FINAL PHASE INVENTORY MANAGEMENT OF SPARE PARTS UNDER NONHOMOGENEOUS POISSON DEMAND RATE

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by
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June 2013

I certify that I have read this thesis and that in my opinion it is full adequate, in scope and in quality, as a dissertation for the degree of Master of Science.

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ABSTRACT

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In product lifecycle, there are three phases, initial phase, normal phase and final phase. Final phase begins when the product is out of production, and ends when the last contract expires. It is generally the longest period in the lifecycle. Although the product is not manufactured any more, spare parts of the product need to be supplied to the market. Firms need to provide these parts at the retailer level until the end of the phase due to legal responsibilities. Because of lack of historical data and unavailability of forecasting, retailers need a systematic policy to decide replenishment quantity and time to prevent excessive holding, backordering, unit and setup costs. In our problem, we assume that demand of the spare part is a non-homogeneous Poisson process where the rate parameter is a non-increasing function of time. We consider all costs and lead time are fixed and known. Due to characteristics of the final phase, the planning horizon is taken as finite and known.

In this study, we developed two alternative heuristics for retailer's problem to minimize total cost during the final phase. First heuristic is a continuous-review policy based on estimation of future replenishments by solving series of deterministic demand sub-problems. Second heuristic is a periodic-review policy with variable period lengths, which solves myopic problems, by selecting subsequent time points to check inventory position. We also developed a simulation model to evaluate performances of the heuristics.

This study provides an efficient way to decide on replenishment quantity and time. Limited numerical results show that heuristics provide near-optimal results for homogeneous cases studied in the literature. Moreover, this is one of the initial studies that considers final phase with non-homogeneous demand rate. In that sense, it makes a contribution to the literature of final phase problems and provides a systematic way of replenishment decisions for the retailers.

Keywords: Inventory Control, Final Phase, Spare Part

ÖZET

HOMOJEN OLMAYAN POISSON TALEP DAĞILIMLI YEDEK PARÇALARIN SON AŞAMADA ENVANTER YÖNETİMİ

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Bir ürünün yaşam döngüsü üç aşamadan oluşmaktadır; ilk aşama, normal aşama ve son aşama. Son aşama, ürünün üretimden kaldırıldığı anda başlayıp, son müşteri sözleşmesi bitene kadar devam eder. Genel olarak bu süreç ürünün yaşam döngüsündeki en uzun aşamadır. Bu aşamada ürün üretilmemesine karşın yedek parçaları sağlanmaya devam edilmelidir. Bu yedek parçalar perakendeci seviyesinde, yasal zorunluluklar bitene kadar tutulmalıdır. Talep geçmişi ve tahminin yapılamamasından ötürü doğabilecek aşırı bekletme, ısmarlama, ürün ve sipariş maliyetlerini engellemek için perakendeciler sistematik bir yaklaşıma ihtiyaç duymaktadır. Bu problemde, talebin homojen olmayan Poisson dağılımla geldiği ve talep kurunun artış göstermeyen zamana bağlı bir fonksiyon olduğu varsayılmıştır. Problemdeki tüm maliyet parametrelerinin sabit ve bilindiği varsayımı altında sınırlı bir zaman aralığı için çözüm geliştirilmiştir.

Bu çalışmada, perakendecinin problemini çözmek için iki adet sezgisel yaklaşım geliştirilmiştir. İlk yaklaşım bir sürekli envanter yöntemi olup, gelecek zamana ait talebin değerlendirmesine ve bir dizi deterministik probleminin çözümüne dayanmaktadır. İkinci yaklaşım bir aralıklı envanter yöntemi olup, aralık uzunluğu miyop olarak çözülen küçük problemlerin sonucuna göre değişiklik göstermektedir. Geliştirmil olduğumuz bir simülasyon aracıyla bu çözümler test edilmiştir.

Bu çalışma, perakendecinin sipariş zamanı ve büyüklüğü konusunda etkili bir çözüm önermektedir. Yapmış olduğumuz sınırlı sayıdaki sayısal sonuçlara göre homojen talep dağılımı için optimal çözüme yakın sonuçlar vermektedir. Ayrıca bu çalışma, son aşamada homojen olmayan talep dağılımını kullanan ilk çalışmalardan biridir. Bu açıdan son aşama problemleri literatürüne bir katkıda bulunup, perakendeciler için sistematik bir sipariş yönetimi önermiştir.
Anahtar Kelimeler: Enventer Kontrolü, Son Aşama, Yedek Parça Yönetimi

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1

Introduction

In manufacturing and logistics, spare part management (SPM) is an important component to achieve desired service level at minimum cost. Besides their usage for repairing, spare parts can also be used to replace failed components, thus extending lifetime of the products. Decision in SPM includes different aspects from forecasting to inspection. Due to its large range of decisions, in industry more than 50% of the maintenance costs are due to spare parts. Moreover in some sectors more than half of the down times are due to unavailability of adequate spare parts [24]. In 2011 press release, Technology Services Industry Association (TSIA) stated that average value of spare parts inventory is 17% of total service revenue and spare parts are critical to delivering prompt quality service [29]. Obviously, SPM is a vital factor for success in manufacturing and business today.

Spare part management consists of different phases throughout the production process. In each phase, supply and demand structure changes and often these phases are studied separately. A major one of these life periods is called Final Phase, which is also known as End-of-life (EOL) phase in the spare part management literature. This phase starts when the product is out of production line and continues until last costumer contract or warranty expires. In this phase, although part is no longer manufactured, the service requirements still continue, hence the spare parts should be supplied until the end. There are also some legal stipulates to firms to provide spare parts until last customer contract expires. Therefore, unavailability of sufficient spare parts inventory can lead some penalty costs which could be more than product value (due to replacement). On the other hand, excessive inventory can lead huge disposal costs at the end of the final phase. Final Phase is known to be the longest period in a product life-cycle in general [32]. For instance in European Union every goods need to have two-years of guarantee at minimum [9]. Moreover, based on Supply of Goods and Services Act 1982, spare parts for motor companies should be provided at least for 10 years [24]. These instances prove that inventory control of spare parts during final phase is a vital decision for enterprises.

Spare parts can be stored in different levels in a multi-echelon inventory system. Based on the industry, parts may be needed to be available at retailer level to provide fast response and lower backordering cost. Especially if production of spare parts is costly for the company, they may want to produce spare parts in large batches, as in the case of serial production. Therefore, retailers need to order spare parts to keep their inventory at a reasonable level.

As described above, this thesis focuses on retailer-level inventory management of spare parts during final phase. This problem is originally discussed through a forecasting-based approach by Moore [19] in 1971, where they define all-time requirements of consumable spare parts for motor-car industry. In his thesis, Pourakbar [21] provides a comprehensive analysis on problem, discussing different approaches.

This study evaluates problem on retailer's behalf. Therefore, our objective is minimizing total cost of the retailer in a decentralized system. The total cost is consisting of unit, setup, holding, and backorder costs. Since we study on a decentralized system, retailer need to decide own replenishment times and quantities, which are our decision variables. Since inventory management varies much based on product type, industry type and other conditions, we decided to focus on the following setting:

- Time horizon is finite and known. This is a common setting in final phase studies because expiration of last contract is known beforehand. Time is considered as a continuous variable over planning horizon.

- At the end of the planning horizon, all backordered demands should be satisfied with a single last order. This one is a part of legal requirements.

On top of this setting, we made the following assumptions to work on a clearer problem:

- Unit, setup, holding and backorder costs are constant and known at time zero.

- Lead time is constant and known.

- Unit demands are unit-sized.

- Demand is a Non-Homogeneous Poisson Process with a non-increasing demand rate over time.

In this study we proposed two heuristics from different perspectives to solve the retailers' inventory management problem.

Following sections in this thesis as follows; in second chapter, problem definition and literature review about spare parts inventory management and final phase is given. In third chapter, our proposed solution methodology is presented. Paper follows with computations in fourth chapter and conclusion in fifth chapter.

Chapter 2

Problem Definition and Literature Review

2.1. Problem Definition

Spare part management consists of different echelon levels in general. Each level requires strategic, tactical and operational decisions. These decisions could be made either by a single decision maker or each level may have its own. In this thesis, we focused on a decentralized system with a focus on single retailer. Problem is based on retailer's controlling spare part inventory in the final phase. As stated before, retailer is the only decision maker and so the purpose of this study is minimizing its total cost in a finite horizon.

In management of the spare part inventory, retailer faces with some challenges. One of the biggest challenges in this management is unavailability of data for forecasting the future demand. This is generally the case in the final phase. Since retailer has limited or inadequate data for forecasting and demand is unknown, retailer should estimate the future demand which makes inventory management much more difficult. On top of that, a certain customer service level may be desired for either cost minimization or customer satisfaction. Service level is especially vital in case of non-zero lead time. Little tardiness in replenishment decision time may lead unexpected high costs for the retailer. Yet another vital decision appears on replenishment quantity. Underestimation of the future demand leads smaller replenishment quantities which may increase the total number of the setups, thus total setup costs. On the other hand overestimation of the demand may lead higher holding costs and moreover, excess inventory could be available at the end of the final phase.

In this thesis, we defined the retailer's problem with the following assumptions;

- Planning horizon is finite and known. This assumption is based on the fact that expiration of the last customer contract and legal responsibilities are known by the retailer.
- Unit, setup, holding and backorder cost parameters are fixed and known.
- Lead time for the supplier is fixed and known. Lead time is independent from replenishment quantity and time. Thus we assume supplier does not spend time for production; there is always adequate inventory at supplier level.
- Demand is a Non-Homogeneous Poisson process with a time-dependent rate. This rate is assumed to be a non-increasing function of time. In some cases, we also assume that rate of the NHPP reaches zero at the end of the planning horizon.
- Time is a continuous variable in the planning horizon.

- All demands are unit-sized.
- Backordering is allowed. Demands are met whenever inventory is available.
- If horizon ends with some backorders, a last order is given to meet all backordered demands.
- There is no salvage cost at the end of the horizon. Therefore, if inventory position is positive at the end of the horizon, all items are disposed.

In this thesis we focused on a retailer's problem within a single echelon system and there is single type of product. Hence our objective is minimizing total cost of the retailer.

Before going into details, we know that retailer has two different extreme solutions. First extreme solution is backordering all demand during final phase and meets all these orders at the end of horizon. Second extreme solution is placing a huge replenishment order at the beginning of the phase.

Assume that retailer applies the first extreme solution and backorders all the demand during time horizon. Due to legal responsibilities, he needs to meet all these demands in a single order and he pays backordering (penalty) costs. If penalty cost is sufficiently small, this extreme solution could be the best choice for the retailer. Otherwise, systematic planning of the replenishments may balance the holding, setup and backordering costs.

There are two decisions need to be taken by the retailer. First one of these decisions is "When I need to place a replenishment order?" Second question is "How much I need to order for each replenishment order?" Correct answers to these two decisions are affected by total cost components: setup, unit, holding and backorder costs. Moreover, these questions needed to be answered throughout the horizon. So our policies should be capable of answering these questions at any time during the horizon.

Due to nature of the final phase, our planning horizon is the time between End of Production (EOP) and End of Service (EOS) where last customer contract expires. In this thesis, the problem starts just before EOP to start horizon with a sufficiently large inventory. Therefore both heuristics starts at First Installment (FI) point, which is lead time length before EOP.

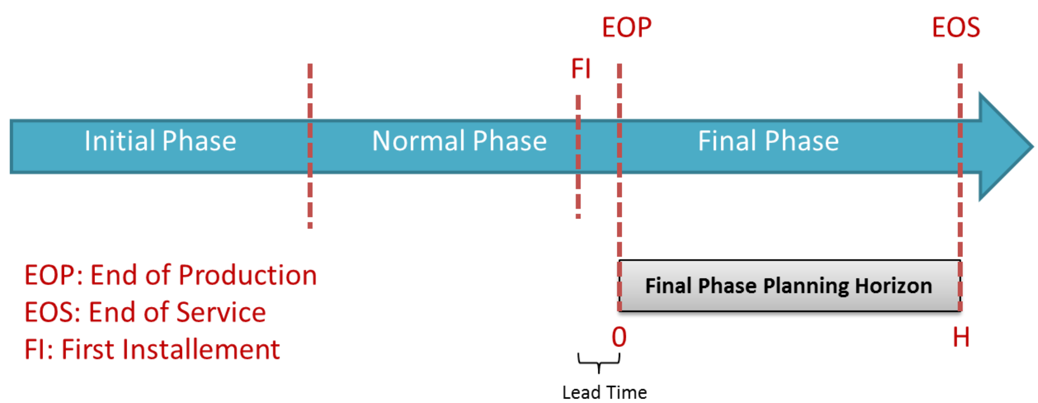

Figure 2.1 Planning horizon of the problem

In order to find satisfying answers to retailer's problem we developed two heuristics. These heuristics answers these questions in a systematic way for the retailer. We detailed these heuristics in the Chapter 3.

2.2. Literature Review

The problem considered in this thesis can be classified under different stream of literature of inventory management, such as Spare Part, Obsolescence, Product Life-Cycle and Final Phase. In this subsection, there are numerous studies that are related with more than one topic among these streams. We try to show the importance of this study in these streams while defining the problems and classifying previous studies, in given order.

Spare Part Management

Spare Part Inventory Management (SPIM) is a broad topic that includes various aspects. The more relevant studies in SPIM are conducted by Fortuin [11, 10] in 1980 and 1981, which define all-time requirements (final order) of spare part inventories. He studies on management of spare parts of a product that have risk of failure, such as electronic products, for the “service after sales” department. He defines the product life-cycle consist of three phases. There are initial, repeat and the final phases. He assumes exponentially decreasing demand in his study.

Another relevant study in SPIM literature is presented by Geurts and Moonen [13] in 1992. In their paper, they analyze and present how ‘insurance type’ spare parts are needed to be keep. They use Dynamic Programming (Markov Programming) approach in this paper, which is also supported by numerical examples. While deciding on uncertainty parameters, they also utilize their approach to measure how good the decision strategy is.

Obsolescence

The main stream of our thesis, Final Phase studies are also related with finite horizon inventory problems with obsolescence. Hadley and Whitin [14] provides the classical obsolescence problem in 1963. In this study demand is a random variable and occurs in time periods independently. Obsolescence time is known or a finite number of possible obsolescence times are given with their respective probabilities. They solve this problem with a dynamic programming approach.

In 1997, David et al. [5] provides the continuous version of the classical obsolescence problem defined by Hadley Whitin. They provide a dynamic programming model for the finite horizon problem, where demand rate is fixed while lifetime of the items follows a known random distribution. In this problem, they observe that there must be a time where ordering to the end is the optimal option. This corresponds to "last-order problem" in our heuristic. This study is also provides structural properties of the problem.

Life Cycle

Elements such as demand direction, length of horizon and stochasticity of the parameters lead different problem definitions in inventory studies. Hence, even for the very same product, we need to apply different inventory policies for the different phases during its life cycle. Often, inventory studies encapsulate a certain time interval in the product life. Such as, our heuristics described in this thesis are useful for a specific time interval in the product life-cycle due to its features and assumptions. There are numerous studies that emphasize these differences. For instance, Solomon et al. [27] showed the life-cycle phases and their distinct features of electronic equipment. They divided electronic product's life-cycle in six phases. There are defined as introduction, growth, maturity, decline, phase-out and obsolescence phases. In this study, they mention on last-buy decision in the obsolescence phase, which is relevant to time-period we interest in this thesis and it will be discussed later in detail. One of the earliest studies that focus on the time interval we interest is performed by Cohen and Whang [4] in 1997. Similar to our study, they focus on the service after sales operations. On top of the management of spare parts, they consider an independent service operator which leads competition. Hence they used a 'game-theoretic' approach in their study. Their decision variables are completely different from ours, they decide product price, after-sales service quality and after-sales price in their problem.

Another interesting research that focuses on product life-cycle is performed by Bradley and Guerrero [2] in 2008. This paper focuses on product design to a better utilization of life-cycle mismatch of the components in a product. In this paper, one of the alternatives that are used to manage life-cycle mismatch is called "life-time buy" or "last-time buy" which corresponds the final decision of inventory operations. This is exactly the same topic that we cover in this thesis.

Another study of Bradley and Guerrero [1] published in 2009 deals with lifetime buy (last-time buy) decision for multiple obsolete parts. Lifetime buy policy is argued to be the necessary when product life-cycle mismatch occurs on spare parts of a product. They prove the existence and the uniqueness of the solution to the problem, however since the solution cannot be expressed in closed-form, they suggest two heuristics which gives upper and lower bounds on the solution. They show results for stationary and non-stationary demand, and suggest that the heuristics give accurate results for the stationary demand case.

In their 2011 paper, Dekker et al. [6] inspect the various aspects of life-cycle phases of spare part management. They mention about unique and difficult cases on managing the spare part inventory and focus on forecasting strategies. Life-cycle phases mentioned above are also available in this study, while they give an importance on the life-cycle of spare part demand. Interested readers may look for the case studies in Fokker Services, IBM, IHC Merwede and Voestalpine Railpro companies, presented in this paper.

Spengler and Schröter [28] developed tools for information management on a closed-loop supply chain at the End-of-Life service period. They model the management of production and recovery system of spare parts and emphasize the importance of several strategies. This study is important since it combines end-of-life service period with product design with a different view. In their paper, they show the difficulties to manage spare part inventory during end-of-life service period. It is known that final phase lasts for many years for electronic equipment [30]. These are the loss of economies of scale since the product is no longer manufactured, possible differences between product generations (hence spare parts may differ), limited flexibility of the spare parts and possible problems on providing materials for spare parts. On a case study, they provide the output of their "system-dynamics" model for spare parts.

Final Phase

Now, we will focus on more relevant studies to our work which focus on spare part inventory management in the final phase. Final phase is also called with different definitions such as "end-of-life service", "post-product life" and "after-sales service period". The problem considers the inventory in the Final Phase period is also known as "End-of-Life Inventory Problem" (EOL), "Final Buy Problem" (FBP) and the "End-of-Production Problem" (EOP) [23]. In this part we will review these relevant studies and emphasize the similarities and differences of our work with them. Note that the terms describing final phase are used interchangeably.

