# ROUTING, SPECTRUM ALLOCATION AND REGENERATOR PLACEMENT IN FLEXIBLE-GRID OPTICAL NETWORKS

A THESIS

SUBMITTED TO THE DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

AND THE GRADUATE SCHOOL OF ENGINEERING AND SCIENCE

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

By

ALPER KAHYA

August 2013

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Prof. Dr. Ezhan Karaşan(Advisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Sinan Gezici

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Oya Ekin Karaşan

Approved for the Graduate School of Engineering and Science:

Prof. Dr. Levent Onural
Director of the Graduate School

# ABSTRACT

## ROUTING, SPECTRUM ALLOCATION AND REGENERATOR PLACEMENT IN FLEXIBLE-GRID OPTICAL NETWORKS

ALPER KAHYA
M.S. in Electrical and Electronics Engineering
Supervisor: Prof. Dr. Ezhan Karaşan
August 2013

Tremendous increase in the number of wireless devices has been resulting in huge growth in the Internet traffic. This growth necessitates efficient usage of resources in the optical networks, which form the backbone of the Internet. Recently proposed flexible optical networks can adjust the optical layer transmission parameters to take advantage of existing channel conditions thereby increasing the resource utilization efficiency. Therefore, flexible optical network is a promising solution to fulfill growing future demand of IP traffic. Apart from efficient usage of the optical spectrum, the degradation of the optical signal as it propagates over the fiber is another problem. In such cases, the optical signal must be regenerated when a lightpath travels longer than the maximum optical reach. However, regenerators are expensive devices with high operational costs. Therefore, they should be placed carefully to reduce the capital and operational network costs. In this dissertation, we deal with the joint routing, spectrum allocation and regenerator placement (RSA-RP) problem for flexible optical networks. Our aim is to find the route and allocate spectrum for each traffic demand by assigning minimum number of nodes as regenerator sites. Firstly, we introduce a novel mixed integer linear programming (MILP) formulation for the joint RSA-RP problem. Since this formulation is not practical for large networks, we propose a decoupled formulation where the RSA-RP problem is decomposed into two phases. In the first step, we find routes and locations of regenerators assuming a full wavelength converting network. Then, we allocate the spectrum to each demand in the second phase. The decoupled model can be used to solve the RSA-RP problem for reasonably sized optical networks. We show that the decoupled model can find optimum solutions for 92% of the all cases tested for the NSFNET topology and 99% of the all cases tested for the Deutsche Telecom topology. We also show that the locations of regenerator sites significantly depend on network parameters such

as the node degree and lengths of the links adjacent to the node.

*Keywords:* Flexible Optical Networks, Regenerator Placement, Routing, Spectrum Allocation.

# ÖZET

## ESNEK OPTİK AĞLARDA YÖNLENDİRME, SPEKTRUM TAHSİSİ VE YENİLEYİCİ YERLEŞTİRİLMESİ

ALPER KAHYA
Elektrik ve Elektronik Mühendisliği, Yüksek Lisans
Tez Yöneticisi: Prof. Dr. Ezhan Karaşan
Ağustos 2013

Kablosuz cihazların sayısındaki muazzam artış internet trafiğinin ciddi bir şekil de artmasına neden oluyor. Bu büyüme bizi internetin altyapısını oluşturan optik ağların kaynaklarını daha verimli kullanılmasına zorluyor. Son zamanlarda araştırmacıların ortaya koyduğu esnek optik ağ kavramı mevcut kanal koşullarından yararlanabilmek için iletim parametrelerini ayarlayıp kaynakların verimli bir şekilde kullanılmasını sağlıyor. Bu nedenle, esnek optik ağlar IP trafiğinin artan gelecekteki talebini karşılamak için umut verici bir çözümdürler. Optik spektrumun etkin kullanımı yanında, optik sinyalin fiber optik kabloda ilerlerken, bozulması da başka bir sorundur. Bu gibi durumlarda, eğer optik sinyalin gitmesi gereken yol, gidebileceği maksimum optik erişim sınırından daha uzun ise, optik sinyalin yeniden oluşturulması gerekir. Ancak, yenileyiciler pahalı cihazlardır ve bunların işletme giderleri de oldukça yüksektir. Bu nedenle, ağın yatırım maliyetini ve operasyonel maliyetini azaltmak için yenileyicilerin dikkatli bir şekilde yerleştirilmesi gerekir. Bu tezde, biz esnek optik ağlarda her talep için yönlendirme, spektrum tahsisi ve yenileyicilerin yerleştirme problemlerinin birlikte çözümüyle ilgilendik. Amacımız her talep için yönlendirme, spektrum tahsisini en az sayıda yenileyiciler grubunu topolojinin düğüm noktalarına yerleştirerek yapmak. Öncelikle, bu sorun için yeni bir karma tamsayı doğrusal programlama formülasyonu önerdik. Ancak, bu formülasyon büyük ağlar için pratik olmadığı için, bir ayrılmış sezgisel algoritma önerdik. Bu sezgisel algoritmada, ana problemi iki faza böldük. İlk fazda, biz ağın tam dalga boyu dönüştürebildiğini varsayarak, yenileyicilerin yerlerini belirledik ve talepler için yönlendirme yaptık. Sonra, ikinci aşamada her talep için spektrum tahsisini yaptık. Ayrılmış model, makul büyüklükte optik ağlar için RSA-RP sorunu çözmek için kullanılabilir. Biz ayrılmış modelin, NSFNET topoloji için tüm vakaların % 92'de ve Deutsche Telekom topoloji için tüm vakaların % 99'da ideal çözümleri bulduğunu gösterdik. Ayrıca yenileyici gruplarının yerlerinin önemli ölçüde düğüm derecesi ve bitişik bağlantı uzunlukları gibi ağ parametrelerine bağlı olduğunu gösterdik.

*Anahtar sözcükler*: Esnek Optik Ağlar, Yenileyici Yerleştirilmesi, Yönlendirme, Spektrum Tahsisi.

# Acknowledgement

I will always be indebted to my advisor Prof. Dr. Ezhan Karaşan for his guidance through out my master courses, my career and especially in this thesis. He greatly enriched my knowledge with his exceptional insights into network and communication areas. His suggestions, comments and additional guidance were invaluable to the completion of this work.

I would like to thank to the members of my thesis jury, Assist. Prof. Sinan Gezici and Assoc. Prof. Dr. Oya Ekin Karaşan for reviewing this dissertation and providing helpful feedback.

Financial support of The Scientific and Technological Research Council of Turkey (TUBITAK) for the Graduate Study Scholarship Program is gratefully acknowledged.

Finally, I would like to express my thanks to my family, who have always supported me with the choices I have made.

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# Chapter 1

# Introduction

As bandwidth-hungry applications such as high-definition video distribution and e-science are becoming widely adopted, the network traffic demand has been growing exponentially. Figure 1.1 reveals that IP traffic is expected to grow at a compound annual growth rate (CAGR) of 23% from 2012 to 2017. In other words, global IP traffic will increase threefold over the next 5 years [1]. However, the physical capacity of the optical fiber is rapidly approaching its maximum limit [2]. Consequently, optical spectrum must be used efficiently to meet this tremendous growth.

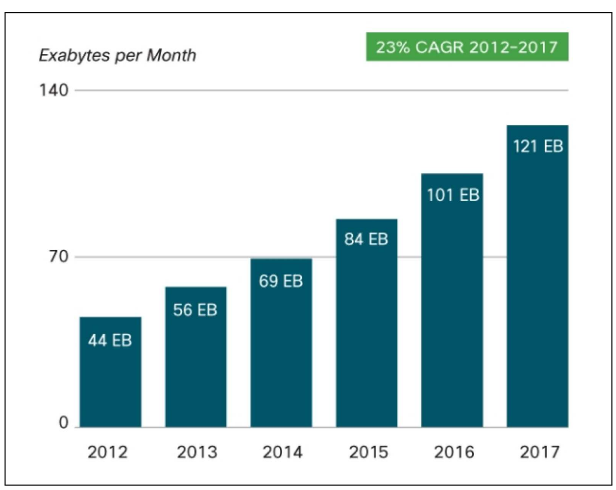



Figure 1.1: Cisco VNI forecasts 120.6 exabytes per month of IP traffic in 2017

Wavelength-division multiplexing (WDM) has considerably boosted the network capacity by multiplexing number of wavelengths into a fiber. Moreover, by efficiently grooming low-speed traffic streams onto high-capacity optical channels, the network performance has been improved. In addition to that, lightpaths at different bit rates have been set up in order to support increasing heterogeneity. Such networks are called mixed line-rate (MLR) optical networks. However, all improvements at WDM optical networks cannot provide promising solutions to satisfy future needs of the optical networks due to some characteristic drawbacks of WDM such as rigid worst case design and mismatch of granularities . Therefore, significant breakthroughs on the spectrum utilization are needed to improve the capacity of optical networks.

Recently, researchers have introduced ways, named flexible, elastic or gridless, to efficiently utilize the available optical spectrum resources. Flexible optical networks are networks that can dynamically arrange their resources (wavelength channels, bandwidth, transmission format, data rate, etc.) in an optimum and elastic way according to the continuously varying traffic conditions and demands, by considering the quality of transmission (QoT) of both the pre-established and newly arriving connections [3]. In other words, flexible optical networks can adjust the transmission parameters to take advantage of existing channel conditions. Therefore, flexible optical networks use spectrum much more efficiently than conventional WDM networks. This is because of the fact that WDM networks are designed by considering the worst case optical path in the network. For example, modulation format and spectral width are arranged for the worst lightpath. After that, every path occupies the same spectral width, regardless of each path's distance. Apart from rigid worst case design, another limitation of network efficiency is the mismatch of granularities between the client layer and physical wavelength in WDM networks. For instance, when client traffic is so small to fill the entire capacity of a wavelength, surplus bandwidth is wasted. In contrast, if connection capacity requires capacity more than one wavelength, a group of wavelengths are allocated. However, there are empty frequency bands between these wavelengths to demultiplex them. These bands cannot be used actively by connection requests in communication so they are wasted. Figure 1.2 shows the advantages of the flexible optical networks over conventional fixed grid optical networks. For example, the fixed grid system can support 400Gb/s data rate by only demultiplexing it to smaller data rates like 100GB/s or 40GB/s. Therefore, more spectrum is used than one contiguous spectrum. Moreover, flexible optical networks have tighter channel spacing as shown in Figure 1.2. In addition to all these advantages of flexible optical networks, it can also dynamically change spectrum according to needs of the demands. In other words, allocated bandwidth can expand or contract in order to satisfy needs of the demand. Furthermore, elastic optical networks can adjust modulation type properly to have longer optical reach or use spectrum more efficiently. For instance, if the lightpath is short, it uses a high-order modulation format, occupying less spectrum. However, if the lightpath is long, it uses a small-order modulation format, occupying more spectrum but more suitable for low optical signal to noise ratio (OSNR) resulting from long transmission distance.

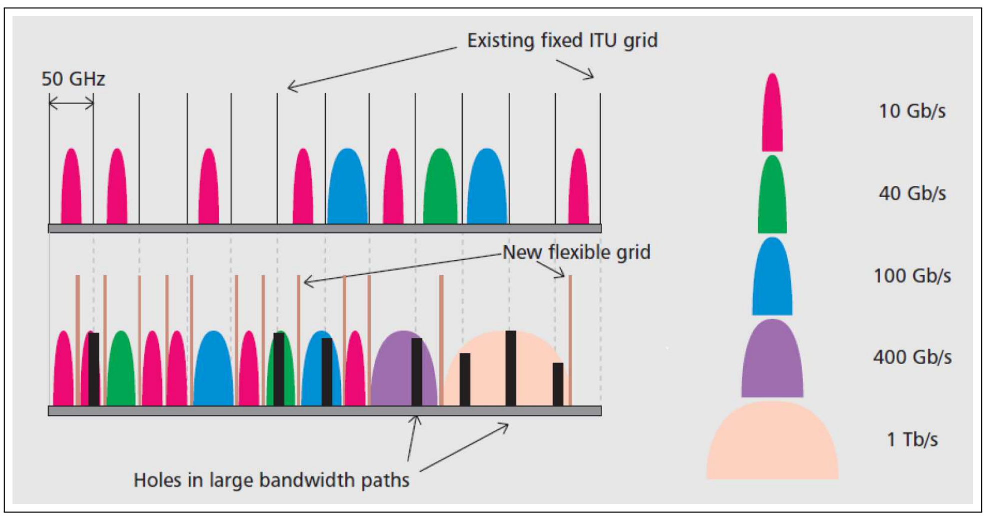



Figure 1.2: Flexible grid versus fixed grid (for a given modulation format) [4]

After introduction of the flexible optical networks, researchers have proposed a novel network architecture, spectrum-sliced elastic optical path network (SLICE) as shown in Figure 1.3. SLICE provides contiguous concatenation of the optical spectrum that allows creation of custom-size bandwidth. SLICE divides the optical spectrum into $|S|$ numbers of slots. Each slot has fixed size bandwidth, called the slot width. Contrary to the rigid bandwidth of the conventional fixed-bandwidth optical path, an optical path in SLICE expands and contracts according to the traffic volume and user request, if necessary. Consequently, spectrum is utilized efficiently by the help of the elastic bandwidth variation. In addition to efficiency, elastic bandwidth variation provides advantages for path restoration. A failed optical path cannot be recovered unless the available bandwidth on the detour route equals or exceeds the original path bandwidth when link failure occurs in conventional fixed-bandwidth optical networks. On the other hand, the bandwidth of the failed working optical path is squeezed to ensure the minimum connectivity in SLICE, when the available bandwidth on the detour route is not sufficient [5]. The SLICE concept for building flexible optical networks is used in this thesis.

