# A SUPPLY SIDE LIMITED PARTICIPATION MODEL OF MONETARY TRANSMISSION MECHANISM

A Master's Thesis

by

ZEYNAL KARACA

DEPARTMENT OF ECONOMICS
BiLKENT UNIVERSITY
ANKARA

July 25, 2001

*To my Dear Fiancée,*

# A SUPPLY SIDE LIMITED PARTICIPATION MODEL OF MONETARY TRANSMISSION MECHANISM

The Institute of Economics and Social Sciences of Bilkent University

by

ZEYNAL KARACA

In Partial Fulfillment of the Requirements for the Degree of
MASTER OF ARTS

in

THE DEPARTMENT OF ECONOMICS
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ANKARA
July 25, 2001

I certify that I have read this thesis and have found that it is fully adequate, in scope and quality, as a thesis for the degree of Master of Arts in Economics.

---

Assistant Professor Dr. Erdem Basci
Supervisor

I certify that I have read this thesis and have found that it is fully adequate, in scope and quality, as a thesis for the degree of Master of Arts in Economics.

---

Assistant Professor Dr. Neil Arnwine
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and quality, as a thesis for the degree of Master of Arts in Economics.

---

Assistant Professor Dr. Zeynep Onder
Examining Committee Member

Approval of the Institute of Economics and Social Sciences:

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Professor Kursat Aydogan
Director of Institute of Economics and Social Sciences

**ABSTRACT**

A SUPPLY SIDE LIMITED PARTICIPATION MODEL OF MONETARY TRANSMISSION MECHANISM

Karaca, Zeynal

MA, Department of Economics

Supervisor: Assistant Prof. Dr. Erdem Basci

July 25, 2001

This thesis is a theoretical investigation of how money growth affects output, employment, consumption and real wages from a supply side channel. We analyze the effects of monetary shocks under deterministic and stochastic environments in a limited participation model with competitive and sticky wages. We find that anticipated money growth decreases output, employment, consumption, working capital and real wages, but increases profitability of the firms. Unanticipated money growth under sticky wages increases employment, output and consumption, decreases price and profits. The main contribution of this thesis to the literature is that when sticky nominal wages are included in a limited participation model with inelastic labor supply stylized business cycle facts can be obtained.

**Keywords:** Limited participation models, Sticky wages, Supply side monetary transmission mechanism.

**ÖZET**

PARASAL AKTARIM MEKANİZMASININ ARZ YÖNLÜ SINIRLI KATILIMLI BİR MODELİ



Karaca, Zeynal

Yüksek Lisans, İktisat Bölümü

Tez Yöneticisi: Yrd. Doç. Dr. Erdem Başçı



Temmuz 25, 2001



Bu tez, para büyümesinin üretimi, istihdamı, tüketimi ve reel ücretleri arz kanalıyla nasıl etkileyebileceğini teorik olarak incelemektedir. Bir kısıtlı katılım modelinde, firmaya uygulanan parasal transferlerin etkilerini deterministik ve stokastik durumlarda ele almaktadır. Stokastik durumlar için, rekabetçi ücretler ile nominal sabit ücret sözleşmeleri ayrı ayrı ele alınmaktadır. Beklenen para büyümelerinin üretimi, istihdamı, tüketimi, işletme sermayesini ve reel ücretleri düşürdüğü, fakat firmaların karlılığını arttırdığı gözlenmektedir. Ücretler sabitken, beklenmeyen para büyümesi istihdamı, üretimi ve tüketimi arttırmakta, fiyat ve karları düşürmektedır. Bu tezin literature ana katkısı, sabit nominal ücret sözleşmeleri altında, esnek olmayan işgücü arzına rağmen iş dalgalarının empirik özellikleri elde edilebilmektedir.



