OUTER APPROXIMATION ALGORITHMS FOR THE CONGESTED p-MEDIAN PROBLEM

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By
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July, 2011

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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In this thesis, we study a generalization of the p-median problem, which is a well-known facility location problem. Given a set of clients, a set of potential facilities, and a positive integer p, the p-median problem is concerned with choosing p facilities and assigning each client to an open facility in such a way that the sum of the travel times between each client and the facility that serves that client is as small as possible. The classical p-median problem takes into account only the travel times between clients and facilities. However, in many applications, the disutility of a client is also closely related to the waiting time at a facility, which is typically an increasing function of the demand allocated to that facility. In an attempt to address this issue, for a given potential facility, we define the disutility of a client as a function of the travel time and the total demand served by that facility. The latter part reflects the level of unwillingness of a client to be served by a facility as a function of the level of utilization of that facility. By modeling this relation using an increasing convex function, we develop convex mixed integer nonlinear programming models. By exploiting the fact that nonlinearity only appears in the objective function, we propose different variants of the well-known outer approximation algorithm. Our extensive computational results reveal that our algorithms are competitive in comparison with the off-the-shelf solvers.

Keywords: facility location problem, p-median, outer approximation, linear constraints, disutility, waiting time, MINLP.

ÖZET

KALABALIK p-ORTANCA PROBLEMİ İÇİN DIŞ YAKLAŞIM ALGORİTMASI

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Bu tez kapsamında, bilinen bir tesis yerleşimi problemi olan p-ortanca probleminin genelleştirilmiş hali üzerinde çalışılmaktadır. Problem; verilen müşteri grubu, potansiyel tesis kümesi ve pozitif p tamsayısı bilgileri doğrultusunda p tane tesisi seçmeyi ve müşteriler ile hizmet aldıkları tesisler arasındaki ulaşım sürelerinin toplamını en azlayacak şekilde her müşteriyi seçilen bir tesise atamayı amaçlamaktadır. Klasik p-ortanca problemi sadece müşteriler ve tesisler arasındaki ulaşım sürelerini dikkate almaktadır. Ancak, bir çok uygulamada müşterilerin memnuniyetsizliği, tesislerdeki bekleme süresiyle yakından ilgilidir. Bekleme süresi, tesise atanan toplam insan sayısının artan bir fonksiyonudur. Bu duruma dikkat çekmek amacıyla, belirli bir tesis için, müşteri memnuniyetsizliğini müşterinin söz konusu tesise olan ulaşım süresine ve bu tesisin hizmet verdiği toplam insan sayısına bağlı bir fonksiyon olarak tanımlıyoruz. İkinci kısım, bir müşterinin o tesisten hizmet alma isteksizliğinin seviyesini, tesisin kullanım derecesine bağlı bir fonksiyon olarak yansıtmaktadır. Bu ilişkiyi artan dışbükey bir fonksiyon kullanarak modellediğimiz için modelimiz dışbükey karışık tamsayılı doğrusal olmayan programlama modelidir. Sadece amaç fonksiyonunun doğrusal olmaması gerçeğini göz önünde bulundurarak iyi bilinen dış yaklaşım algoritmasının farklı türlerini önermekteyiz. Kapsamlı hesaplama sonuçlarımız algoritmalarımızın var olan çözümleyiciler ile rekabet edebilecek durumda olduğunu ortaya koymaktadır.

Anahtar sözcükler: tesis yerleşimi problemi, p-ortanca, dış yaklaşım, doğrusal kısıtlar, memnuniyetsizlik, bekleme süresi, karışık tamsayılı doğrusal olmayan problem.

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Chapter 1

Introduction

The classical p-median problem is to select the locations of p facilities among a given set of possible locations and to allocate the clients to these facilities with the aim of minimizing the total distance or travel time between the clients and the facilities that serve them. The underlying assumption here is that the utility or disutility of the clients is simply a function of a distance related measure. Such a definition may be insufficient when we consider applications where service time is critical, such as location of emergency services. Even in many applications where the service time is not critical, clients may not be happy to wait or to be in crowded places.

In this study, we propose a generalization of the p-median problem by extending the definition of disutility as follows: the disutility of a client is a function of the distance/travel time and the amount of demand served by its facility. The latter part reflects the dislike of the client due to waiting time, crowdedness or poor service quality due to overutilization of capacity. Hence, we define the *Congested p-median problem (CpMP)* as the problem of deciding on the locations of p facilities and the allocations of clients to these facilities in order to minimize the total disutility or dislike of clients.

Our disutility function has two terms. The first term is a linear function of the distance between the client and the facility and the second term is a convex function of the amount of demand covered by the facility that serves this client. As a result, *CpMP* belongs to the class of convex nonlinear mixed integer programming problems.

Mixed-integer nonlinear programming (MINLP) has recently received significant attention from researchers due to the added modeling capabilities. Optimization problems with nonlinear functions and a mix of discrete and continuous variables naturally arise in many diverse settings such as electricity transmission, process systems engineering, portfolio optimization, and the design of water distribution networks. MINLP applications can be found in [1] and [2]. Due to the existence of discrete decision variables, MINLPs are inherently nonconvex. In particular, MINLP generalizes mixed integer linear programming, which is a fairly well-studied class of difficult optimization problems. Furthermore, nonlinearity clearly increases the complexity of the problem.

Due to the computational complexity of general MINLP problems, it is not reasonable to expect to have efficient algorithms that can compute a globally optimal solution for every instance. However, for certain subclasses of MINLP problems, one can hope to make further progress. Convex MINLP problems constitute one such class. A convex MINLP problem is a minimization problem in which the objective function and each inequality constraint is a convex function of the decision variables, each equality constraint is a linear function of the decision variables, and a nonempty subset of decision variables can only take integer values. The relaxation of the integrality constraints gives rise to a convex optimization problem. A large class of convex optimization problems can be solved in polynomial time by using interior-point methods (see [3]). Therefore, the branch-and-bound algorithm, which is one of the most prominent algorithms for mixed integer linear programming, can be extended in a straightforward way to solve an MINLP problem since each problem in each node of the branch-and-bound tree can be solved efficiently. However, despite convexity, nonlinear problems are typically harder to solve than linear programming problems and as the size of the branch-and-bound tree increases, the number of nonlinear subproblems can grow exponentially.

Another class of algorithms developed for convex MINLP problems are centered around the idea of linearizations. Since the first-order (linear) Taylor approximation to a convex function at any point is an underestimator for that function, replacing the convex constraints by their linearizations at a finite set of points provides an outer polyhedral approximation of the feasible region. By refining the set of points at which linear approximations are generated, a better approximation of the feasible region can be achieved. It follows that one can attempt to solve a convex MINLP problem by solving a sequence of mixed integer linear programming problems. Such methods, known as outer approximation methods, provide a lot of flexibility in terms of generating new linearization points. Under certain assumptions, certain variants of outer approximation methods are guaranteed to terminate after generating and adding a finite number of linearization points. More detailed information about the solution methodologies for MINLPs can be obtained from [4].

1.1 Contribution

As mentioned before, in this thesis we propose a generalization of the p-median problem referred as *CpMP*. The difference between two problems is that we consider the dislike of the client due to the crowdedness in the facility besides the travel time. Therefore *CpMP* is a more realistic problem for the applications with service time. In order to take the dislike of the client into consideration, we define the disutility of a client with two terms; the travel time of the client and the demand served by the facility. With the addition of disutility function, the congested p-median problem becomes a linearly constrained convex MINLP.

For the solution methodologies, by exploiting the fact that *CpMP* is a linearly constrained MINLP, we propose three algorithms. The proposed algorithms are the variants of the well-known Outer Approximation Algorithm. They are all centered around the idea of linearizations but they differ on the selection of the linearization points.

The proposed algorithms are implemented and tested. The computational results of the algorithms are analysed and the effects of some parameters on the algorithms are discussed. We also compared the proposed algorithms and report the results. Moreover in order to test the efficiency of the algorithms, the same instances are solved with CPLEX12.1 using the interface OPL IDE. The running times of the algorithms and CPLEX12.1 are compared. The results show that the proposed algorithms outperforms CPLEX12.1 for large instances.

To conclude, in this thesis, a new facility location problem, *CpMP* is proposed and three new algorithms are developed to solve this problem. As the algorithms are not problem dependent, they may be used to solve any linearly constrained convex MINLP. However as the algorithms are developed specially for the linearly constrained problems, they can not be used as a solution methodology for the general case MINLP problems. The efficiency of the algorithms are tested and the results are reported.

1.2 Contents

The rest of the thesis is organized as follows. In the second chapter literature reviews about the facility location problem and solution methodologies for MINLP problems are presented. In Chapter 3, problem definition, the model and properties of the disutility function are introduced. The solution methodologies for the problem are also included in this chapter. First we describe the well-known Outer Approximation Algorithm. Then the three proposed algorithms are explained. In Chapter 4, we give the details of the implementation of the proposed algorithms. Moreover the computational results and the discussion about these results are reported in this chapter. Finally, Chapter 5 consists of the conclusions of this thesis and the possible future studies.

Chapter 2

Literature Review

In this thesis, two different subjects are studied; facility location problem and solution methodologies for MINLPs. Literature for each of these subjects will be analyzed in different sections.

2.1 Facility Location Literature

The p-median problem is one of the oldest facility location problems proposed by Hakimi. [5] The problem locates p uncapacitated facilities in order to serve n demand points. Each demand point is assigned to exactly one facility and the aim of the problem is to minimize the total distance between the demand points and the facilities they are assigned to.

There are also covering location problems. The first one of the covering problems is the set covering problem which minimizes the number of facilities to be located in order to "cover" all the demand points. Another covering problem is maximal covering which aims to cover maximum demand by locating p facilities.

Maximal capture problem referred as MAXCAP is another facility location problem proposed by Revelle [6]. In this problem, a firm seeks to locate p facilities in a spatial market to compete with the already existing firms.

All of the problems described so far have a common point that they only consider the distance from the facilities to demand nodes they serve. However in real-life customers take different factors into consideration while choosing the facility they patronize such as the waiting time at the facility. The following studies consider the waiting time of a customer at a facility as a factor in choosing the facility to patronize.

Marianov and Serra [7] study two problems similar to set covering problem. The first one aims to minimize the number of located facilities and allocates the least number of servers to them so that all the population is served within a standard distance and nobody waits more than a given time-limit. In the second problem, there is no pre-specified number of servers for each facility so the model also tries to minimize the number of servers allocated in each facility. If needed, both models can be modified to serve people from the closest facility. In order to solve these problems, they develop a new heuristic that is suitable for large examples.

Silva and Serra [8] study a different version of MAXCAP problem. Rather than just concentrating on the effect of travel time as in the original MAXCAP problem, they also consider the effect of waiting time to maximize the market capture. To solve their problem, they propose metaheuristics which find a solution close to optimal and save significant amount of running time. Furthermore they compare their problem with the original MAXCAP problem and show that the new model decreases the waiting time of individuals.

In the work of Marianov, Ríos and Icaza [9] a new model is proposed to locate the facilities of a firm such that the market share of the firm is maximized. In this model customers are allowed to choose the facilities they patronize depending on the travel time to the facility and the waiting time at the facility. The customers belonging to the same demand node can be served from different facilities. Moreover each customer can be assigned to different facilities. Satisfying all of these conditions creates a demand equilibrium and it is given by a nonlinear equation. Since the solution of the model requires to solve a nonlinear equation (finding the demand equilibrium), they develop a new heuristic to solve their problem. The proposed heuristic solves the problem quickly and gives a near-optimal solution.

Verter and Lapierre [10] propose a problem of locating preventive health care facilities to maximize the participation of people. The problem has three assumptions. The first one is that each facility should have a minimum number of customers as the quality of the facility is directly related to the number of people served by the facility. Second one is that the people tend to go to the nearest facility and lastly the probability of participation of an individual decreases with distance. They gave the model of the problem and evaluate alternative solution strategies. Furthermore with the re-formulation of the problem, they solve two real-life problems.

Another study requiring a minimum number of customers to be served by each facility belongs to Carreras and Serra [11]. They propose a p-median like model that locates the maximum number of facilities that need a minimum catchment area. The objective of the model is minimizing average distance from the population to the facilities. In order to solve the problem a tabu heuristic is proposed in the study and applied to locate pharmacies in a rural region of Spain.

In the work of Fang, Bian and Xuefeng [12], a new model is proposed to locate multiple-server facilities when the demand is elastic and customers choose the facilities they patronize by maximizing their utilization. In order to represent the utilization, they used a cost function consisting of travel cost and service time cost. The travel cost is an input to the model whereas the service time cost is determined by the current local demand and the capacity of the facility. The effect of congestion is built into the model by the service time cost. They also proposed a greedy heuristic to solve their problem. The computational results show that the heuristic is faster than the enumeration method.

Drezner and Wesolowsky [13] study an extended version of the p-median problem. In the problem, customers choose the facilities according to the transportation cost and the cost charged by the facility. The cost charged by the facility is a function of the total number of users patronizing the facility. Due to the fact that facilities pass on their cost to the users, as the number of users increases the cost charged by the facility decreases. It is assumed that the customers choose the “cheapest” facility. They proposed solution methodologies for the problem and tested the algorithms on the problems with a convex region of demand.

Desrochers, Marcotte and Stan [14] study a congested facility location problem. They propose a utility function with two factors; travel time and waiting time. As the travel time is calculated by the distance between two nodes, it is constant. However the waiting time is calcualted as a function of the total demand served by a facility. They propose a model including this utility function and try to solve this problem by using column generating algorithms.

Further information about the facility location problems can be found in [15], [16], [17].

Similar to the studies above, in our model we also consider other factors than the distance between the demand node and the facility it is assigned to. The model that we study is very similar to that of Desrochers' [14]. We have a disutility function with two components; the first one is the distance and the second one is the effect of how crowded the facility is. We assume that the disutility of an individual increases as the total number of people assigned to the same facility with that individual increases. In our model individuals belonging to the same demand point may be served by different facilities. Depending on the applications, this is more realistic since not all individuals act necessarily in the same way. Furthermore most of the studies above propose heuristics for solution methodologies. However we developed three exact algorithms to solve our problem.

2.2 Methods for MINLPs Literature

As the solution techniques for MILPs and NLPs are developed, new algorithms to solve MINLP problems are proposed. One of them is the Outer Approximation (OA) Algorithm proposed by Duran and Grossmann [18]. The algorithm basically iterates between MILPs and NLPs in order to solve MINLP. The MILPs are the linear relaxations of the original problem which get stronger in each iteration and give a lower bound to the problem. NLPs are the original problems for fixed integer values or the feasibility problem. NLPs give upper bounds to the problem.

Lots of modified versions of OA algorithm are developed. One of them is Branch and Cut Based Outer Approximation(BB-OA) proposed by Queseda and Grossmann [19]. In this algorithm different from the original OA algorithm, sequential solution of several MILPs are avoided and one single tree search is performed. Later the hybrid algorithm of OA and BB-OA algorithm is developed. Bonami et al. [22] implemented these different versions of the OA algorithm and the effectiveness of the framework is reported.

In the work of Gunluk and Linderoth [20], perspective reformulations of mixed integer programs with indicator variables are proposed. Each indicator variable controls a different subset of the decision variables. According to the value of the indicator variable, the decision variables are forced to have fixed values or belong to a convex set. The proposed reformulation method produces extended formulations for the simple sets they analyzed and yields strong relaxations. Pespective cuts are studied also by Frangioni and Gentile [21].

In our solution methodologies, we modified the outer approximation algorithm and developed three different algorithms to solve our problem. Our algorithms are not problem specific, meaning that every linearly constrained problem can be solved with these algorithms.

Chapter 3

Problem Definition and Solution Methodologies

This chapter consists of two sections; Problem Definition and Solution Methodologies. In the Problem Definition section, we first define our problem and the parameters. Then, a mixed integer nonlinear model is presented. The properties of the disutility function included in the model are also discussed in this section. For the solution methodologies, we first describe an already existing algorithm, that of Outer Approximation(OA) algorithm, and show how it works on an example. Later we describe the three algorithms; OA1, OA2, OA3 that we developed by modifying the OA algorithm to solve linearly constrained MINLPs.

3.1 Problem Definition

We are given a set of possible locations to locate facilities, the number of facilities to be opened, a set of population zones, the number of individuals in each population zone and the disutility function which gives the disutility of an individual. Two aspects affect the disutility function. The first one is the crowdedness of the facility that the individual is assigned to. The more the facility gets crowded, the more the disutility of individuals assigned to that facility increases. The second aspect is the distance between the facility that the individual is assigned to and the population zone of the individual. Our goal is to minimize the sum of each individual's disutility by locating the given number of facilities and assigning all the individuals in population zones to the located facilities. Individuals of the same population zone may be assigned to different facilities.

3.2 The *CpMP* Model

In this section, we define the parameters and the variables of the problem. Then we present the mixed integer nonlinear model.

3.2.1 Parameters

M: set of population zones
$M = \{1,...m\}$
N: possible locations to locate facilities
$N = \{1,...n\}$
d_i: number of individuals at population zone $i \in M$
p: number of facilities to locate
$U_{ij}(z_j)$: disutility of an individual at population zone $i \in M$ if that individual is assigned to a facility at location $j \in N$
The disutility function is defined as:

$$U_{ij}(z_j) = f_j(z_j) + b_{ij} \ \forall i \in M \ , \forall j \in N$$

where $f_j(z_j)$ is the contribution of facility crowdedness and b_{ij} is the distance between the facility j and population zone i. Here z_j is the total number of people assigned to the facility j.

3.2.2 Variables

We use the following decision variables:
To decide if a facility is located at the possible location $j \in N$, we use a discrete variable y_j $\forall j \in N$.

$$y_j = \begin{cases} 1 & \text{if a facility is located at location } j \in N \\ 0 & \text{otherwise} \end{cases}$$

For each pair of $i \in M$ and $j \in N$, x_{ij} gives the fraction of the population zone $i \in M$ assigned to facility $j \in M$. Note that the x_{ij} variables are continuous since the individuals in the same population zone can be assigned to different facilities.
The variable z_j is the total number of individuals assigned to facility $j \in N$.

3.2.3 Model

$$\min \sum_{i \in M} \sum_{j \in N} U_{ij}(z_j) d_i x_{ij} \tag{3.1}$$

$$\text{s.t.} \sum_{j \in N} x_{ij} = 1 \quad \forall i \in M \tag{3.2}$$

$$\sum_{k \in M} d_k x_{kj} = z_j \quad \forall j \in N \tag{3.3}$$

$$\sum_{j \in N} y_j = p \tag{3.4}$$

$$x_{ij} \leq y_j \quad \forall i \in M, \forall j \in N \tag{3.5}$$

$$x_{ij} \geq 0 \quad \forall i \in M, \forall j \in N \tag{3.6}$$

$$y_j \in \{0,1\} \forall j \in N \tag{3.7}$$

The objective function (3.1) is the total disutility of all the individuals.
Constraints (3.2) ensure that the entire demand of each population zone is satisfied.
Constraints (3.5) ensure that a facility is available to serve the demand of population zones if and only if it is open.

Constraints (3.3) compute the total number of individuals assigned to a facility. Constraint (3.4) guarantees that exactly p facilities are located.

3.2.4 Properties of Disutility Functions

The disutility function $U_{ij}(z_j) = f_j(z_j) + b_{ij}$ where $z_j = \sum_{i \in M} d_i x_{ij}$ should have the following properties:

1. $f_j(z_j)$ is a nondecreasing function for $z_j \geq 0$.
2. $f_j(z_j)z_j$ is convex.

These properties are necessary to have a reasonable problem with a convex objective function. The first property provides that the disutility of an individual in the population zone $i \in M$ assigned to facility $j \in N$ is nondecreasing as the total number of individuals assigned to facility j is increasing. The second one is needed to have a convex MINLP.

Since linear functions are convex, all the constraints of our model are convex. Therefore we only need to guarantee that the objective function is convex.

Proposition 1. *If $f_j(z_j)z_j$ is convex $\forall j \in N$, then the objective function is convex.*

Proof. Since the utility function is defined as $U_{ij}(z_j) = f_j(z_j) + b_{ij}$, the objective function can be written as:

$$
\begin{aligned}
\sum_{i\in M}\sum_{j\in N} U_{ij}(z_j)d_i x_{ij} &= \sum_{i\in M}\sum_{j\in N}\left(f_j(z_j)+b_{ij}\right)d_i x_{ij} \\
&= \sum_{j\in N} f_j(z_j)\sum_{i\in M} d_i x_{ij} + \sum_{i\in M}\sum_{j\in N} b_{ij}d_i x_{ij} \\
&= \sum_{j\in N} f_j(z_j)z_j + \sum_{i\in M}\sum_{j\in N} b_{ij}d_i x_{ij}
\end{aligned}
$$

We know that the sum of convex functions is convex. Therefore as $\sum_{i\in M}\sum_{j\in N} b_{ij}d_i x_{ij}$ is convex by linearity, having $\sum_{j\in N} f_j(z_j)z_j$ convex guarantees that the objective function is convex.

□

3.3 Outer Approximation Algorithm

The Outer Approximation(OA) algorithm is a method to solve convex MINLP problems, (P) defined as:

$$
(P)\begin{cases}
\min h(x,y) \\
\ \text{s.t. } \ g(x,y) \le 0 \\
\qquad x \in \mathbb{R}^m \ , \ y \in \mathbb{Z}^n
\end{cases}
$$

where the functions $h : \mathbb{R}^{m+n} \to \mathbb{R}$ and $g : \mathbb{R}^{m+n} \to \mathbb{R}^r$ are twice continuously differentiable and $Y = \{y \in \mathbb{Z}^n : \exists x \in \mathbb{R}^m \text{ s.t } g(x,y) \le 0\}$ is bounded. It is assumed that the continuous relaxation of (P), referred as $\tilde{P}$ is also bounded. By defining a new variable, (P) can be reformulated as a nonlinear optimization problem with a linear objective function.

$$
(P)\begin{cases}
\min \alpha \\
\ \text{s.t. } \ h(x,y) \le \alpha \\
\qquad g(x,y) \le 0 \\
\qquad x \in \mathbb{R}^m \ , \ y \in \mathbb{Z}^n, \ \alpha \in \mathbb{R}
\end{cases}
$$

The OA algorithm alternates between solving MILP problems and convex NLP problems. The MILP problems are the relaxations of (P) that are obtained by replacing all the nonlinear constraints with their linearizations at some chosen points. We will refer to these relaxed problems as $P^{OA}(T)$ where T is the set of the chosen points.

$$P^{OA}(T) \begin{cases} \min \alpha \\ \text{ s.t. } \nabla h(x^k, y^k)^T \begin{pmatrix} x - x^k \\ y - y^k \end{pmatrix} + h(x^k, y^k) \leq \alpha \quad \forall (x^k, y^k) \in T \\ \qquad \nabla g(x^k, y^k)^T \begin{pmatrix} x - x^k \\ y - y^k \end{pmatrix} + g(x^k, y^k) \leq 0 \quad \forall (x^k, y^k) \in T \\ \qquad x \in \mathbb{R}^m \ , \ y \in \mathbb{Z}^n \ , \ \alpha \in \mathbb{R} \end{cases}$$

The convex NLP problems are either $P_{\tilde{y}}$ which is the original problem (P) with fixed $y = \tilde{y}$ or the feasibility problem $P^F_{\tilde{y}}$ with fixed $y = \tilde{y}$.

$$P_{\tilde{y}} \begin{cases} \min h(x, \tilde{y}) \\ \text{ s.t. } g(x, \tilde{y}) \leq 0 \\ \qquad x \in \mathbb{R}^m \end{cases} \qquad P^F_{\tilde{y}} \begin{cases} \min \sum_{i=1}^{r} u_i \\ \text{ s.t. } g(x, \tilde{y}) - u \leq 0 \\ \qquad u \geq 0 \\ \qquad x \in \mathbb{R}^m \ , \ u \in \mathbb{R}^r \end{cases}$$

The OA algorithm starts with $T = \{(x^0, y^0)\}$ where (x^0, y^0) is an optimal solution of $\tilde{P}$. Then each iteration k starts with solving the $P^{OA}(T)$ problem and obtaining an optimal solution $(\tilde{\alpha}^k, \tilde{x}^k, \tilde{y}^k)$. Let $y^k = \tilde{y}^k$. If P_{y^k} is feasible, let x^k be an optimal solution of P_{y^k} and add (x^k, y^k) to the set T. Note that (x^k, y^k) is a feasible solution for (P), so $h(x^k, y^k)$ gives an upper bound for (P) as it is a minimization problem. But if P_{y^k} is not feasible, then (x^k, y^k) is added to the set T where x^k is an optimal solution of the feasibility problem $P^F_{y^k}$. In each iteration since T gets larger, the relaxation gets stronger and gives better lower bounds for (P). The algorithm terminates when the gap between the lower bound and the upper bound is not more then some tolerance $\epsilon > 0$. The algorithm finds an optimal solution for (P) in a finite number of iterations provided that the assumptions on (P) are satisfied (see Theorem 1 in [22]). The algorithm is described in Figure 3.1. Next we illustrate the steps of the algorithm on an example.

$UB = +\infty$
$LB = -\infty$
(x^0, y^0) = optimal solution of $\tilde{P}$
T = $\{(x^0, y^0)\}$
$k = 1$; Choose a convergence tolerance ϵ
while $UB - LB > \epsilon$ and $P^{OA}(T)$ is feasible **do**
  Let $(\tilde{\alpha}^k, \tilde{x}^k, \tilde{y}^k)$ be the optimal solution of $P^{OA}(T)$
  $LB = \tilde{\alpha}^k$
  if $P_{\tilde{y}^k}$ is feasible **then**
    Let x^k be an optimal solution to P_{y^k}
    $UB = \min(UB, h(x^k, \tilde{y}^k))$
  else
    Let x^k be an optimal solution to $P^F_{y^k}$
  end if
  $y^k = \tilde{y}^k$
  $T = T \cup \{(x^k, y^k)\}$
  $k = k + 1$
end while

Figure 3.1: Outer Approximation Algorithm

3.3.1 Example

$$(P) \begin{cases} \min h(x) = (x - 3/4)^2 + (y - 1)^2 \\ \text{s.t. } \; g_1(x, y) = y + x^2 - 1 \leq 0 \\ \qquad g_2(x, y) = -y \leq 0 \\ \qquad x \in \mathbb{Z}, \; y \in \mathbb{R} \end{cases}$$

From KKT conditions:
$\nabla h(x, y) + \nabla g(x, y)u = 0 \;\; u \geq 0 \;,\; g(x, y) \leq 0 \;,\; u^T g(x, y) = 0$

The optimal solution of the NLP relaxation of (P), $\tilde{P}$, is $(x^*, y^*) = (\frac{1}{2}, \frac{3}{4})$

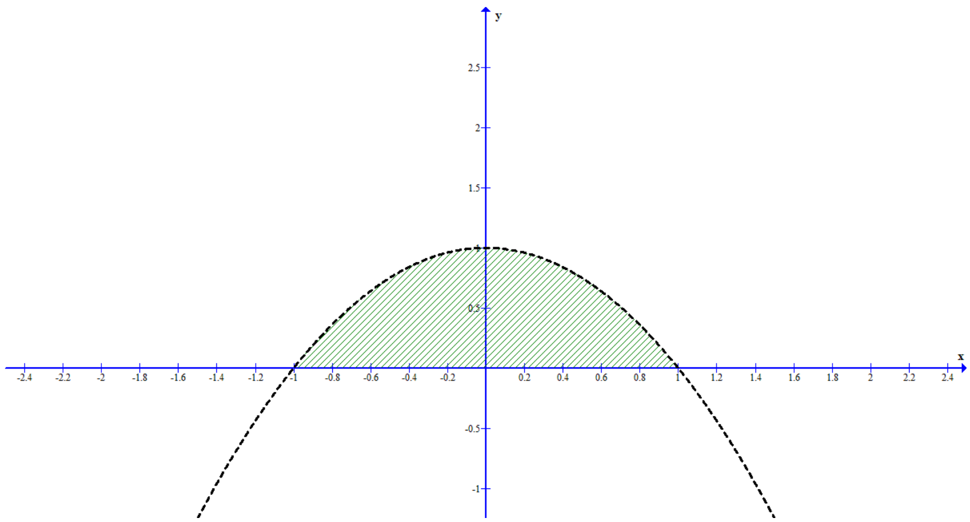


Figure 3.2: Feasible region of the original problem.

The initial feasible region is shown in Figure 3.2.

Applying the Outer Approximation algorithm:

First linearization point: $(x^0, y^0) = (\frac{1}{2}, \frac{3}{4})$

Linearizations:

$$\nabla h(x^0, y^0)^T \begin{pmatrix} x - x^0 \\ y - y^0 \end{pmatrix} + h(x^0, y^0) \leq \alpha \qquad \Rightarrow \qquad \frac{-x}{2} + \frac{-y}{2} + \frac{3}{4} \leq \alpha$$

$$\nabla g_1(x^0, y^0)^T \begin{pmatrix} x - x^0 \\ y - y^0 \end{pmatrix} + g_1(x^0, y^0) \leq 0 \qquad \Rightarrow \qquad x + y - \frac{5}{4} \leq 0$$

After first linearizations;

$T = \{(\frac{1}{2}, \frac{3}{4})\}$

$$P^{OA}(T) \begin{cases} \min \alpha \\ \text{s.t. } \frac{-x}{2} + \frac{-y}{2} + \frac{3}{4} \leq \alpha \\ \quad x + y - \frac{5}{4} \leq 0 \\ \quad -y \leq 0 \\ \quad x \in \mathbb{Z} \; y \in \mathbb{R} \end{cases}$$

Opt Solutions: $\{(x, y) : x \in \mathbb{Z}, x + y = 5/4\}$

Let $(\tilde{x}, \tilde{y}) = (-2, \frac{13}{4})$

$z^*_{P^{OA}(T)} = \tilde{\alpha} = 1/8$ is a lower bound on z^*_P

$z^L = 1/8$

Let S_0 denote the feasible set of the above problem.(S_0 is shown in Figure 3.5)

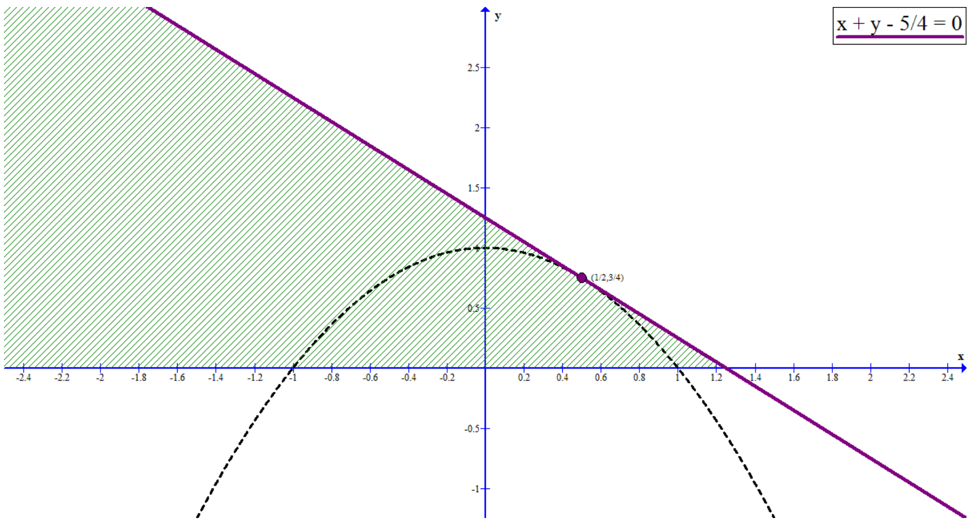


Figure 3.3: Feasible region after the first linearization.

Since $(-2, \frac{13}{4})$ is not feasible for P, continue with the algorithm.
As $P_{\tilde{x}}$ is infeasible solve the feasibility problem for $x = -2$, i.e solve $P^F_{\tilde{x}}$.

$$P^F_{\tilde{x}} \begin{cases} \min u_1 + u_2 \\ \text{ s.t. } \; y + 3 - u_1 \leq 0 \\ \qquad -y - u_2 \leq 0 \\ \qquad y \in \mathbb{R} \end{cases} \qquad \Rightarrow y^* = 0$$

Second linearization point: $(x^1, y^1) = (-2, 0)$
Linearizations:

$$\nabla h(x^1, y^1)^T \begin{pmatrix} x - x^1 \\ y - y^1 \end{pmatrix} + h(x^1, y^1) \leq \alpha \qquad \Rightarrow \qquad \frac{-11x}{2} - 2y - \frac{39}{16} \leq \alpha$$

$$\nabla g_1(x^1, y^1)^T \begin{pmatrix} x - x^1 \\ y - y^1 \end{pmatrix} + g_1(x^1, y^1) \leq 0 \qquad \Rightarrow \qquad -4x + y - 5 \leq 0$$

After second linearizations;
$T = \{(\frac{1}{2}, \frac{3}{4}), (-2, 0)\}$

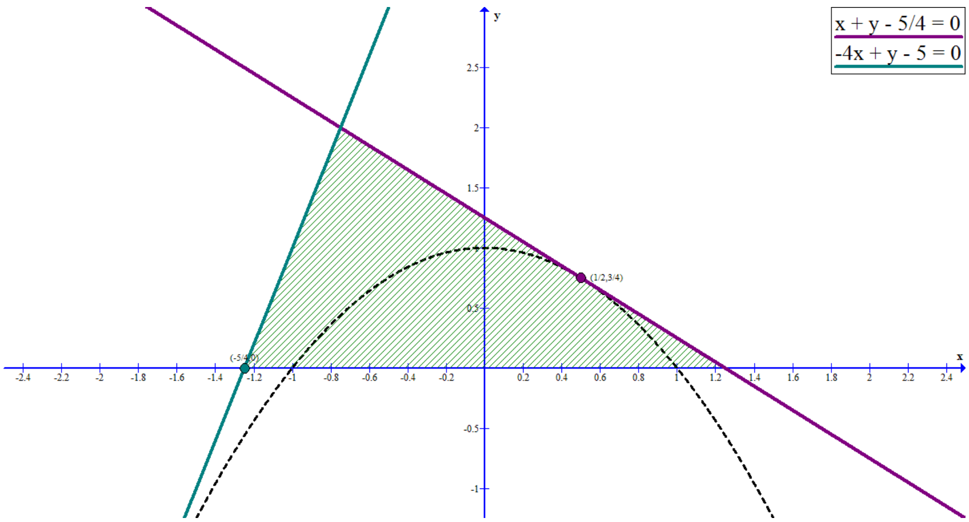


Figure 3.4: Feasible region after the second linearization.

$$P^{OA}(T)\begin{cases}\min \alpha \\ \text{s.t. } \frac{-11x}{2} - 2y - \frac{39}{16} \leq \alpha \\ \quad -4x + y - 5 \leq 0 \\ \quad (x, y) \in S_0\end{cases}$$

Opt Solutions: $\{(x, y) : x \in \mathbb{Z}, x + y = 5/4\}$

Let $(\tilde{x}, \tilde{y}) = (0, \frac{5}{4})$

$z^*_{POA(T)} = \tilde{\alpha} = 1/8$ is a lower bound on z^*_P

$z^L = 1/8$

Let S_1 denote the feasible set of the above problem. (S_1 is shown in Figure 3.4)

Since $(0, \frac{5}{4})$ is not feasible for P continue with the algorithm.
As $P_{\tilde{x}}$ is feasible solve $P_{\tilde{x}}$.

$$P_{\tilde{x}}\begin{cases}\min (-3/4)^2 + (y-1)^2 \\ \text{s.t. } y - 1 \leq 0 \\ \quad -y \leq 0 \\ \quad x \in \mathbb{Z}\ y \in \mathbb{R}\end{cases}$$

$y^* = 1$

$z^*_{P_{x=0}} = 9/16$ is an upper bound on z^*_P

$z^U = 9/16$

Third linearization point: $(x^2, y^2) = (0, 1)$
Linearizations:

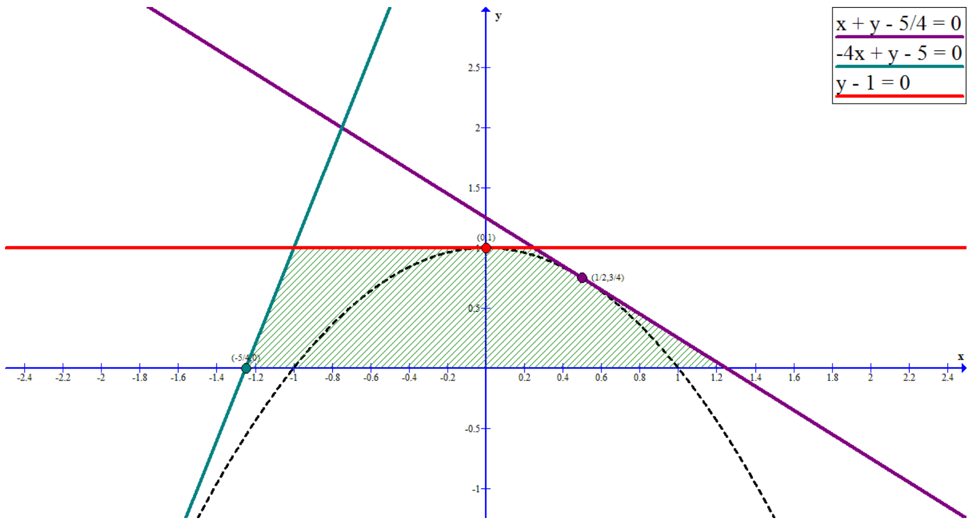


Figure 3.5: Feasible region after the third linearization.

$$\nabla h(x^2, y^2)^T \begin{pmatrix} x - x^2 \\ y - y^2 \end{pmatrix} + h(x^2, y^2) \leq \alpha \qquad \Rightarrow \qquad \frac{-3x}{2} + \frac{9}{16} \leq \alpha$$

$$\nabla g_1(x^2, y^2)^T \begin{pmatrix} x - x^2 \\ y - y^2 \end{pmatrix} + g_1(x^2, y^2) \leq 0 \qquad \Rightarrow \qquad y - 1 \leq 0$$

After third linearizations;

$T = \{(\frac{1}{2}, \frac{3}{4}), (-2, 0), (0, 1)\}$

$$P^{OA}(T) \begin{cases} \min \alpha \\ \text{s.t. } \frac{-3x}{2} + \frac{9}{16} \leq \alpha \\ \quad y - 1 \leq 0 \\ \quad (x, y) \in S_1 \end{cases}$$

Opt Solution: $(\tilde{x}, \tilde{y}) = (0, 1)$

$z^*_{P^{OA}(T)} = \tilde{\alpha} = 9/16$ is a lower bound on z^*_P

$z^L = 9/16$

Since $(0, 1)$ is feasible for (P) and $z^U = z^L = 9/16$, STOP.

