DYNAMIC IMPLICATIONS OF PROSPECT UTILITY IN AN OVERLAPPING GENERATIONS MODEL

A Master's Thesis

by
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September 2014

To my mother

DYNAMIC IMPLICATIONS OF PROSPECT UTILITY IN AN OVERLAPPING GENERATIONS MODEL

Graduate School of Economics and Social Sciences
of
İhsan Doğramacı Bilkent University

by

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

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This thesis studies an overlapping generations model in the presence of prospect theory which has scarcely been addressed in macroeconomic growth models. The set up in this thesis provides us a unique steady state with global convergence and multiple steady states with local convergence. The presence of prospect preferences in the utility form leads to the multiplicity even under convex technology. Numerical analysis supports us that cross country income divergence can also be explained by a mechanism in which preference component is altered.

Keywords: Prospect utility, Overlapping generations, Threshold dynamics.

ÖZET

BEKLENTI KURAMı FAYDA FONKSIYONUNUN ARDıŞıK KUŞAKLAR MODELI ÜZERINE DINAMIK ETKILERI

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Bu tez, makro iktisadi büyüme modellerinde nadir kullanılan beklenti kuramı fayda fonksiyonunun varlığıyla ardışık kuşaklar modelini çalışmaktadır. Bu tez içinde ki kurgu bize bir tane kararlı durum noktası ile global yakınsama ve birden fazla kararlı durum noktası ile lokal yakınsama sağlamaktadır. Fayda fonksiyonu içindeki beklenti kuramı tercihlerinin varlığı konveks teknoloji altında bile çeşitlilik doğurur. Nŭmerik analizler ülkeler arasındaki gelir ayrımının tercih bileşenin değiş açıklanabildiği bir mekanizmayı desteklemektedir.

Anahtar sözcükler: Beklenti kuramı fayda fonksiyonu, Ardışık kuşaklar, Eşik dinamikleri.

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CHAPTER 1

INTRODUCTION

The classical optimal growth models deem convex technology as the assurance of monotonic convergence of capital stocks to a unique steady state. Nevertheless, this model structure cannot be used to apprehend why different development patterns exist in the long term. To explain divergence in growth paths, models in which market frictions available that affect the technology can cause increasing returns have been presented. Optimal paths determined by Dechert and Nishimura (1983) and Mitra and Ray (1984) prove the existence of critical dynamics that lead poverty and development traps in models with convex-concave technology. In such models, the initial level of capital stocks turn out to be decisive in directing an economy converges to a higher or a lower steady state.

Studies which deliver multiple steady states concentrate on the technology component leaving the preference component essentially unaltered and they concentrate on convex regimes. This thesis advocates another mechanism, *prospect utility* to explain different long term development patterns among countries by showing the existence of multiple steady states with convex technology.

Prospect theory is introduced by Kahneman and Tversky (1979) as an alternative representation of preferences instead of expected utility theory; which is considered to be the standard model of individual decision making. They motivate their ideas by using their experimental evidence. In these experiments, they observe that people sometimes violate some of the expected utility theory axioms, especially independence of irrelevant alternatives. The importance of prospect theory is its descriptive ability because it is successful in predicting the decision makers' behavior consistently. Prospect theory is counted as one of the most influential theories for behavioral decisions under risk; therefore, this paper relies on this theory.

The prospect theory is suggested as a modification of classical expected utility mainly in the following two points:

1. Prospect theory builds on the situation that agents value their prospects in gains and losses relative to a reference point whereas the expected utility theory determines the final wealth. Boulding (1981) emphasizes the importance of reference dependence: "...the perception of potential threats to survival may be much more important in determining behavior than the perceptions of potential profits, so that profit maximization is not really the driving force: it is fear of loss rather than hope of gain that limits our behavior."

2. Prospect utility incorporates a kink at a reference point which shaped the utility function is convex in losses and concave in gains.

This thesis focuses on these two features to add an alternative explanation to cross country income divergence. It is now worth observing the development patterns of a country which has prospect preference.

