A HYBRID INVENTORY CONTROL POLICY FOR MEDICAL SUPPLIES IN HOSPITALS

A THESIS

SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL

ENGINEERING

AND THE INSTITUTE OF ENGINEERING AND SCIENCE

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

By

Gökçe Akın

July, 2010

I certify that I have read this thesis and that in my opinion it is full adequate, in scope and in quality, as a dissertation for the degree of Master of Science.

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ABSTRACT

A HYBRID INVENTORY CONTROL POLICY FOR MEDICAL SUPPLIES IN HOSPITALS

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July, 2010

In this thesis, we consider the inventory control problem of medical supplies that arises in a particular hospital environment. The items are stored in nursing stations from where they are retrieved by the nurses and used for the needs of in-patients or out-patients. The nursing stations are replenished from a central warehouse. Items are moved between the hospital's central warehouse and the nursing stations by a capacitated porter cart. In the representative nursing station that we analyze, the need for the medical supplies by the in-patients can arise at any time during day or night. It is possible to replenish the nursing stations during the day time on a continuous scale; however, this is not possible after-hours because the warehouse operates only during regular working hours. For this particular setting, we propose a hybrid inventory control policy which consists of a continuous review joint replenishment policy to manage the day time demand and a periodic review policy to manage the night time demand. The prior performance measure is set to satisfy the target service levels in the nursing stations. For a special case of the problem with a single item, we develop exact expressions to estimate the policy parameters. For the multi-item case, we analyze the impact of the policy parameters on the service level targets by simulating a representative system under different scenarios. Finally, we analyze a sample data collected from a nursing station and prescribe methods to determine the policy parameters.

Keywords: Medical supplies, inventory control, joint replenishment, service level, health care system

ÖZET

HASTANELERDEKİ TIBBİ SARF MALZEMELERİ İÇİN KARMA ENVANTER KONTROL POLİTİKASI

Gökçe Akın
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Tez Yöneticisi: Yrd. Doç. Dr. Osman Alp
Temmuz, 2010

Bu tezde, incelediğimiz bir hastane ortamında kullanılan tıbbi sarf malzemeleri için bir envanter kontrol problemi incelenmiştir. Bu hastanede tıbbi sarf malzemeleri her katta bulunan hasta bakım istasyonlarında belirli miktarlarda tutulmakta ve kat hemşireleri tarafından yatan veya ayakta tedavi gören hastalar için kullanılmaktadır. Hasta bakım istasyonları kapasiteli bir el arabası kullanılarak ana depodan yeniden doldurulmaktadır. Ele alınan örnek katta tıbbi sarf malzemeleri için gün içerisinde veya gece herhangi bir saatte talep görülebilmektedir. Gün içerisinde hasta bakım istasyonları herhangi bir zamanda sürekli olarak yeniden doldurulabilirken; çalışma saatleri dışında ana depo kapalı olduğu için bu mümkün olmamaktadır. Böyle bir sisteme uygun olarak karma bir envanter politikası önerilmiştir. Bu politikada gün içerisindeki envanter kontrolü için sürekli yeniden gözden geçirilen toplu sipariş politikası önerilirken; gece için bir dönemsel gözden geçirme politikası önerilmiştir. Performans kriteri, hedeflenen hizmet düzeyinin sağlanması olarak belirlenmiştir. Özel bir durum olarak tek ürünlü sistemde politika parametrelerin elde edilebilmesi için kesin ifadeler türetilmiştir. Çok ürünlü sistem için ise politika parametrelerinin hizmet düzeyi üzerindeki etkileri gözlemlemek için sistem farklı senaryolar altında simüle edilmiş ve parametre kestirimi için yöntemler önerilmiştir. Son olarak, hastaneden alınan örnek data incelenmiş ve önerilen kestirim yöntemleri uygulanıp değerlendirilmiştir.

Anahtar sözcükler: Tıbbi sarf malzemeler, envanter kontrolü, toplu sipariş, hizmet düzeyi, sağlık sistemi

Acknowledgement

First of all, I would like to express my sincere gratitude to my supervisor Asst. Prof. Dr. Osman Alp for his invaluable guidance and support during my graduate study. He has supervised me with everlasting interest and motivation throughout this study. I would like to thank once more for his encouraging advices on the other academical issues, especially for the last two years.

I am also grateful to Asst. Prof. Dr. Murat Fadıloğlu and Asst. Prof. Dr. Banu Yüksel Özkaya for accepting to read and review this thesis and for their invaluable suggestions.

I would like to thank to Ankara Güven Hospital for letting me to analyze the hospital and providing me the representative data for this study.

I would like to express my sincere thanks to Prof. Dr. İhsan Sabuncuoğlu, Assoc. Prof. Dr. Bahar Yetiş Kara and (once more) Asst. Prof. Dr. Osman Alp, since they have always trusted in me and appreciated my work as their teaching assistant for two years.

I am indebted to my fiance Korhan Aras for his incredible support and encouragement for six years. I am also lucky to have Uğur Cakova as one of my best friends who is ready to listen to me, encourages me with his advices and comes up instant solutions all the time. Additionally, I am thankful to Pelin Damcı (and Mehmet Can Kurt), Gülşah Hançerlioğulları, Hatice Çalık, Ece Demirci, Efe Burak Bozkaya (and Füsun Şahin Bozkaya), Esra Koca, Burak Paç, Can Öz, Yiğit Saç, Emre Uzun and all other friends that I failed to mention here, for their invaluable support and friendship during my graduate study.

Most importantly, I would like to express my deepest gratitude to my family for their endless love and support throughout my life.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Health care sector supply chains are known to be structurally complicated and are characterized by two chains: an external and an internal chain as shown in Figure 1.1 (Rivard-Royer, et al., 2002). Within this thesis, we do not consider the external chain, instead we mainly focus on the internal chain which consists of the main warehouse, the nursing stations and patients. In the internal chain, the nurses retrieve the necessary items from the nursing stations for the patient needs and the nursing stations are replenished by the main warehouse. Within hospitals, it is the top priority to meet the patient needs on time, and hence inventory management plays a significant role in hospital supply chains. As it is also underlined by Burns et al. (2002), in the hospital supply chains the end users are not customers who are directly paying for the item, but are the patients who create a demand at the nursing stations due to clinical preference. For this reason, cost-benefit analysis or the budget constraints are not considered as the main issue in the inventory management of medical items.

