  values are the extreme values. Based on this we can define the optimal policy of the channel for different ranges of  $\kappa$ ; The important issue to be considered is that, what the policy must be when the  $\kappa$ ; is between  $Q_b/\beta$  and  $Q_s/\beta$ . Note that in this case the optimal order quantity will be  $\kappa/\beta$ ; therefore the manufacturer needs nor to sell neither to buy any carbon emission limit. Based on these results, the policy is as given.

Proof (b): Result follows from part (a) when we let  $p_s = p_b$ .

**Remark:**

As shown in figure 3.1, the policy for case (a) implies that if the order amount is less than  $Q_b$  then the retailer has to increase the order quantity up to  $Q_b$  amount, and if the order size is greater than  $Q_s$  then the order size have to be decreased to  $Q_s$  amount. If the  $\kappa$  value offered by the authorities is such that  $\frac{\kappa}{\beta}$  is between  $Q_b$  and  $Q_s$  amount, then the manufacturer has to follow the given production limit. In other words this range corresponds to neither buying nor selling any carbon amount.

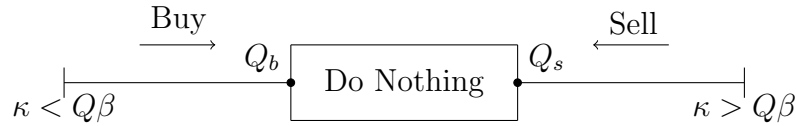


Figure 3.1: The Optimal Policy for The Channel

# Chapter 4

## Coordination Under Return Contract

### Introduction

This chapter considers the pricing decision faced by the manufacturer. According to the return contract, the retailer may return some or all the unsold items at the end of the selling season. This percentage is denoted by  $R$ .

The manufacturer's policy involves three decision variables, namely the wholesale price  $c_1$ , return price  $c_2$  and the return percentage  $R$ . Recall that we assume  $c_2 \leq c_1$ . In the remaining part of this chapter, the setting which results in coordination in terms of the expected profit between manufacturer and retailer will be discussed.

## 4.1 Independent Retailer with Returns

Expected profit function of the independent retailer with returns:

$$\begin{aligned}
 EP_R(Q) = & -c_1Q + \int_0^{(1-R)Q} [xp + RQc_2 + ((1-R)Q - x)c_3]f(x)dx \\
 & + \int_{(1-R)Q}^Q [xp + (Q-x)c_2]f(x)dx + \int_Q^\infty [pQ - (x-Q)g]f(x)dx
 \end{aligned} \tag{4.1}$$

Using the first order condition (FOC):  $\frac{dEP_R(Q)}{dQ} = 0$

$$-c_1 + p + g - F(Q)[p + g - c_2] - [(1-R)(c_2 - c_3)]F((1-R)Q) = 0$$

which implies that optimal ordering quantity must satisfy the following

$$F(Q)[p + g - c_2] + [(1-R)(c_2 - c_3)]F((1-R)Q) = p + g - c_1 \tag{4.2}$$

## 4.2 Channel Coordination

To achieve channel coordination the retailer's order amount must be equal to the optimal production amount of integrated channel. In other words, we search for the parameter setting that will result in the same optimal quantity of the integrated channel and the retailer. Since the manufacturer's optimal policy has three regions where optimal production quantities are given by  $Q_s$ ,  $Q_b$  and  $\frac{\kappa}{\beta}$ , we will consider coordination in these three regions separately. In the remaining of this section we used coordination and optimal in the same meaning.

### 4.2.1 Coordination when Manufacturer Sells Emission Allowance

Consider the case when the manufacturer sells the extra emission allowance. Recall that in this action the optimal production quantity is given as  $Q_s = F^{-1}\left(\frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3}\right)$  in (3.2). Substituting this into (4.2) results in (4.3):

$$\begin{aligned} [(1-R)(c_2 - c_3)]F((1-R)Q_s) &= p + g - c_1 - \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3}[p + g - c_2] \\ F((1-R)Q_s) &= \frac{1}{[(1-R)(c_2 - c_3)]} \left[ p + g - c_1 - \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3}[p + g - c_2] \right] \end{aligned} \quad (4.3)$$

The quantity of unsold items that manufacture allows to be returned,  $R$ , is between zero and one, inclusive. To analyze the above equation with different  $R$ , first note that if  $0 < R < 1$ , from (4.3) the problem will be coordinated and (4.1) gives the retailer's expected profit. In this case the manufacturer's expected profit will be:

$$\begin{aligned} EP_M(Q_s) &= Q_s(c_1 - c) - \int_0^{(1-R)Q_s} RQ_s(c_2 - c_3)f(x)dx - \int_{(1-R)Q_s}^{Q_s} [(Q_s - x) \\ &\quad (c_2 - c_3)]f(x)dx - \int_{Q_s}^{\infty} (x - Q_s)g_1f(x)dx + (\kappa - Q_s\beta)p_s \end{aligned}$$

Recall that we have assumed  $c_3 < c < c_1 < p$  and  $c_3 < c_2 \leq c_1 < p$ . For the special cases of  $R = 0$  and  $R = 1$  we have the following results.

**THEOREM 4.1.** *The policy of a manufacturer allowing unlimited returns, i.e.,  $R = 1$  for full credit, i.e.,  $c_1 = c_2$  do not coordinate the channel.*

Proof:

If  $R = 1$  and returns are accepted with full credit, i.e.,  $c_1 = c_2$ , then (4.3) will

turn into the following form:

$$c_1 - p - g = \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3} [c_2 - p - g]$$

If  $c_1 = c_2$ ,  $p + g_2 - c_3 = p + g_2 - c - \beta p_s$  must hold which implies  $c_3 = c + \beta p_s$ . This means that  $c_3 > c$ , which is a contradiction to the assumption that  $c > c_3$ .

**THEOREM 4.2.** *A policy which allows for unlimited returns,  $R = 1$ , at partial credit can achieve coordination if  $g_1 > \beta p_s$ .*

Proof:

We will show that if the above condition holds and  $R = 1$  then there exists a  $c_2$  such that  $c_3 < c_2 < c_1$  so that the channel is coordinated.

First from (4.3) we see that for  $R = 1$  we must have

$$c_1 = p + g - \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3} (p + g - c_2).$$

For  $c_1 \in (c, p)$  we show that there exists  $c_2 < c_1$  such that the above expression holds. This follows directly since  $c + \beta p_s > c_3$ .

Next we show that there exist a  $c_2$  such that  $c_3 < c_2$ . From (4.3) we had

$$c_1 = p + g - \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3} (p + g - c_2)$$

Since  $c < c_1$ , then  $c < p + g - \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3} (p + g - c_2)$ . This is equivalent to

$$c - p - g < -\frac{p + g_2 - c - \beta p_s}{p + g - c} (p + g - c_2)$$

$$p + g - c > \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3} (p + g - c_2).$$

If we assume  $\frac{p + g_2 - c - \beta p_s}{p + g - c} > 1$ , then the following relation between both sides of the equation must hold:

$$p + g_2 - c - \beta p_s > p + g - c$$

$$g + g_1 - c - \beta p_s > g - c$$

which is satisfied according to  $g_1 > \beta p_s$ .

So assuming that  $g_1 > \beta p_s$  appropriately chosen  $c_1$  and  $c_2$  for the following expression can achieve channel coordination.

**THEOREM 4.3.** *The policy of a manufacturer allowing no returns, i.e.,  $R = 0$  coordinates the channel if  $\frac{g_1(c_1 - c_3)}{p + g - c_3} < \beta p_s$  holds.*

Proof:

With  $R = 0$ , (4.3) will be

$$c_1 = p + g - \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3} [p + g - c_2] - (c_2 - c_3)F(Q_s).$$

Substituting  $Q_s$  given by (3.2) in the above formulation, we have

$$c_1 = p + g - \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3} [p + g - c_2] - (c_2 - c_3) \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3}$$

$$(c_1 - p - g)(p + g_2 - c_3) = -(p + g_2 - c - \beta p_s)(p + g - c_3)$$

Given that  $g_2 = g + g_1$ , rewrite the above equation:

$$(c_1 - p - g)(p + g + g_1 - c_3) = -(p + g + g_1 - c - \beta p_s)(p + g - c_3)$$

Simplifying and rewriting this equation leads to the following steps:

$$(c_1 - c) = \frac{g_1(c_1 - c_3)}{(c_3 - p - g)} + \frac{\beta p_s(c_3 - p - g)}{(c_3 - p - g)}$$

$$c_1 = c - \left[ \frac{g_1(c_1 - c_3)}{p + g - c_3} - \beta p_s \right].$$

Since  $c_1$  is assumed to be greater than  $c$ , channel coordination is achieved if

$$\frac{g_1(c_1 - c_3)}{p + g - c_3} < \beta p_s$$

## 4.2.2 Coordination when Manufacturer Buys Emission Allowance

Now suppose the optimal action of the manufacturer is to buy carbon allowance. Then, recall that the optimal production size for the channel is given by  $Q_b = F^{-1}\left(\frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3}\right)$  in (3.3). Substituting  $Q_b$  into the (4.2) will finally result in (4.4).

$$c_1 = p + g - \frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3} [p + g - c_2] - [(1 - R)(c_2 - c_3)] F[(1 - R)Q_b]$$

$$F[(1 - R)Q_b] = \frac{1}{(1 - R)(c_2 - c_3)} \left[ p + g - c_1 - \frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3} (p + g - c_2) \right] \quad (4.4)$$

Similar to the selling region, if  $0 < R < 1$  then the channel will be coordinated. The corresponding profit of the retailer will be given by (4.1). In this case the manufacturer's profit will be given by the following:

$$EP_M(Q_b) = Q_b(c_1 - c) - \int_0^{(1-R)Q_b} RQ_b(c_2 - c_3)f(x)dx - \int_{(1-R)Q_b}^{Q_b} [(Q_b - x)(c_2 - c_3)]f(x)dx - \int_{Q_b}^{\infty} (x - Q_b)g_1f(x)dx + (\kappa - Q_b\beta)p_b$$

Special cases of  $R$  are considered in the following three theorems.