Optimal Solution of P is $(x^*, y^*) = (0, 1)$ and Objective Value$= z^*_P = 9/16$.

3.4 Proposed Algorithms

Our problem is a linearly constrained MINLP. Therefore rather than using existing algorithms for MINLPs, we developed three different algorithms by exploiting the fact that our problem is a linearly constrained MINLP. These algorithms are modified versions of OA algorithm. Each algorithm is described separately in the following subsections.

3.4.1 OA1

OA1 algorithm is very similar to OA algorithm. As the problem we are trying to solve is linearly constrained, all the constraints of (P) will be included in $P^{OA}(T)$ problems as they are. Therefore for an optimal solution $(\tilde{\alpha}^k, \tilde{x}^k, \tilde{y}^k)$ of $P^{OA}(T)$ problem, $(h(\tilde{x}^k, \tilde{y}^k), \tilde{x}^k, \tilde{y}^k)$ is feasible for (P) (i.e., $P_{\tilde{y}^k}$ is feasible). Under these circumstances, the feasibility problem $P^F_{\tilde{y}^k}$ is never solved. Note that $\tilde{x}^k$ may not be an optimal solution for $P_{\tilde{y}^k}$ as $P^{OA}(T)$ is a relaxation of (P).

In the k^{th} iteration of OA1, the $P^{OA}(T)$ problem is solved. If it is not feasible the algorithm terminates with infeasibility. Otherwise we obtain an optimal solution $(\tilde{\alpha}^k, \tilde{x}^k, \tilde{y}^k)$ and lower bound is set to $\tilde{\alpha}$. Then the linearization point (x^k, y^k) is added to set T where x^k is an optimal solution of $P_{\tilde{y}^k}$ and $y^k = \tilde{y}^k$. Before moving to the next iteration, the upper bound value is updated if $h(x^k, y^k)$ is less than the current upper bound value. The algorithm stops when the gap between the upper bound and the lower bound is less than or equal to the chosen tolerance. The algorithm is described in Figure 3.6.

3.4.2 OA2

In OA2 algorithm, the aim is to find an optimal solution for MINLPs by solving only MILPs. The motivation behind the OA2 algorithm is that solving NLPs may be expensive. Therefore by eliminating NLPs, we may decrease the time to

$UB = +\infty$
$LB = -\infty$
$(x^0, y^0) =$ optimal solution of $\tilde{P}$
$T = \{(x^0, y^0)\}$
$k = 1$; Choose a convergence tolerance ϵ
while $\frac{UB-LB}{\max\{|LB|,1\}} > \epsilon$ **do**
  Let $(\tilde{\alpha}, \tilde{x}, \tilde{y})$ be an optimal solution of $P^{OA}(T)$
  $LB = \tilde{\alpha}$
  Let $y^k = \tilde{y}$ and x^k be an optimal solution to $P_{\tilde{y}}$
  $UB = \min(UB, h(x^k, y^k))$
  $T = T \cup \{(x^k, y^k)\}$
  $k = k + 1$
end while

Figure 3.6: OA1

solve our problem. For this purpose, we made some changes in OA1 algorithm.

In the k^{th} iteration of the OA2 algorithm, after obtaining an optimal solution $(\tilde{\alpha}^k, \tilde{x}^k, \tilde{y}^k)$ of $P^{OA}(T)$ (if it is feasible), $P_{\tilde{y}^k}$ problem is not solved. Instead the point $(\tilde{x}^k, \tilde{y}^k)$ is added to the set T directly (i.e., we don't choose the "best" x^k values for $\tilde{y}^k$). Lower bound values are updated similar to OA1 however the update of upper bound is different. If $h(\tilde{x}^k, \tilde{y}^k)$ is less than the current upper bound, then the upper bound value is updated. Note that $\tilde{\alpha}$ underestimates $h(\tilde{x}^k, \tilde{y}^k)$. Similar to OA1, the algorithm terminates with infeasibility if $P^{OA}(T)$ is infeasible in some iteration, and terminates with optimality if the gap between the upper bound and the lower bound is less than or equal to the chosen tolerance. The algorithm is described in Figure 3.7. Note that this algorithm may converge to an optimal solution in the limit for $\epsilon = 0$.

Example: Consider the following MINLP

$$\begin{cases} \min x^2 - 25y \\ \text{ s.t. } \; x + 6y \leq 1 \\ \qquad x \geq -2 \\ \qquad y \geq 0 \\ \qquad x \in \mathbb{R} \;, \; y \in \mathbb{Z} \end{cases}$$

The optimal solution of the NLP Relaxation problem is $(x, y) = (-2, 1/2)$ hence the first linearization point is $(-2, 1/2)$. Observe that the only integer solution for the problem is 0 (i.e. , $y = 0$). Therefore the problem is equivalent to

$$\begin{cases} \min x^2 \\ \text{ s.t. } \; -2 \leq x \leq 1 \\ \qquad x \in \mathbb{R} \end{cases}$$

When we iteratively solve the MILP problems of this problem, we have the following x values for $y = 0$:
$x = 1, -1/2, 1/4, -1/8, 1/16...$ and the corresponding lower and upper bound values are:
$UB = 1, 1/4, 1/16, 1/64, 1/256...$, $LB = -8, -2, -1/2, -1/8, -1/32...$
The gap between the upper bound and the lower bound is 0 in the limit.

However OA1 algorithm converges to the optimal solution with two iterations for the same problem. The steps followed by the OA1 algorithm are similar to those of OA2 until the selection of the second linearization point. Instead of choosing (1,0) as the second linearization point, OA1 finds the "best" x for $y = 0$. As the best x for y = 0 is 0, the second linearization point is selected to be (0,0). Then the upper bound is updated with the objective function value at point (0,0) (i.e., UB = 0). After the choice of the second linearization point, the MILP problem becomes;

$$
\begin{cases}
\min \alpha \\
\text{ s.t. } \ -4x - 4 - 25y \leq \alpha \\
\quad -25y \leq \alpha \\
\quad x + 6y \leq 1 \\
\quad x \geq -2 \\
\quad y \geq 0 \\
\quad x \in \mathbb{R}\ ,\ y \in \mathbb{Z}
\end{cases}
$$

Optimal solutions: $\{(x, y) : x \in [-2, 1] \text{ and } y = 0\}$
Optimal Objective Function Value = 0.
As we update the LB at the end of each MILP iteration, LB is set to 0. After this operation, LB = UB. Then we terminate with the optimal objective value 0 and optimal solution (0,0).

$UB = +\infty$
$LB = -\infty$
$(x^0, y^0) =$ optimal solution of $\tilde{P}$
$T = \{(x^0, y^0)\}$
$k = 1$; Choose a convergence tolerance ϵ
while $\frac{UB-LB}{\max\{|LB|,1\}} > \epsilon$ **do**
  Let $(\tilde{\alpha}, \tilde{x}, \tilde{y})$ be an optimal solution of $P^{OA}(T)$
  $LB = \tilde{\alpha}$
  Let $y^k = \tilde{y}$ and $x^k = \tilde{x}$
  $UB = \min(UB, h(x^k, y^k))$
  $T = T \cup \{(x^k, y^k)\}$
  $k = k + 1$
end while

Figure 3.7: OA2

3.4.3 OA3

The example in Section 3.4.2 shows that OA2 may solve more than one MILPs that have the same optimal integer solution. This is a weakness of the OA2

$UB = +\infty$
$LB = -\infty$
$(x^0, y^0) =$ optimal solution of $\tilde{P}$
$T = \{(x^0, y^0)\}$
$Y = \emptyset$
$k = 1$; Choose a convergence tolerance ϵ
while $\frac{UB-LB}{\max\{|LB|,1\}} > \epsilon$ **do**
  Let $(\tilde{\alpha}, \tilde{x}, \tilde{y})$ be an optimal solution of $P^{OA}(T)$
  $LB = \tilde{\alpha}$
  if $\tilde{y} \in Y$ **then**
    Let $y^k = \tilde{y}$ and x^k be an optimal solution to $P_{\tilde{y}}$
  else
    Let $y^k = \tilde{y}$ and $x^k = \tilde{x}$
  end if
  $UB = \min(UB, h(x^k, y^k))$
  $T = T \cup \{(x^k, y^k)\}$
  $Y = Y \cup \{y^k\}$
  $k = k + 1$
end while

Figure 3.8: OA3

algorithm compared to OA1 algorithm. Therefore in order to prevent this weakness, hybrid algorithm OA3 is developed. Different from OA2 algorithm, OA3 algorithm keeps track of the y^k values of the points chosen for the set T. If the same y^k appears in an optimal solution of one of the $P^{OA}(T)$ problems then the algorithm continues exactly the same with the OA1 algorithm. (x^k, y^k) is added to set T where x^k is an optimal solution of P_{y^k}. Otherwise similar to OA2 we just add the optimal solution of $P^{OA}(T)$ to set T. This guarantees that the algorithm will terminate in a finite time. The algorithm is described in Figure 3.8.

Chapter 4

Implementation and Computational Results

In this chapter, we explain the implementation details of the proposed algorithms OA1, OA2 and OA3. Then, the computational results of the algorithms are reported. Moreover we discuss the performance and the efficiency of the algorithms for several aspects.

4.1 Implementation

We implemented 3 different algorithms:

OA1 - OA algorithm for linearly constrained problems as presented in Section 3.4.1.

OA2 - Iteratively solving MILP problems as presented in Section 3.4.2.

OA3 - Enhanced version of OA2 algorithm as presented in Section 3.4.3.

4.1.1 Existing Software Components

During the implementation, we used some existing software components; for the NLPs, we used the interior point solver, Ipopt from COIN-OR and for the MILPs we used Cplex12.1. In order to use these solvers, we needed to input the problems with a specific format.

For Cplex, we create a .lp file which includes our model. Then by using the methods of cplex.h, Library Class of Cplex, we solve our problem. The solutions are written to arrays. Writing the model to an .lp file is enough for Cplex however for Ipopt some extra information is needed. For each different NLP problem, other than the model, the Hessian matrix, Jacobian matrix and the gradient of the objective function is needed. In order to solve P which is a MINLP, two NLP problems are implemented. First one is for the relaxation of P, $\tilde{P}$,and the second one is for the original problem for fixed y values, P_y. We will refer to these problems as *NlpRelaxation* and *NlpFixedY*, respectively.

4.1.2 Implementation Details

In this section, the variables and the arrays used in the implementation of the algorithms are given. Then the implementation details of the algorithms are explained through flow charts.

4.1.2.1 Variables

- UB - current upper bound to the problem
- LB - current lower bound to the problem
- zMIP - the objective function value of MILP problems (Result of Cplex)
- zNLP - the objective function value of NLP problems (Result of Ipopt)
- z - the objective function value of the original problem for the chosen linearization point $(\tilde{x}, \tilde{y})$.

- yValue - String representation of discrete variables. For example: $y =$ (1,0,0,1) is stored as ”1001”.

4.1.2.2 Arrays

In our implementation, we used the following arrays.

1. Optimal Solution of MILPs (Output of Cplex)
   - yMip, xMip
2. Optimal Solution of NLPs (Output of Ipopt)
   - yNlp, xNlp
3. Optimal Solution of the Problem (Output of the Algorithm)
   - yOpt, xOpt

xMip, xNlp and xOpt stores the continuous variables, whereas yMip, yNlp and yOpt stores the discrete variables.

4.1.2.3 Definitions for the Flow Charts

The implementation details of the algorithms are explained through the flow charts. The steps included in the flow charts are explained in this section.

1. INITIALIZE
   All three algorithms start with an initialization phase. In this phase, all the parameters are defined as well as the arrays and the variables to be used during the algorithms. Moreover the *UB* and *LB* variables get their initial values infinity and minus infinity, respectively.
2. NLP-RELAXATION
   In order to have the first linearization point $(\tilde{x}, \tilde{y})$, we solve the $NlpRelaxation$. The continuous variables are stored in $xNlp$ array.

3. UPDATE-MILP

   In the first execution of this phase which is after the $NLP-RELAXATION$ phase, we create our first MILP problem from scratch, but in the later executions we just update the MILP problem by adding new constraints which are the linearizations of the objective function for the chosen linearization point. At the end of this phase, new MILP problem is stored in *milp.lp* file.

4. CPLEX

   In this phase, the MILP problem created in the $UPDATE-MILP$ phase is solved by using Cplex. The *milp.lp* file is converted to a cplex problem and optimized by Cplex. Optimal solution (x^{mip}, y^{mip}) is stored in the $xMip$ and $yMip$ arrays and the objective function value is stored in $zMIP$.

5. UPDATE-LB

   In each iteration, since we add new constraints to our MILP problem in $UPDATE-MILP$ phase, our objective function value can not get smaller. So we directly update the LB and equalize to $zMIP$.

6. NLP-FIXED

   We solve $NlpFixedY$ problems in this phase in order to get the "best" x values for the given y values. This problem is always solved for the y^{mip} values and optimal solution of the problem x^{nlp} is stored in $xNlp$ array. Moreover the objective function value of the problem, $h(x^{nlp}, y^{mip})$ is stored in $zNLP$.

7. UPDATE-INCUMBENT

   If the objective function value for our new linearization point $(\tilde{x}, \tilde{y})$ is smaller than the current UB, output arrays and UB value are updated.
   $UB = h(\tilde{x}, \tilde{y})$, *xOpt* stores the $\tilde{x}$ values and *yOpt* stores the $\tilde{y}$ values.
   If this phase is executed after the $NLP - FIXED$ phase then the linearization point $(\tilde{x}, \tilde{y}) = (x^{nlp}, y^{mip})$ whereas if it is executed after $EVALUATE - Z$ phase then $(\tilde{x}, \tilde{y}) = (x^{mip}, y^{mip})$.

8. EVALUATE-Z

   Since we choose (x^{mip}, y^{mip}) as a linearization point in OA2, and in OA3 for some cases (i.e if the current integer solution was not optimal for the previous MILPs), we need $h(x^{mip}, y^{mip})$ value in order to update the UB value. This phase evaluates that value. Note that $zMIP$ underestimates this value.

9. EVALUATE-Y

   In this phase we convert the values stored in $yMip$ array to a string $yValue$.

10. CHECK-Y

    In this phase, we search through the linked list and check if the same integer solution was optimal for any of the previous MILP problems.

11. ADD-Y

    We add the current $yValue$ value to the linked list.

12. STOPPING-CRITERIA

    The algorithm stops when the gap between LB and UB value is less than or equal to ϵ. $(\frac{UB-LB}{\max\{|LB|,1\}} > \epsilon)$

4.1.2.4 OA1 Implementation

The OA1 algorithm starts with defining the parameters and variables of the algorithm in the $INITIALIZATION$ phase which is followed by $NLP-RELAXATION$ phase. After first linearization point $(\tilde{x}, \tilde{y})$ is found at the end of the $NLP-RELAXATION$ phase as an optimal solution of $NlpRelaxation$, we update our MILP as described in $UPDATE-MILP$. Until the stopping criterion is satisfied, current MILP is solved in the $CPLEX$ phase and an optimal solution (x^{mip}, y^{mip}) is stored in the arrays $xMip$ and $yMip$. If the MILP is not feasible, then the algorithm terminates by infeasibility. Otherwise, in the $UPDATE-LB$ phase we set LB value to the objective function value of the MILP, stored in $zMIP$ at the end of the $CPLEX$ phase and continue with the $NLP-FIXED$ phase in order to get the "best" x values for y^{mip}. If the objective function of the $NlpFixedY$ stored in $zNLP$ is less than the current UB value, we update the incumbent in $UPDATE-INCUMBENT$ phase. Finally, we update our MILP with the linearization point $(\tilde{x}, \tilde{y}) = (x^{nlp}, y^{mip})$ where x^{nlp}

is an optimal solution of the $NlpFixedY$. If the stopping criterion is satisfied the algorithm terminates and returns the current incumbent as optimal solution. The flow chart of OA1 is in Figure 4.1.

4.1.2.5 OA2 Implementation

OA2 algorithm implementation is exactly the same as the OA1 algorithm until the $NLP - FIXED$ phase of OA1. Instead of solving $NlpFixedY$ for y^{mip}, in OA2 $EVALUATE - Z$ phase is executed in which $z = h(x^{mip}, y^{mip})$ value is evaluated. If z is less than the current UB value, we update the incumbent in $UPDATE - INCUMBENT$ phase. Finally, we update our MILP with the linearization point $(\tilde{x}, \tilde{y}) = (x^{mip}, y^{mip})$. If the stopping criterion is satisfied the algorithm terminates and returns the current incumbent as optimal solution. The flow chart of OA2 is in Figure 4.2.

4.1.2.6 OA3 Implementation

Recall that OA3 solves $NlpFixedY$ if the same integer values happen to be optimal. Therefore we need to keep track of all the integer solutions in each iteration. The total number of different integer solutions of the problem is known (i.e., $\binom{n}{p}$). But how many of them will be optimal for the MILPs until the termination is not known. Thus, linked list is a better choice than array for storing the integer solutions. Each cell of the linked list has a string variable, $yValue$.

OA3 is a hybrid implementation of OA1 and OA2 algorithm. This algorithm works similar to the OA1 and OA2 algorithms up to $NLP - FIXED$ phase. After the $UPDATE - LB$ phase, in this algorithm, in order to check if the current y^{mip} solution was optimal for the previous MILPs, we first convert our solution into a string variable, $yValue$, in the $EVALUATE - Y$ phase and then check if $yValue$ is in the linked list or not. If it is in the linked list then the algorithm works like OA1. We solve $NlpFixedY$ problem for y^{mip}, update the

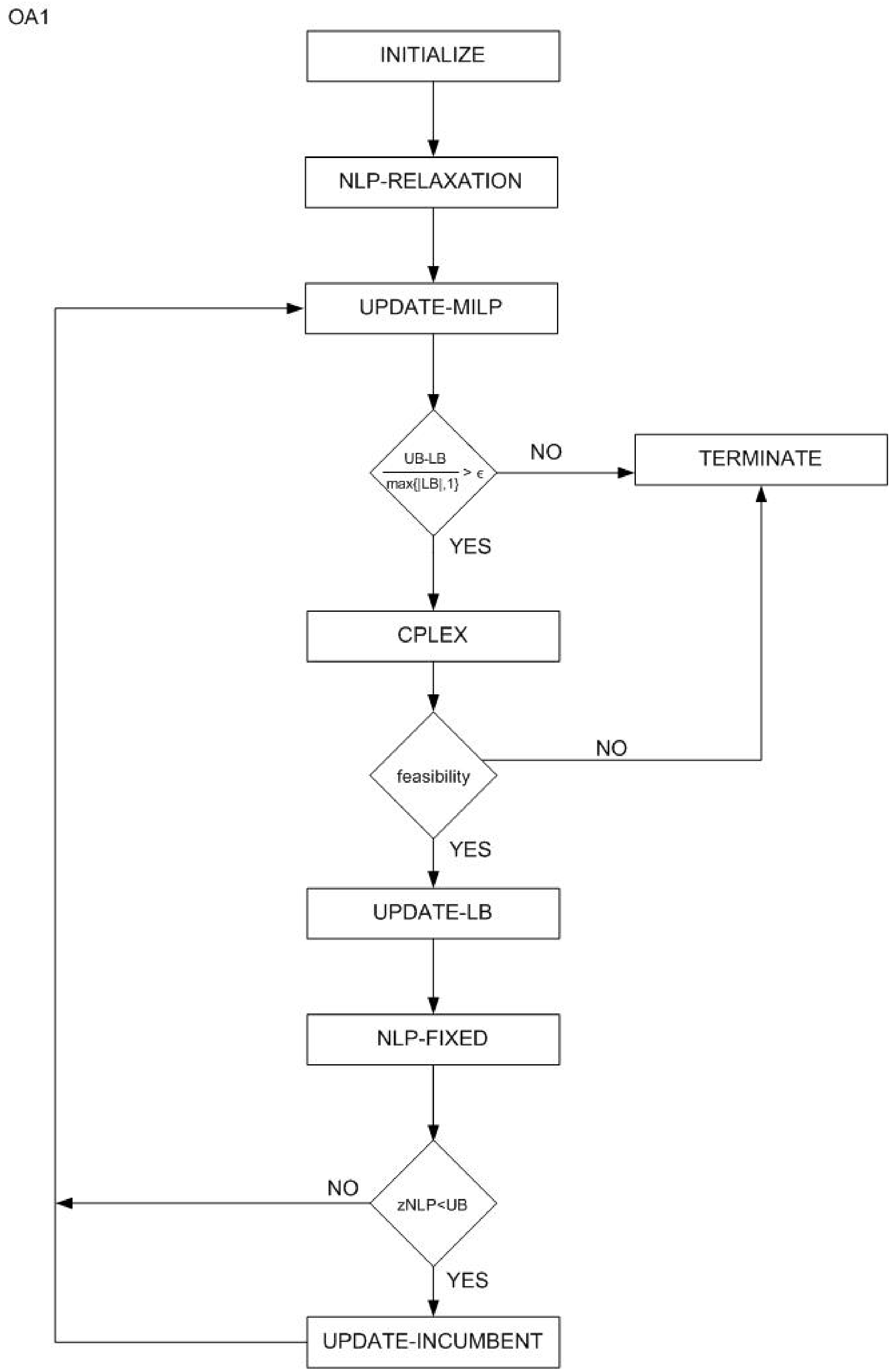


Figure 4.1: OA1 Flow Chart.

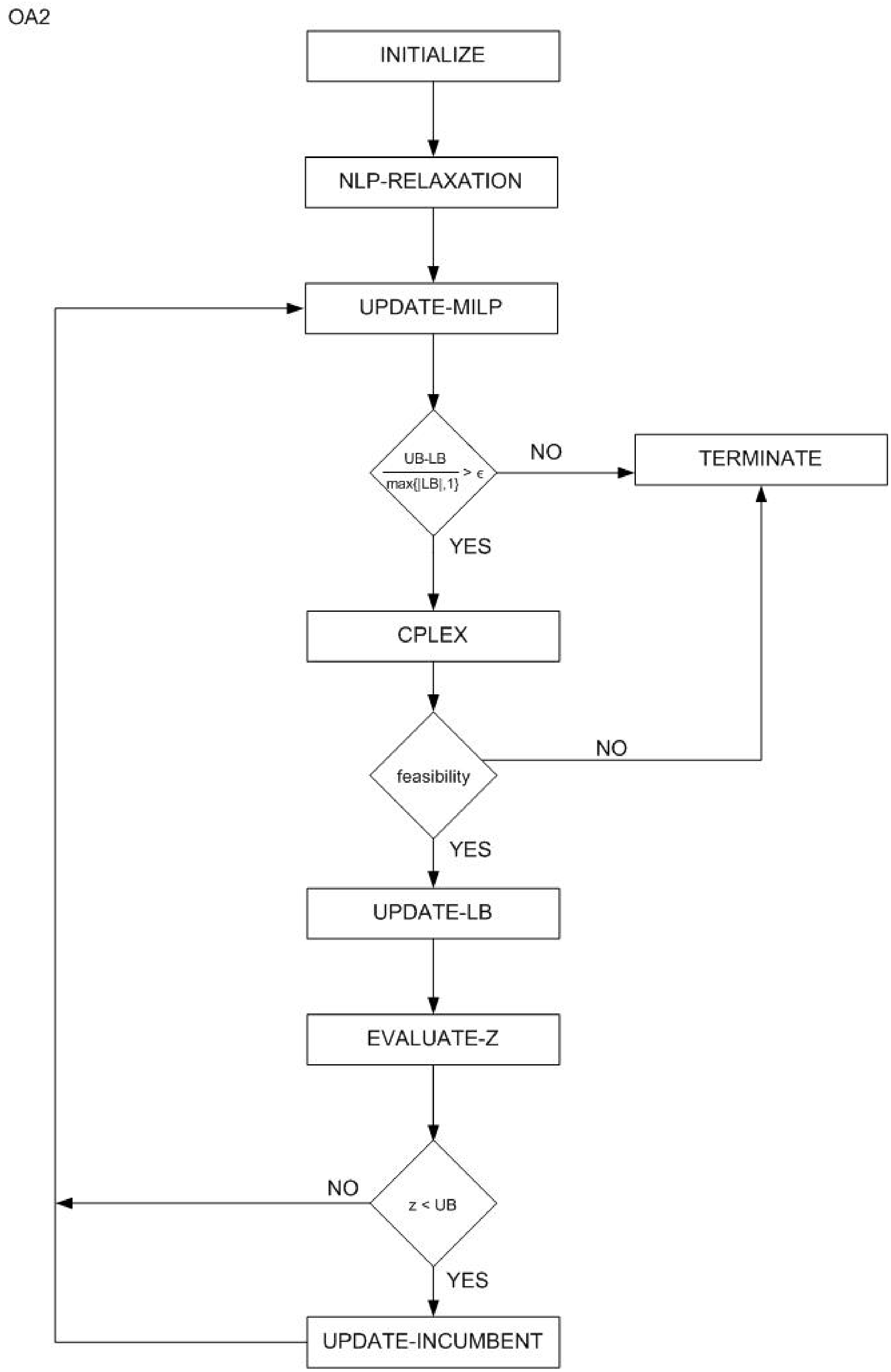


Figure 4.2: OA2 Flow Chart.

incumbent if $zNLP$ is less than the current UB value and the linearization point is (x^{nlp}, y^{mip}). Otherwise if $yValue$ is not in the linked list, then the algorithm works like OA2. We evaluate $z = h(x^{mip}, y^{mip})$, update the incumbent if z is less than the current UB and the linearization point is (x^{mip}, y^{mip}). Additionally we add y^{mip} to the list if it was not in the list. If the stopping criterion is satisfied the algorithm terminates and returns the current incumbent as optimal solution. The flow chart of OA3 is in Figure 4.3.

4.1.2.7 *CpMP* Specific Details

The implemented algorithms can be adopted to all linearly constrained convex MINLPs. However for each problem, the necessary information for using Ipopt should be provided. These are described in Section 4.1.1. Moreover how to linearize the objective function should also be given to the program. For *CpMP*, we linearize each nonlinear term $z_j f_j(z_j)$ in the objective function separately by introducing new variables $t_j \quad \forall j \in N$.
New objective function: $\sum_{j \in N} t_j + \sum_{i \in M} \sum_{j \in N} b_{ij} d_i x_{ij}$
Additional Constraints: $z_j f_j(z_j) \leq t_j \quad \forall j \in N$
Linearizations of the additional constraints at point $(\tilde{z}, \tilde{t})$:
$(f_j'(\tilde{z}_j)\tilde{z}_j + f_j(\tilde{z}_j))z_j - t_j \leq f_j'(\tilde{z}_j)\tilde{z}_j^2 \quad \forall j \in N$

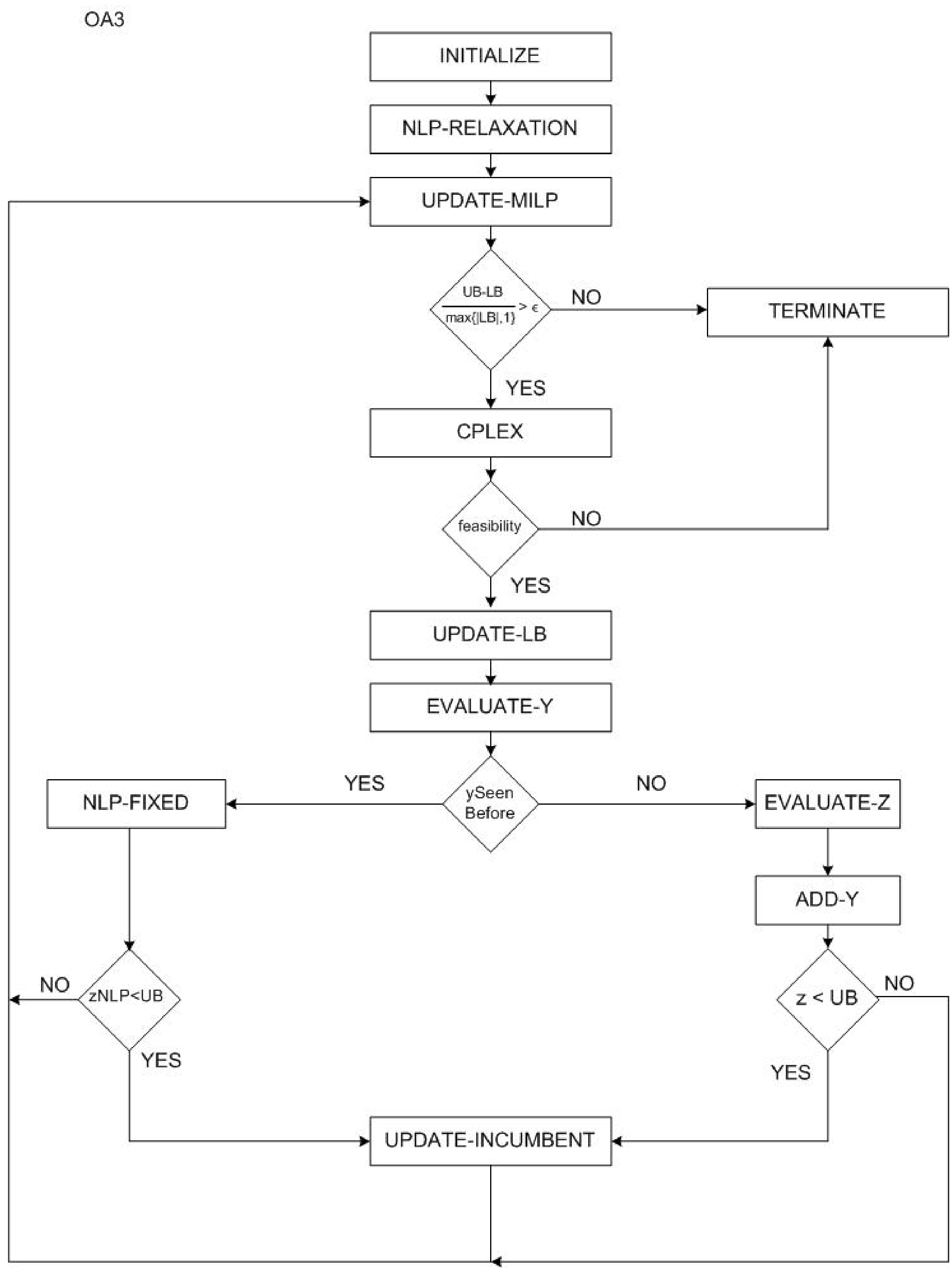


Figure 4.3: OA3 Flow Chart.

4.2 Computational Results

In this section, we report our computational results. All the experiments are carried out on a Intel(R) Core(TM) i7 CPU Q720 processor with a clock speed of 1.60GHz and a 8 GB RAM running under Windows 7 Home Premium. The algorithms are implemented, executed and run in the C++ environment using Microsoft Visual Studio 2008.

4.2.1 Computational Setup

We initially tested the efficiency of our algorithms with the disutility function:

$$U_{ij}(z_j) = a_j z_j + b_{ij} \text{ with } a_j \geq 0$$

We will refer to this disutility function as the *Linear Disutility Function*. The a_j value is the rate of penalty of facility j for the congestion and the b_{ij} is the distance from population zone $i \in M$ to a facility located in $j \in N$.

a_j is a facility dependent parameter due to the fact that the tolerance of each facility to the congestion may change. In our computational work in order to decide the a_j values, we used two different factors; the facility types and the facility constants. The facility type stands for how well a facility tolerates the congestion. The facility constant is for the observation of the effect of travel time and the congestion to the disutility function. There are three different facility types and four different facility constants. The types are 1, 2, 3 and the constants are 0, 0.1, 0.01, 0.001. The a_j value is obtained by the multiplication of the facility constant with the facility type. We considered three different kinds of instances; in the first one all facilities are type 1, in the second one a facility can be either type 1 or 2 and in the third one a facility may be one of the three different types. After we randomly choose the types of the facilities in all three problems, we tested each of them with three different facility constants. This results in 9 different sets of a_j values which are shown in Table 4.1. We will refer to each set as Ai for $i = 1, \ldots 9$. Furthermore we tested the instance with all a_j values equal to 0, which corresponds to p-median problem.