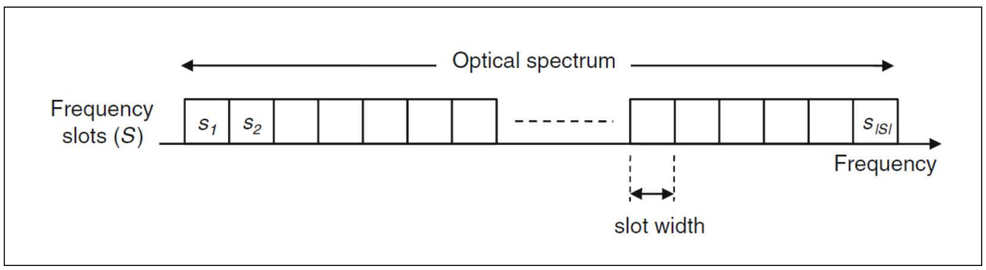



Figure 1.3: Optical spectrum divided into frequency slots [6]

In addition to efficient usage of the optical spectrum, the degradation of the optical signal is another problem. Optical signal deteriorates due to transmission impairments. The amount of the degradation depends on the length of the fiber optic cable, number of hops on route of the signal and other factors. These impairments change structure of the optical signal so that receiver side cannot decode signal properly. Consequently, the distance which optical signal can propagate while satisfying the quality of transmission (QoT) requirements is limited. This distance is called the optical reach. Beyond this length, the optical signals are degraded so that bit error rate (BER) becomes too high to maintain healthy communication. In such cases the optical signal must be regenerated to increase the transmission range of the optical network. However, regenerators are expensive devices and operating expense of them is also considerably high. Therefore, they should be placed carefully to reduce network cost.

The problem studied in this thesis is joint routing, spectrum allocation and regenerator placement (RSA-RP) for flexible optical networks under static demand scheme and signal regeneration, spectrum contiguity, and spectrum continuity and slice capacity constraints. In the RSA problem, each traffic demand is mapped onto an optical lightpath which is routed over a subset of links interconnecting the source and destination nodes on each link along the optical path and sufficient spectrum is allocated for the lightpaths depending on the bandwidth requirement of the traffic demand. On the other hand, the RP problem determines the locations of the minimum number of regenerators to be placed in the optical network in order to guarantee that the optical reach constraint is satisfied by all lihtpaths. We use SLICE as the flexible network architecture. Signal regeneration constraints impose that total traveled distance by optical signal before destination node or next regenerator site must be lower than the optical reach. Consequently, it must be regenerated at some intermediate nodes. Spectrum contiguity enforces to assign contiguous spectrum to each demand. Spectrum continuity forces that same portion of the spectrum must be allocated to the demand on each link traveled by it. Slice capacity imposes to allocate each slice in each network link to one demand at most. Our objective is to minimize the number of regenerator sites.

Firstly, we introduce a novel mixed integer linear programming (MILP) formulation for the joint RSA-RP problem. However, this formulation is not practical for large networks. Therefore, we propose a decoupled heuristic algorithm. In this heuristic, RSA-RP problem is decomposed into two phases. Firstly, routing and regenerator placement problems are solved simultaneously. Then, second part assigns slices to each demand for given regenerators locations and optical paths. We propose MILP and integer linear programming (ILP) models for first and second parts, respectively. Decoupled model finds the optimum solution for the joint problem, if the solution obtained in the first part gives a feasible solution in the second part. This is due to fact that the solution of the first part is actually a lower bound for the joint MILP formulation of the problem. Our aim is to find the minimum number of regenerator sites. In order to evaluate the performance of the proposed decoupled model, we use two optical network topologies: 14-node

NSFNET topology [7] and 14-node Deutsche Telecom topology [8]. Our results show that the decoupled model can find optimum solutions for 92% of all cases considered for the NSFNET topology and 99% of all cases considered for the Deutsche Telecom topology.

The remainder of the thesis is organized as follows. In Chapter 2, related works on routing and wavelength assignment, regenerator placement problem and flexible optical networks are examined. In Chapter 3, the MILP models for the joint and decoupled RSA-RP problem will be presented. The performance of the decoupled model is evaluated through numerical studies in Chapter 4. Finally, the conclusions will be drawn in Chapter 5.

# Chapter 2

# Literature Review

This chapter presents a brief introduction to routing and wavelength assignment and regenerator placement problems, and provides the motivation with some of the related work on flexible optical networks .

## 2.1 Routing and Wavelength Assignment Problem

The problem of providing routes to the lightpath requests and assigning a wavelength on each of the links along the path in optical networks is called the routing and wavelength assignment (RWA) problem. RWA is one of the main problems in WDM optical networks. Hence, researchers investigate many approaches to solve this problem optimally. These approaches consider distinct wavelength conversion capabilities, traffic types and objectives.

- **Wavelength Conversion Capability**

  WDM networks can be classified into three types of networks in terms of wavelength conversion capability.

  **Networks with full wavelength conversion capability:** Wavelength converter can convert an arriving wavelength on a link into another wavelength at an intermediate node before forwarding it on the next link. If every node of the network has wavelength converter, this type of network is called wavelength-convertible network with full wavelength conversion capability. These types of networks, where the only limiting factor is the number of available wavelengths on each link, are similar to circuit-switched networks. Since wavelength assignment is not an issue anymore, only the routing problem is considered [9].

**Networks with wavelength continuity constraint:** In these optical networks, connection must satisfy the wavelength continuity constraint which means that the connection must have the same wavelength on all links along its path. A variety of heuristics has been proposed to assign wavelengths to lightpaths in the literature. In [9], ten different heuristics are compared from performance and complexity standpoint. Some of these heuristics are First-Fit, Random, Least-Used and Most-Used Wavelength Assignment. Most-Used Wavelength Assignment outperforms other algorithms in terms of blocking probability. Although First-Fit is not as good as Most-Used, its communication and computational costs are less than Most-Used. We can see that each algorithm is good at different points but not sufficiently good other points. There is always trade off between computational complexity and network performance. In other words, algorithms which have best performance generally require much more computational power and information related to network.

**Networks with sparse wavelength conversion capability:** In some optical networks, wavelength converters are placed sparsely but strategically. Therefore, the number of wavelength converters may decrease significantly, which results in substantial cost savings.

- **Traffic Type**

  WDM networks can be classified into two types of networks depending on whether lifetime of the lightpath demands is permanent or not.

  **Static Lightpath Demands (SLD):** In the static traffic case, entire connection requests are known in advance and are permanent. Hence, the problem is to build up appropriate lightpath for each demand while minimizing the number of wavelengths or number of fibers in the network. Alternately, given a fixed number of wavelengths or fibers, one may try to minimize blocking probability [9].

**Dynamic Lightpath Demands (DLD):** In the dynamic traffic case lightpaths dynamically enter and leave the networks after a finite connection holding duration. Dynamic lightpath demands can be decomposed into two variants, Scheduled Lightpath Demands (SLD) and Ad-Hoc Lightpath Demands (ALD) [10]. In SLD, the set up and holding times of the connections are known beforehand. In ALD, on the other hand, the connection requests arrive and depart in a random fashion or follow a certain pattern. Therefore, the aim in ALD is to minimize the amount of connection blocking. Due to the challenging nature of the DLD, heuristic methods are commonly utilized rather than exact ones. Moreover, RWA generally is separated into two subproblems, routing and wavelength assignment. Many efficient routing methods have been proposed [11], [12], [13]. Also, a number of heuristics have been suggested in order to solve the wavelength assignment problem [14], [15], [16].

- **Objective**

  Researchers pay attention to different objectives. Some of them try to minimize the total number of utilized wavelengths, after all lightpath requests are provisioned over the network. Some works focus on reducing probability of the blocking or equivalently maximizing the number of carried lightpath requests.

## 2.2 Regenerator Placement Problem in WDM Networks

In transparent WDM networks, data is transmitted from its source to its destination in optical form, without experiencing any optical-to-electrical and electrical to optical conversion. Transparent optical channel should allow end to end communication regardless of bit rates, signal formats and modulation type. Nevertheless, optical signals encounter some impairments as they are switched, multiplexed and amplified along its route during propagation. These impairments can be classified into linear and nonlinear. Some of the important linear impairments are fiber attenuation, component insertion loss, amplifier spontaneous emission (ASE) noise, chromatic dispersion (CD) (or group velocity dispersion (GVD)), polarization mode dispersion (PMD). Some of the important non-linear impairments are self-phase modulation (SPM), cross phase modulation (XPM), four wave mixing (FWM), stimulated brillouin scattering (SBS), and stimulated Raman scattering (SRS) [16]. Due to these impairments, the distance which an optical signal can propagate while satisfying the quality of transmission (QoT) requirements is limited. This distance is called the optical reach. Consequently, an optical signal has to be regenerated in order to go beyond the optical reach. Regenerators are used to regenerate the optical signal by converting it to an electrical signal,regenerating the electrical signal and then converting it back to an optical signal. This process is called the optical-electrical-optical (OEO) conversion. OEO conversion enables the optical signal to reach long distances; however this process is quite expensive due to power consumption, heat dissipation, high equipment and maintenance costs of the regenerators. Therefore, opaque networks that contain signal regenerators at every node are not economically viable. Hence, [17] gives an alternative to fully transparent and fully opaque networks which is called translucent optical networks. In translucent optical networks, regenerators are located sparsely but strategically so the optimum balance between network design cost and service provisioning performance is established by using fewer signal regenerators than opaque optical networks. In addition to regeneration of the optical signals, regenerators also have wavelength conversion capability. Therefore, they can effectively reduce wavelength collisions, thereby improving wavelength resource utilization.

The aim of the network designers is to minimize the number of regenerator used in a network because of the capital expenditures(CAPEX) and operating expense (OPEX) of the regenerators. This problem is called in literature as Regenerator Placement Problem (RPP). This problem has been widely studied in the literature.

In [18], RPP is studied. The authors try to minimize the number of regenerator sites by guaranteeing a path between every pair of nodes. This problem is formulated as a static lightpath establishment (SLE) problem under the signal quality constraints. By using distance-based regeneration, the authors evaluate the signal quality as a criterion for determining the regeneration sites. This perspective is useful when the details of the fiber layout are not provided at the initial stages of the design. Therefore, we also use this approach in this thesis. The authors also prove that this problem is NP-complete. RPP is formulated as the problem of computing the minimum connected dominating set in labeled graphs (LCDS). They compare this heuristic approach with integer linear programming (ILP) solution of the problem. Contrary to the existing literature, writers consider edge-disjointness among path segments in their heuristics. In this paper, only routing and regenerator placement problems are considered. They assume that enough resources exist to assign wavelength to each demand after routing and regenerator placement problems are solved.

The problem of placing the minimum number of regenerators in WDM networks to accommodate all requests with the consideration of physical impairments is studied in [19]. The authors consider two main linear impairments, PMD, and ASE, since they can be practically applied to constrain optical transmission. They firstly establish an ILP formulation of the problem to obtain the optimal solution and to compare it with a proposed heuristic algorithm. Although, optimal solution can be attained with ILP, it takes long time to compute. Therefore, they suggest a fast heuristic algorithm. This algorithm is especially suitable for large networks with large demand sets. Finally, they investigate the effect of their heuristic in terms of the blocking probability of the connection requests and they conclude that regenerator placement significantly reduce the blocking probability. Even though, many aspects of the regenerator placement are studied in this paper, the authors do not examine the wavelength conversion capability of the regenerators.

Yang and Ramamurthy address the problem of survivable lightpath provisioning in WDM mesh networks with shared path protection and sparse OEO regeneration in [20]. This problem is formulated into a SLE problem under the signal quality constraints, the wavelength continuity constraints, and the path diversity constraints. An ILP based solution approach is used to find optimal solutions. However, it is not practical for large scaled networks. Hence, the authors propose a local optimization heuristic approach and a tabu search heuristic approach to solve this problem for large mesh networks. One of the different approaches in this paper is that with given regenerator placement, the authors try to minimize amount of active regenerator modules. In other words, number of regenerator modules is fixed at each node and the aim is to minimize actively used regenerators. Also, wavelength conversion capability of the regenerators is not neglected. Moreover, the authors assume, as in the [18], that PMD and ASE are enough to consider impairments. In addition to minimize the number of active regenerator modules, the authors also try obtain resource-efficient provisioning solutions by maximizing resource sharing. Suggested heuristics achieve a high level of resource-sharing rates (over 60% for OEO modules and over 30% for wavelength links).

In [21], the authors propose four regenerator placement algorithms based on network topology and traffic prediction. Then, given sparse regenerator placement, they address the problem of wavelength routing by incorporating two regenerator allocation strategies with heuristic wavelength routing algorithms. They consider wavelength routing under the DLE scheme and try to minimize blocking probability. They use the bit error rate (BER) to evaluate the signal quality and as a criterion for generating the regeneration demands. Therefore, a number LNmax, which denotes that a transparent optical signal can traverse at most LNmax links without having its BER exceed the BER threshold, is determined for each topology. LNmax is used in heuristic wavelength routing algorithms in order to assign route and wavelength to each demand without violating physical impairment constraints. The authors also show that compared to their opaque counterparts, translucent networks with a slightly compromised performance in terms of blocking probability save up to 76% and 88% network cost for regenerators under light and heavy traffic loads, respectively. This indicates once again why translucent networks are one of the most promising solutions to reduce CAPEX and OPEX of the optical networks.

Ezzahdi et al. [22] present a heuristic for routing and wavelength assignment with regenerator placement taking into account physical layer impairments. The suggested algorithm, LERP, minimizes the lightpath demands rejection ratio and the number of required regenerators under the SLE scheme. The authors develop in their previous work a BER-Predictor tool which is used to predict, for any lightpath, the BER value at intermediates nodes. By using this tool, they evaluate the signal quality as a criterion for generating the regeneration site. BER-Predictor takes into account the simultaneous effect of the four impairments, namely, CD, PMD, ASE and non-linear phase shift. In addition to that, RWA and RPP are not easily solved concurrently. Therefore, LERP decomposes problem into two parts, RWA and RPP. Firstly, RWA is performed. Then, regenerator sites are determined. As we have done in this thesis, this work tries to minimize the number of regenerator sites rather than amount of regenerator modules at nodes. After, regenerator sites are minimized, they assume that sufficient regenerator modules are located at each regenerator site.

Klinkowski et al. [23] investigate the offline problem of RWA and RP in translucent networks, by minimizing the lightpath blocking and number of regenerator module. The authors present two variants of the problem, which correspond to two different types of QoT estimators, called linear and nonlinear. In the linear QoT, the effects of the nonlinear impairments are overestimated and accumulated to the rest of the impairments in the QoT calculation. As a result, the QoT estimation of a lightpath solely depends on its route. In a nonlinear QoT, nonlinear impairments like crosstalk or cross-phase modulation, which account for the interferences from neighboring lightpaths in the network, are explicitly computed. Then, the QoT estimated for a lightpath depends on the routes of other lightpaths in the network. For the linear case, the authors suggest an ILP model and two heuristics, LS and Three-Step Heuristic. For the nonlinear case, the authors propose a heuristic iterative regenerator placement algorithm (IRP). Both Three-Step Heuristic and IRP are designed to guarantee no lightpath blocking due to signal degradation and wavelength conversion requirements. The authors compare LS and Three-Step Heuristic with LERP, algorithm in [15], in terms of computation time, blocking probability and average number of utilized regenerator modules. These algorithms perform much better than LERP in every aspect. As a contribution of this paper, the relation between the number of regenerator modules and network size is also examined. Same topologies are tested after lengths of their links are multiplied by constant numbers. So the authors obtain the conclusion that as network sizes become larger, the number of required regenerator modules also increases inevitably.