**Keywords:** Kısıtlı katım modelleri, sabit ücret sözleşmeleri, arz yönlü parasal aktarım mekanizması.

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# Contents

# 1 Introduction

This thesis is a theoretical investigation of how money growth affects output, prices, employment, consumption and real wages. This question has been studied in the literature under the heading of *monetary transmission mechanism.* The typical Keynesian monetary transmission mechanism in textbooks, (e.g. Romer, 1996), indicates that the monetary injections increase output via a demand side transmission mechanism. Money growth stimulates total spending, which in turn bids up prices. Under sticky wages, as a result of the increase in the price level, real wages decrease and hence employment and total output increase. In our model, we show the effects of monetary injections from a supply side channel, becasue the government injects the money to the economy through the firm.

The stylized empirical observations about monetary shocks indicate that if a contractionary shock on money is imposed, then aggregate output, profits and real wages decrease, prices and interest rates increase (e.g. Christiano et al., 1997). These observations are inconsistent with textbook monetary transmission theories. In order to explain the movements in actual data, limited participation models have been studied by Christiano and Eichenbaum (1992), Fuerst (1992), Christiano et al (1997), Evans and Marshall (1998), Christiano and Gust (1999), Cooley and Nam (1998), Cooley and Vincenzo (1999).

To assess the effects of exogenous shocks to monetary policy, we use a simplified version of the limited participation model developed by Fuerst (1992). Fuerst (1992) presents a competitive equilibrium model that captures the effects of monetary and productivity shocks on interest rates, output, investment, consumption, prices and employment level. He argues that monetary injections decrease the cost of using cash to finance production through the interest rate channel, because monetary injections are made through loan market to the producers. In our thesis, there is no loan market and money injections are directly made to the producers.

The limited participation model in Christiano et al. (1997), indicates that if there is an unplausibly high labor supply elasticity (e.g. 2 percent) and a very high markup rate (e.g. 40 percent), then, the limited participation models can account for all stylized facts. Christiano et al. (1997, p.1204) conclude that: '...It seems important to embed labor market frictions,which have the effects of mimicking a high elasticity of labor supply into the current generation of limited participation models.' although they propose no formal model of labor market frictions. In this dissertation we take a step to fill this gap.

Sticky wages have been widely considered in the literature in setups other than the textbook Keynesian model as well. For example, Cho and Cooley (1997), Cooley and Hansen (1998) and Folkertsma (1999) have studied the effects of money shocks under sticky wages on output, consumption, prices and employment level.

In our thesis, we analyze the effects of monetary shocks under deterministic and stochastic environments. In deterministic case, we find that the prices grow in the same direction as money growth, but in the stochastic case we find the reverse effects of money shocks on prices under sticky wages. Kydland and Prescott (1990, p.5) says, quoted in Den Haan (2000), '...any theory in which procyclical prices figure crucially in accounting for postwar business fluctuations is doomed to failure.' In our thesis, we find the countercyclical price movements due to monetary injections.

The remaining part of this thesis is organized as follows: In section 2, we discuss the model under deterministic money growth. In section 3, we analyze a stochastic representative agent model with flexible wages and competitive equilibrium, sticky wages with persistent unemployment and sticky wages with occasional unemployment in sequence. In section 4, we discuss the results and make concluding remarks.

## 2 A Deterministic Representative Agent Model

Let us consider a representative family composed of three members; a father, a mother and a son. At the beginning of the day, the family has a fixed amount of money, $M$. The average money amount across all families will be denoted by $\bar{M}$. The father takes fraction, $n$ of this money as working capital and the mother takes the remaining part of it for shopping at the beginning of the day. The son goes out for work with no money. Father has a factory but no labor force. Therefore, the father hires labor for production. The son supplies labor and at the beginning of the day he receives a nominal wage at rate $W$. Then, the produced goods are sold at the goods market. On the other hand, the government steadily injects $x\bar{M}$ amount of money every morning to the economy through the firm. The exact amount of monetary injection is assumed to be known in this section. At the end of the day, when the family members come back home, the mother comes home with some amount of purchased goods and the money left over, if any. The son comes home with his wage earnings and the father comes home with the profit earnings.

