CIRCUIT THEORY BASED MODELING
AND ANALYSIS OF CMUT ARRAYS

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By
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ABSTRACT

CIRCUIT THEORY BASED MODELING AND ANALYSIS OF CMUT ARRAYS

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Many ultrasonic technology applications require capacitive micromachined ultrasonic transducers (CMUTs) to be used in the form of large arrays to attain better performance in terms of powerful, broadband and beam-formed radiated acoustic signals. To entirely benefit from its important characteristics, it is necessary to use analysis tools that are capable of handling multiple CMUT cells. In this regard, finite element analysis (FEA) tools become unfit for use because in arrays with large number of cells it is computationally very cumbersome and often practically impossible. Although, some simplification had been done by assuming long 1-D CMUT array elements as infinitely long, the results of these FEA simulations are misleading. In these models only a single periodic portion is modeled and rigid boundary conditions are applied at the symmetry planes. All the cells are assumed to be electrically driven in phase with the rest of the cells and the solution obtained for this portion is extended over the entire element. However, these simple models are not exact, because they exclude the important effects of mutual acoustic interactions between the cells.

In this work, we developed an accurate nonlinear equivalent circuit model for circular uncollapsed CMUT cells. We investigated the effects of mutual acoustic interactions in uncollapsed CMUT arrays and showed that the performance of the array is highly influenced with this phenomenon. These mutual acoustic interactions rise through the immersion medium caused by the pressure field generated by each cell acting upon the others. To study its effects, we connected each cell in the array to a radiation impedance matrix that contains the mutual radiation impedance between every pair of cells, in addition to their self radiation impedances. Hence, analysis of the performance of a large array became a circuit theory problem and can be scrutinized with circuit simulators.
Surface micromachining technology enables batch fabrication of large CMUT arrays, which resolves cost issues and many physical limitations. Designers have to consider a great number of different array configurations. For nearly two decades, the lack of appropriate design and analysis tools prevented the investigation of array performance. By using the proposed model, one can very rapidly obtain the linear frequency and nonlinear transient responses of arrays with a large number of uncollapsed CMUT cells.

Although, we use rapid circuit theory techniques, efficient analysis of very large arrays is still challenging, since a typical CMUT array may contain many tens of elements with hundreds of cells in each, which makes it computationally cumbersome. To partition the problem, we electrically drive a small number of elements in the array and keep the rest undriven but biased and with their electrical ports terminated with a load. The radiation impedance matrix can be partitioned and rearranged to represent these loads in a reduced form. In this way, only the driven elements can be simulated by coupling their cells through this reduced radiation impedance matrix. Under small signal regime, the separately calculated responses of element clusters can be added by using the superposition principle to find the total response. This method considerably reduces the number of cells and the size of the actual radiation impedance matrix, at the expense of calculating the inverse of a large complex symmetric matrix.

Keywords: CMUT, lumped element nonlinear equivalent circuit model, large signal model, small signal model, uncollapsed mode of operation, array, mutual acoustic coupling, self and mutual radiation impedances, reduced radiation impedance matrix.
ÖZET

CMUT DİZİNLERİNİN DEVRE TEORİSİ TABANLI MODELLENMESİ VE ANALİZİ

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Bu çalışmamızda, çökmemiş dairesel CMUT hücreleri için doğrusal olmayan bir eşdeğer devre modeli geliştirdik. CMUT dizinlerindeki karşılıklı akustik etkileşimleri incelendik ve bu olgunun dizin performansı üzerinde oldukça etkili olduğunu gösterdik. Bu etkileşimler, dizindeki her hücrenin bulunduğu ortamda yarattığı basınç alanlarının diğer hücrelerin üzerine etkimesi ile gerçekleşmektedir. Bu etkiyi araştırmak için dizindeki her bir hücrenin eşdeğer devresini, bütün hücre
çiftleri arasındaki karşılıklı radyasyon empedanslarına ek olarak, öz radyasyon empedanslarını da içeren bir matrise bağladık. Böylece geniş dizinlerin performans analizi devre simülatörleri ile kolaylıkla çözelbilen bir devre teorisi problemi haline dönüştü müs oldu.

Yüzey mikroisleme teknolojisi sayesinde maliyet ve birçok fiziksel sınırlama sorunu çözülecek geniş CMUT dizinlerinin ürünlere halinde üretilmesi mümkün kılınmıştır. Tasarımçılardan olası birçok farklı dizin konfigürasyonunu dikkate almak durumundadır. Yaklaşık son yirmi yıldır uygun tasarım ve analiz araclarının eksikliği yüzden dizin performansının tam olarak incelenebilmesi mümkün olamamıştır. Önerilen bu model ile çok sayıda çökmemiş CMUT hücresinin oluşan dizinlerin doğrusal frekans ve doğrusal olmayan geçici rejim analizleri hızlı bir şekilde yapılabilmesidir.


Anahtar sözcükler: CMUT, toplu öğeli doğrusal olmayan eşdeğer devre modeli, büyük işaret modeli, küçük işaret modeli, çökmemiş çalışma modu, dizin, karşılıklı akustik bağlam, öz ve karşılıklı radyasyon empedansları, indirgenmiş radyasyon empedans matrisi.
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Chapter 1

Introduction

The need for an accurate lumped element equivalent circuit model for capacitive micromachined ultrasonic transducers (CMUT) is extensively discussed [2, 3, 4, 5, 6, 7, 8]. The finite element method (FEM) is a powerful technique for the analysis of CMUTs, when the number of CMUT cells in an array is low [9, 10, 11, 12]. The CMUT operation can be accurately simulated and information on the nonlinear effects, medium loading, crosstalk, and the effect of the higher order harmonics can be obtained.

An iterative approach must be adopted, however, to design CMUTs using FEM. This approach is very computation intensive and can take very long. Getting results with FEM analysis for arrays which contain large number of CMUT cells is practically impossible. On the other hand, realization of arrays comprising large number of cells at low cost is one of the fundamental advantages of CMUT technology [13, 14, 15, 16].

Design and analysis of CMUTs using a lumped element equivalent circuit model provides the rapid insight gained with analytical modeling methods. It requires the knowledge of radiation impedance. Therefore, the radiation interface must be accurately included in the model. The equivalent circuits of single CMUT cells can then be used to model arrays by appropriately terminating each cell with respective impedance. There has been a significant improvement on this topic,
both for single cells and arrays, recently [17, 18, 19].

In this thesis, we first present an accurate nonlinear equivalent circuit model for a circular, uncollapsed CMUT cell. The model is derived from the physics of the device and is based on two principal factors; (i) the mechanical power on the membrane is derived, from which the membrane velocity and the associated transduction force delivered to the mechanical section is obtained, and (ii) the force equilibrium in the mechanical section is used to model the collapse behavior. The analytical approach is similar to the approaches in [2] and [3].

In this thesis, we present a force definition that is consistent with the choice of the through variable that represents the membrane velocity, such that they are directly linked through an energy relation. We discuss the dependence of equivalent circuit on the choice of through variable employed in the linear mechanical section and provide the results for three possible velocity definitions. We quantify the collapse voltage analytically as a function of the external static force, such as atmospheric pressure, and the cell parameters gap height, clamp capacitance and membrane compliance, as a direct consequence of the model. The model is for uncollapsed mode of operation: it very accurately predicts the behavior of CMUT until the membrane touches the substrate. The force equilibrium on the membrane before and beyond the collapsing displacement is derived, again in terms of model parameters. The relevant equations for analytical design and implementation in circuit simulators are given. The radiation interface is completely modeled, and dependence of the radiation medium variables and circuit variables of the mechanical section is discussed and clarified. Because most receivers are operated under small signal conditions, a linear small signal equivalent circuit is derived from the large signal model and presented.

The primary purpose of transducer arrays used in many ultrasound applications is to radiate powerful, beam-formed acoustic signals. The acoustic crosstalk that occurs between the closely packed cells of CMUT arrays is considered important, because it impairs both beam-forming and powerful radiation. The effect of crosstalk had been assessed extensively by experiments and measurements. It is hypothesized that the crosstalk is caused by either one of the two phenomena.
The first one is the waves propagating in the silicon substrate [20, 21, 13]. The latter is a result of the acoustic interactions that occur when the sound pressure fields of the transducers exert force on each other through the immersion medium. This phenomena has been recognized in sonar transducer arrays for many decades, and its significant effects on array performance have been studied by means of the mutual radiation impedance between the transducers [22, 23, 24]. In this work, we are concerned only with the second crosstalk mechanism, which has an evident effect on CMUT arrays [25, 26, 18, 27], and ignore the first one.

CMUT membranes have low mechanical impedance, which makes them inherently suitable for immersion applications. With this major advantage, CMUTs are capable of transmitting and receiving wideband acoustic signals. On the other hand, a low mechanical impedance means low quality factor (Q) of mechanical resonance. It brings with it severe effects due to mutual acoustic interactions, which are manifested in the operational bandwidth of the transducer [28, 26, 18]. As is the case for other electromechanical transducer types, this drawback can cause degradation in acoustic power radiation [29, 30, 17], distortion in sound beam patterns [24] and sometimes even failure of the electronic amplifiers that drive the transducers [31].

Surface micromachining technology has made possible batch fabrication of large CMUT arrays, offering the opportunity to integrate driving electronics. Depending on the precision of the manufacturing process, array designers may be presented with a great number of different configurations to attempt. However, the lack of appropriate design and analysis tools prevents investigation of the mutual acoustic coupling effects occurring in large arrays.

FEM tools are widely used in the analysis of acoustic transducers. However, it is not feasible to analyze large arrays without making some simplifying assumptions in the FEM model. There exist reduced FEM models that are used to simulate long 1-D CMUT arrays [28, 32, 13, 9]. In these models, the array structure is assumed to be infinitely long, so that only a single periodic portion is modeled to be electrically driven in phase with the rest of the cells. However, these simple reduced FEM models are not exact, and the important effects of
acoustic interactions cannot be accurately investigated [28]. Therefore, FEM is not suitable for use in designing CMUT arrays, although it can be employed to verify a particular design with a small number of cells.

It is common to use Mason’s linear electrical equivalent circuit when analyzing single as well as multiple CMUT cells. Once the self and mutual radiation impedances of the cells are known and taken into account, an equivalent circuit for an immersed CMUT array can be built [26, 18, 33]. However, the predictions of this equivalent model are not satisfactory [34, 26].

In this thesis, we use the nonlinear equivalent circuit model that we developed for single CMUT cells as a building block for CMUT arrays. The equivalent circuit including all cells in the array is coupled at their acoustic terminals through an impedance matrix. The matrix contains the self radiation impedance of each cell and the mutual radiation impedance between every pair of cells. We present an accurate and easy-to-compute approximation for the mutual radiation impedance derived by Porter [35]. The approximate expression can be applied to large arrays. Employing the proposed model, we discuss some aspects of mutual acoustic interactions in CMUT cell clusters and elements. Where possible, we perform FEM simulations and show that the results obtained by the equivalent circuit model and FEM are very consistent.
Chapter 2

Lumped Element Nonlinear
Circuit Model for a Circular
CMUT Cell

This chapter presents an accurate nonlinear equivalent circuit model for a circular uncollapsed CMUT cell. The force model is derived so that the energy and power is preserved in the equivalent circuit model. The model is able to predict the entire behavior of CMUT until the membrane touches the substrate. Many intrinsic properties of CMUT cell such as the collapse condition, collapse voltage, the voltage-displacement interrelation and the force equilibrium before and after collapse voltage in presence of external static force are obtained as a direct consequence of the model. The small signal equivalent circuit for any bias condition is obtained from the large signal model. The model can be implemented in circuit simulation tools and model predictions are in excellent agreement with FEM simulations.
Figure 2.1: Two-dimensional view and the dimensional parameters of the circular capacitive micromachined ultrasonic transducer (CMUT) geometry.