In their paper dated 1998, Teunter and Haneveld [33] described the final order problem. They solve the problem of "last-order" for the client, who will give a final order of spare parts from manufacturer due to discontinuity of spare part supply of the manufacturer. Client is assumed to have a machine which needs these critical spare parts to operate. Client wants to use this machine at least for a certain amount of time. Therefore, client should keep a sufficient inventory of these critical spare parts. They suggest an order-up-to level policy for this last order quantity. They found it by minimizing the total discounted cost. On 3 different examples they show that their model provides near-optimal results. There are some features of their problem, which is significantly different from the problem considered in this thesis. Teunter and Haneveld consider the time-period where service agreement ends, while we consider time between End-of-Production and End-of-Service (Final Phase) period. Moreover, they solve this problem for only one final order, while we allow replenishments during the time horizon which leads different assumptions.

In another study of Teunter and Fortuin [30], they study on the same problem with supplier's perspective. In this case the decision maker is the service department of the supplier. However, the ordering structure is the same as Teunter and Haneveld's study: only one last order is allowed to make for decision maker [33]. For given cost parameters, they reach near to optimal solutions of the quantity of the last order. They both provide 'optimal' final order by using stochastic dynamic programming and a 'near-optimal' final order level by using an explicit cost formulation. They show that the explicitly defined final order level is near to optimal final order level, which is practical to compute. Moreover, in this study they suggest a "remove-down-to" level, where they defined discrete time intervals and remove some spare parts from the stock if the inventory level is above the "remove-down-to" level. This study is important, because they take decisions after final phase started due to "remove policy". Although they only remove items from the stock instead of replenish it as we cover in this thesis, this paper is closely related with ours since they allow actions during the time horizon.

In their 1999 paper, Fortuin and Martin [12] define phases of the spare part life-cycle. It's one of the earliest study that use term "final phase" by referencing Teunter and Fortuin's definition of End-of-Life service (EOL) [31]. This is a comprehensive study that shows different aspects of management of spare part inventory. It covers logistics, demand and delivery, management concepts of spare parts and also devotes a section to show differences between spare part inventory management with traditional approaches. They emphasize the distinction between phases of the spare part life-cycle, which are defined as initial, normal and final phase. In the following paragraphs, we will also review the work of Teunter and Haneveld [32] in 2002, which use the same final phase definition as in this paper.

Cattani and Souza [3] consider the effect of delaying the end of life buy in their 2002 paper. Their study is slightly different from Teunter and Haneveld's research in terms of time of the final order (end-of-life buy) [33]. By using the information obtained by delaying the final order decision, they argue that the underage and overage costs can be reduced. For different settings they show that the cost benefit of delaying the decision is non-decreasing function of time and concave. This is a remarkable result for the manufacturers who can delay their final order decision. They used the newsvendor problem as a basis to calculate costs of initial problem. On numerical experiments they provide how effective their model is.

Draper and Suanet's work in 2005 includes various and detailed information about IBM's inventory operations [7]. They stated that IBM divided inventory life-cycle into three phases: Early-Life, Mid-Life and End-of-Life. Their definition of End-of-Life phase is precisely the same as we define final phase. They note that this phase takes 7 years on average, although it varies a lot for different PC parts. They also stated that Service Parts Logistics organization is responsible for the actions in the end-of-life phase and used a 'last-order' at the beginning of this phase. This is precisely the problem that we mentioned above. They indicate that specialized algorithms are being used for this decision where historical data and demand forecast play a significant role. They refer the paper of Teunter and Haneveld for more information on last-buy problem [32].

Inderfurth and Mukherjee [17] consider different approaches in the final phase in their paper dated 2008. They differentiate the different phases of the product life-cycle similar to studies mentioned above. They stated that the managing the spare part inventory between end-of-production (EOP) to end-of-service (EOS) is especially challenging for many industries. This time period corresponds to final phase (or post-product life cycle) in our study. Assumptions and observations in this paper are very close to our problem. They show how the problem can be modeled as a Decision Tree and can be solved by Stochastic Dynamic Programming procedure. Moreover, they propose a relatively simpler heuristic by inspired by the solution of the dynamic programming.

Another study on the spare parts inventory management in the final phase is conducted by van Kooten and Tan [34] on parts under condemnation. Their model includes repairs of the spare parts. They suggest a continuous-time transient Markovian Model with certain repair probability and repair lead time.

Pinçe and Dekker [20] deal with the inventory of slow moving items subject to obsolescence in their paper dated 2011. They consider a continuous review inventory system and works in a similar environment to this problem. They assume that the demand rate drops a lower rate in a known time during the horizon. In this study, policy changing is proposed and an approximate solution of time to shift to new control policy is given. Advantages of such a shift are also described in the paper. During all horizon demand is assumed to follow Poisson Process with a constant rate, which drops to another constant rate at a known time. In that sense, their demand definition is one of the studies that are close to our problem. The policy used in the paper is one-for-one replenishment policy for both policies (initial policy and new policy) with different parameters. Our problem is slightly different from their definition and includes setup cost, which makes one-for-one replenishment policy an undesired alternative. For the problem they consider, they achieve satisfying numerical results that show the superiority of the switching.

There are also some studies that cover the different aspects of the final phase problem. Pourakbar et al. [23] suggests alternative decisions in the final phase such as offering a new product. They discuss the effects of such alternatives and show how they are more cost-efficient than keeping spare parts inventory at some point in the final phase. Hence, their study examines the cost trade-offs of such policies and give an exact expression represents expected total cost. They also show that such an expression leads the solution of last-order quantity and time to switch policies simultaneously. Their study is based on a real-life study of a major consumer electronic goods manufacturer, which is common in final phase studies. They developed two models, first, an alternative service policy and second, a more sophisticated model for the cost function which is closer to real-life cases. In the study, demand is assumed to follow non-stationary Poisson process, which is also an assumption in this thesis. Moreover, horizon is finite and cost parameters are fixed and known similar to ours. However, they consider only “one-time buy” policies with review and scrapping options. Since we allow multiple orders during the final phase and associate a setup cost for this operation, the total cost structure and the behavior of the solutions to the problems are different from each other, respectively.

In 2012, Pourakbar and Dekker [22] combine customer differentiation with the final phase inventory problem. Note that their study is different from other studies in the final phase literature, where procurement (replenishment) is an available option as we assume and they also use non-stationary demand rate. They show that their model reaches remarkable cost improvements on the problem.

Now, we will cover two researches that are very close to our problem, in detail.

In the study of Inderfurth and Kleber [16] in 2013, alternative management of spare part inventory in End-of-Production phase is studied. Due to challenges in managing the spare part inventory at this phase, they argue that options such as extra production and remanufacturing provide flexibility to the manufacturer. For this problem, they provide order-up-to levels for extra production and remanufacturing options, very similar to our model in this thesis. The decisions are told to be simple compared the complexity of the problem. They show that the problem can be modeled as a stochastic dynamic optimization problem. However, the policy to minimize average total cost is found to be too complex. Therefore, they suggest simple order-up-to policies, which are shown to be worked well for most of the cases when policy parameters are chosen appropriately. Their research is a great contribution to the literature, considering the number of studies about the final phase that considers extra production.

Note that, our study has similarities to the problem they worked. First of all, in both studies the time horizon is defined as the final phase (end-of-production phase) and assumed to be finite and known. Second, cost parameters are assumed to be fixed and known. Third, demand is assumed to be stochastic. Fourth, extra production is possible with a major setup cost. And lastly, the objective is to minimize total cost. Although the working environment is defined very similar, our approaches differentiate in the modeling phase. First of all, they discretize the time intervals into periods; hence their model suggests a periodic review policy. As we will see in following sections, our heuristics are continuous-review policies, indeed. Second, they update the estimation of the demand along the horizon while we assume that the distribution of the demand is known due to historical data beforehand. Third, their application area is automotive sector; hence they benefit from easiness of remanufacturing which does not take major setup time and setup cost. Our study focus on general cases hence remanufacturing is not an option. Lastly, they stated that extra production is only available with a minimum order quantity. We allow extra production for any quantity during the final phase.

The other research that is close to our work in the literature is conducted by Teunter and Haneveld [32] in 2002. Actually our study is inspired by the problem they defined in their paper. Hence, we will extensively cover the details of this study in here and describe the similarities and differences with ours. We also used this study as a benchmark in our numerical experiments.

They study on manufacturer's spare part inventory problem in the final phase. Since the expiration of last contract is known, they assume that the planning horizon is finite and known. There is no setup cost in the study; hence the replenishments are unit sized. Note that, they allow replenishments after the beginning of the final phase, but with a higher price. Demand is assumed to be stationary Poisson process. They propose an initial order-up-to level for the initial order, which is also known as the "last-order", "final buy" and "lifetime buy" in the literature, and then provide order-up-to levels for the remaining horizon. Since there is no setup cost, it is an $(S-1, S)$ inventory policy, where “S-1” is considered as a reorder level. Manufacturer should place unit-sized replenishment orders whenever the inventory position drops below the order-up-to level of the current time. They solved this problem optimally and provide a method which gives (1) initial order quantity and (2) time period length where order-up-to level is constant. By using this information, one can calculate the order-up-to level for any given time.

The problem we consider shows similarities to theirs in the following aspects. (1) The planning horizon is finite and known. (2) The cost parameters are known and fixed (holding, backorder). (3) Replenishments are allowed during the final phase. (4) Demand follows Poisson process. (5) A reorder level-order up to level policy is suggested.

We can also list the different aspects of our solution method as follows. (1) Setup cost exists and fixed. (2) Lead time is non-zero and fixed. (3) Poisson demand rate can be defined as non-stationary. (4) Unit cost is fixed and same during planning horizon.

Note that among they suggest (1) and (3) as an extension to their model. In our problem, we assume that demand of the spare part is a nonhomogeneous Poisson process where the rate parameter is a non-increasing function of time.

In this study, we developed two heuristics for retailer’s problem to minimize total cost during the final phase. One of these heuristics is a continuous-review policy while the second one is a periodic-review policy. Due to complexity of the problem, we provide near-optimal results with these heuristics. This is one of the initial studies that considers final phase with non-homogeneous demand rate with replenishment option. In that sense, it makes a contribution to the literature of final phase problems and provides a systematic way of replenishment decisions for the retailers.

Chapter 3

Solution Methods

To solve a finite horizon problem, there are two types of policies based on the time of the decision. First type of approach is providing a static policy, where the problem is solved at the beginning of the horizon and applied thoroughly. For instance, in their paper, Teunter and Haneveld find the optimal order-up-to levels before the time horizon is started and these decisions are applied throughout the horizon [32]. All orders are given based on these optimal order-up-to levels. Second approach is constructing a rolling policy, where the decisions are given in continuous time. Such rolling policies are usually applied when the system changes over time. An order-up-to level can be used if applicable.

This problem, due to its very nature, is hard to solve optimally. Scarf showed that finite horizon problems can be solved with an optimal (s,S) inventory policy by using dynamic programming [25]. To the best of author's knowledge, there is no optimal solution to our problem in the literature.

In this study, two heuristics are provided to solve retailer's problem. Both of these heuristics are rolling policies with an approximation to the unknown optimal solution. There are two decisions variables in this problem, reorder level and order-up to level. Both policies use same reorder level mechanism however their selection of order-up-to level varies.

Our first heuristics provides a continuous review policy with look-ahead capability into remaining time horizon. Replenishment decisions are independent from past decisions and affected by the residual time. For each decision, a deterministic subproblem, between current time and end of horizon, is solved to estimate future orders. Solution to the deterministic subproblem is obtained by using Johnson and Montgomery's notes on the "Continuous Review Lot Size Problem" [18]. Deterministic subproblems will be explained in detail in section 3.2.2. This estimation helps us to decide on replenishment quantity because when the number of remaining orders $-N-$ is known (or fixed), then the deterministic demand subproblem problem can be solved optimally (Lot Size Problem) [18]. Therefore, based on the best possible choice of N, one can choose a replenishment quantity to minimize expected total cost until end of horizon. Therefore, solution of deterministic subproblem is solely the effective parameter on ordering quantity. On the other hand, replenishment time is chosen based on the inventory position. By using a reorder level, decision points can be found easily. Different reorder levels can be used based on the structure of the system. In this thesis we used both Type-1 and Type-2 service level. Notice that, since demand is a non-increasing function of time, reorder level (r) is also formulated as a non-increasing function of time. This definition comes with a benefit that retailers can avoid unnecessary and costly operations and review inventory only at discrete times.

Second heuristic can be categorized as a periodic review policy with variable period lengths. Instead of considering residual time horizon, this policy has a myopic look over the problem. Hence, instead of estimating all remaining orders as we do in 1st policy, we are only looking for the next expected replenishment time. The objective in each decision point is minimizing the total cost per unit time. It uses same reorder level definition as in first heuristic. However, ordering quantity is determined to minimize total holding and backordering cost for small steps. In each decision point we need to select a period length, which gives minimum cost per unit time. Since such a search can be exhaustive, it is assumed that a set of possible candidates for next replenishment time is provided. So the selection is based on the minimization of total cost between $t_{now}\ or\ t_i$ and estimated next order point $-t'_{i+1}-$ which resembles applications in real life. Then, ordering quantity is found as the expected demand during next phase. Inventory is checked only at the end of each period.

To sum up, first policy is a variant of well-known reorder level – order up to level (s, S) policy, while second one is a variant of reorder point – reorder level – order up to level (R, s, S) policy. Different from classical approaches, the parameters of these policies change throughout the time horizon.

3.1. Notations and Parameters

Following notations are used in this study:

$[0, H]$: planning horizon

t : continuous time variable, where $t \in [0, H]$

K : setup cost (per replenishment)

u : unit cost

h : holding cost (per item per unit time)

b : backorder cost (per item per unit time)

L : lead time

$\lambda(t)$: demand rate, which is a non-increasing function of time

$D(t)$: demand between [0,t], which is a random variable;

$$D(t) \sim \text{Nonhomogeneous Poisson}\big(\lambda(t)\big)$$

$N(t)$: expected demand between $[0, t]$, where;

$$N(t) = E[D(t)] = \int_0^t \lambda(t).\,dt$$

$I(t)$: Inventory position at time t

As described in 2nd Chapter, horizon length (H), cost parameters (K, u, h, b) and lead time (L) are fixed and known before the horizon.

We denote t_n and Q_n as the time and order quantity of n^{th} replenishment respectively. These are our decision variables. Decision parameters; reorder level and order up to level are denoted as $r(t)$ and $S(t)$, respectively.

3.2. Decision Variables and Levels

Without loss of generality, retailer needs to decide on two variables: time t_i and quantity Q_i for replenishment i. Defining a reorder level helps us to decide about replenishment times. Similarly, an order-up-to level may be beneficial to decide about replenishment quantity. In that sense, we define reorder and order-up-to levels for both policies. However, since our horizon is finite and demand follows a non-increasing rate over time, we need update parameters and levels for these decisions frequently. Best selection of these levels for a decision point may be different from previous decision.

Reorder level is defined same for both policies. Therefore we will start with selection of reorder level and then, selection of order-up-to level will be discussed.

3.2.1. Replenishment Time and Reorder Level

For the selection of replenishment types, we used a reorder level definition which helps to prevent unnecessary setups and loss of service level. An order is placed if inventory position drops below reorder level. Since inventory structure can differ among different business types, it is possible to select this reorder level in several ways. However, for the rest of this study we restrict ourselves to two types of reorder levels for the sake of simplicity: Type-1 (α) and Type-2 (β) service levels during lead time. Here, we used a different Type-1 and Type-2 service level than their traditional definition. We denote r_1, r_2 respectively for Type-1 and Type-2 service measure during lead time.

Reorder levels can be easily calculated by using given parameters as provided in the following subsections.

3.2.1.1. Reorder Level with Type-1 Service Measure

By definition, Type-1 (α service level) leads a reorder level, which satisfies the probability of not seeing any stock-out. Here we use a different service measure and focus only demands during replenishment lead time. Here, at any time $t \epsilon [0, H]$, our reorder level is the smallest integer, whose probability of no stock-out is higher than known and fixed probability level $-\alpha-$.

$$r_1(\alpha, t) = \min_{\rho}\{\rho \mid Prob(Demand\ During\ Lead\ Time \leq \rho) \geq \alpha,\ \ \rho \epsilon \mathbb{Z}^{+}\} \quad (1)$$

Lead time and demand rate can dropped from the parameters of reorder level function since these are fixed throughout the study. Let $\delta(t) = \int_t^{t+L} \lambda(t).dt$ is the expected demand during lead time. Then, probability of no stock-out during replenishment lead time is;

$$Prob(Demand\ During\ Lead\ Time \leq \rho) = \sum_{x=0}^{\rho} \frac{e^{-\delta(t)}.\,\delta(t)^x}{x!} \qquad (2)$$

So we can simply use the following inequality;

$$r_1(\alpha, t) = \min_{\rho} \left\{ \rho \mid \sum_{x=0}^{\rho} \frac{e^{-\delta(t)}.\,\delta(t)^x}{x!} \geq \alpha\ , \rho \epsilon \mathbb{Z}^+, \delta(t) = \int_{t}^{t+L} \lambda(t).\,dt \right\} \qquad (3)$$

3.2.1.2. Reorder Level with Type-2 Service Measure

Type-2 (β service level) is often called as fill-rate alias fraction of demand met on time. For this service measure, probability of not backordering a demand (satisfied demand) should be more than β. In other words, fraction of demand not met on time during lead time should be less than $1 - \beta$. Again, we restrict ourselves to demand during lead time to apply this service level.

By using same t definition, we can define $r_2(\beta, t)$ as;

$$r_2(\beta, t) = \min_{\rho} \{\rho | Fraction\ of\ demand\ not\ met\ on\ time(\rho, t) \leq 1 - \beta\} \qquad (4)$$

One can define the fraction of demand not met on time as;

$$Fraction\ of demand\ not\ met(\rho, t)\ = \frac{Expected\ \#\ of backordered\ demand}{Total\ \#\ of\ expected\ demand}$$

$$= \frac{\sum_{x=\rho+1}^{\infty}(x-\rho).P(N(t+L)-N(t)=x)}{\sum_{x=0}^{\infty}x.P(N(t+L)-N(t)=x)}$$

$$= \frac{\sum_{x=\rho}^{\infty}(x-\rho).\frac{e^{-\delta(t)}.\delta(t)^x}{x!}}{\sum_{x=0}^{\infty}x.\frac{e^{-\delta(t)}.\delta(t)^x}{x!}} \tag{5}$$

$$= \frac{\sum_{x=\rho}^{\infty}(x-\rho).\frac{e^{-\delta(t)}.\delta(t)^x}{x!}}{\delta(t)}$$

where $\delta(t)$ is expected demand during lead time $\delta(t) = \int_t^{t+L}\lambda(t).dt$ as used above. Therefore, similar to Type-1 reorder level, ρ is the smallest integer where fraction of demand not met on time is less than $1-\beta$.