Input	# of types	Facility Constant	Possible a_j values
A1	1	0.1	0.1
A2	2	0.1	0.1, 0.2
A3	3	0.1	0.1, 0.2, 0.3
A4	1	0.01	0.01
A5	2	0.01	0.01, 0.02
A6	3	0.01	0.01, 0.02, 0.003
A7	1	0.001	0.001
A8	2	0.001	0.001, 0.002
A9	3	0.001	0.001, 0.002, 0.003

Table 4.1: Input sets for a_j values.

For all of our instances, we assumed that a facility can be located to any of the population zones (i.e., $n = m$). Our original data has 80 population zones to be served. However in order to assess the performance of our algorithms with respect to input size, we generated four data sets with sizes $m \in \{20, 40, 60, 80\}$. Our data is taken from [23] which originally has 84 population zones. Four of the zones in the original data were divided into two for the sake of their study. Before using the data, we merged those four pairs ((13-14), (41-42), (51-52), (82-83)). Then we renumbered the zones.

For each data set we generated three different instances with p equal to the $5\%, 10\%$ and 20% of m. Note that the b_{ij} and d_i values for $i \in M$ and $j \in N$ are fixed for different instances with the same n value. With these different sets of parameters we define 27 different instances. These instances are generated with the combinations of Ai and p values. Three consecutive instances have the same Ai but different p values. The parameters of the instances are shown in Table 4.2.

Other than the problem parameters, there is also an algorithmic parameter; the tolerance of the algorithm. To measure the impact of the tolerance for the

PARAMETERS of INSTANCES								
1	2	3	4	5	6	7	8	9
A1,5%	A1,10%	A1,20%	A2,5%	A2,10%	A2,20%	A3,5%	A3,10%	A3,20%
10	11	12	13	14	15	16	17	18
A4,5%	A4,10%	A4,20%	A5,5%	A5,10%	A5,20%	A6,5%	A6,10%	A6,20%
19	20	21	22	23	24	25	26	27
A7,5%	A7,10%	A7,20%	A8,5%	A8,10%	A8,20%	A9,5%	A9,10%	A9,20%

Table 4.2: Parameters of each instance.

proposed algorithms, we tested our algorithms for three different tolerances; $\epsilon = 0.01, 0.001$ and 0.0001.

Furthermore, we tested the efficiency of the proposed algorithms on the nonlinearity of the problem with a new disutility function referred as *Nonlinear Disutility Function*.

$$U'_{ij}(z_j) = a_j z_j^3 + b_{ij} \text{ and } a_j \geq 0$$

We tested the *Nonlinear Disutility Function* with the same input set of *Linear Disutility Function* with a slight difference of scaling a_j values by 10^6 (i.e, the new facility constants are $10^{-7}, 10^{-8}, 10^{-9}$). The reason is that as we take the third exponent of the total demand served by a facility in the *Nonlinear Disutility Function*, the linear part is dominated by the nonlinear part. In other words, travel time has no impact in the disutility function. We set the tolerance to 0.001 for testing the *Nonlinear Disutility Function*.

4.2.2 Experimental Results and Discussion

The results of the computational experiments are summarized in this section. We will compare the performances of the algorithms for several aspects. A discussion about the p-median and the *CpMP* problem is also included. Finally an overall discussion will be given.

4.2.2.1 Linearization of the Nonlinear Terms

As the proposed algorithms work with the linearizations of nonlinear terms, how we linearize the objective function may have an impact on the proposed algorithms. Any subset of the nonlinear terms can be linearized by a single variable but we only look at two extreme cases. First one is linearizing each term separately which generates p linearizations in each iteration whereas the second case is linearizing the summation of the terms by a single variable and this creates only one linearization. The first case results in stronger but larger MILPs after each iteration. On the other hand in the second case MILP grows slowly but the linearizations are weak.

In order to observe the trade-off between the number of linearizations and the strength of them, we solved our instances with respect to the first and second linearization cases. The results show that it takes days to converge to an optimal solution with the single linearization case while all the instances are solved within 30 minutes with separate linearizations case. We used the instances with n = 60 for this experiment and put a time limit of 10 minutes for each problem. The maximum computation time with separate linearization case for $n = 60$ is 10 minutes. The results are shown in Tables 4.3, 4.4, 4.5. In the tables, for the separate linearization case, only the running time of the algorithm is shown. But for the single linearization case, if the algorithm did not terminate before the time limit then the gap at the end of 10 minutes is reported. Otherwise the running time is presented in the tables.

When the results in Tables 4.3, 4.4, 4.5 are interpreted, we observed that the interrupted instances are more than the instances with running times less than 10 minutes. Additionally even though the single linearization case terminates before the time limit, the running time of the algorithm is much more than the running time of the separate case. As a consequence of these results, we decided to linearize each nonlinear term separately for the rest of our experiments.

A	p	Algorithm	Single Linearization	Seperate Linearization
A1	3	OA1	4,944%	94,677
		OA2	4,961%	471,838
		OA3	4,961%	472,696
	6	OA1	3,197%	175,048
		OA2	3,249%	222,426
		OA3	3,249%	222,082
	12	OA1	1,840%	188,068
		OA2	1,905%	238,432
		OA3	1,905%	242,915
A4	3	OA1	0,594%	65,982
		OA2	0,597%	71,163
		OA3	0,597%	74,620
	6	OA1	0,474%	42,223
		OA2	0,474%	37,738
		OA3	0,474%	40,309
	12	OA1	0,304%	24,639
		OA2	0,302%	20,936
		OA3	0,302%	23,319
A7	3	OA1	119,412	11,035
		OA2	87,715	7,609
		OA3	87,232	7,020
	6	OA1	105,280	11,985
		OA2	78,322	9,951
		OA3	78,440	10,637
	12	OA1	61,811	9,095
		OA2	35,251	5,155
		OA3	34,919	5,631

Table 4.3: Single Linearization Results for n=60, A1-A4-A7

A	p	Algorithm	Single Linearization	Seperate Linearization
A2	3	OA1	4,553%	260,497
		OA2	4,554%	333,886
		OA3	4,554%	334,968
	6	OA1	2,906%	152,960
		OA2	2,945%	200,841
		OA3	2,945%	203,128
	12	OA1	1,710%	99,178
		OA2	1,733%	91,230
		OA3	1,733%	87,518
A5	3	OA1	0,577%	73,726
		OA2	0,572%	84,443
		OA3	0,572%	85,520
	6	OA1	0,471%	49,717
		OA2	0,473%	54,241
		OA3	0,473%	56,425
	12	OA1	0,303%	22,012
		OA2	0,327%	20,405
		OA3	0,327%	20,811
A8	3	OA1	79,848	11,575
		OA2	59,525	7,535
		OA3	59,514	7,924
	6	OA1	0,017%	25,397
		OA2	0,017%	14,913
		OA3	0,017%	16,443
	12	OA1	212,854	8,346
		OA2	118,735	5,787
		OA3	118,858	6,568

Table 4.4: Single Linearization Results for n=60, A2-A5-A8

A	p	Algorithm	Single Linearization	Seperate Linearization
A3	3	OA1	4,208%	183,818
		OA2	4,195%	262,891
		OA3	4,195%	252,887
	6	OA1	2,580%	125,688
		OA2	2,627%	154,461
		OA3	2,627%	156,472
	12	OA1	1,501%	119,730
		OA2	1,419%	69,126
		OA3	1,419%	69,518
A6	3	OA1	0,574%	86,424
		OA2	0,563%	78,749
		OA3	0,563%	78,640
	6	OA1	0,484%	46,363
		OA2	0,464%	46,801
		OA3	0,464%	47,643
	12	OA1	0,368%	29,203
		OA2	0,346%	19,594
		OA3	0,346%	19,157
A9	3	OA1	480,210	43,041
		OA2	418,565	37,237
		OA3	418,341	37,846
	6	OA1	0,012%	21,075
		OA2	0,009%	14,228
		OA3	0,009%	13,509
	12	OA1	0,017%	13,213
		OA2	0,016%	11,598
		OA3	0,016%	10,983

Table 4.5: Single Linearization Results for n=60, A3-A6-A9

4.2.2.2 Experimental Results

The software package for the proposed algorithms output various results to the users. These are the running time of the algorithm, located facilities, optimal assignments, total demand served by each facility, optimal objective value and the contribution of linear and nonlinear part to the objective. Moreover the number of NLP and MILP iterations and the total time spent on solving these iterations are generated separately by the software package.

Located facilities, optimal assignments and the total demand served by each facility can be used to observe different behaviors of the p-median and *CpMP* problems while selecting the facilities to be located and assigning the population zones to the located facilities. Equivalently, from these results, we can deduce the effectiveness of the disutility function.

Linear and nonlinear part of the objective function can give an idea about if the travel time dominates the waiting time or vise versa. Besides, these results may enable us to interpret the performances of the algorithms depending on nonlinearity.

The number of NLP and MILP iterations shows the increase in the number of MILPs as a result of eliminating NLP iterations in the OA2 algorithm. Similarly, these results can also indicate the impact of reducing the NLP iterations in OA3 algorithm. Furthermore, the number of NLP iterations for the OA3 algorithm represents the number of different facility sets that were optimal for two different MILP problems. For the OA1 algorithm, the time spent on solving MILP and NLP iterations can give an insight about the hardness of the MILPs and NLPs. The reason is that the number of iterations for each problem type is equal.

Lastly, the running time corresponds to the speed of algorithms. These results can be used to compare the performances of the three proposed algorithms. While comparing two algorithms, we will use the ratio of their running times. The ratio will be shown as OAi/OAj for $i, j \in \{1, 2, 3\}$.

Some of the results are given during the discussions and the rest of them are in Appendix.

4.2.2.3 Proposed Algorithms vs. CPLEX 12.1

In order to assess the efficiency of the proposed algorithms, we solved our instances with an off-the-shelf solver CPLEX12.1 using OPL IDE as an interface. We set the tolerance of the algorithms equal to the tolerance of CPLEX12.1 which is 0.0001. In order to measure the performance of CPLEX12.1, as a start we used a small sample which contains instances of all sizes 20, 40, 60, 80. The results of this sample show that for the larger inputs, CPLEX12.1 can not converge to a solution within hours. Therefore we put a time limit of 10 minutes for $n = 20, 40, 60$ and 30 minutes for $n = 80$. These limits are the maximum running time of the proposed algorithms for the corresponding instance sets.

After the experiment is completed for all of our instances, it is observed that CPLEX12.1 solves almost all the instances with $n = 20$ faster than the proposed algorithms. Yet, it is difficult to reach a conclusion regarding the efficiency of the algorithms for instances with $n = 40$. The reason is that the results are variable in aspect of running time comparison of CPLEX12.1 and the proposed algorithms. In addition most of the instances with $n = 60, 80$ can not be solved by CPLEX12.1 within their time limit. Therefore the proposed algorithms are more efficient than CPLEX12.1 for these instances due to the selection criteria of the time limit. According to these observations, we can declare that the performance of CPLEX12.1 decreases as the input size increases. As a result, the proposed algorithms are more efficient than CPLEX12.1 for large instances. Figure 4.4 represents these results visually for instances $n = 20, 40, 60, 80$. The instances on the figure are as described in Section 4.2.1.

It can be recognized from Figure 4.4 that the running time of CPLEX12.1 gets closer to the running time of the proposed algorithms for the instances from 19 to 27. In fact the Figure 4.5 shows that CPLEX12.1 beats the proposed algorithms for these instances. The common characteristic of these instances is

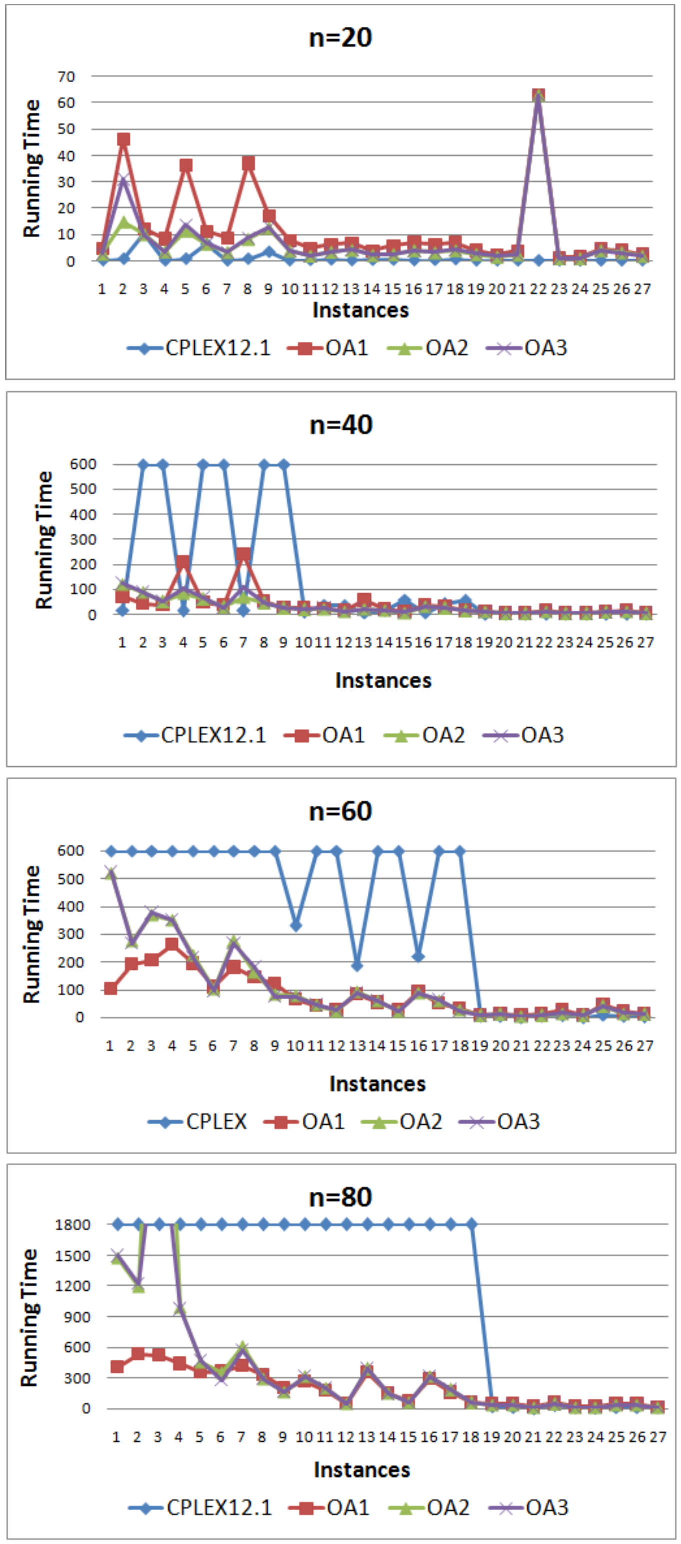


Figure 4.4: CPLEX12.1 vs. Proposed Algorithms

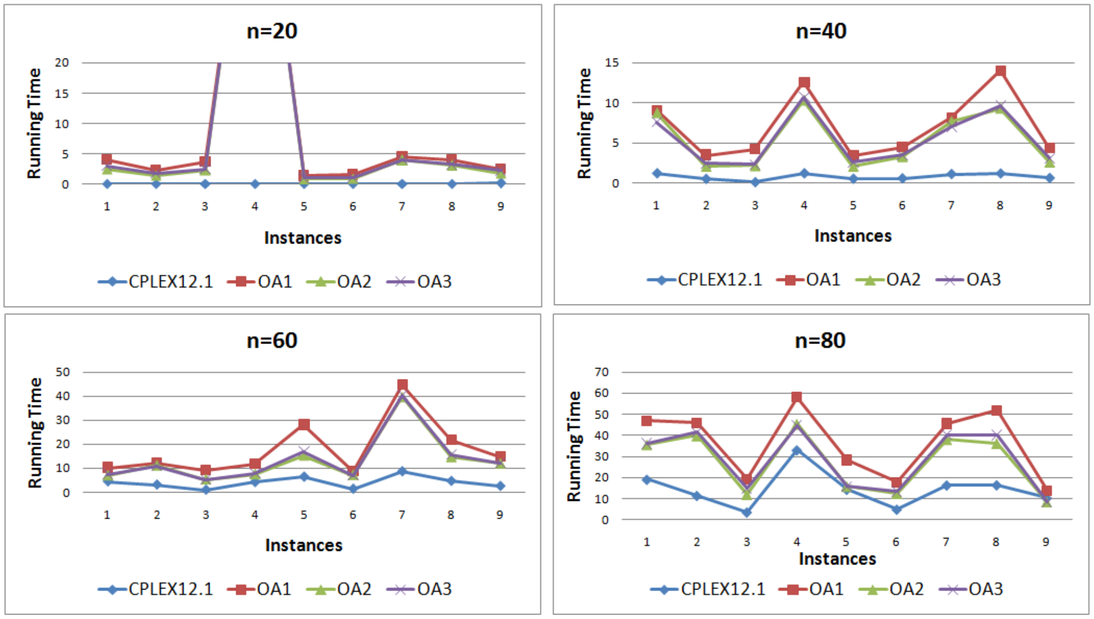


Figure 4.5: CPLEX12.1 vs. Proposed Algorithms for instances 19 - 27

that of having the facility constant equal to 0.001. Since the facility constant is so small, the contribution of the nonlinear part to the objective function is relatively minor. Hence, the *CpMP* problem behaves similar to p-median problem which is an MILP.

4.2.2.4 Effect of Facility Constant

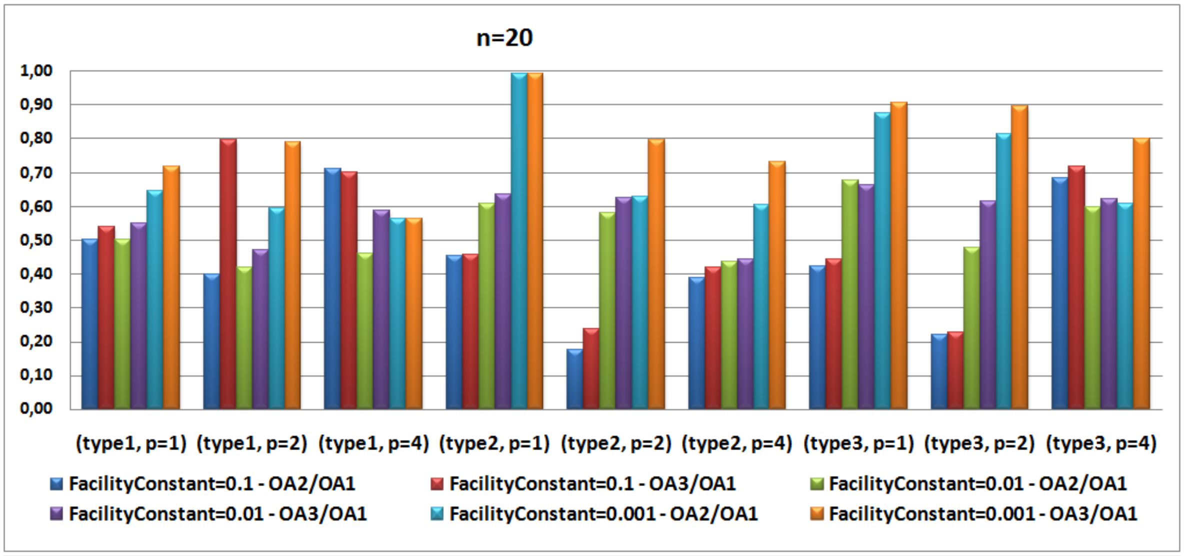
In this section, the impact of facility constant on the proposed algorithms is examined. The figure showing the results for instances with $n = 20, 40, 60, 80$ is 4.2.2.4. The figure is structured in a way that the impact of facility constants can be analysed easily. For each (facility type, p) pair OA3/OA1 and OA2/OA1 values are plotted on the graph for three different facility constants.

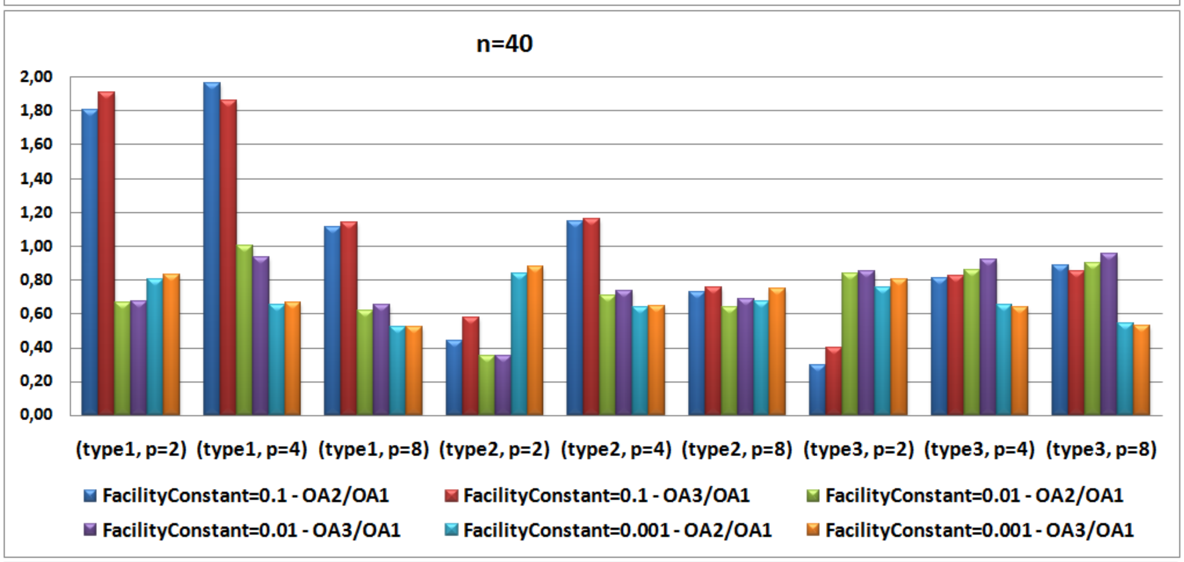
For all the instances with n = 20, the OA3 and OA2 algorithms converges to an optimal solution before the OA1 algorithm. When the results are analysed, no specific pattern about the performances of the algorithms is deduced from the Figure 4.2.2.4 as the data varies a lot.

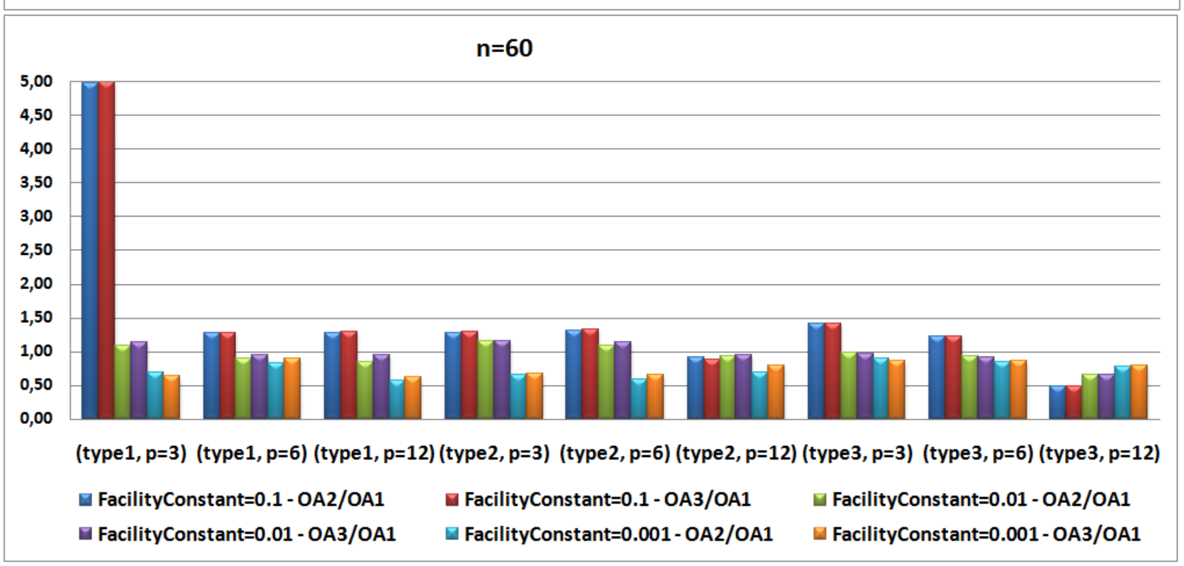
However for the instances with n = 40, 60 and 80, we may have a comment on subsets of the instances. In order to observe the impact of just the facility constant, we analysed the instances that have only one facility type. (These are referred in the figures as the ones that have type 1 in their definitions.) The analysis indicates that as the facility constant decreases, OA2 and OA3 performs better than OA1. Likewise for the instances (type1, p=4), (type 2, p=4) and (type 3, p=4), the running time of OA2 and OA3 algorithms gets closer to the running time of the OA1 algorithm for smaller facility constants. Lastly, observe that the most of the instances OA1 beats OA2 and OA3 are the ones with facility constant 0.1.

4.2.2.5 Effect of p

In this section, we examined the impact of p value. We tested our instances for three different p values. For each different value of n, p gets the values 5%, 10%, 20% of n. When the results are analysed, it is seen that as p increases, the number of iterations decreases. Accordingly the running time of the algorithms also decreases. As we mentioned before the running time of the algorithms represents the hardness of the instances. Therefore we may say that the instances with larger p values are easier than the ones with small p values. This is due to the fact that as the p value increases, the contribution of the nonlinear part to the objective function decreases. For each different algorithm, we present the number of iterations of the instances with $n = 80$ in Figure 4.7. This sample is representative of all instances witn $n = 40, 60$ and 80. It is difficult to comment on instances with $n = 20$ as the results are various. In this figure, for the same Ai parameter, three different p values are shown consecutively. Note that for all of the algorithms only the number of MILP iterations are reported. (For OA3 the number of NLP iterations are negligible compared to MILP iterations. For OA1, total number of NLP iterations are exactly the same with MILP iterations. There is no NLP iterations in OA2 algorithm.)


n=20
1,00
0,90
0,80
0,70
0,60
0,50
0,40
0,30
0,20
0,10
0,00
(type1, p=1) (type1, p=2) (type1, p=4) (type2, p=1) (type2, p=2) (type2, p=4) (type3, p=1) (type3, p=2) (type3, p=4)
FacilityConstant=0.1 - OA2/OA1
FacilityConstant=0.1 - OA3/OA1
FacilityConstant=0.01 - OA2/OA1
FacilityConstant=0.01 - OA3/OA1
FacilityConstant=0.001 - OA2/OA1
FacilityConstant=0.001 - OA3/OA1


n=40
2,00
1,80
1,60
1,40
1,20
1,00
0,80
0,60
0,40
0,20
0,00
(type1, p=2) (type1, p=4) (type1, p=8) (type2, p=2) (type2, p=4) (type2, p=8) (type3, p=2) (type3, p=4) (type3, p=8)
FacilityConstant=0.1 - OA2/OA1
FacilityConstant=0.1 - OA3/OA1
FacilityConstant=0.01 - OA2/OA1
FacilityConstant=0.01 - OA3/OA1
FacilityConstant=0.001 - OA2/OA1
FacilityConstant=0.001 - OA3/OA1


n=60
5,00
4,50
4,00
3,50
3,00
2,50
2,00
1,50
1,00
0,50
0,00
(type1, p=3) (type1, p=6) (type1, p=12) (type2, p=3) (type2, p=6) (type2, p=12) (type3, p=3) (type3, p=6) (type3, p=12)
FacilityConstant=0.1 - OA2/OA1
FacilityConstant=0.1 - OA3/OA1
FacilityConstant=0.01 - OA2/OA1
FacilityConstant=0.01 - OA3/OA1
FacilityConstant=0.001 - OA2/OA1
FacilityConstant=0.001 - OA3/OA1

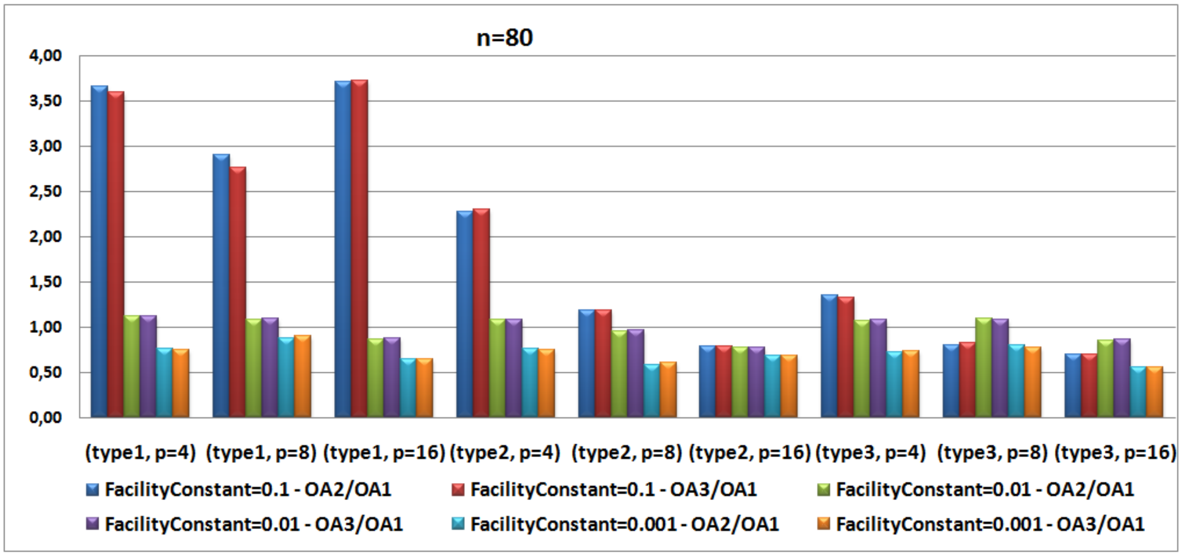


Figure 4.6: Running Time Comparison For Different Facility Constants

4.2.2.6 Effect of Nonlinear Utility Function

As mentioned in Section 4.2.1, for the *Nonlinear Disutility Function*, we scaled the a_j values by 1.000.000. As a result, there is not any common instance solved for both disutility functions. Hence, in this section we will examine the *Nonlinear Disutility Function* in itself. We discuss the effect of p and the input size.

When we analysed the effect of p on *Nonlinear Disutility Function*, the results showed that as p increases, the number of iterations decreases. This observation is exactly the same with the *Linear Disutility Function* case which is explained in Section 4.2.2.5. Furthermore, we noticed that as the input size increases the OA1 algorithm performs better than OA2 and OA3 algorithms. The results of the instances solved by using the *Nonlinear Disutility Function* are shown in tables from A.41 to A.52 in Appendix.

4.2.2.7 Effect of Tolerance

In this section, the effect of the tolerance of the algorithms is analysed. As already mentioned before we tested our instances with three different ϵ values;

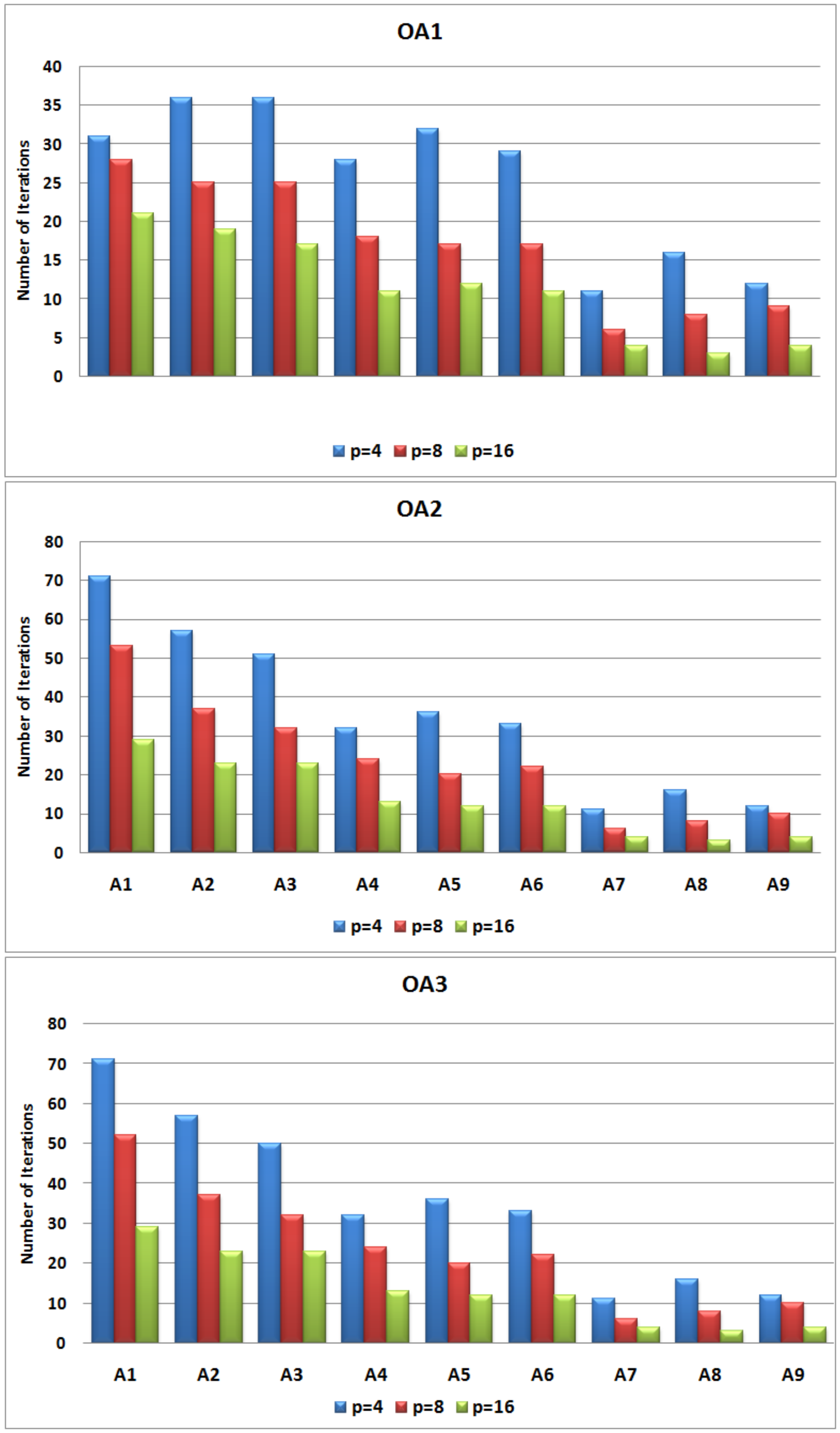


Figure 4.7: Number of Iterations for instances n=80

	Extra MILPs				Extra NLPs		Total Increase in Running Time
	1	2	3	4	1	2	
OA1	2,227	2,403	-	-	0,39	1,123	6,143
OA2	2,995	2,902	3,073	3,276	-	-	12,246
OA3	3,027	2,886	3,089	3,307	0,842	1,077	14,228

Table 4.6: Running time of extra iterations for n=60, A5, p=6.