2.1 Defining the Through and Across Variables

The basic geometry of a circular CMUT with a partial electrode is given in Fig. 2.1. The displacement profile for thin clamped plates or membranes obtained using plate theory [36, 37], when depressed by uniform pressure, is

\[ x(r, t) = x_p(t) \left( 1 - \frac{r^2}{a^2} \right)^2 \text{ for } r \leq a, \quad (2.1) \]

where \( a \) is the radius of the aperture, \( r \) is the radial position, and \( x_p \) is the displacement at the center of the membrane; positive displacement is toward the bottom electrode\(^1\). It is shown that CMUTs with full electrodes, with thin plate membranes also have the same profile [2]. The capacitance, \( \delta C(r, t) \), of a concentric narrow ring on the membrane of radius \( r \) and width \( dr \) can be expressed as

\[ \delta C(r, t) = \frac{\epsilon_0 2\pi r \, dr}{t_{ge} - x(r, t)} = \frac{\epsilon_0 2\pi r \, dr}{t_{ge} - x_p(t) \left( 1 - \frac{r^2}{a^2} \right)^2}, \quad (2.2) \]

where \( \epsilon_0 \) is the permittivity of the gap and \( t_{ge} = t_g + t_i/\epsilon_r \) is the effective gap height. Here, \( t_i \) and \( t_g \) are the thicknesses of the insulating layer and the vacuum gap height, respectively, and \( \epsilon_r \) is the relative permittivity of the insulating material. The capacitance, \( C(t) \), of the deflected membrane with a partial electrode of an inner radius \( (a_i) \) and an outer radius \( (a_o) \) can be found by an integration:

\[ C(t) = \int_{a_i}^{a_o} \delta C(r, t) \, dr = C_0 \, g \left( \frac{x_p(t)}{t_{ge}} \right), \quad (2.3) \]

\(^1\)Throughout the thesis, the first subscripts \( R, A \) and \( P \) of mechanical variables refer to rms, average and peak quantities, respectively.
where the function \( g(\cdot) \) is defined by
\[
g(u) = \frac{\tanh^{-1}(K_i \sqrt{u}) - \tanh^{-1}(K_o \sqrt{u})}{\sqrt{u}},
\tag{2.4}
\]
where \( K_i = (1 - a_i^2/a^2) \), \( K_o = (1 - a_o^2/a^2) \) and \( C_0 = \epsilon_0 \pi a^2 / t_{ge} \).

If a voltage \( V(t) \) is applied across the terminals, the instantaneous energy stored on the capacitance is given by \( E(t) = \frac{1}{2} C(t) V^2(t) \).

Suppose we choose the rms membrane velocity defined by
\[
v_R(t) = \frac{dx_R(t)}{dt} = \frac{d}{dt} \sqrt{\frac{1}{\pi a^2} \int_0^a 2\pi r x^2(r,t) dr}
\tag{2.5}
\]
as the through variable of the equivalent circuit, which is defined in [38] as the *spatial rms velocity*. For the membrane profile in (2.1), we have \( x_R(t) = x_P(t) / \sqrt{5} \). To preserve the energy, the corresponding across variable for force, \( f_R(t) \), should be written as
\[
f_R(t) = \sqrt{5} C_0 V^2(t) \frac{g'(x_P(t))}{t_{ge}},
\tag{2.6}
\]

where
\[
g'(u) = \frac{1}{2u} \left( \frac{K_i}{1 - K_i^2 u} - \frac{K_o}{1 - K_o^2 u} - g(u) \right).
\tag{2.8}
\]

We also need the second derivative of (2.4) in this work, which is
\[
g''(u) = \frac{1}{2u} \left( \frac{K_i^3}{(1 - K_i^2 u)^2} - \frac{K_o^3}{(1 - K_o^2 u)^2} - 3g'(u) \right).
\tag{2.9}
\]

To ensure that (2.7) actually satisfies energy conservation, let us consider the power on the concentric narrow ring on the membrane:
\[
\delta P = \frac{\partial (\delta E)}{\partial t} = V(t) \delta C \frac{\partial}{\partial t} \delta V(t) + \frac{1}{2} V^2(t) \frac{\partial (\delta C)}{\partial t}.
\tag{2.10}
\]
The electrical and mechanical components of power on the ring are well separated in (2.10). The second term is the mechanical power on the ring:
\[
\delta P_M = \frac{1}{2} V^2(t) \frac{\partial}{\partial x} \left( \frac{\epsilon_0 2\pi r dr}{t_{ge} - x(r,t)} \right) \frac{\partial x(r,t)}{\partial t}.
\tag{2.11}
\]
The total instantaneous mechanical power, $P_M(t)$, is obtained by integrating $\delta P_M$ across the membrane surface, to obtain

$$P_M(t) = \int_0^a \delta P_M = \left[ \frac{\sqrt{5}C_0V^2(t)}{2t_{ge}} g' \left( \frac{x_p(t)}{t_{ge}} \right) \right] \frac{dx_R(t)}{dt}. \quad (2.12)$$

The through variable for the instantaneous mechanical power given in (2.12) is clearly the time rate of change of $x_R(t)$, defined in (2.5). We recognize that $P_M(t)$ is the product of $v_R(t)$ and a term which corresponds to the force on the membrane inducing this velocity, which is $f_R(t)$ given in (2.7).

For $a_i/a \leq 0.25$ and for $a_o/a \geq 0.8$ the displacement profile agrees well with the assumed profile and the material presented in this thesis is applicable to such CMUTs. The profile deviates from (2.1) for other choices of $a_i$ and $a_o$ and the accuracy of the model deteriorates; however, the model predictions still provide good guidance for design.

For CMUTs with full electrodes (2.4), (2.8), (2.9) and (2.7) simplify to

$$g(u) = \frac{\tanh^{-1}(\sqrt{u})}{\sqrt{u}}$$

$$g'(u) = \frac{1}{2u} \left( \frac{1}{1-u} - g(u) \right) \quad (2.13)$$

$$g''(u) = \frac{1}{2u} \left( \frac{1}{(1-u)^2} - 3g'(u) \right)$$

$$f_R(t) = \frac{\sqrt{5}C_0V^2(t)}{4x_p(t)} \left[ \frac{t_{ge}}{t_{ge} - x_p(t)} - \frac{\tanh^{-1} \left( \frac{x_p(0)}{t_{ge}} \right)}{\sqrt{\frac{x_p(t)}{t_{ge}}}} \right]. \quad (2.14)$$

The series expansion of $g(u)$ around $u = 0$ is

$$g(u) = \left( K_i + \frac{K^3_i}{3} u + \frac{K^5_i}{5} u^2 + \frac{K^7_i}{7} u^3 \right)$$

$$- \left( K_o + \frac{K^3_o}{3} u + \frac{K^5_o}{5} u^2 + \frac{K^7_o}{7} u^3 \right) \quad (2.15)$$

from which its derivatives around $u = 0$ can also be calculated. These are useful in circuit simulator applications when $u \ll 1$. 

8
Figure 2.2: A comparison of $F_{\text{tot}}$ and $f_R$ normalized with $C_0V^2(t)/4t_{ge}$ for a full electrode membrane.

The force in (2.14) is not the same as the total force on the membrane, $F_{\text{tot}}$, given in [2] as the across variable found using Mason’s approach:

$$F_{\text{tot}}(t) = \frac{C_0V^2(t)}{4t_{ge}} \left[ \frac{t_{ge}}{t_{ge} - x_P(t)} + \frac{\tanh^{-1}\left(\frac{x_P(t)}{t_{ge}}\right)}{\sqrt{\frac{x_P(t)}{t_{ge}}}} \right] \tag{2.16}$$

Fig. 2.2 is a comparison of these two force values as a function of $x_P/t_{ge}$. In Eq.(10) of [2], if the derivative had been taken with respect to $x_P$, similar to the approach in [3], rather than $x$, there would have been an additional $(1 - r^2/a^2)^2$ term inside the integral and the two results would have been identical.

### 2.2 Large Signal Equivalent Circuit

The circuit variables on the electrical side can be found by considering the time rate of change of the instantaneous charge, $Q(t) = C(t)V(t)$, on the CMUT capacitance:

$$\frac{\partial Q(t)}{\partial t} = C(t) \frac{\partial V(t)}{\partial t} + \frac{\partial C(t)}{\partial t} V(t) = i_{C_{ap}}(t) + i_V(t), \tag{2.17}$$
similar to the notation in [2]. Hence the current components are

\[ i_{Cap}(t) = C(t) \frac{dV(t)}{dt} = C_0 \frac{dV(t)}{dt} + i_C(t), \tag{2.18} \]

where

\[ i_C(t) = (C(t) - C_0) \frac{dV(t)}{dt}. \tag{2.19} \]

The velocity current is given by

\[ i_V(t) = \frac{\partial C(t)}{\partial t} V(t) = \frac{\partial C(t)}{\partial x_R} \frac{\partial x_R}{\partial t} V(t). \tag{2.20} \]

Using (2.6), (2.7) and \( C(t) = 2E(t)/V^2(t) \) we find

\[ i_V(t) = \frac{2f_R(t)}{V(t)} v_R(t) = \sqrt{5} C_0 V(t) \frac{x_P(t)}{t_{ge}} g' \left( \frac{x_P(t)}{t_{ge}} \right) v_R(t). \tag{2.21} \]

Eqs. (2.19) and (2.21) are the same as the corresponding equations in [2]. We can form the large signal equivalent circuit as depicted in Fig. 2.3. \( C_{Rm} \) and \( L_{Rm} \) are the compliance of the membrane and the inductance corresponding to the mass of the membrane suitable for the \( \{f_R, v_R\} \) rms model. For the same model \( Z_{RR} \) is the radiation impedance of the CMUT cell given in [2], which can also be found in Appendix A.1.

Because the direction of \( x_P \) is chosen towards the bottom electrode and the particle velocity of the acoustic signal propagating into the medium is in the opposite direction, we denote the polarity of the transmitted force, \( f_{RO} \), across the radiation impedance as shown in the figure. Similarly, any dynamic and static external force, such as an incident acoustic signal or atmospheric pressure, must appear in the form of \( f_{RI} \) and \( F_{Rb} \), respectively, in the model.

Figure 2.3: Large signal equivalent circuit referred to as the \( \{f_R, v_R\} \) model, since, the through variable in the mechanical section is \( v_R \).
For the velocity profile given by (2.1), the average velocity, \( v_A(t) \), across the membrane is equal to \( v_A(t) = v_P(t)/3 \). If \( v_A(t) \) is the through variable, the across variable is \( f_A(t) = 3f_R(t)/\sqrt{5} \), which preserves energy in the \( \{f_A,v_A\} \) model. Similarly, if \( v_P(t) = dx_P(t)/dt \) is used as through variable, \( f_P(t) = f_R(t)/\sqrt{5} \) is the force variable. In all cases, the mechanical circuit components must be scaled properly in order to be consistent and equivalent. The circuit components for all these models are listed in Table 2.1.

### 2.2.1 Collapse

In order to quantify the collapse phenomenon, we consider the circuit of Fig. 2.4 for \( \{f_P,v_P\} \) peak model to examine the static behavior under collapse conditions when an external static force \( F_{Pb} \) is present. We apply a voltage of \( V_{DC} \) to get the force \( F_P \) and the static displacement \( X_P \). The static force equilibrium in the mechanical section can be written as:

\[
F_P + F_{Pb} = X_P/C_{Pm},
\]

which yields

\[
\frac{V_{DC}}{V_r} = \sqrt{\frac{3(X_P/t_{ge} - F_{Pb}/F_{Pg})}{2g'(X_P/t_{ge})}} \text{ for } \frac{X_P}{t_{ge}} \geq \frac{F_{Pb}}{F_{Pg}},
\]

where we define \( V_r \) as

\[
V_r = \sqrt{\frac{4t_{ge}^2}{3C_{Pm}C_0}} = 8t_{m} t_{ge}^{3/2} t_{ge}^{1/2} \sqrt{\frac{Y_0}{27\epsilon_0(1-\sigma^2)}}
\]

and \( F_{Pg} = t_{ge}/C_{Pm} \) is the force required to deflect the membrane until center displacement reaches gap height, \( x_P = t_{ge} \). \( V_{DC}/V_r \) for a CMUT with full electrodes is plotted in Fig.2.5 with respect to \( X_P/t_{ge} \) for \( F_{Pb}/F_{Pg} \) = 0, 0.1, 0.5, 0.7 and 0.9.