3.2.1.3. Change in Reorder Level

In this subsection, we will introduce a useful observation, which leads tracking the inventory position only when a demand occurs become sufficient instead of tracking it continuously.

In first type of service level (Type-1), for any time t, we have a reorder level as;

$$r_1(\alpha, t) = \min_{\rho}\left\{\rho \mid \sum_{x=0}^{\rho}\frac{e^{-\delta(t)}.\delta(t)^x}{x!} \geq \alpha\ , \rho\epsilon\mathbb{Z}^+\right\} \tag{6}$$

Here, if we increase time t, then $\delta(t)$ decreases. We can prove it for any $t_1, t_2\epsilon[0, H]$ where $t_1 \leq t_2$, then;

$$\lambda(t_1) \geq \lambda(t_2) \tag{7}$$

this follows,

$$\delta(t_1) = \int_{t_1}^{t_1+L} \lambda(t).dt \geq \int_{t_2}^{t_2+L} \lambda(t).dt = \delta(t_2)$$
$$\delta(t_1) \geq \delta(t_2) \tag{8}$$

Since $\delta(t_1) \geq \delta(t_2)$, it directly follows that

$$r_1(\alpha, t_1) \geq r_1(\alpha, t_2) \quad \text{if } t_1 \leq t_2 \text{ where } t_1, t_2 \in [0, H] \tag{9}$$

Same can be applied for Type-2 service measure. Taking same t_1 and t_2 we can easily show for same ρ, fraction of demand not met on time will be $P_1 \geq P_2$ respectively for t_1 and t_2. Then we get;

$$r_2(\beta, t_1) \geq r_2(\beta, t_2) \qquad \text{if } t_1 \leq t_2 \text{ where } t_1, t_2 \in [0, H] \tag{10}$$

This condition is useful in terms of applying the policies. Because, obviously r_1 and r_2 both are non-increasing functions of time. Therefore, necessary condition for a replenishment, where inventory position is below any of service level could only happen when a demand arrives. Therefore, checking reorder levels only when a demand arrives will be sufficient.

3.2.2. Replenishment Quantity and Order-up-to Levels

Based on the selection of replenishment time which is obtained by using the reorder level, retailer needs to decide about quantity of replenishment. This replenishment quantity $Q(t)$ is a major decision for retailer which affects remaining horizon heavily. If replenishment is overestimated, then holding cost increases. On the other hand if it is underestimated then retailer may need additional replenishment which may increase total setup cost.

There are some measures which is extremely important for selection of replenishment quantity. These are residual time $H - t$, holding, backorder and setup costs and demand rate. Although cost parameters are fixed and known, changes in residual time and demand rate affects replenishment quantity. Since replenishment quantity decision is independent from past decisions, we can evaluate the remaining time horizon and demand rate and provide a level which helps to determine the replenishment quantity. Therefore, we used two order-up-to level definitions which are used to decide replenishment quantity.

Replenishment quantity and next replenishment time affects each other. Therefore, one can select an estimated time for next replenishment and then calculate order-up-to level. Our heuristics are differentiated at this point. In order to provide an estimate time for next replenishment we can make an exhaustive search in a continuous interval and find the best candidate. Instead, we can limit ourselves to a finite set consists of various time periods and select the best one among them, which is time-efficient.

As described above, first alternative takes residual time into consideration while second alternative concerns only with the given time period. Our first heuristic uses the first method described above while second heuristic applies the other one. Therefore we can say that our first heuristic takes the remaining time horizon into consideration and thus it is a policy with look-ahead capability. Our second heuristic, in that sense, is a myopic policy. In here, we would like to describe how these two different structures are applied. First we will describe how first policy consider residual time to select replenishment quantity. We will present a subtopic, deterministic subproblem, which is used for this task. Then quantity decision of our myopic policy, second heuristic, will be explained.

3.2.2.1. Order-up-to Level Decision with Look Ahead Capability

We know that our selection of replenishment quantity will affect the expected next replenishment time and expected number of residual orders. The relation is shown at Figure 3.1.

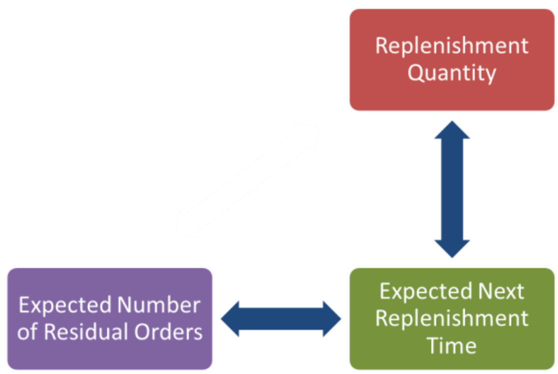

Figure 3.1 Relation between replenishment quantity decision and its effects.

Every replenishment order affects the remaining replenishments, hence to find replenishment quantity for only one interval needs to solve the consecutive problems as well. Hence decision to replenishment quantity needs the residual time into consideration. As we see every decision to replenishment quantity needs solving the subproblem between t_{now} and H.

Note that the replenishment quantity belongs to a large set and the expected next replenishment time is continuous, hence we can find and set the expected number of residual orders to find others.

Note that subproblem between t_{now} and H is a smaller version of the original problem, where we are deciding demand at time 0, when the time horizon is between t_{now} and H. Solving this subproblem optimally has same complexity to solve the real problem. Therefore, we simplified the subproblems as follows: If we assume that the demand during residual time $[t_{now}, H]$ is *deterministic* and equal to demand rate $\lambda(t)$, we can solve the subproblem. Hence for an arbitrary time interval (t, u), the demand is known and fixed to $\int_t^u \lambda(s)ds$. We know that the deterministic subproblem (DS) can be solved optimally if the total number of remaining replenishment is fixed. Therefore, starting from $n = 1$ to a sufficiently large number N, we can calculate the total cost for deterministic subproblem and then select the one which gives minimum total cost. This approach is suggested by Johnson and Montgomery [18] to solve Continuous Review Lot Size Problem. Trying various n values is also suggested in their study. Note that, we are looking for the best selection of "expected number of residual orders" and then finding the expected time of next replenishment and finally the replenishment quantity.

Deterministic Subproblem

Assume that total number of remaining orders is n.

In this step, we will find the optimal solution to the deterministic subproblem (DS) between $t_{now} + L$ to H with deterministic demand rate. Define $v \in [t_{now} + L, H]$ represents time.

We denote $TC_{t_{now}}^D(n)$ is the total cost of optimal selection of n replenishment times for deterministic subproblem, starting from t_{now}. For any selection of replenishment times $\{ v_{1n}, v_{2n}, \ldots, v_{nn}\}$ the total cost is denoted as $TC_{t_{now}}^D(n, v_{1n}, \ldots, v_{nn})$ where superscript D represents deterministic problem. By using expectation on demand rate, expected total cost between t_{now} and H becomes;

$$TC^{D}_{t_{now}}(n, v_{1n}, \dots, v_{nn}) = Setup\ Cost + Unit\ Cost + Holding\ Cost$$

$$\begin{aligned} TC^{D}_{t_{now}}(n, v_{1n}, \dots, v_{nn}) \ &= n.K + u.\sum_{j=1}^{n-1}\Big(N\big(v_{(j-1)n} + \mathrm{L}\big) - N\big(v_{jn} + \mathrm{L}\big)\Big) \\ &+ u.\big(N(H) - N(v_{nn} + L)\big) \\ &+ h.\sum_{j=1}^{n-1}\Big(\big(v_{(j+1)n} - v_{jn}\big).N\big(v_{(j+1)n} + L\big) - \int_{i=v_{jn}+L}^{v_{(j+1)n}+L} N(i).di\Big) \\ &+ h.\Big((H - v_{nn} - L).N(H) - \int_{i=v_{nn}+L}^{H} N(i).di\Big) \end{aligned} \tag{11}$$

Taking partial derivatives of this term with respect to v_{jn}'s where $j \in \{2, \dots, n\}$ gives optimality conditions. In optimal solution of ordering times $v^*_{2n}, \dots, v^*_{nn}$ the resulting terms must be equal to zero. This gives $n - 1$ nonlinear conditions.

For $j = \ 2, \dots, n - 1$

$$\lambda\big(v^*_{jn} + \mathrm{L}\big).\big(v^*_{jn} - v^*_{(j-1)n}\big) = N\big(v^*_{(j+1)n} + \mathrm{L}\big) - N\big(v^*_{jn} + \mathrm{L}\big) \tag{12}$$

For $j = n$

$$\lambda\big(v^*_{jn} + \mathrm{L}\big).\big(v^*_{jn} - v^*_{(j-1)n}\big) = N(\mathrm{H}) - N\big(v^*_{jn} + \mathrm{L}\big) \tag{13}$$

There are $n - 1$ unknowns with $n - 1$ equality conditions since we set $v_{1n} = t_{now}$ for all solutions. Then, these equations will have a unique solution. These solutions could be found by using mathematical software. Johnson and Montgomery suggest setting a value for v_{2n} and then solving all remaining variables. If last condition does not satisfy equality another value of v_{2n} should be selected [18]. These conditions can be solved by using mathematical software.

Since we know the optimal solution for the deterministic subproblem with n orders, we can simplify the total cost term. Now, best total cost for deterministic subproblem with n orders $TC_{t_{now}}^{D}$ can be defined as;

$$TC_{t_{now}}^{D}(n) = \inf_{v_{1n}, v_{2n}, \dots, v_{nn}} \left\{ TC_{t_{now}}^{D}(n,\ v_{1n}, v_{2n}, \dots, v_{nn}) \right\} \tag{14}$$

Determining Replenishment Quantity based on DS Solutions

As discussed before, we can solve DS optimally for any given n. Iterating from $n = 1$ to a sufficiently large upper bound **N** gives the optimal number of orders, which is denoted by $n_{t_{now}}^{*}$. A lousy selection of **N** can be calculated as follows:

$$\mathbf{N} = \frac{b(H - t_{now})\big(N(H) - N(t_{now})\big)}{K} \tag{15}$$

where we compare total setup cost with the total backordering cost of extreme solution where all residual orders are backordered.

We can write;

$$n_{t_{now}}^{*} = \underset{n}{\operatorname{argmin}} \left\{ TC_{t_{now}}^{D}(n), n \in \mathbb{Z}^{+} \right\} \tag{16}$$

After selecting best solution to expected number of residual orders, one can calculate the ordering quantity. For optimal n^*, we have $v_{1n^*}, v_{2n^*}, \ldots$. Then replenishment quantity for order i is;

$$\begin{aligned} Q(t_{now}) &= N(v_{2n^*} + L) - N(v_{1n^*} + L) \\ &= N(v_{2n^*} + L) - N(t_{now} + L) \end{aligned} \tag{17}$$

which corresponds to expected demand until expected next immediate replenishment time.

3.2.2.2. Myopic Order-up-to Level Decision

As discussed before, we can select the review period length among a set of finite candidates. This alternative may represent real-life conditions better since most of business applies periodic replenishments.

For each period length in the candidate set, we will define and solve a subproblem. Since these subproblems are considerably smaller than the subproblems we solved before, we don't need to assume deterministic demand for these problems. We denote $\mathcal{L}$ for the candidate set. For each candidate $l \in \mathcal{L}$, define the subproblem between $[t_{now} + L, t_{now} + L + l]$. Order-up-to level for any l will be the smallest integer, which satisfies the service level derived by holding and backordering cost parameters. Denote

$$\delta(t_{now} + L) = \int_{t_{now}+L}^{t_{now}+L+l} \lambda(t)dt$$

as the expected demand during review period. We will assume that the inventory position at the beginning of the period is equivalent to smallest integer, that satisfies service level derived by holding and backorder cost parameters. Let l_j is the jth candidate in the set $\mathcal{L}$. Let, $Q_j(t)$ is the replenishment quantity for candidate j at time t. Then,

$$Q_j(t) = \min_{Q} \left\{ Q \mid \sum_{x=0}^{Q} \frac{e^{-\delta(t)}.\,\delta(t)^x}{x!} \geq \frac{b}{b+h}, Q \in \mathbb{Z} \right\} \tag{18}$$

which is the same solution to newsvendor problem assuming demand follows Poisson distribution with rate $\delta(t)$. After solving all subproblems we will select the one give the minimum cost per unit time. Then, for l_j we can find expected total cost by using following equation.

$$TC(t, l_j) = K + u.\,Q_j(t) + \sum_{d=0}^{\infty} \begin{array}{c} P\big(D(t+L+l_j) - D(t+L) = d\big). \\ EHB\left(t+L, t+L+l_j, Q_j(t)\right) \end{array} \tag{19}$$

where $EHB(u, v, Q)$ is the expected holding and backordering cost between u and v where starting inventory level is Q. This cost can be calculated by using order statistics of the Non-homogeneous Poisson Process.

Then total cost per unit time is simply

$$TCPU(t, l_j) = \frac{TC(t, l_j)}{l_j} \tag{20}$$

For each element l in the candidate set $\mathcal{L}$, we get $TCPU(t, l)$.

Here, we will select the best $l \in \mathcal{L}$ as l_t^* which satisfies

$$l_t^* = \left\{ l \mid l \in \mathcal{L}, TCPU(t,l) \leq TCPU\left(t,l^{'}\right)\ \forall l^{'} \in \mathcal{L} \right\} \tag{21}$$

Now, we have the best solution for the myopic subproblem. Let j^* is the index of selected period length. Then the order-up-to level is,

$$S(t) = Q_{j^*}(t) \tag{22}$$

After finding our order-up-to level, now we can define the real replenishment quantity, such as:

$$Q(t_i) = S(t_i) - I(t_i) \tag{23}$$

3.2.3. Last Order Problem

In section 3.2.2.1, we see how replenishment quantity can be chosen by solving deterministic subproblem for the residual time horizon. Remember that, we were solving deterministic subproblems for fixed number of residual orders. When we are sufficiently close to end-of-horizon $H,$ we can solve the stochastic subproblem without simplifying the stochasticity. We will call this problem as "Last Order Problem" (LOP).

As we prove in section 3.2.1.3, reorder levels are non-increasing function of time. On top of that, if we assume that demand rate is a continuous, non-increasing function of time, then these reorder levels are step functions with certain break points.

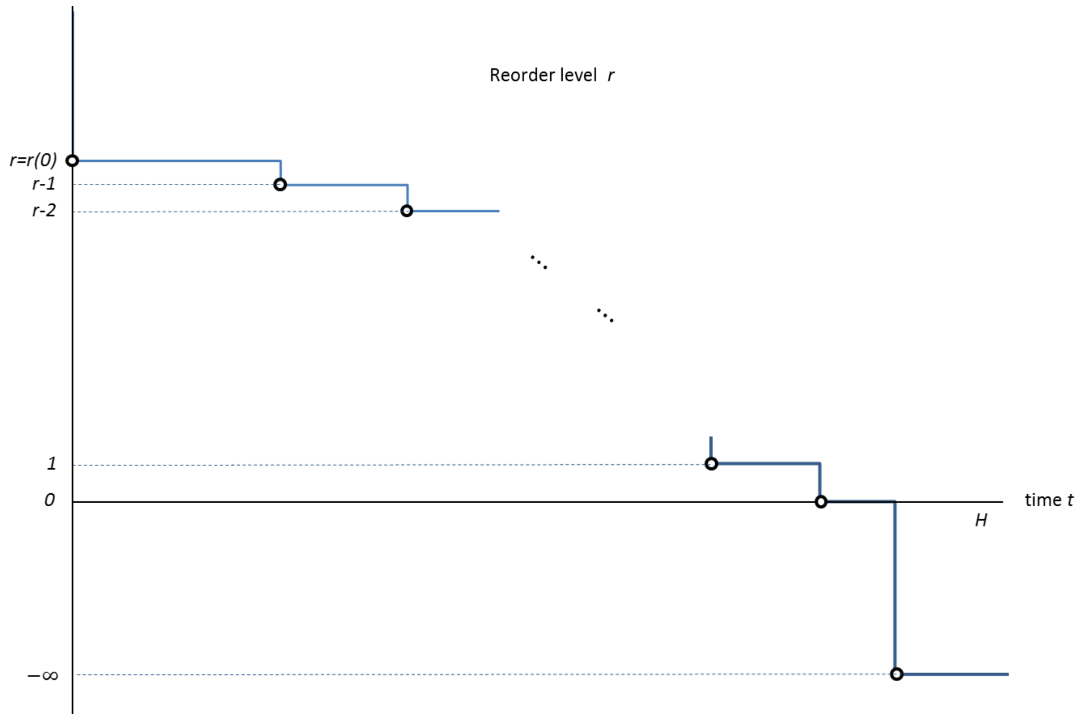

Figure 3.2 Behavior of Reorder Level during Final Phase when Demand Rate is Non-Increasing Function of Time

At time $t = H - L$, the order up to level will become minus infinity with optimal number of estimated replenishments $n_t^* = 1$ for sure. Therefore, instead of issuing an order, retailer may want to wait until the end of horizon and simply meet the all backordered demands with a single replenishment. Note that, the start of the last phase where $\mathrm{r} = -\infty$ may become earlier than H-L.

In their study, Teunter and Haneveld show a similar effect while describing the optimal $(S - 1, S)$ policy [32]. He shows that optimal order-up-to level function S will reach zero, and eventually become minus infinity, where not placing any order is the best option.

The reason for order up to level function takes minus infinity value can be reviewed as follows; assume that an arbitrary demand occurred in t', which is smaller than $H - L$.