First studies on the impairment-aware routing and wavelength assignment (IA-RWA) problem for translucent optical networks propose to divide the optical core network into several islands of transparency or optically transparent domains. An island consists of a part of the physical topology in which any lightpath can be established without intermediate signal regeneration. If a connection traverses several islands, the island boundary nodes carry out the signal regeneration. Karasan and Arisoylu address the transparent domain partition by employing an ILP model and heuristic to minimize the number of total divided transparent islands in [24]. The authors show that the regenerator cost is dramatically reduced in translucent networks compared to opaque networks.

Savasini et al. [25] study the problem of minimizing the number of nodes equipped with signal regeneration while promising an arbitrary degree of end-to-end connectivity in the optical networks. Therefore, the authors propose a two-step algorithm that is compared to a coverage algorithm for mobile and ad hoc wireless networks. This heuristic offers a substantial efficiency improvement (up to 40% in topologies with up to 250 nodes) over the already existing decentralized algorithms, when it comes to finding sub-optimal solution to the problem of minimizing the k-connected-dominating node.

## 2.3 Routing and Spectrum Assignment Problem

The emergence of flexible optical networking introduces new challenges for the design of the optical networks. These challenges can be decomposed into two titles:

- **Routing and Spectrum Assignment:** Traditional routing and wavelength assignments algorithms cannot be used anymore. This is due to the fact that, a number of contiguous slices or subcarriers are assigned rather than assigning a certain wavelength to each connection request. Consequently, new routing and spectrum allocation (RSA) algorithms should be studied. These algorithms have to consider four constraints:

  1. **Signal Regeneration:** Each lightpath exceeding the optical reach must be regenerated.
  2. **Spectrum Contiguity (SCG):** Contiguous portion of the frequency spectrum must be assigned to each demand.
  3. **Spectrum Continuity (SC):** In networks without the spectrum conversion capability, same spectrum segment must be assigned to each demand on each link traveled by it. This constraint is similar to wavelength continuity constraint.
  4. **Slice Capacity:** Each slice in each network link can be allocated to one demand at most.

- **Modulation Level Selection:** Another challenge is the selection of the appropriate modulation constellation for each demand in order to satisfy the required QoS. This selection mainly depends on the required bit-rate of the traffic demand, distance between source and destination nodes and how many hops are on the lightpath.

We now review the literature to understand the RSA problem more deeply.

Duran et al. address the routing and spectrum assignment (RSA) problem in flexible optical networks under the dynamic traffic in [26]. Firstly, the authors present the adaptation of four classic wavelength assignment heuristics, First-Fit, Random, Most-Used and Least-Used, to this new problem. Then, they propose a new method, called LUSF as a spectrum assignment heuristic algorithm that jointly solves the routing and the spectrum assignment subproblems using an exhaustive search. This method is called AUR-ESS. Their novel algorithms perform much better than classical ones in terms of blocking probability. They use SLICE as the flexible network architecture. Spectrum assignment problem is solved under spectrum contiguity, spectrum continuity and slice capacity constraints.

Klinkowski and Krzysztof investigate the problem of off-line routing and spectrum allocation in fexible grid optical networks with dedicated path protection in [27]. They formulate the problem as an ILP problem and a novel AFADPP heuristic algorithm which provides near optimal solutions for larger networks. Objective of the ILP and AFADPP is to minimize the total width of spectrum. AFADPP heuristic algorithm is compared with other algorithms in the literature. Simulations show that AFADPP surpass other algorithms in terms of the number of slices, required in the network and it provides results close to the optimal ones. Klinkowski et al. use the concept of channel while proposing the ILP formulation. Each channel consists of a subset of adjacent slices, and the RSA problem consists in finding a route and assigning a channel to a demand. Therefore, both ILP formulation is solved easier and spectrum contiguity constraint is not involved into constraints of ILP. They do not employ different modulation levels by considering the condition of optical channels. Therefore, allocated bandwidth to demands does not change with their path's distance or the number of transmitted hop nodes.

Christodoulopoulos et al. [28] investigate the routing, modulation level and spectrum allocation (RMLSA) problem in OFDM-based elastic optical networks. This problem is formulated into a static lightpath establishment (SLE). The authors propose ILP formulation of the problem. They decompose (RMLSA) into two part (RML+SA) and formulate each part as ILP. They also suggest and a sequential heuristic algorithm combined with appropriate ordering policies. They compare these algorithms under different topologies in terms of spectrum utilization and running times. The proposed sequential heuristic with an appropriate ordering discipline can give close to optimal solutions in low running times. The authors also examine benefits of the OFDM-based networks over fixed-grid networks. Results show that OFDM-based networks save more than 720GHz spectrum with the help of flexible spectrum allocation and adaptive modulation levels. These results indicate one more time that flexible optical networks offer a promising solution for future high capacity transport networks. NP-completeness of the (RMLSA) problem is also proved.

Klinkowski et al [29] formulate a Multi-Hour Routing and Spectrum Allocation (MHRSA) optimization problem and solve it with the help of both Integer Linear Programming (ILP) and efficient BRKGA-based heuristic algorithm by minimizing un-served traffic rate. Three Spectrum Allocation (SA) schemes of different grades of flexibility are proposed. They are Fixed, Semi-Elastic and Elastic schemes. Elastic scheme is also divided into Expansion/Reduction and Reallocation schemes. Simulation results indicate that BRKGA-based heuristic algorithm provides results close to the optimal ones in more than 95% of the instances. When SA schemes are compared in terms of un-served traffic rate, Expansion/Reduction and Reallocation schemes outperform. Elastic scheme allows spectrum sharing among connections which enables the statistical multiplexing of traffic over the same network. However, performance difference between Expansion/Reduction and Reallocation schemes is lower than 5%. The Expansion/Reduction approach should involve lower hardware and control plane complexity with respect to Reallocation. Consequently, the performance tradeoff of spectrum Expansion/Reduction is low and it can be considered as an attractive approach for elastic SA. Even though modulation level selection is not considered, un-served traffic rate is considerably reduced by means of adaptive spectrum allocation.

In the research done by Kozicki et al in [30], the concept and experimental realization of distance-adaptive spectrum allocation schemes are investigated. They adjust the modulation format, cross-connection bandwidth and the number of OFDM subcarriers to optimize performance with respect to transmission distance. The authors show that distance-adaptive spectrum allocation schemes reduce required spectral resources by more than 60% when compared to the traditional traffic allocation scheme based on ITU-T grid. They also experimentally show the effectiveness of distance-adaptive spectrum allocation schemes. Furthermore, in their experiments, they prove applicability of these schemes in optical networks based on Bandwidth-Variable Wavelength Cross-Connects (BV-WXC).

Different from most of the works in literature, the spectrum is represented by segments rather than fixed slices in the research done by Wan et al. [31]. Hence this method can also support fully gridless networks. Segment representation of spectrum is new concept in literature. It simplifies formulation of the RSA problem to same extent. Wan et al. study the dynamic complete RSA problem under transmission distance constraint. They formulate RSA problem that jointly takes into consideration the subproblems of signal format selection, path routing, and spectrum segment assignment. Due to changeable modulation level, RSA problem can only be formulated by using a nonlinear programing model. Consequently, they decompose this nonlinear model into three parts, selecting the modulation format, solving the linear basic RSA problem, and checking the transmission distances. The authors suggest heuristics to solve linear basic RSA. Simulations indicate that variable modulation level improves significantly performance of the network in terms of blocking probability.

Velasco et al. introduce the concept of channels for the representation of contiguous spectral resources in [6]. Hence, the contiguity constraint is handled implicitly. They show that the use of a pre-computed set of channels allows considerably reducing the problem complexity. The authors address an off-line RSA problem in which enough spectrum needs to be allocated for each demand of a given traffic matrix. They formulate novel ILP formulations of RSA that are based on the assignment of channels. The evaluation results reveal that the proposed approach allows solving the RSA problem much more efficiently than previously proposed ILP based methods. This is due to fact that channel based ILP formulations have considerably fewer variables. The authors also suggest relaxed RSA formulations. We also propose similar relaxations to solve the regenerator placement problem.

Regenerators can also change modulation level of the optical signal during regeneration. Yang et al. use this property of the regenerators in [32]. They study the impairment-aware dynamic routing and subcarrier allocation problem in translucent SLICE networks. They propose an impairment-aware routing algorithm that tries to balance traffic flows evenly across the network to reduce the blocking probability. The authors consider three cases. In the first case, same modulation level is selected for each connection and it does not change along path. In the second case, one of the two modulation levels is selected for each connection and does not change along the path. In the third case, one of the two modulation levels is selected for each connection and can be changed during regeneration at the regenerator nodes on the path. For all cases, the spectrum conversion capability of OEO regenerator nodes is also considered. The simulation results indicate that the last case substantially outperforms first two cases with respect to blocking probability. Hence, it can be again noticed that in flexible optical networks, selecting the modulation level of the optical signal by considering the channel condition significantly improves network performance.

## 2.4 Regenerator Placement in Flexible Optical Networks

Regenerator placement in flexible optical networks is a very interesting problem due to capabilities of the regenerators. They can change modulation level, extend optical reach and reallocate spectrum. Consequently, the use of regenerators at intermediate nodes of the lightpath and the application of adequate modulation levels, according to transmission characteristics of optical links, brings significant savings in the spectrum utilization in opaque optical networks [33]. Indeed, shorter links which can support transmission with higher modulation levels will require less spectrum resources than longer links. Similar property is applicable to translucent EON. Klinkowski et al. study the effect of the use of regenerators and their placement on spectrum usage in a translucent EON in [34]. They formulate an offline problem of Spectrum Allocation with Regeneration Placement (SA-RP) and propose a heuristic algorithm to solve it. Obtained results show that the use of regenerators brings considerable savings in the utilization of spectrum resources in EON when comparing to the network with minimal regeneration capability.

## 2.5 Contributions of the Thesis

In this thesis, we simultaneously solve the RSA and RP problems in the flexible optical networks. Our aim is to minimize regenerator sites in network. Based on this literature review, it can be seen that these two problems we are considering, have been studied in the literature for conventional WDM networks. However, regenerator placement problem in flexible optical networks is a new problem, which is examined by only a few studies in the literature. To the best of our knowledge, none of the works in the literature solve RSA and RP problem concurrently. The joint consideration of these two problems has the advantage of preventing sub-optimal solutions acquired by optimizing the two problems separately. We formulate the joint RSA-RP problem as a mixed integer linear programming problem. However, this formulation is not appropriate for large networks. Consequently, we also suggest a two-step decoupled algorithm. In the first step, we first find routes and locations of regenerators assuming a full wavelength converting network. Then we allocate the spectrum to each demand in the second phase. We show that this algorithm can find optimum solutions even for large networks for the great majority of test cases considered in the thesis.

The subsequent chapters are organized as follows: In Chapter 3, we firstly propose a novel MILP formulation for the joint RSA/RP problem. Then, we introduce a two-phase heuristic algorithm which separates the joint RSA/RP problem into two-phases where routing and regenerator placement problems are solved in the first phase and the spectrum assignment problem is solved in the second phase. We then present numerical results in Chapter 4. Finally, Chapter 5 concludes the thesis.

# Chapter 3

# Joint Routing, Spectrum Allocation and Regenerator Placement Problem

In this chapter, we formulate the optimization framework for joint routing, spectrum allocation and regenerator placement (RSA-RP) problem for flexible optical networks as a mixed integer linear program (MILP). We then propose a decoupled two-phase model to solve this problem with less computational complexity.

## 3.1 Problem Overview

Most studies on routing and spectrum allocation assume that the optical medium can carry data signals without any bit error as we have seen from the literature review. However, physical impairments of the optical signal introduced by optical fibers and components, for instance power loss, noise, and dispersions, enforce fundamental constraints in flexible optical networks and must be taken into consideration in RSA problems of flexible optical networks. Due to these constraints, the maximum reach that an optical signal can travel before the signal quality degrades below a level necessary for successful communication is limited. Consequently, to overcome these impairments and to travel long distances lightpath must be recovered through 3R (reamplification, reshaping, retiming) regeneration with optical-electrical-optical (OEO) conversion. In addition to regeneration of the optical signals, regenerators can also reallocate spectrum. Therefore, they can effectively relax the spectrum continuity constraint (SC), thereby lowering blocking probability of the network and improving the spectrum utilization. Moreover, the modulation level of optical signal can be changed during regeneration by taking consideration into transmission characteristics of optical links. Hence, best modulation type is selected for each semi-lightpath[1]. Nevertheless, regenerators are costly devices. Therefore, to reduce both CAPEX and OPEX of the network, regenerators should be placed efficiently and effectively. Extensive studies reveal that even if only a few nodes are equipped with regenerators, network can achieve an acceptable performance close to that of a fully regeneration capability network [21],[24]. Consequently, regenerator placement in flexible optical network is a significant problem that should be examined in depth.

In this work, we focus on solving the routing and spectrum allocation (RSA) problem jointly with the regenerator placement (RP) problem under static demand scheme. We use SLICE as the flexible network architecture. We use fixed modulation level for all requests. We use the length of the lightpath as a criterion for regeneration. If the distance traveled by a lightpath without regeneration exceeds the optical reach distance, the demand must be regenerated at least at one of the intermediate nodes on its path. Actually regeneration depends on the underlying technology of the network system such as the type of fibers and spacing between regenerators. Therefore, distance-based regeneration is used as an approximation [35].

Given a network and demand set, our aim is to assign route and spectrum to each demand under signal regeneration, spectrum contiguity, and spectrum continuity and slice capacity constraints. The objective is to minimize number of regenerator sites. After, minimizing regenerator sites, we assume that each site

[1]A semi-lightpath is an optical signal traversing a sequence of fiber links without going through any signal regeneration and any wavelength conversion [23].

has sufficient number of regenerator modules.

## 3.2 Problem Statement

The offline RSA-RP problem can be formally stated as follows:

A flexible optical network is represented by a graph $G = (V, E)$, where the set of nodes is denoted as $V$ and the set of bidirectional fiber links connecting two nodes in $V$ is denoted as $E$. A set $D$ consists of lightpaths to be transported. Each lightpath demand $d$ is represented by a tuple $(s_d, t_d, n_d)$, where $s_d$ and $t_d$ are the source and the destination nodes respectively and $n_d$ is the requested number of slots. The objective is to make route and spectrum assignments for each demand by using the minimum number of regenerator sites.