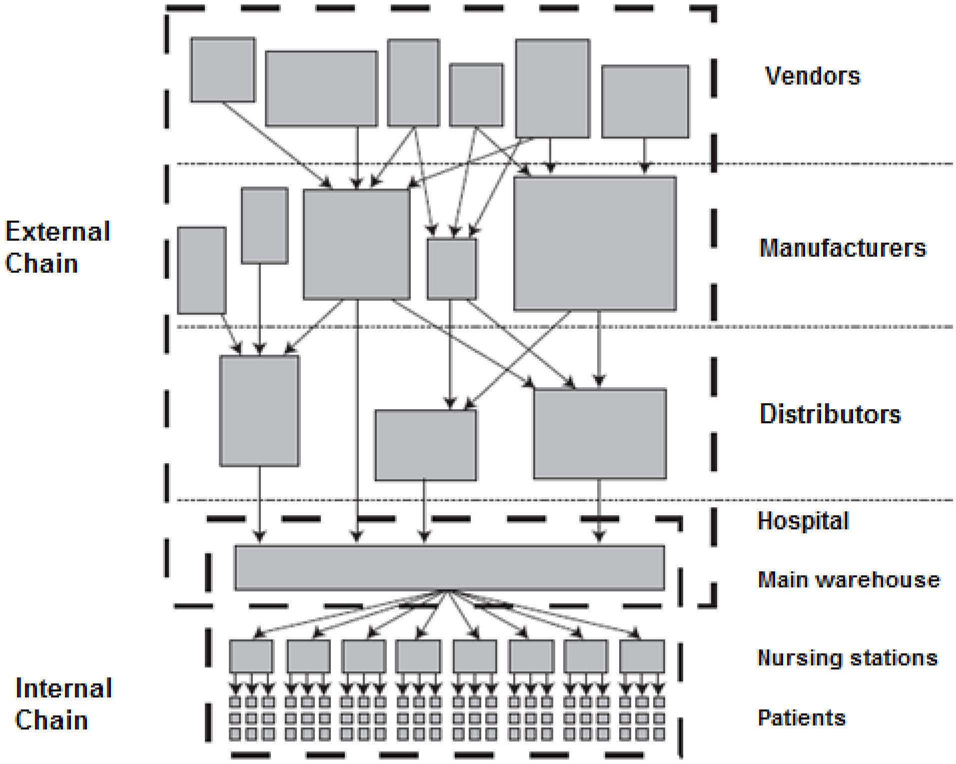

Figure 1.1: Health care sector supply chain
(Adapted from Rivard-Royer, et al., 2002)

In hospitals, since a patient's health is the main concern, it is essential to satisfy the patient needs on time. For this reason, throughout this thesis we mainly focus on satisfying required service levels rather than minimizing the cost. Type 2 service level (i.e. fill-rate), which is the proportion of unsatisfied demand, is taken as a performance indicator. In a typical hospital, there are three types of items that can be demanded at anytime: medical supplies, pharmaceuticals and textile items. Injector, syringe, etc. can be given as the examples of medical supplies. Pharmaceuticals include medicines and hygienic liquids, whereas textile items are composed of towels, bed sheets, etc. Certain amounts of medical supplies should be kept close to the point of delivery (nursing stations in our case), from where the nurses retrieve the items for the patient needs. In these places, a variety of these medical supplies need to be hold so that demand, which can occur at anytime during a day, can be met on time.

CHAPTER 1. INTRODUCTION

Within our study, we consider the inventory system in a local hospital in Ankara, namely Ankara Guven Hospital. In this hospital, a material tracking system is being used for the medical supplies, since the medical supplies are the most demanded group of items. With the existing material tracking system in the hospital, each medical supply's actual inventory level throughout the day can be tracked easily. However, neither the responsible personnel in the hospital know, nor this system includes the information of exactly when and how the replenishments should be done at the nursing stations, in order to satisfy the target service levels while not holding excess amount of inventory due to limited storage places. For this reason, we motivate our study on this inventory control problem of Guven Hospital.

Ankara Guven Hospital, founded 36 years ago, is one of the leading institutions of the Turkish health care sector (Ankara Guven Hospital, 2010). The hospital has the certification of ISO 9001:2000 and the certification of Joint Commission International (JCI) accreditation standards. In order to meet the expectations of patients and their companions, the hospital employs over 850 personnel. The hospital was established on 20,000 m^2 area. It consists of eight operating rooms and 156 beds. There are five nursing stations, from which in-patients and out-patients are served, and one central warehouse, from which the nursing stations are replenished. Within this thesis, we select the nursing station at Block A 3rd Floor as the representative station, in which 195 different medical supplies are used. For these medical supplies, at the representative nursing station the total daily demand is 240 on the average.

Based on the inventory control problem that arises in Guven Hospital, we consider a multi-item system in which a capacitated porter cart is used for moving items from the main warehouse to the nursing stations. Controlling multiple items in a system, where high service levels are required, is a challenge itself. Additionally, another challenge of our problem is that due to the on- and off-periods of the central warehouse, nursing stations can be replenished on a continuous scale while it is not possible to make any replenishment during night-time. This structure is not a typical structure examined in the inventory literature that considers a typical supply chain, such as a retail supply chain. We suggest a hybrid policy to be used for these two distinct time frames. According to our proposed policy, the continuous review joint replenishment policy $(\boldsymbol{s}, Q)$ is adapted during day-time, and periodic review $(T, \boldsymbol{S})$ policy is used for the night-time in order to satisfy the target service levels, where T is set to one lead time ahead of the end of the day. The $(\boldsymbol{s}, Q)$ policy, is proposed by Tanrikulu et al. (2009), and works in a way that, a fixed size of joint order Q is given whenever an item's inventory position drops below its reorder point (s). In this policy, the joint order size is allocated to other items so that all items' inventory positions in excess of their reorder points are equalized as far as they can be. This policy leads to fully utilized porter cart (when Q is set to the cart capacity) at each ordering instances, along with satisfying the target service levels by reviewing each item's inventory position continuously.