Assume our inventory position drops to -1. Retailer has two alternatives. First one is not giving any order at time t' and pays the backordering cost until end of horizon, which corresponds;

$$C_1(t) = b.(H - t) \tag{24}$$

The second option is giving an order at t' and meets the item at $t' + L$. Then, the cost becomes;

$$C_2 = b.L + K + u \tag{25}$$

If $C_1(t') < C_2$ then the best policy is not giving any replenishment order. Therefore, it is obvious that the optimal order up to level for t' is strictly below zero. In order to find optimal reorder level, let there is another demand arrives where inventory position drops to -2 at time t''. Since $t'' \geq t'$ and $C_1(t') < C_2$, we get $C_1(t'') \leq C_1(t') < C_2$. Therefore best policy for this singular order is same as previous: do not issue a replenishment order. Clearly, it is same for all demand after here and it is easy to see that, procedure can continue until minus infinity. Therefore optimal reorder level is minus infinity. When the retailer approaches near to end of the planning horizon, best estimation for remaining orders will get closer to zero.

Based on this observation, when optimal number of estimated replenishment is less than 2, time until H is sufficiently close for considering not giving any order until end of final phase. Moreover the residual stochastic subproblem can be solved.

In this step we will compare expected total cost of two alternatives. First one is placing any order at time $t = t_{now}$. Without loss of generality, assume that we will place an order with size of x. Then, expected total cost between t and H will be;

$$\begin{aligned} &Expected\ Total\ Cost \\ &\qquad = E\ [Setup\ Cost + Unit\ Cost + Holding\ Cost \\ &\qquad + Backorder\ Cost] \end{aligned} \tag{26}$$

Denote the demand between $t + L$ and H is

$$D(t + L, H) \sim NHPP\left(\int_{t+L}^{H} \lambda(t).\, dt\right)$$

Then we can expand total cost formulation as;

$$\begin{aligned} E[TC(x)] = K + K.\, Prob\Big(\big(D(t + L, H) - x - I(t)\big) < 0\Big) + u.\, x \\ + u.\Big(E\Big[\big(D(t + L, H) - x - I(t)\big)^{+}\Big]\Big) \\ + EHBC\big(t + L, H, x + I(t)\big) \end{aligned} \tag{27}$$

where $EHBC$ represent $Expected\ Holding\ and\ Backorder\ Cost$ between $t + L$ and H with an inventory position X at $t + L$. We can expand $EHBC$ as;

$$\begin{aligned} &EHBC\big(t + L, H, x + I(t)\big) \\ &\qquad = \sum_{d=0}^{\infty} P(D(t + L, H) = d).\, EHB(t + L, H, x + I(t), d) \end{aligned} \tag{28}$$

and by using order statistics, we know that d demands will be distributed over horizon where demand rate-time areas between consecutive demand times are equal.

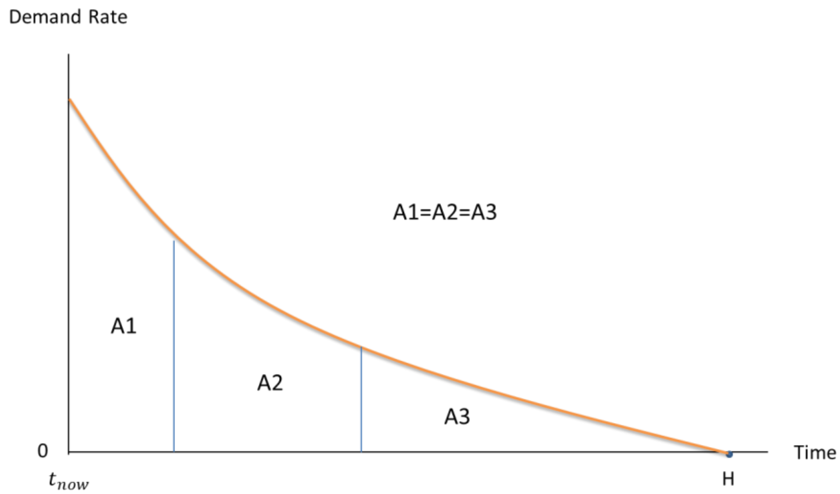

Figure 3.3 Order Statistics of Arrival Times of Demands

Then, once can calculate EHB for any input using bisection method.

Let $m = \int_{t+L}^{H} \lambda(u).du$. We can rewrite (28) as the following;

$$\begin{aligned} TC^{Last-Order}(x) \\ &= K + K.\left(1 - \sum_{d=0}^{x+I(t)} \frac{e^{-m}.m^d}{d!}\right) + u.x \\ &+ u.\left(\sum_{d=x+I(t)}^{\infty} \big(d - x - I(t)\big).\frac{e^{-m}.m^d}{d!}\right) \\ &+ \sum_{d=0}^{\infty} \frac{e^{-m}.m^d}{d!}.EHB(t+L, H, x+I(t), d) \end{aligned} \tag{29}$$

As discussed above, our second alternative is not placing any order until H. Modifying (30) we can write;

$$TC^{Last-Order}(0) = K.\left(1 - e^{-m}\right) + u.\left(\sum_{d=0}^{\infty} d.\frac{e^{-m}.m^{d}}{d!}\right) + \sum_{d=0}^{\infty} \frac{e^{-m}.m^{d}}{d!}.EHB\left(t+L, H, I(t), d\right) \tag{30}$$

Note that, first term in the cost expression (31) represents the setup cost at the end of the horizon which is dependent to demand between $t + L$ and H which follows Poisson Process with rate m.

3.3. Heuristics

3.3.1. First Heuristic; Based on the Expected Number of Residual Orders

Since we are dealing with a finite horizon problem, replenishment times and quantities affect the remaining orders. In finite horizon inventory problems, generally, replenishments are correlated with each other. Logic behind this policy is based on this observation. In order to shape our policy, we are solving a deterministic subproblem for the remaining time horizon.

We know that deterministic demand variation of the problem can be solved optimally for fixed number of residual orders as discussed in section 3.2.2.1. If it is known that there will be n ordering points, then it leads $n - 1$ optimality conditions using first derivatives. These conditions have a unique solution. This observation constitutes a basis for determining the replenishment quantity in our heuristic. Shortly, in our first heuristic, a deterministic subproblem at each ordering time is solved by using optimality conditions, and results of the problem used for determining ordering quantity.

We assume that, retailer start the final phase with sufficient (optimal) inventory level. In other words, assume the inventory is ordered at time '$-L$'.

As stated before, this heuristic is a continuous review policy, which is a variant of classical reorder level, order up to level (s, S) inventory policy.

A scheme of the heuristic can be seen in the Figure 3.4.

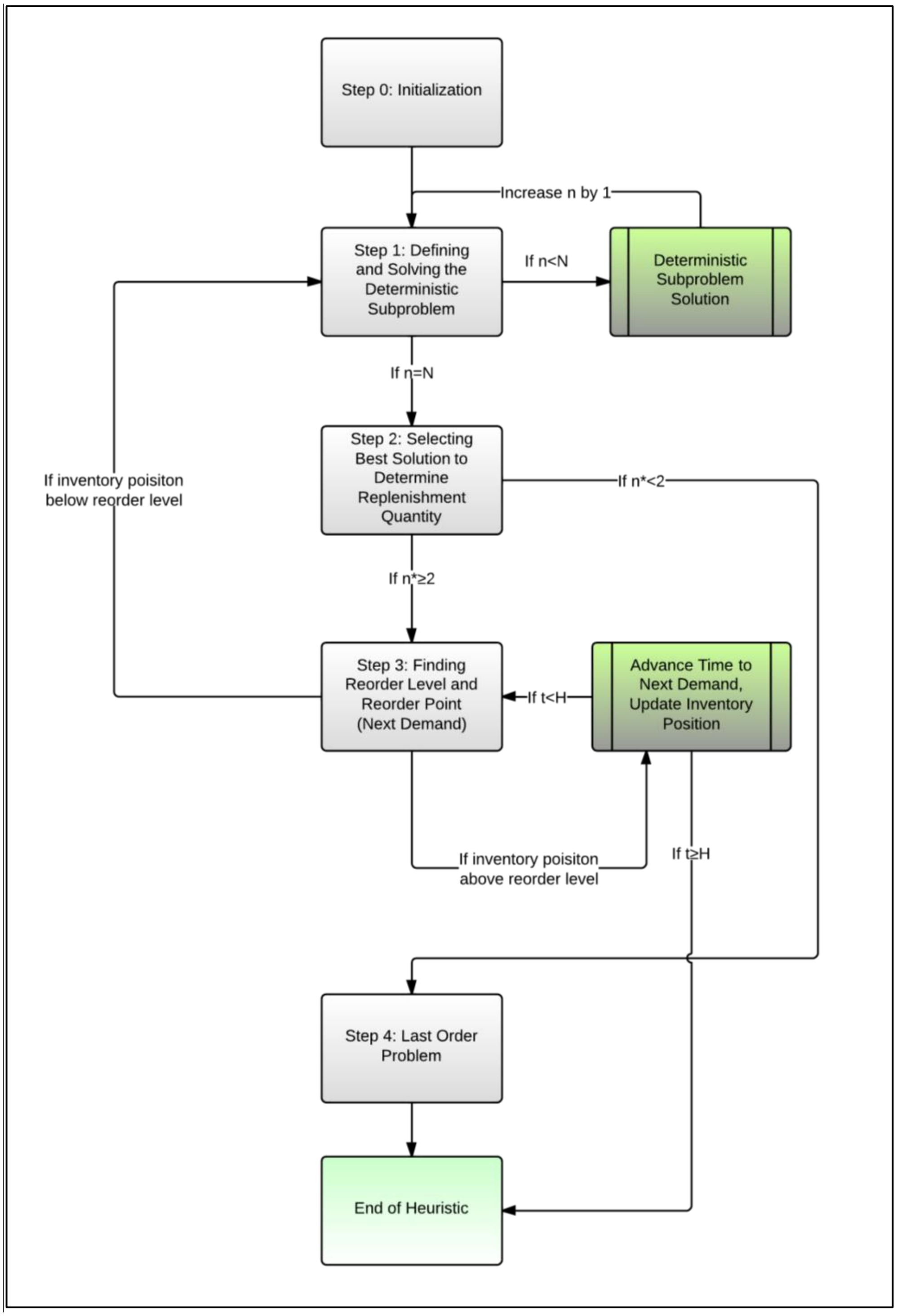

Figure 3.4 Flow chart of the 1st Heuristic

Here, we will show steps of the first heuristic.

Step 0 - Initialization

Set $t_{now} = -L$ and $i = 0$. t_{now} represents the current time and i is used to represent iteration number (namely, the number of current replenishment).

Step 1 – Defining and Solving the Deterministic Subproblem

Set $\overline{t_i} = t_{now}$, where t_i is the order time of i^{th} order while $\overline{t_i}$ is a candidate for t_i. Here, if $i = 0$, it is considered as the "first order".

In order to estimate optimal number of residual orders, we will consider time interval between $[\mathrm{t_{now}} + L, \mathrm{H}]$. We will solve deterministic subproblem in this interval for different selection of number of residual replenishments.

Set $n = 1$ initially and solve DS as discussed in section 3.2.2.1. Then increase n by 1 and solve DS again. Repeat this process until $n = N$, where N is a sufficiently large upper bound of n.

In practice, the total cost decreases while n increases at first, and then increase after some point, which is close to optimal value of n. Therefore, in practice N can be chosen based on the observation on the increment in total cost while n is getting larger. We can say that change in cost is not always convex but close to have a convex shape.

For each subproblem with residual orders n, we get the total cost with the optimal selection of replenishment times over the remaining horizon:

$$\begin{aligned} TC^D_{t_{now}}(n) &= \min_{v_{1n}, v_{2n}, \dots, v_{nn}} \left\{ TC^D_{t_{now}}(n,\ v_{1n}, v_{2n}, \dots, v_{nn}) \right\} \\ TC^D_{t_{now}}(n) &= TC^D_{t_{now}}(n, v^*_{1n}, v^*_{2n}, \dots, v^*_{nn}) \end{aligned} \tag{31}$$

After obtaining total cost for each value $n \in \{1,2,\dots,N\}$ go to step 2.

Step 2 – Selecting Best Solution to Decide Replenishment Quantity

Now we have expected total costs of N different selection of total number of residual orders. Among these costs, we need to select the best possible n to minimize total cost for residual time horizon. Let $n^*_{t_{now}}$ is the best selection of total number of residual orders, such that;

$$n^*_{t_{now}} = \underset{n}{\operatorname{argmin}}\left\{TC^D_{t_{now}}(n), n \in \mathbb{Z}^+\right\} \tag{32}$$

After selecting best solution to expected number of residual orders, one can calculate the ordering quantity.

If $n^*_{t_{now}} \leq 2$ it means we enter the "last-order problem" which is introduced in Section 3.2.3. In this condition, we are sufficiently close to the end-of-horizon H and therefore, remaining problem can be solved optimally. Go to step 4.

If $n^*_{t_{now}} > 2$ set $t_i = t_{now}$. It means, we decide to issue a replenishment order at t_{now} as i^{th} order. Therefore, we are fixing the value of t_i.

As calculated in the Step 1, for optimal n^*, we have v_{1n^*}, v_{2n^*}, ... Then replenishment quantity for order i is;

$$\begin{aligned} Q(t_i) &= N(v_{2n^*} + L) - N(v_{1n^*} + L) \\ &= N(v_{2n^*} + L) - N(t_i + L) \end{aligned} \tag{33}$$

After selection of replenishment quantity, now retailer places a replenishment order with a size $Q(t_i)$. Go to Step 3.

Step 3 – Finding Reorder Level and Reorder Point

In this step, we need to wait until inventory position drops below reorder level. By using an important observation introduced in Section 3.2.1.3, we can check inventory only when a demand arrives rather than tracking inventory position continuously, which may be costly. Here, we may use one of the two different reorder level explained in section 3.2.1. For each type of service levels, we can easily show that reorder level is a non-increasing function of time which is similar to demand rate function.

The observation leads us to define this step as follows: assume that, a demand arrives at t. Then, inventory position is updated and we will check the reorder level. Based on the selected service measure, reorder level for time t is calculated by using either (3) or (4).

If inventory position is above the reorder level, then repeat step 3. The procedure must be repeated for every until inventory position drops below reorder level, as observation suggests. Otherwise, if inventory position drops below reorder level for any $t \leq H - L$, then go to step 1.

Step 4 – Last Order Problem

Until this step of the heuristic, estimation of best solution for the total number of residual replenishment is used. As described in Section 3.2.3, we can solve the remaining subproblem when we are close to end of the planning horizon. This is our last decision in the problem. Based on the solution of the LOP we can give a last order at t_{now} or skip this decision point. Either way, we need to satisfy any backordered demands at the end of horizon and this concludes the heuristic.

3.3.2. Second Heuristic; Based on the Minimization of Myopic Period

While looking for alternative of the first heuristics, we came up with an idea, which resembles applications in the real life. Assume there is a retailer, which is about to issue an order but undecided about the quantity of the replenishment. A basic solution to this complex problem is setting replenishment quantity to an amount which is most probably cover the demand until next week or next month or next two months, etc. - average time between consecutive replenishments based on historical data. Here, his estimation of next order is can be chosen among meaningful candidates. Note that, this selection is done automatically in the first heuristic by looking ahead to remaining time horizon. Here, our objective is to select best candidate which will minimize total cost per unit time until next estimated order.

A major drawback of this heuristic is, residual time is not being considered while deciding on replenishment quantity. Choosing replenishment quantity based on total cost per period length is a suitable approach for infinite horizon. However, we may reflect the effect of residual time into the heuristic with some extensions.

This policy can be considered as a variant of well-known Silver-Meal heuristic proposed by Silver and Meal [26]. We are minimizing total cost per period, in a finite set of variable period lengths. In that sense this heuristic is a periodic review policy with variable period lengths.

Assume that, set of candidate periods $\mathcal{L}$ is already given or known before the problem. If not provided, this set can be constructed easily based on the nature of the given problem.

Steps of the heuristic are shown in Figure 3.5.

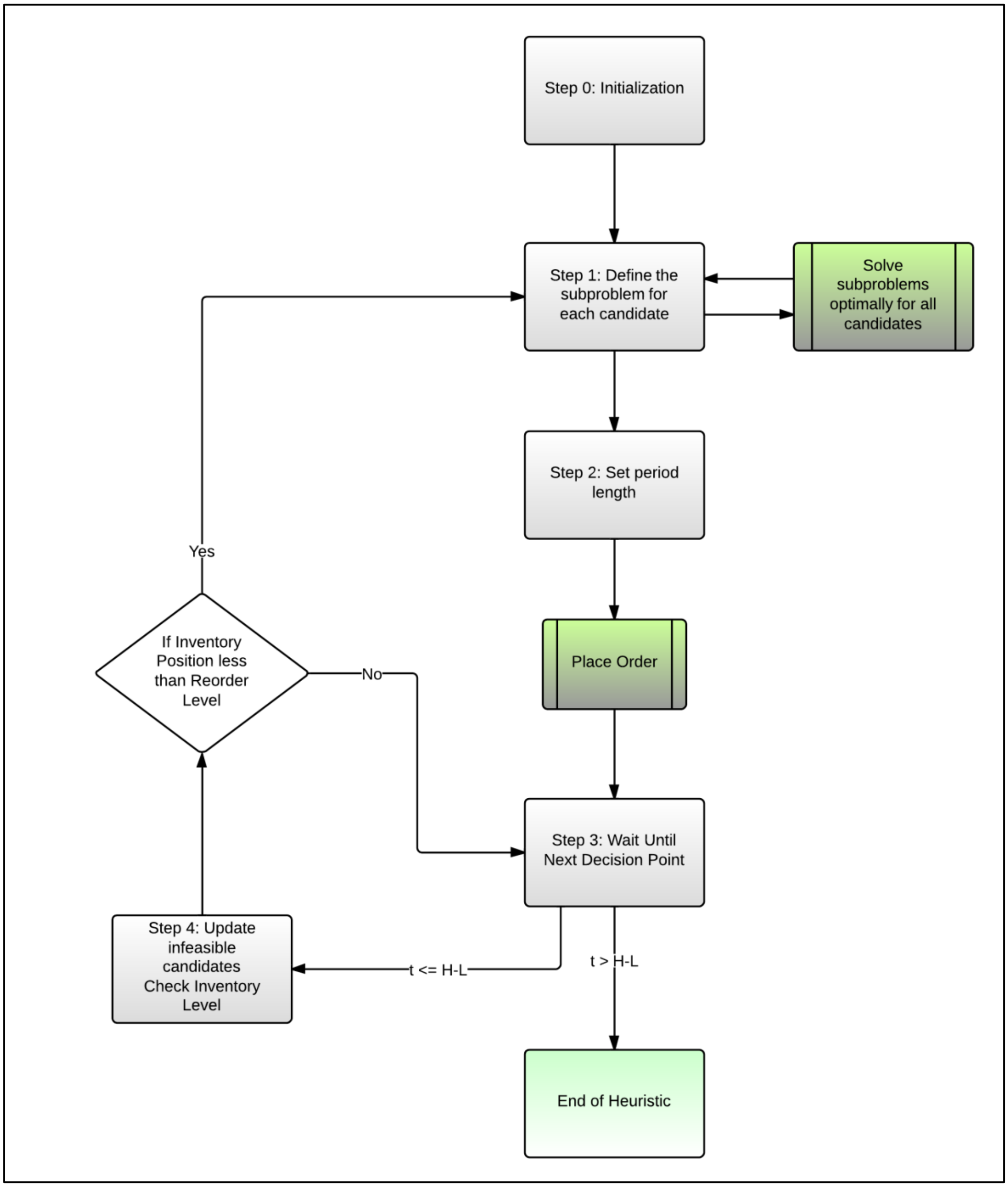

Figure 3.5 Steps of the 2^{nd} heuristic

Step 0: Initialization

Set $t_{now} = -L$ and $i = 0$.