In order to model this economy, we assume the family has a utility function $U(c)+V(1-L)$ where $U(c)$, utility from consumption, is increasing and strictly concave ($U'(c) > 0$, $U''(c) < 0$) and $V(1-L)$, utility from leisure, is also increasing and concave ($V'(1-L) > 0$, $V''(1-L) \leq 0$). Here, $c$ and $L$ denote consumption and hours worked respectively. Bellman's equation for the representative family for given wages, prices, initial money balances and money growth rate can be written as follows:

$$J(m) = \max_{n}\{\max_{c,L,H}\{U(c) + V(1-L) + \beta J(m')\}\}$$

subject to

$$m \geq n \geq 0$$

$$m - n \geq pc$$

$$n + x \geq wH$$

$$1 \geq L \geq 0$$

$$m' = \frac{m + x + p(f(H) - c) + w(L - H)}{1 + x}$$

where $m$ and $m'$ denote normalized money balances of current and next period respectively, $p$ and $w$ are normalized prices and wage respectively. $x$ denotes money growth rate, $H$ denotes labor demand, $f(H)$ denotes the production function, $f'(H) > 0$ and $f''(H) \leq 0$ and $J(m)$ denotes the level of maximum lifetime of utility of a representative family with initial money balance $M$. The normalization is done by dividing the relevant nominal variable by the per family average beginning of period money balances, $\bar{M}$, i.e. $w = \frac{W}{\bar{M}}$, $p = \frac{P}{\bar{M}}$, $m = \frac{M}{\bar{M}}$, $m' = \frac{M'}{\bar{M}'}$ where $\bar{M}'$ is the average money balance per family in the next period.

For a given constant money growth rate $x$, a *competitive equilibrium* for this economy is a list of functions of $m$, $\{J, n, c, m', H, L\}$, a price $p$ and a wage $w$, such that

**1.** $J$ solves the Bellman Equation given above,

**2.** $n, c, m', H, L$ solves the right hand side of the same Bellman Equation,

**3.** Money market is in equilibrium, $m(1) = m'(1) = 1$, goods market is in equilibrium, $c(1) = f(H(1))$, and labor market is in equilibrium, $L(1) = H(1)$.

The Lagrangean of this maximization problem can be written as,

$$L(c, L, H, n, \lambda_1, \lambda_2) = U(c) + V(1 - L) + \beta J(\frac{m + x + p(f(H) - c) + w(L - H)}{1 + x})$$

$$+\lambda_1(m - n - pc) + \lambda_2(n + x - wh)$$

where $\lambda_1$ and $\lambda_2$ are the shadow values of money in consumption and production respectively. In this formulation, we guess that only two choice variables, $n$ and $L$, will take on interior values and hence, $\lambda_1 > 0$ and $\lambda_2 > 0$ are the Lagrangean multipliers of the second and third constraints respectively. Let $U(c) = ln(c)$, the production function $f(H) = H$, and the leisure utility function $V(1 - L) = 1 - L$. There are three markets in this economy: the money market, the labor market and the goods market. In equilibrium, all these must clear. Hence we have the following equilibrium conditions: $m = m' = 1, \quad f(H) = c, \quad H = L$. Solving the Envelope Condition, together with the first order conditions with respect to choice variables and shadow variables will provide us the following solution:

$$n = \frac{\beta - x(1 + x)}{1 + x + \beta} \tag{1}$$

$$w = \frac{(1 - n)(1 + x)}{\beta} = \frac{(1 + x)^3}{(\beta)(1 + x + \beta)} \tag{2}$$

$$H = \frac{n + x}{w} = (\frac{\beta}{1 + x})^2 \tag{3}$$

$$L = H = (\frac{\beta}{1+x})^2 \tag{4}$$

$$p = \frac{1-n}{c} = \frac{1-n}{H} = \frac{(1+x)^4}{\beta^2(1+x+\beta)} \tag{5}$$