Before analyzing the inventory system under our proposed policy, we make a detailed analysis of the transaction data collected from the representative nursing station and classify the items in three classes A, B and C according to total demand proportion of each item within the inventory system. Then, for a sample set of items, we obtain the detailed statistics and probability distributions of item demands, in order to use in our numerical analysis.

We derive exact expressions to find optimal policy parameters for the single item case. However, since it is computationally hard to derive these expressions for the multi item case, $(\boldsymbol{s}, Q) + (T, \boldsymbol{S})$, we analyze the impact of the policy parameters on the service level targets by simulating a representative system under different scenarios. We simulate the inventory system by using Arena and starting our search from the initial values that we estimate by considering the service levels, we obtain the optimal parameter values. We find out that the method for estimating the initial S values gives the optimal values, while the method for estimating the initial s values gives overestimated values.

The remaining of this thesis is organized as follows. In Chapter 2, the related literature is examined in three main parts, which are OR in health care, inventory control of medical supplies, and joint replenishment policies with an emphasis on service levels. Afterwards, we describe the system under consideration along with the data analysis in detail in Chapter 3. In Chapter 4, after explaining the inventory problem on hand, solution approaches are proposed for this problem. After discussing the optimal policies, the proposed hybrid policy for our system is introduced. Also in Chapter 4, the single-item case is analyzed and expressions are derived for obtaining the optimal policy parameters for this special case. Policy parameters estimation methods are introduced in Chapter 5, so that estimated values close to the optimal values of the parameters can be obtained easily, without dealing with complicated structures for the multi-item case. We test these methods by simulating the system in Arena under different scenarios as they are explained in Chapter 5. Finally, by Chapter 6 we conclude the thesis with our conclusion and possible future extensions.

Chapter 2

Literature Review

2.1 OR in Health Care Literature

OR in Health Care literature is composed of three main areas: Health Care Operations Management, Clinical Applications, and Health Care Policy and Economic Analysis (Brandeau, et al., 2004). Health Care Operations Management includes OR applications on managerial issues in hospitals such as designing services, designing and managing the health care supply chain, facility planning and designing, equipment evaluation and selection, process selection, capacity planning and management, demand and capacity forecasting, scheduling and workforce planning, resource allocation in medical environments, etc. Few examples in this area are as follows: Green (2004) introduces the OR applications on hospital capacity planning, Henderson et al. (2004) propose a decision making model for ambulance service, and Daskin et al. (2004) explain how the facility location models can be applied in health care. The second area, Clinical Applications, involves topics like designing and planning of treatments for the patients, assessing how a disease is likely to progress in a patient and choosing drugs, determining dosages and designing other aspects, etc. In this area, studies are mostly focused on applying OR methods to the cancer detection and various type of therapies. Some examples in this area can be given as follows: Maillart et al. (2008) study the breast cancer screening policies dynamically, Lee et al. (2008) work on the planning of dialysis theraphy and Lee and Zaider (2008) introduce a dynamic method for the treatment of prostate cancer. The third area, which is Health Care Policy and Economic Analysis, includes topics such as coordination of influenza vaccination, prediction of Health Care costs for a government, drug policy etc. Some studies within this area can be given as, Chick et al. (2008)'s work on supply chain coordination of the influenza vaccines and the study of Bertsimas et al. (2008) on the prediction of health care costs. In this thesis, we focus on the inventory control problem of medical supplies that arises in hospitals. For this reason, our study falls into Health Care Operations Management area.

2.2 Inventory Control of Medical Supplies

Even the inventory control literature has a wide range, the studies on specifically the inventory control of medical supplies (e.g. syringe, mask, etc.) are limited. To begin with, there are some studies on the general structure of hospital supply chains for the medical supplies. Nicholson et al. (2004) analyze the cost and service level effect of reducing the three echelon system, which involves item movements between suppliers, main warehouse, nursing stations and patients, to a two echelon system in which the main warehouse is removed. In their proposed system, an outside company manages, holds and distributes the items to the nursing stations. They find out that by outsourcing the inventory management a better system can be obtained in terms of efficiency, high service levels and low level of inventory throughout the hospital. There are other studies on structural changes in the hospital inventory systems, such as implementing just-in-time or stockless systems (Rivard-Royer, et al., 2002). As an example, a case study is conducted on this topic by Kumar et al. (2008) to the health care industry of Singapore. In that study they propose a new structure for the hospital supply chains in which just-in-time applications are used and the total cost in the supply chain is reduced.

DeScioli et al. (2005) conduct a research on inventory control in a hospital, in which Automated Point of Use system is proposed to be used for tracking each item's inventory level automatically. In that research, they consider inventory carrying costs, ordering costs, stockout costs and replenishment lead time while deciding on the inventory control policy. Firstly, they propose a standard base stock policy with periodic review. In order to achieve the required service level, they make sure that the order up to level of each item is high enough to satisfy the total demand over the review period and the lead time. Secondly, they propose an (r, Q) policy with periodic review, where Q is the economic order quantity for each item. At a periodic review instance, for an item i whose inventory position is below their reorder level (r_i), an order amount of Q_i is given. Note that in both of these policies, they consider the items individually and they do not use the joint replenishment in terms of setting a total ordering quantity at an ordering instance. In the hospital that we analyze, there is also a tracking system to control each item's inventory level. However, our study differs from that research in way that we consider a continuous review joint replenishment system with a capacitated cart, which is used for moving the items between the warehouse and the nursing stations.