## 3.3 Joint MILP Model of RSA-RP Problem

After this brief introduction, we propose a novel MILP formulation for this problem in this section.

### 3.3.1 Sets, Parameters and Constants

The first set corresponds to nodes denoted $V$. The nodes selected from this set will be assigned as the source-destination pair for each demand. $|V|$ corresponds to the number of nodes in the network topology.

$$V = \{v_1, v_2, \ldots, v_{|V|}\}$$

The second set corresponds to demands denoted by $D$. Each demand in this set is represented by tuple $(s_d, t_d, n_d)$.

$$D = \{d_1, d_2, \ldots, d_{|D|}\}$$

Ordered set $S$ corresponds to slots. All spectrum is divided into $|S|$ numbers of slots. These slots are assigned to each demand under spectrum contiguity, spectrum continuity, slice capacity constraints. $s_1$ and $s_{|S|}$ represents first and last slots respectively. We use slice and slot interchangeably in this thesis.

$$S = \{s_1, s_2, \ldots, s_{|S|}\}$$

Parameters used in the formulation are:

$L \equiv [L_{ij}]$ is the distance matrix among nodes, where $L_{ij}$ is the distance between node $i$ and node $j$, $\forall \{i, j\} \in E$ and $L_{ii} = 0$, $\forall i \in V$.

$\delta_{cl}^{dm}$ is the indicator function, where $c$ and $l$ are the starting slices of the demands $d$ and $m$ respectively. $n_d$ and $n_m$ are the requested number of slots of the demands $d$ and $m$, respectively. $\delta$ indicates whether allocated slices of two demands collide with each other or not.

$$\delta_{cl}^{dm} = \begin{cases} 1 & \text{if} \quad \Big((c \le l) \wedge (l < c + n_d)\Big) \vee \Big((l \le c) \wedge (c < l + n_m)\Big), \\ 0 & \quad \text{otherwise.} \end{cases}$$

$\gamma_{ij}$ is another indicator function. It shows whether or not there exists a physical link between node $i$ and node $j$.

$$\gamma_{ij} = \begin{cases} 1 & \text{if} \quad \{i, j\} \in E, \\ 0 & \quad \text{otherwise.} \end{cases}$$

The constant $R_{max}$ indicates the optical reach of the signal.

### 3.3.2 Decision Variables

In the formulation, we need variables to decide which links are used by demand $d$ and which part of the spectrum is allocated to demand $d$. Therefore, we introduce decision variable $x_{ij}^{dl}$.

$$x_{ij}^{dl} = \begin{cases} 1 & \text{if} \quad \text{starting slot of the demand } d \text{ is } l \text{ on link } \{i, j\}, \\ 0 & \quad \text{otherwise.} \end{cases}$$

$w_i^d$ denotes path length for the demand $d$ out of node $i$. In other words, unless $i$ is a regenerator node, it indicates total traveled distance by demand $d$ from its source node or last regenerator node up to node $i$. If node $i$ is a regenerator node, $w_i^d$ is set to 0.

$y_i$ is another binary decision variable which shows whether node $i$ is a regenerator node or not.

$$y_i = \begin{cases} 1 & \text{if} \quad \text{node } i \text{ is} \quad \text{regenerator site,} \\ 0 & \quad \text{otherwise.} \end{cases}$$

### 3.3.3 Constraints

RSA-RP problem consists of many constraints, which can be listed as flow conservation, signal regeneration, spectrum and capacity constraints.

$$\sum_{l,j:\gamma_{ij}=1} x_{ij}^{dl} - \sum_{l,j:\gamma_{ji}=1} x_{ji}^{dl} = \begin{cases} 1 & \text{if } s_d = i, \\ -1 & \text{if } t_d = i, \\ 0 & \quad \text{otherwise.} \end{cases} \qquad \forall d \in D,\ \forall i \in V \tag{3.1}$$

(3.1) is the flow conservation constraint. (3.2), (3.3), (3.4) are signal regeneration constraints.

$$\sum_{i:i=s_d} w_i^d = 0 \quad \forall d \in D \tag{3.2}$$

(3.2) ensures that $w_i^d$ is equal to zero at source node $s_d$.

$$w_j^d \geq w_i^d + L_{ij} * \sum_c x_{ij}^{dc} - R_{max} * (1 - \sum_c x_{ij}^{dc}) - (R_{max} + L_{ij}) * y_j$$
$$\forall d \in D, \forall \{i,j\} \in E : i \neq t_d \text{ and } j \neq t_d \tag{3.3}$$

(3.3) determines the value of $w_j^d$ after node $i$. If demand $d$ is at node $i$ and then goes to regenerator node $j$, $w_j^d$ becomes zero. However, if node $j$ is not a regenerator node, $w_j^d$ must be $w_i^d + L_{ij}$ which traveled distance from last regenerator node or source node. Although we do not strictly impose equalities, optimization framework selects values 0 or $w_i^d + L_{ij}$ for $w_j^d$.

$$w_i^d + \sum_{c,j} x_{ij}^{dc} * L_{ij} \leq R_{max} \quad \forall d \in D, \forall i \in V \backslash \{t_d\} \tag{3.4}$$

(3.4) imposes that all $w_i^d$ variables must not exceed $R_{max}$.

$$\sum_{m:m \neq d} \sum_{c:\delta_{cl}^{dm} \neq 0} (x_{ij}^{mc} + x_{ji}^{mc}) \leq n_d * \big(1 - (x_{ij}^{dl} + x_{ji}^{dl})\big)$$
$$\forall d \in D, \forall \{i,j\} \in E, \forall l \in S \tag{3.5}$$

(3.5) is the spectrum capacity constraint. It imposes that each slice in each network link can be allocated to one demand at most.

$$-|S| * y_i \leq \sum_{c,j} c * x_{ji}^{dc} - \sum_{l,j} l * x_{ij}^{dl} \leq |S| * y_i$$
$$\forall d \in D, \forall i \in V : i \neq s_d \text{ and } i \neq t_d \tag{3.6}$$

(3.6) relaxes spectrum continuity (SC) constraint if node $i$ has regenerator. If node $i$ does not have a regenerator, it imposes, the SC constraint.

We do not explicitly impose spectrum contiguity (SCG) constraint. However, we allocate implicitly contiguous slices for each demand by the help of (3.5) and (3.6).

$$\sum_{c} (x_{ij}^{dc} + x_{ji}^{dc}) \leq \gamma_{ij} \quad \forall d \in D, \forall \{i,j\} \in E \tag{3.7}$$

(3.7) enforces each demand to use each link only for one direction $i \rightarrow j$ or $j \rightarrow i$. It also allows each demand to use only one contiguous part of the spectrum on each link.

$$x_{ij}^{dl} = 0 \quad \forall \{i, j\} \in E, \forall d \in D, \forall l \in S, \text{ where } l + n_d \geq |S| + 2 \tag{3.8}$$

(3.8) sets some of $x_{ij}^{dl}$'s to zero again if last slot of demand $d$ goes beyond last slot, $s_{|S|}$.

Each regenerator site contains many regenerator modules depending on how many lightpaths pass through on it. Each demand uses one regenerator module, if it requires regeneration at that node. After some of nodes are assigned as a regenerator site, we assume that each regenerator site has enough regenerator modules. Our objective function is minimizing the total number of regenerator sites:

$$\text{Minimize} \quad \sum_{i} y_i \tag{3.9}$$

## 3.4 Decoupled RSA-RP Model

The joint MILP formulation of the RSA-RP problem has a very large size, hence this approach is not applicable in practice except for small networks. Consequently, heuristic approaches are needed for the solution of regenerator placement problem. In this section, we present a decoupled heuristic algorithm. In this heuristic, RSA-RP problem is decomposed into two phases. Routing and regenerator placement problems are solved concurrently in the first part. After placing regenerators and finding routes, second part allocates spectrum to each demand. We propose MILP and ILP models for first and second parts, respectively. If solution found in the first part gives a feasible solution in the second part, solution of the problem is the optimum solution for joint problem. This is due to the fact that the solution of the first part is actually a lower bound for the joint MILP formulation of the problem. Figure 3.1 shows a flowchart of the Decoupled Model.


Figure 3.1: Flow chart of decoupled model

### 3.4.1 Routing and Regenerator Placement Problems

In the first part of the decoupled model, we assign a route for each demand and determine regenerator sites. We formulate routing and regenerator placement as a MILP problem. Formulation of the first part contains flow conservation and signal regeneration constraints. Although variables and constraints of the first section are almost the same with the joint MILP formulation of the RSA-RP problem, it is beneficial to redefine them to emphasize difference between, the decoupled and joint MILP models for the RSA-RP problem.

#### 3.4.1.1 Variable Declarations

In the formulation, we need variable to decide which links are used by demand $d$. Therefore, we introduce decision variable $x_{ij}^d$. We also use predefined variables, $w_i^d$, $y_i$ without any modification. We also use parameters, $\delta_{cl}^{dm}$ and $\gamma_{ij}$ and constant $R_{max}$, as defined in section 3.3.1.

$$x_{ij}^d = \begin{cases} 1 \quad \text{if} \quad \text{demand } d \text{ uses link } \{i,j\}, \\ 0 \qquad \text{otherwise.} \end{cases}$$

#### 3.4.1.2 Constraints

The first part of the decoupled model contains flow conservation, signal regeneration and capacity constraints.

$$\sum_{j:\gamma_{ij}=1} x_{ij}^d - \sum_{j:\gamma_{ji}=1} x_{ji}^d = \begin{cases} 1 & \text{if } s_d = i, \\ -1 & \text{if } t_d = i, \qquad \forall d \in D, \forall i \in V \\ 0 & \quad \text{otherwise.} \end{cases} \tag{3.10}$$

(3.10) is the flow conservation constraint. (3.11), (3.12), (3.13) are signal regeneration constraints.

$$\sum_{i:i=s_d} w_i^d = 0 \quad \forall d \in D \tag{3.11}$$

(3.11) ensures that $w_i^d$ is equal to zero at source node $s_d$.

$$w_j^d \geq w_i^d + L_{ij} * x_{ij}^d - R_{max} * (1 - x_{ij}^d) - (R_{max} + L_{ij}) * y_j$$
$$\forall d \in D, \forall \{i,j\} \in E : i \neq t_d \text{ and } j \neq t_d \tag{3.12}$$

(3.12) determines the value of $w_j^d$ after node $i$. If demand $d$ is at node $i$ and then goes to regenerator node $j$, $w_j^d$ becomes zero. However, if node $j$ is not a regenerator node, $w_j^d$ must be $w_i^d + L_{ij}$ which traveled distance from last regenerator node or source node. Although we do not strictly impose equalities, optimization framework selects values 0 or $w_i^d + L_{ij}$ for $w_j^d$.

$$w_i^d + \sum_j x_{ij}^d * L_{ij} \leq R_{max} \quad \forall d \in D, \forall i \in V \backslash \{t_d\} \tag{3.13}$$

(3.13) imposes that all $w_i^d$ variables must not exceed $R_{max}$.

$$x_{ij}^d + x_{ji}^d \leq \gamma_{ij} \quad \forall d \in D, \forall \{i,j\} \in E \tag{3.14}$$

(3.14) enforces each demand to use each link only for one direction $i \to j$ or $j \to i$.

$$\sum_d n_d * (x_{ij}^d + x_{ji}^d) \leq |S| \quad \forall \{i,j\} \in E \tag{3.15}$$

(3.15) is the link capacity constraint.

$$\text{Minimize} \quad \sum_i y_i + \varepsilon * \text{ secondary objective} \tag{3.16}$$

The objective function (3.16) consists of two objectives. The first and main objective is to minimize the number of regenerator sites. Secondary objective is to find routing such that the likelihood of finding a feasible solution in the second phase is higher. We propose three different secondary objectives. The small constant $\varepsilon$ gives precedence to the first objective. Consequently, the second objective cannot change the number of regenerator sites in the optimum solution. In other words, the first objective determines the number of regenerators sites and the second objective finds best routings by using fixed number of regenerators sites which is determined in the first part. In order to define the secondary objectives, some variables must be introduced.

$uti_{ij}$ denotes utilization of the $\{i,j\}$ link. $c_{ij}$ is difference between $uti_{ij}$ and $Threshold$ on link $\{i,j\}$. $c_{ij}$ is needed to find utilization of the links which have higher utilization than $Threshold$ value, shown in (3.17). We select this ratio as $Threshold$ value since it is good $Threshold$, which ranges between 70% and 90% for tested cases, to determine highly utilized links.

$$Threshold = \frac{|S| - 3}{|S|} \tag{3.17}$$

$$a_{ij} = \begin{cases} 1 & \text{if link } \{i,j\} \text{ has utilization higher than } Threshold, \\ 0 & \quad \text{otherwise.} \end{cases}$$

$$b_{ij} = \begin{cases} 1 & \text{if link } \{i,j\} \text{ has utilization lower than } Threshold, \\ 0 & \quad \text{otherwise.} \end{cases}$$

$$uti_{ij} = \frac{1}{|S|} \sum_{d} n_d * (x_{ij}^d + x_{ji}^d) \quad \forall\{i,j\} \in E \tag{3.18}$$

(3.18) finds the value of the of utilization of link $\{i, j\}$ where $|S|$ is link capacity.

(3.19),(3.20) and (3.21) find which links have utilization higher than $Threshold$.

$$a_{ij} \geq uti_{ij} - Threshold \quad \forall\{i,j\} \in E \tag{3.19}$$

$$b_{ij} \geq Threshold - uti_{ij} \quad \forall\{i,j\} \in E \tag{3.20}$$

$$1 = a_{ij} + b_{ij} \quad \forall\{i,j\} \in E \tag{3.21}$$

$$c_{ij} \geq uti_{ij} - Threshold \quad \forall\{i,j\} \in E \tag{3.22}$$

(3.22) is constraint to find $c_{ij}$ variables.

Secondary Objectives are critical for finding solution at second part of the decoupled model. One of the most significant points in this heuristic algorithm is to find the best routing and regenerator arrangement. The secondary objectives increase the probability of finding a feasible solution at the second phase where we assign contiguous slots to each demand. We will compare the performances of different secondary objectives in Chapter 4. Now, we introduce the three secondary objectives.