$\epsilon \in \{0.01, 0.001, 0.0001\}$ and the input size is $n = 60$. The results are illustrated in Tables 4.7, 4.8, 4.9. The OA2/OA1, OA3/OA1 values as well as the number of iterations for all of the algorithms are shown for all three tolerances. The OA2/OA1 and OA3/OA1 values in the tables represents the proportion of the algorithms' running times. Expectedly the number of iterations solved for each problem increases as the tolerance decreases in all of the three algorithms. Moreover it can be understood from the tables that the number of instances OA1 performs better than OA2 and OA3 increases as the tolerance decreases. This may be due to the running time of extra MILPs, or the increase on the total number of iterations of the algorithms. It generally takes more time to solve the extra MILPs of OA2 and OA3 algorithms as they are larger than the extra MILPs of OA1. A proper instance to show this relationship between the algorithms is the instance with parameters n=60, A5, p=6. OA2 and OA3 performs better than OA1 for $\epsilon = 0.01$ but they perform worse for $\epsilon = 0.001$ or 0.0001 for this specific example. The total number of extra iterations of the algorithms OA1, OA2 and OA3 for $\epsilon = 0.001$ is 4, 4 and 6 respectively. The running times of them are shown in Table 4.6.

			$\epsilon = 0.01$				$\epsilon = 0.001$				$\epsilon = 0.0001$			
A	p	Alg	OA2/OA1	OA3/OA1	MIP	NLP	OA2/OA1	OA3/OA1	MIP	NLP	OA2/OA1	OA3/OA1	MIP	NLP
A1	3	OA1			25	25			26	26			27	27
		OA2	4,841		59	0	4,984		61	0	4,992		62	0
		OA3		4,817	59	0		4,993	61	1		5,023	62	1
	6	OA1			20	20			23	23			24	24
		OA2	1,535		37	0	1,271		40	0	1,436		44	0
		OA3		1,540	37	0		1,269	40	0		1,390	43	2
	12	OA1			17	17			21	21			22	22
		OA2	1,132		27	0	1,268		31	0	1,800		37	0
		OA3		1,142	27	0		1,292	31	1		1,837	37	2
A2	3	OA1			37	37			37	37			38	38
		OA2	1,162		49	0	1,282		52	0	1,333		53	0
		OA3		1,159	49	0		1,286	52	1		1,338	53	2
	6	OA1			24	24			26	26			28	28
		OA2	1,364		35	0	1,313		37	0	1,158		39	0
		OA3		1,287	34	1		1,328	37	2		1,112	38	2
	12	OA1			15	15			17	17			18	18
		OA2	0,891		22	0	0,920		26	0	0,931		27	0
		OA3		0,888	22	0		0,882	25	1		0,875	26	2
A3	3	OA1			35	35			37	37			37	37
		OA2	1,415		46	0	1,430		48	0	1,523		49	0
		OA3		1,414	46	0		1,376	47	1		1,471	48	2
	6	OA1			20	20			24	24			26	26
		OA2	1,221		31	0	1,229		35	0	1,143		36	0
		OA3		1,220	31	0		1,245	35	1		1,255	37	3
	12	OA1			20	20			24	24			26	26
		OA2	1,221		31	0	1,229		35	0	1,143		36	0
		OA3		1,220	31	0		1,245	35	1		1,255	37	3

Table 4.7: Results for Different Tolerances - 1/3

			$\epsilon = 0.01$				$\epsilon = 0.001$				$\epsilon = 0.0001$			
A	p	Alg	OA2/OA1	OA3/OA1	MIP	NLP	OA2/OA1	OA3/OA1	MIP	NLP	OA2/OA1	OA3/OA1	MIP	NLP
A4	3	OA1			26	26			28	28			28	28
		OA2	0,999		27	0	1,079		30	0	1,207		32	0
		OA3		1,004	27	0		1,131	30	2		1,110	30	2
	6	OA1			15	15			16	16			16	16
		OA2	0,924		19	0	0,894		20	0	1,123		23	0
		OA3		0,897	19	0		0,955	20	1		1,056	21	2
	12	OA1			10	10			10	10			11	11
		OA2	0,783		12	0	0,850		13	0	0,917		15	0
		OA3		0,767	12	0		0,946	13	1		0,958	14	2
A5	3	OA1			27	27			27	27			29	29
		OA2	0,964		28	0	1,145		30	0	1,122		31	0
		OA3		0,960	28	0		1,160	30	2		1,069	30	2
	6	OA1			16	16			18	18			18	18
		OA2	0,965		20	0	1,091		24	0	1,125		25	0
		OA3		0,963	20	0		1,135	24	2		1,097	24	2
	12	OA1			11	11			11	11			12	12
		OA2	0,804		13	0	0,927		14	0	0,952		15	0
		OA3		0,828	13	0		0,945	14	1		0,939	15	2
A6	3	OA1			29	29			31	31			31	31
		OA2	0,977		31	0	0,911		31	0	0,965		32	0
		OA3		0,974	31	0		0,910	31	0		0,960	32	1
	6	OA1			16	16			17	17			18	18
		OA2	0,928		19	0	1,009		21	0	1,221		24	0
		OA3		0,909	19	0		1,028	21	1		1,241	24	2
	12	OA1			11	11			12	12			13	13
		OA2	0,658		11	0	0,671		12	0	0,817		15	0
		OA3		0,649	11	0		0,656	12	0		0,673	13	1

Table 4.8: Results for Different Tolerances - 2/3

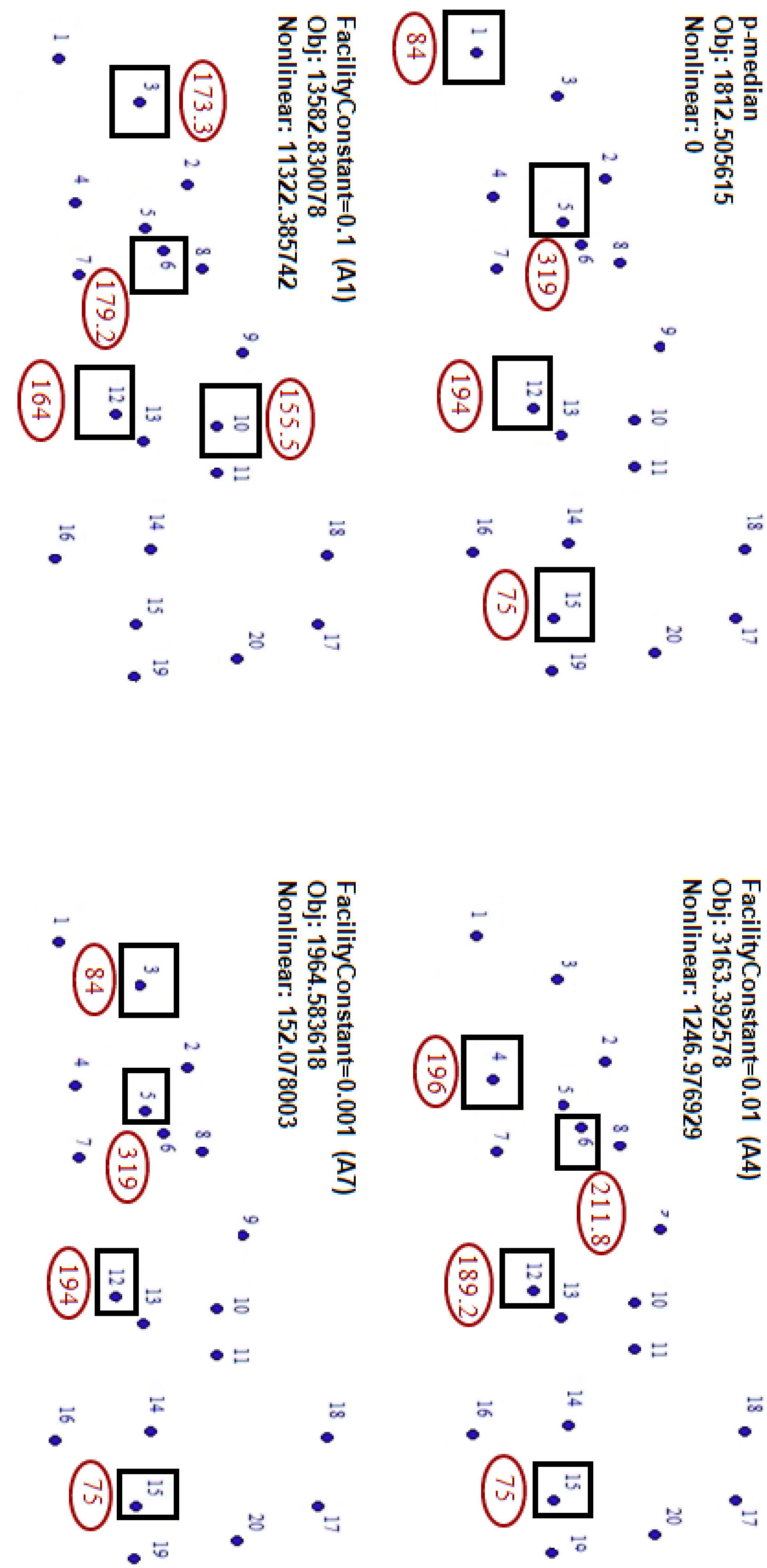
			$\epsilon = 0.01$				$\epsilon = 0.001$				$\epsilon = 0.0001$			
A	p	Alg	OA2/OA1	OA3/OA1	MIP	NLP	OA2/OA1	OA3/OA1	MIP	NLP	OA2/OA1	OA3/OA1	MIP	NLP
A7	3	OA1			4	4			7	7			7	7
		OA2	0,658		4	0	0,690		7	0	0,685		7	0
		OA3		0,758	4	0		0,636	7	1		0,738	7	1
	6	OA1			2	2			3	3			3	3
		OA2	0,882		2	0	0,830		3	0	0,909		3	0
		OA3		0,875	2	0		0,888	3	1		0,891	3	1
	12	OA1			1	1			4	4			4	4
		OA2	0,791		1	0	0,567		4	0	0,571		4	0
		OA3		0,776	1	0		0,619	4	0		0,565	4	0
A8	3	OA1			2	2			6	6			6	6
		OA2	0,820		2	0	0,651		6	0	0,635		6	0
		OA3		0,808	2	0		0,685	6	1		0,659	6	1
	6	OA1			9	9			11	11			11	11
		OA2	0,663		10	0	0,587		11	0	0,546		11	0
		OA3		0,660	10	0		0,647	11	1		0,600	11	1
	12	OA1			1	1			4	4			4	4
		OA2	0,796		1	0	0,693		4	0	0,817		4	0
		OA3		0,787	1	0		0,787	4	1		0,790	4	1
A9	3	OA1			10	10			12	12			12	12
		OA2	0,887		10	0	0,865		12	0	0,892		12	0
		OA3		0,867	10	0		0,879	12	1		0,904	12	1
	6	OA1			5	5			12	12			12	12
		OA2	0,843		6	0	0,675		13	0	0,672		13	0
		OA3		0,872	6	0		0,641	12	1		0,718	13	2
	12	OA1			3	3			4	4			4	4
		OA2	0,780		3	0	0,878		4	0	0,815		6	0
		OA3		0,786	3	0		0,831	4	1		0,818	5	1

Table 4.9: Results for Different Tolerances - 3/3

4.2.2.8 Comparison between p-median and Congested p-median

As mentioned in the computational setup section, instances with $a_j = 0\ \forall j \in N$ corresponds to the original p-median problem. In this section we will compare the *CpMP* with p-median problem both in terms of their solutions and the running time. In order to compare the solutions, we analysed some instances for each $n \in \{20, 40, 60\}$. (The instances for n=80 are not included due to the space constraints.) Since in the p-median problem, all the facilities are type 1 and their facility constant is 0, we choose the instances with only one type of facility (i.e A1, A4, A7). For each value of n, we compare four instances with the same p value (20% of n); the p-median problem and the instances with a_j sets A1, A4 and A7. The Figures 4.8 , 4.9, 4.10 show the comparison of these four instances for $n = 20$, $n = 40$ and $n = 60$ respectively. The dots on the figures represents population zones and the ones that are in a square box are the located facilities. The circled numbers near the located facilities are the number of people assigned to that facility. Observe that in the p-median problem the number of people assigned to each facility varies a lot. However in the *CpMp* instance with set A1 since there is a high penalty for congestion, the people are assigned to facilities more homogenously. It is also seen from the figures that, as the facility constant decreases, *CpMP* converges to p-median problem. This is expected since as the facility constant decreases, the nonlinear part of the disutility function decreases and dominated by the linear part that represents the travel time.

The other aspect to be considered in the comparison between *CpMP* and p-median problem is their running time which determines the hardness of the problem. Figure 4.11 shows the running time of the proposed algorithms separately for the instances with $n = 60$. These instances are representative of all of the instances. The first three instance on the figure corresponds to the p-median problem for three different p values and the remaining 27 represents the instances as described in Section 4.2.1. It can be deduced from the figure that p-median is an easy problem compared to *CpMP* problem. The reason behind this fact is that after the facilities are chosen, assigning people to the located facilities is easy in


p-median
Obj: 1812.505615
Nonlinear: 0
FacilityConstant=0.01 (A4)
Obj: 3163.392578
Nonlinear: 1246.976929
FacilityConstant=0.1 (A1)
Obj: 13582.830078
Nonlinear: 11322.385742
FacilityConstant=0.001 (A7)
Obj: 1964.583618
Nonlinear: 152.078003

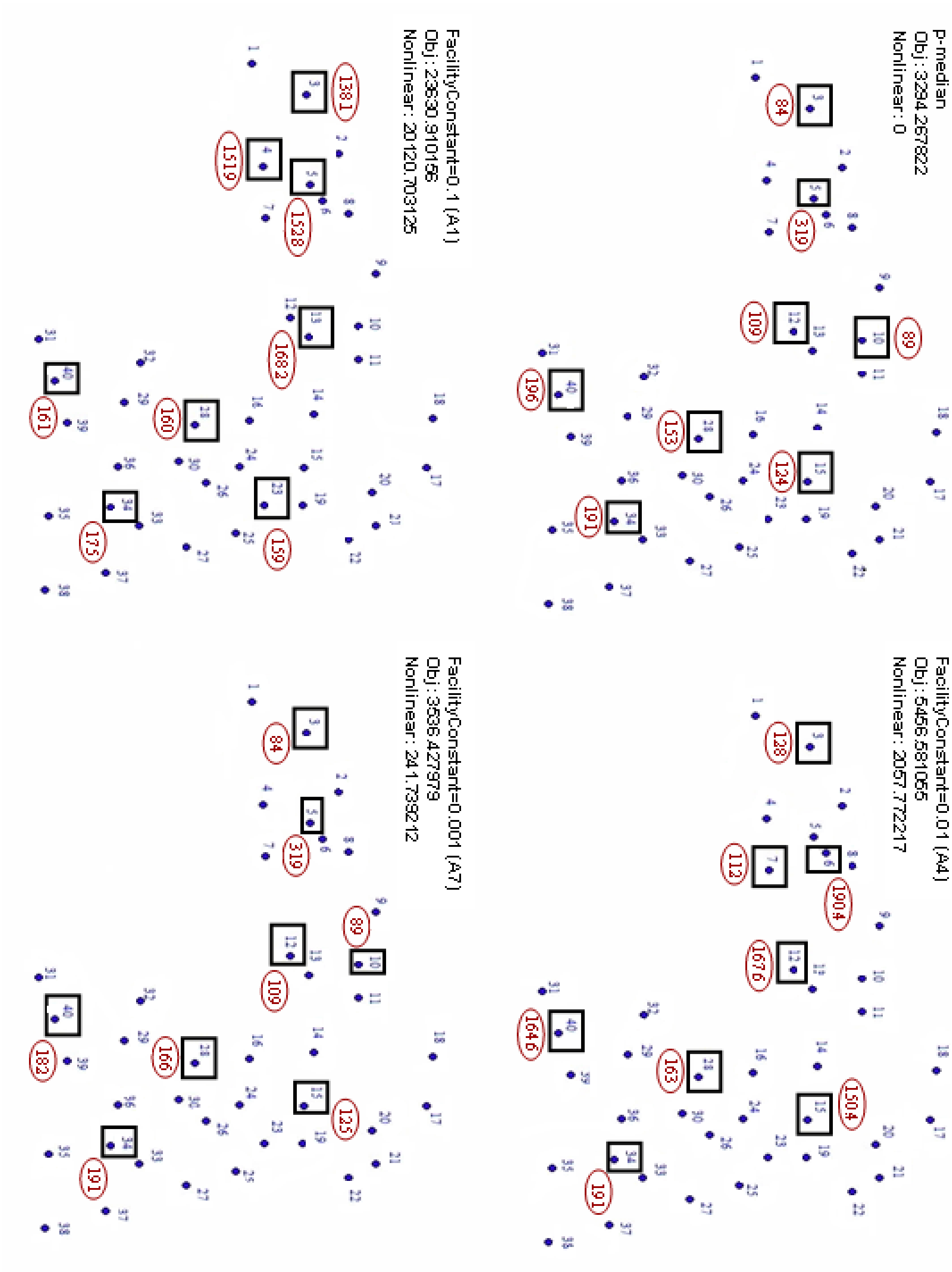


Figure 4.9: p-median $CpMP$ comparison for n = 40.

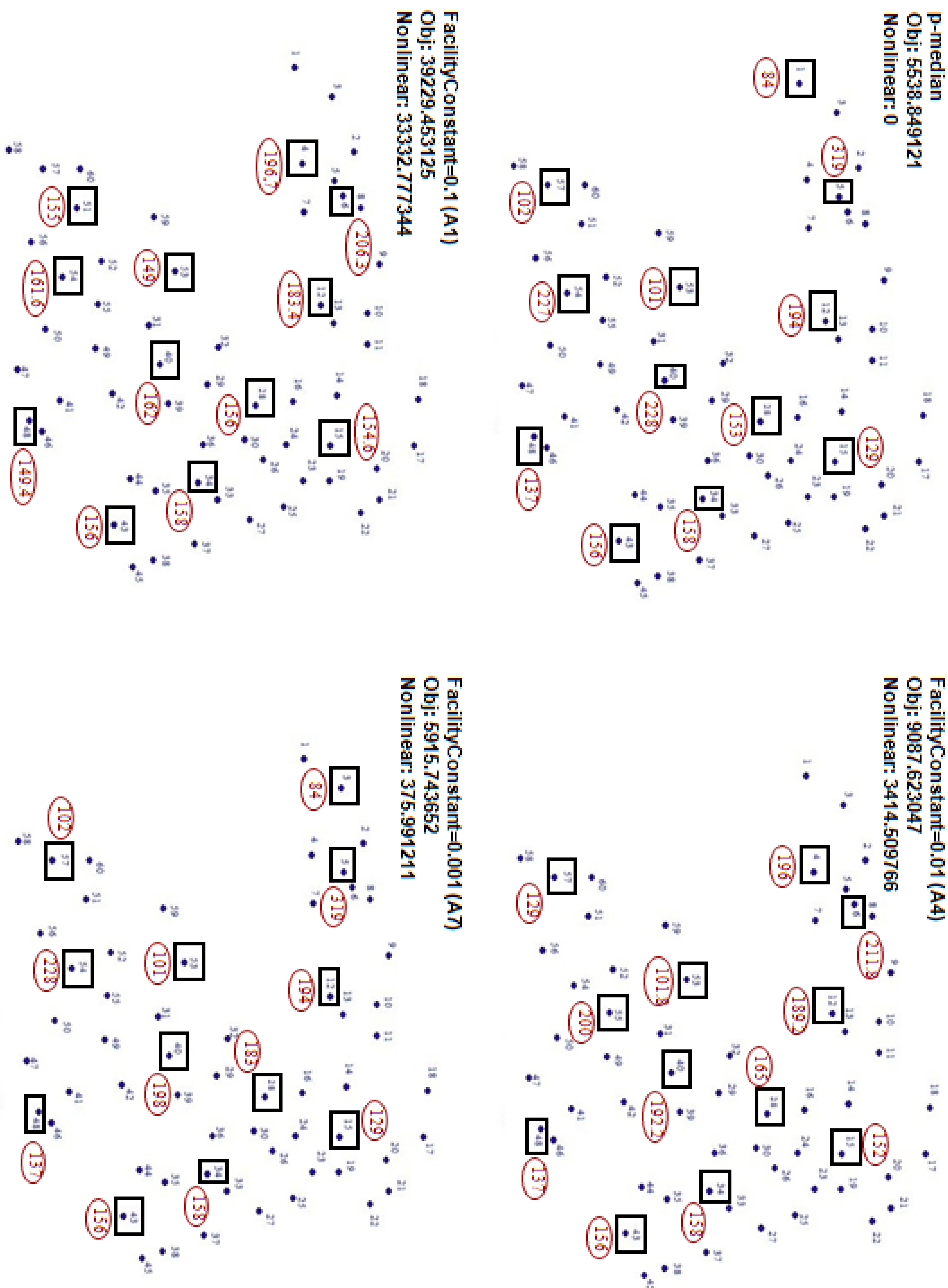


Figure 4.10: p-median $CpMP$ comparison for n =60.

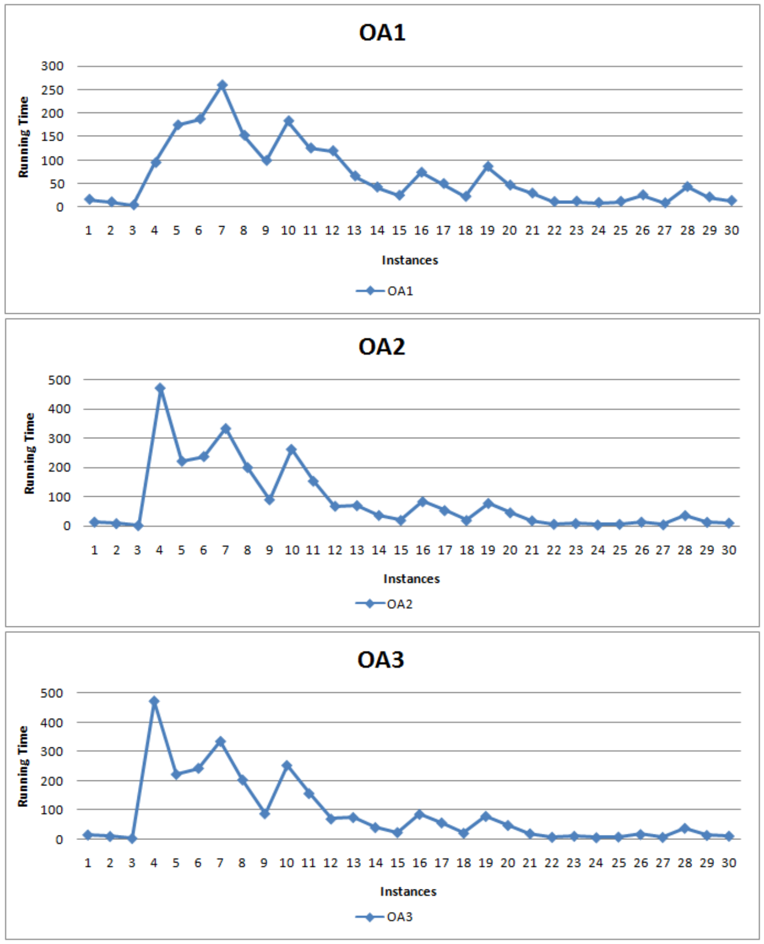


Figure 4.11: Running time of the algorithms for n = 60

p-median problem. All the clients are assigned to the nearest facility as the only concern is the travel time. However in *CpMP* problem, assignment is an NLP problem due to the definition of the disutility function. Therefore it is expected to have less running time for the p-median problem. Moreover observe that as the facility constant decreases, *CpMP* converges to the p-median problem which results in a decrease in the running time of the algorithms.

4.2.2.9 General Discussion

In this section, we will give a general discussion about the proposed algorithms.

The comparison between the proposed algorithms and CPLEX12.1 proves the efficiency of the algorithms. Yet we can compare the proposed algorithms within themselves in aspect of their running time.

OA2 algorithm aims to solve MINLPs by iteratively solving MILPs. This is done by eliminating the NLP iterations of OA1 algorithm. NLP iterations refer to finding an optimal assignment for the located facilities. This attempt increases the number of MILPs solved for each problem. The motivation behind the OA2 algorithm is that NLPs are expensive and solving extra MILPs instead of NLPs will decrease the running time of the algorithm. However the results show that, in many of the instances even though the number of MILPs solved in OA2 algorithm is less than the total number of iterations in OA1 (both the MILPs and NLPs), the running time of OA2 is not reduced as expected. The reason is that the NLPs of *CpMp* are solved in a reasonable time. Furthermore as the MILPs solved in OA2 algorithm grows in each iteration, the running time of MILPs solved towards the end of the algorithm increase enormously. A perfect instance showing this situation is n=80, A1, p=16. The OA1 algorithm solves 21 MILPs and 21 NLPs in order to converge to an optimal solution whereas OA2 solves only 29 MILPs. Yet, OA1 performs better than OA2 for this particular instance. The running time of the iterations solved by OA1 and OA2 algorithms are shown in Table 4.10.

The idea behind the OA3 algorithm is to decrease the number of MILPs solved in OA2 algorithm. Since, OA2 eliminates the NLP iterations, the best assignment for chosen facilities is not found. Hence the same facility set may appear in an optimal solution of subsequent MILPs. In order to avoid this, after a facility set is optimal for the second time, OA3 solves the NLP problem for the facility set. OA3 algorithm seems to perform better than OA2 algorithm. However as the results are interpreted, apparently OA3 performs very close to OA2 for our instances. The reason is that in most of our instances same facility set is never optimal for another MILP or if it is, the solution is optimal for the problem. In the former case no NLP is solved and OA3 performs exactly the same with OA2, in the latter case only one NLP is solved in addition to the MILPs which are same as OA2 MILPs. Furthermore in some cases there are more than one NLPs solved but they either increase the total number of iterations or cost too much.

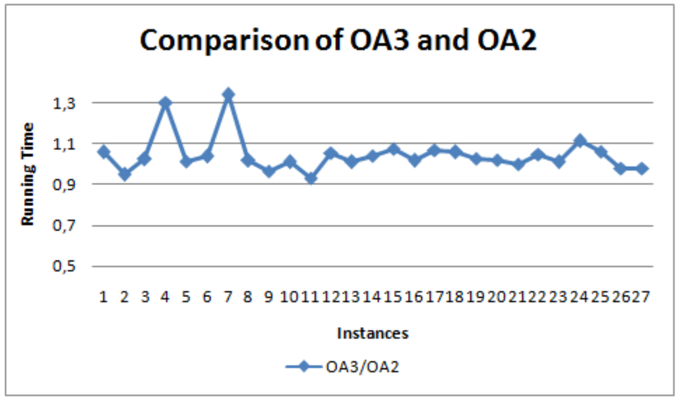


Figure 4.12: Comparison of OA2 and OA3 performances for n=40

The comparison between the OA2 and OA3 algorithms for n=40 is represented in Figure 4.12. The horizontal line correspond to the instances as described in Section 4.2.1 and the vertical line shows the ratio between the running time of OA3 and OA2.

Iteration	OA2	OA1	
	MILP time	MILP time	NLP time
1	1,618	1,342	8,783
2	1,999	1,584	1,435
3	1,928	1,219	1,061
4	3,532	3,221	1,045
5	3,437	2,36	18,252
6	5,204	4,399	0,998
7	4,119	3,364	16,349
8	5,19	9,877	12,292
9	4,377	4,535	5,678
10	4,978	5,566	9,625
11	6,484	5,961	8,455
12	6,239	7,429	5,039
13	14,435	8,913	1,419
14	16,471	21,972	1,794
15	10,091	28,758	7,613
16	11,971	36,673	1,388
17	26,43	51,559	3,385
18	23,958	40,363	0,53
19	23,577	37,864	2,091
20	14,219	31,014	0,422
21	43,908	47,778	2,324
22	42,733		
23	103,822		
24	76,515		
25	291,035		
26	279,297		
27	194,166		
28	229,579		
29	382,757		

Table 4.10: Running time of iterations solved by OA1 and OA2 algorithms for instance with parameters $n = 80$, A1, p=16

Chapter 5

Conclusion and Future Research

In this thesis, we define the generalized version of the p-median problem. This problem defines the disutility of a client with two factors; the travel time and the unwillingness of the client due to the congestion in the facility. With addition of the disutility function, the problem becomes more realistic for the applications with service time.

In order to solve this problem, three algorithms are proposed. Each one of the algorithms is a different modification of the Outer Approximation Algorithm. The proposed algorithms are implemented and tested for different instances. In order to test the efficiency of the algorithms, we solved same instances with an off-the-shelf solver, CPLEX12.1. The computational results show that, the proposed algorithms are competitive with CPLEX12.1.

The solutions of the instances indicates that, *CpMP* problem assigns the clients to facilities homogeneously. However as the p-median problem assigns each client to the nearest facility, the total demand assigned to each facility varies a lot.

Furthermore, the comparison of the algorithms shows that the performance of the OA2 and OA3 algorithm decreases as the input size increases for this problem. Therefore the OA1 algorithm is more efficient for most of the instances with large input sizes.

For the future studies, as the proposed algorithms are not problem dependent, we may search for other problems to test the algorithms. As the results point out that the OA1 algorithm is more efficient than the OA2 and OA3 algorithms for large instances, we may try to find a problem that OA2 and OA3 performs better than OA1.

Another extension for this study could be trying to improve the proposed algorithms. As we discussed in Section 4.2.2.9, the running time of MILPs grow enormously as the iteration number increases. Therefore removing cuts from the MILP could be a solution to decrease the running time of MILPs. This modification may change the efficiency of the algorithms. Hence OA2 and OA3 may perform better than OA1 algorithm.

Additionally a special improvement may be done for the OA3 algorithm. In the proposed version of OA3, the NLP problem is solved if the same integer solution is optimal for a second MILP. The OA3 algorithm may be improved by changing the check condition to solve the NLP problem.

Lastly, the model could be improved and become further realistic. In *CpMP* the effect of congestion in a facility is assumed to be the same for all customers (i.e., $f_j(z_j)$ is only facility dependent). However in real life this may not be the case. An individual may have a personal problem with a facility or a personal interest to a facility. By changing the definition of the disutility function, we may incorporate this extension into the model.

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Appendix A

Computational Results

The computational results of our experiments are reported here. The meanings of the column names in the tables are as follows.

- Interrupt - Shows if the algorithm is terminated by the user or not.
- Alg - Algorithm name.
- OA2/OA1 - Running Time of OA2 / Running Time of OA1
- OA3/OA1 - Running Time of OA3 / Running Time of OA1
- MILP Time - Total time spent to solve all MILPs.
- NLP Time - Total time spent to solve all NLPs.
- MILP Iter. - Number of MILP iterations.

- NLP Iter. - Number of NLP iterations.

- Linear Part - Contribution of travel time to the objective function.

- Nonlinear Part - Contribution of congestion to the objective function.

- Ai* $\forall i \in \{1, \ldots, 9\}$ - A1 values scaled by 1.000.000.