For another standard hospital setting, which also includes a main warehouse and nursing stations, an inventory policy for the medical supplies by considering space restrictions is proposed by Little et al. (2008). In that study, the proposed inventory control policy is a standard base stock policy, whose parameters are found by taking service levels, frequency of deliveries, space constraints and criticality constraints into account. They assume that the replenishment lead time of a nursing station is zero and demand of each item is normally distributed. They obtain results for replenishing every day, every three days and every five days. They state two objectives for the service levels: maximizing the minimum service level and maximizing the average service level. With this in mind, they analyze their results according to the percentage of items at each service level, the average service level and total amount of space used. According to their analysis demand is a more important guide to obtain high service levels rather than the unit volumes. They also show that the same service levels can be reached by delivering everyday with a low space usage than delivering every three or five days with a high space usage. In our research, we do not consider the space constraints explicitly; instead we try to minimize the inventory level in the system while finding the joint replenishment policy parameters. Moreover, in our study we consider each item's service level requirements separately and we make sure that each item is available with a required service level by using a continuous review policy. Another difference of our study from this research is that we also take the replenishment lead time into account.

2.3 Joint Replenishment Policies with an Emphasis on Service Levels

In this section, we review the inventory control literature with an emphasis on joint replenishment policies and service levels. Among the joint replenishment policies, in $(Q, \boldsymbol{S})$ policy, which is firstly proposed by Renberg et al. (1967), an order is placed to raise each item's inventory position to its own order-up-to level (S), whenever the total consumption reaches Q. Pantumsinchai (1992) compares this policy with another joint ordering policy, the can order policy $(\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{S})$ suggested by Balintfy (1964). In that comparison paper, it is stated that $(Q, \boldsymbol{S})$ policy is appropriate for the inventory system in which the stockout costs are low and ordering costs are high. Note also that depending on the problem parameters, $(\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{S})$ has a strong advantage when the high service levels are considered. This is because, by using the can order policy, the inventory position of each item can be tracked and whenever an item's inventory position drops below its must order point (s), a joint order is given for all of the items, whose inventory positions are below their can order points (c). Tracking each item's inventory positions is not possible for $(Q, \boldsymbol{S})$ policy and once an item's inventory position drops below zero, if the total consumption is not Q at that time, then that item may need to stay below zero for a long time, until the total consumption reaches Q. Pantumsinchai (1992) also states that for most of the problems with small lead times with large penalty costs $(Q, \boldsymbol{S})$ policy ends up with negative savings. Large penalty costs can be thought as high service levels, for this reason $(Q, \boldsymbol{S})$ policy may not be appropriate for a system in which high service levels are required. Moreover, Pantumsinchai (1992) compares $(Q, \boldsymbol{S})$ policy with the $(\boldsymbol{R}, T)$ policies, which are proposed by Atkins et al. (1988), as well. According to $(\boldsymbol{R}, T)$ policy, each item's inventory position is review every T periods and a joint order is given for increasing each item's inventory position to its R value. Pantumsinchai (1992) states that the $(\boldsymbol{R}, T)$ policy is comparable to $(Q, \boldsymbol{S})$ policy.

Then, Viswanathan (1997) proposes another inventory control policy $T(\boldsymbol{s}, \boldsymbol{S})$, which involves $(\boldsymbol{s}, \boldsymbol{S})$ policy with periodic ordering instances at every T periods. In this policy, every T periods, inventory positions of all items are reviewed and a joint order is given for the items, whose inventory positions are below their reorder points (s) in order to raise them up to their order up to values (S). Later on Nielsen et al. (2005) suggest another policy in which the $(\boldsymbol{s}, \boldsymbol{S})$ policy is used and inventory positions are reviewed when the total consumption of all items reaches Q. They also show that in all cases this $Q(\boldsymbol{s}, \boldsymbol{S})$ policy is better than the periodic $(\boldsymbol{s}, \boldsymbol{S})$ policy. Moreover, for most of the cases it also outperforms $(Q, \boldsymbol{S})$ policy (Nielsen, et al., 2005).

Aside from these policies, Fung et al. (2001) proposes a periodic review policy (T, S), which considers the coordinated replenishments for multi-item systems with service level constraints as well as positive replenishment lead times. Note that in the policy that is proposed by Atkins et al. (1988) the service level constraints are not considered. For this reason $(T, \boldsymbol{S})$ policy of Fung et al. (2001) is better than $(\boldsymbol{R}, T)$ policy of Atkins et al. (1988) when the high service levels are required. Fung et al. (2001) also compare the $(T, \boldsymbol{S})$ policy with $(\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{S})$ policy and they find out that there are significant savings of $(T, \boldsymbol{S})$ policy over $(\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{S})$ for high service levels and by using $(T, \boldsymbol{S})$ policy, positive lead time can easily be handled compared to $(\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{S})$ policy.

In a recent study, Ozkaya et al. (2006) suggest a joint replenishment policy, in which a joint order is given to raise each item's inventory position to its S, when the total consumption reaches to Q, or a total period of time T passes (whichever is the first).

They show that this policy is performing better than $(\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{S})$, $(Q, \boldsymbol{S})$, $T(\boldsymbol{s}, \boldsymbol{S})$ and $Q(\boldsymbol{s}, \boldsymbol{S})$ in most of the cases.

Lately, Tanrikulu et al. (2009) propose a new continuous review joint replenishment policy $(\boldsymbol{s}, Q)$. In this policy, a joint order size of Q is triggered whenever an item's inventory position drops below its reorder point (s). Q is a fixed order amount, which is set to the capacity of a truck or cart, depending on the environment in which the policy is used. This fixed order size of Q, is allocated to all items in a way that each item's inventory position in excess of the reorder point are balanced. Since capacitated equipment is used for the replenishment in the system under consideration within this thesis, it is important to consider the capacity of that equipment at each ordering instance. $(Q, \boldsymbol{S})$ policy that is suggested by Cachon (2001) also employs a fixed order size of Q at each replenishment and involves continuous review. Comparison results of Tanrikulu et al. (2009) show that $(\boldsymbol{s}, Q)$ policy outperforms the $(Q, \boldsymbol{S})$ policy especially with the high backorder costs and small lead time.