Step 1: Define the Subproblem for Each Candidate

In this step, for each candidate in the period set, we will define the subproblem. Unlike previous deterministic subproblems, the subproblems that will be presented in this policy will be solved optimally.

By using myopic order up to level described in the section 3.2.2.2, we can find total cost per unit time for each candidate. Denote $TCPU(t, l_j)$ is the total cost per unit time for candidate $l_j \in \mathcal{L}$. All $TCPU$s can be calculated easily by using (19). After solving subproblems optimally for each candidate, go to step 2.

Step 2: Set Period Length and Place Replenishment Order

By using information obtained in Step 1, we can denote

$$l_t^* = \operatorname*{argmin}_{l \in \mathcal{L}} \; TCPU(t, l) \tag{34}$$

Now as we decide on our period length, we can set $t_i = t_{now}$. Now we need to specify replenishment quantity. As we have the best solution for the myopic subproblem, we can set the order-up-to level as,

$$S(t) = \lceil N(t + L + l_t^*) - N(t + L) \rceil \tag{35}$$

Based on our selection, we can issue a replenishment order with a size of $Q(t_i)$ at time t_i which is the difference between order-up-to level and current inventory position, as defined in subsection 3.2.2.2. Then, go to step 3.

Step 3: Wait Until Next Decision Point

In first heuristic we checked our inventory whenever a demand arrives instead of tracking it continuously. Although same conditions are still available for this policy, we may limit ourselves to our earlier decisions. Remember that, in second step period length is fixed. To be consistent, we will check inventory position when this period ends. Hence for any replenishment$i$, we will until $t_i + L + l_i$ where l_i is the length of i^{th} period. When $t_{now} = t_i + L + l_i$, if $t_{now} \leq H - L$ go to step 4, otherwise it is end of the problem.

Step 4: Update Candidate Set and Check Inventory Position

Since problem horizon is finite, at some point $t_{now} + l_j$ may be larger than H for any $l_j \in \mathcal{L}$, where period length becomes infeasible since $\lambda(t)$ is defined only between $[0, H]$. In order to prevent infeasible periods, we will update our candidates.

For any $l \in \mathcal{L}$, if $t + l > H$, than update that member as $l = H - t$. As an additional step, we can remove any duplicate candidate in the set $\mathcal{L}$. After this update, all elements in the candidate set become feasible.

Now we will check if our inventory position is less than our reorder level. Here, we are using same reorder level as we used for first heuristic. By using reorder level for Type-1 (1) or Type-2 (4) service measure, if $I(t_{now}) \leq r(t_{now})$ go to Step 1.

An additional step is needed otherwise. If our inventory position is larger than reorder level, retailer shouldn't issue a replenishment order. Therefore, we will solve our subproblems as described in Step 1, with an exception: starting inventory of the period will be equal to current inventory position. Hence; for candidate $l_j \in \mathcal{L}$

$$TC(t, l_j) = K + u.I(t) + \sum_{d=0}^{\infty} \begin{matrix} P\big(D(t + L + l_j) - D(t + L) = d\big). \\ EHB\left(t + L, t + L + l_j, I(t)\right) \end{matrix} \tag{36}$$

and

$$TCPU(t, l_j) = \frac{TC(t, l_j)}{l_j} \tag{37}$$

By using total cost per unit time for each candidate, we can similarly set

$$l_t^* = \underset{l \in \mathcal{L}}{\text{argmin}}\ TCPU(t, l) \tag{38}$$

Finally we decided for our next decision point. Go to Step 3 with new period length.

3.4. Effect of Residual Time on Solutions

Until here, we described two different heuristics to solve retailer's problem. One of these policies is a policy with look-ahead capability, where residual time has an effect on our decisions. Other one focuses on myopic decisions and tries to minimize cost per unit time in every decision point.

One of the very natures of retailer's final phase problem is the finiteness of the time horizon. As discussed in section 2, for most of business types, end of final phase is known and deterministic. Therefore, retailer's problem is a finite horizon problem, as its effect is obvious on our heuristics.

Recall that we have two major driving forces that define our problem and thus affects our heuristics: horizon length and demand rate. In general, while horizon length is effective in our decision on order-up-to level, demand rate affects reorder level. At this point, note that time horizon has no effect on reorder level, which may lead some troubles as described below.

Suppose our demand rate is constant over time horizon, thus demand follows Homogeneous Poisson Process and let its rate is λ. In this case reorder level will be constant for both Type-1 and Type-2 service measure. For Type-1 service measure, rewrite (2) such as:

$$Prob(Demand\ During\ Lead\ Time \leq \rho) = \sum_{x=0}^{\rho} \frac{e^{-\delta(t)}.\,\delta(t)^x}{x!} \tag{39}$$

where $\delta(t)$ is defined as expected demand during lead time. Here, $\delta(t) = \int_t^{t+L} \lambda.\,dt = \lambda.\,L$ thus it's constant and not dependent on t. For constant (40) reorder level

$$r_1(\alpha, t) = \min_{\rho} \left\{ \rho \mid \sum_{x=0}^{\rho} \frac{e^{-\delta(t)}.\,\delta(t)^x}{x!} \geq \alpha\ , \rho \epsilon \mathbb{Z}^+, \delta(t) = \lambda L \right\} \tag{40}$$

becomes constant for any arbitrary t.

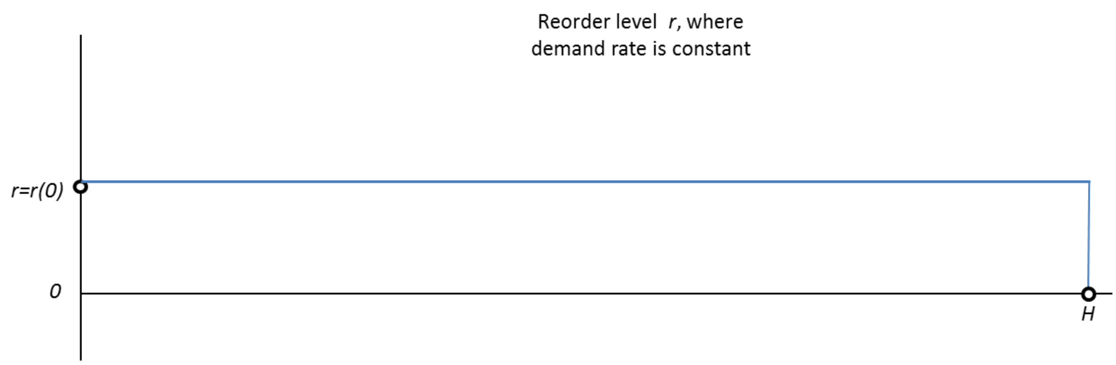

Figure 3.6 Effect of constant demand rate on reorder level

However, in optimal policy, reorder level should decrease towards end of horizon due to risk of overstocking. Such a behavior is incorrect for both practical and theoretical purposes. We know that reorder level should be as low as 0 when time is $H - L$ and it should be decrease gradually towards it.

In order to reflect the effect of residual time on our heuristics we define an adjustment rate for reorder level. This rate uses the following observations:

- Effect of residual horizon at starting point should be zero.
- Adjustment rate should decrease reorder level to zero when $t \geq H - L$.
- Based on problem parameters, gradual decrease may be slow or fast.

Such a rate can be defined as a function of time and horizon length, dependent to rate $\frac{t}{H-L}$ where it's zero at first and reaches 1 when $t = H - L$. Therefore denote adjusted demand rate as

$$\lambda'^{(t)} = \lambda(t).\left(1 - \left(\frac{t}{H - L}\right)^{a}\right) \tag{41}$$

where a is adjustment parameter that defines shape of adjusted demand rate.

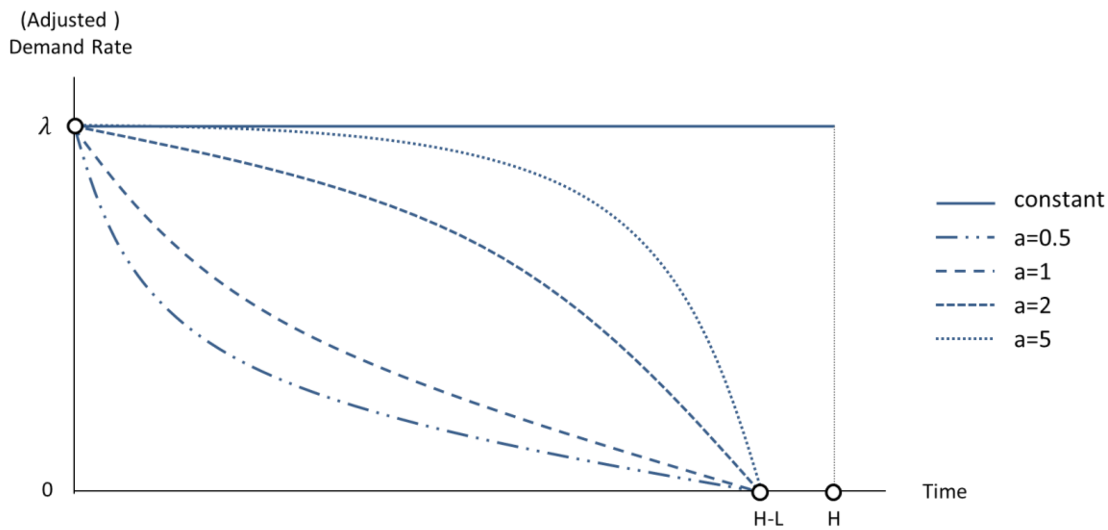

Figure 3.7 Behavior of Adjusted Demand Rates for Various Selection of the Parameter

As shown in figure above, if $a = 1$ and demand rate is constant, adjusted demand decrease linearly. If $a > 1$, adjusted demand rate increase slowly at first and then decrease sharply towards end of horizon. If a is set to be infinity, than adjusted demand rate is equal to demand rate.

Actually, this adjustment works well for homogeneous demand cases as described in the following subsection.

Following subsections are organized as follows. In subsection 3.4.1, we will show that the usage of adjustment parameter provides near-optimal results for the homogeneous case. Then, we will show how the adjustment parameter can be selected for a given problem. In the last subsection, we suggest Power Approximation method to find a sufficiently good selection of the adjustment parameter and show the calculation steps.

3.4.1. Comparison with Teunter and Haneveld's Method for Homogeneous Cases

In 2002, Teunter and Haneveld dealt with a similar problem [32]. In their work, an optimal policy for a homogeneous-demand rate spare part inventory is proposed. In the study, setup cost is discarded; hence an $(S-1, S)$ policy is proved to be optimal. Moreover, for this policy, break points throughout the horizon are given explicitly. Their method will be denoted as THM.

Compared to our study, the demand rate is different. THM considers homogeneous Poisson demand rate for the spare parts while in our study, demand rate is distributed with a non-homogeneous Poisson demand rate. Moreover, we have a fix setup cost, although setup cost is not considered in the study. Therefore, if we set our setup cost as zero, and homogenized our demand rate, than it will be the same problem. Since their problem is similar to the problem described here, we compared performance of our heuristic with his method.

Since our first heuristic provides a closer solution to the THM, we used it for the comparison. Thus, we can examine effect of adjustment parameter by comparing his optimal policy with ours.

Our first heuristic performs as the THM suggests when the setup cost is zero; order size will be unit sized for all orders. It is precisely the same policy of THM $(S-1, S)$ optimal policy. The $S-1$ level corresponds to reorder level in our heuristic, while S level is order up to level as same.

Here, following parameters are used which are defined by Teunter and Haneveld [32].

u=1, b=20, h=0.2, K=0, λ=4, L=0.25 and let H=10. Now we can plot the order-up-to level with given parameters as follows:

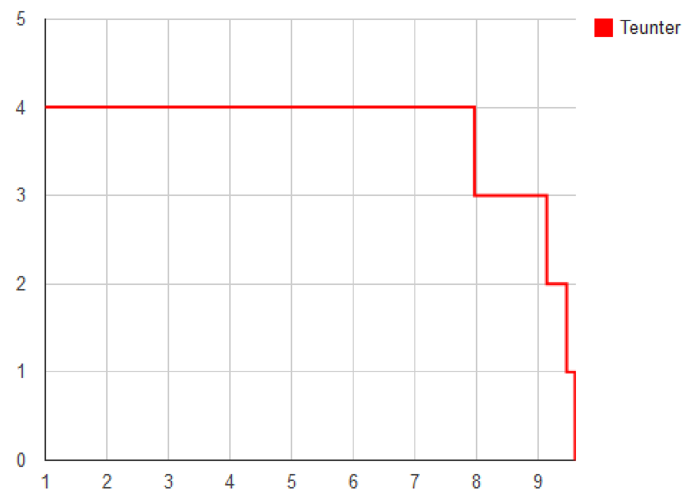

Figure 3.8 Order-up-to level by THM for Teunter and Haneveld's problem

Order-up-to level (reorder level plus one unit) can be plot as follows if adjustment rate is not used.

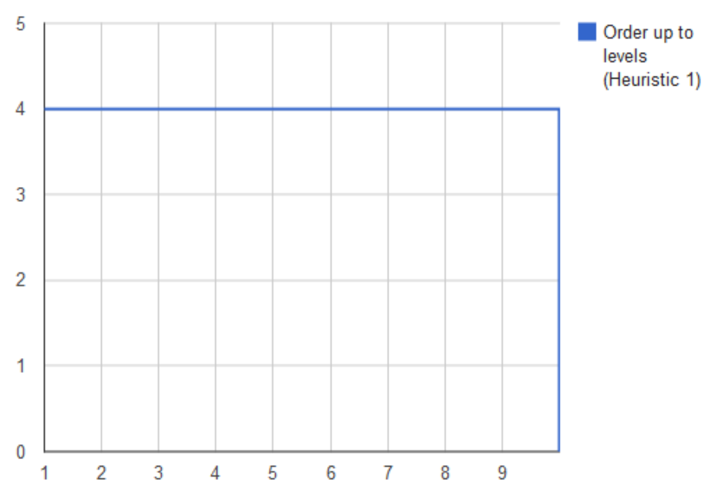

Figure 3.9 Order-up-to level by First Heuristic for Homogeneous Case

As data provides, our heuristic does not perform well for homogeneous case. As described previously, an adjustment is needed to reflect the effect of the residual time.

Let select an adjustment parameter $a = 4.9$. Then we get the following order-up-to level for our first heuristic:

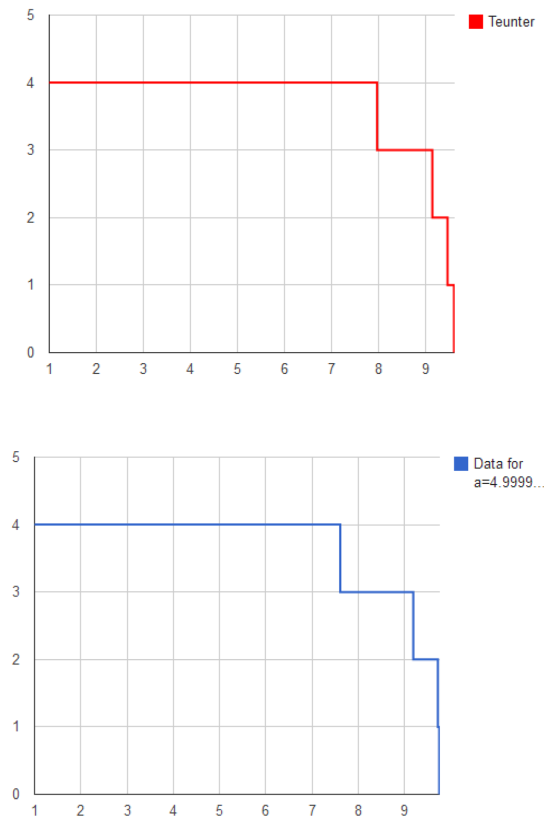

Figure 3.10 Side by side comparison of THM and First Policy's Order-Up-To Level with Adjusted Demand Rate

As shown, adjustment parameter is capable to imitate behavior of reorder level of the optimal solution. The next subsection is devoted to describe which parameters affect the selection of adjustment parameter and how it can be selected.

3.4.2. Selection of the Adjustment Parameter

We observe that selection of parameter a should be related with backordering (b), holding (h) and setup (K) costs, lead time (L) and horizon length (H). If holding cost is high and backorder cost is significantly small, then obviously a should be less because otherwise a may not be sufficient to decrease the adjusted demand rate, which may lead excessive holding cost. In the opposite case, if backorder cost is high and holding cost is small, then a should be high to maintain a higher adjusted demand rate for escaping excessive backorder cost. Due to these observations a should be proportional with b/h. Moreover, a should increase when horizon length and lead time are increasing.

Based on our observation in change of a, we came up with three different alternatives. First alternative gives a mild adjustment parameter, where adjusted demand rate stays high until end-of-horizon and decrease sharply at the end. For this selection a should be selected relatively higher $(a > 1)$. Second alternative is selecting an aggressive adjustment parameter, where adjusted rate decrease at first and its acceleration becomes slower as time passes by or decreases linearly $(a \leq 1)$. Third selection is moderate selection, which is a linear combination of two alternatives.