1. **Minimize Total Link Utilization (MTLU):**

   (3.23) minimizes the number of regenerator sites and the total utilization of the system. $L$ indicates the total number of links. We divide total utilization by $L$ in order to give priority to the first part of the objective function.

$$\text{Minimize} \quad \sum_i y_i + \frac{1}{L} \sum_{i,j:\{i,j\} \in E} uti_{ij} \tag{3.23}$$

2. **Minimize Total Utilization of Highly Utilized Links (MTUHUL):**

   Highly Utilized Links are links which have utilization higher than threshold. *Threshold* is shown in equation (3.17). (3.24) minimizes the amount of regenerator sites and total utilization of highly utilized links. We again divide total utilization by $L$ in order to give priority to the first part of the objective function.

$$\text{Minimize} \quad \sum_i y_i + \frac{1}{L} \sum_{i,j:\{i,j\} \in E} (c_{ij} + Threshold * a_{ij}) \tag{3.24}$$

3. **Minimize Number of Highly Utilized Links (MHUL):**

   (3.25) minimize amount of regenerator sites and number of highly utilized links.

$$\text{Minimize} \quad \sum_i y_i + \frac{1}{L} \sum_{i,j:\{i,j\} \in E} a_{ij} \tag{3.25}$$

### 3.4.2 Spectrum Assignment

In this section, we present the second phase of the decoupled formulation where we assign slots to each demand. The aim of this part is to only find feasible spectrum assignment without maximizing or minimizing any thing. Before this phase, each demand is firstly decomposed into segmented demands in the intermediate step. Each segmented demand corresponds to one of the semi-lightpath of the original demand. The path of each segmented demand is a semi-lightpath of the corresponding demand. For instance in Figure 3.2, we see a demand whose source and destination nodes are 1 and 6, respectively. It passes through one regenerators site, node 3. So the lightpath of this demand, turquoise line, is divided into two segmented demands for each semi-lightpath, red and brown lines. So this demand decomposes into two segmented demands. Therefore, new demand set $D'$ is defined which contains all segmented demands. Due to the fact that each demand[2] in $D'$ does not pass through any regenerators sites, we only need to consider both spectrum contiguity and spectrum continuity for each segmented demand.

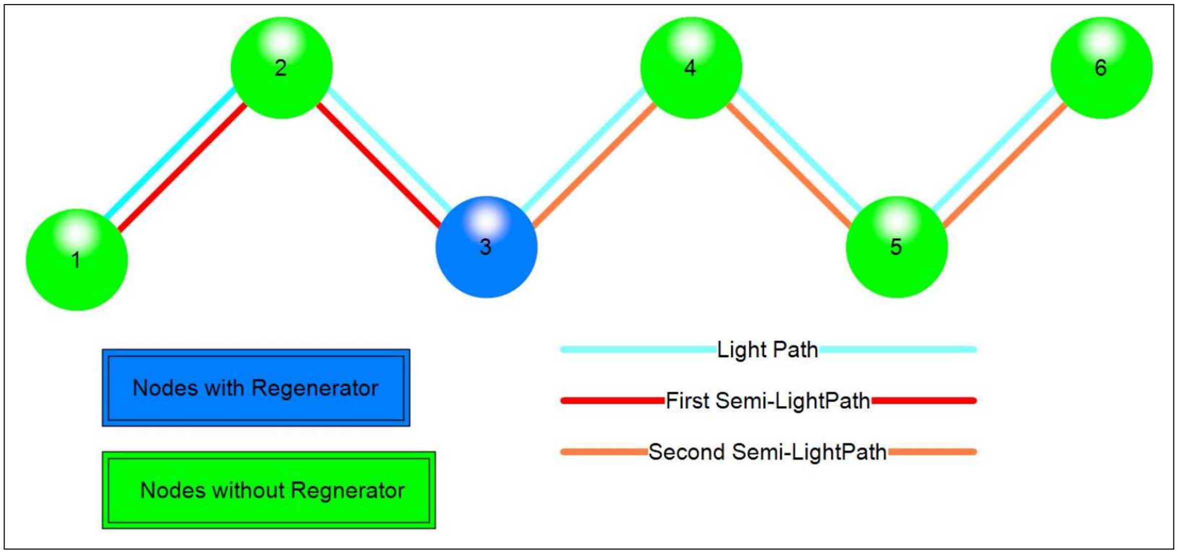



Figure 3.2: Lightpath from node 1 to node 6

Figure 3.3 reveals detailed flow chart of the decoupled model. Firstly, we find the minimum slot number for each traffic matrix by only using path flow constraints since for arbitrarily large slot number $|S|$, RSA-RP problem reduces to routing and regenerator placement problem. As a result, decoupled MILP formulation always find optimal regenerator sites. Consequently, to see real performance of the heuristic algorithm, we simulate all traffic matrices for minimum slot number. Secondly, we find regenerator sites and routes of each demand for a given slot number. Then, each demand is decomposed into segmented demands in the intermediate step. Each segmented demand corresponds to one of the semi-lightpath of the original demand. The path of each segmented demand is a semi-lightpath of the corresponding demand. Therefore, each demand in new demand set does not pass through any regenerator sites. Due to fact that each demand does not pass through any regenerator sites, we only need to consider both spectrum contiguity and spectrum continuity for each segmented demand.

[2]From now on, we use demand instead of quasi-demand for elements of set $D'$ in order to sustain fluency of the text.

After the segmentation part, for given new demands and their routes and regenerators sites, we try to allocate the required number of slots to each demand at the last part. If slot allocation can be done without violating any constraints, optimal solution is obtained. After that, if spectrum allocation gives feasible result, routing,spectrum information of each demand and regenerator sites are obtained.

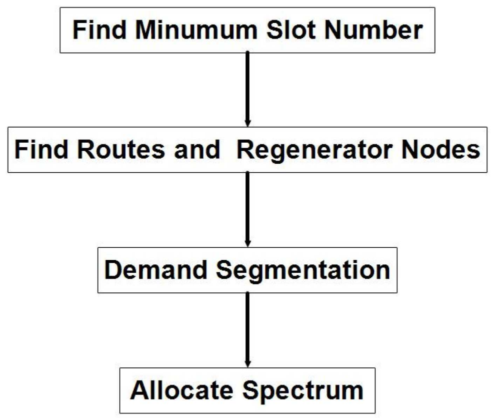



Figure 3.3: Detailed flow chart of the decoupled model

Parameters $\gamma_{ij}$ and $\delta_{cl}^{dm}$ are also used in this formulation. New parameter $Path_{ij}^{d}$ is indicator which shows whether segmented demand $d$ uses link $\{i, j\}$ or not. It is defined as

$$Path_{ij}^{d} = \begin{cases} 1 & \text{if demand } d \text{ uses link } \{i,j\}, \\ 0 & \text{otherwise.} \end{cases}$$

$x'_{dl}$ variable is defined to assign starting slot $l$ to each demand $d$.

$$x'_{dl} = \begin{cases} 1 & \text{if starting slot of the demand } d \text{ is } l, \\ 0 & \text{otherwise.} \end{cases}$$

$$\sum_{l} x'_{dl} = 1 \quad \forall d \in D' \tag{3.26}$$

(3.26) ensures to assign starting spectrum to each demand.

$$\sum_{\substack{m:m\neq d \\ Path_{ij}^{m}\neq 0}} \sum_{c:\delta_{cl}^{dm}\neq 0} x'_{mc} \leq n_d * (1 - x'_{dl})$$

$$\forall d \in D', \forall l \in S, \forall \{i,j\} \in E : Path_{ij}^{d} \neq 0 \quad (3.27)$$

(3.27) imposes that each slice in each link can be allocated to one demand at most.

$$x'_{dl} = 0 \quad \forall d \in D', \forall l \in S, \text{ where } l + n_d \geq |S| + 2 \tag{3.28}$$

(3.28) sets some of $x'_{dl}$'s to zero if the last slot of demand $d$ goes beyond the last slot, $s_{|S|}$.

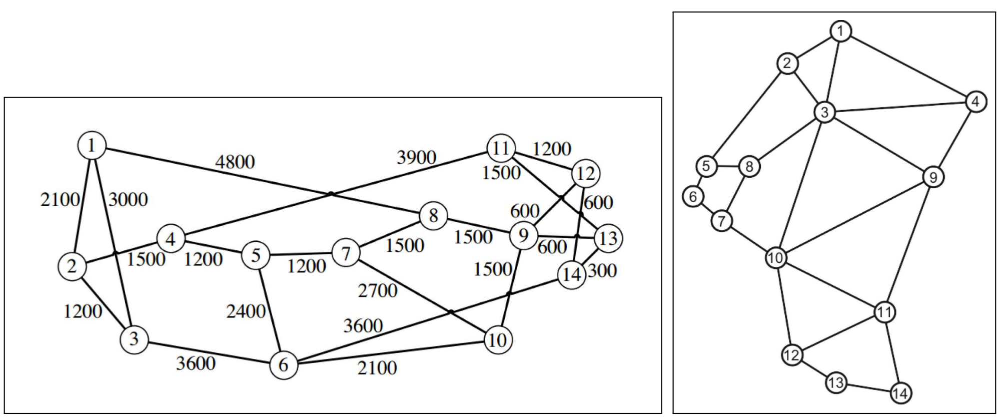
The objective of this part is to find feasible solution. Therefore, the objective can be any arbitrary function which does not have any effect on the optimum solution.

In the next chapter, we present the results of our numerical studies for the decoupled formulation conducted over two optical network topologies.

# Chapter 4

# Numerical Results

This chapter contains the results of the numerical studies conducted using the Decoupled Model. The numerical results are obtained using GAMS with Gurobi 5.5 as the solver. 14-node NSFNET topology [7] and 14-node Deutsche Telecom topology [8], depicted in Figure 4.1, are used in our numerical studies. Table 4.1 contains information on the topologies tested. Detailed information about topologies is presented in Tables 4.2 and 4.3.




(a) 14-node NSFNET Topology

(b) 14-node Deutsche Telecom Topology

Figure 4.1: Tested topologies

Table 4.1: Information on topologies tested

| | 14-node Deutsche Telecom | 14-node NSFNET |
|---|---|---|
| Total Node Number | 14 | 14 |
| Total Bidirectional Links | 23 | 23 |
| Average Distance (in km) | 186,26 | 1819,56 |
| Average Nodal Degree | 3,286 | 3,0714 |

Table 4.2: NSFNET topology link distances

| Node | Nodal Degree | Maximum | Average | Total |
|---|---|---|---|---|
| 1 | 3 | 4800 | 3300 | 9900 |
| 2 | 3 | 2100 | 1600 | 4800 |
| 3 | 3 | 3600 | 2600 | 7800 |
| 4 | 3 | 3900 | 2200 | 6600 |
| 5 | 3 | 2400 | 1600 | 4800 |
| 6 | 4 | 3600 | 3100 | 9300 |
| 7 | 3 | 2700 | 1800 | 5400 |
| 8 | 3 | 4800 | 2600 | 7800 |
| 9 | 4 | 1500 | 1050 | 4200 |
| 10 | 3 | 2700 | 2100 | 6300 |
| 11 | 3 | 3900 | 2200 | 6600 |
| 12 | 3 | 1500 | 1100 | 3300 |
| 13 | 3 | 1500 | 800 | 2400 |
| 14 | 3 | 3600 | 1500 | 4500 |

Table 4.3: Deutsche Telecom topology link distances

| Node | Nodal Degree | Maximum | Average | Total |
|---|---|---|---|---|
| 1 | 3 | 306 | 194 | 582 |
| 2 | 3 | 279 | 171,66 | 515 |
| 3 | 6 | 314 | 228 | 1368 |
| 4 | 3 | 306 | 258 | 774 |
| 5 | 3 | 279 | 171,66 | 353 |
| 6 | 2 | 41 | 39 | 78 |
| 7 | 3 | 182 | 102,33 | 307 |
| 8 | 3 | 220 | 113,66 | 341 |
| 9 | 4 | 353 | 264,5 | 1058 |
| 10 | 5 | 353 | 256 | 1280 |
| 11 | 4 | 275 | 217,25 | 869 |
| 12 | 3 | 207 | 161 | 483 |
| 13 | 2 | 146 | 116,5 | 233 |
| 14 | 2 | 181 | 163,5 | 327 |

We generate three different groups of traffic matrices for each topology. Each group consists of fifteen traffic matrices. Traffic matrices in the first, second and third groups contains of 30, 50 and 70 lightpath demands, respectively. We tested each group with different $R_{max}$ values as illustrated in Table 4.4. Each lightpath demand $d$ is represented by a tuple $(s_d, t_d, n_d)$, where $s_d$ and $t_d$ are the source and the destination nodes respectively and $n_d$ is the requested number of slots. $s_d$ and $t_d$ are selected according to a uniform distribution. $n_d$ can take equally likely values among 1, 2 and 3.

Table 4.4: $R_{max}$ values in km

| Case | 14-nodes Deutsche Telecom | 14-nodes NSFNET |
|---|---|---|
| 1 | 400 | 5000 |
| 2 | 500 | 6500 |
| 3 | 700 | 8000 |

## 4.1 Numerical Results

In this section, we present results of the numerical studies conducted over two topologies. For each given number of traffic demands, we randomly generated 15 sets of lightpaths to be established. In Tables 4.5 - 4.22, the first column shows IDs of traffic matrices. The second column indicates the minimum number of slots necessary for each traffic matrix. The consecutive three columns show regenerator sites for each applied secondary objectives in Section 3.4.1.2. These regenerator sites are obtained in the first part of the heuristic. If the solution of the first part gives a feasible solution in the second part where spectrum assignment is done, this solution is the optimum solution for the joint problem. This is due to fact that the solution of the first part is actually a lowerbound for the joint MILP formulation of the problem. Font color of the cases, which the decoupled model cannot find the optimum solution, are red. All results are presented in Tables 4.5 - 4.22.