To sum up, this thesis contributes to both the inventory control literature and OR in health care literature. By considering two distinct time frames, each requires high service level; we suggest a hybrid policy, which consists of a continuous review policy $(\boldsymbol{s}, Q)$ for the first time frame and a periodic review policy $(T, \boldsymbol{S})$ for the second time frame, to be used. This part forms the main contribution of our thesis to the inventory control literature. Moreover, since we find out that there is no study on medical supplies inventory control which includes continuous review joint replenishment policies, which utilize the capacitated cart at each ordering instance, our study is also innovative for the OR in health care literature.

Chapter 3

System Description and Data Analysis

3.1 System Description

In a typical hospital, there are mainly three types of items to be controlled: medical supplies, pharmaceuticals and textile items. Medical supplies consist of materials such as injector, needle, cotton, bandage, etc. Pharmaceuticals are composed of medicines and hygienic liquids like oxygen peroxide. For the textile items, towels, bed sheets and patient cloths can be given as examples. In Guven Hospital, there are three separate warehouses for these items. Medical supplies are stored in the central warehouse, pharmaceuticals are stored in the pharmacy, and textile items are stored in the textile warehouse. In this particular setting, we focus on the inventory control of the medical supplies.

In Guven Hospital, medical supplies are controlled through a two echelon inventory system. The upper echelon is the central warehouse and the lower echelon consists of five nursing stations that are located at Block A 1st Floor, Block A 3rd Floor, Block B 3rd Floor, Emergency Room and Intense Care Unit. The corresponding inventory system can be seen at Figure 3.1.

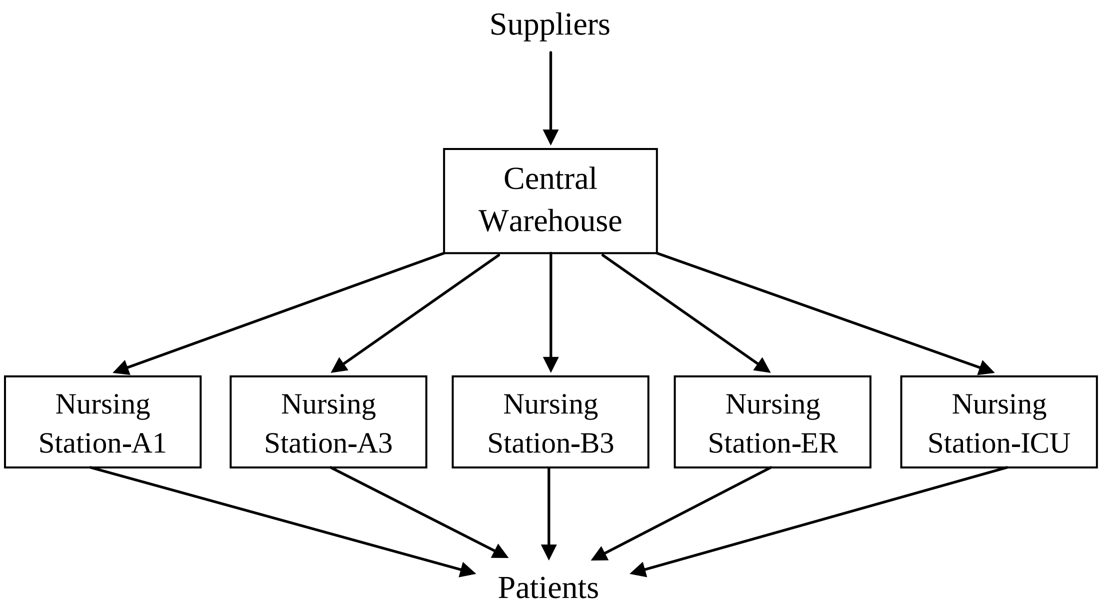

Figure 3.1: Medical Supplies Inventory System in the Hospital

In this system, inventories at the central warehouse are reviewed continuously by using the inventory tracking system and orders for medical supplies are given to the related suppliers whenever necessary. The procurement department of the hospital makes contractual agreements with the suppliers. According to such contracts, shipment related costs are covered by the suppliers and the suppliers agree to deliver the items within a certain amount of time. After the arrival of the medical supplies, the warehouse personnel enters the quantities of each item to the inventory tracking system. In the central warehouse the medical supplies are stored until they are requested and retrieved for the nursing stations. The central warehouse is operating between 9:00 am and 6:00 pm. After 6:00 pm until 9:00 am next day, this warehouse is closed and there cannot be any replenishment during this period.

The nursing stations serve directly to the patients. The patients to be served by the stations can be either in-patient or out-patient. For example, Nursing Station-A3 serves in-patients while Nursing Station-ER serves mostly out-patients. All of the nursing stations can observe demand throughout the day (24-hour period), regardless of the time. Once a medical item is retrieved by a nurse for a patient need, the nurse enters that record into the system by using the item’s barcode, so that they can track the remaining amount of inventory as well as the information of which item is used for which patient. Nursing stations can be replenished by the central warehouse at any time during central warehouse's operating hours (9:00 am-6:00 pm), which we call as the "day-time", however, replenishment is not possible during its after hours (6:00 pm-9:00 am next day), which we denote as the "night-time". Whenever a replenishment occurs at the nursing stations, each responsible nurse enters the quantities of each item to the system and stores the items.

A porter cart is being used for the delivery of items from the central warehouse to the nursing stations. The cart has a capacity in terms of volume. The same porter serves more than one nursing station during day-time. After an order is given by a nursing station, it takes about one hour to prepare and load the medical supplies on the cart, and move these medical supplies to nursing stations. With this in mind, during the last one hour just before the night-time begins, the nursing station does not place any orders, since delivery is not possible in less than one hour and the nursing station cannot be replenished before the end of the day.

Within this thesis, we focus on the inventory control operations at the nursing stations and keep the operations at the central warehouse out of the scope. We take the nursing station at Block A 3rd Floor as the representative station, since it observes demand for the highest variety of medical supplies compared to other stations. Totally, there are 195 independent medical supplies used in this station. The analysis of the types and demand rates of these supplies will be explained in the next section.