In order to finding best estimation of parameter a, we get some simulation runs. For evaluating the performance of the selected a level, we consider Teunter and Haneveld's problem with homogeneous demand rate with K=0. (Appendix 1)

For Teunter and Haneveld's original problem, we found that, if parameter a is taken 5.5 for 1st heuristic, then average total cost for 100 replications is 1.87% away from optimal, which is promising.

For the variations of the problem solved in Teunter and Haneveld's paper [32], we search best a values. Assume the standard problem has following values.

$$\lambda = 4, H = 10, L = 0.25, h = 0.2, b = 20, K = 0, u = 1 \tag{42}$$

Best a levels are found by an exhaustive search algorithm. We found the following a values for the variations of the standard problem as follows:

Table 3-1 Best selection of a for various cases.

Variation		RESULT (100 Replications Each)		
	Best a Level	THM Total Cost	Adjusted Heuristic 1 Total Cost	Gap to THM (%)
H=20	6.5	94.027 ± 1.45	94.301 ± 1.469	0.2920
L=0.5	5.5	50.548 ± 1.545	50.852 ± 1.46	0.6022
b=5	0.8	44.451 ± 1.242	45.197 ± 1.238	1.6788
b=10	1.0	46.176 ± 1.415	46.66 ± 1.197	1.0476
h=0.4	1.0	52.77 ± 1.686	53.191 ± 1.313	0.7971

- For standard problem with horizon length is 20, best a level is 6.5, where total cost is very close the optimal result, gap between costs is just 0.29%.
- For standard problem with lead time 0.5, we get a solution 0.60% away from optimal cost.
- With a backorder cost 5, best level for a is significantly low, 0.8 where we satisfy to be far from optimal solution just 1.67%.
- For the original problem with only difference backorder cost is 10, best selection of a is 1, where average total cost 1.04% away from average total cost with Teunter and Haneveld's optimal policy, THM.
- And lastly, for original problem with only difference holding cost is 0.4, best selection of a is again 1, where total cost is 0.79% away from optimal result.

We conclude that a is depending on the following parameters;

- Holding / backorder cost ratio
- Lead Time
- Horizon Length

We set up 20 different scenarios and get 100 replications for more than 30 different a levels each to better estimate it. We reach the following selections of a, where unit cost is fixed to 1;

Table 3-2 Best selection of adjustment parameter

Parameters					
Horizon Length	Backorder Cost	Holding Cost	Lead Time	Setup Cost	Adjustment Parameter
H	b	h	L	K	Best a
5	18	0.6	0.50	2	0.2
5	15	0.8	0.75	1	0.8
5	15	0.7	0.75	0	5.4
5	25	0.9	1.25	0	7.2
5	18	0.5	0.75	2	0.5
10	19	0.5	0.50	2	0.3
10	15	0.2	0.50	3	2.1
10	21	0.7	0.50	0	7.9
10	19	0.1	1.25	2	3.5
10	25	0.2	0.75	0	6
20	23	1.0	1.25	0	57.5
20	16	0.9	1.25	0	53.1
20	17	0.1	0.50	4	8.2
20	24	0.4	1.25	0	51.2
20	30	0.1	0.25	5	8.1
30	26	0.2	1.25	3	12.6
30	16	0.9	0.25	0	30.4
30	15	0.1	0.25	2	7
30	20	0.2	0.50	0	44.5
30	22	1.0	1.25	0	58.5

3.4.3. Power Approximation of Adjustment Parameter "a"

By using simulation results given in Section 3.4.2, we apply power approximation method, defined by Ehrhardt in 1979 [8]. This approximation assumes that the parameter to be adjusted is a multiplicative function of the factors. Let, the optimal selection of adjustment parameter "a" has the following relation with the problem parameters:

$$a = c_1 H^{c_2} \left(\frac{b}{h}\right)^{c_3} (L+1)^{c_4} (K+1)^{c_5} \tag{43}$$

where c_1, c_2, c_3, c_4, c_5 are the parameters to be approximated. Taking the logarithm of both sides gives us

$$\ln a = \ln c_1 + c_2 \ln H + c_3 \ln\left(\frac{b}{h}\right) + c_4 \ln(L+1) + c_5 \ln(K+1) \tag{44}$$

which can be approximated by linear regression. We take some simulation runs for the approximation (Table 3-2). By using the linear least-squares approach as suggested by the author, the approximations of the parameters are listed below.

Table 3-3 Regression Results for Power Approximation

Parameter	*Approximation*
c_1	0.02
c_2	1.71
c_3	0.34
c_4	1.73
c_5	-1.43

The statistical results obtained from Linear Regression are given in Appendix 4.

Later, we conclude that parameter a should be dependent only to horizon length. Moreover, backordering and holding cost ratio and setup costs are not sufficient to represent the behavior. We know that a should increase when H increases while other relations of problem parameters with a is unclear. Hence we ignore the effects of the other parameters and apply Power Approximation where a is defined as

$$a = \mathrm{c}_1 H^{c_2} \tag{45}$$

and apply Power Approximation. Statistical details are provided in Appendix 5. Approximation is represented as

$$a = 0.05\, H^{1.86} \tag{46}$$

Unfortunately, this approximation explains the behavior of adjustment parameter with adjusted R-Square value of 0.52. Although our Power Approximation is unable to explain adjustment parameter, we use this approximation for the computations in Chapter 4. Such an approximation may lead under or overestimation of adjustment parameter.

By using these results, we get the approximations of a for our experiment set:

Table 3-4 Comparison of Best and Approximated Adjustment Parameter

Parameters						
Horizon Length	Backorder Cost	Holding Cost	Lead Time	Setup Cost	Adjustment Parameter	Adjustment Parameter
H	b	h	L	K	Best a	Approximation
5	18	0.6	0.50	2	0.2	1.09
5	15	0.8	0.75	1	0.8	1.09
5	15	0.7	0.75	0	5.4	1.09
5	25	0.9	1.25	0	7.2	1.09
5	18	0.5	0.75	2	0.5	1.09
10	19	0.5	0.50	2	0.3	3.95
10	15	0.2	0.50	3	2.1	3.95
10	21	0.7	0.50	0	7.9	3.95
10	19	0.1	1.25	2	3.5	3.95
10	25	0.2	0.75	0	6	3.95
20	23	1.0	1.25	0	57.5	14.35
20	16	0.9	1.25	0	53.1	14.35
20	17	0.1	0.50	4	8.2	14.35
20	24	0.4	1.25	0	51.2	14.35
20	30	0.1	0.25	5	8.1	14.35
30	26	0.2	1.25	3	12.6	30.53
30	16	0.9	0.25	0	30.4	30.53
30	15	0.1	0.25	2	7	30.53
30	20	0.2	0.50	0	44.5	30.53
30	22	1.0	1.25	0	58.5	30.53

3.5. Ending Remarks

Residual time is an effective element in finite horizon problems, in general. Most of the time, decisions are affected by the residual time. For our problem, even if the demand rate stays constant, retailers may want to reduce the reorder level to minimize risk of paying unnecessary setup costs. Teunter and Haneveld's optimal policy for the homogeneous demand rate case with zero setup cost shows that reorder level should decrease towards the end of horizon [32]. Therefore, applying our two heuristics on the problems without any modification may lead some excessive setup costs, since reorder level defined by either Type-1 or Type-2 service level does not consider residual time on calculations. However, we know that at some point of time, not giving a replenishment order until the end is the best option for the retailer as mentioned in Section 3.2.3.

Since we know that reorder level should decrease gradually to zero, and then to minus infinity, we decided to use an adjustment parameter, which leads underestimating the demand during lead time:

$$\lambda'^{(t)} = \lambda(t).\left(1 - \left(\frac{t}{H - L}\right)^{a}\right) \tag{47}$$

This underestimation is dependent to residual time until end-of-horizon and can be adjusted by the parameter a. Based on problem parameters, we would like to change our underestimation of the demand during lead time. For instance, when horizon length is sufficiently large, we would like to increase adjustment parameter a, since the relatively lower values of a leads a sharp decrease at the beginning of the horizon. As holding and backorder cost parameters are effective in THM, we know that these two parameters should affect our selection of a. Also, setup cost is another parameter that should be considered.

As the best selection of parameter a seems unclear, we decided to use Power Approximation method, where the parameter a is defined as a multiplicative function of parameters mentioned. We decided that only horizon length is effective on adjustment parameter. Then by using Linear Regression on the logarithm of both sides, we can estimate the parameters in the Power Approximation method. Note that for a good estimation of a we need a sufficiently big sample size. Small number of experiments may lead errors in regression of parameters. Even if we approximate this parameter, such a relation between adjustment parameter a and the other parameter of the problem is not certain. We suggest using this approximation for the decision maker, for a relatively better selection of the adjustment parameter. Decision maker can also find another way to search for the best value of a by taking simulation runs.

Power Approximation provides a value for adjustment parameter a and we use this technique for all simulation results in Chapter 4 unless otherwise stated. We also show why this adjustment is needed in the numerical experiments.

Chapter 4

Computations & Results

In this chapter, we used our both heuristics for various setups and measure the effectiveness of these policies. In 4.1 we will introduce the computation platform, simulation software and system specifications. In 4.2 we provide verification of software by using simple cases and parameters. Finally in section 4.3 we provide comprehensive results for both heuristics and compare results.

4.1. Computation Platform

Since our heuristic is designed for a stochastic problem, in order to evaluate performance of the policies we need simulation. However, available simulation softwares are not flexible to solve the deterministic subproblems. Therefore, we create a user-friend simulation tool, Inventory System Simulator (Insys) for measuring and comparing performance of the policies.

Insys is developed on Object-Oriented Java language (Java JDK 1.7.0) and able to use mathematical software MATLAB® for calculations. There are 3 external libraries in Insys. First external library is exp4j (ver. 0.2.8) which enables the usage of symbolic definition of demand rates by using variables. Second external library is JSC (Java Statistical Classes, ver. 1.0) which is used for demand distribution, such as generating demand points according to Non-homogeneous Poisson Distribution. Last external library is matlabcontrol (ver. 4.0.0) for connecting MATLAB® functions to Insys.

Insys is capable of simulating 100 replications in less than 2 minutes for most cases (homogeneous case with no setup cost). All mathematical operations, such as solving (9) and (10) optimality conditions and calculating long mathematical expressions (8) are done via MATLAB®.

Insys has also well-designed user interface for saving/loading problems and tracking inventory position in continuous time. Both inventory position and level could be tracked in continuous time. After getting runs simulation graphs (inventory movements) are recorded as image files to the computer for detailed analysis. The simulation tool is capable of running THM, 1st Heuristics and 2nd Heuristic. For THM, tool can also provide optimal order-up-to levels.

For numerical experiments in this chapter, Type-2 service level during lead time is applied unless otherwise stated. Service level is fixed to $b/_{b+h}$.

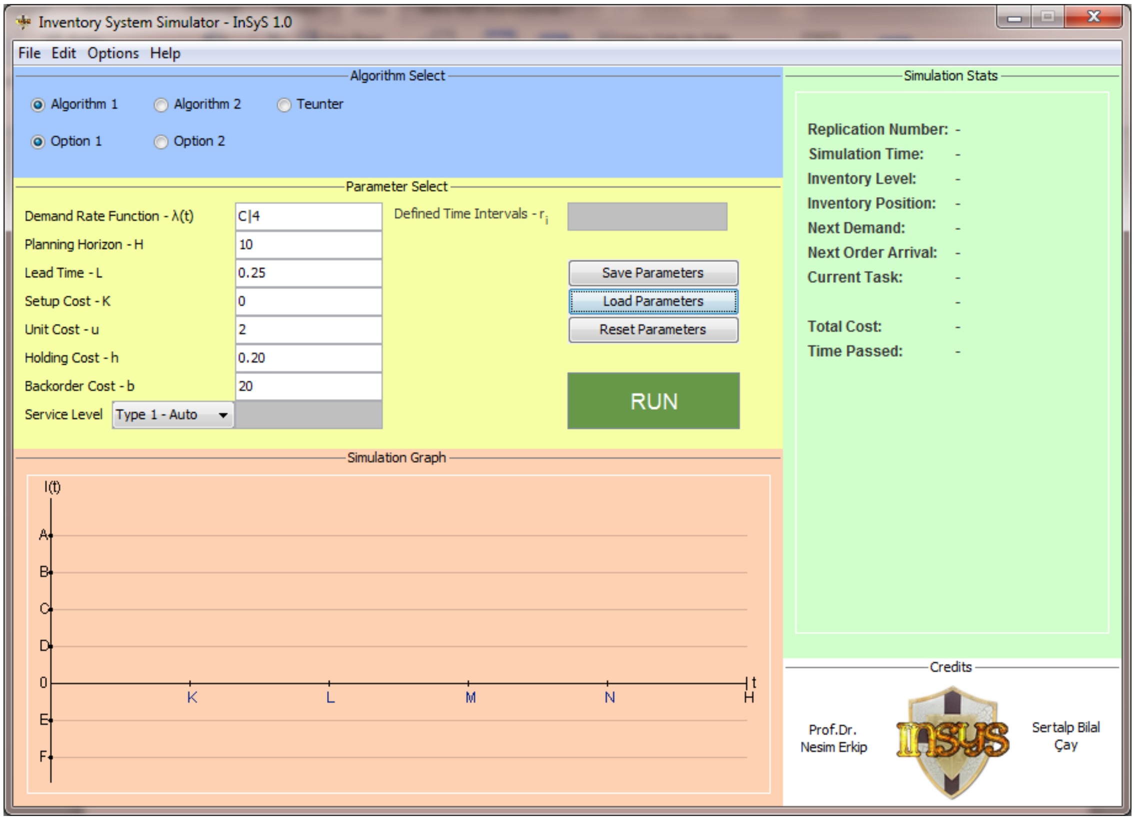

Figure 4.1 User Interface of Insys Simulation Tool

Usage of Insys

Insys is capable to solve Final Phase Problem by using 1st heuristic, 2nd heuristic and THM. Note that, it set some of pre-defined values for the selection of the algorithm. For instance, THM works only if K=0, hence it sets setup cost value at the time of selection.

User can set the non-homogeneous Poisson demand rate in three ways. First, the constant rate is defined by letter "C" and written as "$C|4$" for $\lambda = 4$. Second, piecewise linear cases can be set by letter "P" such as "$P|3{,}3*x{:}\,6, x{:}\,10{,}2*x$" represents

$$\lambda(t) = \begin{cases} 3t & if\ 0 \leq t < 3 \\ t & if\ 3 \leq t < 6 \\ 2t & if\ 6 \leq t < 10 \end{cases}$$

Lastly, user can enter any other function with letter "O" such as "$O|0.2 * (40 - x)$" which corresponds $\lambda(t) = 0.2\ (40 - t)$ (Figure 13). Note that, there is no any restriction for given function, however demand rate should be a non-increasing function to take meaningful results.

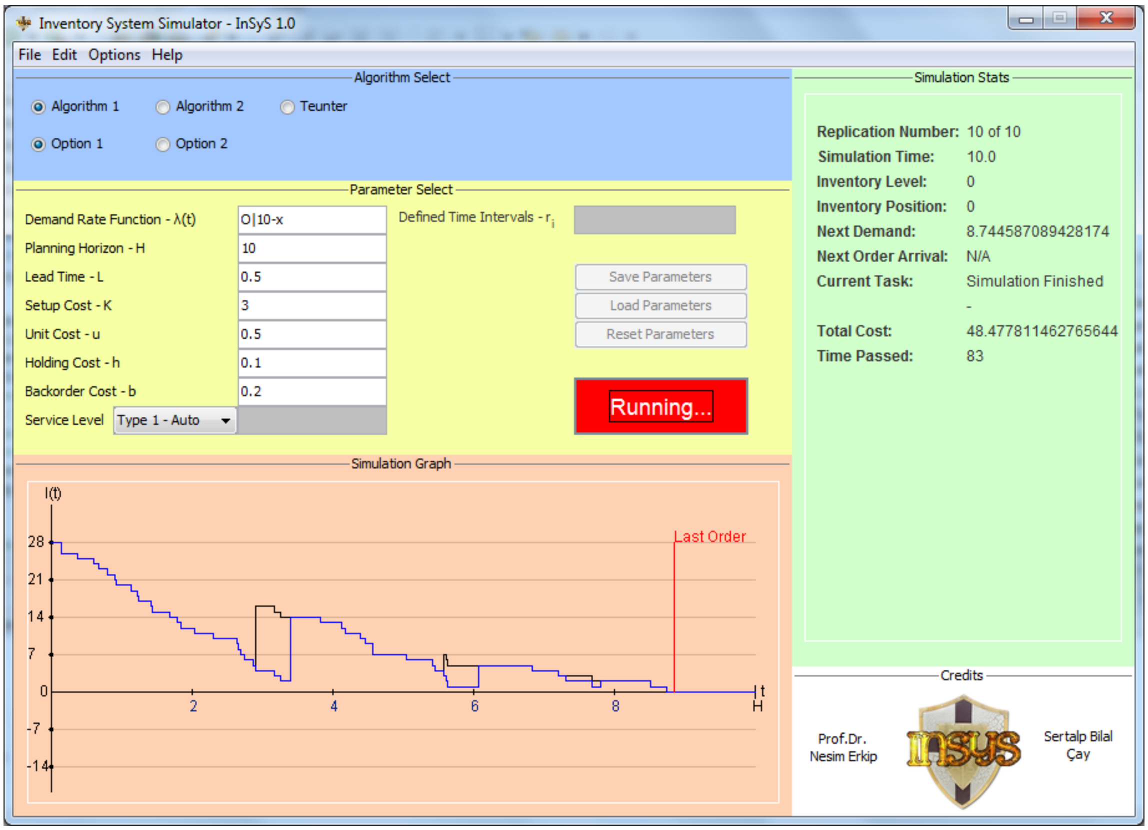

Figure 4.2 Insys is simulating a case, where demand rate is a decreasing linear function of time.

Parameter should be set in numerical format. Only for 2nd heuristic, Defined time intervals should be separated with comma, such as "1,2,4,10" which defines the candidate period length for the 2nd heuristic.