Table 4.5: Results for NSFNET topology when $R_{max}$ is 5000 km and demand number is 30

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 9 | 6,8,11,12 | 6,8,11,13 | 6,8,11,13 |
| 2 | 10 | 1,4,6,8,11 | 1,4,6,8,11 | 1,4,6,8,11 |
| 3 | 8 | 4,6,11,13 | 1,6,11,13 | 4,6,11,12 |
| 4 | 8 | 1,3,4,6,8,11 | 1,3,4,6,8,11 | 1,3,4,6,8,11 |
| 5 | 9 | 6,7,8,11,13 | 6,7,8,11,13 | 6,7,8,11,13 |
| 6 | 8 | 1,4,6,8,11 | 1,4,6,8,11 | 1,4,6,8,11 |
| 7 | 9 | 4,6,8,11 | 4,6,8,11 | 4,6,8,11 |
| 8 | 8 | 1,3,4,6,8,10,11 | 1,3,4,6,8,10,11 | 1,3,4,6,8,10,11 |
| 9 | 9 | 4,6,7,8 | 4,6,8,10 | 4,6,7,8 |
| 10 | 7 | 1,4,6,7,8,11,12 | 1,4,6,7,8,11,14 | 1,4,6,8,10,11,14 |
| 11 | 9 | 6,8,11,13 | 6,8,11,13 | 6,8,11,13 |
| 12 | 10 | 1,4,6,8,11 | 1,4,6,8,11 | 1,4,6,8,11 |
| 13 | 8 | 4,6,11,13 | 4,6,11,12 | 4,6,11,12 |
| 14 | 8 | 1,3,4,6,8,11 | 1,3,4,6,8,11 | 1,3,4,6,8,11 |
| 15 | 8 | 1,4,6,8,11 | 1,4,6,8,11 | 1,4,6,8,11 |

Table 4.6: Results for NSFNET topology when $R_{max}$ is 6500 km and demand number is 30

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 9 | 6,8 | 6,8 | 6,9 |
| 2 | 10 | 6,9 | 6,9 | 6 |
| 3 | 8 | 1,6,9 | 1,6,8 | 1,6,8 |
| 4 | 8 | 1,4,6,8 | 1,4,6,8 | 1,4,6,9 |
| 5 | 9 | 6,8,9 | 6,8,9 | 6,7,9 |
| 6 | 8 | 1,4,6 | 1,4,6 | 1,4,6 |
| 7 | 9 | 4,6 | 4,6 | 4,6 |
| 8 | 8 | 1,3,4,6 | 1,3,4,6 | 1,3,4,6 |
| 9 | 9 | 6,8 | 6,9 | 6,8 |
| 10 | 7 | 1,6,8 | 1,3,5 | 1,3,6 |
| 11 | 9 | 6,9 | 6,8 | 6,9 |
| 12 | 10 | 6,9 | 6,9 | 6,9 |
| 13 | 8 | 1,6,8 | 1,6,8 | 1,6,8 |
| 14 | 8 | 1,4,6,8 | 1,4,6,8 | 1,4,6,9 |
| 15 | 8 | 1,4,6 | 1,4,6 | 1,4,6 |

Table 4.7: Results for NSFNET topology when $R_{max}$ is 8000 km and demand number is 30

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 9 | 0 | 0 | 0 |
| 2 | 10 | 3 | 6 | 6 |
| 3 | 8 | 6 | 3 | 3 |
| 4 | 8 | 2 | 3 | 2 |
| 5 | 9 | 6 | 6 | 3 |
| 6 | 8 | 1,6 | 1,6 | 1,6 |
| 7 | 9 | 0 | 0 | 0 |
| 8 | 8 | 4 | 4 | 2 |
| 9 | 9 | 6 | 3 | 6 |
| 10 | 7 | 8 | 8 | 8 |
| 11 | 9 | 0 | 0 | 0 |
| 12 | 10 | 3 | 6 | 6 |
| 13 | 8 | 6 | 3 | 3 |
| 14 | 8 | 2 | 3 | 2 |
| 15 | 8 | 1,6 | 1,6 | 1,6 |

Table 4.8: Results for NSFNET topology when $R_{max}$ is 5000 km and demand number is 50

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 15 | 1,4,6,8 | 1,4,6,8 | 4,6,8,11 |
| 2 | 13 | 1,4,6,7,8,11,13 | 1,4,6,7,8,11,13 | 1,4,6,7,8,11,13 |
| 3 | 13 | 1,4,6,8,11 | 1,4,6,8,11 | 1,4,6,8,11 |
| 4 | 12 | 1,4,6,8,11 | 1,4,6,8,11 | 1,4,6,8,14 |
| 5 | 13 | 1,4,6,8,11,14 | 1,4,6,8,11,12 | 1,4,6,8,11,13 |
| 6 | 12 | 1,3,4,6,8,11,13 | 1,3,4,6,8,11,12 | 1,3,4,6,8,10,11 |
| 7 | 17 | 4,6,7,8 | 4,6,7,8 | 4,6,7,8 |
| 8 | 14 | 1,4,6,11 | 1,4,6,11 | 4,6,7,8 |
| 9 | 13 | 1,4,6,8,11,14 | 1,4,6,8,11,14 | 1,4,6,8,11,12 |
| 10 | 12 | 1,3,4,6,8,10,11 | 1,3,4,6,8,10,11 | 1,3,4,6,8,10,11 |
| 11 | 14 | 1,4,6,8,10 | 4,6,8,10,11 | 1,4,6,8,10 |
| 12 | 12 | 1,4,6,8,10,11 | 1,4,6,8,11,13 | 1,4,6,8,10,11 |
| 13 | 16 | 1,6,8,11 | 4,6,8,11 | 4,6,8,11 |
| 14 | 11 | 1,4,6,7,8,11,12 | 1,4,6,7,8,11,12 | 1,4,6,7,8,11,14 |
| 15 | 12 | 1,6,8,11,14 | 1,6,8,11,14 | 1,6,8,11,14 |

Table 4.9: Results for NSFNET topology when $R_{max}$ is 6500 km and demand number is 50

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 15 | 5,8 | 5,8 | 4,6 |
| 2 | 13 | 1,3,4,8 | 1,3,4,8 | 1,3,4,8 |
| 3 | 13 | 1,4,6 | 1,4,6 | 1,4,6 |
| 4 | 12 | 1,3,6 | 1,3,6 | 1,6,8 |
| 5 | 13 | 1,4,6 | 1,6,7 | 1,6,11 |
| 6 | 12 | 1,3,4,6,9 | 1,3,4,6,9 | 1,3,4,6,9 |
| 7 | 17 | 4,6,8 | 4,6,8 | 4,6,8 |
| 8 | 14 | 4,6 | 4,6 | 4,6 |
| 9 | 13 | 1,3,4,6 | 1,3,4,6 | 1,3,4,6 |
| 10 | 12 | 1,3,6,11 | 1,3,6,11 | 1,3,6,11 |
| 11 | 14 | 1,6,8 | 1,6,8 | 1,6,8 |
| 12 | 12 | 1,6,8 | 1,6,8 | 1,6,8 |
| 13 | 16 | 4,6,8 | 4,6,8 | 4,6,8 |
| 14 | 11 | 1,3,4,6 | 1,3,4,6 | 1,3,4,6 |
| 15 | 12 | 6,9 | 6,9 | 6,9 |

Table 4.10: Results for NSFNET topology when $R_{max}$ is 8000 km and demand number is 50

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 15 | 8 | 8 | 8 |
| 2 | 13 | 3,8 | 3,8 | 3,8 |
| 3 | 13 | 1,4 | 1,2 | 1,2 |
| 4 | 12 | 10 | 10 | 10 |
| 5 | 13 | 1 | 1 | 1 |
| 6 | 12 | 2,8 | 1,2 | 1,2 |
| 7 | 17 | 0 | 0 | 0 |
| 8 | 14 | 6 | 3 | 3 |
| 9 | 13 | 6 | 6 | 6 |
| 10 | 12 | 3 | 3 | 3 |
| 11 | 14 | 8 | 6 | 3 |
| 12 | 12 | 1 | 8 | 8 |
| 13 | 16 | 0 | 0 | 0 |
| 14 | 11 | 1,6 | 1,6 | 1,6 |
| 15 | 12 | 8 | 1 | 1 |

Table 4.11: Results for NSFNET topology when $R_{max}$ is 5000 km and demand number is 70

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 19 | 1,4,6,8,10,11 | 1,4,6,8,10,11 | 1,4,6,8,10,11 |
| 2 | 22 | 4,6,10,11 | 4,6,10,11 | 4,6,10,11 |
| 3 | 20 | 1,4,6,8,11 | 1,4,6,8,11 | 1,4,6,8,11 |
| 4 | 17 | 1,3,4,6,8,11,14 | 1,3,4,6,8,11,14 | 1,3,4,6,8,11,14 |
| 5 | 20 | 4,6,8,11 | 4,6,8,11 | 4,6,8,11 |
| 6 | 21 | 1,4,6,8,11 | 1,4,6,8,11 | 1,4,6,8,11 |
| 7 | 29 | 4,6,7,8 | 4,6,7,8 | 4,6,7,8 |
| 8 | 16 | 1,4,6,8,11 | 1,4,6,8,11 | 1,4,6,8,11 |
| 9 | 15 | 1,3,4,6,8,11,14 | 1,3,4,6,8,11,14 | 1,3,4,6,8,11,14 |
| 10 | 21 | 4,6,8,10,11 | 4,6,8,10,11 | 4,6,8,10,11 |
| 11 | 16 | 1,4,6,8,11,14 | 1,4,6,8,9,11 | 1,4,6,8,9,11 |
| 12 | 24 | 4,6,10,11 | 4,6,10,11 | 4,6,7,11 |
| 13 | 19 | 1,4,6,11,12 | 1,4,6,11,13 | 1,4,6,11,14 |
| 14 | 17 | 1,4,6,8,10,11 | 1,4,6,8,10,11 | 1,4,6,8,10,11 |
| 15 | 19 | 1,4,6,8,11 | 1,4,6,8,11 | 1,4,6,8,11 |

Table 4.12: Results for NSFNET topology when $R_{max}$ is 6500 km and demand number is 70

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 19 | 1,4,6,9 | 1,4,6,8 | 1,4,6,8 |
| 2 | 22 | 4,6 | 4,6 | 4,6 |
| 3 | 20 | 1,6,8 | 1,4,6 | 1,6,8 |
| 4 | 17 | 1,3,4,6,8 | 1,3,4,6,8 | 1,3,4,6,8 |
| 5 | 20 | 4,6,8 | 4,6,8 | 4,6,8 |
| 6 | 21 | 1,6,9 | 1,6,8 | 1,6,8 |
| 7 | 29 | 6,9 | 6,8 | 6,9 |
| 8 | 16 | 1,4,6 | 1,4,6 | 1,4,6 |
| 9 | 15 | 1,3,4,6,7 | 1,3,4,6,8 | 1,3,4,6,8 |
| 10 | 21 | 6,8 | 6,8 | 6,8 |
| 11 | 16 | 1,4,6 | 1,4,6 | 1,4,6 |
| 12 | 24 | 4,6 | 4,6 | 4,6 |
| 13 | 19 | 1,4,6 | 1,4,6 | 1,4,6 |
| 14 | 17 | 1,2,6,11 | 1,2,6,11 | 1,2,6,11 |
| 15 | 19 | 1,4,6,8 | 1,4,6,8 | 1,4,6,8 |

Table 4.13: Results for NSFNET topology when $R_{max}$ is 8000 km and demand number is 70

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 19 | 1,3 | 1,3 | 1,3 |
| 2 | 22 | 3 | 3 | 3 |
| 3 | 20 | 8 | 8 | 8 |
| 4 | 17 | 2,8 | 2,8 | 4,8 |
| 5 | 20 | 8 | 8 | 8 |
| 6 | 21 | 0 | 0 | 0 |
| 7 | 29 | 0 | 0 | 0 |
| 8 | 16 | 3 | 2 | 3 |
| 9 | 15 | 6,8 | 3,8 | 3,8 |
| 10 | 21 | 6 | 1 | 1 |
| 11 | 16 | 2,8 | 1,2 | 1,3 |
| 12 | 24 | 0 | 0 | 0 |
| 13 | 19 | 3 | 2 | 6 |
| 14 | 17 | 3,8 | 3,8 | 3,8 |
| 15 | 19 | 3,8 | 1,6 | 1,6 |

Table 4.14: Results for Deutsche Telecom topology when $R_{max}$ is 400 km and demand number is 30

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 10 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 2 | 9 | 3,4,9,10,11 | 2,3,9,10,11 | 3,5,9,10,11 |
| 3 | 10 | 1,3,5,9,10,11 | 1,3,5,9,10,11 | 1,3,5,9,10,11 |
| 4 | 9 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 5 | 12 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 6 | 10 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 7 | 11 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 8 | 10 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 9 | 9 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 10 | 8 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 11 | 13 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 12 | 13 | 3,4,9,10,11,13 | 3,4,9,10,11,12 | 3,4,9,10,11,13 |
| 13 | 11 | 3,9,10,11,12 | 3,9,10,11,13 | 3,9,10,11,13 |
| 14 | 11 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 15 | 12 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |

Table 4.15: Results for Deutsche Telecom topology when $R_{max}$ is 500 km and demand number is 30

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 10 | 3,9,10 | 3,9,10 | 3,9,10 |
| 2 | 9 | 3,9,10 | 3,9,10 | 3,9,10 |
| 3 | 10 | 3,9,10 | 3,9,10 | 3,9,10 |
| 4 | 9 | 3,9,10,11 | 2,9,10,14 | 2,9,10,14 |
| 5 | 12 | 3,9,10 | 3,9,10 | 3,9,10 |
| 6 | 10 | 3,9,11 | 3,9,10 | 3,9,10 |
| 7 | 11 | 3,9,10,14 | 3,9,10,11 | 3,9,10,11 |
| 8 | 10 | 9,10 | 9,10 | 9,10 |
| 9 | 9 | 3,9,10 | 1,9,10 | 1,9,10 |
| 10 | 8 | 3,9,10 | 3,9,10 | 3,9,10 |
| 11 | 13 | 3,9,10 | 3,9,10 | 3,9,10 |
| 12 | 13 | 3,9,10 | 3,9,10 | 3,9,10 |
| 13 | 11 | 3,9,10 | 3,9,10 | 3,9,10 |
| 14 | 11 | 3,9,10 | 3,9,10 | 3,9,10 |
| 15 | 12 | 3,10,11 | 3,10,11 | 3,10,11 |

Table 4.16: Results for Deutsche Telecom topology when $R_{max}$ is 700 km and demand number is 30

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 10 | 9 | 9 | 9 |
| 2 | 9 | 9 | 9 | 9 |
| 3 | 10 | 9 | 9 | 9 |
| 4 | 9 | 9 | 9 | 9 |
| 5 | 12 | 3 | 9 | 9 |
| 6 | 10 | 11 | 10 | 11 |
| 7 | 11 | 9 | 9 | 9 |
| 8 | 10 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 |
| 10 | 8 | 9 | 9 | 9 |
| 11 | 13 | 10 | 10 | 10 |
| 12 | 13 | 9 | 9 | 9 |
| 13 | 11 | 10 | 9 | 9 |
| 14 | 11 | 9 | 9 | 9 |
| 15 | 12 | 11 | 9 | 9 |