**THEOREM 4.4.** *The policy of a manufacturer allowing unlimited returns*



$(R = 1)$  for full credit  $(c_1 = c_2)$  is not optimal.

Proof: If  $R = 1$  and return policy is full credit  $c_1 = c_2$ , then from (4.4) we have

$$c_1 - p - g = \frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3} [c_2 - p - g]$$

Since  $c_1 = c_2$ , then  $p + g_2 - c_3 = p + g_2 - c - \beta p_b$ . However this requires  $c_3 > c$ , which is a contradiction to the assumption that  $c > c_3$ .

**THEOREM 4.5.** *A policy which allows for unlimited returns at partial credit is system optimal if  $g_1 > \beta p_b$ .*

Proof:

We will show that if the above condition holds and  $R = 1$ , then there exists a  $c_2$  such that  $c_3 < c_2 < c_1$  so that the channel is coordinated.

First Substituting  $R = 1$  into (4.4) will result in

$$c_1 = p + g - \frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3} (p + g - c_2)$$

For  $c_1 \in (c, p)$  we show that there exists  $c_2 < c_1$  such that the above expression holds. This follows directly since  $c + \beta p_b > c_3$ .

Now we need to show that there exist a  $c_2$  such that  $c_3 < c_2$ . Since  $c < c_1$ , then  $c < p + g - \frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3} (p + g - c_2)$ .

Or  $p + g - c > \frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3} (p + g - c_2)$ , which is satisfied according to  $g_1 > \beta p_b$ .

**THEOREM 4.6.** *The policy of a manufacturer allowing no returns i.e  $R = 0$  is optimal if*

$$\frac{g_1(c_1 - c_3)}{p + g - c_3} < \beta p_b$$

Proof:

From (4.4) we have

$$c_1 - p - g = -\frac{p + g_2 - c - \beta p_b}{p + g_2 - c_3}[p + g - c_3]$$

Given that  $g_2 = g + g_1$

$$(c_1 - p - g)(c_3 - p - g - g_1) = (c + \beta p_b - p - g - g_1)(c_3 - p - g)$$

Rewriting the above equation results in

$$c_1 = c - \left[ \frac{g_1(c_1 - c_3)}{p + g - c_3} - \beta p_b \right]$$

Which implies that in order to achieve channel coordination the following condition must be satisfied.

$$\frac{g_1(c_1 - c_3)}{p + g - c_3} < \beta p_b$$

### 4.2.3 Coordination when Optimal Order is $\kappa/\beta$

Substituting  $F\left(\frac{\kappa}{\beta}\right)$  into (4.2) results in

$$-c_1 + p + g - F\left(\frac{\kappa}{\beta}\right)[p + g - c_2] - [(1 - R)(c_2 - c_3)]F\left[\left(1 - R\right)\frac{\kappa}{\beta}\right] = 0$$

Rewriting it

$$\begin{aligned} [(1 - R)(c_2 - c_3)]F\left[\left(1 - R\right)\frac{\kappa}{\beta}\right] &= p + g - c_1 - F\left(\frac{\kappa}{\beta}\right)[p + g - c_2] \\ &= (p + g)(1 - F\left(\frac{\kappa}{\beta}\right)) - c_1(1 - F\left(\frac{\kappa}{\beta}\right))c_2 \end{aligned} \tag{4.5}$$

When  $0 < R < 1$  the channel is coordinated. Similar to the two previous cases the retailer's expected profit will be given by (4.1). The manufacturer's expected profit will be:

$$\begin{aligned}
EP_M\left(\frac{\kappa}{\beta}\right) &= \frac{\kappa}{\beta}(c_1 - c) - \int_0^{(1-R)\frac{\kappa}{\beta}} \left[R\frac{\kappa}{\beta}(c_2 - c_3)\right]f(x)dx \\
&\quad - \int_{(1-R)\frac{\kappa}{\beta}}^{\frac{\kappa}{\beta}} \frac{\kappa}{\beta} \left[\left(\frac{\kappa}{\beta} - x\right)(c_2 - c_3)\right]f(x)dx - \int_{\frac{\kappa}{\beta}}^{\infty} \left[\left(x - \frac{\kappa}{\beta}\right)g_1\right]f(x)dx
\end{aligned}$$

The cases when  $R = 0$  and  $R = 1$  lead to the following theorems.

**THEOREM 4.7.** *If manufacturer allows for full return and full credit, the channel will not be coordinated.*

$$\begin{aligned}
-c_1 + p + g - F\left(\frac{\kappa}{\beta}\right)[p + g - c_2] &= 0 \\
F\left(\frac{\kappa}{\beta}\right) &= \frac{p + g - c_1}{p + g - c_2}
\end{aligned}$$

since  $c_1 = c_2$ , then  $F\left(\frac{\kappa}{\beta}\right) = 1$ , since we assume the demand random variable unbounded the channel is not coordinated.

**THEOREM 4.8.** *If manufacturer allows for full return, and  $c_2 < c_1$  appropriately chosen  $c_1$  and  $c_2$  values if*

$$c_1 = (p + g) \left[ \overline{F}\left(\frac{\kappa}{\beta}\right) \right] + F\left(\frac{\kappa}{\beta}\right) c_2$$

*can achieve channel coordination.*

Proof:

If  $R = 1$ , (4.5) will turn into the following form:

$$-c_1 + p + g - F\left(\frac{\kappa}{\beta}\right)[p + g - c_2] = 0$$

rewriting it will lead to

$$c_1 = (p + g) \left[ \bar{F} \left( \frac{\kappa}{\beta} \right) \right] + F \left( \frac{\kappa}{\beta} \right) c_2$$

**THEOREM 4.9.** *If manufacturer allows for no returns, channel is coordinated if  $c_1 = p + g - (p + g - c_3)F \left( \frac{\kappa}{\beta} \right)$*

Proof:

If  $R = 0$ , (4.5) will turn to:

$$c_1 = (p + g) \left[ \bar{F} \left( \frac{\kappa}{\beta} \right) \right] + c_3 F \left( \frac{\kappa}{\beta} \right)$$

From the above derivation, it is possible to check the retail pricing policy  $c_1$  for the given  $\kappa$ .

$$F \left( \frac{\kappa}{\beta} \right) = \frac{p + g - c_1}{p + g - c_3}$$

The following table presents the coordination in each region together with the conditions required for each case in selling, buying and "Do Nothing" regions. We have compared the coordination with and without a carbon cap (Pasternack).

	No Cap	With Cap	
$R = 0$	Not Optimal	$Q_s$	Optimal conditioned that $\frac{g_1(c_1 - c_3)}{p + g - c_3} < \beta p_s$
		$Q_b$	Optimal conditioned that $\frac{g_1(c_1 - c_3)}{p + g - c_3} < \beta p_b$
		$\frac{\kappa}{\beta}$	Optimal With $c_1 = (p + g) \left[ \bar{F} \left( \frac{\kappa}{\beta} \right) \right] + F \left( \frac{\kappa}{\beta} \right) c_3$
$R = 1$ and $c_1 = c_2$	Not Optimal	$Q_s$	Not optimal
		$Q_b$	Not optimal
		$\frac{\kappa}{\beta}$	Not optimal
$R = 1$ and $c_2 < c_1$	Optimal For appropriately chosen $c_1$ and $c_2$ from $c_1 = p + g - \frac{p + g_2 - c}{p + g_2 - c_3} (p + g - c_2)$	$Q_s$	Optimal if $g_1 > \beta p_s$
		$Q_b$	Optimal if $g_1 > \beta p_b$
		$\frac{\kappa}{\beta}$	Optimal For appropriately chosen $c_1$ and $c_2$ from $c_1 = (p + g) \left[ \bar{F} \left( \frac{\kappa}{\beta} \right) \right] + F \left( \frac{\kappa}{\beta} \right) c_2$

Table 4.1: The optimality of the policy for different settings of return rate

The results show that contrary to the findings of Pasternack, it is possible to achieve coordination even if the manufacturer allows for no returns. The possibility of trading the allowance increases the flexibility of the problem and enables

the manufacturer to have more options in pricing policies, in the sense that while the manufacturer allows for no returns, he can reduce the wholesale price in order to motivate the retailer for the channel optimal ordering policy. Our findings show that for the special cases of wholesale price the corresponding profit of both parties can be increase after the coordination practice. This can be seen in our numerical results.