Instance		CPLEX12.1			Proposed Algorithms Running Time		
A	p	Interrupt	Gap	RunningTime	OA1	OA2	OA3
A1	1	0	0	0,19	4,63	2,67	2,87
A1	2	0	0	0,72	46,16	14,76	31,09
A1	4	0	0	10,25	12,20	10,00	9,94
A2	1	0	0	0,11	8,47	3,54	3,39
A2	2	0	0	0,73	36,49	11,40	13,84
A2	4	0	0	6,43	11,26	6,47	6,82
A3	1	0	0	0,11	8,78	3,23	3,53
A3	2	0	0	0,72	37,08	8,14	8,92
A3	4	0	0	3,42	16,97	12,42	12,95
A4	1	0	0	0,17	7,61	3,71	4,01
A4	2	0	0	0,28	4,60	1,86	2,22
A4	4	0	0	0,44	6,15	3,31	3,70
A5	1	0	0	0,19	6,71	4,06	4,28
A5	2	0	0	0,33	3,68	2,14	2,29
A5	4	0	0	0,37	5,66	2,47	2,50
A6	1	0	0	0,19	7,00	4,01	4,09
A6	2	0	0	0,31	6,14	3,08	3,66
A6	4	0	0	0,56	7,02	3,93	4,53
A7	1	0	0	0,09	3,99	2,51	2,96
A7	2	0	0	0,08	2,36	1,50	1,81
A7	4	0	0	0,09	3,74	2,31	2,45
A8	1	0	0	0,11	63,06	62,64	62,63
A8	2	0	0	0,09	1,36	0,84	1,05
A8	4	0	0	0,08	1,58	0,93	1,10
A9	1	0	0	0,11	4,51	3,96	4,07
A9	2	0	0	0,13	3,97	3,11	3,25
A9	4	0	0	0,17	2,47	1,79	2,25

Table A.1: Proposed Algorithms and CPLEX12.1 for n=20, $\epsilon = 0.0001$

Instance		CPLEX12.1			Proposed Algorithms Running Time		
A	p	Interrupt	Gap	RunningTime	OA1	OA2	OA3
A1	2	0	0	15,18	70,01	120,15	122,80
A1	4	1	51,99	600	40,86	90,01	85,94
A1	8	1	52,05	600	37,66	50,22	52,79
A2	2	0	0	13,93	208,29	81,89	101,93
A2	4	1	42,67	600	47,83	60,51	64,37
A2	8	1	44,69	600	34,83	29,02	27,55
A3	2	0	0	13,92	239,65	68,25	106,71
A3	4	1	21,99	600	52,65	44,90	45,35
A3	8	1	35,53	600	27,36	24,01	25,05
A4	2	0	0	7,13	23,72	17,74	19,60
A4	4	0	0	37,07	21,11	21,21	23,85
A4	8	0	0	36,96	14,33	11,15	10,92
A5	2	0	0	6,58	54,98	20,31	19,01
A5	4	0	0	15,77	18,22	14,65	15,62
A5	8	0	0	58,98	10,40	6,80	7,62
A6	2	0	0	6,04	36,23	33,01	31,28
A6	4	0	0	38,45	29,15	23,55	27,00
A6	8	0	0	56,21	13,94	13,95	14,72
A7	2	0	0	1,2	9,05	8,76	7,57
A7	4	0	0	0,51	3,44	2,09	2,40
A7	8	0	0	0,1	4,21	2,16	2,29
A8	2	0	0	1,2	12,57	10,30	10,66
A8	4	0	0	0,53	3,40	2,08	2,53
A8	8	0	0	0,58	4,40	3,30	3,46
A9	2	0	0	1,11	8,13	7,68	7,02
A9	4	0	0	1,18	13,98	9,29	9,59
A9	8	0	0	0,62	4,28	2,60	3,03

Table A.2: Proposed Algorithms and CPLEX12.1 for n=40, $\epsilon = 0.0001$

Instance		CPLEX12.1			Proposed Algorithms Running Time		
A	p	Interrupt	Gap	RunningTime	OA1	OA2	OA3
A1	3	1	74,01	600	103,98	519,02	522,28
A1	6	1	72,25	600	192,05	275,77	267,03
A1	12	1	61,85	600	204,99	368,92	376,55
A2	3	1	71,44	600	262,89	350,45	351,78
A2	6	1	69,17	600	193,36	223,90	215,06
A2	12	1	56,55	600	109,80	102,24	96,11
A3	3	1	67,7	600	180,75	275,30	265,90
A3	6	1	66,83	600	143,54	164,07	180,17
A3	12	1	52,37	600	120,33	83,22	74,57
A4	3	0	0	330,57	66,12	79,81	73,41
A4	6	1	16	600	42,19	47,39	44,54
A4	12	1	9,55	600	26,67	24,45	25,55
A5	3	0	0	186,69	84,12	94,38	89,92
A5	6	1	14,56	600	53,99	60,73	59,22
A5	12	1	8,17	600	25,42	24,20	23,88
A6	3	0	0	220,83	92,28	89,08	88,63
A6	6	1	13,48	600	50,43	61,59	62,60
A6	12	1	10,79	600	32,86	26,85	22,13
A7	3	0	0	4,48	10,14	6,95	7,48
A7	6	0	0	3,12	12,28	11,17	10,94
A7	12	0	0	0,94	9,23	5,28	5,22
A8	3	0	0	4,2	11,71	7,43	7,72
A8	6	0	0	6,41	27,96	15,27	16,78
A8	12	0	0	1,36	8,77	7,17	6,92
A9	3	0	0	8,7	44,56	39,74	40,29
A9	6	0	0	4,63	21,77	14,64	15,63
A9	12	0	0	2,53	14,69	11,97	12,03

Table A.3: Proposed Algorithms and CPLEX12.1 for n=60, $\epsilon = 0.0001$

Instance		CPLEX12.1			Proposed Algorithms Running Time		
A	p	Interrupt	Gap	RunningTime	OA1	OA2	OA3
A1	4	1	81,71	1800	403,12	1474,06	1493,10
A1	8	1	75,11	1800	531,83	1195,15	1216,51
A1	16	1	64,18	1800	525,05	5255,74	2959,94
A2	4	1	79,71	1800	437,86	991,53	972,35
A2	8	1	72,18	1800	352,43	458,39	474,03
A2	16	1	58,29	1800	366,77	358,17	283,60
A3	4	1	77	1800	422,93	613,25	571,66
A3	8	1	68,58	1800	330,26	289,23	292,57
A3	16	1	54,25	1800	205,26	163,38	168,21
A4	4	1	30,33	1800	270,79	319,36	315,01
A4	8	1	24,13	1800	176,56	199,89	195,68
A4	16	1	15,55	1800	52,66	53,80	51,68
A5	4	1	28,65	1800	357,33	389,43	389,01
A5	8	1	21,94	1800	152,33	152,50	144,66
A5	16	1	17,08	1800	75,19	66,09	66,71
A6	4	1	24,9	1800	293,84	313,94	312,77
A6	8	1	23,67	1800	157,77	184,82	185,64
A6	16	1	17,46	1800	58,87	58,38	55,06
A7	4	0	0	19,06	47,04	35,58	35,97
A7	8	0	0	11,42	46,03	39,84	41,33
A7	16	0	0	3,53	19,38	11,91	14,73
A8	4	0	0	33,04	58,17	45,48	44,55
A8	8	0	0	14,31	28,43	15,69	15,54
A8	16	0	0	4,88	17,71	12,37	13,30
A9	4	0	0	16,35	45,70	38,06	40,19
A9	8	0	0	16,43	51,95	36,20	39,99
A9	16	0	0	10,23	13,87	8,34	8,28

Table A.4: Proposed Algorithms and CPLEX12.1 for n=80, $\epsilon = 0.0001$

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1	1	OA1	5,29			2,21	2,42	21	21	49791,17	4632,77	45158,40
A1	1	OA2	2,65	0,50		2,03	0,00	21	0	49791,17	4632,77	45158,40
A1	1	OA3	2,86		0,54	2,06	0,11	21	1	49791,17	4632,77	45158,40
A1	2	OA1	35,80			2,89	32,45	13	13	25605,85	2995,83	22610,02
A1	2	OA2	14,27	0,40		13,70	0,00	29	0	25605,85	2996,26	22609,60
A1	2	OA3	28,55		0,80	12,12	15,87	26	4	25605,85	2995,83	22610,02
A1	4	OA1	12,18			5,01	6,77	17	17	13582,83	2260,44	11322,39
A1	4	OA2	8,66	0,71		8,27	0,00	22	0	13583,63	2259,43	11324,20
A1	4	OA3	8,53		0,70	8,05	0,00	22	0	13583,63	2259,43	11324,20
A2	1	OA1	8,21			3,00	4,52	21	21	49947,43	4789,03	45158,40
A2	1	OA2	3,71	0,45		2,98	0,00	21	0	49947,43	4789,03	45158,40
A2	1	OA3	3,76		0,46	2,85	0,27	21	1	49947,43	4789,03	45158,40
A2	2	OA1	63,90			3,65	59,69	15	15	25605,85	2995,83	22610,02
A2	2	OA2	11,22	0,18		10,72	0,00	24	0	25607,02	3007,82	22599,20
A2	2	OA3	15,15		0,24	9,30	5,29	23	3	25605,85	2995,83	22610,02
A2	4	OA1	13,31			3,43	9,41	14	14	13591,26	2275,84	11315,42
A2	4	OA2	5,15	0,39		4,68	0,00	17	0	13594,36	2264,13	11330,22
A2	4	OA3	5,57		0,42	4,39	0,75	17	1	13591,26	2275,84	11315,42
A3	1	OA1	8,42			2,34	4,96	21	21	50222,94	5064,53	45158,40
A3	1	OA2	3,56	0,42		2,43	0,00	21	0	50222,94	5064,53	45158,40
A3	1	OA3	3,74		0,44	2,33	0,25	21	1	50222,94	5064,53	45158,40
A3	2	OA1	37,19			4,51	32,15	14	14	26223,67	3624,47	22599,20
A3	2	OA2	8,21	0,22		7,68	0,00	19	0	26223,67	3624,47	22599,20
A3	2	OA3	8,46		0,23	6,97	0,86	18	2	26223,67	3624,47	22599,20
A3	4	OA1	16,77			3,38	9,23	14	14	15679,05	2761,43	12917,62
A3	4	OA2	11,47	0,68		7,33	0,00	24	0	15680,12	2765,07	12915,05
A3	4	OA3	12,01		0,72	6,90	0,81	23	2	15679,05	2761,43	12917,62

Table A.5: Linear Utility Function $\epsilon = 0.001$, n=20 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4	1	OA1	7,25			2,40	3,73	15	15	9148,61	4632,77	4515,84
A4	1	OA2	3,63	0,50		2,68	0,00	15	0	9148,61	4632,77	4515,84
A4	1	OA3	3,98		0,55	2,57	0,25	15	1	9148,61	4632,77	4515,84
A4	2	OA1	4,46			1,34	2,70	8	8	5072,48	2724,78	2347,70
A4	2	OA2	1,87	0,42		1,41	0,00	8	0	5072,48	2724,78	2347,70
A4	2	OA3	2,11		0,47	1,31	0,30	8	1	5072,48	2724,78	2347,70
A4	4	OA1	6,15			1,84	3,67	9	9	3163,39	1916,42	1246,98
A4	4	OA2	2,84	0,46		2,26	0,00	10	0	3163,86	1918,60	1245,26
A4	4	OA3	3,60		0,59	2,31	0,78	10	2	3163,39	1916,42	1246,98
A5	1	OA1	6,69			2,76	2,50	15	15	9304,87	4789,03	4515,84
A5	1	OA2	4,08	0,61		2,62	0,00	15	0	9304,87	4789,03	4515,84
A5	1	OA3	4,26		0,64	2,70	0,10	15	1	9304,87	4789,03	4515,84
A5	2	OA1	3,66			1,30	1,54	8	8	5072,48	2724,78	2347,70
A5	2	OA2	2,13	0,58		1,31	0,00	8	0	5072,48	2724,78	2347,70
A5	2	OA3	2,29		0,63	1,35	0,14	8	1	5072,48	2724,78	2347,70
A5	4	OA1	5,78			1,86	3,44	9	9	3214,68	1969,42	1245,26
A5	4	OA2	2,52	0,44		2,08	0,00	9	0	3214,68	1969,42	1245,26
A5	4	OA3	2,57		0,44	2,09	0,00	9	0	3214,68	1969,42	1245,26
A6	1	OA1	5,92			3,03	1,88	15	15	9580,38	5064,53	4515,84
A6	1	OA2	4,01	0,68		3,00	0,00	15	0	9580,38	5064,53	4515,84
A6	1	OA3	3,92		0,66	2,83	0,10	15	1	9580,38	5064,53	4515,84
A6	2	OA1	5,93			2,22	2,98	10	10	5847,28	3533,01	2314,27
A6	2	OA2	2,84	0,48		2,14	0,00	11	0	5847,28	3533,71	2313,57
A6	2	OA3	3,64		0,61	2,06	0,86	11	2	5847,28	3533,01	2314,27
A6	4	OA1	7,05			2,32	4,18	10	10	3888,21	2260,60	1627,61
A6	4	OA2	4,21	0,60		3,68	0,00	12	0	3888,21	2260,60	1627,61
A6	4	OA3	4,38		0,62	3,44	0,37	12	1	3888,21	2260,60	1627,61

Table A.6: Linear Utility Function $\epsilon = 0.001$, n=20 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7	1	OA1	3,88			0,95	1,31	5	5	5084,35	4632,77	451,58
A7	1	OA2	2,51	0,65		0,86	0,00	5	0	5084,35	4632,77	451,58
A7	1	OA3	2,79		0,72	0,92	0,27	5	1	5084,35	4632,77	451,58
A7	2	OA1	2,31			0,30	0,91	2	2	2959,55	2724,78	234,77
A7	2	OA2	1,37	0,59		0,30	0,00	2	0	2959,55	2724,78	234,77
A7	2	OA3	1,83		0,79	0,33	0,45	2	1	2959,55	2724,78	234,77
A7	4	OA1	3,78			0,87	1,58	4	4	1964,58	1812,51	152,08
A7	4	OA2	2,12	0,56		0,83	0,00	4	0	1964,58	1812,51	152,08
A7	4	OA3	2,12		0,56	0,86	0,00	4	0	1964,58	1812,51	152,08
A8	1	OA1	62,07			1,12	0,61	5	5	5240,62	4789,03	451,58
A8	1	OA2	61,63	0,99		1,13	0,00	5	0	5240,62	4789,03	451,58
A8	1	OA3	61,51		0,99	0,90	0,15	5	1	5240,62	4789,03	451,58
A8	2	OA1	1,31			0,35	0,41	2	2	2959,55	2724,78	234,77
A8	2	OA2	0,83	0,63		0,26	0,00	2	0	2959,55	2724,78	234,77
A8	2	OA3	1,05		0,80	0,29	0,22	2	1	2959,55	2724,78	234,77
A8	4	OA1	1,55			0,27	0,64	3	3	1971,64	1812,51	159,13
A8	4	OA2	0,93	0,60		0,31	0,00	3	0	1971,64	1812,51	159,13
A8	4	OA3	1,13		0,73	0,29	0,22	3	1	1971,64	1812,51	159,13
A9	1	OA1	4,50			0,79	0,55	5	5	5516,12	5064,53	451,58
A9	1	OA2	3,93	0,87		0,78	0,00	5	0	5516,12	5064,53	451,58
A9	1	OA3	4,07		0,90	0,81	0,12	5	1	5516,12	5064,53	451,58
A9	2	OA1	3,86			1,32	0,77	5	5	3356,73	2724,78	631,95
A9	2	OA2	3,14	0,81		1,38	0,00	5	0	3356,73	2724,78	631,95
A9	2	OA3	3,45		0,89	1,56	0,13	5	1	3356,73	2724,78	631,95
A9	4	OA1	2,37			0,90	1,10	5	5	2222,06	1924,53	297,53
A9	4	OA2	1,44	0,61		1,10	0,00	6	0	2222,06	1924,53	297,53
A9	4	OA3	1,90		0,80	1,11	0,41	6	2	2222,06	1924,53	297,53

Table A.7: Linear Utility Function $\epsilon = 0.001$, n=20 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1	2	OA1	64,52			20,33	42,62	27	27	88483,95	8335,70	80148,24
A1	2	OA2	116,52	1,81		114,51	0,00	56	0	88485,56	8343,94	80141,63
A1	2	OA3	123,22		1,91	115,91	5,23	56	7	88483,95	8335,70	80148,24
A1	4	OA1	39,69			21,37	16,55	22	22	45941,02	5837,27	40103,75
A1	4	OA2	77,91	1,96		75,97	0,00	37	0	45944,92	5851,21	40093,71
A1	4	OA3	73,77		1,86	71,44	0,39	36	1	45941,02	5837,27	40103,75
A1	8	OA1	35,72			18,07	15,52	18	18	23630,91	3510,21	20120,70
A1	8	OA2	39,70	1,11		37,64	0,00	27	0	23634,42	3514,86	20119,57
A1	8	OA3	40,62		1,14	37,66	0,80	27	1	23630,91	3510,21	20120,70
A2	2	OA1	172,79			40,89	129,26	36	36	89393,49	9221,89	80171,60
A2	2	OA2	76,36	0,44		73,91	0,00	47	0	89415,48	9188,90	80226,58
A2	2	OA3	99,39		0,58	73,93	22,93	47	3	89423,70	9181,74	80241,96
A2	4	OA1	46,85			26,60	18,25	24	24	46168,23	6065,80	40102,42
A2	4	OA2	53,65	1,15		51,51	0,00	36	0	46194,62	6057,31	40137,31
A2	4	OA3	54,32		1,16	49,66	2,31	35	3	46168,23	6065,80	40102,42
A2	8	OA1	32,15			10,66	20,08	13	13	24462,18	4380,06	20082,13
A2	8	OA2	23,48	0,73		22,07	0,00	21	0	24463,44	4388,38	20075,07
A2	8	OA3	24,37		0,76	21,87	0,97	21	2	24462,18	4380,06	20082,13
A3	2	OA1	207,78			24,51	181,37	31	31	90291,09	10103,96	80187,13
A3	2	OA2	61,88	0,30		59,94	0,00	45	0	90294,63	10130,37	80164,25
A3	2	OA3	82,85		0,40	58,20	22,56	44	3	90291,09	10103,96	80187,13
A3	4	OA1	46,05			21,31	22,57	23	23	47555,38	7419,80	40135,57
A3	4	OA2	37,30	0,81		35,50	0,00	28	0	47559,78	7388,17	40171,62
A3	4	OA3	37,91		0,82	35,43	0,45	28	1	47555,38	7419,80	40135,57
A3	8	OA1	24,18			13,17	9,19	16	16	25370,63	5241,59	20129,05
A3	8	OA2	21,39	0,88		19,78	0,00	22	0	25371,73	5239,90	20131,83
A3	8	OA3	20,64		0,85	18,48	0,58	21	1	25370,63	5241,59	20129,05

Table A.8: Linear Utility Function $\epsilon = 0.001$, n=40 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4	2	OA1	23,12			12,09	8,16	25	25	16335,67	8315,38	8020,28
A4	2	OA2	15,37	0,66		12,56	0,00	24	0	16336,17	8319,01	8017,16
A4	2	OA3	15,52		0,67	12,48	0,16	24	1	16335,67	8315,38	8020,28
A4	4	OA1	19,03			8,82	8,38	15	15	9654,06	5502,00	4152,06
A4	4	OA2	19,03	1,00		17,19	0,00	22	0	9655,91	5426,33	4229,58
A4	4	OA3	17,75		0,93	14,98	0,75	20	2	9654,06	5502,00	4152,06
A4	8	OA1	13,29			4,82	5,46	10	10	5456,58	3398,81	2057,77
A4	8	OA2	8,22	0,62		5,68	0,00	11	0	5457,79	3397,24	2060,55
A4	8	OA3	8,66		0,65	5,73	0,33	11	1	5456,58	3398,81	2057,77
A5	2	OA1	53,32			16,69	34,51	28	28	17192,26	9158,00	8034,26
A5	2	OA2	18,53	0,35		16,81	0,00	26	0	17192,26	9158,00	8034,26
A5	2	OA3	18,70		0,35	16,88	0,00	26	0	17192,26	9158,00	8034,26
A5	4	OA1	17,72			7,86	6,66	15	15	9983,59	5875,78	4107,81
A5	4	OA2	12,56	0,71		9,52	0,00	16	0	9983,60	5874,33	4109,26
A5	4	OA3	13,04		0,74	9,75	0,25	16	1	9983,59	5875,78	4107,81
A5	8	OA1	9,30			4,23	3,81	10	10	6139,76	3864,00	2275,76
A5	8	OA2	5,93	0,64		4,81	0,00	10	0	6139,76	3864,00	2275,76
A5	8	OA3	6,37		0,68	4,83	0,33	10	1	6139,76	3864,00	2275,76
A6	2	OA1	34,30			18,25	11,34	29	29	18011,64	9927,78	8083,86
A6	2	OA2	28,81	0,84		24,31	0,00	30	0	18012,21	9940,38	8071,83
A6	2	OA3	29,28		0,85	24,44	0,42	30	2	18011,64	9927,78	8083,86
A6	4	OA1	25,10			12,64	10,84	19	19	11174,38	6327,07	4847,31
A6	4	OA2	21,64	0,86		20,20	0,00	26	0	11174,88	6328,89	4845,99
A6	4	OA3	23,10		0,92	20,37	1,29	26	2	11174,88	6328,89	4845,99
A6	8	OA1	13,20			6,62	4,66	11	11	6319,90	3542,44	2777,47
A6	8	OA2	11,92	0,90		10,03	0,00	15	0	6320,00	3547,40	2772,60
A6	8	OA3	12,61		0,96	10,14	0,52	15	1	6319,90	3542,44	2777,47

Table A.9: Linear Utility Function $\epsilon = 0.001$, n=40 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7	2	OA1	8,66			1,76	1,48	6	6	9111,75	8308,92	802,84
A7	2	OA2	6,99	0,81		1,70	0,00	6	0	9111,75	8308,92	802,84
A7	2	OA3	7,18		0,83	1,72	0,17	6	1	9111,75	8308,92	802,84
A7	4	OA1	2,61			0,39	0,77	2	2	5843,46	5419,03	424,43
A7	4	OA2	1,70	0,65		0,42	0,00	2	0	5843,46	5419,03	424,43
A7	4	OA3	1,73		0,66	0,45	0,00	2	0	5843,46	5419,03	424,43
A7	8	OA1	3,68			0,67	1,64	3	3	3536,43	3294,69	241,74
A7	8	OA2	1,92	0,52		0,70	0,00	3	0	3536,44	3294,63	241,81
A7	8	OA3	1,92		0,52	0,70	0,00	3	0	3536,44	3294,63	241,81
A8	2	OA1	11,92			1,86	1,67	6	6	9478,99	8308,92	1170,07
A8	2	OA2	10,00	0,84		1,87	0,00	6	0	9478,99	8308,92	1170,07
A8	2	OA3	10,45		0,88	1,86	0,33	6	1	9478,99	8308,92	1170,07
A8	4	OA1	2,56			0,28	0,80	2	2	6005,87	5419,03	586,84
A8	4	OA2	1,64	0,64		0,25	0,00	2	0	6005,87	5419,03	586,84
A8	4	OA3	1,65		0,65	0,27	0,00	2	0	6005,87	5419,03	586,84
A8	8	OA1	4,51			0,56	1,47	4	4	3696,90	3361,65	335,26
A8	8	OA2	3,04	0,67		0,94	0,00	5	0	3696,90	3361,65	335,26
A8	8	OA3	3,39		0,75	1,00	0,23	5	1	3696,90	3361,65	335,26
A9	2	OA1	8,57			3,03	2,42	9	9	9538,04	8319,01	1219,03
A9	2	OA2	6,47	0,76		3,43	0,00	9	0	9538,04	8319,01	1219,03
A9	2	OA3	6,86		0,80	3,53	0,30	9	1	9538,04	8319,01	1219,03
A9	4	OA1	13,20			4,17	4,77	11	11	6258,29	5640,11	618,19
A9	4	OA2	8,60	0,65		4,48	0,00	12	0	6258,32	5640,89	617,43
A9	4	OA3	8,41		0,64	4,04	0,25	11	1	6258,29	5640,11	618,19
A9	8	OA1	4,32			0,78	1,67	4	4	3681,59	3370,46	311,13
A9	8	OA2	2,34	0,54		0,62	0,00	4	0	3681,73	3371,37	310,35
A9	8	OA3	2,29		0,53	0,59	0,00	4	0	3681,73	3371,37	310,35

Table A.10: Linear Utility Function $\epsilon = 0.001$, n=40 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1	3	OA1	94,68			55,33	35,27	26	26	145713,97	13943,63	131770,33
A1	3	OA2	471,84	4,98		466,54	0,00	61	0	145715,52	13956,50	131759,02
A1	3	OA3	472,70		4,99	466,91	0,53	61	1	145713,97	13943,63	131770,33
A1	6	OA1	175,05			60,56	108,81	23	23	75806,17	9890,54	65915,63
A1	6	OA2	222,43	1,27		216,63	0,00	40	0	75816,50	9911,93	65904,57
A1	6	OA3	222,08		1,27	216,34	0,00	40	0	75816,50	9911,93	65904,57
A1	12	OA1	188,07			114,78	68,36	21	21	39229,45	5896,68	33332,78
A1	12	OA2	238,43	1,27		233,54	0,00	31	0	39240,73	5894,88	33345,85
A1	12	OA3	242,91		1,29	233,14	4,78	31	1	39232,77	5883,90	33348,87
A2	3	OA1	260,50			106,21	148,27	37	37	145713,97	13943,63	131770,33
A2	3	OA2	333,89	1,28		328,45	0,00	52	0	145747,66	13964,78	131782,88
A2	3	OA3	334,97		1,29	328,37	0,60	52	1	145713,97	13943,63	131770,33
A2	6	OA1	152,96			69,20	76,40	26	26	76175,55	10264,75	65910,80
A2	6	OA2	200,84	1,31		194,28	0,00	37	0	76196,12	10277,48	65918,64
A2	6	OA3	203,13		1,33	194,70	1,88	37	2	76175,55	10264,75	65910,80
A2	12	OA1	99,18			61,49	32,58	17	17	39518,82	6391,91	33126,90
A2	12	OA2	91,23	0,92		86,49	0,00	26	0	39516,04	6376,06	33139,97
A2	12	OA3	87,52		0,88	81,97	0,90	25	1	39513,54	6375,99	33137,54
A3	3	OA1	183,82			103,63	75,72	37	37	146789,31	14978,35	131810,97
A3	3	OA2	262,89	1,43		258,49	0,00	48	0	146792,28	14951,32	131840,95
A3	3	OA3	252,89		1,38	247,65	0,72	47	1	146789,31	14978,35	131810,97
A3	6	OA1	125,69			64,85	57,11	24	24	76531,59	10535,76	65995,84
A3	6	OA2	154,46	1,23		150,58	0,00	35	0	76506,01	10545,38	65960,63
A3	6	OA3	156,47		1,24	151,90	0,58	35	1	76505,12	10533,83	65971,29
A3	12	OA1	119,73			43,79	69,92	17	17	39940,86	6856,33	33084,52
A3	12	OA2	69,13	0,58		63,51	0,00	22	0	39941,49	6860,51	33080,98
A3	12	OA3	69,52		0,58	63,31	0,38	22	1	39940,86	6856,33	33084,52

Table A.11: Linear Utility Function $\epsilon = 0.001$, n=60 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4	3	OA1	65,98			46,52	14,02	28	28	27048,83	13846,57	13202,26
A4	3	OA2	71,16	1,08		66,25	0,00	30	0	27049,56	13848,40	13201,16
A4	3	OA3	74,62		1,13	67,82	1,52	30	2	27048,83	13846,57	13202,26
A4	6	OA1	42,22			24,46	14,15	16	16	16240,56	9531,02	6709,53
A4	6	OA2	37,74	0,89		34,34	0,00	20	0	16241,37	9524,33	6717,04
A4	6	OA3	40,31		0,95	35,64	1,18	20	1	16240,56	9531,02	6709,53
A4	12	OA1	24,64			10,39	8,28	10	10	9087,62	5673,51	3414,11
A4	12	OA2	20,94	0,85		15,49	0,00	13	0	9088,18	5671,90	3416,29
A4	12	OA3	23,32		0,95	15,74	1,18	13	1	9087,62	5673,51	3414,11
A5	3	OA1	73,73			44,79	11,59	27	27	27048,83	13846,57	13202,26
A5	3	OA2	84,44	1,15		67,43	0,00	30	0	27048,96	13847,34	13201,62
A5	3	OA3	85,52		1,16	67,28	1,25	30	2	27048,83	13846,57	13202,26
A5	6	OA1	49,72			28,80	15,96	18	18	16573,64	9818,81	6754,82
A5	6	OA2	54,24	1,09		49,53	0,00	24	0	16575,76	9807,04	6768,73
A5	6	OA3	56,42		1,13	49,68	1,92	24	2	16573,64	9818,81	6754,82
A5	12	OA1	22,01			11,03	7,79	11	11	9375,56	5911,46	3464,10
A5	12	OA2	20,41	0,93		17,55	0,00	14	0	9376,02	5913,84	3462,19
A5	12	OA3	20,81		0,95	17,55	0,36	14	1	9375,56	5911,46	3464,10
A6	3	OA1	86,42			59,84	21,49	31	31	28063,13	14856,89	13206,24
A6	3	OA2	78,75	0,91		74,47	0,00	31	0	28063,13	14856,89	13206,24
A6	3	OA3	78,64		0,91	74,54	0,00	31	0	28063,13	14856,89	13206,24
A6	6	OA1	46,36			26,27	16,72	17	17	16888,72	10201,44	6687,28
A6	6	OA2	46,80	1,01		43,94	0,00	21	0	16889,50	10200,94	6688,56
A6	6	OA3	47,64		1,03	43,87	0,80	21	1	16888,72	10201,44	6687,28
A6	12	OA1	29,20			13,73	11,04	12	12	10009,94	6368,02	3641,92
A6	12	OA2	19,59	0,67		15,85	0,00	12	0	10011,04	6369,60	3641,43
A6	12	OA3	19,16		0,66	15,52	0,00	12	0	10011,04	6369,60	3641,43

Table A.12: Linear Utility Function $\epsilon = 0.001$, n=60 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7	3	OA1	11,04			3,62	4,29	7	7	15152,10	13822,46	1329,64
A7	3	OA2	7,61	0,69		4,23	0,00	7	0	15152,10	13822,46	1329,64
A7	3	OA3	7,02		0,64	3,99	0,47	7	1	15152,10	13822,46	1329,64
A7	6	OA1	11,99			2,21	2,01	3	3	9994,44	9283,21	711,22
A7	6	OA2	9,95	0,83		2,37	0,00	3	0	9994,47	9282,04	712,42
A7	6	OA3	10,64		0,89	2,26	0,74	3	1	9994,44	9283,21	711,22
A7	12	OA1	9,10			0,88	3,85	4	4	5915,74	5539,75	375,99
A7	12	OA2	5,16	0,57		0,89	0,00	4	0	5915,76	5539,66	376,11
A7	12	OA3	5,63		0,62	1,14	0,00	4	0	5915,76	5539,66	376,11
A8	3	OA1	11,58			3,95	3,77	6	6	15152,10	13822,46	1329,64
A8	3	OA2	7,54	0,65		3,77	0,00	6	0	15152,10	13822,46	1329,64
A8	3	OA3	7,92		0,68	3,84	0,42	6	1	15152,10	13822,46	1329,64
A8	6	OA1	25,40			10,65	9,00	11	11	10377,34	9544,59	832,74
A8	6	OA2	14,91	0,59		9,42	0,00	11	0	10377,34	9544,59	832,74
A8	6	OA3	16,44		0,65	9,36	1,51	11	1	10377,34	9544,59	832,74
A8	12	OA1	8,35			1,69	2,34	4	4	6030,87	5606,11	424,76
A8	12	OA2	5,79	0,69		1,79	0,00	4	0	6030,87	5606,11	424,76
A8	12	OA3	6,57		0,79	1,84	0,75	4	1	6030,87	5606,11	424,76
A9	3	OA1	43,04			7,35	6,35	12	12	16177,51	14856,89	1320,62
A9	3	OA2	37,24	0,87		8,19	0,00	12	0	16177,51	14856,89	1320,62
A9	3	OA3	37,85		0,88	8,27	0,59	12	1	16177,51	14856,89	1320,62
A9	6	OA1	21,08			10,50	7,60	12	12	10401,00	9326,47	1074,53
A9	6	OA2	14,23	0,68		11,72	0,00	13	0	10401,00	9326,47	1074,53
A9	6	OA3	13,51		0,64	10,54	0,34	12	1	10401,00	9326,47	1074,53
A9	12	OA1	13,21			1,42	3,45	4	4	6277,83	5555,71	722,12
A9	12	OA2	11,60	0,88		1,84	0,00	4	0	6277,97	5555,88	722,09
A9	12	OA3	10,98		0,83	1,83	0,67	4	1	6277,83	5555,71	722,12

Table A.13: Linear Utility Function $\epsilon = 0.001$, n=60 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1	4	OA1	361,04			201,72	149,73	31	31	207829,03	20257,80	187571,23
A1	4	OA2	1319,95	3,66		1309,18	0,00	71	0	207867,44	20269,66	187597,78
A1	4	OA3	1296,08		3,59	1285,49	0,00	71	0	207867,44	20269,66	187597,78
A1	8	OA1	329,30			225,40	95,75	28	28	107224,46	13379,58	93844,89
A1	8	OA2	954,75	2,90		946,61	0,00	53	0	107290,70	13445,15	93845,55
A1	8	OA3	911,03		2,77	893,02	9,53	52	2	107255,30	13426,03	93829,27
A1	16	OA1	457,77			339,08	108,30	21	21	55481,71	8416,79	47064,93
A1	16	OA2	1700,87	3,72		1691,78	0,00	29	0	55498,12	8355,03	47143,09
A1	16	OA3	1706,16		3,73	1696,17	0,00	29	0	55498,12	8355,03	47143,09
A2	4	OA1	387,54			238,59	131,76	36	36	208841,25	21220,81	187620,44
A2	4	OA2	881,87	2,28		866,84	0,00	57	0	208888,88	21319,07	187569,80
A2	4	OA3	892,80		2,30	871,81	6,24	57	1	208888,88	21319,07	187569,80
A2	8	OA1	299,46			163,01	127,00	25	25	107860,13	14044,19	93815,94
A2	8	OA2	353,98	1,18		344,55	0,00	37	0	107878,23	13913,54	93964,69
A2	8	OA3	354,89		1,19	345,69	0,00	37	0	107878,23	13913,54	93964,69
A2	16	OA1	275,65			161,37	102,94	19	19	56664,42	9540,75	47123,67
A2	16	OA2	216,98	0,79		208,25	0,00	23	0	56677,45	9535,36	47142,09
A2	16	OA3	216,98		0,79	208,29	0,00	23	0	56677,45	9535,36	47142,09
A3	4	OA1	398,32			254,99	135,24	36	36	208916,50	21208,17	187708,31
A3	4	OA2	538,61	1,35		530,34	0,00	51	0	209025,66	21178,43	187847,23
A3	4	OA3	526,02		1,32	516,50	1,11	50	1	208916,50	21208,17	187708,31
A3	8	OA1	311,47			138,50	164,61	25	25	108925,01	15082,19	93842,82
A3	8	OA2	250,13	0,80		242,55	0,00	32	0	108933,99	15056,23	93877,76
A3	8	OA3	258,15		0,83	242,71	7,63	32	1	108925,01	15082,19	93842,82
A3	16	OA1	184,91			88,68	88,84	17	17	59311,88	12262,14	47049,74
A3	16	OA2	129,26	0,70		123,01	0,00	23	0	59320,94	12253,63	47067,32
A3	16	OA3	129,90		0,70	123,74	0,00	23	0	59320,94	12253,63	47067,32

Table A.14: Linear Utility Function $\epsilon = 0.001$, n=80 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4	4	OA1	248,41			118,13	26,36	28	28	38736,22	19867,62	18868,61
A4	4	OA2	278,58	1,12		175,06	0,00	32	0	38738,69	19879,66	18859,03
A4	4	OA3	277,85		1,12	174,80	0,00	32	0	38738,69	19879,66	18859,03
A4	8	OA1	152,23			55,82	27,52	18	18	22501,44	12900,52	9600,92
A4	8	OA2	165,47	1,09		97,08	0,00	24	0	22505,07	12903,84	9601,23
A4	8	OA3	166,09		1,09	97,19	0,53	24	1	22501,44	12900,52	9600,92
A4	16	OA1	49,56			24,91	12,38	11	11	12810,12	7943,92	4866,20
A4	16	OA2	42,76	0,86		31,12	0,00	13	0	12810,36	7948,86	4861,50
A4	16	OA3	43,51		0,88	31,28	0,48	13	1	12810,12	7943,92	4866,20
A5	4	OA1	347,84			154,18	36,08	32	32	39745,97	20821,08	18924,89
A5	4	OA2	375,74	1,08		220,81	0,00	36	0	39748,42	20806,93	18941,49
A5	4	OA3	375,68		1,08	220,72	0,00	36	0	39748,42	20806,93	18941,49
A5	8	OA1	144,05			55,71	27,16	17	17	23088,01	13586,91	9501,10
A5	8	OA2	138,44	0,96		77,81	0,00	20	0	23089,24	13581,89	9507,35
A5	8	OA3	138,72		0,96	78,06	0,32	20	1	23088,01	13586,91	9501,10
A5	16	OA1	64,44			34,43	12,61	12	12	13745,55	8603,16	5142,39
A5	16	OA2	50,36	0,78		33,93	0,00	12	0	13744,31	8602,68	5141,63
A5	16	OA3	50,13		0,78	34,87	0,00	12	0	13744,31	8602,68	5141,63
A6	4	OA1	283,13			142,83	28,93	29	29	39551,55	20472,20	19079,35
A6	4	OA2	304,43	1,08		193,79	0,00	33	0	39551,70	20464,55	19087,15
A6	4	OA3	304,99		1,08	193,69	0,34	33	1	39551,55	20472,20	19079,35
A6	8	OA1	156,79			56,89	26,36	17	17	23961,32	13759,38	10201,95
A6	8	OA2	172,01	1,10		99,52	0,00	22	0	23964,04	13759,04	10204,99
A6	8	OA3	170,66		1,09	98,28	0,00	22	0	23964,04	13759,04	10204,99
A6	16	OA1	58,24			29,29	13,74	11	11	14462,17	8666,66	5795,51
A6	16	OA2	49,50	0,85		35,18	0,00	12	0	14463,91	8685,17	5778,74
A6	16	OA3	50,31		0,86	35,46	0,47	12	1	14462,17	8666,66	5795,51