Table 4.17: Results for Deutsche Telecom topology when $R_{max}$ is 400 km and demand number is 50

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 15 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 2 | 17 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 3 | 15 | 2,3,9,10,11 | 2,3,9,10,11 | 3,5,9,10,11 |
| 4 | 16 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 5 | 16 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 6 | 20 | 3,9,10,11,12 | 3,9,10,11,12 | 3,9,10,11,13 |
| 7 | 18 | 3,4,9,10,11,12 | 3,4,9,10,11,12 | 3,4,9,10,11,12 |
| 8 | 15 | 3,4,5,9,10,11 | 3,4,5,9,10,11 | 3,4,5,9,10,11 |
| 9 | 14 | 2,3,9,10,11 | 3,4,9,10,11 | 3,5,9,10,11 |
| 10 | 18 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 11 | 18 | 3,4,5,9,10,11 | 3,4,5,9,10,11 | 3,4,5,9,10,11 |
| 12 | 20 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 13 | 21 | 3,5,10,11 | 3,5,10,11 | 3,5,10,11 |
| 14 | 15 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 15 | 15 | 3,4,5,9,10,11 | 2,3,4,9,10,11 | 3,4,5,9,10,11 |

Table 4.18: Results for Deutsche Telecom topology when $R_{max}$ is 500 km and demand number is 50

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 15 | 3,9,10 | 3,9,10 | 3,9,10 |
| 2 | 17 | 3,9,10 | 3,9,10 | 3,9,10 |
| 3 | 15 | 3,9,10 | 3,9,10 | 3,9,10 |
| 4 | 16 | 3,9,10 | 3,9,10 | 3,9,10 |
| 5 | 16 | 3,9,10 | 3,9,10 | 3,9,10 |
| 6 | 20 | 3,9,10 | 3,9,10 | 3,9,10 |
| 7 | 18 | 3,9,10 | 3,9,10 | 3,9,10 |
| 8 | 15 | 3,9,10,14 | 3,9,10,11 | 3,9,10,14 |
| 9 | 14 | 9,10 | 9,10 | 9,10 |
| 10 | 18 | 3,10,11 | 3,10,11 | 3,10,11 |
| 11 | 18 | 3,9,10,11 | 3,9,10,14 | 3,9,10,11 |
| 12 | 20 | 3,10,11 | 3,10,11 | 3,9,10 |
| 13 | 21 | 3,10 | 3,10 | 3,10 |
| 14 | 15 | 3,9,10 | 3,9,10 | 3,9,10 |
| 15 | 15 | 3,9,10 | 3,9,10 | 3,9,10 |

Table 4.19: Results for Deutsche Telecom topology when $R_{max}$ is 700 km and demand number is 50

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 15 | 9 | 9 | 9 |
| 2 | 17 | 3 | 3 | 9 |
| 3 | 15 | 9 | 9 | 9 |
| 4 | 16 | 9 | 9 | 9 |
| 5 | 16 | 9 | 9 | 9 |
| 6 | 20 | 9 | 9 | 9 |
| 7 | 18 | 9 | 9 | 9 |
| 8 | 15 | 9 | 9 | 9 |
| 9 | 14 | 10 | 10 | 9 |
| 10 | 18 | 9 | 9 | 9 |
| 11 | 18 | 9 | 9 | 9 |
| 12 | 20 | 9 | 9 | 9 |
| 13 | 21 | 10 | 10 | 9 |
| 14 | 15 | 9 | 9 | 9 |
| 15 | 15 | 3 | 3 | 3 |

Table 4.20: Results for Deutsche Telecom topology when $R_{max}$ is 400 km and demand number is 70

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 22 | 3,5,9,10,11 | 3,5,9,10,11 | 3,5,9,10,11 |
| 2 | 20 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 3 | 23 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 4 | 20 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 5 | 22 | 3,4,9,10,11,12 | 3,4,9,10,11,13 | 3,4,9,10,11,12 |
| 6 | 22 | 3,5,9,10,11 | 3,5,9,10,11 | 3,5,9,10,11 |
| 7 | 20 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 8 | 23 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 9 | 20 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 10 | 22 | 3,4,9,10,11,12 | 3,4,9,10,11,13 | 3,4,9,10,11,12 |
| 11 | 22 | 3,5,9,10,11 | 3,5,9,10,11 | 3,5,9,10,11 |
| 12 | 20 | 3,9,10,11 | 3,9,10,11 | 3,9,10,11 |
| 13 | 23 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 14 | 20 | 3,4,9,10,11 | 3,4,9,10,11 | 3,4,9,10,11 |
| 15 | 22 | 3,4,9,10,11,12 | 3,4,9,10,11,13 | 3,4,9,10,11,12 |

Table 4.21: Results for Deutsche Telecom topology when $R_{max}$ is 500 km and demand number is 70

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 22 | 3,9,10 | 3,9,10 | 3,9,10 |
| 2 | 20 | 3,9,10 | 3,9,10 | 3,9,10 |
| 3 | 23 | 3,9,10 | 3,9,10 | 3,9,10 |
| 4 | 20 | 3,9,10 | 3,9,10 | 3,9,10 |
| 5 | 22 | 3,9,10 | 3,9,10 | 3,9,10 |
| 6 | 22 | 3,9,10 | 3,9,10 | 3,9,10 |
| 7 | 20 | 3,9,10 | 3,9,10 | 3,9,10 |
| 8 | 23 | 3,9,10 | 3,9,10 | 3,9,10 |
| 9 | 20 | 3,9,10 | 3,9,10 | 3,9,10 |
| 10 | 22 | 3,9,10 | 3,9,10 | 3,9,10 |
| 11 | 22 | 3,9,10 | 3,9,10 | 3,9,10 |
| 12 | 20 | 3,9,10 | 3,9,10 | 3,9,10 |
| 13 | 23 | 3,9,10 | 3,9,10 | 3,9,10 |
| 14 | 20 | 3,9,10 | 3,9,10 | 3,9,10 |
| 15 | 22 | 9 | 9 | 9 |

Table 4.22: Results for Deutsche Telecom topology when $R_{max}$ is 700 km and demand number is 70

| Problem # | Minimum Slot # | MTLU | MTUHUL | MHUL |
|---|---|---|---|---|
| 1 | 22 | 9 | 9 | 9 |
| 2 | 20 | 9 | 9 | 9 |
| 3 | 23 | 9 | 9 | 9 |
| 4 | 20 | 9 | 9 | 9 |
| 5 | 22 | 9 | 9 | 9 |
| 6 | 22 | 9 | 9 | 9 |
| 7 | 20 | 9 | 9 | 9 |
| 8 | 23 | 9 | 9 | 9 |
| 9 | 20 | 9 | 9 | 9 |
| 10 | 22 | 9 | 9 | 9 |
| 11 | 22 | 9 | 9 | 9 |
| 12 | 20 | 10 | 9 | 9 |
| 13 | 23 | 9 | 9 | 9 |
| 14 | 20 | 9 | 9 | 9 |
| 15 | 22 | 3,9,10 | 3,9,10 | 3,9,10 |

We can see from these results that each secondary objective can find optimum solution for at least one case where other secondary objectives do not provide a feasible solution. So three secondary algorithms can be applied consecutively to find the optimum solution. In addition to that, if all secondary objectives find the same result and optimal solution also cannot be found, we try to push optimization framework to find different regenerator sites by prohibiting each regenerator site which has been found before one by one. For instance, every secondary objective cannot find optimum solution but find same nodes 4, 6, 7 and 8 for problem 7 at Table 4.8. We firstly prevent optimization framework from selecting node 4 as regenerator site. Then we apply each secondary objective one by one to find regenerator sites which are different from node 4. If we do not find optimum solution for every secondary objective, we secondly prevent optimization framework from selecting node 6 as regenerator site. We apply each secondary objective one by one to find regenerator sites which are different from node 6. We do same procedure until optimum solution is found for all previously founded nodes. For this problem, we found optimum result, when node 7 is prohibited. We found optimum solutions for three problems for the NSFNET topology. Table 4.23 shows founded optimal results of this local improvement procedure.

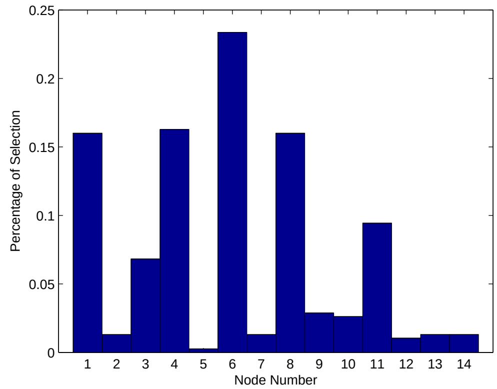
Table 4.23: Local improvement results

| Table | Problem # | Nodes |
|---|---|---|
| 4.8 | 7 | 4,6,8,10 |
| 4.10 | 14 | 3,8 |
| 4.13 | 14 | 1,6 |

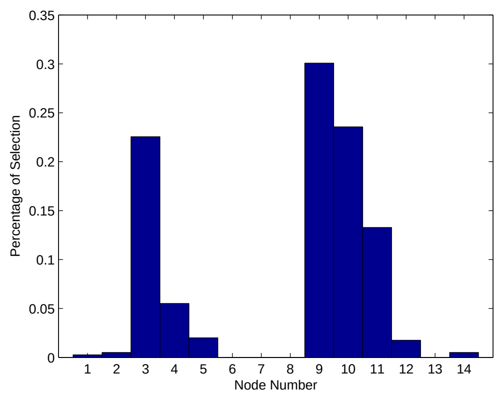
We have tested two topologies with 135 different cases for each of them. The secondary objective, MTLU, finds the optimum solution for 83% of all cases for NSFNET topology and 89% of all cases for Deutsche Telecom topology. The secondary objective, MTUHUL, finds the optimum solution for 79% of all cases for NSFNET topology and 93% of all cases for Deutsche Telecom topology. The secondary objective, MHUL, finds the optimum solution for 71% of all cases for NSFNET topology and 87% of all cases for Deutsche Telecom topology. On the other hand, if we use all three secondary objectives such that a feasible solution is obtained for at least one of the solutions provided by the three secondary objective functions or by local improvement, the decoupled model can find optimum solutions for 92% of all cases for NSFNET topology and 99% of all cases for Deutsche Telecom topology.

## 4.2 Analysis of the Results

In this section, we study results to understand which parameters are significant to determine regenerator sites. Therefore, we obtain histograms of the results by considering different parameters. Data used in these histograms are feasible results which are obtained by using the first, second and third secondary objectives respectively until the optimum solution is found. Therefore, histograms are distributions of the feasible results. In addition to that, each histogram bar shows average percentage of selection.



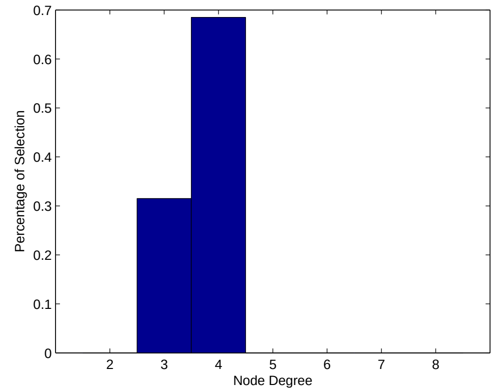

(a) 14-nodes NSFNET Topology



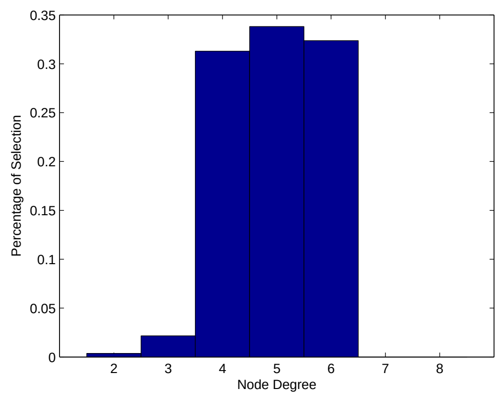

(b) 14-nodes Deutsche Telecom Topology

Figure 4.2: Histograms of the selected nodes

Figure 4.2 shows normalized distribution of the selected nodes. Node 1, 4, 6, 8 and 11 are frequently selected for the NSFNET Topology. Total percentage of them is more than 80%. This is mainly due the fact that these nodes are neighbour to at least one long link. Therefore, lightpaths pass through these nodes need to be regenerated before or after these long links. For Deutsche Telecom Topology, nodes 3, 9 and 10 are selected commonly whose total percentage is more than 88%. These nodes have high node degree. Consequently, many lightpaths are passed through these nodes to minimize regenerator sites. In other words, optimization framework selects nodes which have high node degree to regenerate as many lightpaths as possible. In addition to node degree, nodes 3, 9 and 10 also are neighbour to long links which is also an influential reason to select them.



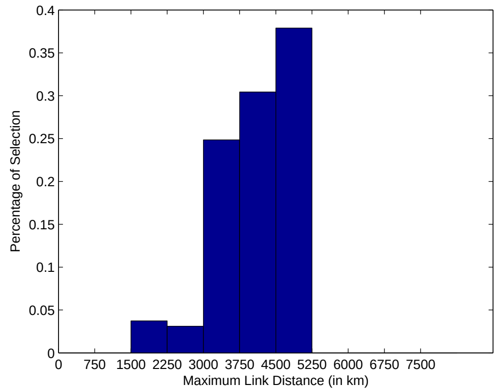

(a) 14-nodes NSFNET Topology



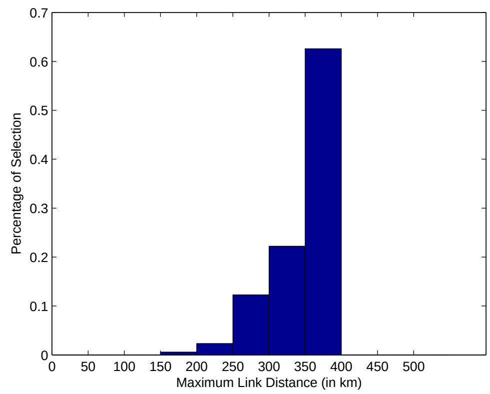

(b) 14-nodes Deutsche Telecom Topology

Figure 4.3: Histograms of node degree of the selected nodes

Figure 4.3 presents normalized distribution of node degree of the selected nodes. For NSFNET Topology, nodes with node degree 4 are selected much more than nodes with node degree 3. For Deutsche Telecom Topology, nodes with node degree 4,5 or 6 consist of 97% of selected nodes. So nodes with higher node degree are selected as regenerator nodes more commonly. However, it is not only factor. Also link distances are critical for regenerator site selection.