It can be observed from the figure that, the bias voltage can be increased until it reaches a maximum for a particular external static force and the equilibrium is stable in this region. If the voltage is increased beyond the maximum, the transduction force exceeds the restoring force and collapse occurs. Bias voltage must
be decreased in order to maintain equilibrium in this region. This equilibrium is unstable. In [39], a similar equilibrium curve is also obtained for an electrostatic parallel-plates actuator.

The figure reveals the relation of collapse phenomena, the bias voltage, the static force, and $V_r$. For example, there is no static force in vacuum and the bias voltage maximum is 1.000476$V_r$, hence the collapse voltage of a CMUT in vacuum can be taken as $V_r$. In the presence of a static force, such as atmospheric pressure, membrane is pre-depressed by this force and collapse occurs at a bias voltage less than $V_r$. 

Figure 2.4: Generic large signal equivalent circuit model with parameters given in Table 2.1.

Figure 2.5: The voltage at the stable (solid) and unstable (dashed) static equilibrium as a function of $F_{Pb}/F_{Pg}$ for different $X_P$ values for membrane with full electrodes with the properties given in Section 2.4. The straight line shows the variation of the voltage required to reach collapse point for all $F_{Pb}/F_{Pg}$. In the static FEM analysis results (dotted) the stress stiffening effects are ignored.
It is clear from Fig. 2.5 and (2.23) that the displacement threshold for collapse for any \( \frac{F_{Pb}}{F_{Pg}} \) is reached when \( \frac{V_{DC}}{V_r} \) is maximum. Hence, the displacement at collapse point, \( X_{Pc} \), is obtained from

\[
\frac{d}{dX_P} \left( \frac{V_{DC}}{V_r} \right) \bigg|_{X_P=X_{Pc}} = 0, \tag{2.25}
\]

while equilibrium condition in (2.23) is maintained. \( X_{Pc} \) can be evaluated from (2.23) readily. For membranes with full electrodes a very accurate approximation is,

\[
\frac{X_{Pc}}{t_{ge}} \approx 0.4648 + 0.5433 \frac{F_{Pb}}{F_{Pg}} - 0.01256 \left( \frac{F_{Pb}}{F_{Pg}} - 0.35 \right)^2 - 0.002775 \left( \frac{F_{Pb}}{F_{Pg}} \right)^9. \tag{2.26}
\]

The voltage, \( V_c \), required to reach \( X_{Pc} \) can be obtained by using (2.26) in (2.23). The variation of \( V_c \) with respect to \( \frac{F_{Pb}}{F_{Pg}} \) is essentially a straight line and can be approximated as

\[
\frac{V_c}{V_r} \approx 0.9961 - 1.0468 \frac{F_{Pb}}{F_{Pg}} + 0.06972 \left( \frac{F_{Pb}}{F_{Pg}} - 0.25 \right)^2 + 0.01148 \left( \frac{F_{Pb}}{F_{Pg}} \right)^6. \tag{2.27}
\]

Eq. (2.27) versus (2.26) is also plotted in Fig. 2.5 as the collapse threshold. Similarly, \( F_{Pb}/F_{Pg} \) ratio can also be approximated very accurately in terms of \( V_c/V_r \) as

\[
\frac{F_{Pb}}{F_{Pg}} \approx 0.9891 - 1.037 \frac{V_c}{V_r} + 0.2083 \left( \frac{V_c}{V_r} - 0.229 \right)^2 - 0.0755 \left( \frac{V_c}{V_r} \right)^3. \tag{2.28}
\]

Solution of (2.23) for \( X_P/t_{ge} \) in terms of \( \frac{V_{DC}}{V_r} \) and \( \frac{F_{Pb}}{F_{Pg}} \) is also useful for modeling in circuit simulators. There is a very good approximation for \( g'(u) \),

\[
g'(u) \approx \frac{1}{2(1-u)} - \frac{5}{6} \frac{1}{5-3u}, \quad \text{for} \quad 0 \leq u \leq 1, \tag{2.29}
\]

which can be used to find \( X_P/t_{ge} \) for all bias conditions except in the vicinity of collapse threshold. The exact expression for \( g'(u) \) given in (2.13) must be used
near collapse threshold. Thus, $X_P/t_{ge}$ is found as

$$X_P = \begin{cases} 
2\sqrt{-\frac{p}{3}} \cos \left[ \frac{1}{3} \cos^{-1} \left( \frac{3q}{2p} \sqrt{-\frac{3}{p}} \right) - \frac{2\pi}{3} \right] + \frac{8}{9} + \frac{F_{Pb}}{3F_{Pg}} , & \text{if } 0 \leq X_P \leq X_{Pc} - 0.1(1 - F_{Pb}/F_{Pg}) . \\
\frac{X_{Pc}}{t_{ge}} \mp \sqrt{-\frac{g''(u)}{g'''(u)}} \left( \frac{\nu^2}{V_c^2} - 1 \right) \bigg|_{u=X_{Pc}/t_{ge}}, & \text{if } |X_P - X_{Pc}| < 0.1(1 - F_{Pb}/F_{Pg}) . \\
2\sqrt{-\frac{p}{3}} \cos \left[ \frac{1}{3} \cos^{-1} \left( \frac{3q}{2p} \sqrt{-\frac{3}{p}} \right) - \frac{2\pi}{3} \right] + \frac{8}{9} + \frac{F_{Pb}}{3F_{Pg}} , & \text{if } X_{Pc} + 0.1(1 - F_{Pb}/F_{Pg}) < X_P \leq t_{ge} . 
\end{cases} \tag{2.30}
$$

where

$$p = \left( 9d - c^2 \right)/27 ,$$

$$q = \left( 2c^3 - 27cd + 243e \right)/729 ,$$

$$c = - \left( 8 + 3F_{Pb}/F_{Pg} \right) ,$$

$$d = \left( 5 + 8F_{Pb}/F_{Pg} + 2\gamma \right) ,$$

$$e = -5 \left( F_{Pb}/F_{Pg} + \gamma \right) ,$$

$$\gamma = (2/9) \left( V_{DC}/V_r \right)^2 . \tag{2.31}$$

### 2.2.2 Received and Transmitted Pressure

$f_I$ and $f_O$ are received and transmitted forces of the model, respectively. It is more convenient if these are expressed in terms of the pressure at the surface of the membrane. When an equivalent model is produced, transducers of any kind are converted into a rigid piston transducer with uniformly distributed velocity and displacement, $v$ and $x$, respectively, across its radiating surface. All power and energy conversion at the radiating interface is expressed by these lumped variables.

CMUTs cannot produce a static output pressure in infinite fluid volume. There is no radiation impedance for static signals. When a static pressure $P_0$
Table 2.1: Relations between the mechanical variables of different models for the equivalent circuit given in Fig. 2.4, and turns ratio and spring softening compliance in the small signal model.

<table>
<thead>
<tr>
<th>Model</th>
<th>RMS</th>
<th>Average</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>{f_R, v_R}</td>
<td>{f_A, v_A}</td>
<td>{f_P, v_P}</td>
</tr>
<tr>
<td>f</td>
<td>f_R</td>
<td>\frac{3}{\sqrt{5}}f_R</td>
<td>\frac{1}{\sqrt{5}}f_R</td>
</tr>
<tr>
<td>v</td>
<td>v_R</td>
<td>\frac{\sqrt{5}}{3}v_R</td>
<td>\sqrt{5}v_R</td>
</tr>
<tr>
<td>C_M</td>
<td>C_{Rm} = \frac{9}{5\pi}a^2 &amp; C_{Am} = \frac{5}{9}C_{Rm} &amp; C_{Pm} = 5C_{Rm}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L_M</td>
<td>L_{Rm} = \rho\pi a^2 t_m &amp; L_{Am} = \frac{9}{5}L_{Rm} &amp; L_{Pm} = \frac{1}{5}L_{Rm}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z_R</td>
<td>Z_{RR} &amp; Z_{AR} = \frac{9}{5}Z_{RR} &amp; Z_{PR} = \frac{1}{5}Z_{RR}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f_I</td>
<td>\pi a^2 p_m &amp; \frac{3}{\sqrt{5}}\pi a^2 p_m &amp; \frac{1}{\sqrt{5}}\pi a^2 p_m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f_O</td>
<td>\pi a^2 p_o &amp; \frac{3}{\sqrt{5}}\pi a^2 p_o &amp; \frac{1}{\sqrt{5}}\pi a^2 p_o</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F_b</td>
<td>\sqrt{\frac{5}{3}}\pi a^2 P_0 &amp; \pi a^2 P_0 &amp; \frac{1}{3}\pi a^2 P_0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>n_R &amp; n_A = \frac{3}{\sqrt{5}}n_R &amp; n_P = \frac{1}{\sqrt{5}}n_R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_S</td>
<td>C_{RS} &amp; C_{AS} = \frac{5}{9}C_{RS} &amp; C_{PS} = 5C_{RS}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$p_m$ and $p_o$ are the incident and transmitted wave pressures at the radiation interface, respectively.

When a static pressure is present in the medium, the total force on the membrane is $\pi a^2 P_0$. The work done on a narrow ring by this static pressure can be obtained as:

$$\delta E = (P_0 2\pi r dr) \int_0^{x(r)} dx = (P_0 2\pi r dr)x(r). \quad (2.32)$$

Then the total work is found as

$$E = P_0 2\pi X_P \int_0^{a} \left(1 - \frac{r^2}{a^2}\right)^2 r dr = \frac{\pi a^2 P_0}{3} X_P. \quad (2.33)$$

It is clear from (2.33) that $\pi a^2 P_0$ corresponds to the input static force in the average model $\{f_A, v_A\}$.

For dynamic signals, we consider the power relation at the radiation interface. The acoustic power intercepted by a receiving transducer from an incident plane wave can be expressed in terms of the particle velocity in the medium, $v_m$, as $\pi a^2 \rho_c (v_m v_m^\ast)$. The power emitted to the medium from a rigid piston with a large
aperture compared to the wavelength, can be expressed similarly. The same power written in terms of velocity distribution on the membrane yields:

\[ \rho c \int_0^a v_i P_i^* \left( 1 - \frac{r^2}{a^2} \right) 2\pi r dr = \pi a^2 \rho c (v_R v_R^*) . \] (2.34)

Therefore, when the radiated power is expressed in terms of the through variable only, \( \text{rms} \) velocity maintains the consistency between transducer output and radiated power. We conclude that the forces obtained by multiplying the equivalent uniform dynamic pressures on the membrane surface by membrane area are the lumped forces at the output terminals of the \( \text{rms} \) equivalent circuit. The forces obtained from dynamic field pressures must be scaled when used in other two models. These relations are given in Table 2.1.

The power delivered to the medium is the same in all three models. However, the force, hence the equivalent uniform pressure delivered to the medium is scaled in \( \{ f_P, v_P \} \) and \( \{ f_A, v_A \} \) models according to the associated through variable and is different than the product of the area and the pressure in the field. The only through variable which produces an equivalent circuit whose dynamic output pressure is compatible with the field pressure is \( v_R \). Consequently, there is no need to scale the received dynamic pressure in the \( \text{rms} \) model. For example, when the output velocity is used in beam-forming, the particle velocity at the output of the \( \text{rms} \) equivalent circuit corresponds to the physical particle velocity of the CMUT cell. In [40], \( v_R \) is shown to be the suitable reference lumped velocity for diffraction constant calculations in transducers.

### 2.2.3 Spring Constant of the Membrane

The accuracy of equivalent circuit presented in this work depends on two factors: the agreement of actual velocity profile with the assumed one and the accuracy of mechanical circuit elements and the radiation impedance. A CMUT cell with a circular membrane, \( a/t_m \geq 80 \), is assumed and the compliance of the membrane, \( C_m \), is taken as in Table 2.1. It can be shown by FEM analysis that although both profile and expression in Table 2.1 are excellent models at very low center displacement, they deviate from these as center displacement increases. Particularly
$C_m$ decreases significantly because the membrane gets stiffer [41], [42].

The membranes of CMUTs often fall into the “plate” class ($a/t_m < 80$) [41]. As the plate gets thicker its compliance becomes softer compared to the value calculated from Table 2.1.