When the results are analyzed, it will be easily noticed that $n$ is decreasing in $x$. Note that as the money growth rate is reduced towards the *Friedman rule*, that is $x = \beta - 1$, working capital, $n$, increases to its maximum, in order to compensate the taxation required for negative money growth. Similarly, the effects of $x$ on real wages $w/p$ and labor demand $L$ can be determined. We have obtained from the solution of equilibrium that $\frac{\beta}{1+x} f'(L) = \frac{w}{p}$ and $L = (\frac{\beta}{1+x})^2$. As it can be seen from these two equations, anticipated money growth, $x$, has negative effects both on real wages and equilibrium employment.

Likewise, the profit of the firm is also affected by the monetary surprise injections. Let $\Pi$ denote the profit function such that $\Pi = p.f(H) - wH$. We can find the profit by using the equation (2), (4), (5) as follows:

$$\pi = \frac{(1+x)(1+x-\beta)}{(1+x+\beta)} \tag{6}$$

As it can be seen from equation (6), a one time monetary injection increases the equilibrium profit at the expense of real wages.

# 3 A Stochastic Representative Agent Model

In this section, we will assume a random component to monetary injections (or taxes), $X_t$. The exact amount of $X_t$ will be learned by the family members after they have seperated from home with their money balances. We will analyze the stochastic case of our model under two subsections. In the first part, the case of *competitive equilibrium* will be analyzed. This will be a simplified version of Fuerst (1992). In the second part, we will impose the sticky wages. This second part will be studied under two sections; the first section will investigate the case of *persistent unemployment* that is the case of relatively high wages and the other section will examine the case of *occasional unemployment*.

## 3.1 Competitive Equilibrium

Let us assume the monetary shocks are unanticipated. Unlike the deterministic case, let's assume further that the leisure is not valued by any agent so that labor is supplied inelastically and therefore anticipated money growth does not effect employment and output.

Let $s$ denote the state of nature and $\pi(s)$ denote the probability density function of $s$. We will assume monetary policy, $x(s)$, will depend on $s$. Then the representative agent's maximization problem can be written as follows:

$$J(m) = \max_{n}\{\sum_{s} \pi(s)\{\max_{c(s),H(s)}\{U(c(s)) + \beta J(m'(s))\}\}$$

subject to, for all s

$$m - n(m) \geq p(s)c(s)$$

$$n(m) + x(s) \geq w(s)H(s)$$

$$m'(s) = \frac{m + x(s) + p(s)(f(H(s)) - c(s)) + w(s)(L(s) - H(s))}{1 + x(s)}$$

Notice here that the consumption and labor demand are determined after the monetary injection while $n$ is determined before the uncertainity is resolved. Assuming both cash in advance constraints will bind, let us put the values of $c(s) = \frac{m-n(m)}{p(s)}$ and $H(s) = \frac{n(m)+x(s)}{w(s)}$ into $m'$ and then plug these new values in objective function. Also imposing inelastic labor supply $L(s) = 1$ for all $S$, the objective function will become:

$$J(m) = \max_{n}\{\sum_{s} \pi(s)\{U(\frac{m - n(m)}{p(s)}) + \beta J(\frac{p(s)f(\frac{n(m)-x(s)}{w(s)}) + w(s)}{1 + x(s)}\}\}$$

Then the first order condition of the representative agent's problem is

$$\sum_{s} \pi(s)\{\frac{-U'(c(s))}{p(s)} + \frac{\beta p(s)}{1 + x(s)}\frac{1}{w(s)}f'(\frac{n + x(s)}{w(s)})J'(m')\} = 0$$

and the envelope condition is as follows:

$$J'(m) = \sum_{s} \pi(s)\frac{U'(c(s))}{p(s)}$$

Let $U(c(s)) = ln(c(s)),\ f(H(s)) = \theta H(s)$. After imposing the equlibrium conditions, $m = m' = 1,\quad f(H(s)) = c(s),\quad H(s) = L(s) = 1$, in the first order condition, together with the envelope condition, we have the following result:

$$1 \quad = \quad \beta \sum_s \pi(s) \{ \frac{1 - n(1)}{(n(1) + x(s))(1 + x(s))} \}$$