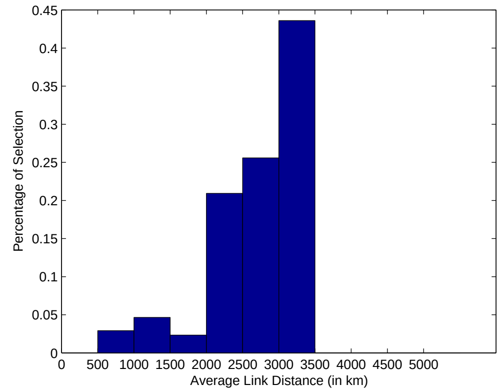

(a) 14-nodes NSFNET Topology



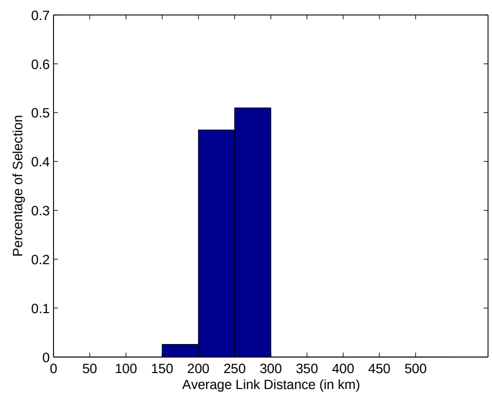

(b) 14-nodes Deutsche Telecom Topology

Figure 4.4: Histograms of maximum neighbour link distance of the selected nodes

Figure 4.4 indicates normalized distribution of the maximum neighbour link distance of the selected nodes. For both topologies, nodes with longer maximum neighbour link distance are selected in most cases since optical signals, which pass through these nodes, need to be regenerated before or after these links to satisfy signal regeneration constraints. Consequently, nodes which are neighbour to these links are selected as regenerator nodes more commonly.



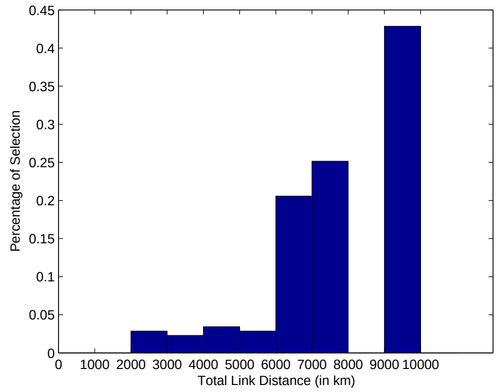

(a) 14-nodes NSFNET Topology



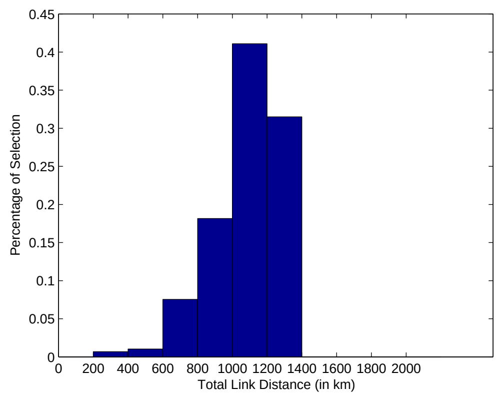

(b) 14-nodes Deutsche Telecom Topology

Figure 4.5: Histograms of average neighbour link distance of the selected nodes

Figure 4.5 shows normalized distribution of average neighbour link distance of the selected nodes. As in distribution of the maximum neighbour link distance, longer distances are selected more commonly. For NSFNET Topology, nodes with average link distance, between 2000 and 3500, constitute of 90% of all selected nodes. For Deutsche Telecom Topology, nodes with average link distance, between 200 and 300, constitute of 97% of all selected nodes. Consequently, average link distance is critical point to determine regenerator sites.




(a) 14-nodes NSFNET Topology




(b) 14-nodes Deutsche Telecom Topology

Figure 4.6: Histograms of total neighbour link distance of the selected nodes

Figure 4.6 demonstrates normalized distribution of total neighbour link distance of the selected nodes. Nodes with longer total neighbour link distance are selected more frequently. However, for NSFNET Topology, nodes with total link distance between 8000 and 9000 km are not selected as regenerator node since no node has total distance between 8000 and 9000 km.

In addition to link distances, position of the node is also very important. Consequently, position of the node, neighbour link distances and node degree should be evaluated simultaneously to find proper regenerator sites.

# Chapter 5

# Conclusion

In this thesis, we have investigated the RSA and RP problems jointly in the flexible optical networks. We have proposed an MILP optimization framework to solve this joint problem. However, this formulation is not computationally tractable for large networks. Consequently, we have presented a two-phase decoupled model which separates the joint RSA/RP problem into two-phases where routing and regenerator placement problems are solved in the first phase and the spectrum assignment is solved in the second phase. Three secondary objective functions are proposed in order to increase the likelihood that the decoupled model provides optimum solutions for the joint problem. We have investigated the performance of this decoupled approach.

The first secondary objective, MTLU, finds optimum solution for 83% of all cases considered for NSFNET topology and 89% of all cases considered for Deutsche Telecom topology. MTUHUL, the second secondary objective, finds optimal solution for 79% of all cases considered for NSFNET topology and 93% of all cases considered for Deutsche Telecom topology. The third secondary objective, MHUL, finds optimum solution for 71% of all cases considered for NSFNET topology and 87% of all cases considered for Deutsche Telecom topology. On the other hand, if we use three secondary objectives sequentially until optimal solution is found, decoupled model can find optimal solutions for 92% of all cases considered for NSFNET topology and 99% of all cases considered for Deutsche

Telecom topology.

We have also examined distribution of nodes which are selected as regenerator sites. Histograms show that nodes with higher node degree have been selected more commonly. Also, nodes which have long adjacent link distances have been selected more frequently.

For future research, selection of the appropriate modulation type for each demand along each semi-lightpath on its route can also be analyzed together with the RSA-RP problem. Due to the fact that regenerators can change the modulation type of the optical signal, the most appropriate modulation type can be selected for each semi-lightpath. Consequently, maximum reach becomes longer and spectrum is used more efficiently. Moreover, path recovering in the case of link failure can be also examined with RP problem for flexible optical networks. Bandwidth squeezed restoration is possible for flexible optical networks. In the case of link failure, connectivity is ensured by squeezing bandwidth. Therefore, flexible optical network with appropriately placed regenerators ensures a highly survivable restoration.

# Bibliography

[1] C. V. N. Index, "Forecast and methodology, 2012 - 2017," *Cisco white paper*, 2013.

[2] R.-J. Essiambre, G. Kramer, P. J. Winzer, G. J. Foschini, and B. Goebel, "Capacity limits of optical fiber networks," *Journal of Lightwave Technology*, vol. 28, no. 4, pp. 662–701, 2010.

[3] I. Tomkos, E. Palkopoulou, and M. Angelou, "A survey of recent developments on flexible/elastic optical networking," in *14th International Conference on Transparent Optical Networks (ICTON)*, pp. 1–6, 2012.

[4] O. Gerstel, M. Jinno, A. Lord, and S. B. Yoo, "Elastic optical networking: A new dawn for the optical layer?," *IEEE Communications Magazine*, vol. 50, no. 2, pp. s12–s20, 2012.

[5] M. Jinno, H. Takara, B. Kozicki, Y. Tsukishima, Y. Sone, and S. Matsuoka, "Spectrum-efficient and scalable elastic optical path network: architecture, benefits, and enabling technologies," *IEEE Communications Magazine*, vol. 47, no. 11, pp. 66–73, 2009.

[6] L. Velasco, M. Klinkowski, M. Ruiz, and J. Comellas, "Modeling the routing and spectrum allocation problem for flexgrid optical networks," *Photonic Network Communications*, vol. 24, no. 3, pp. 177–186, 2012.

[7] J. Li and C. Qiao, "Schedule burst proactively for optical burst switched networks," *Computer Networks*, vol. 44, no. 5, pp. 617–629, 2004.

[8] K. Manousakis, K. Christodoulopoulos, and E. Varvarigos, "Impairment-aware offline RWA for transparent optical networks," in *INFOCOM 2009*, pp. 1557–1565, 2009.

[9] H. Zang, J. P. Jue, B. Mukherjee, *et al.*, "A review of routing and wavelength assignment approaches for wavelength-routed optical wdm networks," *Optical Networks Magazine*, vol. 1, no. 1, pp. 47–60, 2000.

[10] D. Banerjee and B. Mukherjee, "A practical approach for routing and wavelength assignment in large wavelength-routed optical networks," *IEEE Journal on Selected Areas in Communications*, vol. 14, no. 5, pp. 903–908, 1996.

[11] K.-m. Chan and T.-s. P. Yum, "Analysis of least congested path routing in wdm lightwave networks," in *INFOCOM'94*, pp. 962–969, 1994.

[12] H. Harai, M. Murata, and H. Miyahara, "Performance of alternate routing methods in all-optical switching networks," in *INFOCOM'97*, vol. 2, pp. 516–524, 1997.

[13] R. Barry and S. Subramaniam, "The max sum wavelength assignment algorithm for WDM ring networks," in *OFC 97*, pp. 121–122, 1997.

[14] E. Karasan and E. Ayanoglu, "Effects of wavelength routing and selection algorithms on wavelength conversion gain in wdm optical networks," *IEEE/ACM Transactions on Networking*, vol. 6, no. 2, pp. 186–196, 1998.

[15] S. Subramaniam and R. A. Barry, "Wavelength assignment in fixed routing WDM networks," in *ICC 97*, vol. 1, pp. 406–410, 1997.

[16] S. Azodolmolky, M. Klinkowski, E. Marin, D. Careglio, J. S. Pareta, and I. Tomkos, "A survey on physical layer impairments aware routing and wavelength assignment algorithms in optical networks," *Computer Networks*, vol. 53, no. 7, pp. 926–944, 2009.

[17] B. Ramamurthy, H. Feng, D. Datta, J. P. Heritage, and B. Mukherjee, "Transparent vs. opaque vs. translucent wavelength-routed optical networks," in *OFC/IOOC'99*, vol. 1, pp. 59–61, 1999.

[18] A. Sen, S. Murthy, and S. Bandyopadhyay, "On sparse placement of regenerator nodes in translucent optical network," in *GLOBECOM 2008*, pp. 1–6, 2008.

[19] W. Zhang, J. Tang, K. Nygard, and C. Wang, "Repare: Regenerator placement and routing establishment in translucent networks," in *GLOBECOM 2009*, pp. 1–7, 2009.

[20] X. Yang, L. Shen, and B. Ramamurthy, "Survivable lightpath provisioning in WDM mesh networks under shared path protection and signal quality constraints," *Journal of Lightwave Technology*, vol. 23, no. 4, p. 1556, 2005.

[21] X. Yang and B. Ramamurthy, "Sparse regeneration in translucent wavelength-routed optical networks: Architecture, network design and wavelength routing," *Photonic Network Communications*, vol. 10, no. 1, pp. 39–53, 2005.

[22] M. A. Ezzahdi, S. Al Zahr, M. Koubaa, N. Puech, and M. Gagnaire, "Lerp: a quality of transmission dependent heuristic for routing and wavelength assignment in hybrid wdm networks," in *ICCCN 2006*, pp. 125–136, 2006.

[23] B. Garcia-Manrubia, P. Pavon-Marino, R. Aparicio-Pardo, M. Klinkowski, and D. Careglio, "Offline impairment-aware RWA and regenerator placement in translucent optical networks," *Journal of Lightwave Technology*, vol. 29, no. 3, pp. 265–277, 2011.

[24] E. Karasan and M. Arisoylu, "Design of translucent optical networks: Partitioning and restoration," *Photonic Network Communications*, vol. 8, no. 2, pp. 209–221, 2004.

[25] M. S. Savasini, P. Monti, M. Tacca, A. Fumagalli, and H. Waldman, "Regenerator placement with guaranteed connectivity in optical networks," in *Optical Network Design and Modeling*, pp. 438–447, 2007.

[26] R. Duran, I. Rodriguez, N. Fernandez, I. de Miguel, N. Merayo, P. Fernandez, J. Aguado, T. Jimenez, R. Lorenzo, and E. Abril, "Performance comparison of methods to solve the routing and spectrum allocation problem," in *14th*

*International Conference on Transparent Optical Networks (ICTON)*, pp. 1–4, 2012.

[27] M. Klinkowski and K. Walkowiak, "Offline RSA algorithms for elastic optical networks with dedicated path protection consideration," in *4th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT)*, pp. 670–676, 2012.

[28] K. Christodoulopoulos, I. Tomkos, and E. Varvarigos, "Elastic bandwidth allocation in flexible OFDM-based optical networks," *Journal of Lightwave Technology*, vol. 29, no. 9, pp. 1354–1366, 2011.

[29] M. Klinkowski, M. Ruiz, L. Velasco, D. Careglio, V. Lopez, and J. Comellas, "Elastic spectrum allocation for time-varying traffic in flexgrid optical networks," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 1, pp. 26–38, 2013.

[30] B. Kozicki, H. Takara, T. Tanaka, S. Yoshiaki, A. Hirano, K. Yonenaga, and M. Jinno, "Distance-adaptive path allocation in elastic optical path networks," *IEICE Transactions on Communications*, vol. 94, no. 7, pp. 1823–1830, 2011.

[31] X. Wan, N. Hua, and X. Zheng, "Dynamic routing and spectrum assignment in spectrum-flexible transparent optical networks," *Journal of Optical Communications and Networking*, vol. 4, no. 8, pp. 603–613, 2012.

[32] M. Klinkowski, "On the effect of regenerator placement on spectrum usage in translucent elastic optical networks," in *14th International Conference on Transparent Optical Networks (ICTON)*, pp. 1–6, 2012.

[33] M. Kiese and M. Schuster, "Exploiting transponder performance in optical ofdm networks," in *National Fiber Optic Engineers Conference*, 2009.

[34] S. Yang and F. Kuipers, "Impairment-aware routing in translucent spectrum-sliced elastic optical path networks," in *17th European Conference on Networks and Optical Communications (NOC)*, pp. 1–6, 2012.

[35] J. M. Simmons, "Network design in realistic," *IEEE Communications Magazine*, vol. 44, no. 11, pp. 88–94, 2006.