When full returns for full credit is allowed, manufacturer is always the losing part. In this case for the finite number of order quantities the coordination is not achievable.

Similar to the results of Pasternack, when full returns with a return price less than the wholesale price is allowed the manufacturer and retailer can have the same order size. For the selling and buying cases of emission allowance, the condition on the goodwill cost of manufacturer implies that for one unit of emission per production, the cost of one unit of unsatisfied demand is larger than the profit/cost of selling/buying one unit of emission allowance.

### 4.3 Numerical Analysis

In this chapter, we present a set of computational experiments in which we evaluate the optimal policies for the return contract and discuss the sensitivity of the optimal policy parameters with respect to various system parameters. Suppose that the end customer demand follows a normal distribution with mean 400 and standard deviation of 100 units.

$c = 1.5$	$g = 1.2$	$N(\mu, \sigma) = N(400, 100)$	$p = 4.5$
$c_1 = 3$	$g_1 = 1.1$	$R = 0, 1$	$p_b = 1$
$c_3 = 0.5$	$g_2 = 2.3$	$\beta = 1$	$p_s = 0.7$

Table 4.2: Input Data for The Return Contract

When  $R = 0$  and the retailer is profit maximizer (before coordination), we can find the retailer's optimal production quantity:  $Q_R^*$

$$-c_1 + p + g - F(Q_R^*)(p + g - c_2) - F((1 - R)Q_R^*)((1 - R)(c_2 - c_3)) = 0$$

which results in  $F(Q_R^*) = 0.519$ ., and the corresponding  $z$  value will be  $z = 0.05$ .  $z = \frac{Q_R^* - \mu}{\sigma}$ . Therefore  $Q_R^* = 405$ . The expected profit of the retailer with his optimal order quantity will be:  $EP_R(405) = 392.78$ . To specify the expected profit of the manufacturer when retailer orders 405, first the  $\kappa$  value must be declared. Because if the assigned cap is less than the retailer's order, 405 units, then the manufacturer must purchase the shortage from the market. Obviously  $\beta = 1$  means that one unit of  $\kappa$  corresponds to one unit production. Suppose there is no cap on the emission amount by manufacturer. That is, the manufacturer can produce any order quantity which is placed by the retailer(In this case 405). Then, regardless of the loss or profit incurred due to the cap, the manufacturer's expected profit will be:  $EP_M(405) = 566.3$ .

Let's look at the channel case. Previously we have derived a policy with three regions for the case that the manufacturer acts as his own retailer.

$F(Q_s) = \frac{p + g_2 - c - \beta p_s}{p + g_2 - c_3} = 0.73$ . For  $F(z) = 0.73$ ,  $z$  is approximately 0.61. So  $Q_s^* = 461$ . With the same setting  $Q_b$  will be  $Q_b^* = 447$

The optimal policy for the manufacturer will be:  $Q^* = \begin{cases} 447 & \text{if } \kappa < 447 \\ \kappa & \text{if } 447 < \kappa < 461 \\ 461 & \text{if } \kappa > 461 \end{cases}$

The policy and the current scenario is displayed in the following figure.

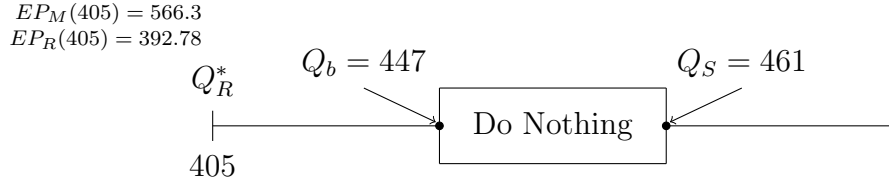


Figure 4.1: The Scenario Without Returns

### Coordination when manufacturer buys emission allowance

(a)  $\kappa < Q_R^*$

Suppose at the beginning of the production period, the government has announced the  $\kappa$  to be 400. After the announcement of  $\kappa = 400$ , if the retailer orders his optimal 405 units, manufacturer has to pay  $(405 - 400) * 1$  to buy 5 units of emission allowance to fulfill the demand.

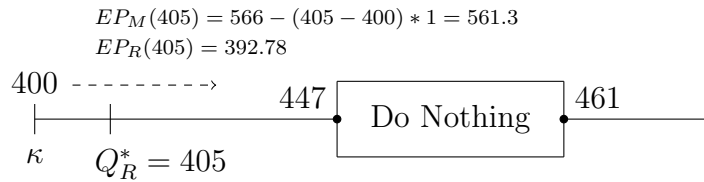


Figure 4.2: The Optimal Policy at Buying Region with  $\kappa < Q_R^*$

For the purpose of coordination, according to the policy, the manufacturer has to increase the cap up to 447 units. The possibility of achieving coordination between two parties happens in two cases:

1. When  $R = 0$  with the condition:  $\frac{g_1(c_1 - c_3)}{p + g - c_3} < \beta p_b$ . It follows that  $c_1 < 5.2$ . Hence in this case in order to achieve coordination in order quantities an upper bound for the wholesale price  $c_1$  is 5.2. When wholesale price is less than 5.2 the manufacturer and retailer may have the same order quantity. However, the



profit shares will be different with different wholesale prices. In our problem we have looked for the wholesale prices with a win-win outcome for both retailer and manufacturer.

Suppose  $c_1 = 2.96$ , then the retailer will get almost the same profit when he ordered his optimal 405 and manufacturer will get all the profit due to coordination. In this case the expected profits will be  $EP_R(447) = 392.62$  and  $EP_M(447) = 582.82$ .

Suppose that the manufacturer gives all the profit due to coordination to the retailer. By adjusting  $c_1 = 2.913$  the expected profits will be  $EP_R(447) = 413.63$  and  $EP_M(447) = 561.82$ .

In order to make coordination favorable for both, we set  $c_1 = 2.94$ . Consequently, both retailer and manufacturer benefit from coordination. The expected profit of retailer and manufacturer will be:  $EP_R(447) = 401.56$  and  $EP_M(447) = 573.88$ .

One can see the influence of the coordination on the expected profits by comparing the expected profits before and after the practice of coordination. That would be,  $401.56 - 392.78 = 8.78$  and  $573.88 - 561.3 = 12.58$  increase in profits for retailer and manufacturer respectively.

The following summarizes this results:

$$\begin{aligned}
 c_1 = 2.913 & \begin{cases} EP_R(447) = 413.63 \\ EP_M(447) = 561.82 \end{cases} \\
 c_1 = 2.96 & \begin{cases} EP_R(447) = 392.62 \\ EP_M(447) = 582.82 \end{cases} \\
 c_1 = 2.94 & \begin{cases} EP_R(447) = 401.56 \\ EP_M(447) = 573.88 \end{cases}
 \end{aligned}$$

2. When  $R = 1, c_1 > c_2$ . Assuming  $g_1 > \beta p_b$ , for the appropriately chosen  $c_1$  and  $c_2$  values according to the relation based on them i.e.,  $c_1 = 1.81 + 0.68c_2$

Since  $g_1 = 1.1 > \beta p_b = 1$  the condition is satisfied. One possible combination of  $c_1$  and  $c_2$  is  $(c_1, c_2) = (3.19, 2.03)$ . In this setting the expected profits will be:  $EP_R(447) = 393.22$  and  $EP_M(447) = 582.22$ . The manufacturer benefit from coordination. One other setting is  $(c_1, c_2) = (3.13, 1.94)$ . The expected profits

will be:  $EP_R(447) = 414.13$  and  $EP_M(447) = 561.32$ . In which the retailer takes all the profit of coordination. To make them both benefit from coordination, consider  $(c_1, c_2) = (3.1632, 1.99)$ . The expected profits will be:  $EP_R(447) = 402.67$  and  $EP_M(447) = 572.77$ . The increase in expected profits compared to the uncoordinated case will be 9.89 and 11.47 for retailer and manufacturer respectively.

The results are summarized as follows(after rounding to two digit decimals):

$$\begin{aligned} (c_1, c_2) = (3.13, 1.94) & \begin{cases} EP_R(447) = 414.13 \\ EP_M(447) = 561.32 \end{cases} \\ (c_1, c_2) = (3.19, 2.03) & \begin{cases} EP_R(447) = 393.22 \\ EP_M(447) = 582.22 \end{cases} \\ (c_1, c_2) = (3.16, 1.99) & \begin{cases} EP_R(447) = 402.67 \\ EP_M(447) = 572.77 \end{cases} \end{aligned}$$

(b)  $Q_R^* < \kappa$

The carbon cap  $\kappa$  could also be larger than 405, say 435 units. In this case, the retailer's profit does not change. However, when the independent retailer orders his optimal 405 units, the manufacturer could sell the extra  $(435 - 405)$  units with  $p_s = 0.7$  and produce retailer's optimal order. This case refers to uncoordinated system and the expected profits will be:  $EP_R(447) = 392.78$  and  $EP_m(447) = 566.3 + 21 = 587.3$ .