From here we can find the value of $n(1)$. Once the value of $n(1)$ is found, the rest of the equilibrium will be easily solved. After imposing the equilibrium conditions, we obtain prices as $p(s) = \frac{1-n(1)}{\theta}$, consumption as $c(s) = \theta H(s) = \theta$ and the wages as $w(s) = n(1) + x(s)$. We can also find the profit function in terms of $s$. Let $\Pi(s)$ denote the profit function. Then $\Pi(s) = p(s)f(H(s)) - w(s)H(s) = 1 - 2n(1) - x(s)$. As it can be seen from here, the profit depends negatively on the value of $x(s)$.

Let us solve for numerical values for $x(s)$. Let $x(s) = 0.1 \ \ with \ \ probability \ \ 0.5$ and $x(s) = -0.1 \ \ with \ \ probability \ \ 0.5$. Then we can find the value of $n$ as follows:

$$n(1) \approx \{ \frac{\beta}{2(1 + \beta)} \} + \{ \frac{\sqrt{\beta^2 - 0.04\beta(1 + \beta)}}{2(1 + \beta)} \}$$

Then, the price, consumption, wages and the profit can be easily calculated from the equations derived above for these numeric values of $n(1)$. Notice here that output, prices, consumption, employment are not affected by surprise money growth, while nominal and real wages are positively dependent on money shocks.

## 3.2 Persistent Unemployment

If nominal wages are negotiated at the beginning of each period before knowing the monetary shock, then $W$ cannot depend on $s$. We will call such wage contracts as sticky wages. In this subsection, we will suppose that the sticky wages are set so high that persistent unemployment occurs. We will relax this relatively high sticky wage assumption in the next section. The representative agent's maximization problem will be as follows:

$$J(m) = \max_{n}\{\sum_{s} \pi(s)\{\max_{c(s),H(s)}\{U(c(s)) + \beta J(m')\}\}\}$$

subject to, for all s

$$m - n = p(s)c(s)$$

$$n(m) + x(s) = \bar{w}H(s)$$

$$1 - H(s) \geq 0$$

$$m' = \frac{m + x(s) + p(s)(f(H(s)) - c(s)) + \bar{w}(L(s) - H(s))}{1 + x(s)}$$

Let $\lambda_3(s)$ be the langrange multipliers of the third constraint, i.e. the full employment constraint. Since third constraint is taken as strict inequality, it is obvious that $\lambda_3(s)$ will be zero. If we eliminate $c(s)$ and $H(s)$ using the first two cash in advance constraints, our choice variable will only be $n(m)$. After imposing the first and the second equations of our constraint into $m'$ and objective function, we can easily solve for the equilibrium.

The first order condition of the representative agent's problem, evaluated at equilibrium, is;

$$\sum_s \pi(s)\{-\frac{U'(c(s))}{p(s)} + \frac{p(s)\beta\theta}{(\bar{w})(1+x(s))}J'(m')\} = 0 \tag{7}$$

The envelope condition, then will be;

$$J'(m) = \sum_s \pi(s)\frac{U'(c(s))}{p(s)} \tag{8}$$

Let $U(c(s)) = ln(c(s))$, $f(H(s)) = \theta H(s)$. There are again three markets in this economy: the money market, the labor market and the goods market. In equilibrium, money and goods market clear but labor supply is rationed at the level of labor demand. That is, we have: $m = m' = 1$, $f(H(s)) = c(s)$, $H(s) = L(s) \leq 1$. The similar disequilibrium models are studied in Bennasy (1995) and Folkerstma (1999).