The prospect theory has been actively employed in many fields such as; in finance, Barberis and Huang (2008) show that how pricing of financial securities change in accordance to probability weighting parameter; in insurance, Sydnor (2010) concludes that most agents have over insurance on risky choices; in industrial organization, Heidhues and Koszegi (2008) states that firms differentiate prices in case of customers have prospect theory preferences. In financial economics literature, Li and Yang (2013) build a general equilibrium model to investigate the effects of prospect theory for the disposition effect, asset prices and trading volume. Moreover, Koszegi and Rabin (2009) and Foellmi, Rosenblatt-Wisch and Schenk-Hoppe (2011) study consumption-saving decisions under prospect utility in an optimal growth model in presence of habit formation. Nevertheless, none of these studies determine the dynamic implications of prospect utility in growth theory, specifically, in an overlapping generations economy.

In this thesis, we adapt a two-period overlapping generations model to include such a preference structure which is embodied with prospect theory to analyze the equilibrium dynamics. To observe the dynamics, the representative young household's maximization problem at period t is first solved then the saving locus is obtained and pass to the long run dynamics. Based on the policy function which links the capital stock at time t and $t + 1$, this thesis provides a model that can explain why some countries encounter development trap. The thesis, even under a convex technology, could explain persistent cross country income differences in a standard two-period overlapping generations model in which prospect theory is augmented.

The key reason of our results is that the importance of reference dependence in a manner that the solution of representative young household's maximization problem which has prospect utility. Incorporating such an hypothesis on preferences, our model supports unique optimal steady state with global convergence and multiple steady states with local convergence. The presence of prospect preferences in the utility leads to the multiplicity even under a convex technology. This thesis also analyzes how initial level of capital stock behave in response to a change in preference parameters.

This thesis is organized as follows: In part 2, model of the economy is specified and saving locus of representative young household is derived. In part 3, analysis of dynamics under convex technology and implications of benchmark parametrization are presented. Part 4 includes summary of findings and conclusion.

CHAPTER 2

THE MODEL

We consider an economy with two period overlapping generations, one young household and one old household. Each household is alive for two periods and at each point in time these two generations overlap. When young, each household is endowed with one unit of labor, which are inelastically supplied to the labor market. In return for their supplied labor, they earn a wage rate of w_t. This amount of income is allocated into current consumption, c_t, and savings, s_t. The budget constraint of the representative young agent born at period t,

$$c_t + s_t = w_t$$

When he gets old at time $t+1$, in his second period of life, the agent gets retired. At this time, they owe their income to their savings made at time t. Not only the savings but also the return to savings,R^e_{t+1}, are entirely consumed in period 2 because old agents do not care about happenings after their death. So, in second period of life, the income of an old agent is $R^e_{t+1}s_t$, where $R^e_{t+1} = (1 + r_{t+1})$ and budget constraint is

$$d^e_{t+1} = R^e_{t+1}s_t$$

In our model, agents value their prospects in terms of gains and losses based on a reference point. Households are more averse to losses than gains.

With this in mind, our model displays an implementation of prospect theory in the utility function of a representative young household born at period t. To see the effects of prospect preference by ϕ, representative young agents solves the following problem

$$\max_s u\left(c_t\right) + \beta\left[\left(1-\phi\right)u\left(d^e_{t+1}\right) + \phi\nu\left(d^e_{t+1} - c_t\right)\right] \tag{0.1}$$

subject to

$$c_t + s_t = w_t \tag{0.2}$$

$$d^e_{t+1} = R^e_{t+1}s_t \tag{0.3}$$

$$c_t \geq 0,\ d^e_{t+1} \geq 0, s_t \geq 0. \tag{0.4}$$

The theory suggests that loss aversion at the kink of the value function is more relevant than the degree of curvature away from the kink. For easiness, we make $\nu(d_{t+1} - c_t)$ linear over both gains and losses.