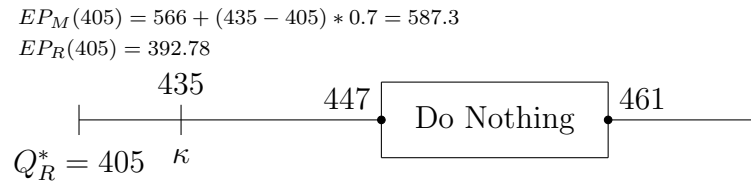


Figure 4.3: The Optimal Policy at Buying Region with  $Q_R^* < \kappa$

When coordination is applied, the objective of both manufacturer and retailer would be 447 units. For the case of  $R = 0$  if  $c_1 = 2.94$  is accepted, the expected profits would be  $EP_R(447) = 401.56$  and  $EP_m(447) = 608.88$ . Again an increase in both of the expected profits compared to the uncoordinated model is observed.

For the case of  $R = 1$  for  $(c_1, c_2) = (3.1632, 1.97)$  the expected profits would be  $EP_R(447) = 402.67$  and  $EP_M(447) = 607.77$ .

The results are summarized as follows:

When  $R = 0$  and  $c_1 < 5.2$

$$c_1 = 2.94 \begin{cases} EP_R(447) = 401.56 \\ EP_M(447) = 608.88 \end{cases}$$

When  $R = 1$  and  $c_1 = 1.81 + 0.68c_2$

$$(c_1, c_2) = (3.16, 1.97) \begin{cases} EP_R(447) = 402.67 \\ EP_M(447) = 607.77 \end{cases}$$

### Coordination when manufacturer sells emission allowance

Suppose the assigned cap is  $\kappa = 500$ ,

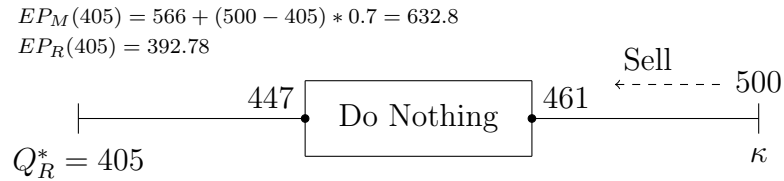


Figure 4.4: The Optimal Policy at Selling Region

The possibility of achieving coordination between two parties happens in two cases:

1. When  $R = 0$  conditioned that  $\frac{g_1(c_1 - c_3)}{p + g - c_3} < \beta p_s$ . It follows that an upper bound for  $c_1$  is:  $c_1 < 3.81$ .

If the manufacturer takes all the profit incurred due to coordination, he would set  $c_1 = 2.93$ . The resulting expected profits would be  $EP_R(461) = 393.47$  and  $EP_M(461) = 668.27$ . If he gives all the profit of coordination to the retailer he would set the wholesale price as  $c_1 = 2.855$  which results in  $EP_R(461) = 428.05$  and  $EP_M(461) = 633.7$ . Looking for the appropriate wholesale price  $c_1$  which

could benefit both parties, we found:  $c_1 = 2.88$ . The expected profit of retailer and manufacturer will be:  $EP_R(461) = 416.5$  and  $EP_M(461) = 645.22$ . The additional profits after the coordination would be  $645.22 - 632.8 = 12.42$  for the manufacturer and  $416.5 - 392.78 = 23.72$  for retailer.

Special cases which result in increase in profits are summarized as follows:

$$c_1 = 2.93 \begin{cases} EP_R(461) = 393.47 \\ EP_M(461) = 668.27 \end{cases}$$

$$c_1 = 2.85 \begin{cases} EP_R(461) = 428 \\ EP_M(461) = 633.7 \end{cases}$$

$$c_1 = 2.88 \begin{cases} EP_R(461) = 416.5 \\ EP_M(461) = 645.22 \end{cases}$$

2. When  $R = 1, c_1 > c_2$ , assuming  $g_1 > \beta p_s$ , then for the appropriately chosen  $c_1$  and  $c_2$  values according to the relation based on them i.e.  $c_1 = 1.54 + 0.73c_2$  The condition is satisfied since  $g_1 = 1.1 > \beta p_s = 0.7$

Suppose  $c_2 = 2.326$  and  $c_1 = 3.24$ . The expected profits will be:  $EP_R(461) = 393.1$  and  $EP_M(461) = 668.6$ . If  $c_2 = 2.188$  and  $c_1 = 3.137$ , the expected profits will be:  $EP_R(461) = 429$  and  $EP_M(461) = 632.75$ .

Letting  $c_2 = 2.25$ , then  $c_1 = 3.182$ . In this setting the expected profits will be:  $EP_R(461) = 412.83$  and  $EP_M(461) = 648.9$ . The additional profit due to coordination will be 20.05 and 16.1 respectively.

The results are summarized as follows(after rounding to two digit decimals):

$$(c_1, c_2) = (3.24, 2.32) \begin{cases} EP_R(461) = 393.1 \\ EP_M(461) = 668.6 \end{cases}$$

$$(c_1, c_2) = (3.13, 2.19) \begin{cases} EP_R(461) = 429 \\ EP_M(461) = 632.75 \end{cases}$$

$$(c_1, c_2) = (3.18, 2.25) \begin{cases} EP_R(461) = 412.83 \\ EP_M(461) = 648.9 \end{cases}$$

**Remark:** What if the parties do not act as what policy suggests? to answer

this question we consider the above case. With  $\kappa = 500$  they have to accept  $Q_s = 461$  as the optimal. Suppose they agree on  $Q_b = 447$  which is the optimal for the channel but in buying region. Intuitively we can find a  $c_1$  which results in coordination with increase in expected profits with respect to the retailer's optimal order  $Q_R^*$ . However the increase would be less than the case when they accept 461. To see this case in details, we proceed with a coordinating  $c_1 = 2.91$ . For 447 units the expected profits will be  $EP_R(447) = 414.97$  and  $EP_M(447) = 644.57$ .

$$c_1 = 2.91 \begin{cases} EP_R(447) = 414.97 \\ EP_M(447) = 644.57 \end{cases}$$

for which the overall profit is  $EP_R(447) + EP_M(447) = 1059.55$ . However the overall profit with 461 units is  $416.5 + 645.22 = 1061.72$ .

If they act based on a order size even less than 447 units, the overall channel profit will decrease accordingly. For instance, ordering 420 units will result in 1042.64 overall channel profit for  $c_1 = 2.96$

$$c_1 = 2.96 \begin{cases} EP_R(420) = 407.21 \\ EP_M(420) = 635.44 \end{cases}$$

If they agree on 500 units, with  $c_1 = 2.8$ , expected profits will be  $EP_R(500) = 406.67$  and  $EP_M(500) = 640.83$ . The overall profit will be  $406.67 + 640.83 = 1047.5$ .

$$c_1 = 2.8 \begin{cases} EP_R(500) = 406.67 \\ EP_M(500) = 640.83 \end{cases}$$

### **Coordination when quantity ordered is in $\kappa/\beta$ region**

Suppose the assigned  $\kappa$  is 455.

$$EP_M(405) = 566 + (455 - 405) * 0.7 = 601.3$$

$$EP_R(405) = 392.78$$

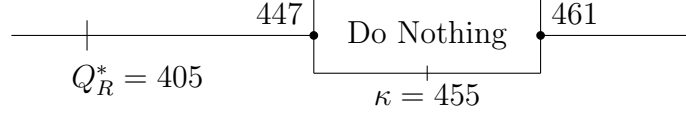


Figure 4.5: The Optimal Policy at "Do Nothing" Region

1. When  $R = 0$

$$c_1 = (p + g) \left[ \bar{F} \left( \frac{\kappa}{\beta} \right) \right] + F \left( \frac{\kappa}{\beta} \right) c_3$$

$$c_1 = (5.7 * 0.291) + (0.709 * 0.5) = 2.013$$

With the wholesale price  $c_1 = 2.013$  the expected profits will be  $EP_R(455) = 816.52$  and  $EP_M(455) = 213.3$ .

$$c_1 = 2.01 \begin{cases} EP_R(455) = 816.52 \\ EP_M(455) = 213.3 \end{cases}$$

If  $\kappa = 460$  then  $\bar{F} \left( \frac{\kappa}{\beta} \right) = 0.73$  and  $c_1 = 1.9$ . Therefore,  $EP_R(460) = 868.28$  and  $EP_M(460) = 165.44$ .

$$c_1 = 1.9 \begin{cases} EP_R(460) = 868.28 \\ EP_M(460) = 165.44 \end{cases}$$

If  $\kappa = 448$  then  $\bar{F} \left( \frac{\kappa}{\beta} \right) = 0.68$  and  $c_1 = 2.14$  then  $EP_R(448) = 759.17$  and  $EP_M(448) = 264.27$ .

$$c_1 = 2.14 \begin{cases} EP_R(448) = 759.17 \\ EP_M(448) = 264.27 \end{cases}$$

2. When  $R = 1, c_2 < c_1$ . If full returns is allowed, the following represents the relation between  $c_1$  and  $c_2$ . With  $\kappa = 455$  and  $F \left( \frac{\kappa - \mu}{\sigma} \right) = 0.709$  the wholesale price would be  $c_1 = 1.659 + 0.709c_2$ .