When we impose these equilibrium conditions into (8), after some calculations we obtain $J'(m) = \frac{1}{1-n(1)}$ *and* $J'(1) = \frac{1}{1-n(1)}$. Using this result together with (7), we have

$$1 = \sum_s \pi(s)\{\frac{1-n(1)}{(1+x(s))(n+x(s))}\} \tag{9}$$

Once we found the value of $n(1)$ from the equation (9), then the rest of the equilibrium will be easly solved. After imposing the equilibrium conditions, we can find the following results:

$$H(s) = \frac{n(1)+x(s)}{\bar{w}} \tag{10}$$

$$c(s) = \theta H(s) = \theta \, \frac{n(1) + x(s)}{\bar{w}} \tag{11}$$

$$p(s) = \frac{1 - n(1)}{n(1) + x(s)} \frac{\bar{w}}{\theta} \tag{12}$$

$$\Pi(s) = p(s) f(H(s) - \bar{w} H(s) = 1 - 2n(1) - x(s) \tag{13}$$

As it can be seen from the equation (9), the demand for working capital, $n$, does not depend on $\bar{w}$ since $p$ also increases with $\bar{w}$ to leave the real wage $\frac{\bar{w}}{p}$, unaffected by $\bar{w}$. In contrast to working capital $n(1)$, employment, output and consumption depend negatively on the nominal wage rate. For employment to be feasible, $H(s) \leq 1$, for all $s$. This requires a high wage contract, $\bar{w} \geq n(1) + max(x(s))$.

The decrease in price level with money injections is obtained from equation (12). To determine behaviour of consumption, labor demand and profit against monetary injections, let's look at their partial derivatives with respect to $x(s)$.

$$\frac{\partial H(s)}{\partial x(s)} = \frac{1}{\bar{w}} > 0, \ \forall \ s \tag{14}$$

$$\frac{\partial c(s)}{\partial x(s)} = \frac{\theta}{\bar{w}} > 0, \ \forall \ s \tag{15}$$

$$\frac{\partial \Pi(s)}{\partial x(s)} < 0, \ \forall \ s \tag{16}$$

The reason for an increase in labor demand is quite intiutive. We know that there is a persistent unemployment due to highly setted wage contracts. If the producers are provided with subsidies, that is $x(s)$ is positive, then they will have more resource to hire more labor force. Hence, the labor demand will increase and the unemployment level will decrease.

Similar to the labor demand, the consumption will also move in the same direction with the movement in $x(s)$. The higher the amount of money injection, $x(s)$, the more labor force is employed and therefore the more goods are produced and sold pulling down the price level. The source of decrease in profit is mainly coming from labor market. The total labor cost has increased, but there is no increase in nominal sales revenue, because the consumers have the same amount of money. The prices decrease at the same percentage rate of increase in produced goods, hence the total sales revenue remains intact. Therefore profits are decreasing with positive money injections and increasing with negative money injections.

Let $x(s) = 0.1 \;\; with \;\; probability \;\; 0.5$ and $x(s) = -0.1 \;\; with \;\; probability \;\; 0.5$. From the equation (10) we can find $n(1)$ as follows :

$$n(1) \approx \{\frac{\beta}{2(1+\beta)}\} + \{\frac{\sqrt{\beta^2 - 0.04\beta(1+\beta)}}{2(1+\beta)}\} \tag{17}$$

Therefore, even under sticky wages, working capital has the same level as competitive equilibrium, in the previous section. Then, the price, consumption, wages and the profit can be easily calculated from the equations derived above for these numeric values of $n(1)$. Real wages and profits turn out to be the same as those in competitive equilibrium. But here, prices fluctuate instead of nominal wages.