We define a piecewise-linear prospect utility function to identify the asymmetry between gains and losses

$$v\left(\Delta_t\right) = \{\Delta_t \quad if \;\; \Delta_t \geq 0, \lambda\Delta_t \;\; if \;\; \Delta_t < 0,$$

where $\Delta_t = d_{t+1} - c_t$ and $\lambda > 1$ captures the loss aversion.

The utility function is not differentiable at the kink;however, it has to be continuously differentiable for the entire region of gains and losses. To obtain differentiability, we should modify the utility function in a way that the loss aversion coefficient, λ, and the utility part form an entity. Hence, constructing loss aversion coefficient as a switching function is needed. Considering the assumption of the loss aversion coefficient in piecewise-linear form, λ should be greater than 1 to weigh losses more than the gains and equal to 1 to weigh gains more than losses. Hence, its value should switch whenever as close as possible to the reference point. With this regard, such a switching function for λ can be defined as

$$\Omega\left(\Delta\right) = 1 + \frac{\gamma}{1 + \exp^{\mu\Delta}}$$

where $\Omega(\Delta) \in [1, \gamma + 1]$ and μ is a parameter which indicates speed of switching. As the vale of μ increases, the speed of switching around zero increases as well. The value range of the loss aversion coefficient λ is in direct proportion to γ. For our model, function $\Omega(\Delta)$ provides us to have a smooth function to express the loss aversion coefficient λ. Now, to obtain a twice continuously differentiable utility function with a similar shape to our previously expressed piecewise-linear utility function, we substitute above switching function for our loss aversion coefficient, λ, in the piecewise-linear function. Then, we have

$$v\left(\Delta_t\right)=\Delta_t\left(1+\frac{\gamma}{1+\exp^{\mu\Delta_t}}\right)$$

Then, the maximization problem of the representative young agent turns out to be

$$\begin{aligned}\max_s\, & u\left(w_t-s_t\right)+\beta\left(1-\phi\right)u\left(R_{t+1}^e s_t\right)+\\ & \beta\phi\left(\left(R_{t+1}^e+1\right)s_t-w_t\right)\left(1+\frac{\gamma}{1+\exp^{\mu\left(R_{t+1}^e+1\right)s_t-\mu w_t}}\right)\end{aligned}\tag{0.5}$$

Assuming that the felicity function is in logarithmic form, taking derivative of young household's objective function with respect to s_t and setting it equal to 0 ends up with

$$\begin{aligned}&-\frac{1}{w_t-s_t}+\frac{\beta\left(1-\phi\right)}{s_t}+\beta\phi\left(R_{t+1}^e+1\right)\left(1+\frac{\gamma}{1+\exp^{\mu\left(R_{t+1}^e+1\right)s_t-\mu w_t}}\right)+\\ &\beta\phi\left(\left(R_{t+1}^e+1\right)s_t-w_t\right)\left(-\frac{\mu\gamma\left(R_{t+1}^e+1\right)\exp^{\mu\left(R_{t+1}^e+1\right)s_t-\mu w_t}}{\left(1+\exp^{\mu\left(R_{t+1}^e+1\right)s_t-\mu w_t}\right)^2}\right)=0\end{aligned}\tag{0.6}$$

Dividing both sides by $\beta(R_{t+1}^e+1)$ and rearranging the above equation gives us

$$\frac{(1-\phi)}{\left(R^e_{t+1}+1\right)s_t}+\phi+\frac{\phi\gamma}{1+\exp^{\mu\left(R^e_{t+1}+1\right)s_t-\mu w_t}}=$$
$$\frac{1}{\beta\left(R^e_{t+1}\right)(w_t-s_t)}+\frac{\phi\mu\gamma(\left(R^e_{t+1}\right)(w_t-s_t)\exp^{\mu\left(R^e_{t+1}+1\right)s_t-\mu w_t}}{\left(1+\exp^{\mu\left(R^e_{t+1}+1\right)s_t-\mu w_t}\right)^2} \quad (0.7)$$