3.2 Data Analysis

As it is mentioned in the previous section, we choose the nursing station at Block A 3rd Floor as the sample station. There are 195 different medical supplies that are used in this station. We obtained detailed demand data of these medical items for six months (April 1, 2009 - September 1, 2009). The demand data include the time of each retrieval (by the nurses) instance of an item and the amount of items per retrieval. Before analyzing this data, we prune them so that the outlier data, which occur due to system failures, are removed.

First of all, we classify the items into three classes A, B and C according to total demand proportion of each item within the inventory system. The related total demand data are given in the Table A.1 in Appendix A. According to this classification, A items constitute nearly 8% of the total number of items but represents nearly 83% of the total demand in six months while B items make up 22% of the total number of items but represents nearly 15% of the total demand. C items constitute 70% of the total number of items but represents 2% of the total demand.

In Table A.1, A items average daily demand is within the range of 3 and 45. This is a wide range because the first four items in this group have much higher demand than the remaining items. The range of average daily demand for the B items is 0.17 and 3, and the rest of the items belong to the group C. Note that, the total daily demand of all items is 240 on the average. We analyze the daily demands of some fast moving items and obtain their descriptive statistics for the observed demand per day by using Minitab. In Table A.2 in Appendix A, the related statistics are given for the first 42 items. Observe that the daily demands of A items have high level of variances, and Gamma distribution turns out to be the best fitted distribution for these items' daily demands. Some A items' histograms with distribution fits are given in Appendix B. For the B items, none of the distributions can be fitted to the daily demands. Thus, for this group of items we accumulate the daily demands into demands per 10 days and then we fit Normal distribution to the total demand per 10 days. The C items have very low daily demands, which may be observed once in a week or even once in six months. Daily demand distribution fitting is not proper for these items. For this reason, we analyze the time between demands and find out that the best distribution which fits to the interarrival times of C items is the Exponential distribution. In other words, during a day a C item's demand is Poisson distributed. Since C items constitute only 2% of all items and it is straightforward to set a policy under Poisson demand, we exclude these items from further analysis.

Next, we analyze the daily demands to see if there is a correlation between items by using Minitab. For this analysis we search for the dependency within the classes. In Table 3.1 the correlation coefficients and p-values for the most demanded 10 items from group A.

Table 3.1: Correlation analysis for A items (Correlation coefficient, p-values)

ID	41	51	42	52	122	146	20	95	44	74
41	-	(0.201, 0.005)	(0.236, 0.001)	(0.209, 0.004)	(0.239, 0.001)	(0.044, 0.554)	(0.190, 0.009)	(0.113, 0.125)	(0.215, 0.003)	(0.125, 0.089)
51		-	(0.15, 0.042)	(0.478, 0.000)	(0.46, 0.000)	(0.234, 0.001)	(0.500, 0.000)	(0.402, 0.000)	(0.032, 0.661)	(0.393, 0.000)
42			-	(0.303, 0.000)	(0.169, 0.021)	(-0.033, 0.652)	(0.061, 0.411)	(0.128, 0.081)	(0.241, 0.001)	(0.003, 0.968)
52				-	(0.396, 0.000)	(0.247, 0.001)	(0.396, 0.000)	(0.316, 0.000)	(0.102, 0.167)	(0.252, 0.001)
122					-	(0.188, 0.010)	(0.358, 0.000)	(0.235, 0.001)	(0.039, 0.602)	(0.166, 0.024)
146						-	(0.345, 0.000)	(0.355, 0.000)	(0.003, 0.963)	(0.247, 0.001)
20							-	(0.464, 0.000)	(-0.017, 0.819)	(0.437, 0.000)
95								-	(0.046, 0.530)	(0.587, 0.000)
44									-	(0.033, 0.658)
74										-

According to this table, if p-values are smaller than 0.01, then there is sufficient evidence that the correlations are not zero, otherwise the correlations between the items are zero. With this in mind, there are 28 out of 45 pairs of items which are correlated. Among these correlated pairs, the maximum absolute correlation coefficient is 0.587 and the minimum absolute correlation coefficient is 0.188. In Table 3.2 the correlation coefficients and p-values for a sample set of B items are given. Among these items, only items ID75 and ID2 are correlated with each other with a correlation coefficient of 0.211. For the C items we did not analyze the correlations, since the demand rates for those items are so low and there is no sufficient data for such analysis. Even we find out that there is a certain level of correlation between some items, within this thesis we do not consider the dependencies between the medical supplies.

Table 3.2: Correlation analysis for B items (Correlation coefficient, p-values)

ID	75	112	69	2	54
75	-	(0.039, 0.592)	(0.003, 0.970)	(0.211, 0.003)	(0.081, 0.263)
112		-	(-0.016, 0.828)	(0.139, 0.053)	(0.050, 0.490)
69			-	(-0.010, 0.895)	(0.008, 0.917)
2				-	(0.007, 0.926)
54					-

The system that we explain in the previous section makes it necessary for us to obtain distinct demand structures for the day-time and night-time. Since a continuous review policy is suitable for the day-time inventory control, we need to find the distributions for the time between demands (i.e. interarrival times) and compounding parts of these demands. On the other hand, since a periodic review policy is appropriate for the night-time inventory control, the distribution for the demand per night need to be obtained for each item. For this reason, we choose some representative items from classes A and B, and analyze their demand structure in detail by using Minitab. From Class A, we analyze the items ID51, ID52, ID122 and ID95. From Class B, we analyze the items ID75 and ID112. Note that while deciding the best fitted distribution for the related demand data, we use Minitab's Anderson Darling test and select the distribution with the sufficiently large p-value.

Firstly, we begin our analysis with A items. For item ID51, the interarrival times during day-time are best fitted to the Gamma distribution with shape 0.4874 and scale 10168. There is also compound part for the day-time demand of this item. We set an empirical distribution for this part as it is shown in Table 3.3. For the total demand per night the best fitted distribution that we obtain is Gamma distribution with shape 3.171 and scale 5.950. Histograms of item ID51 day-time and night-time demands with the related distributions are given in Figures 3.2 and 3.3. The distribution of the interarrival times of item ID52 is also Gamma distribution with shape 0.5462 and scale 23176. Compound part of the day-time demand is shown in Table 3.3. The total demand per night is distributed with Gamma distribution with shape 2.523 and scale 4.142. Histograms with these distributions can be seen in Figures 3.4 and 3.5. The items ID122 and ID95 have also Gamma distributed interarrival times with shape parameters 0.6403 and 0.4544, and scale parameters 19437 and 34706 respectively. Demand per arrival distributions are shown in Table 3.3. The total demand per night has Gamma Distribution as well, with shape parameters 2.298 and 2.636 and scale parameters 2.602 and 1.326. These items' histograms with the related distributions are shown in Figures 3.6, 3.7, 3.8 and 3.9.