Table A.15: Linear Utility Function $\epsilon = 0.001$, n=80 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7	4	OA1	44,62			15,09	9,81	11	11	21700,75	19793,55	1907,20
A7	4	OA2	33,84	0,76		15,32	0,00	11	0	21700,75	19793,55	1907,20
A7	4	OA3	33,67		0,75	15,24	0,00	11	0	21700,75	19793,55	1907,20
A7	8	OA1	43,26			5,34	5,07	6	6	13672,67	12681,76	990,92
A7	8	OA2	38,02	0,88		5,52	0,00	6	0	13672,67	12681,76	990,92
A7	8	OA3	38,88		0,90	5,49	0,91	6	1	13672,67	12681,76	990,92
A7	16	OA1	16,61			2,18	4,90	4	4	8376,67	7877,87	498,81
A7	16	OA2	10,81	0,65		2,11	0,00	4	0	8376,69	7877,79	498,90
A7	16	OA3	10,78		0,65	2,07	0,00	4	0	8376,69	7877,79	498,90
A8	4	OA1	57,57			31,03	13,50	16	16	22685,11	20200,75	2484,37
A8	4	OA2	43,81	0,76		31,24	0,00	16	0	22685,71	20204,44	2481,27
A8	4	OA3	43,35		0,75	31,89	0,00	16	0	22685,71	20204,44	2481,27
A8	8	OA1	27,22			8,91	10,77	8	8	14025,05	12900,62	1124,43
A8	8	OA2	15,72	0,58		8,76	0,00	8	0	14025,05	12900,62	1124,43
A8	8	OA3	16,67		0,61	8,74	0,00	8	0	14025,05	12900,62	1124,43
A8	16	OA1	15,13			2,48	4,47	3	3	8564,05	7883,13	680,91
A8	16	OA2	10,40	0,69		2,57	0,00	3	0	8564,85	7878,96	685,89
A8	16	OA3	10,45		0,69	2,59	0,00	3	0	8564,85	7878,96	685,89
A9	4	OA1	41,70			19,51	10,43	12	12	22185,08	20130,53	2054,56
A9	4	OA2	30,21	0,72		19,56	0,00	12	0	22185,08	20130,53	2054,56
A9	4	OA3	30,93		0,74	19,64	0,00	12	0	22185,08	20130,53	2054,56
A9	8	OA1	41,42			10,88	10,54	9	9	14327,77	13062,13	1265,64
A9	8	OA2	33,26	0,80		12,66	0,00	10	0	14329,81	13061,97	1267,84
A9	8	OA3	31,99		0,77	12,56	0,00	10	0	14329,81	13061,97	1267,84
A9	16	OA1	14,88			2,69	5,05	4	4	8755,85	7988,78	767,08
A9	16	OA2	8,35	0,56		2,90	0,00	4	0	8755,85	7988,78	767,08
A9	16	OA3	8,26		0,55	2,86	0,00	4	0	8755,85	7988,78	767,08

Table A.16: Linear Utility Function $\epsilon = 0.001$, n=80 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1	1	OA1	4,63			2,28	1,71	21	21	49791,17	4632,77	45158,40
A1	1	OA2	2,67	0,58		1,98	0,00	21	0	49791,17	4632,77	45158,40
A1	1	OA3	2,87		0,62	2,03	0,11	21	1	49791,17	4632,77	45158,40
A1	2	OA1	46,16			2,92	42,76	13	13	25605,85	2995,83	22610,02
A1	2	OA2	14,76	0,32		14,20	0,00	30	0	25605,85	2996,26	22609,60
A1	2	OA3	31,09		0,67	12,57	17,92	26	4	25605,85	2995,83	22610,02
A1	4	OA1	12,20			5,16	6,66	17	17	13582,83	2260,44	11322,39
A1	4	OA2	10,00	0,82		9,52	0,00	24	0	13583,12	2257,33	11325,79
A1	4	OA3	9,94		0,81	8,88	0,52	23	1	13582,83	2260,44	11322,39
A2	1	OA1	8,47			2,92	4,87	21	21	49947,43	4789,03	45158,40
A2	1	OA2	3,54	0,42		2,90	0,00	21	0	49947,43	4789,03	45158,40
A2	1	OA3	3,39		0,40	2,65	0,06	21	1	49947,43	4789,03	45158,40
A2	2	OA1	36,49			3,75	32,21	15	15	25605,85	2995,83	22610,02
A2	2	OA2	11,40	0,31		10,84	0,00	25	0	25605,92	2993,00	22612,92
A2	2	OA3	13,84		0,38	9,13	4,12	23	3	25605,85	2995,83	22610,02
A2	4	OA1	11,26			3,87	6,97	15	15	13591,26	2275,84	11315,42
A2	4	OA2	6,47	0,57		6,04	0,00	20	0	13591,47	2277,51	11313,96
A2	4	OA3	6,82		0,61	5,20	1,12	18	2	13591,26	2275,84	11315,42
A3	1	OA1	8,78			2,17	5,43	21	21	50222,94	5064,53	45158,40
A3	1	OA2	3,23	0,37		2,11	0,00	21	0	50222,94	5064,53	45158,40
A3	1	OA3	3,53		0,40	2,15	0,25	21	1	50222,94	5064,53	45158,40
A3	2	OA1	37,08			4,18	32,29	14	14	26223,67	3624,47	22599,20
A3	2	OA2	8,14	0,22		7,58	0,00	19	0	26223,67	3624,47	22599,20
A3	2	OA3	8,92		0,24	7,43	0,83	18	2	26223,67	3624,47	22599,20
A3	4	OA1	16,97			3,49	9,25	14	14	15679,05	2761,43	12917,62
A3	4	OA2	12,42	0,73		8,14	0,00	26	0	15679,13	2762,69	12916,44
A3	4	OA3	12,95		0,76	7,50	1,16	24	3	15679,05	2761,43	12917,62

Table A.17: Linear Utility Function $\epsilon = 0.0001$, n=20 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4	1	OA1	7,61			2,72	3,81	15	15	9148,61	4632,77	4515,84
A4	1	OA2	3,71	0,49		2,72	0,00	15	0	9148,61	4632,77	4515,84
A4	1	OA3	4,01		0,53	2,73	0,25	15	1	9148,61	4632,77	4515,84
A4	2	OA1	4,60			1,45	2,70	8	8	5072,48	2724,78	2347,70
A4	2	OA2	1,86	0,40		1,39	0,00	8	0	5072,48	2724,78	2347,70
A4	2	OA3	2,22		0,48	1,48	0,28	8	1	5072,48	2724,78	2347,70
A4	4	OA1	6,15			1,89	3,62	9	9	3163,39	1916,42	1246,98
A4	4	OA2	3,31	0,54		2,72	0,00	12	0	3163,39	1916,33	1247,06
A4	4	OA3	3,70		0,60	2,39	0,78	10	2	3163,39	1916,42	1246,98
A5	1	OA1	6,71			2,74	2,51	15	15	9304,87	4789,03	4515,84
A5	1	OA2	4,06	0,60		2,61	0,00	15	0	9304,87	4789,03	4515,84
A5	1	OA3	4,28		0,64	2,71	0,10	15	1	9304,87	4789,03	4515,84
A5	2	OA1	3,68			1,32	1,54	8	8	5072,48	2724,78	2347,70
A5	2	OA2	2,14	0,58		1,34	0,00	8	0	5072,48	2724,78	2347,70
A5	2	OA3	2,29		0,62	1,36	0,12	8	1	5072,48	2724,78	2347,70
A5	4	OA1	5,66			1,78	3,39	9	9	3214,68	1969,42	1245,26
A5	4	OA2	2,47	0,44		2,03	0,00	9	0	3214,68	1969,42	1245,26
A5	4	OA3	2,50		0,44	2,04	0,00	9	0	3214,68	1969,42	1245,26
A6	1	OA1	7,00			3,76	2,21	15	15	9580,38	5064,53	4515,84
A6	1	OA2	4,01	0,57		3,00	0,00	15	0	9580,38	5064,53	4515,84
A6	1	OA3	4,09		0,58	3,01	0,10	15	1	9580,38	5064,53	4515,84
A6	2	OA1	6,14			2,44	2,98	10	10	5847,28	3533,01	2314,27
A6	2	OA2	3,08	0,50		2,38	0,00	12	0	5847,28	3533,71	2313,57
A6	2	OA3	3,66		0,60	2,12	0,84	11	2	5847,28	3533,01	2314,27
A6	4	OA1	7,02			2,31	4,15	10	10	3888,21	2260,60	1627,61
A6	4	OA2	3,93	0,56		3,40	0,00	12	0	3888,21	2260,60	1627,61
A6	4	OA3	4,53		0,64	3,58	0,38	12	1	3888,21	2260,60	1627,61

Table A.18: Linear Utility Function $\epsilon = 0.0001$, n=20 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7	1	OA1	3,99			1,04	1,30	5	5	5084,35	4632,77	451,58
A7	1	OA2	2,51	0,63		0,86	0,00	5	0	5084,35	4632,77	451,58
A7	1	OA3	2,96		0,74	1,11	0,25	5	1	5084,35	4632,77	451,58
A7	2	OA1	2,36			0,39	0,89	2	2	2959,55	2724,78	234,77
A7	2	OA2	1,50	0,64		0,39	0,00	2	0	2959,55	2724,78	234,77
A7	2	OA3	1,81		0,77	0,33	0,44	2	1	2959,55	2724,78	234,77
A7	4	OA1	3,74			0,84	1,58	4	4	1964,58	1812,51	152,08
A7	4	OA2	2,31	0,62		1,02	0,00	4	0	1964,58	1812,51	152,08
A7	4	OA3	2,45		0,65	1,19	0,00	4	0	1964,58	1812,51	152,08
A8	1	OA1	63,06			0,93	0,61	5	5	5240,62	4789,03	451,58
A8	1	OA2	62,64	0,99		1,00	0,00	5	0	5240,62	4789,03	451,58
A8	1	OA3	62,63		0,99	1,01	0,14	5	1	5240,62	4789,03	451,58
A8	2	OA1	1,36			0,39	0,41	2	2	2959,55	2724,78	234,77
A8	2	OA2	0,84	0,62		0,29	0,00	2	0	2959,55	2724,78	234,77
A8	2	OA3	1,05		0,77	0,32	0,18	2	1	2959,55	2724,78	234,77
A8	4	OA1	1,58			0,32	0,63	3	3	1971,64	1812,51	159,13
A8	4	OA2	0,93	0,59		0,31	0,00	3	0	1971,64	1812,51	159,13
A8	4	OA3	1,10		0,70	0,30	0,21	3	1	1971,64	1812,51	159,13
A9	1	OA1	4,51			0,80	0,55	5	5	5516,12	5064,53	451,58
A9	1	OA2	3,96	0,88		0,79	0,00	5	0	5516,12	5064,53	451,58
A9	1	OA3	4,07		0,90	0,81	0,11	5	1	5516,12	5064,53	451,58
A9	2	OA1	3,97			1,47	0,72	5	5	3356,73	2724,78	631,95
A9	2	OA2	3,11	0,78		1,35	0,00	5	0	3356,73	2724,78	631,95
A9	2	OA3	3,25		0,82	1,36	0,13	5	1	3356,73	2724,78	631,95
A9	4	OA1	2,47			0,98	1,11	5	5	2222,06	1924,53	297,53
A9	4	OA2	1,79	0,72		1,36	0,00	6	0	2222,06	1924,53	297,53
A9	4	OA3	2,25		0,91	1,23	0,56	6	2	2222,06	1924,53	297,53

Table A.19: Linear Utility Function $\epsilon = 0.0001$, n=20 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1	2	OA1	70,01			20,32	48,05	27	27	88483,95	8335,70	80148,24
A1	2	OA2	120,15	1,72		118,09	0,00	57	0	88484,03	8333,81	80150,22
A1	2	OA3	122,80		1,75	115,88	4,79	56	7	88483,95	8335,70	80148,24
A1	4	OA1	40,86			22,98	16,16	23	23	45941,02	5837,27	40103,75
A1	4	OA2	90,01	2,20		88,05	0,00	40	0	45941,67	5845,67	40096,00
A1	4	OA3	85,94		2,10	83,23	0,73	39	2	45941,02	5837,27	40103,75
A1	8	OA1	37,66			19,62	15,93	19	19	23630,91	3510,21	20120,70
A1	8	OA2	50,22	1,33		48,02	0,00	30	0	23631,25	3510,26	20120,99
A1	8	OA3	52,79		1,40	47,87	2,68	30	3	23630,91	3510,21	20120,70
A2	2	OA1	208,29			42,40	163,08	37	37	89393,49	9221,89	80171,60
A2	2	OA2	81,89	0,39		79,39	0,00	49	0	89393,49	9221,89	80171,60
A2	2	OA3	101,93		0,49	78,31	20,93	49	5	89393,49	9221,89	80171,60
A2	4	OA1	47,83			26,49	19,28	24	24	46168,23	6065,80	40102,42
A2	4	OA2	60,51	1,27		58,33	0,00	39	0	46169,20	6063,51	40105,70
A2	4	OA3	64,37		1,35	57,74	4,45	38	6	46168,23	6065,80	40102,42
A2	8	OA1	34,83			12,51	20,90	14	14	24462,18	4380,06	20082,13
A2	8	OA2	29,02	0,83		27,51	0,00	23	0	24462,33	4379,33	20083,00
A2	8	OA3	27,55		0,79	24,44	1,52	22	3	24462,18	4380,06	20082,13
A3	2	OA1	239,65			27,28	210,19	32	32	90291,09	10103,96	80187,13
A3	2	OA2	68,25	0,28		66,05	0,00	46	0	90292,45	10087,59	80204,86
A3	2	OA3	106,71		0,45	64,95	39,25	45	4	90291,09	10103,96	80187,13
A3	4	OA1	52,65			24,96	25,07	24	24	47555,38	7419,80	40135,57
A3	4	OA2	44,90	0,85		42,90	0,00	30	0	47555,65	7421,77	40133,88
A3	4	OA3	45,35		0,86	42,06	1,05	30	2	47555,38	7419,80	40135,57
A3	8	OA1	27,36			14,31	10,58	16	16	25370,63	5241,59	20129,05
A3	8	OA2	24,01	0,88		22,30	0,00	23	0	25371,73	5239,90	20131,83
A3	8	OA3	25,05		0,92	21,99	1,28	22	2	25370,63	5241,59	20129,05

Table A.20: Linear Utility Function $\epsilon = 0.0001$, n=40 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4	2	OA1	23,72			12,75	7,72	25	25	16335,67	8315,38	8020,28
A4	2	OA2	17,74	0,75		14,66	0,00	25	0	16335,74	8313,96	8021,78
A4	2	OA3	19,60		0,83	16,04	0,46	25	2	16335,67	8315,38	8020,28
A4	4	OA1	21,11			9,59	9,29	15	15	9654,06	5502,00	4152,06
A4	4	OA2	21,21	1,00		19,20	0,00	23	0	9654,06	5502,00	4152,06
A4	4	OA3	23,85		1,13	20,07	1,48	23	3	9654,06	5502,00	4152,06
A4	8	OA1	14,33			5,14	5,96	10	10	5456,58	3398,81	2057,77
A4	8	OA2	11,15	0,78		8,36	0,00	14	0	5456,65	3399,78	2056,86
A4	8	OA3	10,92		0,76	7,41	0,71	13	2	5456,58	3398,81	2057,77
A5	2	OA1	54,98			17,27	35,56	28	28	17192,26	9158,00	8034,26
A5	2	OA2	20,31	0,37		18,41	0,00	26	0	17192,26	9158,00	8034,26
A5	2	OA3	19,01		0,35	17,21	0,00	26	0	17192,26	9158,00	8034,26
A5	4	OA1	18,22			8,04	6,91	15	15	9983,59	5875,78	4107,81
A5	4	OA2	14,65	0,80		11,51	0,00	17	0	9983,59	5875,40	4108,19
A5	4	OA3	15,62		0,86	11,05	0,51	17	2	9983,59	5875,78	4107,81
A5	8	OA1	10,40			4,77	4,24	11	11	6139,76	3864,00	2275,76
A5	8	OA2	6,80	0,65		5,62	0,00	11	0	6139,76	3864,00	2275,76
A5	8	OA3	7,62		0,73	5,64	0,69	11	2	6139,76	3864,00	2275,76
A6	2	OA1	36,23			19,77	11,50	29	29	18011,64	9927,78	8083,86
A6	2	OA2	33,01	0,91		28,33	0,00	32	0	18011,64	9927,43	8084,21
A6	2	OA3	31,28		0,86	26,25	0,37	30	2	18011,64	9927,78	8083,86
A6	4	OA1	29,15			15,25	12,08	21	21	11174,38	6327,07	4847,31
A6	4	OA2	23,55	0,81		22,08	0,00	27	0	11174,44	6328,29	4846,14
A6	4	OA3	27,00		0,93	23,19	2,14	27	3	11174,38	6327,07	4847,31
A6	8	OA1	13,94			6,82	5,07	11	11	6319,90	3542,44	2777,47
A6	8	OA2	13,95	1,00		11,97	0,00	17	0	6319,91	3540,89	2779,02
A6	8	OA3	14,72		1,06	11,41	1,21	16	2	6319,90	3542,44	2777,47

Table A.21: Linear Utility Function $\epsilon = 0.0001$, n=40 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7	2	OA1	9,05			1,91	1,64	6	6	9111,75	8308,92	802,84
A7	2	OA2	8,76	0,97		1,90	0,00	6	0	9111,75	8308,92	802,84
A7	2	OA3	7,57		0,84	1,86	0,22	6	1	9111,75	8308,92	802,84
A7	4	OA1	3,44			0,86	1,04	3	3	5843,46	5419,03	424,43
A7	4	OA2	2,09	0,61		0,75	0,00	3	0	5843,46	5419,03	424,43
A7	4	OA3	2,40		0,70	0,76	0,32	3	1	5843,46	5419,03	424,43
A7	8	OA1	4,21			0,74	1,97	3	3	3536,43	3294,69	241,74
A7	8	OA2	2,16	0,51		0,78	0,00	3	0	3536,44	3294,63	241,81
A7	8	OA3	2,29		0,54	0,77	0,00	3	0	3536,44	3294,63	241,81
A8	2	OA1	12,57			1,76	1,67	6	6	9478,99	8308,92	1170,07
A8	2	OA2	10,30	0,82		1,75	0,00	6	0	9478,99	8308,92	1170,07
A8	2	OA3	10,66		0,85	1,81	0,34	6	1	9478,99	8308,92	1170,07
A8	4	OA1	3,40			0,41	1,37	3	3	6005,87	5419,03	586,84
A8	4	OA2	2,08	0,61		0,64	0,00	3	0	6005,87	5419,03	586,84
A8	4	OA3	2,53		0,74	0,60	0,50	3	1	6005,87	5419,03	586,84
A8	8	OA1	4,40			0,54	1,58	4	4	3696,90	3361,65	335,26
A8	8	OA2	3,30	0,75		1,00	0,00	5	0	3696,90	3361,65	335,26
A8	8	OA3	3,46		0,79	0,90	0,32	5	1	3696,90	3361,65	335,26
A9	2	OA1	8,13			3,11	1,73	9	9	9538,04	8319,01	1219,03
A9	2	OA2	7,68	0,95		4,09	0,00	9	0	9538,04	8319,01	1219,03
A9	2	OA3	7,02		0,86	3,51	0,27	9	1	9538,04	8319,01	1219,03
A9	4	OA1	13,98			4,31	5,21	11	11	6258,29	5640,11	618,19
A9	4	OA2	9,29	0,66		5,02	0,00	13	0	6258,32	5640,89	617,43
A9	4	OA3	9,59		0,69	4,56	0,69	12	2	6258,29	5640,11	618,19
A9	8	OA1	4,28			0,76	1,62	4	4	3681,59	3370,46	311,13
A9	8	OA2	2,60	0,61		0,75	0,00	5	0	3681,60	3370,09	311,51
A9	8	OA3	3,03		0,71	0,75	0,41	5	1	3681,59	3370,46	311,13

Table A.22: Linear Utility Function $\epsilon = 0.0001$, n=40 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1	3	OA1	103,98			61,99	37,76	27	27	145713,97	13943,63	131770,33
A1	3	OA2	519,02	4,99		513,18	0,00	62	0	145715,52	13956,50	131759,02
A1	3	OA3	522,28		5,02	515,94	0,59	62	1	145713,97	13943,63	131770,33
A1	6	OA1	192,05			70,72	115,29	24	24	75806,17	9890,54	65915,63
A1	6	OA2	275,77	1,44		269,36	0,00	44	0	75806,38	9890,53	65915,85
A1	6	OA3	267,03		1,39	259,51	0,98	43	2	75806,17	9890,54	65915,63
A1	12	OA1	204,99			126,74	72,01	22	22	39229,45	5896,68	33332,78
A1	12	OA2	368,92	1,80		363,60	0,00	37	0	39229,58	5898,39	33331,19
A1	12	OA3	376,55		1,84	365,02	6,02	37	2	39229,45	5896,68	33332,78
A2	3	OA1	262,89			109,53	147,79	38	38	145713,97	13943,63	131770,33
A2	3	OA2	350,45	1,33		344,90	0,00	53	0	145713,98	13942,49	131771,50
A2	3	OA3	351,78		1,34	344,93	1,19	53	2	145713,97	13943,63	131770,33
A2	6	OA1	193,36			81,24	104,62	28	28	76175,55	10264,75	65910,80
A2	6	OA2	223,90	1,16		217,21	0,00	39	0	76176,34	10268,55	65907,79
A2	6	OA3	215,06		1,11	206,32	1,90	38	2	76175,55	10264,75	65910,80
A2	12	OA1	109,80			71,29	32,71	18	18	39513,54	6375,99	33137,54
A2	12	OA2	102,24	0,93		97,34	0,00	27	0	39514,06	6382,90	33131,16
A2	12	OA3	96,11		0,88	89,02	1,98	26	2	39513,54	6375,99	33137,54
A3	3	OA1	180,75			105,79	70,20	37	37	146789,31	14978,35	131810,97
A3	3	OA2	275,30	1,52		270,67	0,00	49	0	146792,28	14951,32	131840,95
A3	3	OA3	265,90		1,47	259,73	1,48	48	2	146789,31	14978,35	131810,97
A3	6	OA1	143,54			76,41	63,01	26	26	76502,88	10529,21	65973,66
A3	6	OA2	164,07	1,14		160,11	0,00	36	0	76504,19	10529,37	65974,82
A3	6	OA3	180,17		1,26	168,10	7,78	37	3	76502,88	10529,21	65973,66
A3	12	OA1	120,33			42,74	70,31	17	17	39940,86	6856,33	33084,52
A3	12	OA2	83,22	0,69		77,40	0,00	25	0	39940,88	6857,32	33083,55
A3	12	OA3	74,57		0,62	68,18	0,60	23	2	39940,86	6856,33	33084,52

Table A.23: Linear Utility Function $\epsilon = 0.0001$, n=60 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4	3	OA1	66,12			46,23	14,63	28	28	27048,83	13846,57	13202,26
A4	3	OA2	79,81	1,21		74,75	0,00	32	0	27048,93	13845,91	13203,02
A4	3	OA3	73,41		1,11	67,09	1,34	30	2	27048,83	13846,57	13202,26
A4	6	OA1	42,19			24,48	13,96	16	16	16240,56	9531,02	6709,53
A4	6	OA2	47,39	1,12		43,80	0,00	23	0	16240,56	9531,98	6708,58
A4	6	OA3	44,54		1,06	38,57	2,34	21	2	16240,56	9531,02	6709,53
A4	12	OA1	26,67			11,36	9,46	11	11	9087,62	5673,51	3414,11
A4	12	OA2	24,45	0,92		18,33	0,00	15	0	9087,75	5674,29	3413,45
A4	12	OA3	25,55		0,96	17,29	2,48	14	2	9087,62	5673,51	3414,11
A5	3	OA1	84,12			52,44	13,52	29	29	27048,83	13846,57	13202,26
A5	3	OA2	94,38	1,12		75,64	0,00	31	0	27048,96	13847,34	13201,62
A5	3	OA3	89,92		1,07	70,83	1,33	30	2	27048,83	13846,57	13202,26
A5	6	OA1	53,99			30,50	18,00	18	18	16573,64	9818,81	6754,82
A5	6	OA2	60,73	1,12		55,71	0,00	25	0	16573,85	9815,09	6758,76
A5	6	OA3	59,22		1,10	52,34	1,89	24	2	16573,64	9818,81	6754,82
A5	12	OA1	25,42			12,91	8,77	12	12	9375,56	5911,46	3464,10
A5	12	OA2	24,20	0,95		21,18	0,00	15	0	9375,67	5913,74	3461,92
A5	12	OA3	23,88		0,94	19,99	0,75	15	2	9375,56	5911,46	3464,10
A6	3	OA1	92,28			63,00	23,97	31	31	28063,13	14856,89	13206,24
A6	3	OA2	89,08	0,97		84,66	0,00	32	0	28063,13	14856,89	13206,24
A6	3	OA3	88,63		0,96	83,70	0,62	32	1	28063,13	14856,89	13206,24
A6	6	OA1	50,43			30,65	16,59	18	18	16888,72	10201,44	6687,28
A6	6	OA2	61,59	1,22		58,51	0,00	24	0	16889,21	10195,80	6693,40
A6	6	OA3	62,60		1,24	57,41	1,91	24	2	16888,72	10201,44	6687,28
A6	12	OA1	32,86			15,81	12,44	13	13	10009,94	6368,02	3641,92
A6	12	OA2	26,85	0,82		22,96	0,00	15	0	10009,96	6368,74	3641,22
A6	12	OA3	22,13		0,67	17,46	0,81	13	1	10009,94	6368,02	3641,92

Table A.24: Linear Utility Function $\epsilon = 0.0001$, n=60 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7	3	OA1	10,14			3,66	3,54	7	7	15152,10	13822,46	1329,64
A7	3	OA2	6,95	0,69		4,41	0,00	7	0	15152,10	13822,46	1329,64
A7	3	OA3	7,48		0,74	4,44	0,50	7	1	15152,10	13822,46	1329,64
A7	6	OA1	12,28			2,28	2,21	3	3	9994,44	9283,21	711,22
A7	6	OA2	11,17	0,91		2,55	0,00	3	0	9994,47	9282,04	712,42
A7	6	OA3	10,94		0,89	2,29	0,80	3	1	9994,44	9283,21	711,22
A7	12	OA1	9,23			0,93	3,89	4	4	5915,74	5539,75	375,99
A7	12	OA2	5,28	0,57		0,89	0,00	4	0	5915,76	5539,66	376,11
A7	12	OA3	5,22		0,57	0,90	0,00	4	0	5915,76	5539,66	376,11
A8	3	OA1	11,71			3,90	3,94	6	6	15152,10	13822,46	1329,64
A8	3	OA2	7,43	0,63		3,56	0,00	6	0	15152,10	13822,46	1329,64
A8	3	OA3	7,72		0,66	3,55	0,44	6	1	15152,10	13822,46	1329,64
A8	6	OA1	27,96			11,31	10,51	11	11	10377,34	9544,59	832,74
A8	6	OA2	15,27	0,55		9,55	0,00	11	0	10377,34	9544,59	832,74
A8	6	OA3	16,78		0,60	9,53	1,54	11	1	10377,34	9544,59	832,74
A8	12	OA1	8,77			1,72	2,54	4	4	6030,87	5606,11	424,76
A8	12	OA2	7,17	0,82		2,27	0,00	4	0	6030,87	5606,11	424,76
A8	12	OA3	6,92		0,79	1,90	0,85	4	1	6030,87	5606,11	424,76
A9	3	OA1	44,56			7,38	6,53	12	12	16177,51	14856,89	1320,62
A9	3	OA2	39,74	0,89		8,48	0,00	12	0	16177,51	14856,89	1320,62
A9	3	OA3	40,29		0,90	9,47	0,63	12	1	16177,51	14856,89	1320,62
A9	6	OA1	21,77			10,78	7,87	12	12	10401,00	9326,47	1074,53
A9	6	OA2	14,64	0,67		12,03	0,00	13	0	10401,00	9326,47	1074,53
A9	6	OA3	15,63		0,72	12,08	0,82	13	2	10401,00	9326,47	1074,53
A9	12	OA1	14,69			1,66	4,20	4	4	6277,83	5555,71	722,12
A9	12	OA2	11,97	0,81		3,40	0,00	6	0	6277,85	5556,06	721,79
A9	12	OA3	12,03		0,82	2,52	0,75	5	1	6277,83	5555,71	722,12

Table A.25: Linear Utility Function $\epsilon = 0.0001$, n=60 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1	4	OA1	403,12			221,65	171,89	32	32	207829,03	20257,80	187571,23
A1	4	OA2	1474,06	3,66		1462,39	0,00	74	0	207833,50	20275,45	187558,05
A1	4	OA3	1493,10		3,70	1462,41	18,96	74	1	207829,03	20257,80	187571,23
A1	8	OA1	531,83			365,84	157,00	33	33	107224,46	13379,58	93844,89
A1	8	OA2	1195,15	2,25		1185,91	0,00	57	0	107226,25	13370,03	93856,22
A1	8	OA3	1216,51		2,29	1194,20	12,06	57	3	107224,46	13379,58	93844,89
A1	16	OA1	525,05			395,09	117,66	22	22	55481,71	8416,79	47064,93
A1	16	OA2	5255,74	10,01		5244,22	0,00	39	0	55481,71	8417,02	47064,70
A1	16	OA3	2959,94		5,64	2943,46	5,52	33	2	55481,71	8416,79	47064,93
A2	4	OA1	437,86			276,95	144,12	38	38	208790,33	21205,63	187584,70
A2	4	OA2	991,53	2,26		975,39	0,00	59	0	208798,70	21217,10	187581,59
A2	4	OA3	972,35		2,22	946,03	9,55	58	2	208790,33	21205,63	187584,70
A2	8	OA1	352,43			199,81	141,38	27	27	107858,08	13939,07	93919,01
A2	8	OA2	458,39	1,30		447,85	0,00	42	0	107859,48	13921,37	93938,11
A2	8	OA3	474,03		1,35	451,15	12,27	42	3	107858,08	13939,07	93919,01
A2	16	OA1	366,77			217,56	136,95	21	21	56664,42	9540,75	47123,67
A2	16	OA2	358,17	0,98		348,61	0,00	28	0	56664,44	9540,98	47123,46
A2	16	OA3	283,60		0,77	270,36	3,85	25	2	56664,42	9540,75	47123,67
A3	4	OA1	422,93			269,93	144,54	36	36	208916,50	21208,17	187708,31
A3	4	OA2	613,25	1,45		604,12	0,00	54	0	208917,05	21209,15	187707,89
A3	4	OA3	571,66		1,35	561,49	1,11	51	1	208916,50	21208,17	187708,31
A3	8	OA1	330,26			148,15	173,17	25	25	108925,01	15082,19	93842,82
A3	8	OA2	289,23	0,88		280,97	0,00	34	0	108925,55	15088,35	93837,20
A3	8	OA3	292,57		0,89	267,43	16,90	33	2	108925,01	15082,19	93842,82
A3	16	OA1	205,26			99,87	97,17	18	18	59311,88	12262,14	47049,74
A3	16	OA2	163,38	0,80		156,49	0,00	26	0	59313,17	12266,50	47046,67
A3	16	OA3	168,21		0,82	156,75	4,49	26	3	59311,88	12262,14	47049,74

Table A.26: Linear Utility Function $\epsilon = 0.0001$, n=80 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4	4	OA1	270,79			132,08	29,40	29	29	38736,22	19867,62	18868,61
A4	4	OA2	319,36	1,18		209,82	0,00	35	0	38736,29	19869,63	18866,66
A4	4	OA3	315,01		1,16	203,45	2,97	34	2	38736,22	19867,62	18868,61
A4	8	OA1	176,56			72,24	30,72	20	20	22501,44	12900,52	9600,92
A4	8	OA2	199,89	1,13		127,29	0,00	27	0	22502,01	12920,67	9581,34
A4	8	OA3	195,68		1,11	118,66	2,58	26	3	22501,44	12900,52	9600,92
A4	16	OA1	52,66			26,53	13,08	11	11	12810,12	7943,92	4866,20
A4	16	OA2	53,80	1,02		41,48	0,00	15	0	12810,17	7946,31	4863,86
A4	16	OA3	51,68		0,98	38,07	0,98	14	2	12810,12	7943,92	4866,20
A5	4	OA1	357,33			163,38	37,40	33	33	39745,97	20821,08	18924,89
A5	4	OA2	389,43	1,09		233,15	0,00	37	0	39745,97	20821,08	18924,89
A5	4	OA3	389,01		1,09	232,80	0,98	37	1	39745,97	20821,08	18924,89
A5	8	OA1	152,33			63,57	27,20	18	18	23088,01	13586,91	9501,10
A5	8	OA2	152,50	1,00		92,23	0,00	22	0	23088,04	13586,99	9501,04
A5	8	OA3	144,66		0,95	83,67	0,37	21	1	23088,01	13586,91	9501,10
A5	16	OA1	75,19			43,59	14,59	14	14	13739,73	8601,69	5138,04
A5	16	OA2	66,09	0,88		49,76	0,00	15	0	13739,75	8601,44	5138,31
A5	16	OA3	66,71		0,89	48,59	1,67	15	2	13739,73	8601,69	5138,04
A6	4	OA1	293,84			152,14	30,32	30	30	39551,55	20472,20	19079,35
A6	4	OA2	313,94	1,07		204,17	0,00	34	0	39551,70	20464,55	19087,15
A6	4	OA3	312,77		1,06	202,81	0,80	34	2	39551,55	20472,20	19079,35
A6	8	OA1	157,77			57,69	26,65	17	17	23961,32	13759,38	10201,95
A6	8	OA2	184,82	1,17		112,40	0,00	24	0	23961,54	13762,46	10199,08
A6	8	OA3	185,64		1,18	111,10	2,38	24	2	23961,32	13759,38	10201,95
A6	16	OA1	58,87			28,44	14,04	11	11	14462,17	8666,66	5795,51
A6	16	OA2	58,38	0,99		42,90	0,00	14	0	14462,40	8667,04	5795,36
A6	16	OA3	55,06		0,94	39,60	1,03	13	2	14462,17	8666,66	5795,51

Table A.27: Linear Utility Function $\epsilon = 0.0001$, n=80 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7	4	OA1	47,04			16,19	10,04	11	11	21700,75	19793,55	1907,20
A7	4	OA2	35,58	0,76		16,26	0,00	11	0	21700,75	19793,55	1907,20
A7	4	OA3	35,97		0,76	16,06	0,00	11	0	21700,75	19793,55	1907,20
A7	8	OA1	46,03			5,67	5,23	6	6	13672,67	12681,76	990,92
A7	8	OA2	39,84	0,87		5,76	0,00	6	0	13672,67	12681,76	990,92
A7	8	OA3	41,33		0,90	5,78	0,86	6	1	13672,67	12681,76	990,92
A7	16	OA1	19,38			2,84	6,55	5	5	8376,67	7877,87	498,81
A7	16	OA2	11,91	0,61		2,70	0,00	5	0	8376,68	7877,88	498,80
A7	16	OA3	14,73		0,76	2,69	1,62	5	1	8376,67	7877,87	498,81
A8	4	OA1	58,17			31,81	13,43	16	16	22685,11	20200,75	2484,37
A8	4	OA2	45,48	0,78		33,67	0,00	17	0	22685,12	20200,39	2484,73
A8	4	OA3	44,55		0,77	32,66	0,32	17	1	22685,11	20200,75	2484,37
A8	8	OA1	28,43			9,34	11,26	8	8	14025,05	12900,62	1124,43
A8	8	OA2	15,69	0,55		8,85	0,00	8	0	14025,05	12900,62	1124,43
A8	8	OA3	15,54		0,55	8,76	0,00	8	0	14025,05	12900,62	1124,43
A8	16	OA1	17,71			2,98	6,50	4	4	8564,05	7883,13	680,91
A8	16	OA2	12,37	0,70		3,72	0,00	4	0	8564,05	7883,33	680,72
A8	16	OA3	13,30		0,75	3,72	1,57	4	1	8564,05	7883,13	680,91
A9	4	OA1	45,70			21,91	11,87	13	13	22185,08	20130,53	2054,56
A9	4	OA2	38,06	0,83		27,37	0,00	15	0	22185,07	20130,52	2054,55
A9	4	OA3	40,19		0,88	28,37	1,05	15	1	22185,08	20130,53	2054,56
A9	8	OA1	51,95			17,02	14,82	12	12	14327,77	13062,13	1265,64
A9	8	OA2	36,20	0,70		16,63	0,00	12	0	14327,77	13062,13	1265,64
A9	8	OA3	39,99		0,77	17,64	2,41	12	1	14327,77	13062,13	1265,64
A9	16	OA1	13,87			2,71	5,12	4	4	8755,85	7988,78	767,08
A9	16	OA2	8,34	0,60		2,86	0,00	4	0	8755,85	7988,78	767,08
A9	16	OA3	8,28		0,60	2,83	0,00	4	0	8755,85	7988,78	767,08