At period t, representative firm maximizes its profits by choosing the labor input paid at a wage rate of w_t and capital output paid at a return to stock R_t:

$$\Pi_t=\max_{L_T}F\left(K_t,L_t\right)-w_tL_t-R_tK_t$$

In this economy, at any period of time firms are assumed to have a Cobb-Douglas type of production function with capital share of output, α, and productivity, A,

$$F\left(K_t,L_t\right)=AK_t^{\alpha}L_t^{1-\alpha} \quad (0.8)$$

As the production function is homogeneous of degree one, it can also be expressed in its intensive form by denoting $k_t=\frac{K_t}{L_t}$, as follows:

$$f\left(k_t\right)=Ak_t^{\alpha} \quad (0.9)$$

Moreover, since all agents are price takers, two factors of production are paid their respective marginal product:

$$w_t = f(k_t) - k_t f^{'}(k_t) = A(1-\alpha) k_t^{\alpha} \tag{0.10}$$

$$R_t = f^{'}(k_t) = A\alpha k_t^{\alpha-1} \tag{0.11}$$

The capital accumulation rule transforms savings of representative young agent at period t into productive capital for period $t+1$:

$$K_{t+1} = I_t = s_t$$

In this economy, agents behave rationally and future rates of return are perfectly forecasted. Therefore, the equilibrium with perfect foresight:

$$R_{t+1}^{e} = R_{t+1} = f^{'}(k_{t+1}) \tag{0.12}$$

$$k_{t+1} = s_t \tag{0.13}$$

After characterizing the equilibrium in this economy with perfect foresight, and defining $x = (R+1)(w-s)$ in the saving function gives us the following steady state equation:

$$\phi + \frac{(1-\phi)}{x+w} - \frac{1}{\beta\,(R+1)\,(Rw-x)} - \frac{\phi\gamma}{1+\exp^{\mu x}}\left(\frac{\phi\mu x\exp^{\mu x}}{1+\exp^{\mu x}} - 1\right) = 0 \quad (0.14)$$

In order to tackle with this equation and to find out the number of steady states we now resort to numerical analysis.

CHAPTER 3

NUMERICAL ANALYSIS

We have obtained the required equation (0.14) in previous part to observe the long run dynamic implications of prospect theory on the optimal path. In this model, one period amounts to 40 years and we assume that one year discount factor β is around 0.98[1], capital share of output α is taken 0.33[2], γ is interval of the loss aversion coefficient and it is not constant. A higher value of γ leads to a higher value for the range of the loss aversion coefficient. μ is switching speed around the reference point. As μ gets higher, switching around zero gets faster. Lastly, ϕ is in between zero (no prospect utility) and one (prospect utility).[3]

Case 1. $\phi = 0$ (*without prospect utility)*

We are analyzing the dynamics of the economy as if preference structure has no prospect.

[1] See David de La Croix and Michel (2002)

[2] See Barro and Sala-i Martin (1995)

[3] See R.Rosenblatt-Wisch (2008)

If there is only one steady state solution, k^s; optimal policy function shows that no matter initial level of capital stock, $k_{0,}$ is lower or higher than the steady state, it will sooner or later converges to the steady state level. This means that this steady state is *globally asymptotically stable*.

To see the dynamic implications numerically, we consider the following set of fairly standard parameterization (also see Figure 3.1):

$$\beta = 0.98^{40};\ \alpha = 0.33;\ \gamma = 10;\ \mu = 2;\ \phi = 0;\ A = 10$$

We have now a unique solution for the steady state. To have a better understanding for this analysis, we can use following policy function as a representative (see Figure 3.2):

The only solution for the steady state value of capital, $k^s = 5.64049$. With this scenario, we cannot explain cross country income differences because no matter how much initial capital stock is available in the economy, it will finally converges to the unique steady state which is *globally asymptotically stable.*

Case 2. $\phi \in (0, 1]$ (with prospect utility)

Now, we are assuming that preference structure has prospect.