Table 3.3: Distributions of compound parts for A items day-time demands

	Probability Distributions			
Demand per arrival	ID51	ID52	ID122	ID95
1	0.51	0.52	0.78	0.91
2	0.26	0.28	0.16	0.08
3	0.09	0.13	0.05	0.01
4	0.05	0.03	0.01	-
5	0.06	0.04	-	-
6	0.02	-	-	-
8	0.01	-	-	-

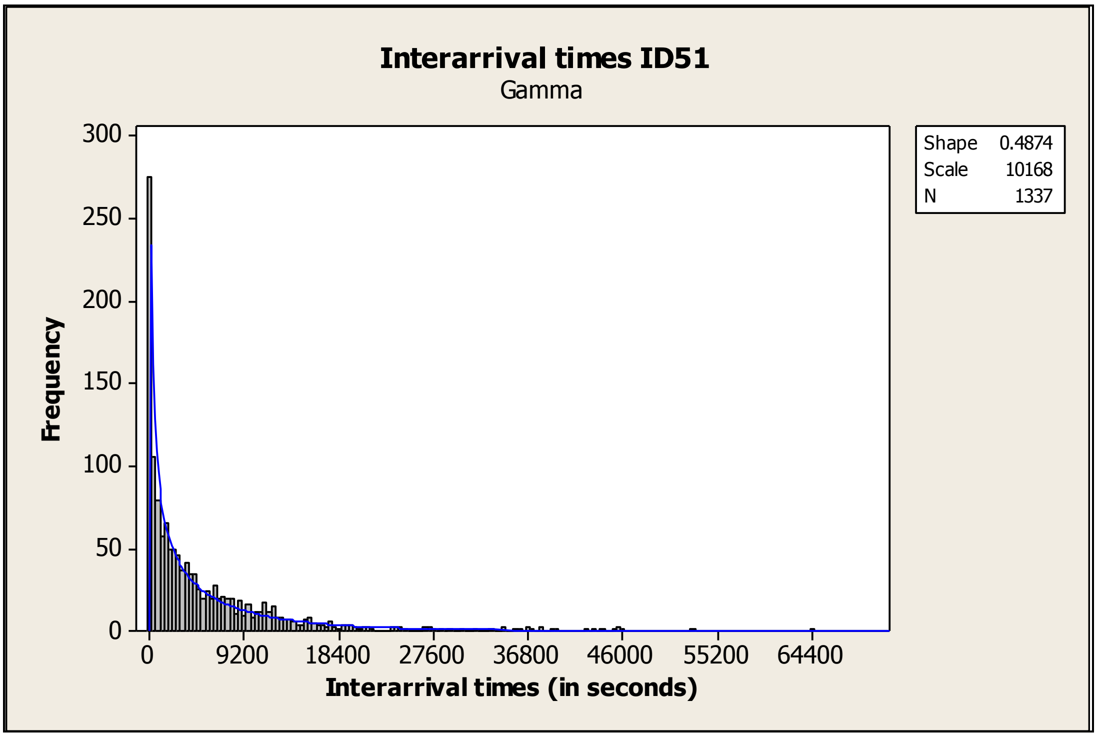

Figure 3.2: Histogram with Gamma fit for interarrival times of item ID51