The lumped element models require only a lump-sum but correct, assessment of the effect. Both the collapse voltage and the resonance dynamics depend on the membrane compliance. If the dependence of this compliance to the physical dimensions of the membrane is adequately modeled and included into the equivalent circuit, the accuracy of the model predictions for thicker plates increases. A comprehensive model for $C_m$ nonlinearity applicable for all possible $a/t_m$ ratios and material properties is not addressed in this work. Nevertheless, it is shown in Section 2.4 that even without any correction for profile or $C_m$ the equivalent circuit produces very accurate results.

2.3 Small Signal Model

Almost all reception operations are small signal applications. A small signal equivalent circuit can be derived from the large signal model. We consider \{\textit{f$_R$, v$_R$}\} model and make the small signal assumptions: we assume that the ac voltage at the device terminal is small and write

\[
V^2(t) = [V_{DC} + V_{ac}(t)]^2 \approx V_{DC}^2 + 2V_{DC}V_{ac}(t),
\]

(2.35)

since $|V_{ac}(t)| \ll V_{DC}$. We write the displacement as

\[
x_R(t) = X_R + x_r(t) \quad \text{with} \quad |x_r(t)| \ll X_R,
\]

(2.36)

\textit{f$_R$}(t) given by (2.7) in the large signal model can be linearized around $X_R$ as

\[
f_R(t) = F_R + f_r(t) = f_R|_{x_R=X_R} + \frac{df_R}{dx_R|_{x_R=X_R}} x_r(t).
\]

(2.37)

\footnote{Capital letters with capital subscripts refer to dc quantities, whereas lowercase letters with lowercase subscripts are small signal quantities.}
Ignoring the second order terms, the force terms become

\[ F_R = \sqrt{5} \frac{C_0 V^2_{DC}}{2t_{ge}} g' \left( \frac{X_P}{t_{ge}} \right) \]  
(2.38)

\[ f_r(t) \approx \frac{2F_R}{V_{DC}} \nu(t) + \sqrt{5} \frac{C_0 V^2_{DC}}{2t_{ge}^2} g'' \left( \frac{X_P}{t_{ge}} \right) \nu(t). \]  
(2.39)

Eq. (2.38) is the dc force which provides the static deflection. \( g''(\cdot) \) is given by (2.9) and \( X_P/t_{ge} \) can be calculated from (2.30). From (2.39) we obtain the linear transduction equation in rms variables as

\[ f_r(t) = n_R \nu(t) + \frac{x_r(t)}{C_{RS}}, \]  
(2.40)

where

\[ n_R = \frac{2F_R}{V_{DC}} \]  
(2.41)

is the electromechanical turns ratio at the operating point and

\[ C_{RS} = \frac{2t_{ge}^2}{5C_0 V^2_{DC} g''(X_P/t_{ge})} \]  
(2.42)

is the spring softening capacitor. We note that a linearization of (2.21) around the operating point gives

\[ i_v = \frac{2F_R}{V_{DC}} \nu(t) = n_R \nu(t), \]  
(2.43)

consistent with the turns ratio definition of (2.41).

The only small signal component on the electrical side is the capacitance of the deflected membrane found when (2.3) is linearized at the operating point:

\[ C_{0d} = C_0 \ g \left( \frac{X_P}{t_{ge}} \right). \]  
(2.44)

The small signal equivalent circuit with these components is depicted in Fig. 2.6.

In order to evaluate circuit parameters \( C_{0d}, n_R, \) and \( C_{RS} \), we first specify \( X_R/t_{ge} \) such that \( X_R/t_{ge} < X_{Pc}/(\sqrt{5} t_{ge}) \) for the operating \( V_{DC} \) and the static force \( F_b \) using (2.26) and then evaluate the circuit parameters using (2.41), (2.42) and (2.44).
We follow the procedures given in Section 2.2 to get the equivalent circuits for other through variables. The turns ratio and the spring softening compliance for peak and average equivalent circuits are given in Table 2.1. $C_{0d}$ remains unchanged.

### 2.4 Comparison with FEM Analysis

The predictions of the equivalent circuit model are examined through FEM analyses. Static, prestressed harmonic and nonlinear transient analyses are performed using the simulation package ANSYS v13 (ANSYS Inc., Canonsburg, PA). In all simulations of this chapter, an immersed CMUT cell with a silicon nitride membrane is used, whose material properties are taken as $\rho = 3.27$ g/cm$^3$, $Y_0 = 320$ GPa and $\sigma = 0.263$. The density and the speed of sound in water are taken as 1 g/cm$^3$ and 1500 m/sec, respectively.

In Fig. 2.7, a comparison is made between the prediction of the equivalent model and the FEM model, based on the conductance of a CMUT cell in water. In FEM simulations, an absorbing boundary layer is employed, which simulates a fluid domain that extends to infinity beyond the boundary. Although it is preferable to use a 2-D axisymmetric FEM model for a single CMUT cell, we used a 3-D FEM model for all prestressed harmonic analyses. We realized that in 2-D FEM models, the resonance frequency and the amplitude of the harmonic response change depending on the distance between the absorbing boundary layer and the CMUT. However, we did not observe this problem in 3-D FEM models, when the absorbing boundary layer is located at least $0.2\lambda + a$ away from the
center of the CMUT cell, as suggested by Ansys. Here, $\lambda$ is taken as the greatest wavelength of the pressure waves for that analysis.

The membrane of this CMUT is quite thick ($a/t_m = 20$). The model employs the thin plate compliance for membrane and this contributes to the difference in the resonance frequency predicted by the model and by FEM analysis.

![Figure 2.7: Small signal conductance of a silicon nitride ($\text{Si}_3\text{N}_4$) membrane CMUT in water with $a=20 \, \mu\text{m}$, $t_{ge}=250 \, \text{nm}$, $t_m=1 \, \mu\text{m}$. 1-V ac signal is applied with 60, 70 and 80 V bias voltages. Finite element method (FEM; solid line) results are acquired from prestressed harmonic analyses and compared with the frequency response of the equivalent circuit model (dashed line).](image)

The large signal performance of the model is compared with the FEM results on the same CMUT cell, but under extreme electrical drive conditions, which emphasize the nonlinear effects. In Fig. 2.8, the model and FEM predictions are depicted for a CMUT biased with 40 V and driven with a sinusoidal signal of 50 V peak amplitude at 1 MHz. For reference, the small signal resonance frequency under 40 V dc bias is 5.3 MHz. Time domain steady state response of the model is compared with the transient analysis in FEM. The nonlinearity is very noticeable, because the amplitude of the ac signal is large and the frequency is approximately one-fifth the resonance frequency of this CMUT. FEM and model predictions are very consistent.
Figure 2.8: Peak displacement of the CMUT cell in water with $a=20\ \mu m$, $t_{ge}=250\ \text{nm}$, $t_m=1\ \mu m$, which is driven with 50 V peak ac voltage and 40 V bias voltage. The frequency of the applied signal is one-fifth the resonance frequency of the immersed transducer. Steady-state time domain response of the model (dashed line) is compared with the one obtained with the finite element method (FEM; solid line).

The large signal performance of the model is further studied and a peak is observed in the real part of the fundamental component of the source current at half the resonance frequency. This can be explained as follows: the generated force is proportional to the square of the applied voltage and second harmonic is inherently present in the generated force. The second-harmonic component increases very significantly at high sinusoidal drive levels. When the second-harmonic frequency of the applied voltage coincides with the resonance frequency, there is an efficient acoustic radiation and the current drawn from the source increases. We repeated this analysis here when 40 V peak sinusoidal voltage and 10 V bias voltage are applied to the same CMUT cell in water, which has a collapse voltage of 95 V. As shown in Fig. 2.9, FEM and lumped element model results agree very well.
Figure 2.9: Real part of the fundamental source current flowing through a silicon nitride (Si$_3$N$_4$) membrane CMUT cell in water with $a=20\ \mu\text{m}$, $t_{ge}=250\ \text{nm}$, $t_m=1\ \mu\text{m}$. A 40 V peak ac voltage is applied on 10 V bias voltage. Large signal response is observed in the finite element method (FEM; solid line) transient analysis and compared with the response of the model shown in Fig. 2.3 (dashed line).
Chapter 3

An Equivalent Circuit Model for CMUT Arrays

CMUTs are usually composed of large arrays of closely packed cells. In this chapter, we use an equivalent circuit model to analyze CMUT arrays with multiple cells. We study the effects of mutual acoustic interactions through the immersion medium caused by the pressure field generated by each cell acting upon the others. To do this, all the cells in the array are coupled through a radiation impedance matrix at their acoustic terminals. An accurate approximation for the mutual radiation impedance is defined between two circular cells, which can be used in large arrays to reduce computational complexity. Hence, a performance analysis of CMUT arrays can be accurately done with a circuit simulator. By using the proposed model, one can very rapidly obtain the linear frequency and nonlinear transient responses of arrays with an arbitrary number of CMUT cells. We performed several FEM simulations for arrays with small numbers of cells and showed that the results are very similar to those obtained by the equivalent circuit model.
Figure 3.1: Configuration of a rectangular array of CMUT cells, where the center-to-center displacement between the $i$th and $j$th cell is denoted.

3.1 Mutual Radiation Impedance Between CMUTs

CMUT array elements consist of multiple cells, which are usually closely packed and electrically driven in parallel. A generic CMUT array is shown in Fig. 3.1, where $a$ is the radius of the cells, $d_{ij}$ is the center-to-center separation between any two cells, and $M$ and $K$ denote the number of cells in the rows and columns. The total radiation impedance of the $i$th cell is defined as

$$Z_i = Z_{ii} + \sum_{\substack{j=1 \atop i \neq j}}^{N} \frac{v_j}{v_i} Z_{ij},$$  \hspace{1cm} (3.1)$$

where $N = MK$ is the number of cells, $Z_{ii}$ is the self radiation impedance of the $i$th cell when it is located on an infinite rigid plane baffle. $v_i$ and $v_j$ are the reference velocities for the $i$th and $j$th cells and $Z_{ij}$ is the mutual radiation impedance between them [31]. For a given pair of cells, the value of $Z_{ij}$ depends only on the radius of each cell and the separation between them normalized with the wavelength in the immersion medium.

The acoustic force at the radiation interface of each cell can be interpreted in
matrix form with
\[
\begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_N
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} & \cdots & Z_{1N} \\
Z_{21} & Z_{22} & \cdots & Z_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{N1} & Z_{N2} & \cdots & Z_{NN}
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
\vdots \\
v_N
\end{bmatrix},
\] (3.2)

where \( F_i \) and \( v_i \) represent the rms force and the rms velocity of the individual cells, respectively. The square matrix, \( Z = [Z_{ij}] \), is the impedance matrix. If all the transducers in the array are identical, the self radiation impedance is the same for all of them. According to the acoustical reciprocity theorem, \( Z_{ji} = Z_{ij} \), so that \( Z \) is a complex symmetric matrix.

![Figure 3.2: Mutual radiation resistance, \( R_{12} \), and reactance, \( X_{12} \), between two clamped circular radiators normalized to \( \rho c S \) when \( ka = 1 \). \( \rho \) and \( c \) are the density and velocity, respectively, of sound in the immersion medium, and \( S = \pi a^2 \) is the surface area of each radiator. The impedance values are referred to spatial rms velocity.](image)

### 3.1.1 An Approximation for the Mutual Radiation Impedance

Porter studied the self and mutual radiation impedances of flexural disks with different boundary conditions, located on an infinite rigid plane baffle [35]. Infinite series expressions for the real and imaginary parts of the mutual radiation impedance, \( Z_{12} \), between two clamped radiators are given in Eqs. 44 and 46.

*The factor \( \frac{1}{i} \) is missing on the right-hand side of the equation.
Figure 3.3: The real and imaginary parts of the $ka$ dependent term, $A(ka)$, of the approximate mutual radiation impedance expression given in (3.3) for $ka < 5.5$.

of [35], respectively. For $ka = 1$, Fig. 3.2 shows the variation of $Z_{12}$ as a function of $kd$, where $k$ is the wavenumber in the immersion medium and $d$ is the center-to-center distance between the two circular radiators. In this work, the impedance values are referred to the spatial rms velocity of the radiator [34]. These values are $5/9$ times the values obtained by Porter [35], where the average velocity rather than rms velocity is chosen as the reference velocity.