## 3.3 Occasional Unemployment

Let us again assume the sticky wages. But now, wages are set at a relatively lower value such that only occasional unemployment may occur conditional upon the money injections. Note that, in our model the nominal wages may fluctuate between each periods, but the normalized wages, $\bar{w} = W/\bar{M}$, remain constant within each period. Then the representative agent's maximization problem will be as follow:

$$J(m) = \max_{n}\{\sum_{s} \pi(s)\{\max_{c(s),H(s)}\{U(c(s)) + \beta J(m')\}\}\}$$

subject to, for all s

$$m - n \geq p(s)c(s)$$

$$n(m) + x(s) \geq \bar{w}H(s)$$

$$1 - H(s) \geq 0$$

$$m' = \frac{m + x(s) + p(s)(f(H(s)) - c(s)) + \bar{w}(L(s) - H(s))}{1 + x(s)}$$

Let $\lambda_1(s)$, $\lambda_2(s)$ and $\lambda_3(s)$ be the Langrange multipliers of the first three constraints respectively. Then the first order conditions of the representative agent's problem are;

$$-\sum_{s} \pi(s)\lambda_1(s) + \sum_{s} \pi(s)\lambda_2(s) = 0 \tag{18}$$

$$U'(c(s)) - p(s)\lambda_1(s) - \frac{p(s)\beta J'(m')}{1 + x(s)} = 0 \tag{19}$$

$$\{\frac{p(s)f'(H(s)) - \bar{w}}{1 + x(s)}\}\beta J'(m') - \bar{w}\lambda_2(s) - \lambda_3(s) = 0 \tag{20}$$

(21) $$m - n(m) - p(s)c(s) \geq 0, \quad with \;\; equality \;\; if \;\; \lambda_1(s) > 0$$

(22) $$n(m) + x(s) - \bar{w}H(s) \geq 0, \quad with \;\; equality \;\; if \;\; \lambda_2(s) > 0$$

(23) $$1 - H(s) \geq 0, \quad with \;\; equality \;\; if \;\; \lambda_3(s) > 0$$

Let $U(c(s)) = ln(c(s))$, $f(H(s)) = \theta H(s)$. There are three markets in this economy: the money market, the labor market and the goods market. In equilibrium, all these except labor market must clear. Hence we have the following equilibrium conditions: $m = m' = 1$, $f(H(s)) = c(s)$, $H(s) = L(s) \leq 1$. The solution of the maximization problem is conditional upon states.[1] There exists $s$, such that $\lambda_1(s) > 0$, $\lambda_2(s) = 0$ $and$ $\lambda_3(s) > 0$ and for other $s$' s, $\lambda_1(s) > 0$, $\lambda_2(s) > 0$ $and$ $\lambda_3(s) = 0$. The envelope condition is

(24) $$J'(m) = \sum_s \pi(s) \frac{U'(c(s))}{p(s)}$$

When we impose the equilibrium conditions into (24), after some manipulation we obtain $J'(m) = \frac{1}{m-n(m)}$ $and$ $J'(m') = \frac{1}{m'-n(m')}$. Using this result together with (19), we have

(25) $$\lambda_1(s) = \frac{1}{m - n(m)} - \frac{\beta}{1 + x(s)} \frac{1}{m' - n(m')}$$

From (9), together with the (24) and the equilibrium conditions, we can find the value

[1]The model that we have developed in this section includes occasinally binding constraints. The solution to such models are studied in Christiano and Fisher (2000).

of $\lambda_2(s)$ as

$$\lambda_2(s) = \frac{p(s)\theta - \bar{w}}{1 + x(s)} \frac{\beta}{\bar{w}} \frac{1}{m' - n(m')} \tag{26}$$

From (18), together with (25)and (26), we can write the following equation

$$\sum_s \pi(s)\{\frac{1}{m - n(m)} - \frac{\beta}{1 + x(s)} \frac{1}{m' - n(m')}\} = \sum_s \pi(s)\{\frac{p(s)\theta - \bar{w}}{1 + x(s)} \frac{\beta}{\bar{w}} \frac{1}{m' - n(m')}\} \tag{27}$$