Suppose that the manufacturer gets all the profit earned due to coordination practice. He will choose  $(c_1, c_2) = (2.196, 3.217)$ . The expected profits would be  $EP_R(455) = 392.96$  and  $EP_M(455) = 636.87$ . If he gives the profit to the

retailer, then  $(c_1, c_2) = (2.055, 3.117)$ . Therefore, the expected profits would be  $EP_R(455) = 428.13$  and  $EP_M(455) = 601.7$

One middle road strategy is to choose  $c_1$  and  $c_2$  such that both will benefit from coordination. For this purpose, the manufacturer sets  $(c_1, c_2) = (2.125, 3.166)$ . Then  $EP_R(455) = 410.96$  and  $EP_M(455) = 618.86$

The results are summarized as follows(after rounding to two digit decimals):

$$\begin{aligned} (c_1, c_2) = (3.21, 2.19) & \begin{cases} EP_R(455) = 392.96 \\ EP_M(455) = 636.87 \end{cases} \\ (c_1, c_2) = (3.12, 2.05) & \begin{cases} EP_R(455) = 428.13 \\ EP_M(455) = 601.7 \end{cases} \\ (c_1, c_2) = (3.17, 2.12) & \begin{cases} EP_R(455) = 410.96 \\ EP_M(455) = 618.86 \end{cases} \end{aligned}$$

### Profit Shares Of the Manufacturer and the Retailer

Let

$$PS_b = \frac{EP_R(447)}{EP_R(447) + EP_M(447)}$$

$$PS_s = \frac{EP_R(461)}{EP_R(461) + EP_M(461)}$$

$$PS_n = \frac{EP_R(455)}{EP_R(455) + EP_M(455)}$$

represent the profit share of retailer from the total channel profit when the order quantity falls within  $Q_b$ ,  $Q_s$  and "Do Nothing" regions respectively.

When  $R = 0$ ,

$$PS_b = \frac{401.56}{401.56 + 573.88} = \frac{401.56}{975.44} = 0.411$$

$$PS_s = \frac{416.5}{416.5 + 645.22} = \frac{416.5}{1061.72} = 0.392$$

$$PS_n = \frac{816.52}{816.52 + 213.3} = \frac{816.52}{1029.82} = 0.793$$

When  $R = 1$ ,

$$PS_b = \frac{402.67}{402.67 + 572.77} = \frac{402.67}{975.44} = 0.413$$

$$PS_s = \frac{412.83}{412.83 + 648.9} = \frac{412.83}{1061.73} = 0.389$$

$$PS_n = \frac{410.96}{410.96 + 618.86} = \frac{410.96}{1029.82} = 0.399$$

It is observed that in every case except when in "Do Nothing" region the manufacturer allows no returns, the profit share of retailer would be approximately 40%. It is also interesting to make a comparison with the case when the retailer orders his optimal. In buying region for instance, the proportion of profit incurred by retailer is 0.411. Coordination increased his profit but no changes happened in the profit share. In selling region this would be 0.383. In case when  $\kappa$  is in between, the proportion of retailer's profit to the total under retailer's optimal is 0.395. So the majority of the cases suggest that it is favorable for retailer to accept coordination.



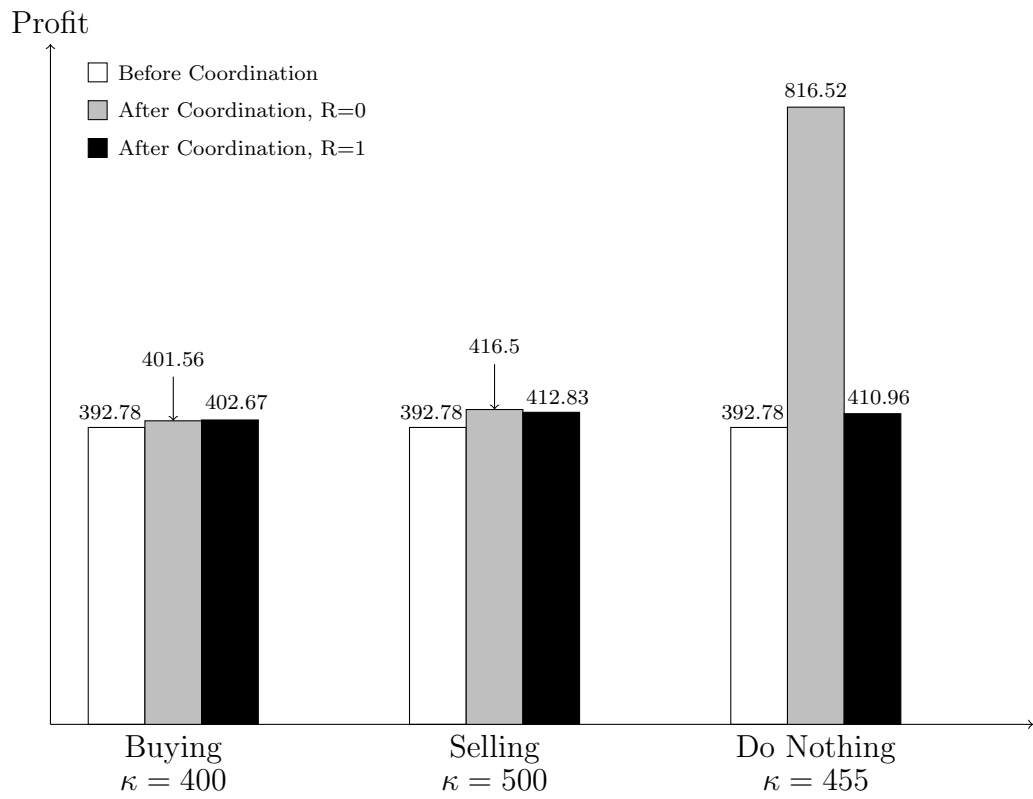


Figure 4.6: Profit of The Retailer Before and After Coordination

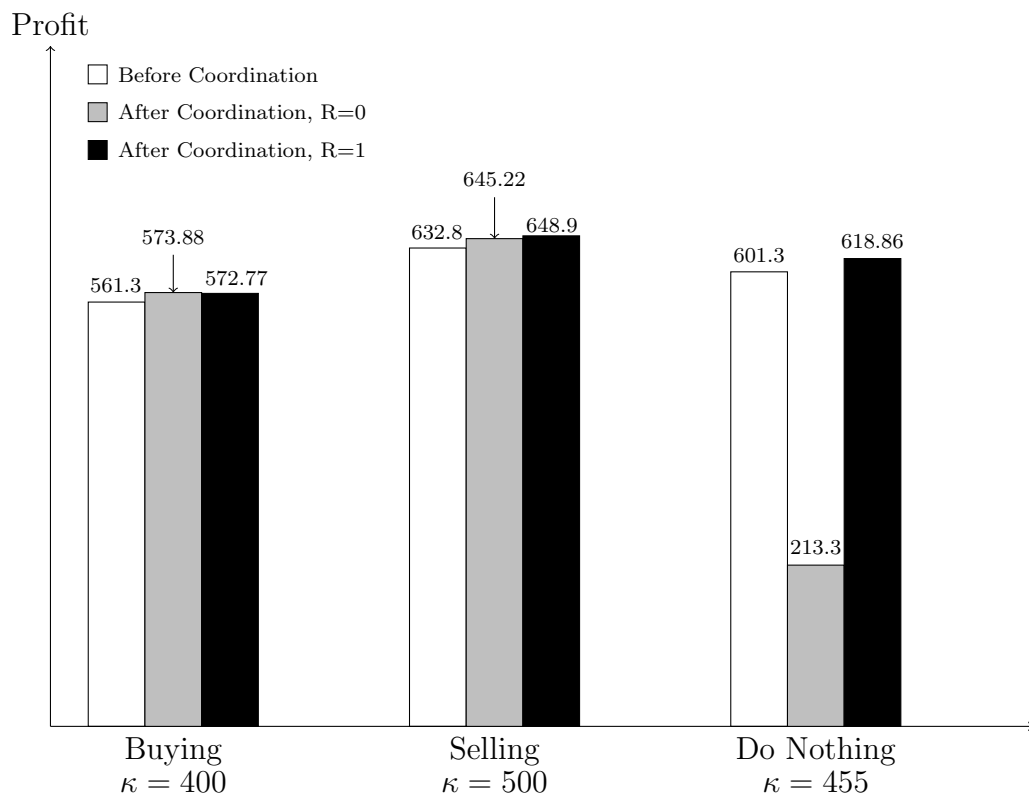


Figure 4.7: Profit of The Manufacturer Before and After Coordination

# Chapter 5

## Coordination Under Rebate Contract

### Introduction

In this chapter, we show that a policy that combines the use of wholesale price and channel rebate for the sales beyond a pre-determined amount coordinates a channel composed of a retailer and a manufacturer where the manufacturer is exposed to restrictions on the amount of carbon emission with trading option. According to the rebate contract, the manufacturer pays the retailer a predetermined credit  $u$  for the sales beyond a predetermined target quantity of  $T$ . We assume that the goodwill cost associated with unsatisfied demand is negligible. Therefore the order quantities in each case of buying and selling carbon allowance will have the following form:

$$Q_b = F^{-1} \left( \frac{p - c - \beta p_b}{p - c_3} \right) \quad (5.1)$$

$$Q_s = F^{-1} \left( \frac{p - c - \beta p_s}{p - c_3} \right) \quad (5.2)$$

The Independent Retailer Without Rebate:

The independent retailer's problem under wholesale price only contract is identical to that of the integrated channel with no cap, except that wholesale price  $c_1$  replaces manufacturing cost  $c$  in the optimal order quantity of integrated channel. Let  $Q_0$  and  $\underline{r}$  denote the retailer's optimal order quantity and profit under wholesale price only contract respectively.

$$Q_0 = F^{-1}\left(\frac{p - c_1}{p - c_3}\right). \quad (5.3)$$

$$\underline{r} = (p - c_3)\Gamma(Q_0) \equiv (p - c_3)\Gamma\left[F^{-1}\left(\frac{p - c_1}{p - c_3}\right)\right] \quad (5.4)$$

where  $\Gamma(Q) = \int_0^Q xf(x)dx$  and denotes the truncated expected value.

## 5.1 Independent Retailer With Rebate

The following discussion follows from Taylor [10]. We restate his results for completeness.

Let  $r(Q|T)$  represent the retailer's expected profit function under a target rebate contract.

$$r(Q|T) = (p - c_1)E[\min(x, Q)] - (c - c_3)E(Q - x)^+ + uE[\min(x, Q) - T]^+ \quad (5.5)$$

Where

$$E[\min(x, Q)] = \int_0^Q xf(x)dx + Q \int_Q^\infty f(x)dx$$

$$E(Q - x)^+ = \int_0^Q (Q - x)f(x)dx$$

$$E[\min(x, Q) - T]^+ = \int_T^Q (x - T)f(x)dx + (Q - T)[1 - F(Q)]$$

Differentiating the expected profit function for two cases of  $Q \leq T$  and  $Q > T$

and rewriting it using the above expressions results in the following:

$$r(Q|T) = \begin{cases} (p - c_1)Q - (p - c_3) \int_0^Q (x - Q)f(x)dx & \text{if } Q \leq T \\ (p - c_1)Q - (p - c_3) \int_0^Q (x - Q)f(x)dx \\ + u \left[ \int_T^Q (x - T)f(x)dx + (Q - T)[1 - F(Q)] \right] & \text{if } Q > T \end{cases} \quad (5.6)$$

Applying first order conditions (FOC) enables us to find the optimal order quantities:

$$\frac{\partial r(Q|T)}{\partial Q} = \begin{cases} (p - c_1) - (p - c_3)F(Q) & \text{if } Q \leq T \\ (p + u - c_1) - (p + u - c_3)F(Q) & \text{if } Q > T \end{cases}$$

$$F(Q_R) = \begin{cases} \frac{p - c_1}{p - c_3} & \text{if } Q \leq T \\ \frac{p + u - c_1}{p + u - c_3} & \text{if } Q > T \end{cases} \quad (5.7)$$

Note that  $r(\cdot|T)$  is concave on  $[0, T)$  and  $[(T, \infty)$ . Let  $Q_0$  and  $Q_1$  denote the retailer's optimal order quantities for the cases of with rebate and no rebate respectively.

$$Q_0 = F^{-1} \left( \frac{p - c_1}{p - c_3} \right) \quad (5.8)$$

$$Q_1 = F^{-1} \left( \frac{p + u - c_1}{p + u - c_3} \right) \quad (5.9)$$

When  $Q \leq T$ , the problem turns to the independent retailer's problem with no rebate under wholesale price only contract. For which we have also defined the optimal order quantity with  $Q_0 = F^{-1} \left( \frac{p - c_1}{p - c_3} \right)$ . Clearly  $Q_0 < Q_1$  for  $u \geq 0$

Define  $f_0(T) \equiv r(Q_0|T) - r(Q_1|T)$  on  $T \in [Q_0, Q_1]$  and define  $\tau_0$  to satisfy  $f_0(\tau_0) = 0$ . This implies that there exist such a sales target between two optimal order size of the retailer  $Q_0$  and  $Q_1$  for which the retailer's expected profit will be the same no matter which optimal order size is placed.

**Theorem 5.1.** (a)  $\tau_0$  exists, is unique, and satisfies  $\tau_0 \in (Q_0, Q_1)$

(b) The optimal ordering policy for the retailer under a target rebate,  $Q_R^*$ , is given by the following:

$$Q_R^* = \begin{cases} Q_1 & \text{if } T < \tau_0 \\ Q_0 & \text{if } T > \tau_0 \\ \{Q_0, Q_1\} & \text{if } T = \tau_0 \end{cases} \quad (5.10)$$

Proof from Taylor [10]:

It is straightforward to show that if  $T \leq Q_0$ , then  $Q_1$  maximizes  $r(\cdot|T)$  and if  $T \geq Q_1$ , then  $Q_0$  maximizes the  $r(\cdot|T)$ .

If  $Q_0 \leq T \leq Q_1$ , then  $r(Q_0|T) = (p - c_3)\Gamma(Q_0)$  and  $r(Q_1|T) = (p + u - s)\Gamma(Q_1) - u[\Gamma(T) + T(1 - F(T))]$ . Because  $f_0(Q_0) < 0 < f_0(Q_1)$ , and  $f_0(\cdot)$  is continuous and increasing, there exist a single valued inverse function  $f_0^{-1}$  and a unique  $\tau_0$ ; further,  $\tau_0 \in (Q_0, Q_1)$ . If  $Q_0 < T < Q_1$ , then  $\lim_{Q \rightarrow T^-} (\partial/\partial Q)r(Q|T) < 0 < \lim_{Q \rightarrow T^+} (\partial/\partial Q)r(Q|T)$ . Because  $Q_0$  maximizes  $r(\cdot|T)$  on  $[0, T)$  and  $Q_1$  maximizes  $r(\cdot|T)$  on  $(T, \infty)$ , then  $Q_R^* = \arg \max_{Q \in [Q_0, Q_1]} r(Q|T)$ . If  $T < \tau_0$ , then  $f_0(T) < 0$  and  $r(Q_1|T) > r(Q_0|T)$ . If  $T > \tau_0$ , then  $f_0(T) > 0$ .

Regarding the above theorem we have the following interpretations.

The retailer's optimal order quantity  $Q_R^*$  is decreasing in  $T$ . Consider the extreme values of  $T$ . If  $T$  is extremely large, then the probability of selling units beyond  $T$  is very small. To receive rebate, the retailer's order size must be larger than  $T$ . However, in such a case the resulting overage cost exceeds the expected revenue from the rebate. As a consequent, the retailer behaves as if there exists no rebate and orders the same quantity in the no rebate case. In other words he will order  $Q_0$ . If  $T = 0$ , then the the retailer receives  $p + u$  for each unit sell. However, in this case the manufacturer will increase the retail price to compensate the loss. Hence the optimal order quantity for the retailer will be the newsvendor quantity where the retail price is  $p + u$ , which is denoted by  $Q_1$ .

For intermediate values of  $T$ , the retailer has to choose whether wants the rebate i.e., orders more than  $T$  or not. In either case, the retailer's optimal order quantity is a function of expected cost and marginal revenue from an incremental sale.

If the retailer orders more than  $T$ , the marginal revenue is  $p + u$ ; if he orders less than  $T$ , the marginal revenue will be  $p$ . As  $T$  increases, the attractiveness of going for the rebate decreases. At the threshold  $\tau_0$  the retailer is indifferent between ordering  $Q_0$  and  $Q_1$ . (See Taylor [10])

The retailer profit under a target rebate is

$$r = \begin{cases} (p + u - s)\Gamma(Q_1) - u[\Gamma(T) + T(1 - F(T))] & \text{if } T < \tau_0 \\ (p - c_3)\Gamma(Q_0) & \text{if } T \geq \tau_0 \end{cases} \quad (5.11)$$

## 5.2 Channel Coordination

Recall that in order to achieve channel coordination the order quantity of two parties must be equal. That is, the order quantity when the manufacturer acts as there exists no retailer (with  $Q_s$ ,  $Q_b$  and  $\kappa/\beta$ ) with the case when there exists an independent retailer ( $Q_R^*$ ) must be equal. In this part, we study the settings for each of contract parameters, i.e., wholesale price  $c_1$ , rebate  $u$  and target sales  $T$ , that may lead to achieve coordination. Similar to the return contract we have three cases: selling, buying the emission allowance and using all available allowance that is "Do Nothing" region.

### 5.2.1 Coordination when Manufacturer Sells Emission Allowance

Let  $\pi_s$  denote the channel's profit corresponding to the optimal order quantity  $Q_s$ .