$Z_{12}$ is an inseparable expression of $ka$ and $kd$. By its nature, it is a slowly decaying function of $kd$ [35]. This implies that for a large CMUT array, the combined interactions from distant cells may become highly effective on the acoustic load impedance experienced by each cell. For this reason, the mutual radiation impedance between all pairs of cells needs to be taken into account, which may introduce a huge $Z$ matrix to compute.

If we carefully analyze the real and imaginary parts of $Z_{12}$, we see that both decays proportionally with $kd$ and the phase difference between them is always nearly 90 degrees. This convinces us to obtain an accurate approximation of the following form:

$$\frac{Z_{12}}{\rho c S} \approx A(ka) \frac{\sin(kd)}{kd} + j \frac{\cos(kd)}{kd} \quad \text{for} \quad ka < 5.5. \quad (3.3)$$

Here, $A(ka)$ is found by curve fitting and it is a complex function as depicted in Fig. 3.3. To obtain the real and imaginary parts of $A(ka)$, tenth-order polynomials
are used, the coefficients of which are given in the Appendix A.2. When $ka \ll 1$ and $ka \ll kd$, $A(ka) = (5/9)(ka)^2/2$ [35]. To calculate $Z_{ij}$ in (3.2), $d$ in (3.3) must be replaced with the corresponding $d_{ij}$.

With (3.3) the dependence of $Z_{12}$ on $ka$ and $kd$ is now separated. For $ka > 5.5$, this approximation is not correct, since in the vicinity of $ka = 2\pi$ the decay of $Z_{12}$ is not simply proportional to $kd$. However, beyond this limit the values of $Z_{12}$ are very small compared to the self radiation resistance and can be ignored†.

### 3.2 Acoustic Interactions Between Closed Packed CMUT Cells

A CMUT array can be built by combining several of the equivalent circuits developed in Chapter 2, through the appropriate $Z$ matrix, as demonstrated in Fig. 3.4. The equivalent circuit model and the mutual radiation impedance described in Section 3.1 are consistent, since both are obtained for circular disks with a clamped edge. There are two important, albeit ordinary assumptions for this model. First, the array is located on an infinite, rigid plane baffle, so that acoustic radiation takes place in the semi-infinite fluid domain [35]. Second, higher order modes of the cells in the array are assumed not to be excited in the frequency region of interest [17].

In this section, we study the impact of acoustic interactions in different CMUT array elements when all the cells are biased and driven in parallel with a 1 V peak ac voltage. The cells are closely packed, with the edge-to-edge separation between them being $a/10$. A commercial circuit simulator (Advanced Design System (ADS), Agilent Technologies, Palo Alto, CA) is used to obtain the linear and nonlinear responses of the equivalent circuit model of CMUT elements with arbitrary number of cells. The effects of cell apertures and their relative locations in the elements are examined. The results are compared with 3-D FEM (ANSYS)

†For example at $ka = 2\pi$, when $kd = 4\pi$, $R_{12}/p c S = 4.32 \times 10^{-4}$ but when $kd = 16\pi$, it decays rapidly and the oscillation amplitude of $R_{12}/p c S$ becomes approximately $7 \times 10^{-6}$.
Figure 3.4: (a) Equivalent circuit of a single CMUT cell. (b) Equivalent circuit representation of an array of \( N \) CMUTs. \( F_b \) is the static external force, such as that due to atmospheric pressure. \( f_t \) is the dynamic external force, such as that due to an incident acoustic signal. For an array element, electrical terminals of all cells are connected in parallel: \( V_1 = V_2 = \ldots = V_N \). For thick membranes with \( a/t_m < 10 \), a correction for the membrane compliance, \( C_m \) [1], is used.

results and shown to be similar to the equivalent model predictions. The circuit simulations are completed within a few seconds, while a single FEM simulation lasts after a few hours. Guidelines for implementing the model in ADS are given in Appendix B.

3.2.1 Effects of Cell Aperture and Location

Three CMUT cells with different radii and thicknesses that resonate around 3.5 MHz in water are considered. They are designated as CMUTs I, II and III, with \( a \approx \lambda/4, \lambda/8 \) and \( \lambda/16 \), respectively, where \( \lambda \) is the wavelength in water at 3.5 MHz. The physical properties of these CMUTs, and the dc bias voltages applied to them are given in Table 3.1. Notice that cells with larger radii have thicker membranes resulting in the same resonance frequency.

We investigate the effects of mutual interactions between the cells for 2-cell,
Table 3.1: Dimensions and bias voltages of the CMUT cells used in the simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CMUT I $(a \simeq \lambda/4)$</th>
<th>CMUT II $(a \simeq \lambda/8)$</th>
<th>CMUT III $(a \simeq \lambda/16)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membrane radius, $a$</td>
<td>104μm</td>
<td>53μm</td>
<td>26.9μm</td>
</tr>
<tr>
<td>Membrane thickness, $t_m$</td>
<td>13μm</td>
<td>4.3μm</td>
<td>1.3μm</td>
</tr>
<tr>
<td>Gap height, $t_g$</td>
<td>150nm</td>
<td>200nm</td>
<td>270nm</td>
</tr>
<tr>
<td>Young’s modulus, $Y_0$</td>
<td>320 GPa</td>
<td>320 GPa</td>
<td>320 GPa</td>
</tr>
<tr>
<td>Density, $\rho$</td>
<td>3270 kg/m$^3$</td>
<td>3270 kg/m$^3$</td>
<td>3270 kg/m$^3$</td>
</tr>
<tr>
<td>Poisson’s ratio, $\sigma$</td>
<td>0.263</td>
<td>0.263</td>
<td>0.263</td>
</tr>
<tr>
<td>Insulator thickness, $t_i$</td>
<td>100nm</td>
<td>100nm</td>
<td>100nm</td>
</tr>
<tr>
<td>Insulator permittivity, $\epsilon_r$</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Collapse voltage, $V_c$</td>
<td>90V</td>
<td>97V</td>
<td>100V</td>
</tr>
<tr>
<td>Bias voltage, $V_{DC}$</td>
<td>0.5$V_c$</td>
<td>0.7$V_c$</td>
<td>0.7$V_c$</td>
</tr>
</tbody>
</table>

$\lambda$ is the wavelength in water at 3.5 MHz.

3-cell and 4-cell elements (Fig. 3.5). Note that for 2-cell and 3-cell elements, cell locations are symmetric. Thus, there are two different values in the $Z$ matrix: the self radiation impedance, $Z_{11}$, and the mutual radiation impedance, $Z_{12}$. For 4-cell case, there is another term, $Z_{14}$, because of the dissimilar distance between the diagonal cells.

Equivalent circuit simulations are done for CMUTs I, II and III to observe the effects of cell radius on the mutual acoustic coupling that takes place in each particular cell configuration. The total electrical conductances, $G$, of the parallel connected cells for all cases are plotted in Fig. 3.6. In each graph, the values for a single cell placed on an infinite rigid baffle are also given for reference. According to the results, each type of CMUT appears to be influenced by the mutual interactions, but in a different manner. Calculated for $a = \lambda/4$, Fig. 3.6(a) shows that there is a noticeable shift in the resonance frequency to higher values as the number of cells increases. Fig. 3.6(b) and Fig. 3.6(c) show that there is a frequency shift to lower values.
There is a prominent effect of mutual acoustic coupling for the 4-cell element. For $a \simeq \lambda/8$ and $\lambda/16$, conductance curves indicate the presence of spurious resonances. It is clear that these resonances are caused by the acoustic coupling of cells through the medium and this phenomena is accurately modeled by using the mutual impedances. Note that the same effect also exists for $a \simeq \lambda/4$, but is quite subtle. To further investigate this anomaly, we observed the magnitude of the peak displacement, $x_P$, of the cells (Fig. 3.7). The behavior of the cells in the same row is substantially different regardless of the cell apertures, because they experience different acoustic loads from the immersion medium. However, the results indicate that this resonance effect is more pronounced when the cell radius is small.

### 3.2.2 6-Cell and 7-Cell Elements

In this section, two hexagonally arranged CMUT array elements with six and seven cells (Fig. 3.8) are discussed. The total radiation impedance experienced by the cell in the middle of the 7-cell element will not be the same as that for those located at the sides. The effect of this difference on element behavior is explored for different cell radii.

The conductances of each element are plotted in the graphs in the upper row of Fig. 3.9, for different cell radii. The presence of the center cell significantly alters the conductance, and its effect becomes quite dramatic as the aperture size decreases. The remaining graphs of Fig. 3.9 depict the magnitude and phase of the peak displacement of each cell. The anomalies in the conductance values are more clear when the displacements of the center cell and the edge cells differ the most. For $a \simeq \lambda/4$, the displacement of the center cell decreases at around 3 MHz, where the responses of the 6-cell and 7-cell elements are very similar, as indicated by their conductances. This is because the center cell of the 7-cell element acts virtually as a rigid plane, which is indeed the case for the 6-cell array. But for $a \simeq \lambda/16$, the cells at the edges of the 7-cell element do not vibrate effectively at around 3.7 MHz. However, for $a \simeq \lambda/8$, neither the peak displacement of the center cell nor that of the edge cells vanishes at the given frequency range.
Figure 3.5: Configuration of CMUT array elements with (a) two, (b) three and (c) four closely packed cells, located on an infinite rigid plane baffle.

Figure 3.6: The electrical conductance of a single cell and 2-cell, 3-cell and 4-cell cases using CMUTs (a) I, (b) II and (c) III for each cell.

Figure 3.7: The magnitude of the peak displacement of the cells in the first row of the 4-cell element using CMUTs (a) I, (b) II and (c) III for each cell.
Figure 3.8: (a) 6-cell and (b) 7-cell hexagonal CMUT array element geometries. All the cells in each element are biased and driven electrically in parallel.

Figure 3.9: The total electrical conductance of 6-cell and 7-cell hexagonal elements, and the amplitude and phase of the peak displacement, $x_P$, of each cell in the elements built with CMUT I, II and III. The cells are located on an infinite rigid plane baffle and they are immersed in water. $\lambda$ is the wavelength in water at 3.5 MHz.
Chapter 4

CMUT Array Elements

4.1 1-D CMUT Array Element

Ordinarily, a 1-D CMUT array element has a length between $L = 10\lambda - 20\lambda$ and a width of $W = \lambda/2$, at the center frequency. It may contain 2 to 8 side by side cells in the width and hence, may involve hundreds of cells in the whole array. It is impractical to build and simulate such an array with FEM. Therefore, reduced FEM models are usually utilized, where 1-D CMUT array elements are assumed to be infinitely long [32, 13, 9]. In a reduced FEM model, only the smallest periodic portion of the element is modeled and the obtained results are assumed to be identical throughout the entire element. This method is misleading, because the mutual coupling effects for the infinitely long and finite elements are not the same [28].

A typical 1-D CMUT array element is shown in Fig. 4.1. We analyzed this element when its length, $L$, is $10\lambda$ and $20\lambda$ at 3.5 MHz in water. The width of the element is $\lambda/2$ with two side by side cells and the edge to edge separation between each pair of cells is $a/10$. We built a reduced 3-D FEM model, where only the dashed cell regions in the figure are modeled that correspond to the smallest periodic portion of the element. Rigid boundary conditions are applied to the FEM model. We compared the FEM model results with the equivalent circuit
simulations, where we modeled each cell separately and coupled them through the impedance matrix, $\mathbf{Z}$. The elements have 88 ($L = 10\lambda$) or 176 ($L = 20\lambda$) cells.

FEM model predicts that cells #1 and #3 exhibit the same behavior as do cells #2 and #4. Fig. 4.2 shows the magnitude of the peak displacements of cells #1 and #2. Both the FEM and the equivalent circuit predict a difference in the response of these two cells. In addition, the circuit model detects many spurious resonances, which are due to the finite size of the elements. It is also important to note that the displacements obtained by the circuit model are different for each cell, whereas it is assumed to be periodic in the reduced FEM model. Fig. 4.3 demonstrates the displacements of the cells for the 10$\lambda$ long element at 3.26 MHz in water. It can be observed that both the magnitude and the phase of the cells are significantly different. This behavior is an indication of a spurious mode excited around this frequency.