Table A.28: Linear Utility Function $\epsilon = 0.0001$, n=80 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1	1	OA1	5,73			2,34	2,69	21	21	49791,17	4632,77	45158,40
A1	1	OA2	2,73	0,48		2,04	0,00	21	0	49791,17	4632,77	45158,40
A1	1	OA3	3,02		0,53	2,18	0,14	21	1	49791,17	4632,77	45158,40
A1	2	OA1	39,13			3,06	35,58	13	13	25605,85	2995,83	22610,02
A1	2	OA2	13,25	0,34		12,68	0,00	25	0	25629,39	2941,96	22687,43
A1	2	OA3	27,89		0,71	11,74	15,58	24	3	25605,85	2995,83	22610,02
A1	4	OA1	10,47			3,90	6,13	14	14	13582,83	2260,44	11322,39
A1	4	OA2	7,25	0,69		6,81	0,00	19	0	13583,63	2259,43	11324,20
A1	4	OA3	7,29		0,70	6,85	0,00	19	0	13583,63	2259,43	11324,20
A2	1	OA1	8,26			2,98	4,55	21	21	49947,43	4789,03	45158,40
A2	1	OA2	3,65	0,44		2,99	0,00	21	0	49947,43	4789,03	45158,40
A2	1	OA3	3,96		0,48	2,99	0,28	21	1	49947,43	4789,03	45158,40
A2	2	OA1	72,64			4,12	67,90	15	15	25605,85	2995,83	22610,02
A2	2	OA2	10,09	0,14		9,52	0,00	21	0	25650,02	3070,60	22579,42
A2	2	OA3	12,63		0,17	9,10	2,93	21	2	25605,85	2995,83	22610,02
A2	4	OA1	14,86			3,84	10,39	14	14	13591,26	2275,84	11315,42
A2	4	OA2	4,72	0,32		4,26	0,00	16	0	13607,24	2299,74	11307,50
A2	4	OA3	4,58		0,31	4,12	0,00	16	0	13607,24	2299,74	11307,50
A3	1	OA1	8,65			2,05	5,38	21	21	50222,94	5064,53	45158,40
A3	1	OA2	3,43	0,40		2,24	0,00	21	0	50222,94	5064,53	45158,40
A3	1	OA3	3,44		0,40	2,03	0,24	21	1	50222,94	5064,53	45158,40
A3	2	OA1	38,03			3,54	33,90	13	13	26223,67	3624,47	22599,20
A3	2	OA2	7,35	0,19		6,74	0,00	18	0	26223,67	3624,47	22599,20
A3	2	OA3	7,52		0,20	6,40	0,50	17	1	26223,67	3624,47	22599,20
A3	4	OA1	18,09			3,69	9,94	13	13	15679,05	2761,43	12917,62
A3	4	OA2	10,75	0,59		6,23	0,00	19	0	15692,65	2737,98	12954,67
A3	4	OA3	10,90		0,60	5,94	0,00	19	0	15692,65	2737,98	12954,67

Table A.29: Linear Utility Function $\epsilon = 0.01$, n=20 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4	1	OA1	7,47			2,48	3,83	14	14	9148,61	4632,77	4515,84
A4	1	OA2	3,64	0,49		2,55	0,00	14	0	9148,61	4632,77	4515,84
A4	1	OA3	3,65		0,49	2,58	0,00	14	0	9148,61	4632,77	4515,84
A4	2	OA1	5,14			1,78	2,87	8	8	5072,48	2724,78	2347,70
A4	2	OA2	2,11	0,41		1,63	0,00	8	0	5072,48	2724,78	2347,70
A4	2	OA3	2,22		0,43	1,44	0,30	8	1	5072,48	2724,78	2347,70
A4	4	OA1	5,79			1,74	3,42	8	8	3163,39	1916,42	1246,98
A4	4	OA2	2,30	0,40		1,71	0,00	8	0	3163,86	1918,60	1245,26
A4	4	OA3	2,25		0,39	1,72	0,00	8	0	3163,86	1918,60	1245,26
A5	1	OA1	8,43			2,66	4,23	15	15	9304,87	4789,03	4515,84
A5	1	OA2	4,36	0,52		2,82	0,00	15	0	9304,87	4789,03	4515,84
A5	1	OA3	4,48		0,53	2,72	0,26	15	1	9304,87	4789,03	4515,84
A5	2	OA1	5,22			1,35	3,01	8	8	5072,48	2724,78	2347,70
A5	2	OA2	2,28	0,44		1,43	0,00	8	0	5072,48	2724,78	2347,70
A5	2	OA3	2,44		0,47	1,34	0,27	8	1	5072,48	2724,78	2347,70
A5	4	OA1	5,56			1,49	3,54	8	8	3220,62	1975,36	1245,26
A5	4	OA2	2,59	0,46		2,09	0,00	9	0	3214,68	1969,42	1245,26
A5	4	OA3	2,47		0,44	2,01	0,00	9	0	3214,68	1969,42	1245,26
A6	1	OA1	8,79			3,18	4,54	15	15	9580,38	5064,53	4515,84
A6	1	OA2	3,98	0,45		2,94	0,00	15	0	9580,38	5064,53	4515,84
A6	1	OA3	4,10		0,47	2,84	0,26	15	1	9580,38	5064,53	4515,84
A6	2	OA1	7,71			2,32	4,59	10	10	5847,28	3533,01	2314,27
A6	2	OA2	3,19	0,41		2,41	0,00	10	0	5851,53	3563,96	2287,57
A6	2	OA3	3,63		0,47	2,41	0,48	10	1	5847,28	3533,01	2314,27
A6	4	OA1	8,25			2,40	5,25	9	9	3888,21	2260,60	1627,61
A6	4	OA2	3,98	0,48		3,09	0,00	11	0	3888,21	2260,60	1627,61
A6	4	OA3	3,81		0,46	3,19	0,00	11	0	3888,21	2260,60	1627,61

Table A.30: Linear Utility Function $\epsilon = 0.01$, n=20 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7	1	OA1	3,22			0,66	0,79	3	3	5084,35	4632,77	451,58
A7	1	OA2	2,36	0,73		0,64	0,00	3	0	5084,35	4632,77	451,58
A7	1	OA3	3,10		0,96	0,89	0,00	3	0	5084,35	4632,77	451,58
A7	2	OA1	1,71			0,10	0,45	1	1	2959,55	2724,78	234,77
A7	2	OA2	1,25	0,73		0,11	0,00	1	0	2959,55	2724,78	234,77
A7	2	OA3	1,22		0,71	0,12	0,00	1	0	2959,55	2724,78	234,77
A7	4	OA1	2,43			0,31	0,75	2	2	1964,58	1812,51	152,08
A7	4	OA2	1,70	0,70		0,31	0,00	2	0	1964,58	1812,51	152,08
A7	4	OA3	1,65		0,68	0,31	0,00	2	0	1964,58	1812,51	152,08
A8	1	OA1	72,17			1,10	1,50	5	5	5240,62	4789,03	451,58
A8	1	OA2	69,19	0,96		1,01	0,00	5	0	5240,62	4789,03	451,58
A8	1	OA3	68,42		0,95	0,90	0,32	5	1	5240,62	4789,03	451,58
A8	2	OA1	1,33			0,10	0,50	1	1	2959,55	2724,78	234,77
A8	2	OA2	0,73	0,55		0,09	0,00	1	0	2959,55	2724,78	234,77
A8	2	OA3	0,73		0,55	0,08	0,00	1	0	2959,55	2724,78	234,77
A8	4	OA1	1,78			0,18	0,89	2	2	1971,64	1812,51	159,13
A8	4	OA2	0,86	0,48		0,17	0,00	2	0	1971,64	1812,51	159,13
A8	4	OA3	0,83		0,47	0,17	0,00	2	0	1971,64	1812,51	159,13
A9	1	OA1	5,68			0,97	1,40	5	5	5516,12	5064,53	451,58
A9	1	OA2	4,10	0,72		0,83	0,00	5	0	5516,12	5064,53	451,58
A9	1	OA3	4,32		0,76	0,76	0,28	5	1	5516,12	5064,53	451,58
A9	2	OA1	4,70			1,25	1,61	5	5	3356,73	2724,78	631,95
A9	2	OA2	3,36	0,71		1,50	0,00	5	0	3356,73	2724,78	631,95
A9	2	OA3	3,46		0,74	1,32	0,28	5	1	3356,73	2724,78	631,95
A9	4	OA1	2,63			0,65	1,56	4	4	2222,06	1924,53	297,53
A9	4	OA2	1,17	0,45		0,77	0,00	5	0	2222,06	1924,53	297,53
A9	4	OA3	1,57		0,60	0,71	0,44	5	1	2222,06	1924,53	297,53

Table A.31: Linear Utility Function $\epsilon = 0.01$, n=20 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1	2	OA1	65,84			21,66	42,83	27	27	88483,95	8335,70	80148,24
A1	2	OA2	107,75	1,64		106,12	0,00	51	0	88658,83	8316,23	80342,60
A1	2	OA3	112,37		1,71	107,41	3,32	51	5	88655,94	8484,33	80171,60
A1	4	OA1	37,32			17,90	17,78	19	19	46121,29	6004,15	40117,14
A1	4	OA2	73,96	1,98		72,11	0,00	35	0	45996,87	5876,91	40119,96
A1	4	OA3	74,16		1,99	72,03	0,29	35	1	45941,02	5837,27	40103,75
A1	8	OA1	30,44			15,29	13,11	16	16	23722,79	3568,57	20154,22
A1	8	OA2	29,68	0,98		27,62	0,00	22	0	23721,19	3528,10	20193,10
A1	8	OA3	28,90		0,95	26,87	0,00	22	0	23721,19	3528,10	20193,10
A2	2	OA1	190,22			44,15	143,63	36	36	89393,49	9221,89	80171,60
A2	2	OA2	73,79	0,39		71,53	0,00	44	0	89727,11	9586,11	80141,00
A2	2	OA3	100,25		0,53	75,20	22,76	45	2	89727,11	9586,11	80141,00
A2	4	OA1	46,04			25,24	18,75	22	22	46258,83	6146,61	40112,21
A2	4	OA2	43,86	0,95		41,83	0,00	30	0	46363,52	6287,17	40076,36
A2	4	OA3	43,12		0,94	41,09	0,00	30	0	46363,52	6287,17	40076,36
A2	8	OA1	32,14			10,16	20,56	12	12	24504,55	4391,94	20112,61
A2	8	OA2	21,29	0,66		19,87	0,00	19	0	24533,09	4347,06	20186,03
A2	8	OA3	21,81		0,68	19,84	0,50	19	1	24504,55	4391,94	20112,61
A3	2	OA1	235,12			27,85	205,48	31	31	90291,09	10103,96	80187,13
A3	2	OA2	65,96	0,28		64,05	0,00	44	0	90373,85	10232,85	80141,00
A3	2	OA3	91,90		0,39	65,30	24,73	44	3	90291,09	10103,96	80187,13
A3	4	OA1	47,32			21,23	24,09	22	22	47597,58	7498,64	40098,94
A3	4	OA2	35,90	0,76		34,08	0,00	26	0	47609,11	7518,66	40090,45
A3	4	OA3	35,98		0,76	34,14	0,00	26	0	47609,11	7518,66	40090,45
A3	8	OA1	27,60			14,41	11,09	16	16	25370,63	5241,59	20129,05
A3	8	OA2	20,03	0,73		18,38	0,00	20	0	25453,30	5273,49	20179,80
A3	8	OA3	20,67		0,75	18,90	0,00	20	0	25453,30	5273,49	20179,80

Table A.32: Linear Utility Function $\epsilon = 0.01$, n=40 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4	2	OA1	27,36			13,86	9,91	25	25	16335,67	8315,38	8020,28
A4	2	OA2	17,17	0,63		13,91	0,00	24	0	16336,17	8319,01	8017,16
A4	2	OA3	16,53		0,60	13,29	0,17	24	1	16335,67	8315,38	8020,28
A4	4	OA1	20,29			8,91	9,27	14	14	9654,06	5502,00	4152,06
A4	4	OA2	17,50	0,86		15,60	0,00	19	0	9686,78	5532,22	4154,56
A4	4	OA3	16,98		0,84	14,77	0,33	19	1	9686,78	5532,22	4154,56
A4	8	OA1	11,35			3,75	4,18	8	8	5466,81	3438,29	2028,52
A4	8	OA2	7,95	0,70		4,75	0,00	9	0	5484,84	3439,76	2045,08
A4	8	OA3	7,40		0,65	4,64	0,00	9	0	5484,84	3439,76	2045,08
A5	2	OA1	66,42			15,79	48,20	26	26	17275,60	9235,90	8039,70
A5	2	OA2	20,92	0,31		19,04	0,00	26	0	17192,26	9158,00	8034,26
A5	2	OA3	20,29		0,31	18,48	0,00	26	0	17192,26	9158,00	8034,26
A5	4	OA1	19,30			7,54	8,30	14	14	9983,59	5875,78	4107,81
A5	4	OA2	12,35	0,64		9,14	0,00	15	0	10007,98	5931,69	4076,29
A5	4	OA3	12,41		0,64	9,20	0,00	15	0	10007,98	5931,69	4076,29
A5	8	OA1	10,20			4,40	4,23	9	9	6139,76	3864,00	2275,76
A5	8	OA2	6,63	0,65		5,36	0,00	10	0	6139,76	3864,00	2275,76
A5	8	OA3	6,97		0,68	5,04	0,36	10	1	6139,76	3864,00	2275,76
A6	2	OA1	38,61			20,66	12,41	29	29	18011,64	9927,78	8083,86
A6	2	OA2	30,19	0,78		24,38	0,00	29	0	18017,12	9888,58	8128,54
A6	2	OA3	29,32		0,76	23,67	0,12	29	1	18011,64	9927,78	8083,86
A6	4	OA1	23,65			12,01	10,10	18	18	11224,06	6456,57	4767,49
A6	4	OA2	18,78	0,79		17,41	0,00	23	0	11237,49	6390,53	4846,97
A6	4	OA3	21,13		0,89	18,17	1,42	23	2	11224,06	6456,57	4767,49
A6	8	OA1	14,53			7,01	5,38	11	11	6319,90	3542,44	2777,47
A6	8	OA2	10,98	0,76		9,10	0,00	13	0	6327,37	3565,25	2762,12
A6	8	OA3	11,14		0,77	9,22	0,00	13	0	6327,37	3565,25	2762,12

Table A.33: Linear Utility Function $\epsilon = 0.01$, n=40 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7	2	OA1	7,67			0,74	0,53	3	3	9111,75	8308,92	802,84
A7	2	OA2	7,17	0,93		0,85	0,00	3	0	9111,75	8308,92	802,84
A7	2	OA3	7,04		0,92	0,70	0,00	3	0	9111,75	8308,92	802,84
A7	4	OA1	2,57			0,41	0,71	2	2	5843,46	5419,03	424,43
A7	4	OA2	1,86	0,73		0,45	0,00	2	0	5843,46	5419,03	424,43
A7	4	OA3	1,88		0,73	0,42	0,00	2	0	5843,46	5419,03	424,43
A7	8	OA1	2,44			0,10	0,88	1	1	3536,43	3294,69	241,74
A7	8	OA2	1,49	0,61		0,10	0,00	1	0	3536,82	3294,27	242,55
A7	8	OA3	1,52		0,62	0,11	0,00	1	0	3536,82	3294,27	242,55
A8	2	OA1	13,73			1,80	2,08	6	6	9478,99	8308,92	1170,07
A8	2	OA2	12,13	0,88		2,11	0,00	6	0	9478,99	8308,92	1170,07
A8	2	OA3	10,93		0,80	1,84	0,33	6	1	9478,99	8308,92	1170,07
A8	4	OA1	2,63			0,27	0,88	2	2	6005,87	5419,03	586,84
A8	4	OA2	1,77	0,67		0,31	0,00	2	0	6005,87	5419,03	586,84
A8	4	OA3	1,72		0,65	0,28	0,00	2	0	6005,87	5419,03	586,84
A8	8	OA1	3,95			0,42	1,25	3	3	3696,90	3361,65	335,26
A8	8	OA2	2,62	0,66		0,38	0,00	3	0	3696,90	3361,65	335,26
A8	8	OA3	2,61		0,66	0,42	0,00	3	0	3696,90	3361,65	335,26
A9	2	OA1	7,61			2,69	1,52	8	8	9538,04	8319,01	1219,03
A9	2	OA2	6,94	0,91		3,65	0,00	9	0	9538,04	8319,01	1219,03
A9	2	OA3	7,24		0,95	3,61	0,29	9	1	9538,04	8319,01	1219,03
A9	4	OA1	11,67			3,00	4,00	7	7	6275,46	5564,82	710,65
A9	4	OA2	7,06	0,61		2,65	0,00	7	0	6260,03	5634,00	626,03
A9	4	OA3	7,00		0,60	2,65	0,00	7	0	6260,03	5634,00	626,03
A9	8	OA1	2,50			0,12	0,47	1	1	3686,92	3363,25	323,67
A9	8	OA2	2,00	0,80		0,12	0,00	1	0	3690,87	3360,67	330,20
A9	8	OA3	1,95		0,78	0,12	0,00	1	0	3690,87	3360,67	330,20

Table A.34: Linear Utility Function $\epsilon = 0.01$, n=40 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1	3	OA1	94,61			53,92	36,61	25	25	146118,41	14355,46	131762,94
A1	3	OA2	458,03	4,84		453,31	0,00	59	0	145809,59	13880,81	131928,78
A1	3	OA3	455,77		4,82	451,06	0,00	59	0	145809,59	13880,81	131928,78
A1	6	OA1	132,07			46,52	79,85	20	20	76192,00	10208,18	65983,82
A1	6	OA2	202,80	1,54		197,25	0,00	37	0	75943,45	9989,80	65953,65
A1	6	OA3	203,33		1,54	197,80	0,00	37	0	75943,45	9989,80	65953,65
A1	12	OA1	130,93			66,75	59,46	17	17	39252,69	5880,77	33371,92
A1	12	OA2	148,26	1,13		143,69	0,00	27	0	39345,27	5840,85	33504,42
A1	12	OA3	149,56		1,14	144,94	0,00	27	0	39345,27	5840,85	33504,42
A2	3	OA1	250,18			104,64	140,63	37	37	145713,97	13943,63	131770,33
A2	3	OA2	290,71	1,16		285,55	0,00	49	0	146489,42	14172,43	132317,00
A2	3	OA3	289,94		1,16	285,27	0,00	49	0	146489,42	14172,43	132317,00
A2	6	OA1	128,18			58,45	62,97	24	24	76365,55	10387,79	65977,75
A2	6	OA2	174,88	1,36		168,70	0,00	35	0	76196,12	10277,48	65918,64
A2	6	OA3	164,98		1,29	158,25	0,51	34	1	76423,09	10475,31	65947,77
A2	12	OA1	72,76			39,18	28,11	15	15	39616,80	6355,41	33261,39
A2	12	OA2	64,82	0,89		60,39	0,00	22	0	39687,49	6278,44	33409,05
A2	12	OA3	64,59		0,89	60,15	0,00	22	0	39687,49	6278,44	33409,05
A3	3	OA1	166,36			92,19	70,02	35	35	146789,31	14978,35	131810,97
A3	3	OA2	235,39	1,41		231,50	0,00	46	0	147186,73	14869,73	132317,00
A3	3	OA3	235,28		1,41	231,52	0,00	46	0	147186,73	14869,73	132317,00
A3	6	OA1	98,51			46,52	48,57	20	20	76531,59	10535,76	65995,84
A3	6	OA2	120,30	1,22		116,87	0,00	31	0	76825,16	10654,07	66171,09
A3	6	OA3	120,15		1,22	116,72	0,00	31	0	76825,16	10654,07	66171,09
A3	12	OA1	113,70			38,74	68,64	16	16	39940,86	6856,33	33084,52
A3	12	OA2	56,06	0,49		50,66	0,00	19	0	40103,93	6904,33	33199,60
A3	12	OA3	55,53		0,49	50,06	0,00	19	0	40103,93	6904,33	33199,60

Table A.35: Linear Utility Function $\epsilon = 0.01$, n=60 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4	3	OA1	57,80			40,34	12,46	26	26	27113,38	13907,14	13206,24
A4	3	OA2	57,75	1,00		53,17	0,00	27	0	27098,51	13902,81	13195,70
A4	3	OA3	58,06		1,00	53,58	0,00	27	0	27098,51	13902,81	13195,70
A4	6	OA1	38,22			20,98	13,06	15	15	16240,56	9531,02	6709,53
A4	6	OA2	35,31	0,92		32,04	0,00	19	0	16243,35	9548,76	6694,59
A4	6	OA3	34,27		0,90	31,07	0,00	19	0	16243,35	9548,76	6694,59
A4	12	OA1	24,65			10,57	8,27	10	10	9087,62	5673,51	3414,11
A4	12	OA2	19,31	0,78		13,64	0,00	12	0	9093,25	5667,24	3426,01
A4	12	OA3	18,91		0,77	13,61	0,00	12	0	9093,25	5667,24	3426,01
A5	3	OA1	81,13			46,40	16,78	27	27	27048,83	13846,57	13202,26
A5	3	OA2	78,17	0,96		60,12	0,00	28	0	27059,91	13853,67	13206,24
A5	3	OA3	77,90		0,96	60,59	0,00	28	0	27059,91	13853,67	13206,24
A5	6	OA1	45,29			24,78	15,54	16	16	16594,44	9803,72	6790,72
A5	6	OA2	43,72	0,97		38,89	0,00	20	0	16590,60	9799,66	6790,94
A5	6	OA3	43,60		0,96	38,90	0,00	20	0	16590,60	9799,66	6790,94
A5	12	OA1	23,74			11,47	8,55	11	11	9375,56	5911,46	3464,10
A5	12	OA2	19,08	0,80		16,13	0,00	13	0	9376,02	5913,84	3462,19
A5	12	OA3	19,66		0,83	16,75	0,00	13	0	9376,02	5913,84	3462,19
A6	3	OA1	83,82			55,37	23,41	29	29	28063,13	14856,89	13206,24
A6	3	OA2	81,89	0,98		77,77	0,00	31	0	28063,13	14856,89	13206,24
A6	3	OA3	81,61		0,97	77,43	0,00	31	0	28063,13	14856,89	13206,24
A6	6	OA1	43,63			24,61	15,41	16	16	16888,72	10201,44	6687,28
A6	6	OA2	40,47	0,93		37,61	0,00	19	0	16923,92	10218,54	6705,38
A6	6	OA3	39,68		0,91	36,84	0,00	19	0	16923,92	10218,54	6705,38
A6	12	OA1	27,10			12,75	10,19	11	11	10017,87	6625,66	3392,21
A6	12	OA2	17,83	0,66		14,08	0,00	11	0	10034,35	6671,07	3363,28
A6	12	OA3	17,60		0,65	13,86	0,00	11	0	10034,35	6671,07	3363,28

Table A.36: Linear Utility Function $\epsilon = 0.01$, n=60 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7	3	OA1	6,25			1,70	1,96	4	4	15152,10	13822,46	1329,64
A7	3	OA2	4,11	0,66		1,68	0,00	4	0	15152,10	13822,46	1329,64
A7	3	OA3	4,74		0,76	1,98	0,00	4	0	15152,10	13822,46	1329,64
A7	6	OA1	9,89			1,15	1,09	2	2	9994,44	9283,21	711,22
A7	6	OA2	8,73	0,88		1,21	0,00	2	0	9994,47	9282,04	712,42
A7	6	OA3	8,65		0,87	1,20	0,00	2	0	9994,47	9282,04	712,42
A7	12	OA1	5,44			0,15	1,03	1	1	5915,74	5539,75	375,99
A7	12	OA2	4,30	0,79		0,15	0,00	1	0	5917,54	5538,85	378,69
A7	12	OA3	4,22		0,78	0,15	0,00	1	0	5917,54	5538,85	378,69
A8	3	OA1	5,59			0,91	0,90	2	2	15152,10	13822,46	1329,64
A8	3	OA2	4,58	0,82		0,89	0,00	2	0	15152,10	13822,46	1329,64
A8	3	OA3	4,51		0,81	0,89	0,00	2	0	15152,10	13822,46	1329,64
A8	6	OA1	21,32			8,36	7,14	9	9	10413,45	9326,47	1086,98
A8	6	OA2	14,15	0,66		8,46	0,00	10	0	10377,34	9544,59	832,74
A8	6	OA3	14,06		0,66	8,46	0,00	10	0	10377,34	9544,59	832,74
A8	12	OA1	5,32			0,19	0,89	1	1	6030,87	5606,11	424,76
A8	12	OA2	4,23	0,80		0,19	0,00	1	0	6030,87	5606,11	424,76
A8	12	OA3	4,19		0,79	0,18	0,00	1	0	6030,87	5606,11	424,76
A9	3	OA1	42,84			6,34	5,25	10	10	16177,51	14856,89	1320,62
A9	3	OA2	37,99	0,89		7,99	0,00	10	0	16177,51	14856,89	1320,62
A9	3	OA3	37,15		0,87	7,24	0,00	10	0	16177,51	14856,89	1320,62
A9	6	OA1	8,32			3,48	2,39	5	5	10401,00	9326,47	1074,53
A9	6	OA2	7,01	0,84		4,63	0,00	6	0	10448,99	9282,04	1166,95
A9	6	OA3	7,26		0,87	4,87	0,00	6	0	10448,99	9282,04	1166,95
A9	12	OA1	12,27			0,78	2,37	3	3	6277,83	5555,71	722,12
A9	12	OA2	9,57	0,78		1,19	0,00	3	0	6279,53	5545,62	733,91
A9	12	OA3	9,64		0,79	1,20	0,00	3	0	6279,53	5545,62	733,91

Table A.37: Linear Utility Function $\epsilon = 0.01$, n=60 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1	4	OA1	293,41			156,85	128,41	27	27	208563,61	20940,29	187623,33
A1	4	OA2	1124,87	3,83		1115,37	0,00	64	0	208212,02	20481,22	187730,80
A1	4	OA3	1122,91		3,83	1113,53	0,00	64	0	208212,02	20481,22	187730,80
A1	8	OA1	229,23			146,19	76,05	23	23	107451,55	13644,25	93807,30
A1	8	OA2	648,43	2,83		640,82	0,00	45	0	107332,17	13436,30	93895,87
A1	8	OA3	644,52		2,81	636,24	0,00	45	0	107332,17	13436,30	93895,87
A1	16	OA1	320,60			196,34	114,53	17	17	55865,55	8877,71	46987,84
A1	16	OA2	1026,76	3,20		1016,62	0,00	26	0	55787,18	8783,57	47003,61
A1	16	OA3	1042,88		3,25	1031,93	0,00	26	0	55787,18	8783,57	47003,61
A2	4	OA1	393,84			233,22	141,90	34	34	208841,25	21220,81	187620,44
A2	4	OA2	562,90	1,43		548,48	0,00	46	0	209115,77	21526,23	187589,53
A2	4	OA3	562,56		1,43	548,10	0,00	46	0	209115,77	21526,23	187589,53
A2	8	OA1	270,53			132,46	128,57	22	22	108623,27	14631,88	93991,38
A2	8	OA2	325,02	1,20		315,60	0,00	34	0	108439,41	14070,41	94368,99
A2	8	OA3	324,95		1,20	315,63	0,00	34	0	108439,41	14070,41	94368,99
A2	16	OA1	189,13			99,00	79,73	16	16	56856,62	9836,30	47020,32
A2	16	OA2	125,79	0,67		116,75	0,00	19	0	56836,45	9513,50	47322,95
A2	16	OA3	125,73		0,66	116,84	0,00	19	0	56836,45	9513,50	47322,95
A3	4	OA1	383,01			232,56	142,38	34	34	208916,50	21208,17	187708,31
A3	4	OA2	516,78	1,35		508,87	0,00	48	0	210495,66	22510,87	187984,78
A3	4	OA3	519,60		1,36	511,82	0,00	48	0	210495,66	22510,87	187984,78
A3	8	OA1	278,48			118,87	151,13	22	22	108925,01	15082,19	93842,82
A3	8	OA2	236,73	0,85		229,13	0,00	30	0	109060,41	15104,72	93955,70
A3	8	OA3	237,02		0,85	229,35	0,00	30	0	109060,41	15104,72	93955,70
A3	16	OA1	186,14			85,10	93,48	16	16	59403,86	12343,64	47060,22
A3	16	OA2	111,28	0,60		104,93	0,00	20	0	59602,23	12373,16	47229,07
A3	16	OA3	111,11		0,60	104,80	0,00	20	0	59602,23	12373,16	47229,07

Table A.38: Linear Utility Function $\epsilon = 0.01$, n=80 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4	4	OA1	254,32			119,51	24,24	27	27	38877,78	19999,37	18878,41
A4	4	OA2	287,41	1,13		177,09	0,00	31	0	38900,44	19989,27	18911,17
A4	4	OA3	286,22		1,13	177,23	0,00	31	0	38900,44	19989,27	18911,17
A4	8	OA1	146,54			47,41	26,85	16	16	22580,41	13006,29	9574,13
A4	8	OA2	144,57	0,99		72,07	0,00	20	0	22653,62	13143,61	9510,01
A4	8	OA3	144,22		0,98	72,10	0,00	20	0	22653,62	13143,61	9510,01
A4	16	OA1	47,83			23,27	11,77	10	10	12810,12	7943,92	4866,20
A4	16	OA2	40,88	0,85		28,67	0,00	12	0	12813,61	7927,44	4886,17
A4	16	OA3	41,12		0,86	28,85	0,00	12	0	12813,61	7927,44	4886,17
A5	4	OA1	341,78			146,33	36,63	31	31	39745,97	20821,08	18924,89
A5	4	OA2	367,91	1,08		211,07	0,00	35	0	40008,27	21187,04	18821,22
A5	4	OA3	366,70		1,07	210,75	0,00	35	0	40008,27	21187,04	18821,22
A5	8	OA1	130,82			43,40	25,65	15	15	23125,67	13489,52	9636,15
A5	8	OA2	125,22	0,96		63,97	0,00	18	0	23177,19	13590,15	9587,04
A5	8	OA3	124,36		0,95	63,81	0,00	18	0	23177,19	13590,15	9587,04
A5	16	OA1	60,53			30,97	12,93	11	11	13788,53	8657,86	5130,67
A5	16	OA2	50,75	0,84		35,06	0,00	12	0	13744,31	8602,68	5141,63
A5	16	OA3	50,65		0,84	34,95	0,00	12	0	13744,31	8602,68	5141,63
A6	4	OA1	279,31			137,74	30,07	28	28	39783,05	20574,97	19208,08
A6	4	OA2	298,96	1,07		187,80	0,00	32	0	39583,28	20367,97	19215,31
A6	4	OA3	298,49		1,07	188,19	0,00	32	0	39583,28	20367,97	19215,31
A6	8	OA1	152,70			52,14	26,03	16	16	23961,32	13759,38	10201,95
A6	8	OA2	172,72	1,13		99,69	0,00	22	0	23964,04	13759,04	10204,99
A6	8	OA3	172,88		1,13	99,65	0,00	22	0	23964,04	13759,04	10204,99
A6	16	OA1	54,17			24,82	14,15	10	10	14540,74	8623,47	5917,27
A6	16	OA2	45,60	0,84		30,90	0,00	11	0	14463,91	8685,17	5778,74
A6	16	OA3	45,47		0,84	30,92	0,00	11	0	14463,91	8685,17	5778,74

Table A.39: Linear Utility Function $\epsilon = 0.01$, n=80 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7	4	OA1	42,87			13,80	9,12	10	10	21755,11	19834,20	1920,92
A7	4	OA2	33,66	0,79		13,99	0,00	10	0	21755,11	19834,20	1920,92
A7	4	OA3	33,52		0,78	13,98	0,00	10	0	21755,11	19834,20	1920,92
A7	8	OA1	38,53			1,61	1,93	2	2	13672,67	12681,76	990,92
A7	8	OA2	36,08	0,94		1,71	0,00	2	0	13672,67	12681,76	990,92
A7	8	OA3	36,05		0,94	1,70	0,00	2	0	13672,67	12681,76	990,92
A7	16	OA1	10,43			0,28	0,78	1	1	8378,67	7879,86	498,81
A7	16	OA2	9,50	0,91		0,28	0,00	1	0	8380,46	7878,96	501,51
A7	16	OA3	9,35		0,90	0,28	0,00	1	0	8380,46	7878,96	501,51
A8	4	OA1	59,14			31,88	14,04	16	16	22685,11	20200,75	2484,37
A8	4	OA2	43,47	0,74		31,78	0,00	16	0	22685,71	20204,44	2481,27
A8	4	OA3	43,47		0,73	31,80	0,00	16	0	22685,71	20204,44	2481,27
A8	8	OA1	15,13			2,98	5,05	4	4	14026,90	12699,97	1326,93
A8	8	OA2	9,59	0,63		2,73	0,00	4	0	14026,90	12699,97	1326,93
A8	8	OA3	9,56		0,63	2,72	0,00	4	0	14026,90	12699,97	1326,93
A8	16	OA1	12,47			1,57	2,59	2	2	8564,05	7883,13	680,91
A8	16	OA2	9,67	0,78		1,57	0,00	2	0	8564,85	7878,96	685,89
A8	16	OA3	9,64		0,77	1,56	0,00	2	0	8564,85	7878,96	685,89
A9	4	OA1	42,34			19,50	10,67	12	12	22185,08	20130,53	2054,56
A9	4	OA2	31,03	0,73		20,14	0,00	12	0	22185,08	20130,53	2054,56
A9	4	OA3	30,86		0,73	20,04	0,00	12	0	22185,08	20130,53	2054,56
A9	8	OA1	34,10			5,73	7,81	6	6	14327,77	13062,13	1265,64
A9	8	OA2	25,72	0,75		5,82	0,00	6	0	14329,81	13061,97	1267,84
A9	8	OA3	25,60		0,75	5,86	0,00	6	0	14329,81	13061,97	1267,84
A9	16	OA1	14,59			2,79	5,37	4	4	8755,85	7988,78	767,08
A9	16	OA2	8,48	0,58		2,96	0,00	4	0	8755,85	7988,78	767,08
A9	16	OA3	8,45		0,58	2,96	0,00	4	0	8755,85	7988,78	767,08