**Proposition 5.1.** *Let  $u_s^*(c_1)$  denote the optimal rebate amount in the selling*

region. For any  $\delta \in (0, \pi_s)$ ,  $u_s^*(c_1)$  is set such that:

$$u_s^*(c_1) = \frac{(p - c_3)(c_1 - c - \beta p_s)}{c + \beta p_s - c_3}$$

Proof:

It follows from the first order condition for the case of  $Q > T$ , using  $F(Q_s) = \frac{p - c - \beta p_s}{p - c_3}$ .

$$\frac{\partial r(Q|T)}{\partial Q} = (p + u - c_1) - (p + u - c_3)F(Q_s) = 0$$

The optimal rebate amount is a function of wholesale price  $c_1$ .

$$u_s(c_1) - u_s(c_1)F(Q_s) = c_1 - p + (p - c_3)F(Q_s)$$

$$u_s(c_1)[1 - F(Q_s)] = c_1 - p + (p - c_3)F(Q_s)$$

$$u_s(c_1) = \frac{(c_1 - p) + (p - c_3)F(Q_s)}{\bar{F}(Q_s)}$$

$$u_s^*(c_1) = \frac{(p - c_3)(c_1 - c - \beta p_s)}{c + \beta p_s - c_3} \quad (5.12)$$

**Remark.**

The equation above represents the relation of two of the contract parameters, namely wholesale price  $c_1$  and target rebate  $u$ . As the equation displays, the target rebate is a function of the wholesale price. The equation, however does not result in a single combination of  $c_1$  and  $u_s^*(c_1)$ . Suppose the manufacturer and retailer agree on a profit sharing structure. When the retailer is a profit maximizer, he orders his optimal  $Q_R^*$ , which amounts to the expected profit of  $\underline{r}$ . Suppose by adding  $\epsilon$  to the retailer's profit, the manufacturer increases the retailer's profit from the current value  $\underline{r}$  to the agreed amount  $\delta$ . Then (5.12) will lead to a single  $c_1, u_s^*(c_1)$  pair which will result in the above profit sharing.

**Proposition 5.2.** For any  $\delta \in (0, \pi_s)$ , (a). The optimal wholesale price  $c_1^*$



is set such that:

$$(p - c_3)\Gamma \left[ F^{-1}\left(\frac{p - c_1}{p - c_3}\right) \right] = \delta - \epsilon \quad (5.13)$$

(b). The optimal sales target is set such that:

$$(p + u_s^* - c_3)\Gamma(Q_s) - u_s^*(\Gamma(T^*) + T^*(1 - F(T^*))) = \delta \quad (5.14)$$

Proof (a): It is straightforward. In order to realize  $\delta$  as the new expected profit of the retailer,  $c_1$  is the only contract parameter which influences the retailer's profit margin. Intuitively, one expects  $c_a$  to be decreased if an increase in retailer's profit is expected.

Proof (b): It is straightforward. We mentioned that the manufacturer convinced the retailer to set the channel optimal order quantity as his optimal. The retailer agreed to the channel optimal  $Q_s$  in return of an increase equal to  $\epsilon$  in his expected profit. With increase in profit his new expected profit will reach to  $\delta$ . As a consequent,  $T$  will be set such that the retailers profit under target rebate will be  $\delta$ . The left hand side of the equality sign refers to the expected profit of retailer after rebate is realized from (5.11).

### Corollary 5.1.

Under the coordinating contract parameters, the resulting profit to the manufacturer and retailer will be  $m_s^* = \pi_s - \delta$  and  $r_s^* = \delta$  respectively.

## 5.2.2 Coordination when Manufacturer Buys Emission Allowance

Let  $\pi_b$  denote the channel's profit with the optimal order quantity  $Q_b$ .

**Proposition 5.4.** *Let  $u_b^*(c_1)$  denote the optimal rebate amount in the buying region. For any  $\delta \in (0, \pi_b)$ ,  $u_b^*(c_1)$  is set such that:*

$$u_b(c_1) = \frac{(p - c_3)(c_1 - c - \beta p_b)}{c + \beta p_b - c_3} \quad (5.15)$$

Proof is similar to the same case in selling region.

**Proposition 5.5.** *For any  $\delta \in (0, \pi_b)$ , (a). The optimal wholesale price  $c_1^*$  is set such that:*

$$\underline{r} = \delta - \epsilon \quad (5.16)$$

(b). *The optimal sales target is set such that:*

$$(p + u_b^* - c_3)\Gamma(Q_b) - u_b^*(\Gamma(T^*) + T^*(1 - F(T^*))) = \delta \quad (5.17)$$

Proofs are similar to the same case in selling region.

**Corollary 5.2.**

Under the coordinating contract parameters, the profit of manufacturer and retailer will be  $m_b^* = \pi_b - \delta$  and  $r_b^* = \delta$  Respectively.

### 5.2.3 Coordination when Manufacturer Produces $\kappa/\beta$ Units

Let  $\pi_n$  denote the channel's profit with the optimal order quantity  $\kappa/\beta$ .

**Proposition 5.7.** *Let  $u_n^*(c_1)$  denote the optimal rebate amount in "Do Nothing" region. For any  $\delta \in (0, \pi_n)$ ,  $u_n^*(c_1)$  is set such that:*

$$u_n(c_1) = \frac{(p - c_3)F(\kappa/\beta)(c_1 - p)}{\bar{F}(\kappa/\beta)} \quad (5.18)$$

Proof is similar to the same case in selling region.

**Proposition 5.8.** *For any  $\delta \in (0, \pi_n)$ , (a). The optimal wholesale price  $c_1^*$*

is set such that:

$$\underline{r} = \delta - \epsilon \quad (5.19)$$

(b). The optimal sales target is set such that:

$$(p + u_n^* - c_3)\Gamma(\kappa/\beta) - u_n^*(\Gamma(T^*) + T^*(1 - F(T^*))) = \delta \quad (5.20)$$

Proof is similar to the same case in selling region.

**Corollary 5.3.**

Under the coordinating contract parameters, the profit of manufacturer and retailer will be  $m_n^* = \pi_n - \delta$  and  $r_n^* = \delta$  respectively.

### 5.3 Numerical Analysis

In this section, for the problem 2 the implementation of the policy obtained through the numerical example is conducted. While in order to make the comparison of the results with the result of problem 1, the values assumed in the problem 1 are utilized.

$c = 1.5$	$g = 0$	$N(\mu, \sigma) = N(400, 100)$	$p = 4.5$
$c_1 = 3$	$g_1 = 0$	$R = 0$	$p_b = 1$
$c_2 = 0$	$g_2 = 0$	$\beta = 1$	$p_s = 0.7$
$c_3 = 0.5$			

Table 5.1: Input Data for The Rebate Contract

The integrated channel case:  $F(Q_b) = \frac{p - c - \beta p_b}{p - c_3} = \frac{4.5 - 1.5 - 1}{4.5 - 0.5} = \frac{2}{4} = 0.5$   
 For  $F(z) = 0.5$ , from the standard normal table we find the corresponding  $z$  value to be  $z = 0$ .  $Q_b = \sigma z + \mu = 400$ . For the case of selling extra limit  $F(Q_s) = \frac{4.5 - 1.5 - 0.7}{4.5 - 0.5} = 0.575$ . Then  $z = 0.19$  for  $F(z) = 0.575$ . Therefore  $Q_s = 0.19 * 100 + 400 = 419$ . This implies that when the shortage is not punished, i.e.  $g_2 = 0$ , the independent manufacturer will produce less than the case when shortage is penalized.

Independent Retailer Without Rebate:

$F(Q_0) = \frac{p - c_1}{p - c_3} = \frac{4.5 - 3}{4.5 - 0.5} = 0.375$ . From the table  $F(z) = 0.375$  and  $z = -0.32$ . Therefore  $Q_0 = 400 - 32 = 368$ .  $\Gamma(Q_0) = 111.89$ . The retailer's profit will be  $\underline{r} = (p - c_3)\Gamma(Q_0) = (4.5 - 0.5) * 111.89 = 447.56$ .

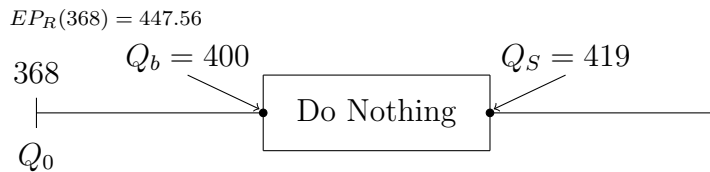


Figure 5.1: The Scenario Without Rebate

Independent Retailer With Rebate:

Channel coordination requires the optimal order sizes of integrated channel and independent retailer to be equal. Suppose the given  $\kappa = 450$ . Since  $\kappa > Q_s\beta$ . We have to reduce the order quantity to 419 units. , We proceed to find the optimal values for system parameters.

$$\begin{aligned} u_s(c_1) &= \frac{(p - c_3)(c_1 - c - \beta p_s)}{c + \beta p_s - c_3} \\ &= \frac{(4.5 - 0.5)(c_1 - 1.5 - 0.7)}{1.5 + 0.7 - 0.5} \\ &= 2.35c_1 - 5.17 \end{aligned}$$

Target rebate is a function of wholesale price  $c_1$  and independent of target sales  $T$ . The retailer will accept the channel optimal order size  $Q_s$  only if the above relation holds. However, there exists no single solution for it with respect to the  $c_1$ . To crack down the problem, we need a measure to decide on the  $c_1$ . As mentioned before, we assume that the concept of coordination is in such a way that the manufacturer increases the profit of the retailer to 600 from the current amount 447.56. That means,  $\delta = 600$ , and  $\underline{r} = 447.56$ , and  $\epsilon = 600 - 447.56 = 152.44$ .