Figure 4.1: A 1-D CMUT array element of finite size located on an infinite rigid plane baffle. The rigid boundary conditions applied for the reduced FEM model are depicted assuming an infinitely long array element.
Figure 4.2: The magnitude of the peak displacement of the two cells that are situated in the middle of the array element given in Fig. 4.1, when $L = 10\lambda$ and $L = 20\lambda$. The equivalent circuit model predictions include the interactions between all cells in the element. The FEM results are for the reduced model with the infinitely long array element assumption.
4.1.1 Frequency Response Under Linear Conditions

We modeled two more elements with CMUT I and CMUT III cells with $L = 10\lambda$ and $W = \lambda/2$ at 3.5 MHz in water. The cells in the elements are placed in the same configuration given in Fig. 3.1, where for the one built with CMUT I cells, $M = 22$ and $K = 1$ and for the one built with CMUT III cells, $M = 88$ and $K = 4$. For the element that was built with CMUT II cells, $M = 44$ and $K = 2$. The edge to edge separation between each pair of cells is $a/10$.

These elements are analyzed by the equivalent circuit model and the total electrical conductance, $G$, of each are obtained as shown in Fig. 4.4. While there is a hardly visible spurious resonance at far end of the response of the element built by CMUT I cells, there are many significant ones for the elements built by CMUT II and III cells, as shown in Fig. 4.4(e) and Fig. 4.4(f), respectively. It is important to note that the displacements obtained by the circuit model are different for each cell. The distinct spurious resonance observed around 3.77 MHz in Fig. 4.4(e)
corresponds to an oscillation along the width of the element. The other spurious resonances correspond to oscillations along the length of the element. Similarly, the two distinct spurious resonances observed around 3.29 MHz and 3.87 MHz in Fig. 4.4(f) correspond to oscillations in the lateral direction along the width of the element built by CMUT III cells.

We would expect the effects of mutual acoustic interactions to diminish as we increase the center-to-center separation between each pair of cells. The total response will begin converge to the superposition of the individual cells located on a rigid plane baffle. As \( d \) increases, spurious resonances that are currently inside the frequency band will shift to lower frequencies and then disappear. However, other spurious resonances will rise from the end of the frequency band. This is demonstrated in Fig. 4.5. It is seen in Fig. 4.5(a) that as \( d \) is increased up to 5\( a \), the insignificant spurious resonance of the element built by CMUT I cells becomes a prominent one. On the other hand, the exact opposite situation happens for the elements built by CMUT II and III cells; the spurious resonances become less prominent as \( d \) is increased up to 5\( a \). This can be observed in Figs. 4.5(b)(c).

Trying to reduce the mutual acoustic effects by separating the cells may not be a practical approach. Alternatively, some mechanical loss may be introduced to each cell in the array element. This is demonstrated in Fig. 4.6. A resistance, \( R_{\text{loss}} \), is added in series with the mechanical \( LC \) section of each cell’s equivalent circuit, which is a fraction of the self radiation resistance, \( R_{RR} \), of each cell at 3.5 MHz in water. In these examples \( R_{RR} \) corresponds to 0.533\( \rho c S \), 0.160\( \rho c S \) and 0.042\( \rho c S \) for CMUTs I, II and III, respectively, where \( S = \pi a^2 \) is the surface area of each. The presence of loss in CMUT I cells reduces efficiency as shown in Fig. 4.6(a). On the contrary, when \( R_{\text{loss}} = R_{RR} \) for CMUT II and III cells, even the distinct spurious resonances of the elements built by these cells are fairly damped without a considerable reduction in the efficiency.
Figure 4.4: Configurations of the $10\lambda$ long elements that are built by CMUTs (a) I, (b) II and (c) III cells, with $M = 22, 44, 88$ and $K = 1, 2, 4$, respectively. The edge to edge separation between each pair of cells is $a/10$. The total electrical conductance of each array element is depicted when it is located on an infinite rigid plane baffle and immersed in water.
Figure 4.5: The total electrical conductances of the 1-D CMUT array element, which consist of CMUT (a) I, (b) II and (c) III cells, are depicted for different center-to-center separations between the cells. Each element is located on an infinite rigid plane baffle and immersed in water.
Figure 4.6: The total electrical conductances of the 1-D CMUT array element, which consists of CMUT (a) I, (b) II and (c) III cells, are depicted when different amounts of mechanical loss is present. A resistance, $R_{\text{loss}}$, is added in series with the mechanical $LC$ section of each cell, which is a fraction of the self radiation resistance, $R_{RR}$, of each cell at 3.5 MHz in water. Each element is located on an infinite rigid plane baffle and immersed in water.
Figure 4.7: (a) Far-field radiation patterns of the 1-D CMUT array element, which consists of CMUT III cells, with $M = 88$ and $K = 4$ ($L = 10\lambda$ and $W = \lambda/2$) at 2.96 MHz and 3 MHz, where (b) and (c) show the corresponding displacement profiles, respectively.

The pressure field produced by $N$ cells that are located on an infinite rigid plane baffle is

$$p(r, \theta, \phi) = j\frac{\rho ckS}{2\pi} D(\theta) \sum_{i=1}^{N} U_{Ai} e^{-jkr_i},$$

(4.1)

where $D(\theta) = 48J_3(ka \sin(\theta))/(ka \sin(\theta))^3$ is the amplitude directivity function of the clamped edge radiator [35], $U_{Ai}$ is the average velocity phasor at the surface of the $i$th cell and $r_i$ is the radial distance of the $i$th cell to the observation point in the medium.

Fig. 4.7(a) depicts the far-field radiation patterns and the transmitting voltage responses (TVRs) at 2.96 MHz and 3 MHz for the $10\lambda$ long element built by CMUT III cells. At these frequencies, the corresponding displacement profiles of the element are given in Fig. 4.7(b) and Fig. 4.7(c), where two different mode shapes can be observed. At each frequency, both the magnitude and the phase of the cells are significantly different. The conductance variation in Fig. 4.4(f)
indicates the existence of a spurious mode at 3 MHz, while 2.96 MHz corresponds to a frequency point between two spurious modes. This behavior enhances the displacement amplitudes of the cells at 3 MHz, but reduces the main lobe of the radiation pattern while increasing the side lobe levels, with respect to 2.96 MHz.

For the 10\(\lambda\) long element, the clarity of the spurious modes becomes less pronounced as the cell radii increase. Many spurious modes still exist when the element consists of CMUT II (\(a \simeq \lambda/8\)) cells. However, with CMUT I (\(a \simeq \lambda/4\)) cells with thicker membranes, the cell displacements are fairly uniform in the frequency band. Fig. 4.4(d) depicts the conductance of this element which is free of any notable spurious mode in the frequency band.

### 4.1.2 Transient Response

By using the equivalent circuit model, the transient responses of 10\(\lambda\) long elements are analyzed when they are excited by a square voltage pulse with 100ns duration. The rise and fall times of the pulse are 10ns.

For the CMUTs I, II and III cells, the voltage is initially kept at 45V, 68V and 70V, and then increased to 65V, 88V and 90V within the pulse duration, respectively. Fig. 4.8(b), Fig. 4.9(b) and Fig. 4.10(b) show the time domain pressure pulses generated at 1 mm away from the center of the elements. The frequency spectrum of these pressure pulses are given in Fig. 4.8(c), Fig. 4.9(c) and Fig. 4.10(c). Notice the presence of oscillations in Fig. 4.9(b), centered around 3.77 MHz, long after the pulse is applied. This is due to the existence of the spurious mode observed for the element built by CMUT II cells. Similarly, the presence of oscillations in Fig. 4.10(b), centered around 3.4 MHz, are due to the existence of the spurious modes observed for the element built by CMUT III cells. The simulations show the existence of spurious modes clearly. They cause oscillations in the impulse response. Although, it causes a little reduction in the amplitude of the focused beam, it results in an increase in the side lobe amplitude as well as an extension of impulse response duration.
Figure 4.8: (a) Configuration of the $10\lambda$ long element that is built by CMUT I cells with $M = 22$ and $K = 1$. (b) The pressure pulse generated at 1 mm away from the center of this element, and (c) its spectrum. The element is located on an infinite rigid plane baffle and it is excited by a square voltage pulse with 0.1$\mu$s duration.
Figure 4.9: (a) Configuration of the $10\lambda$ long element that is built by CMUT II cells with $M = 44$ and $K = 2$. (b) The pressure pulse generated at 1 mm away from the center of this element, and (c) its spectrum. The element is located on an infinite rigid plane baffle and it is excited by a square voltage pulse with $0.1\mu s$ duration.
Figure 4.10: (a) Configuration of the $10\lambda$ long element that is built by CMUT III cells with $M = 88$ and $K = 4$. (b) The pressure pulse generated at 1 mm away from the center of this element, and (c) its spectrum. The element is located on an infinite rigid plane baffle and it is excited by a square voltage pulse with $0.1\mu s$ duration.

CMUT III
$a \approx \lambda/16$
at 3.5 MHz
4.2 Reduced Radiation Impedance Matrix

In CMUT arrays, the acoustic force at the radiation interface of each cell can be interpreted in matrix form as in Fig. 4.11, where \( F_i \) and \( U_i \) represent the \textit{rms} force and \textit{rms} velocity of the individual cells, respectively. The square matrix is the radiation impedance matrix, \( Z \), where the diagonal elements are the self and off-diagonals are the mutual radiation impedances.

\[
\begin{bmatrix}
F_1 \\
\vdots \\
F_n \\
F_{(n+1)} \\
F_{(n+2)} \\
\vdots \\
F_{(n+m)}
\end{bmatrix}
= 
\begin{bmatrix}
Z_{11} & \cdots & Z_{1n} & Z_{1(n+1)} & \cdots & Z_{1(n+m)} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
Z_{n1} & \cdots & Z_{nn} & Z_{n(n+1)} & \cdots & Z_{n(n+m)} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
& & & & & \\
B^T & \cdots & B & \vdots & \ddots & \vdots \\
& & & & & \\
& & & & & \\
& & & & & \\
& & & & & \\
& & & & & \\
& & & & & \\
& & & & & \\
& & & & & \\
& & & & & \\
U_1 \\
\vdots \\
U_n \\
U_{(n+1)} \\
U_{(n+2)} \\
\vdots \\
U_{(n+m)}
\end{bmatrix}
\]

Figure 4.11: Radiation impedance matrix, \( Z \), is depicted, where the equivalent circuit variables \( F_i \) and \( U_i \) represent the force and \textit{rms} velocity of the individual cells in a CMUT array. \( Z \) is partitioned such that \( n \) and \( m \) are the number of cells in the driven and undriven elements, respectively.

A typical CMUT array contains hundreds of cells in each of its elements, which may end up with over 100,000 cells in the entire array. For such a large array, the computation of the radiation impedance matrix and the solution of the circuit theory based model require an excessive time. Ultrasonic arrays are usually actuated by applying voltage pulses to groups of elements located side to side in the array. In this case, we may partition \( Z \) as shown in Fig. 4.11, where \( n \) is the number of cells in the electrically driven elements and \( n + m \) (\( m \gg n \)) is the total number of cells in the array. The circuit variables of the driven elements are aggregated at the upper left corner of \( Z \), which is denoted as \( A \). Note that if the rest of the elements were constrained to be stationary (i.e. \( U_{(n+1)}, U_{(n+2)}, \cdots U_{(n+m)} = 0 \)), then only \( A \) would determine the overall response,
because it is the radiation impedance matrix of the driven elements when they are located on an infinite rigid plane baffle. On the other hand, in the small signal regime, we have the following relation for the undriven elements

\[
\begin{bmatrix}
F_{(n+1)} \cdots F_{(n+m)}
\end{bmatrix}^T = -E\begin{bmatrix}
U_{(n+1)} \cdots U_{(n+m)}
\end{bmatrix}^T,
\] (4.2)

where \( E \) is the electromechanical impedance matrix observed at their radiation interfaces.