If there are three steady state solutions, k_l, k_m, k_h;

i) if $k_0 < k_m$ then sequence of optimal capital stock, $\overrightarrow{k}$ converge to k_l which is *locally asymptotically stable.*

*ii)*if $k_0 > k_m$ then sequence of optimal capital stock, $\overrightarrow{k}$ converge to k_h which is

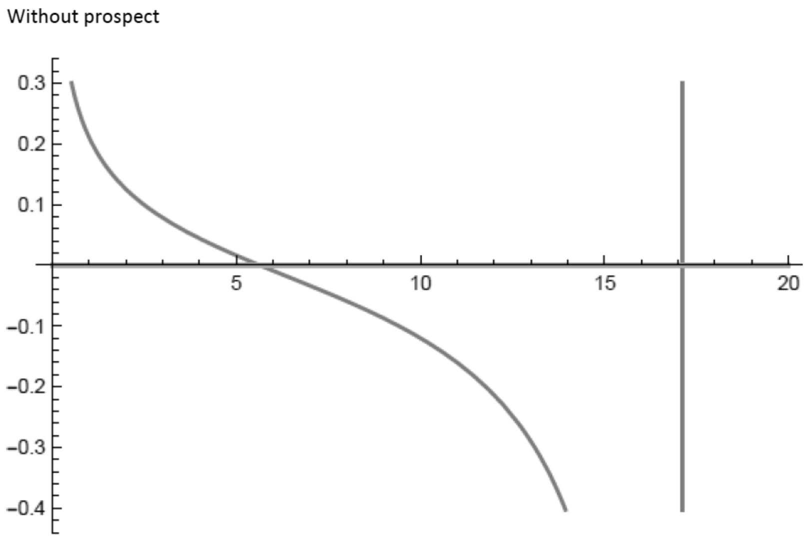

Figure 3.1: Figure 1

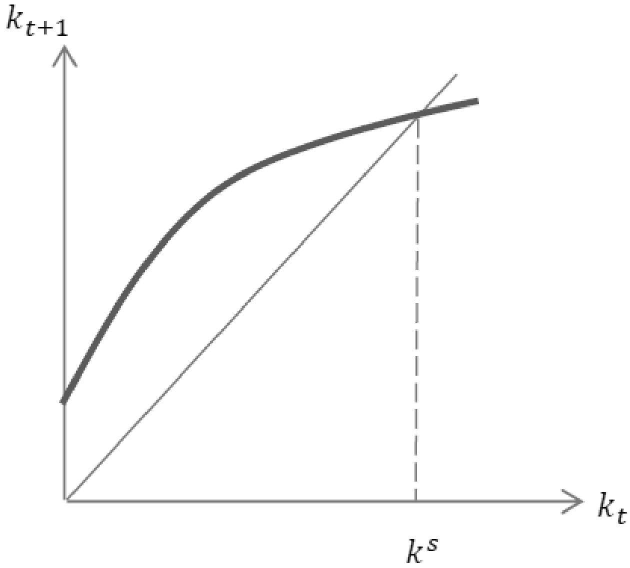

Figure 3.2: Figure 2

locally asymptotically stable.

iii) k_m is unstable and indeed it is the critical (threshold) level of capital stock below which optimal path leads to k_l, development trap, and above which optimal path converges to the high steady state k_h.

We have now prospect in our numerical analysis with the following parameter values (also see Figure 3.3):

$$\beta = 0.98^{40}; \alpha = 0.33; \gamma = 10; \mu = 2; \phi = 0.98; A = 10$$

Again, a representative policy function is helpful for a better understanding of dynamics (see Figure 3.4):

The exact three solutions for the steady state values are $k_l = 7.61835, k_m = 8.38707$, and $k_h = 15.6047$.

For those countries with an initial level of capital stock lower than k_m the sequence of optimal capital stock converges to k_l, and in this situation k_l is development trap. On the other side, those countries with an initial level of capital stock higher than k_m converges to k_h. Both k_l and k_h are locally asymptotically stable and under these circumstances, the model exhibits multiplicity of optimal steady states with local convergence even under a convex technology.