In default settings, Insys provides 10 replications with the given setup and their inventory position-level versus time graphs. (Figure 14) Note that all generations and statistics are also recorded for comparison purposes. (Appendix 2)

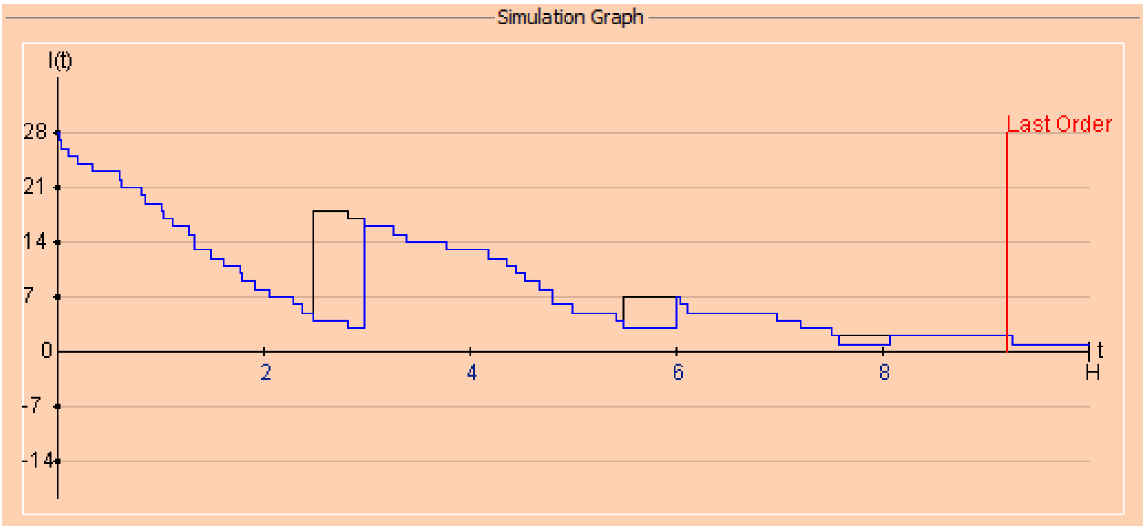

Figure 4.3 A sample output file of Insys.

4.2. Validation and Verification of Software

As shown in the previous subsection Insys provides reasonable results for given inputs. We know that when K=0, the orders should be unit sized, as the solution of deterministic subproblems. For $K = 0$ standard problem, we get the following output Inventory Position vs. time graph:

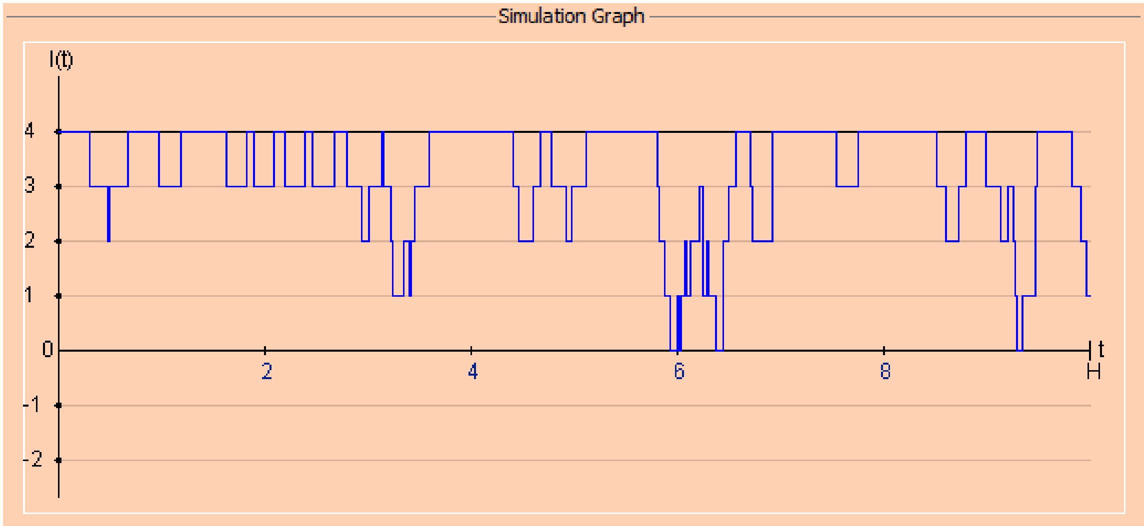

Figure 4.4 Inventory Position-Level vs. Time Graph for Standard Problem by using 1st Heuristic (Appendix 3)

Note that, all orders are unit sized as we expect. See Appendix 3 for the final report on replenishment times and quantities. As seen all orders are unit-sized as expected. It is because, best solution to Deterministic Subproblem is achieved where n equals to expected demand between t_{now} and H and since there is no setup cost. Hence replenishing inventory for every single demand minimizes the total cost. Therefore, order sizes will be unit sized. This theoretic solution is observable in simulation replications, which proves the tool works correctly in terms of (1) solving DS and (2) calculating costs.

We also see that, demand generations of the software are reasonable. For different demand rate functions (λ) we inspect the demand times and verify that times are accurate. On a simple example, when $\lambda = 4$ constant, the expected demand for $H = 10$ should be 40. Over 100 replications, we see that the 95% confidence interval of total demand is 40.0113 ± 0.0345. We assume that the precision we obtain is acceptable.

To verify software, we set up some simple cases and compare the results with the known optimal solutions.

First case we observe is when setup cost is zero and demand rate is constant. We know that THM solution is optimal for this problem. 95% Confidence Interval of Average Total cost is 47.23 ± 1.19 for THM, while our 1st heuristic gives 48.11 ± 1.2 with $a = 5.5$ (Appendix 1).

Second case is performed with THM. We solve standard problem with their algorithm and observe the inventory movements. We see that, software is accurate in terms of calculating order-up-to and reorder levels, defining break points and calculating average total costs.

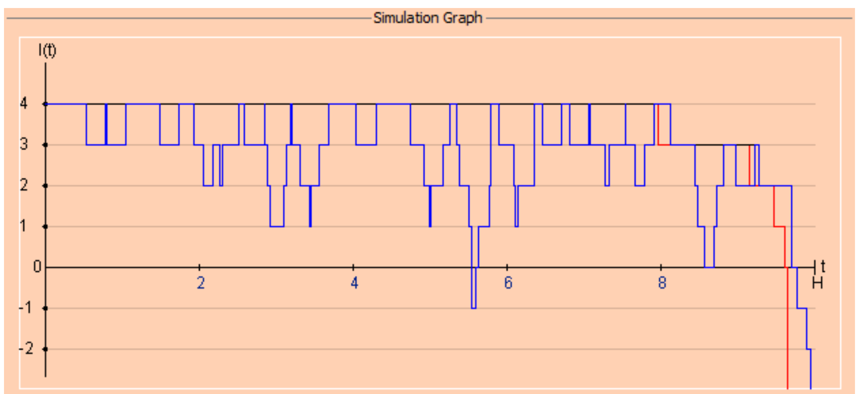

Figure 4.5 Inventory Movement with THM

4.3. Results

For evaluating the performance of the heuristics we set up some pre-defined parameters. The list of these cases as the following;

Table 4-1 Setups for Problem Parameters

Parameter				
Unit Cost		1		
Holding Cost		0.2		
Lead Time		0.25	0.5	
Setup Cost		0	10	
Backordering Cost		5	20	
Horizon Length		10	20	
Demand Rates	(H=10)	4	0.8.(10-t)	-3/50*t^2+6
	(H=20)	4	0.4.(20-t)	-3/200*t^2+6

So, for lead time we use 2 different settings, 2 for setup cost, 2 for backordering cost and 2 for horizon length. For each horizon length, we evaluate (1) homogeneous demand rate case, (2) linear decreasing demand rate case and (3) quadratic decreasing demand rate case for non-homogeneous Poisson process.

By using combinations of these setups, there are totally 48 different cases. By using simulation, we evaluate performances of our heuristics for these cases. In here, we present results of 1st and 2nd Heuristics in order, for homogeneous, linear decreasing and quadratic decreasing order for demand rate.

For each demand rate, we create a demand list by generating non-homogeneous Poisson process demands and apply same demand times to all scenarios.

4.3.1 Results of 1st Heuristic

For the 1st heuristic we take simulation run for homogeneous, linear decreasing and quadratic decreasing demand rates.

For the homogeneous case we use $\lambda = 4$, for linear decreasing $\lambda(t) = 0.8\,(10 - t)$ and for quadratic decreasing we set $\lambda(t) = -\frac{3}{50}.t^2 + 6$ where $\int_0^{10} \lambda(t)\,dt = 40.$ Hence expected total unit cost is equal for all these cases.

Results of constant, linear decreasing and quadratic decreasing demand rate experiments are provided in the Appendix 6, 7 and 8, respectively.

A summary of the results are provided in the following tables.

Table 4-2 Comparison of Simulation Results for 1st Heuristic Different Setups, $\boldsymbol{L = 0.25}$

(L=0.25)		b=5		b=20	
		K=0	K=10	K=0	K=10
H=10	Constant	47.057	109.991	49.332	111.102
	Lin. Dec.	49.009	173.701	52.848	171.102
	Quad. Dec.	48.107	160.088	51.300	158.134
H=20	Constant	93.305	194.615	97.060	195.248
	Lin. Dec.	95.003	303.666	102.252	303.974
	Quad. Dec.	94.770	275.992	100.115	279.418

Table 4-3 Comparison of Simulation Results for 1st Heuristic Different Setups, $\boldsymbol{L = 0.5}$

(L=0.5)		b=5		b=20	
		K=0	K=10	K=0	K=10
H=10	Constant	50.065	106.406	53.655	107.758
	Lin. Dec.	54.035	170.176	64.656	175.686
	Quad. Dec.	52.442	155.380	61.079	156.406
H=20	Constant	99.306	193.686	101.719	195.105
	Lin. Dec.	103.567	302.771	116.253	308.402
	Quad. Dec.	100.887	276.963	111.757	282.822

Note that as we select α rather arbitrarily, comparison of expected costs may not follow expectations.

4.3.2 Results of 2nd Heuristic

We take simulation runs of 2nd heuristic for only homogeneous demand case. We apply the same setups as we used for 1st heuristic. Note that we have two different lead times. Period length candidates are selected as the 2nd degree multipliers of these lead times. So, for L=0.25, candidates are 0.25, 0.5, 1, 2 and 4, while for L=0.5 candidates are 0.5, 1, 2, 4 and 8.

Simulation results are provided in Appendix 9. Comparison with 1st heuristic results is discussed in following subsection.

4.4. Performance Comparisons

We emphasize some of the important comparisons between performances of the policies.

THM – 1st Heuristic

Since THM is only applicable for homogeneous Poisson demand rate and zero setup cost, we evaluate this comparison on standard problem. For the standard problem, defined in (42) we get the following average total cost.

Table 4-4 Comparison of THM and 1st Heuristic with Different Adjustment Parameter on Standard Problem

Method	Adjustment Parameter	Mean (CI 0.95)	Gap (%)
THM	-	48.763 ± 2.168	-
1st Heuristic	-	50.582 ± 2.065	3.730
1st Heuristic	20	49.695 ± 2.034	1.911
1st Heuristic	3.622	49.332 ± 2.232	1.166
1st Heuristic	1	51.036 ± 2.753	4.661

In 100 replications, THM gives a total cost 48.763. We applied 1st Heuristic with no adjustment parameter, and then two extreme adjustment parameter (20 and 1) and finally the adjustment parameter obtained via Power Approximation, 3.622. As we expect, without adjustment, heuristic gives a higher cost compared to approximated value. Extreme solution where $a = 1$ actually gives worse result than not applying adjustment at all, this shows that good selection of the parameter is important for its performance. In another setting, we change the holding cost to 2, while in the original problem it is 0.2. Enlarging holding cost 10 times increased the cost as follows:

Table 4-5 Comparison of THM and 1st Heuristic with Different Adjustment Parameter for $h = 2$

Method	Adjustment Parameter	Mean (CI 0.95)	Gap (%)
THM		82.463 ± 2.397	-
1st Heuristic	-	87.201 ± 2.216	5.740
1st Heuristic	20	86.540 ± 2.276	4.943
1st Heuristic	3.622	86.380 ± 2.275	4.750
1st Heuristic	1	88.365 ± 2.903	7.158

The gap between average total costs (THM vs. Best 1st Heuristic Result) increased in this experiment to 4.75%, while we reach the worst solution when adjustment parameter is fixed to 1.

1st Heuristic, Different Setup Cost Selection

We also compare the effect of adding setup cost to the problem. We know that without setup cost and under homogeneous demand rate, the optimal solution suggests replenishments with unit-size. However, addition of setup cost changes the cost structure. Here, we evaluate how setup cost affects the total cost in the problem. We use 1st Heuristic with three different setup cost parameters, 0, 1 and 5, in order. For each setup cost, we take simulation runs (1) without adjustment operation and (2) adjustment parameter with Power Approximation. We get the following results:

Table 4-6 Effect of Adjustment on Different Setup Cost Settings

Setup Cost	Adjustment Parameter	Mean (CI 0.95)	Decrease (%)
0	-	50.582 ± 2.065	-
0	3.622	49.332 ± 2.232	2.47
1	-	67.129 ± 2.281	-
1	3.622	63.143 ± 2.359	5.93
5	-	93.435 ± 2.903	-
5	3.622	86.102 ± 2.916	7.84

This shows that benefit of applying adjustment is increasing when setup cost is higher. Note that, when we apply THM for the case $K = 1$, the total cost becomes 87.232 and for $K = 5$ it becomes 247.232, since the policy orders a unit for every demand.

1st Heuristic, Different Demand Rate Functions

We compare the performance of the 1st heuristic on different demand rate functions. We choose samples where the total expected demand remains same. Adjustment parameter does not depend on the demand rate function, hence it remains same. We compare cases where $K = 0$ and $K \neq 0$.

Table 4-7 Change in Total Cost for Different Demand Rates

Demand Rate	Setup Cost	Adjustment Parameter	Mean (CI 0.95)	Gap (%)
4	0	3.622	49.332 ± 2.232	-
0.8 (10-t)	0	3.622	52.848 ± 2.556	7.172
6-3 t^2/50	0	3.622	51.3 ± 2.423	3.989
4	10	3.622	111.102 ± 6.074	-
0.8 (10-t)	10	3.622	171.102 ± 11.78	54.004
6-3 t^2/50	10	3.622	158.134 ± 10.773	42.332

As seen from the results, the average total cost increased compared to homogeneous Poisson case. Although confidence intervals are wide, total cost for quadratic decrease case seems slightly better than linear decrease case.

1st Heuristic – 2nd Heuristic

We compare 1st and 2nd Heuristics' results for both when $K \neq 0$ and $K = 0$. Here, candidate set for the 2nd heuristic is defined as the second degree multiples of lead time.

First, we take setup cost as zero ($K = 0$) and also set adjustment parameter to 3.622. We get the following result.

Table 4-8 Comparison of THM, 1st and 2nd Heuristics for Standard Problem

Method	Candidate Set	Mean (CI 0.95)
THM	-	48.763 ± 2.168
1st Heuristic	-	49.332 ± 2.232
2nd Heuristic	0.25, 0.5, 1, 2, 4	53.569 ± 3.438

Then we consider the standard problem with $K = 1$ and $K = 10$. For both setup cost, adjustment parameter is calculated via Power Approximation and set to 3.622.

Table 4-9 Comparison of 1st and 2nd Heuristics for $K = 1$ on Standard Problem

Method	Candidate Set	Mean (CI 0.95)
1st Heuristic	-	63.143 ± 2.359
2nd Heuristic	0.25, 0.5, 1, 2, 4	62.032 ± 2.372

For $K = 10$:

Table 4-10 Comparison of 1st and 2nd Heuristics for $K = 10$ on Standard Problem

Method	Candidate Set	Mean (CI 0.95)
1st Heuristic	-	111.102 ± 6.074
2nd Heuristic	0.25, 0.5, 1, 2, 4	99.435 ± 3.681

Also by comparing the results presented in section 4.3.1 and 4.3.2, we see that 1st heuristic provides slightly better results than 2nd heuristic for zero setup cost case. When we apply non-zero setup cost to the problem, performances of the heuristics becomes closer and especially for higher setup costs 2nd heuristic gives better results in general.

4.5. Remarks and Conclusions

Computational studies give some hints about applications of the heuristics. We will summarize these important results in here.

- On homogeneous demand rate with zero setup cost, 1st heuristic performs best. We compare these results with THM as a benchmark and conclude that it gives near-optimal solutions for most of the cases, at most 2% away from optimal value according to our numerical results.

- On homogeneous demand rate with non-zero setup cost 2nd Heuristic provides better objective values compared to other combinations. Especially for high values of setup cost, 2nd heuristic outperforms 1st heuristic. There are two reasons for this result. (1) Although 1st heuristic benefits from estimation of future replenishments, solving deterministic subproblems involves some errors due to stochasticity. Most of the time, 1st heuristic ends up with more replenishments than estimated at time 0. (2) Inclusion of setup cost in 2nd heuristic pushes retailer to use longer period lengths while holding and backordering cost do the opposite. When setup cost is getting larger, review period lengths are getting longer, which ultimately reduce total setup cost.
- For non-homogeneous demand rate with zero setup cost cases, performance of 1st heuristic is not affected by cost parameters. For instance, for the standard problem with $b = 5$ and 20, average total costs are 47.057 and 49.332, respectively.
- Results of the 1st heuristic on non-zero setup cost cases are heavily affected by the size of setup cost. For higher setup costs, average total cost increased significantly. Same effect is also observable on 2nd heuristic, but not as much as in 1st heuristic results.
- Selection of adjustment parameter is vital for the practical purposes. Although the existence of an explicit way to calculate best a value is unknown, we could explain the its relation with horizon length. Hence, Power Approximation method, only depends on horizon length, is applied and results are compared to THM.
- Candidate set for the 2nd heuristic is always selected as the 2nd degree multipliers of the lead time and this selection provides better values for homogeneous

demand rate with zero setup cost cases. Increasing the size of the candidate set obviously increase the performance of the 2nd heuristic, but may be time-consuming for practical purposes.