To reduce the size of the problem without any loss of accuracy, we may electrically drive a small number of elements in the array and keep the rest undriven (but biased or unbiased), with their electrical ports terminated with a load. Then, by using (4.2), \( Z \) can be rearranged to represent these loads in a reduced form as

\[
\begin{bmatrix}
F_1 \cdots F_n
\end{bmatrix}^T = \left[ A - B(E + D)^{-1}B^T \right] \begin{bmatrix}
U_1 \cdots U_n
\end{bmatrix}^T,
\] (4.3)

where \( B \) and \( D \) are the remaining partitions of \( Z \) as shown in Fig. 4.11. In this way, only the driven elements can be simulated by coupling its cells through this reduced impedance matrix. This method considerably reduces the number of cells and the size of the original radiation impedance matrix at the expense of calculating the inverse of the complex symmetric matrix \((E + D)\). Finally, if needed, the separately calculated responses of the elements can be added by using the superposition principle to find the total response.

Note that, \( E \) depends on the electrical termination and the applied dc bias voltage to the undriven elements. If there is no dc bias voltage, then \( E = Z_{me}I_m \), where \( Z_{me} \) is the mechanical impedance of each cell and \( I_m \) is the identity matrix. When a dc bias voltage is applied, the off-diagonal entries of \( E \) are not zero, unless the cells of the undriven elements are not electrically connected. In practice they are connected in parallel, so \( E \) must be calculated accordingly. Fig. 4.12 shows the small signal equivalent circuits of the cells in the undriven elements when they are unbiased and biased with their electrical ports terminated with a load, \( Z_L \). Derivation of this circuit model is described in detail in Section 2.3.
4.3 Elements in Large Arrays with Undriven Neighbors

We studied the effects of mutual acoustic interactions for different sized elements of 1-D CMUT arrays. In this section, the frequency response of one of those elements is reanalyzed when there are undriven neighbor elements.

We analyzed arrays, which have 3, 5 and 7 elements, and examined their performance in water depending on the surroundings of one of its driven elements. Fig. 4.13 depicts the 7 element array, where each element with 88 cells is nearly $10\lambda$ long and $\lambda/2$ wide at 3.5 MHz in water. Each time only a single element is excited and it is loaded with the reduced radiation impedance matrix to represent the loads due to the undriven elements.

First, all of the elements are dc biased at 70% of the collapse voltage (68V). Fig. 4.14(a) shows the electrical conductance when only the element located in the middle (#4) of the array is driven with 1 V peak ac voltage and there exists one, two and three undriven elements at each side of it. The electrical ports of the undriven elements are loaded with a high impedance electrical load ($Z_L \simeq \infty$). Fig. 4.14(b) plots the response of the 7 element array when the driven element is...
shifted to element #3, #2 and #1, respectively. In each graph, the values for a single element placed on an infinite rigid baffle are also given for reference. These results show that as the number of undriven elements and their positions change the number and rate of the spurious resonances also change. This suggests that assuming nearby undriven elements as rigid boundaries is not correct.

We also considered $Z_L = 50\,\Omega$ termination at the electrical port of the undriven elements, which yielded almost the same results as for $Z_L \simeq \infty$. This is because the reactance of the clamped capacitance is already too low compared to the rest of the series (mechanical) reactance.

On the other hand, when the undriven elements are not dc biased an electromechanical transduction does not occur within them. In this case, the electrical capacitance ($C_0$), the spring softening capacitance ($-C_S$) and the electrical load ($Z_L$) of the undriven cells are not coupled to the mechanical side. Only the
Figure 4.14: Electrical conductances of a single $10\lambda$ long CMUT array element. The variations show the effects of mutual interactions when the undriven elements are biased and their electrical ports are left open. (a) The middle element is driven when there are 1, 2, and 3 undriven elements at each side. (b) Driven element is shifted from center to side in a 7 element array.
Figure 4.15: Electrical conductances of a single $10\lambda$ long CMUT array element. The variations show the effects of mutual interactions when the undriven elements are not biased. (a) The middle element is driven when there are 1, 2, and 3 undriven elements at each side. (b) Driven element is shifted from center to side in a 7 element array.
compliance ($C_m$) and the mass ($L_m$) of the cells in these elements contribute to the overall radiation impedance experienced by the driven element, as shown in Fig. 4.12(b). The analysis made above is repeated to carry this case into effect. The results are plotted in Fig. 4.15. Once again the spurious resonances are highly influenced by the new boundary conditions due to this termination.
4.4 Experimental Measurements

We performed electrical impedance measurements of three different CMUT array elements located on a test die as shown in Fig. 4.16(a). The measurements are performed in a small tank filled with sunflower oil for electrical isolation. The cells in the elements are placed in the same configuration given in Fig. 3.1. The radii of the cells constituting the array elements on the test die range from \( a = 20 \mu m \) to \( 50 \mu m \) and the edge to edge separation between each pair of cells is \( a/10 \). In the absence of a dc bias, the elements with \( a \geq 40 \mu m \) are already in collapsed state and cannot be modeled within the scope of this work. The elements with \( a = 20 \mu m, a = 25 \mu m \) and \( a = 30 \mu m \) are measured, which have 648 (27x24), 418 (22x19) and 288 (18x16) cells, respectively.

These devices were fabricated in Selim’s work [43] at Bilkent University, which involved a low temperature \( (250^\circ C \text{ maximum}) \) surface micromachining technology that utilized a chromium sacrificial layer and silicon nitride dielectric deposition by a Plasma Enhanced Chemical Vapor Deposition (PECVD) system. From the electrical impedance measurements of each element, we approximately deduced the common dimensions of the CMUT cells and the material properties of silicon nitride membrane. They are listed in Table 4.1.

Table 4.1: The material properties of silicon nitride membrane and the dimensions of the CMUT cells.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membrane thickness, ( t_m )</td>
<td>2.1( \mu m )</td>
</tr>
<tr>
<td>Gap height, ( t_g )</td>
<td>80nm</td>
</tr>
<tr>
<td>Insulator thickness, ( t_i )</td>
<td>160nm</td>
</tr>
<tr>
<td>Young’s modulus, ( Y_0 )</td>
<td>115 GPa</td>
</tr>
<tr>
<td>Density, ( \rho )</td>
<td>3100 kg/m³</td>
</tr>
<tr>
<td>Poisson’s ratio, ( \sigma )</td>
<td>0.27</td>
</tr>
<tr>
<td>Insulator permittivity, ( \epsilon_r )</td>
<td>5.4</td>
</tr>
</tbody>
</table>

The silicon die is bonded to a chip holder with epoxy and then wire bonded. The total electrical conductances of each element are obtained with a network analyzer at different dc bias voltages and compared with the simulation results of the equivalent circuit model. The results are plotted in Fig. 4.17. In simulations
Figure 4.16: (a) Configuration of the test die consisting of many CMUT array elements with different cell radii ranging from 20 μm to 50 μm. Array elements with (b) $a=20$ μm, $M=27$ and $K=24$, (c) $a=25$ μm, $M=22$ and $K=19$ and (d) $a=30$ μm, $M=18$ and $K=16$ are used in immersion experiments. Electrical impedances of the elements are measured in a tank filled with sunflower oil.

The density of sunflower oil and the speed of sound are taken as 915 kg/m³ and 1453 m/sec, respectively. To obtain a better fit to the measurements a loss resistance of $R_{loss} = 0.03 \rho c S$ was inserted in series to the mechanical section of each cell. In simulations, the applied dc bias voltage begins from 41, 23 and 13 V for the element with $a=20 \mu m$, $a=25 \mu m$ and $a=30 \mu m$, respectively and then incremented by 1 V steps. In measurements, we observed charging effects and a loss that increases exponentially with frequency. Therefore, it cannot be solely related to dielectric loss, but we suspect that some portion of it can be attributed to backing loss. To obtain the results in Fig. 4.17, we applied a bias voltage that neutralizes the effect of charging and the resulting conductance is regarded as this loss and subtracted from the total conductance.
Figure 4.17: The total electrical conductances of each element in Fig. 4.16, which are obtained with a network analyzer at different dc bias voltages and compared with the simulation results of the equivalent circuit model.
The measured elements are surrounded by many undriven elements as shown in Fig. 4.16(a). Thus, the boundaries of the elements are not similar to that of an infinite rigid baffle. A complete equivalent circuit model including all the elements in the die could not be built, because the mutual radiation impedance between uncollapsed/collapsed CMUT cells with different radii is not studied in this work. However, in simulations, we also included the nearby undriven elements that are identical to the driven element, but did not observe a significant change in its response. The spurious resonances observed in Fig. 4.17 are mostly due to mutual acoustic interactions between the cells of the driven element. As discussed in Section 4.1.1, addition of $R_{\text{loss}}$ in the model considerably damps the spurious resonances and only the distinct ones still remain. The reason of the measured peak in Fig. 4.16(b) around 11 MHz is not exactly known. It is not observed in the simulations, but we think that it may be due to the power loss at the backing. For the element with $a = 30 \mu m$, the measurements (Fig. 4.17(c)) are performed with the data smoothing capability of the network analyzer.
Chapter 5

Conclusion

We presented a lumped element equivalent circuit, which can predict the entire behavior of a circular CMUT cell operated in the uncollapsed mode. Rigid membrane supports and rigid substrate is assumed in deriving the model, hence it does not include the loss to the substrate and cross-talk through the substrate. We found that a correct evaluation of membrane compliance is critical for the accuracy of model predictions and in determining the collapse voltage.

We developed an equivalent circuit model for CMUT arrays, which can be easily employed in circuit simulators. We have shown that by using the proposed model, the effects of the mutual acoustic interactions can be analyzed very rapidly with an accuracy comparable to FEM. We provided an approximate expression for the mutual radiation impedance, which indicates its separability as a function of $ka$ and $kd$. We referred different array configurations and compared them for three different cell radii. The examples show that the design of a single cell and the design of an array cannot be considered separately. However, one can utilize the equivalent circuit model to design and optimize CMUT arrays.

The model lends itself readily for theoretical circuit analysis. The electrical termination of the cells can easily be incorporated to examine its effect on the array performance. Also, the acoustic termination on nonradiating side of the cell, the backing, can be easily included as an appropriate impedance across the
controlled force source, $f_i$, in the model.

The employed circuit simulator must be capable of using frequency domain impedance data. Our circuit simulator can handle arrays with up to about ten thousand cells reasonably well. The number of cells in arrays realized by current design approaches can be quite high. For example, the linear array reported in [44] has more than 121000 cells in 192 elements. To analyze or design such an array, the model must be solved with a circuit simulator capable of handling very large circuits.

We also presented a method to analyze the performance of very large CMUT arrays by partitioning the radiation impedance matrix when there are undriven elements in the array. The process involves an inversion of a large matrix. In the small signal regime, the method is very accurate and it is practical for analyzing the frequency response.

The low mechanical impedance of CMUT arrays makes them suitable for immersion applications, but they also become susceptible to mutual acoustic interactions and many spurious resonances occur. We suspect that introducing loss in the electrical side and using coating layers may alter these resonances, and it must be further investigated on various arrays.
Appendix A

Radiation Impedance

A.1 Self Radiation Impedance

Radiation impedance of a transducer with a certain velocity profile is the ratio of total power radiated from the acoustical terminals to the square of the absolute value of a nonzero reference velocity:

\[ Z_R = \frac{2\pi \int_0^a P(r)v^*(r)r \, dr}{V_i V_i^*} = \frac{P_{TOTAL}}{|V_i|^2} \]  \hspace{1cm} (A.1)

When rms velocity is chosen for \( V_i \), the radiation impedance derived in [36] becomes

\[ Z_{RR} = \pi a^2 \rho_0 c \left( 1 - \frac{20}{(ka)^6} [F_1(2ka) + jF_2(2ka)] \right) \]  \hspace{1cm} (A.2)

where \( \rho_0 \) is the density and \( c \) is the speed of sound of the immersion medium and

\[ F_1(y) = (y^4 - 91y^2 + 504)J_1(y) \]
\[ + 14y(y^2 - 18)J_0(y) - y^5/16 - y^7/768 \]  \hspace{1cm} (A.3)

and

\[ F_2(y) = - (y^4 - 91y^2 + 504)H_1(y) \]
\[ - 14y(y^2 - 18)H_0(y) + 14y^4/15\pi - 168y^2/\pi \]  \hspace{1cm} (A.4)

\( J_n \) and \( H_n \) are the \( n \)th order Bessel and Struve functions, respectively.
For $ka < 0.1$, $Z_{RR} = R_{RR} + jX_{RR}$ can be approximated as

$$Z_{RR} \approx \frac{5}{9} \pi a^2 \rho_0 c \left[ \frac{(ka)^2}{2} + j \frac{2^{16}}{17325\pi} (ka) \right]$$

(A.5)

### A.2 Mutual Radiation Impedance

Porter studied the mutual radiation impedance of a clamped edge radiator, located on an infinite rigid plane baffle [35]. It is an infinite series expression and requires high computational cost when large number of CMUT cells are mutually coupled. In this case, it is more appropriate to use the approximation given in Section 3.1.1. Note that the real and imaginary parts of (3.3) are 5/9 times the values obtained by Porter, where the average velocity rather than rms velocity is chosen as the reference velocity.