Table A.40: Linear Utility Function $\epsilon = 0.01$, n=80 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1*	1	OA1	7,41			4,63	1,76	21	21	25025,58	4632,77	20392,81
A1*	1	OA2	5,34	0,72		4,28	0,00	21	0	25025,58	4632,77	20392,81
A1*	1	OA3	5,55		0,75	4,29	0,14	21	1	25025,58	4632,77	20392,81
A1*	2	OA1	8,61			4,45	3,11	13	13	5561,10	2966,56	2594,53
A1*	2	OA2	9,21	1,07		8,21	0,00	18	0	5561,15	2963,48	2597,67
A1*	2	OA3	8,26		0,96	6,84	0,41	16	3	5561,10	2966,56	2594,53
A1*	4	OA1	6,31			2,08	3,38	9	9	2394,59	1918,60	475,99
A1*	4	OA2	4,49	0,71		3,64	0,00	12	0	2394,59	1918,60	475,99
A1*	4	OA3	4,87		0,77	3,49	0,42	12	1	2394,59	1918,60	475,99
A2*	1	OA1	10,30			4,29	5,18	21	21	25181,84	4789,03	20392,81
A2*	1	OA2	4,99	0,48		4,15	0,00	21	0	25181,84	4789,03	20392,81
A2*	1	OA3	5,30		0,52	4,21	0,25	21	1	25181,84	4789,03	20392,81
A2*	2	OA1	16,07			4,46	10,86	13	13	5561,10	2966,56	2594,53
A2*	2	OA2	7,43	0,46		6,65	0,00	17	0	5561,15	2963,52	2597,63
A2*	2	OA3	7,96		0,50	6,49	0,61	17	2	5561,10	2966,56	2594,53
A2*	4	OA1	6,37			2,06	3,78	9	9	2445,41	1969,42	475,99
A2*	4	OA2	3,71	0,58		3,17	0,00	11	0	2445,41	1969,42	475,99
A2*	4	OA3	4,02		0,63	3,12	0,36	11	1	2445,41	1969,42	475,99
A3*	1	OA1	10,44			3,74	5,87	21	21	25457,35	5064,53	20392,81
A3*	1	OA2	4,63	0,44		3,81	0,00	21	0	25457,35	5064,53	20392,81
A3*	1	OA3	4,84		0,46	3,79	0,28	21	1	25457,35	5064,53	20392,81
A3*	2	OA1	51,65			6,02	44,94	15	15	6187,12	3624,47	2562,65
A3*	2	OA2	7,47	0,14		6,72	0,00	16	0	6187,12	3624,47	2562,65
A3*	2	OA3	7,71		0,15	6,76	0,28	16	1	6187,12	3624,47	2562,65
A3*	4	OA1	8,08			2,87	4,70	11	11	2831,85	2330,76	501,09
A3*	4	OA2	5,91	0,73		5,44	0,00	17	0	2832,43	2324,42	508,01
A3*	4	OA3	6,47		0,80	5,20	0,70	16	2	2831,85	2330,76	501,09

Table A.41: Nonlinear Utility Function $\epsilon = 0.001$, n=20 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4*	1	OA1	7,86			2,19	2,92	11	11	6672,05	4632,77	2039,28
A4*	1	OA2	4,81	0,61		2,14	0,00	11	0	6672,05	4632,77	2039,28
A4*	1	OA3	5,51		0,70	2,53	0,23	11	1	6672,05	4632,77	2039,28
A4*	2	OA1	1,73			0,34	0,83	3	3	3040,91	2724,78	316,13
A4*	2	OA2	0,97	0,56		0,42	0,00	3	0	3040,91	2724,78	316,13
A4*	2	OA3	1,25		0,72	0,47	0,27	3	1	3040,91	2724,78	316,13
A4*	4	OA1	4,37			1,20	2,06	5	5	1931,04	1812,51	118,53
A4*	4	OA2	1,89	0,43		0,80	0,00	4	0	1931,04	1812,51	118,53
A4*	4	OA3	1,97		0,45	0,87	0,00	4	0	1931,04	1812,51	118,53
A5*	1	OA1	6,26			2,15	2,97	11	11	6828,31	4789,03	2039,28
A5*	1	OA2	3,28	0,52		2,15	0,00	11	0	6828,31	4789,03	2039,28
A5*	1	OA3	3,34		0,53	1,98	0,25	11	1	6828,31	4789,03	2039,28
A5*	2	OA1	2,51			0,61	0,81	3	3	3040,91	2724,78	316,13
A5*	2	OA2	1,59	0,63		0,49	0,00	3	0	3040,91	2724,78	316,13
A5*	2	OA3	1,76		0,70	0,45	0,27	3	1	3040,91	2724,78	316,13
A5*	4	OA1	3,53			0,90	1,62	4	4	1931,54	1812,51	119,03
A5*	4	OA2	1,92	0,54		0,92	0,00	4	0	1931,54	1812,51	119,03
A5*	4	OA3	2,22		0,63	0,78	0,45	4	1	1931,54	1812,51	119,03
A6*	1	OA1	7,80			1,87	3,01	11	11	7103,82	5064,53	2039,28
A6*	1	OA2	4,98	0,64		2,12	0,00	11	0	7103,82	5064,53	2039,28
A6*	1	OA3	5,02		0,64	1,95	0,25	11	1	7103,82	5064,53	2039,28
A6*	2	OA1	6,07			1,79	3,29	9	9	3608,97	2813,57	795,40
A6*	2	OA2	4,24	0,70		3,28	0,00	11	0	3608,98	2814,49	794,49
A6*	2	OA3	4,60		0,76	2,75	0,87	10	2	3608,97	2813,57	795,40
A6*	4	OA1	4,02			1,09	2,00	5	5	2038,41	1924,53	113,88
A6*	4	OA2	1,93	0,48		1,08	0,00	6	0	2038,41	1924,53	113,88
A6*	4	OA3	2,28		0,57	0,98	0,44	6	1	2038,41	1924,53	113,88

Table A.42: Nonlinear Utility Function $\epsilon = 0.001$, n=20 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7*	1	OA1	1,95			0,36	0,81	3	3	4836,69	4632,77	203,93
A7*	1	OA2	1,19	0,61		0,39	0,00	3	0	4836,69	4632,77	203,93
A7*	1	OA3	1,44		0,74	0,42	0,27	3	1	4836,69	4632,77	203,93
A7*	2	OA1	1,22			0,08	0,41	1	1	2756,40	2724,78	31,61
A7*	2	OA2	0,80	0,65		0,05	0,00	1	0	2756,40	2724,78	31,61
A7*	2	OA3	0,75		0,62	0,03	0,00	1	0	2756,40	2724,78	31,61
A7*	4	OA1	1,81			0,38	0,81	2	2	1824,36	1812,51	11,85
A7*	4	OA2	0,91	0,50		0,28	0,00	2	0	1824,36	1812,51	11,85
A7*	4	OA3	0,90		0,50	0,34	0,00	2	0	1824,36	1812,51	11,85
A8*	1	OA1	2,51			0,61	1,06	4	4	4992,96	4789,03	203,93
A8*	1	OA2	1,44	0,57		0,62	0,00	4	0	4992,96	4789,03	203,93
A8*	1	OA3	1,65		0,66	0,62	0,23	4	1	4992,96	4789,03	203,93
A8*	2	OA1	1,34			0,05	0,44	1	1	2756,40	2724,78	31,61
A8*	2	OA2	0,89	0,66		0,05	0,00	1	0	2756,40	2724,78	31,61
A8*	2	OA3	0,86		0,64	0,03	0,00	1	0	2756,40	2724,78	31,61
A8*	4	OA1	2,34			0,36	0,84	2	2	1824,41	1812,51	11,90
A8*	4	OA2	1,47	0,63		0,37	0,00	2	0	1824,41	1812,51	11,90
A8*	4	OA3	1,47		0,63	0,38	0,00	2	0	1824,41	1812,51	11,90
A9*	1	OA1	3,85			0,98	1,19	5	5	5244,55	4632,77	611,78
A9*	1	OA2	2,61	0,68		0,94	0,00	5	0	5244,55	4632,77	611,78
A9*	1	OA3	2,90		0,75	0,97	0,27	5	1	5244,55	4632,77	611,78
A9*	2	OA1	1,97			0,16	0,91	2	2	2814,39	2724,78	89,60
A9*	2	OA2	1,05	0,53		0,17	0,00	2	0	2814,39	2724,78	89,60
A9*	2	OA3	1,51		0,77	0,19	0,44	2	1	2814,39	2724,78	89,60
A9*	4	OA1	2,81			0,66	1,22	3	3	1848,02	1812,51	35,51
A9*	4	OA2	1,51	0,54		0,61	0,00	3	0	1848,02	1812,51	35,51
A9*	4	OA3	2,01		0,72	0,72	0,39	3	1	1848,02	1812,51	35,51

Table A.43: Nonlinear Utility Function $\epsilon = 0.001$, n=20 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1*	2	OA1	176,53			27,75	146,16	34	34	40462,58	8347,90	32114,68
A1*	2	OA2	113,08	0,64		109,90	0,00	57	0	40464,80	8354,12	32110,69
A1*	2	OA3	140,56		0,80	96,17	41,32	54	4	40462,58	8347,90	32114,68
A1*	4	OA1	37,89			24,37	11,92	22	22	9862,42	5789,86	4072,55
A1*	4	OA2	57,17	1,51		55,63	0,00	35	0	9863,75	5798,56	4065,18
A1*	4	OA3	56,67		1,50	53,75	1,40	34	2	9862,42	5789,86	4072,55
A1*	8	OA1	10,17			4,18	4,45	10	10	3972,05	3437,59	534,46
A1*	8	OA2	6,97	0,69		5,54	0,00	11	0	3972,06	3437,72	534,34
A1*	8	OA3	7,88		0,77	5,42	0,98	11	2	3972,05	3437,59	534,46
A2*	2	OA1	209,04			31,22	173,78	35	35	41391,64	9262,46	32129,18
A2*	2	OA2	90,79	0,43		87,05	0,00	50	0	41397,15	9282,84	32114,31
A2*	2	OA3	97,09		0,46	89,64	3,56	51	3	41391,64	9262,46	32129,18
A2*	4	OA1	47,78			20,67	25,04	22	22	10091,85	6024,60	4067,25
A2*	4	OA2	32,43	0,68		30,72	0,00	28	0	10091,91	6026,24	4065,67
A2*	4	OA3	32,15		0,67	28,80	1,40	27	3	10091,85	6024,60	4067,25
A2*	8	OA1	12,39			4,73	5,71	10	10	4305,20	3470,75	834,44
A2*	8	OA2	9,78	0,79		8,47	0,00	15	0	4305,36	3470,44	834,92
A2*	8	OA3	10,20		0,82	8,50	0,28	15	1	4306,32	3451,46	854,86
A3*	2	OA1	136,33			40,00	90,37	38	38	42292,44	10161,58	32130,85
A3*	2	OA2	81,34	0,60		75,53	0,00	46	0	42294,73	10147,86	32146,88
A3*	2	OA3	82,18		0,60	75,83	0,42	46	2	42292,44	10161,58	32130,85
A3*	4	OA1	50,62			33,06	14,10	28	28	11423,07	6614,86	4808,20
A3*	4	OA2	70,84	1,40		67,59	0,00	41	0	11423,18	6618,85	4804,33
A3*	4	OA3	70,53		1,39	67,35	0,00	41	0	11423,18	6618,85	4804,33
A3*	8	OA1	8,24			2,92	3,85	7	7	4288,72	3426,28	862,44
A3*	8	OA2	8,60	1,04		7,29	0,00	12	0	4289,45	3437,79	851,66
A3*	8	OA3	9,19		1,12	7,27	0,50	12	1	4288,72	3426,28	862,44

Table A.44: Nonlinear Utility Function $\epsilon = 0.001$, n=40 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4*	2	OA1	31,54			9,33	17,86	21	21	11538,17	8319,01	3219,16
A4*	2	OA2	19,11	0,61		15,01	0,00	23	0	11538,17	8319,01	3219,16
A4*	2	OA3	19,64		0,62	14,92	0,44	23	2	11538,17	8319,01	3219,16
A4*	4	OA1	6,68			1,62	2,25	5	5	5960,14	5426,33	533,81
A4*	4	OA2	4,43	0,66		1,65	0,00	5	0	5960,14	5426,33	533,81
A4*	4	OA3	4,63		0,69	1,75	0,17	5	1	5960,14	5426,33	533,81
A4*	8	OA1	6,52			1,17	2,85	5	5	3415,84	3336,41	79,42
A4*	8	OA2	3,50	0,54		1,11	0,00	5	0	3415,90	3336,11	79,79
A4*	8	OA3	3,60		0,55	1,22	0,00	5	0	3415,90	3336,11	79,79
A5*	2	OA1	51,51			14,09	34,23	25	25	12410,75	9164,65	3246,10
A5*	2	OA2	21,04	0,41		18,39	0,00	25	0	12410,75	9164,65	3246,10
A5*	2	OA3	21,15		0,41	18,51	0,00	25	0	12410,75	9164,65	3246,10
A5*	4	OA1	8,99			2,70	2,84	7	7	6223,91	5426,33	797,57
A5*	4	OA2	7,27	0,81		3,84	0,00	9	0	6223,91	5426,33	797,57
A5*	4	OA3	7,77		0,86	3,90	0,41	9	2	6223,91	5426,33	797,57
A5*	8	OA1	5,09			1,23	1,89	5	5	3473,43	3360,81	112,62
A5*	8	OA2	2,86	0,56		1,40	0,00	5	0	3473,43	3360,81	112,62
A5*	8	OA3	3,01		0,59	1,41	0,00	5	0	3473,43	3360,81	112,62
A6*	2	OA1	51,22			19,19	28,63	28	28	13075,79	8505,09	4570,69
A6*	2	OA2	33,62	0,66		30,53	0,00	34	0	13075,79	8504,86	4570,93
A6*	2	OA3	33,24		0,65	28,99	1,19	33	4	13075,79	8505,09	4570,69
A6*	4	OA1	13,48			4,37	5,34	11	11	6335,48	5647,78	687,70
A6*	4	OA2	9,72	0,72		6,29	0,00	14	0	6335,48	5647,78	687,70
A6*	4	OA3	9,84		0,73	5,91	0,45	13	2	6335,48	5647,78	687,70
A6*	8	OA1	5,51			1,38	2,60	5	5	3460,63	3339,53	121,10
A6*	8	OA2	2,34	0,42		1,27	0,00	5	0	3460,64	3339,37	121,27
A6*	8	OA3	2,48		0,45	1,34	0,00	5	0	3460,64	3339,37	121,27

Table A.45: Nonlinear Utility Function $\epsilon = 0.001$, n=40 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7*	2	OA1	3,81			0,64	0,64	3	3	8633,52	8308,92	324,61
A7*	2	OA2	2,98	0,78		0,65	0,00	3	0	8633,52	8308,92	324,61
A7*	2	OA3	3,20		0,84	0,67	0,20	3	1	8633,52	8308,92	324,61
A7*	4	OA1	3,15			0,06	0,47	1	1	5473,34	5419,03	54,31
A7*	4	OA2	2,56	0,81		0,06	0,00	1	0	5473,34	5419,03	54,31
A7*	4	OA3	2,54		0,81	0,06	0,00	1	0	5473,34	5419,03	54,31
A7*	8	OA1	1,86			0,08	0,64	1	1	3308,48	3294,27	14,21
A7*	8	OA2	1,06	0,57		0,05	0,00	1	0	3308,48	3294,27	14,21
A7*	8	OA3	1,06		0,57	0,05	0,00	1	0	3308,48	3294,27	14,21
A8*	2	OA1	3,60			0,76	0,67	3	3	8768,39	8308,92	459,47
A8*	2	OA2	2,76	0,77		0,76	0,00	3	0	8768,39	8308,92	459,47
A8*	2	OA3	2,81		0,78	0,66	0,13	3	1	8768,39	8308,92	459,47
A8*	4	OA1	2,34			0,06	0,42	1	1	5499,72	5419,03	80,69
A8*	4	OA2	1,86	0,79		0,06	0,00	1	0	5499,72	5419,03	80,69
A8*	4	OA3	1,89		0,81	0,08	0,00	1	0	5499,72	5419,03	80,69
A8*	8	OA1	2,87			0,14	1,22	2	2	3321,92	3294,68	27,24
A8*	8	OA2	1,62	0,56		0,16	0,00	2	0	3321,94	3294,48	27,46
A8*	8	OA3	2,18		0,76	0,14	0,56	2	1	3321,92	3294,68	27,24
A9*	2	OA1	4,27			1,19	1,19	5	5	8815,08	8319,01	496,07
A9*	2	OA2	3,17	0,74		1,28	0,00	5	0	8815,08	8319,01	496,07
A9*	2	OA3	3,37		0,79	1,25	0,20	5	1	8815,08	8319,01	496,07
A9*	4	OA1	6,82			0,41	0,62	2	2	5556,89	5419,03	137,86
A9*	4	OA2	6,05	0,89		0,36	0,00	2	0	5556,89	5419,03	137,86
A9*	4	OA3	6,27		0,92	0,34	0,22	2	1	5556,89	5419,03	137,86
A9*	8	OA1	6,43			0,16	0,94	2	2	3332,87	3294,30	38,58
A9*	8	OA2	5,46	0,85		0,16	0,00	2	0	3332,87	3294,28	38,59
A9*	8	OA3	5,96		0,93	0,19	0,47	2	1	3332,87	3294,30	38,58

Table A.46: Nonlinear Utility Function $\epsilon = 0.001$, n=40 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1*	3	OA1	162,01			102,57	52,27	38	38	71850,78	13981,01	57869,77
A1*	3	OA2	671,13	4,14		662,44	0,00	76	0	71859,30	13990,25	57869,05
A1*	3	OA3	670,57		4,14	661,34	0,61	76	1	71862,85	13977,93	57884,93
A1*	6	OA1	96,19			67,85	24,43	24	24	17147,09	9854,11	7292,98
A1*	6	OA2	332,17	3,45		327,84	0,00	47	0	17148,68	9867,25	7281,42
A1*	6	OA3	307,30		3,19	301,78	0,95	44	2	17147,09	9854,11	7292,98
A1*	12	OA1	22,76			9,52	9,14	10	10	6759,68	5711,64	1048,04
A1*	12	OA2	18,92	0,83		15,44	0,00	11	0	6759,99	5717,00	1042,99
A1*	12	OA3	19,55		0,86	15,54	0,52	11	1	6759,68	5711,64	1048,04
A2*	3	OA1	194,69			128,04	58,94	42	42	71850,78	13981,01	57869,77
A2*	3	OA2	451,25	2,32		443,60	0,00	68	0	71867,92	14013,53	57854,39
A2*	3	OA3	445,32		2,29	434,55	3,17	67	2	71850,78	13981,01	57869,77
A2*	6	OA1	118,39			81,37	29,34	25	25	17517,87	10228,19	7289,68
A2*	6	OA2	247,10	2,09		239,70	0,00	43	0	17519,65	10199,79	7319,85
A2*	6	OA3	250,08		2,11	239,51	3,07	43	1	17518,03	10214,81	7303,23
A2*	12	OA1	23,87			10,53	9,52	10	10	6962,73	5790,16	1172,57
A2*	12	OA2	22,26	0,93		19,01	0,00	14	0	6963,46	5797,32	1166,14
A2*	12	OA3	22,93		0,96	19,14	0,31	14	1	6962,73	5790,16	1172,57
A3*	3	OA1	210,43			136,63	66,49	46	46	72946,25	15068,12	57878,13
A3*	3	OA2	292,14	1,39		285,47	0,00	60	0	72954,63	15046,12	57908,50
A3*	3	OA3	300,88		1,43	287,57	6,58	60	3	72946,25	15068,12	57878,13
A3*	6	OA1	113,32			71,54	28,68	27	27	17816,41	10454,10	7362,31
A3*	6	OA2	219,43	1,94		206,97	0,00	42	0	17819,97	10479,74	7340,22
A3*	6	OA3	232,07		2,05	217,28	2,15	43	2	17816,41	10454,10	7362,31
A3*	12	OA1	36,01			19,58	10,78	12	12	7364,84	6119,38	1245,46
A3*	12	OA2	39,80	1,11		34,79	0,00	17	0	7365,02	6120,43	1244,59
A3*	12	OA3	40,45		1,12	33,82	1,33	17	2	7364,84	6119,38	1245,46

Table A.47: Nonlinear Utility Function $\epsilon = 0.001$, n=60 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4*	3	OA1	83,40			57,14	20,08	32	32	19701,98	13867,45	5834,53
A4*	3	OA2	78,41	0,94		72,51	0,00	34	0	19707,59	13896,37	5811,23
A4*	3	OA3	79,79		0,96	72,71	1,23	34	2	19701,98	13867,45	5834,53
A4*	6	OA1	24,52			13,48	7,48	13	13	10278,81	9320,29	958,52
A4*	6	OA2	20,53	0,84		17,15	0,00	15	0	10279,60	9315,96	963,64
A4*	6	OA3	21,08		0,86	16,38	1,01	14	2	10278,81	9320,29	958,52
A4*	12	OA1	10,44			1,51	2,72	4	4	5731,60	5539,88	191,73
A4*	12	OA2	7,63	0,73		1,48	0,00	4	0	5731,60	5539,88	191,73
A4*	12	OA3	8,21		0,79	1,47	0,31	4	1	5731,60	5539,88	191,73
A5*	3	OA1	112,45			65,04	30,03	32	32	19701,98	13867,45	5834,53
A5*	3	OA2	85,55	0,76		68,65	0,00	33	0	19707,48	13896,31	5811,17
A5*	3	OA3	87,02		0,77	68,91	1,25	33	2	19701,98	13867,45	5834,53
A5*	6	OA1	38,80			19,22	13,81	17	17	10631,07	9718,72	912,35
A5*	6	OA2	40,36	1,04		35,13	0,00	24	0	10631,20	9717,51	913,69
A5*	6	OA3	41,40		1,07	34,93	1,33	24	1	10631,20	9717,51	913,69
A5*	12	OA1	9,17			1,93	3,56	5	5	5805,63	5624,73	180,91
A5*	12	OA2	4,96	0,54		1,84	0,00	5	0	5807,70	5605,64	202,05
A5*	12	OA3	4,82		0,53	1,79	0,00	5	0	5807,70	5605,64	202,05
A6*	3	OA1	100,64			73,07	23,76	32	32	20726,68	14856,89	5869,79
A6*	3	OA2	99,45	0,99		96,38	0,00	36	0	20726,68	14856,89	5869,79
A6*	3	OA3	99,39		0,99	96,33	0,00	36	0	20726,68	14856,89	5869,79
A6*	6	OA1	53,57			26,08	17,92	17	17	10798,28	9520,55	1277,74
A6*	6	OA2	46,43	0,87		37,25	0,00	22	0	10799,43	9522,85	1276,58
A6*	6	OA3	47,47		0,89	37,20	0,98	22	1	10798,28	9520,55	1277,74
A6*	12	OA1	15,62			2,54	2,68	5	5	5886,17	5656,30	229,87
A6*	12	OA2	12,61	0,81		2,90	0,00	6	0	5886,18	5656,43	229,75
A6*	12	OA3	12,57		0,81	2,87	0,00	6	0	5886,18	5656,43	229,75

Table A.48: Nonlinear Utility Function $\epsilon = 0.001$, n=60 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7*	3	OA1	12,53			1,34	1,06	3	3	14427,73	13832,93	594,81
A7*	3	OA2	12,08	0,96		2,00	0,00	4	0	14428,43	13836,36	592,07
A7*	3	OA3	11,67		0,93	1,28	0,36	3	1	14427,73	13832,93	594,81
A7*	6	OA1	12,64			0,11	0,66	1	1	9394,02	9284,04	109,98
A7*	6	OA2	11,83	0,94		0,12	0,00	1	0	9394,11	9282,04	112,07
A7*	6	OA3	11,75		0,93	0,14	0,00	1	0	9394,11	9282,04	112,07
A7*	12	OA1	3,84			0,09	0,66	1	1	5558,63	5538,95	19,68
A7*	12	OA2	2,81	0,73		0,09	0,00	1	0	5558,63	5538,85	19,78
A7*	12	OA3	2,79		0,73	0,09	0,00	1	0	5558,63	5538,85	19,78
A8*	3	OA1	11,33			1,01	1,14	3	3	14427,73	13832,93	594,81
A8*	3	OA2	10,72	0,95		1,89	0,00	4	0	14428,43	13836,36	592,07
A8*	3	OA3	10,78		0,95	1,23	0,45	3	1	14427,73	13832,93	594,81
A8*	6	OA1	7,50			0,75	1,17	2	2	9470,44	9293,00	177,44
A8*	6	OA2	6,60	0,88		1,33	0,00	3	0	9470,44	9293,00	177,44
A8*	6	OA3	6,91		0,92	0,66	0,66	2	1	9470,44	9293,00	177,44
A8*	12	OA1	5,76			0,50	2,59	2	2	5574,51	5538,85	35,66
A8*	12	OA2	3,00	0,52		0,48	0,00	2	0	5574,51	5538,85	35,66
A8*	12	OA3	2,98		0,52	0,50	0,00	2	0	5574,51	5538,85	35,66
A9*	3	OA1	9,77			3,20	2,64	7	7	15230,34	14181,60	1048,74
A9*	3	OA2	6,30	0,65		2,82	0,00	7	0	15230,89	14164,04	1066,84
A9*	3	OA3	6,46		0,66	2,95	0,00	7	0	15230,89	14164,04	1066,84
A9*	6	OA1	5,91			0,72	1,11	2	2	9469,11	9293,00	176,11
A9*	6	OA2	4,84	0,82		1,23	0,00	3	0	9469,11	9293,00	176,11
A9*	6	OA3	5,04		0,85	0,72	0,56	2	1	9469,11	9293,00	176,11
A9*	12	OA1	4,56			0,53	1,56	2	2	5577,85	5538,85	39,00
A9*	12	OA2	2,90	0,64		0,53	0,00	2	0	5577,85	5538,85	39,00
A9*	12	OA3	2,86		0,63	0,52	0,00	2	0	5577,85	5538,85	39,00

Table A.49: Nonlinear Utility Function $\epsilon = 0.001$, n=60 - 3/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A1*	4	OA1	358,99			231,93	114,28	39	39	108228,32	20281,24	87947,08
A1*	4	OA2	1681,92	4,69		1665,93	0,00	88	0	108251,18	20296,02	87955,16
A1*	4	OA3	1681,42		4,68	1665,81	0,00	88	0	108251,18	20296,02	87955,16
A1*	8	OA1	286,03			206,73	67,38	29	29	24411,36	13322,29	11089,07
A1*	8	OA2	674,76	2,36		661,59	0,00	53	0	24413,55	13326,00	11087,55
A1*	8	OA3	658,13		2,30	641,50	3,37	53	2	24413,55	13326,00	11087,55
A1*	16	OA1	67,85			44,60	16,41	13	13	9612,22	8052,96	1559,26
A1*	16	OA2	165,56	2,44		159,22	0,00	17	0	9612,37	8056,89	1555,48
A1*	16	OA3	165,61		2,44	159,29	0,00	17	0	9612,37	8056,89	1555,48
A2*	4	OA1	432,17			287,52	130,82	45	45	109198,05	21246,45	87951,61
A2*	4	OA2	1853,81	4,29		1838,63	0,00	81	0	109201,21	21243,72	87957,49
A2*	4	OA3	1614,45		3,74	1599,15	0,38	77	1	109198,05	21246,45	87951,61
A2*	8	OA1	224,33			141,32	71,45	28	28	25024,45	13899,67	11124,78
A2*	8	OA2	434,54	1,94		423,37	0,00	49	0	25025,75	13905,01	11120,74
A2*	8	OA3	438,61		1,96	419,31	7,75	49	4	25039,97	13912,09	11127,89
A2*	16	OA1	80,90			55,83	18,19	14	14	10030,74	8191,83	1838,90
A2*	16	OA2	122,24	1,51		116,00	0,00	17	0	10031,12	8188,35	1842,77
A2*	16	OA3	124,11		1,53	116,72	1,01	17	1	10030,74	8191,83	1838,90
A3*	4	OA1	425,15			266,40	137,40	44	44	109413,48	21404,77	88008,71
A3*	4	OA2	702,19	1,65		681,78	0,00	67	0	109431,88	21344,58	88087,30
A3*	4	OA3	711,64		1,67	686,34	5,13	67	2	109413,48	21404,77	88008,71
A3*	8	OA1	233,44			156,99	67,96	31	31	26107,24	14982,32	11124,92
A3*	8	OA2	335,57	1,44		328,95	0,00	43	0	26108,05	14987,25	11120,80
A3*	8	OA3	336,91		1,44	328,41	1,95	43	1	26107,24	14982,32	11124,92
A3*	16	OA1	96,52			71,82	16,59	15	15	10489,01	8563,48	1925,53
A3*	16	OA2	173,11	1,79		166,23	0,00	22	0	10490,48	8553,43	1937,05
A3*	16	OA3	174,05		1,80	166,25	0,89	22	1	10489,01	8563,48	1925,53

Table A.50: Nonlinear Utility Function $\epsilon = 0.001$, n=80 - 1/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A4*	4	OA1	233,50			181,74	39,38	38	38	28937,79	19935,34	9002,45
A4*	4	OA2	278,93	1,19		267,35	0,00	49	0	28937,80	19931,40	9006,41
A4*	4	OA3	271,46		1,16	257,71	2,00	48	2	28937,79	19935,34	9002,45
A4*	8	OA1	89,22			56,86	22,93	22	22	14137,52	12774,53	1362,99
A4*	8	OA2	74,19	0,83		65,33	0,00	23	0	14138,38	12780,93	1357,45
A4*	8	OA3	75,05		0,84	65,35	0,78	23	1	14137,52	12774,53	1362,99
A4*	16	OA1	17,36			1,73	3,20	3	3	8073,66	7881,14	192,52
A4*	16	OA2	13,31	0,77		1,42	0,00	3	0	8076,21	7880,01	196,20
A4*	16	OA3	13,26		0,76	1,44	0,00	3	0	8076,21	7880,01	196,20
A5*	4	OA1	259,77			189,01	57,03	39	39	29966,45	21097,60	8868,86
A5*	4	OA2	301,53	1,16		289,65	0,00	48	0	29970,14	21125,25	8844,89
A5*	4	OA3	275,68		1,06	263,71	0,31	46	1	29966,45	21097,60	8868,86
A5*	8	OA1	108,47			59,19	22,88	22	22	14480,27	13031,57	1448,70
A5*	8	OA2	103,68	0,96		78,45	0,00	27	0	14481,45	13041,40	1440,05
A5*	8	OA3	103,83		0,96	78,85	0,00	27	0	14481,45	13041,40	1440,05
A5*	16	OA1	25,65			2,65	5,46	4	4	8136,96	7882,10	254,86
A5*	16	OA2	19,94	0,78		2,65	0,00	4	0	8138,09	7878,96	259,13
A5*	16	OA3	21,54		0,84	2,70	1,44	4	1	8136,96	7882,10	254,86
A6*	4	OA1	240,96			181,69	44,49	38	38	29897,13	20891,23	9005,90
A6*	4	OA2	248,66	1,03		234,90	0,00	45	0	29900,00	20894,14	9005,87
A6*	4	OA3	259,10		1,08	243,31	2,03	46	2	29897,13	20891,23	9005,90
A6*	8	OA1	61,87			29,97	19,46	16	16	14773,90	13146,33	1627,57
A6*	8	OA2	90,32	1,46		78,42	0,00	23	0	14773,90	13146,33	1627,57
A6*	8	OA3	90,07		1,46	78,22	0,00	23	0	14773,90	13146,33	1627,57
A6*	16	OA1	15,88			3,04	5,23	4	4	8218,12	7880,01	338,10
A6*	16	OA2	10,37	0,65		3,11	0,00	4	0	8218,12	7880,01	338,10
A6*	16	OA3	11,84		0,75	3,10	1,50	4	1	8218,12	7880,01	338,10

Table A.51: Nonlinear Utility Function $\epsilon = 0.001$, n=80 - 2/3

A	p	Alg	Running Time	OA2/OA1	OA3/OA1	MILP Time	NLP Time	MIP Iter.	NLP Iter.	Objective Value	Linear Part	Nonlinear Part
A7*	4	OA1	40,36			7,99	4,26	8	8	20760,54	19813,30	947,24
A7*	4	OA2	35,97	0,89		8,32	0,00	8	0	20760,74	19820,47	940,27
A7*	4	OA3	36,82		0,91	8,28	0,73	8	1	20760,54	19813,30	947,24
A7*	8	OA1	21,56			0,95	1,31	2	2	12828,98	12681,76	147,22
A7*	8	OA2	19,92	0,92		0,97	0,00	2	0	12828,98	12681,76	147,22
A7*	8	OA3	19,76		0,92	0,98	0,00	2	0	12828,98	12681,76	147,22
A7*	16	OA1	10,41			0,13	1,87	1	1	7897,19	7877,06	20,13
A7*	16	OA2	8,35	0,80		0,13	0,00	1	0	7897,20	7876,96	20,23
A7*	16	OA3	8,24		0,79	0,13	0,00	1	0	7897,20	7876,96	20,23
A8*	4	OA1	46,79			14,85	7,82	11	11	21454,88	20051,83	1403,04
A8*	4	OA2	41,67	0,89		18,86	0,00	12	0	21454,96	20051,15	1403,81
A8*	4	OA3	42,65		0,91	19,00	0,89	12	1	21454,88	20051,83	1403,04
A8*	8	OA1	9,42			1,90	2,32	3	3	12889,57	12681,76	207,81
A8*	8	OA2	6,74	0,72		1,86	0,00	3	0	12889,57	12681,76	207,81
A8*	8	OA3	7,54		0,80	1,86	0,80	3	1	12889,57	12681,76	207,81
A8*	16	OA1	8,14			0,13	0,84	1	1	7903,44	7876,96	26,48
A8*	16	OA2	7,08	0,87		0,14	0,00	1	0	7903,44	7876,96	26,48
A8*	16	OA3	7,15		0,88	0,13	0,00	1	0	7903,44	7876,96	26,48
A9*	4	OA1	30,95			12,74	4,98	11	11	21273,66	20003,89	1269,77
A9*	4	OA2	32,98	1,07		20,48	0,00	13	0	21275,04	20004,34	1270,69
A9*	4	OA3	33,62		1,09	20,58	0,30	13	1	21273,66	20003,89	1269,77
A9*	8	OA1	15,71			2,92	2,54	4	4	13014,27	12679,92	334,35
A9*	8	OA2	12,73	0,81		2,73	0,00	4	0	13014,69	12680,51	334,19
A9*	8	OA3	13,45		0,86	2,75	0,41	4	1	13014,27	12679,92	334,35
A9*	16	OA1	6,41			0,28	0,80	1	1	7912,39	7878,02	34,37
A9*	16	OA2	5,32	0,83		0,27	0,00	1	0	7912,39	7878,02	34,37
A9*	16	OA3	5,30		0,83	0,28	0,00	1	0	7912,39	7878,02	34,37

Table A.52: Nonlinear Utility Function $\epsilon = 0.001$, n=80 - 3/3