Suppose  $c_1^*$  is set in such a way that  $\underline{r} = 600$

$$(p - c_3)\Gamma \left[ F^{-1} \left( \frac{p - c_1^*}{p - c_3} \right) \right] = 600$$

$$\Gamma \left[ F^{-1} \left( \frac{p - c_1^*}{p - c_3} \right) \right] = \frac{600}{4} = 150$$

$$\Gamma(Q) = 150 \implies \int_0^Q x f(x) dx = 150$$

$$\text{Solving this integral with MATLAB, gives } Q = F^{-1} \left( \frac{p - c_1^*}{p - c_3} \right) = 394$$

$$F(394) = \frac{p - c_1^*}{p - c_3} \implies p(D \leq 394) = \frac{p - c_1^*}{p - c_3}$$

$$p \left( \frac{D - 400}{100} \leq \frac{394 - 400}{100} \right) = \frac{p - c_1^*}{p - c_3} \implies F(-0.06) = \frac{p - c_1^*}{p - s}$$

$$\frac{p - c_1^*}{p - s} = 0.476 \implies c_1^* = 4.5 - 4 * 0.474 = 2.6$$

Back to the optimal rebate amount:  $u_s(c_1) = 2.35c_1 - 5.17 = 2.35 * 2.6 - 5.17 = 0.94$

The target sales is obtained from the following expression.

$$(p + u_s^* - c_3)\Gamma(Q_s) - u_s^*(\Gamma(T^*) + T^*(1 - F(T^*))) = \delta$$

$$(4.5 + 0.94 - 0.5) * 191 - 0.94(\Gamma(T^*) + T^*(1 - F(T^*))) = 600$$

$$343 = 0.94[\Gamma(T^*) + T^*(1 - F(T^*))]$$

$$365 = \Gamma(T^*) + T^*(1 - F(T^*))$$

$$T = 410.$$

Under  $Q_s$  the expected profit of the integrated channel is 764. This is regardless of the profit which incur with allowance selling. If we add  $(450 - 419) * 0.7 = 21.7$  to 764, the overall profit will be 785.7. If the retailer goes for rebate, the manufacturer's profit after giving  $600 - 447.76 = 152.24$  to the retailer will be  $m_s^* = 785.7 - 152.24 = 633.46$ . However if the manufacturer can not convince the retailer for the coordinated order size his expected profit will be  $(p - c_3)\Gamma(368) + (450 - 368)0.7 = 447.56 + 57.4 = 505$

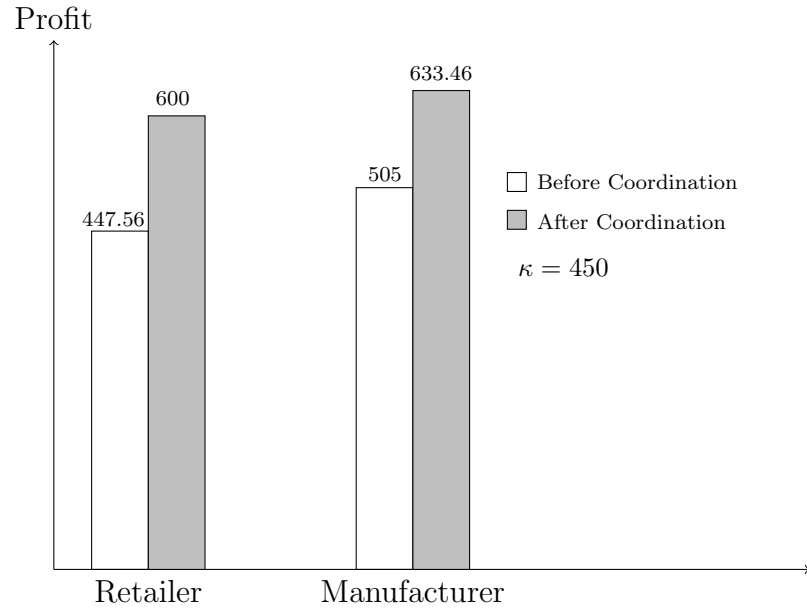


Figure 5.2: Profit of The Manufacturer and Retailer Before and After Coordination when Manufacturer Sells Extra Allowance

# Chapter 6

## Extensions and Conclusion

In this chapter, we review how the findings contribute to the objective of the thesis. We have studied the coordination of a manufacturer and a retailer under the cap and trade regulation of carbon emission. To this end, the conditions of optimality for the parameters of different contract types are uncovered. We have created a conceptual model of each contract type. Return and rebate contracts have been investigated in detail. We have demonstrated how contracting parameters can be derived based on the profit share in order to achieve coordination. We have explored and analyzed supply chain coordination where the drivers of each contract type are discussed and the optimal values are found.

Our results for the return contract show that when no returns is allowed, the manufacturer and retailer may achieve coordination. In this case, the wholesale price is set such that the revenue or loss due to the selling and buying the limit is shared between the members. Besides, when the manufacturer allows for full returns, the channel may be coordinated as well. We looked for such combinations of wholesale price and the return price which increased the expected profit of both manufacturer and retailer compared to the uncoordinated channel and a win-win outcome. The results show that, instead of selling the extra emission allowance to the market, the manufacturer stimulates the retailer to order channel optimal order size rather than retailer's optimal. This way, an increase in the manufacturer's profit margin is observed which exceeds the profit from the selling of extra

allowance to the market directly. In the buying region the intuition is straightforward. The manufacturer benefits from selling the product to the retailer; even if he incurs the cost of buying the required emission allowance. The profit from the unit production is larger than the unit emission cost. Consequently, whatever the retailer's optimal order is, the manufacturer offers the retailer to order channel optimal. Due to the increase in the retailer's expected profit, he will accept the offer. Finally in "Do Nothing" region, the manufacturer neither buys nor sells any allowance. Based on our numerical results, an increase to the expected profits of both parties after the practice of coordination was incurred. However in "Do Nothing" region, the case of no returns was costly for the manufacturing part. In fact, the retailer accepts the channel optimal order size with a wholesale price which is not interesting for the manufacturer.

In short the results of the return contract is as follows:

1. Coordination is achievable even if no returns is allowed.
2. If full returns with full credit is allowed, the channel could not be coordinated.
3. If full returns with partial credit is allowed, the channel could be coordinated.
4. Coordination can be a profit maximizing action in all cases considered, except in  $\kappa/\beta$  region with no returns permission.
5. If selling and buying prices are close together, the  $Q_s$  and  $Q_b$  will approach together. This leads to a single ordering policy for the manufacturer.
6. The more larger the assigned cap is, the more profit will be incurred for both manufacturer and retailer. Similarly small vales of cap decreases the corresponding profits of manufacturer and retailer.
7. If the assigned cap converges to zero, the manufacturer will accept the retailer's optimal instead of motivating him for channel optimal.



For the rebate contract the results are summarized as follows:

1. Coordination is achievable with appropriate choices of target sales, rebate and wholesale price.
2. Manufacturer has to sacrifice a proportion of his expected revenue due to coordination practice in order to motivate the retailer for channel optimal order quantity.
3. As the assigned  $\kappa$  increases, the manufacturer will be able to offer larger increases to retailer's profit by reducing the wholesale price and reducing target sales.
4. Small values of  $\kappa$  restricts the manufacturer to offer more profit to the retailer in order to motivate him for the channel optimal order quantity.

Other directions for future research include the coordination issue with the supply chain composed of a manufacturer, a distributor and a retailer. How to determine the coordination method when the incentive factor for less emission is incorporated, can also be a future work. The competing retailers and manufacturers case could also be an extension for this research. It would also be interesting to extend our single-period setting into a multi-period one, with consideration of ordering and production policies of the retailer and the manufacturer over multiple planning horizons.

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# Appendix A

## Appendix

### Table of Notations:

$c$	Manufacturing Cost
$c_1$	Wholesale Price
$c_2$	Return Price
$c_3$	Salvage Value
$p$	Retail Price
$p_b$	Buying Price of Emission Allowance
$p_s$	Selling Price of Emission Allowance
$g$	Retailer's Goodwill Cost
$g_1$	Manufacturer's Goodwill Cost
$g_2 = g + g_1$	Total Goodwill Cost
$R$	Return Rate
$T$	Target Sales
$u$	Rebate value
$\kappa$	Carbon Emission Cap
$\beta$	Emission Per Unit Product
$X$	Demand Random Variable
$f(X)$	Demand's Density
$F(X)$	Demand's Distribution Function