The function $A(ka)$ can be calculated with the $10^{th}$ order polynomial,

$$A(ka) = \sum_{n=0}^{10} p_n (ka)^n$$

(A.6)

where the real and imaginary parts of its complex coefficients are given in Table A.1.

<table>
<thead>
<tr>
<th></th>
<th>Real Part</th>
<th>Imaginary Part</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{10}$</td>
<td>-5.34237e-7</td>
<td>-2.15428e-7</td>
</tr>
<tr>
<td>$p_9$</td>
<td>9.60487e-6</td>
<td>6.64439e-6</td>
</tr>
<tr>
<td>$p_8$</td>
<td>-1.76278e-5</td>
<td>-8.03963e-5</td>
</tr>
<tr>
<td>$p_7$</td>
<td>-6.79795e-4</td>
<td>4.61167e-4</td>
</tr>
<tr>
<td>$p_6$</td>
<td>5.09681e-3</td>
<td>-1.14002e-3</td>
</tr>
<tr>
<td>$p_5$</td>
<td>-7.47336e-3</td>
<td>7.16523e-4</td>
</tr>
<tr>
<td>$p_4$</td>
<td>-2.50825e-2</td>
<td>-2.72073e-3</td>
</tr>
<tr>
<td>$p_3$</td>
<td>-9.71827e-3</td>
<td>1.47231e-2</td>
</tr>
<tr>
<td>$p_2$</td>
<td>2.87292e-1</td>
<td>-9.01486e-4</td>
</tr>
<tr>
<td>$p_1$</td>
<td>-1.09336e-3</td>
<td>1.92266e-4</td>
</tr>
<tr>
<td>$p_0$</td>
<td>8.62694e-5</td>
<td>-1.42685e-5</td>
</tr>
</tbody>
</table>

Table A.1: Polynomial coefficients of function $A(ka)$. 

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A.3 Matlab Code

```matlab
% Function calculating the self radiation impedance of the clamped edge radiator normalized to pi*a^2*q0*c0, where q0 and c0 are the density and the velocity of sound in the immersion medium. The spatial rms velocity of the membrane is used as the reference velocity.
function Z_RR = Z11(ka)
Z_RR = zeros(1,length(ka));
for k=1:length(ka)
    y = 2*ka(k);
    if (ka(k) >= 0.1)
        F1 = (y^4 - 91*y^2 + 504)*besselj(1,y) + 14*y*(y^2 - 18)*besselj(0,y) - y^5/16 - y^7/768;
        F2 = -(y^4 - 91*y^2 + 504)*struve(1,y) - 14*y*(y^2 - 18)*struve(0,y) + 14*y^4/15/pi - 168*y^2/pi;
        Z_RR(k) = (1 - 2^11*y^9/(F1 + i*F2));
    elseif (ka(k) > 0)
        Z_RR = 5/9*(ka(k)^2)/2 + i*ka(k)*(2^16)/(17325*pi);
    end
end

% Struve function.
function [strv] = struve(order, z)
k = [0:1:50];
for i=1:length(z)
    strv(i) = (z(i)/2)^(order+1)*sum(((z(i)/2)^2).^k./(gamma(k+3/2).*gamma(k+order+3/2)));
end

% Function calculating the normalized mutual radiation impedance between two circular clamped edge radiators. Both the exact and approximate values can be calculated. The spatial rms velocity of the membrane is used as the reference velocity.
function Z12_RR = Z12(ka, kd, approx)
```

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%Polynomial coefficients for A(ka)
\[
p_{\text{real}} = [-5.342367728113970e-07, \\
-9.604871035163506e-06, \\
-1.76278392532377e-05, \\
-6.797953583404895e-04, \\
5.096807036072452e-03, \\
-7.473364955911300e-03, \\
-2.508252144293545e-02, \\
-9.718266445276595e-03, \\
2.872917917003468e-01, \\
-1.093359579861779e-03, \\
8.626944491022083e-05];
\]
\[
p_{\text{imag}} = [-2.154281769741934e-07, \\
6.644393871641362e-06, \\
-8.039629637236947e-05, \\
4.611670878449804e-04, \\
-1.14002201099357e-03, \\
7.16522507150300e-04, \\
-2.720726785397918e-03, \\
1.472308213594988e-02, \\
-9.0148584555729872e-04, \\
1.922659901900687e-04, \\
-1.426845604740463e-05];
\]

%When approximation is used, note that it is valid up to ka=6
\[
\text{if (approx==1 && ka} \leq 6) \\
A_{\text{real}} = p_{\text{real}}(1)\ast(ka^{10}) + p_{\text{real}}(2)\ast(ka^{9}) + \ldots \\
\quad p_{\text{real}}(3)\ast(ka^{8}) + \ldots \\
\quad p_{\text{real}}(4)\ast(ka^{7}) + p_{\text{real}}(5)\ast(ka^{6}) + p_{\text{real}}(6)\ast(ka^{5}) + \ldots \\
\quad p_{\text{real}}(7)\ast(ka^{4}) + p_{\text{real}}(8)\ast(ka^{3}) + \ldots \\
\quad p_{\text{real}}(9)\ast(ka^{2}) + p_{\text{real}}(10)\ast ka + p_{\text{real}}(11); \\
A_{\text{imag}} = p_{\text{imag}}(1)\ast(ka^{10}) + p_{\text{imag}}(2)\ast(ka^{9}) + \ldots \\
\quad p_{\text{imag}}(3)\ast(ka^{8}) + \ldots \\
\quad p_{\text{imag}}(4)\ast(ka^{7}) + p_{\text{imag}}(5)\ast(ka^{6}) + p_{\text{imag}}(6)\ast(ka^{5}) + \ldots \\
\quad p_{\text{imag}}(7)\ast(ka^{4}) + p_{\text{imag}}(8)\ast(ka^{3}) + \ldots \\
\quad p_{\text{imag}}(9)\ast(ka^{2}) + p_{\text{imag}}(10)\ast ka + p_{\text{imag}}(11);
A_{ka} = A_{\text{real}} + i\ast A_{\text{imag}}; \\
\text{if (ka} \geq 0.1) \\
Z_{12,RR} = A_{ka}\ast\left(\sin(\text{kd})/\text{kd} + i\ast\cos(\text{kd})/\text{kd}\right);
\]
\textbf{elseif (ka > 0)}
\begin{align*}
Z_{12 \text{RR}} &= \frac{5}{9} \times (ka^2) \times (\sin(kd) + i \times \cos(kd)) / kd / 2; \\
\end{align*}
\textbf{end}

\%When approximation is used for ka>6 assume Z12=0, because it is very small
\textbf{elseif (approx == 1 && ka > 6)}
\begin{align*}
Z_{12 \text{RR}} &= 0; \\
\end{align*}
\textbf{else}

\%When the exact value of Z12 is desired use the following
\begin{align*}
\textbf{if (ka} & \geq 0.1) \\
\text{m} &= 2; \\
\text{n} &= 2; \\
\text{SUMu} &= 0; \\
\text{for} \text{ u} &= 0:50 \\
\text{SUMv} &= 0; \\
\text{for} \text{ v} &= 0:50 \\
\text{SUMv} &= \text{SUMv} + \gamma(u+v+0.5)/\text{factorial}(u) \times (ka/kd)^{(u+v)} \times \text{besselj}(u+n+1, ka) \times \text{besselj}(v+m+1, ka) \times \text{besselj}(u+v+0.5, kd); \\
\text{SUMu} &= \text{SUMu} + \text{SUMv}; \\
\text{end} \\
\text{end} \\
\text{SUMu} &= \text{SUMu} + \text{SUMv}; \\
\text{end} \\
Z_{12 \text{RR}} &= 2 \times 2^n \times \text{factorial}(n) \times \text{factorial}(m) \times \sqrt{2n+1} \times \sqrt{2m+1} / \sqrt{2kd} / (ka^n) \times \text{SUMu}; \\
\textbf{elseif (ka > 0)} \\
Z_{12 \text{RR}} &= \frac{5}{9} \times (ka^2) \times (\sin(kd) + i \times \cos(kd)) / kd / 2; \\
\textbf{end} \\
\textbf{end}
Appendix B

Guidelines for Implementing the Model in ADS

The equivalent circuit model of CMUT arrays can be implemented by utilizing the available programming languages and circuit simulators. The choice mainly depends on the speed, robustness and complexity of the analysis to be done. For instance, a linear frequency domain analysis can be carried out very rapidly by using a high-level programming language. On the other hand, it is more convenient to perform a large-signal analysis with a circuit simulator.

Advanced Design System (ADS) is the industry’s leading electronic design automation software for RF, microwave, and high speed digital electronic design [45]. In this work, we preferred to use ADS because of the following reasons:

- The employed circuit simulator must be capable of using the frequency domain impedance data.
- The harmonic balance (HB) simulator in ADS provides an accurate estimate of the steady-state response of nonlinear circuits very quickly.

Note that, the first feature is important, because in this work, we defined the radiation impedance expressions in frequency-domain. The second feature is a
practical frequency-domain analysis technique for simulating inherently nonlinear devices like CMUTs.

In the following subsections, it is demonstrated how to implement the large and small-signal equivalent circuits of a single CMUT cell, as components in ADS. The self radiation impedance is realized as a 1-port Z-parameters component. CMUT arrays compose of multiple array elements, which are individual components consisting of many single cell subcomponents. Creating an array element component may require placement of thousands of single cell subcomponents into the same schematic. This task is easily automated by using a macro in ADS. The radiation impedance matrix is built as a multi-port user-compiled model written in C programming language. A detailed guideline explaining how to implement the array model and an up-to-date example workspace for ADS can be found at http://www.ee.bilkent.edu.tr/~cmut. This workspace includes the equivalent circuit models of a single CMUT cell, the self radiation impedance component, the macro for creating CMUT arrays, the C-code for building the user-compiled model of the radiation impedance matrix, etc.

B.1 Large-Signal Equivalent Circuit Component

The large-signal equivalent circuit model of a single circular CMUT cell for the \( \{f_R, v_R\} \) model, can be implemented in ADS as shown in Fig. B.1. The symbolically-defined device (SDD) is a nonlinear component in ADS, which is used to specify algebraic relationships that relate the port voltages, currents, and their derivatives, as well as the currents from other devices. In this model, we used a 4-port SDD (SDD4P) to calculate the instantaneous nonlinear force, \( f_R \), and currents, \( i_{Cap} \) and \( i_V \). Note that, this is the \( \{f_R, v_R\} \) rms model. The peak displacement, \( x_p \), is calculated across the compliance of the membrane, \( C_{Rm} \), with a voltage controlled current source and constantly fed back to SDD4P. The force and current expressions are defined as functions of the applied voltage, its time derivative, \( v_R \) and \( x_p/t_{ge} \).
B.2 Small-Signal Equivalent Circuit Component

The small-signal equivalent circuit model of a single circular CMUT cell for the \{f_R,v_R\} model, can be implemented in ADS as shown in Fig. B.2. This model is more self explanatory than the large-signal model, because only linear circuit components are used and it is compatible with the one given in Fig. 2.6.
Bibliography


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