- Both heuristics provided results in reasonable times as expected. Moreover, addition of extra information such as indefinite integral of the demand rate function is observed to be useful for numerical operations.

Chapter 5

Conclusion

Final phase is generally the longest phase in the lifecycle of a product. It starts when the product is out of production and continues until last contract expires. In this phase, companies have to supply spare parts due to legal responsibilities in the contracts. Therefore, management of inventory of spare parts becomes an issue for retailers; since these parts often need be keep in the retailer level. Due to uncertainty of demand and risk of obsolescence at the end of the horizon, retailers must manage spare part inventory careful to avoid excessive holding, backorder, setup and unit costs.

In this study we focus on a retailer's problem in the final phase. Due to nature of the final phase, we define the horizon is finite and known. We also assume that demand is distributed with Non-Homogeneous Poisson Distribution over the horizon with a non-increasing function of time rate. All cost are taken as fixed and known as the lead time.

Optimal solution of this problem could be obtained via Dynamic Programming, however up to authors' knowledge; there isn't any study on the optimal solution of the problem considered here. Moreover, structure of the optimal solution via Dynamic Programming could be difficult to capture.

In order to provide a fast and applicable solution to retailer's problem, we came up with two heuristics. Our first heuristic is a continuous review heuristic, which uses the solution of the deterministic subproblem for ordering quantity and time decisions. This policy has look-ahead capability over the residual horizon, which is based on estimating future orders. On the opposite, second heuristic uses a myopic look for solving the problem. It is a periodic review policy, where the lengths of the periods are variable and selected among a candidate set. It is more realistic and applicable to real life than first policy, because it needs less data for calculations and faster in terms of CPU time. Remarks on numerical computations and suggestions on application of the heuristics are summarized in subsection 4.5.

We provide three contributions to the literature. First, the heuristics provide near-optimal solution to homogeneous demand case, at most around 2% away from optimal value. Without needing long calculations for optimality, it is a solution for the retailer which is applicable during the final phase. Second, it is one of first studies which consider non-homogeneous Poisson demand distribution for the final phase. Although it is not providing an optimal solution, it is applicable to real life due to its flexibility to apply for decreasing demand cases. Indeed, assumption of decreasing demand rate is common in real life in final phase. We even show the performance of the heuristics for the quadratic decreasing case, which is hard to solve optimally. Our third contribution is that we use the idea of estimating the future replenishments to decide replenishment quantity in a final phase problem. Hence, this study is a new application of look-ahead capability on inventory problems.

In conclusion, this study is not only good at solving the retailer's problem for the final phase, but also useful for academic perspective. As a future study, one can apply the idea of estimating future replenishments to other phases of the lifecycle of products and find new key points to interpret the effect of estimation in the finite horizon problems. Moreover, better ways to select adjustment parameter and period lengths in the myopic heuristic can be found. Also adjustment parameter could be changed dynamically during the planning horizon.

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APPENDIX

Appendix 1 Performance of Adjustment Parameter compared to THM on Homogeneous Case - 1

$$\lambda = 4, H = 10, L = 0.25, h = 0.2, b = 20, K = 0, u = 1$$

Problem	Algorithm	Parameters	RESULT (100 Replications Each)			
		a Level	Average Total Cost	Variance	Confidence Interval (0.95)	Gap to THM (%)
		a				
Constant						
Teunter and Haneveld Original	THM		47.23	36.01	47.23 ± 1.19	
	Alg 1	4	48.35	44.13	48.35 ± 1.31	2.3706
		4.5	48.43	42.01	48.43 ± 1.28	2.5436
		5	48.27	39.94	48.26 ± 1.25	2.1964
		5.5	48.12	36.98	48.11 ± 1.2	1.8757
		6	48.16	37.02	48.16 ± 1.2	1.9736
		6.5	48.16	37.02	48.16 ± 1.2	1.9736
	Alg 1	4	50.13	41.44	50.12 ± 1.27	6.1299
		4.5	50.18	38.12	50.17 ± 1.22	6.2357
		5	50.02	35.86	50.01 ± 1.18	5.9005
		5.5	49.99	33.59	49.99 ± 1.15	5.8442
		6	50.11	33.39	50.1 ± 1.14	6.0834
		6.5	50.12	33.07	50.12 ± 1.14	6.1212

Appendix 2 Sample Output Report of 1 Iteration of Insys (1st Heuristic)

```
C:\Users\Raion\Desktop\Albatross Run\1371509482462.ins  Simulation Start
        Upper Bound= 42
        Cost for 1 is: 66.0 where next order is: 4.799999994581396
        Cost for 2 is: 65.33333333333334 where next order is: 3.1333333129586993
        Cost for 3 is: 66.0 where next order is: 2.2999999992275217
        Cost for 4 is: 71.0 where next order is: 1.800000001636399
        Cost for 5 is: 73.66666666666667 where next order is: 1.4666666666666668
        Cost for 6 is: 77.71428571428571 where next order is: 1.2285714285714282
        Optimal Residual Number for -0.2 is 2
        Order is given at -0.2 :13.0
        Upper Bound= 35
        Cost for 1 is: 52.688428364008665 where next order is: 5.663511703316419
        Cost for 2 is: 55.12561890933911 where next order is: 4.284682266832034
        Cost for 3 is: 57.84421418200432 where next order is: 3.5952675501860383
        Cost for 4 is: 60.47537134560346 where next order is: 3.181618720909901
        Cost for 5 is: 65.56280945466956 where next order is: 2.905852832198355
        Cost for 6 is: 68.9109795325739 where next order is: 2.708877198110855
        Optimal Residual Number for 1.527023400248051 is 1
        Order is given at 1.527023400248051 :17.0
        Upper Bound= 17
        Cost for 1 is: 23.999346609562796 where next order is: 7.863719219308917
        Cost for 2 is: 29.999564406375196 where next order is: 7.218292290786801
        Cost for 3 is: 32.499673304781396 where next order is: 6.895578827272509
        Cost for 4 is: 41.199738643825114 where next order is: 6.701950751497466
        Cost for 5 is: 43.9997822031876 where next order is: 6.572865366416478
        Cost for 6 is: 51.8569561741608 where next order is: 6.480661519380515
        Optimal Residual Number for 5.9274384384733185 is 1
        Order is given at 5.9274384384733185 :8.0
        Upper Bound= 9
        Cost for 1 is: 13.616280451570857 where next order is: 8.922303833901157
        Cost for 2 is: 19.41085363438057 where next order is: 8.629738441643502
        Cost for 3 is: 23.308140225785422 where next order is: 8.483455746954649
        Cost for 4 is: 30.246512180628347 where next order is: 8.395686130098749
        Cost for 5 is: 37.205426817190286 where next order is: 8.337173052367161
        Cost for 6 is: 44.17608012902023 where next order is: 8.295377996129812
        Optimal Residual Number for 8.044607662699214 is 1
        Order is given at 8.044607662699214 :4.0
        Upper Bound= 5
        Cost for 1 is: 9.117240867312974 where next order is: 9.417180089489065
        Cost for 2 is: 16.07816057820865 where next order is: 9.289573452427765
        Cost for 3 is: 19.058620433656486 where next order is: 9.225770133897116
        Cost for 4 is: 25.04689634692519 where next order is: 9.187488142778726
        Cost for 5 is: 31.039080289104316 where next order is: 9.161966815366467
        Optimal Residual Number for 9.034360178305167 is 1
        Order is given at 9.034360178305167 :2.0
        Upper Bound= 2
        Cost for 1 is: 7.000008146242315 where next order is: 9.79680894957918
        Cost for 2 is: 13.000005430828207 where next order is: 9.795745266093727
        Optimal Residual Number for 9.793617899122818 is 1
        Order is given at 9.793617899122818 :0.0
FINAL REPORT
Orders
Number  Time            Size
0       -0.2    13.0
1       1.527023400248051       17.0
2       5.9274384384733185      8.0
3       8.044607662699214       4.0
4       9.034360178305167       2.0
5       9.793617899122818       0.0
Total cost: 91.91376367051015
```

Appendix 3 Insys Report for the Teunter and Haneveld's Problem by using 1st Heuristic

```
FINAL REPORT

Orders
Number Time               Size
0      -0.2               4.0
1      0.2930020217119735 1.0
2      0.4731844563503971 1.0
3      0.9776772203873425 1.0
4      1.6201510576913924 1.0
5      1.8895247000547173 1.0
6      2.191626479640092  1.0
7      2.4643093440363453 1.0
8      2.8033822769579397 1.0
9      2.9319699259491223 1.0
10     3.1429880837001107 1.0
11     3.2124336299514025 1.0
12     3.2468367054926714 1.0
13     3.3897457398533746 1.0
14     4.406194326235035  1.0
15     4.46526556057783   1.0
16     4.77198027963532   1.0
17     4.920924006699904  1.0
18     5.805435064435843  1.0
19     5.82881289078628   1.0
20     5.873487814027821  1.0
21     5.921198517647042  1.0
22     6.009210887889984  1.0
23     6.090851983156553  1.0
24     6.243755021939867  1.0
25     6.245613237983178  1.0
26     6.293172709077642  1.0
27     6.364969877396664  1.0
28     6.714528991536235  1.0
29     6.72190868800894   1.0
30     7.546414491208504  1.0
31     8.519889399787354  1.0
32     8.601495458327408  1.0
33     8.99588774449514   1.0
34     9.137422579252412  1.0
35     9.260614625516673  1.0
36     9.275349901286159  1.0
37     9.285398467436716  1.0
----
Total cost: 47.45927377068042
```

Appendix 4 Linear Regression obtained from Power Approximation Method

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.929
R Square	0.863
Adjusted R Square	0.827
Standard Error	0.734
Observations	20.000

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4.000	51.093	12.773	23.720	0.000
Residual	15.000	8.077	0.538		
Total	19.000	59.171			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-3.922	1.077	-3.642	0.002	-6.217	-1.627	-6.217	-1.627
ln(H)	1.708	0.268	6.364	0.000	1.136	2.280	1.136	2.280
ln(b/h)	0.339	0.266	1.276	0.221	-0.228	0.907	-0.228	0.907
ln(L+1)	1.726	0.826	2.089	0.054	-0.035	3.487	-0.035	3.487
ln(K+1)	-1.431	0.350	-4.086	0.001	-2.178	-0.685	-2.178	-0.685

Appendix 5 Linear Regression obtained from Power Approximation Method

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.739
R Square	0.547
Adjusted R Square	0.521
Standard Error	1.221
Observations	20.000

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1.000	32.337	32.337	21.692	0.000
Residual	18.000	26.833	1.491		
Total	19.000	59.171			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-2.913	1.066	-2.733	0.014	-5.152	-0.674	-5.152	-0.674
ln(H)	1.862	0.400	4.658	0.000	1.022	2.701	1.022	2.701

Appendix 6 Simulation Results for 1st Heuristic, Homogeneous Rate Case

Parameters								RESULT	
Holding Cost	Unit Cost	Demand Rate	Horizon Length	Lead Time	Backorder Cost	Setup Cost	a Level	Average Total Cost	0.95 Confidence Interval
h	u	λ	H	L	b	K	a		
0.2	1	4	10	0.25	20	0	3.622	49.332	49.332 ± 2.232
0.2	1	4	10	0.25	20	10	3.622	111.102	111.102 ± 6.074
0.2	1	4	10	0.25	5	0	3.622	47.057	47.057 ± 2.136
0.2	1	4	10	0.25	5	10	3.622	109.991	109.991 ± 5.695
0.2	1	4	10	0.5	20	0	3.622	53.655	53.655 ± 3.067
0.2	1	4	10	0.5	20	10	3.622	107.758	107.758 ± 5.857
0.2	1	4	10	0.5	5	0	3.622	50.065	50.065 ± 2.706
0.2	1	4	10	0.5	5	10	3.622	106.406	106.406 ± 5.065
0.2	1	4	20	0.25	20	0	13.149	97.060	97.06 ± 2.483
0.2	1	4	20	0.25	20	10	13.149	195.248	195.248 ± 4.736
0.2	1	4	20	0.25	5	0	13.149	93.305	93.305 ± 2.435
0.2	1	4	20	0.25	5	10	13.149	194.615	194.615 ± 5.041
0.2	1	4	20	0.5	20	0	13.149	101.719	101.719 ± 2.765
0.2	1	4	20	0.5	20	10	13.149	195.105	195.105 ± 4.075
0.2	1	4	20	0.5	5	0	13.149	99.306	99.306 ± 3.254
0.2	1	4	20	0.5	5	10	13.149	193.686	193.686 ± 4.02

Appendix 7 Simulation Results for 1st Heuristic, Linear Decreasing Rate Case, h=0.2, u=1

Parameters						RESULT		
Demand Rate	Horizon Length	Lead Time	Backorder Cost	Setup Cost	a Level	Average Total Cost	0.95 Confidence Interval	% Diff compared to Constant Case
λ	H	L	b	K	a			
0.8*(10-t)	10	0.25	20	0	3.622	52.848	52.848 ± 2.556	7.127
0.8*(10-t)	10	0.25	20	10	3.622	171.102	171.102 ± 11.78	54.004
0.8*(10-t)	10	0.25	5	0	3.622	49.009	49.009 ± 2.116	4.148
0.8*(10-t)	10	0.25	5	10	3.622	173.701	173.701 ± 11.122	57.923
0.8*(10-t)	10	0.5	20	0	3.622	64.656	64.656 ± 4.893	20.503
0.8*(10-t)	10	0.5	20	10	3.622	175.686	175.686 ± 13.211	63.038
0.8*(10-t)	10	0.5	5	0	3.622	54.035	54.035 ± 2.699	7.930
0.8*(10-t)	10	0.5	5	10	3.622	170.176	170.176 ± 12.095	59.931
0.4*(20-t)	20	0.25	20	0	13.149	102.252	102.252 ± 2.802	5.349
0.4*(20-t)	20	0.25	20	10	13.149	303.974	303.974 ± 9.988	55.686
0.4*(20-t)	20	0.25	5	0	13.149	95.003	95.003 ± 2.313	1.820
0.4*(20-t)	20	0.25	5	10	13.149	303.666	303.666 ± 9.803	56.034
0.4*(20-t)	20	0.5	20	0	13.149	116.253	116.253 ± 4.419	14.288
0.4*(20-t)	20	0.5	20	10	13.149	308.402	308.402 ± 11.602	58.070
0.4*(20-t)	20	0.5	5	0	13.149	103.567	103.567 ± 2.649	4.291
0.4*(20-t)	20	0.5	5	10	13.149	302.771	302.771 ± 10.23	56.321

Appendix 8 Simulation Results for 1st Heuristic, Quadratic Decreasing Rate Case, h=0.2, u=1

Parameters						RESULT		
Demand Rate	Horizon Length	Lead Time	Backorder Cost	Setup Cost	a Level	Average Total Cost	0.95 Confidence Interval	% Diff compared to Constant Case
λ	H	L	b	K	a			
-3/50*t^2+6	10	0.25	20	0	3.622	51.300	51.3 ± 2.423	3.989
-3/50*t^2+6	10	0.25	20	10	3.622	158.134	158.134 ± 10.773	42.332
-3/50*t^2+6	10	0.25	5	0	3.622	48.107	48.107 ± 2.074	2.231
-3/50*t^2+6	10	0.25	5	10	3.622	160.088	160.088 ± 10.08	45.546
-3/50*t^2+6	10	0.5	20	0	3.622	61.079	61.079 ± 4.408	13.837
-3/50*t^2+6	10	0.5	20	10	3.622	156.406	156.406 ± 11.816	45.146
-3/50*t^2+6	10	0.5	5	0	3.622	52.442	52.442 ± 2.55	4.748
-3/50*t^2+6	10	0.5	5	10	3.622	155.380	155.38 ± 10.334	46.026
-3/200*t^2+6	20	0.25	20	0	13.149	100.115	100.115 ± 2.566	3.148
-3/200*t^2+6	20	0.25	20	10	13.149	279.418	279.418 ± 9.258	43.109
-3/200*t^2+6	20	0.25	5	0	13.149	94.770	94.77 ± 2.236	1.570
-3/200*t^2+6	20	0.25	5	10	13.149	275.992	275.992 ± 8.955	41.814
-3/200*t^2+6	20	0.5	20	0	13.149	111.757	111.757 ± 4.043	9.868
-3/200*t^2+6	20	0.5	20	10	13.149	282.822	282.822 ± 10.773	44.959
-3/200*t^2+6	20	0.5	5	0	13.149	100.887	100.887 ± 2.573	1.592
-3/200*t^2+6	20	0.5	5	10	13.149	276.963	276.963 ± 9.678	42.996

Appendix 9 Simulation Results for 2nd Heuristic, Homogeneous Rate Case, h=0.2, u=1

Parameters						RESULT		
Demand Rate	Horizon Length	Lead Time	Backorder Cost	Setup Cost	a Level	Average Total Cost	0.95 Confidence Interval	% Diff compared to 1st Heuristic
λ	H	L	b	K	a			
4	10	0.25	20	0	3.622	53.569	53.569 ± 3.438	8.589
4	10	0.25	20	10	3.622	99.435	99.435 ± 3.681	-10.501
4	10	0.25	5	0	3.622	50.092	50.092 ± 2.74	6.450
4	10	0.25	5	10	3.622	100.897	100.897 ± 3.817	-8.268
4	10	0.5	20	0	3.622	65.596	65.596 ± 6.631	22.255
4	10	0.5	20	10	3.622	100.790	100.79 ± 4.068	-6.466
4	10	0.5	5	0	3.622	55.903	55.903 ± 3.836	11.661
4	10	0.5	5	10	3.622	95.227	95.227 ± 3.928	-10.506
4	20	0.25	20	0	13.149	107.069	107.069 ± 4.538	10.312
4	20	0.25	20	10	13.149	193.593	193.593 ± 4.996	-0.848
4	20	0.25	5	0	13.149	99.810	99.81 ± 3.31	6.972
4	20	0.25	5	10	13.149	194.380	194.38 ± 4.409	-0.121
4	20	0.5	20	0	13.149	125.319	125.319 ± 8.525	23.201
4	20	0.5	20	10	13.149	189.937	189.937 ± 5.464	-2.649
4	20	0.5	5	0	13.149	109.364	109.364 ± 4.622	10.128
4	20	0.5	5	10	13.149	181.182	181.182 ± 4.335	-6.456