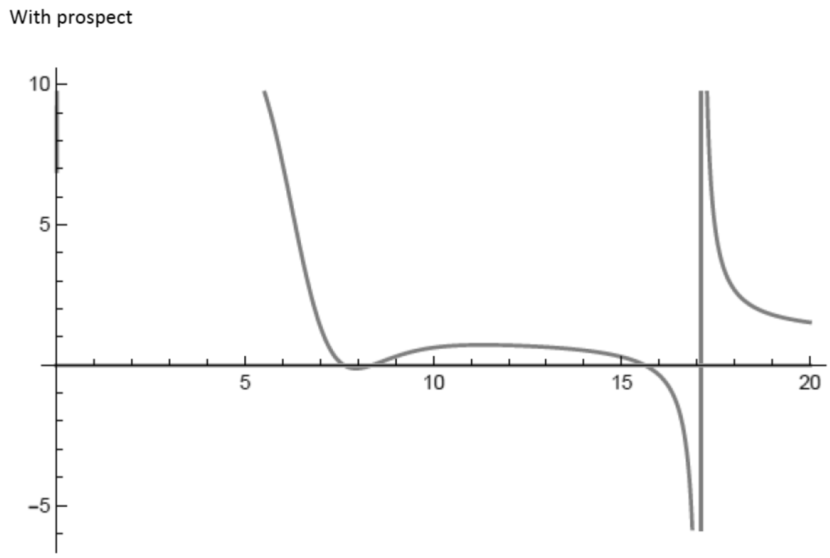

Figure 3.3: Figure 3

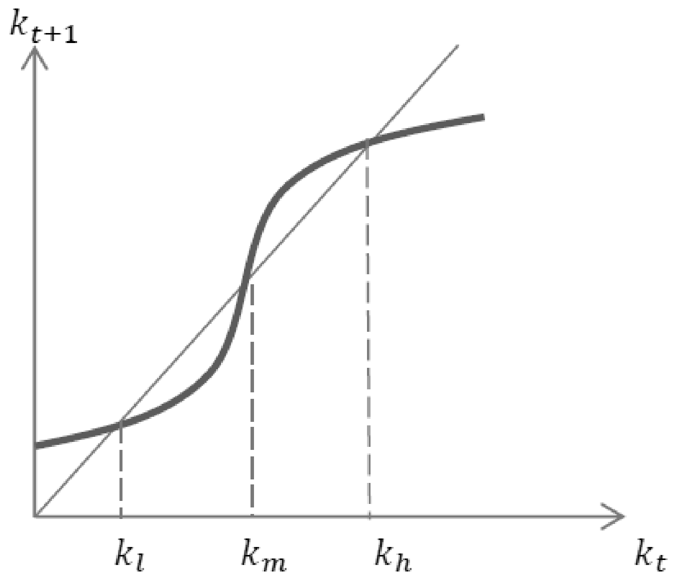

Figure 3.4: Figure 4

CHAPTER 4

CONCLUSION

In discrete time models, following papers by Clark (1971) ;Majumdar and Mitra (1982) and Dechert and Nishimura (1983) and in continuous time Skiba (1978) and Askenazy and Le Van (1999) considers the role of critical value in determining the dynamics of developing patterns. While these models rely on technology to obtain multiplicity, our model achieve multiple steady states and exhibits traps even under convex technology with only a change in preference structure.

In this thesis, we present the dynamic implications of prospect utility in a standard two-period overlapping generations model. Based on the numerical parametrization, the model constructed in this thesis is successful to answer whether different development patterns can be achieved due to a change in preference structure of a representative young household. Since the model includes capital accumulation we can reflect the saving decision of the representative agent to whole economy. While available literature in explaining cross country income differences devoted their analysis to technology component leaving the preference structure unaltered and reach multiplicity, our set up considers an alteration in preferences also delivers local convergence and multiplicity even leaving the technology be convex. Indeed, it also shows that there is an unstable critical stock of capital below which optimal path converges to development trap and above which optimal path converges to high steady state level. As such, even with convex technology, standard two-period overlapping generations set-up augmented with prospect utility could explain persistent cross country income differences.

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