From the first constraint we can find $p(s) = \frac{m-n(m)}{C(s)}$. After imposing the equlibrium conditions, $m = m' = 1$, $f(H(s)) = c(s)$, $H(s) = L(s)$, with the obtained value $p(s)$, we can find from the equation (27):

$$\sum_s \pi(s)\{1 - \frac{\beta}{1 + x(s)}\} = \sum_s \pi(s)\{\frac{1 - 2n(1) - x(s)}{(1 + x(s))(n(1) + x(s))}\beta\} \tag{28}$$

When we solve for the value of $n(1)$ from the equation (28), we can easily characterize the equilibrium of this economy. We can easily express prices, labor demand, consumption and profit of the firms in terms of the monetary injections, $x(s)$. These values will be as follows:

$$H(s) = min\{\frac{n(1) + x(s)}{\bar{w}},\ 1\} \tag{29}$$

$$p(s) = \frac{1 - n(1)}{c(s)} \tag{30}$$

$$c(s) = \theta H(s) \tag{31}$$

$$\Pi(s) = p(s)f(H(s)) - \bar{w}H(s) \tag{32}$$

Let $x(s) = 0.1$ *with probability* 0.5 and $x(s) = -0.1$ *with probability* 0.5. From the equation (28)we can find $n(1)$ as follows:

$$n(1) \approx \frac{1.02 - 0.202\beta}{3.64 - 2.02\beta} \tag{33}$$

Then, the price, consumption, wages and the profit can be easily calculated from the equations derived above for this numerical value of $n(1)$. In contrast with the persistent unemployment case, here output and prices fluctuate less. Other qualitative features of equilibrium are the same.

In our thesis, we analyzed the effects of monetary shocks under deterministic and stochastic environments. In section 2, we discussed the model under deterministic money growth. Here, we found that the prices grow in the same direction as money growth. In section 3, we analyzed a stochastic representative agent model with flexible wages and competitive equilibrium, sticky wages with persistent unemployment and sticky wages with occasional unemployment in sequence. In the stochastic case we found the reverse effects of money shocks on prices under sticky wages.

# 4 Conclusion

In this thesis, we studied the effects of monetary growth on output, prices, employment, consumption and real wages in a general equilibrium framework. When the money growth is anticipated, we found real wages, employment, output, working capital, consumption are all decreasing with money growth while profitability is increasing. Money injections decrease labor supply and hence aggregate output. When money injection rate is reduced to Friedman rule, then, the first best is attained.

In the third part, we analyzed the effects of unanticipated money shocks on output, employment, prices, wages and profit. We found that under the competitive equilibrium case, prices of goods, consumption level and labor demand are unaffected by monetary shocks. Nominal wages move in the same direction with the monetary shocks. Money injections increase nominal wages and hence the real wages. As a result of increase in total labor cost the profit of the firms decreases.

Under sticky nominal wages, when the wage rate is set relatively high before the uncertainity of money shocks resolved, we observed persistent unemployment. If persistent unemployment occurs, then, positive money shocks increase the labor demand, consumption level and total output. Suprisingly the profit is decreasing with suprise money, because the sales revenue does not change, the prices decrease and real wages increase. Nevertheless, the value of real wages here is the same as the value of real wages under competitive equilibrium.

On the other hand, when the wages are set relatively at lower value, then, there exixsts occasional unemployment. Labor demand is rationed after high money shocks. It takes the minimum value of $\{\frac{n(1)+x(s)}{\bar{w}},\ 1\}$. In the range of low suprise money growth, money injections increase the employment level, hence the total output increases. Prices decrease and consumption level increases. However, in the range of relatively higher suprise money growth, money injections do not alter the employment level, consumption level, prices and profit rate.

In an influential empirical study, Cochrane (1998) points out that, in interpreting the evidence from VARs, with a view point that anticipated money growth reduces output while unanticipated money schocks increase it, makes more sense than a model without these attributes. In this dissertation, we introduce a model with such a feature. Moreover, the model allows for involuntary unemployment over business cycle fluctuations, a need pointed out in the literature on limited participation models of the monetary transmission mechanism.

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