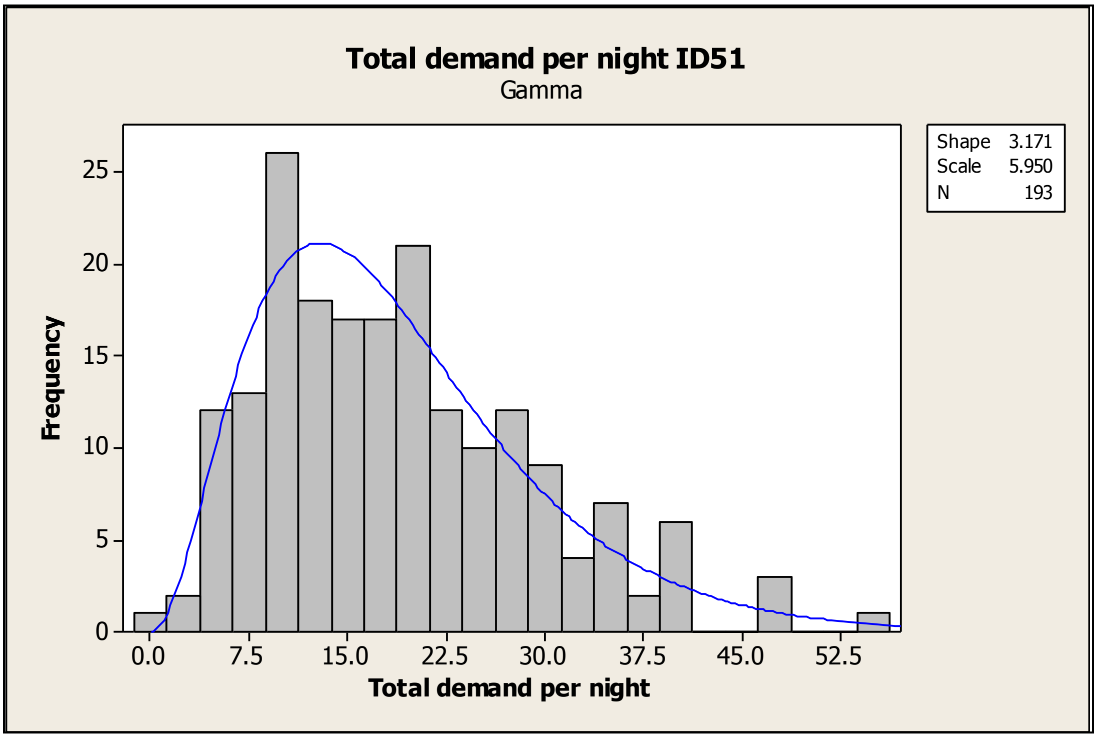

Figure 3.3: Histogram with Gamma fit for item ID51 total demand per night

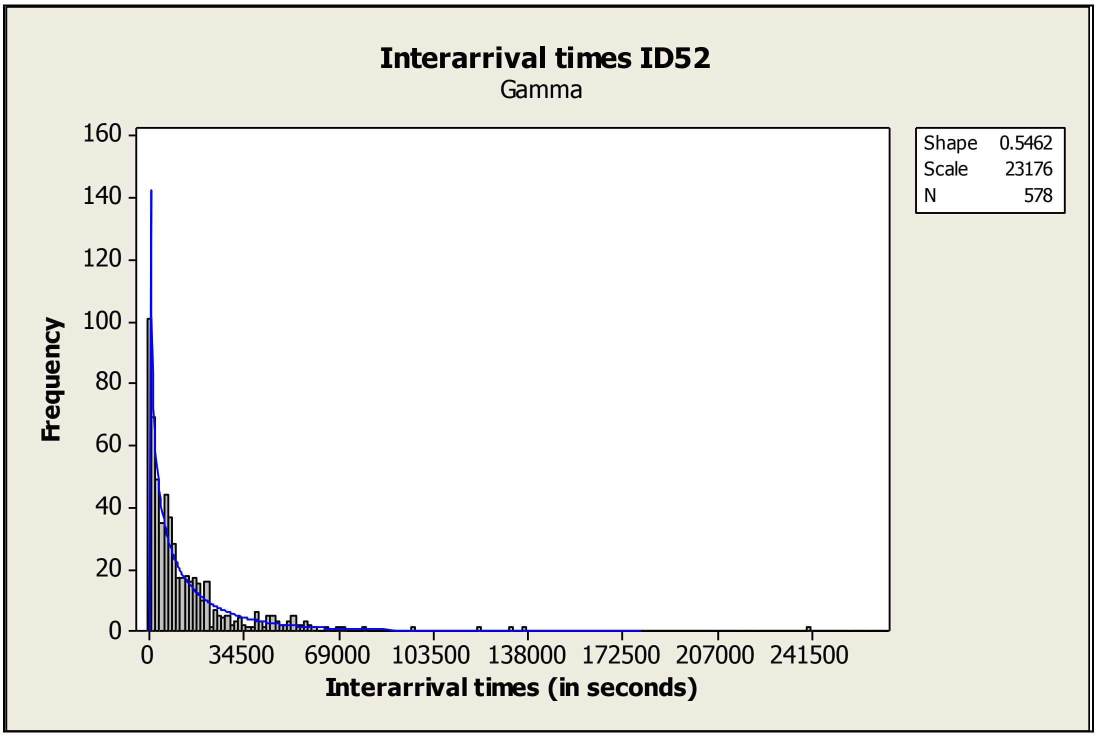

Figure 3.4: Histogram with Gamma fit for interarrival times of item ID52

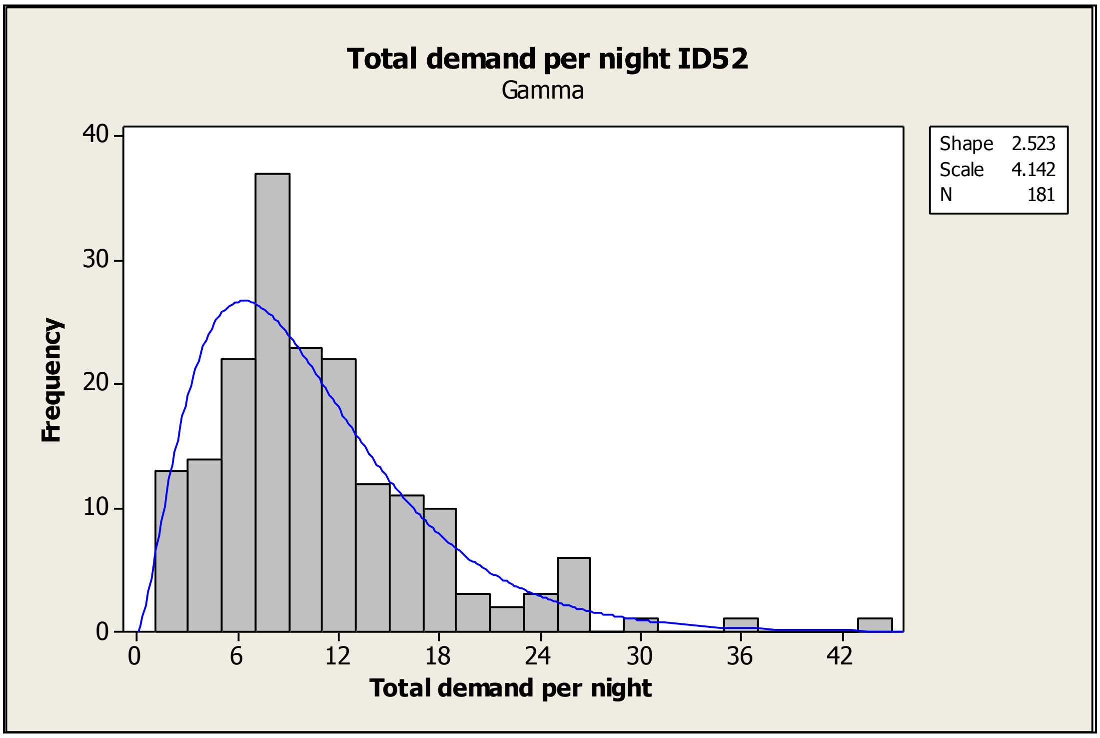

Figure 3.5: Histogram with Gamma fit for item ID52 total demand per night

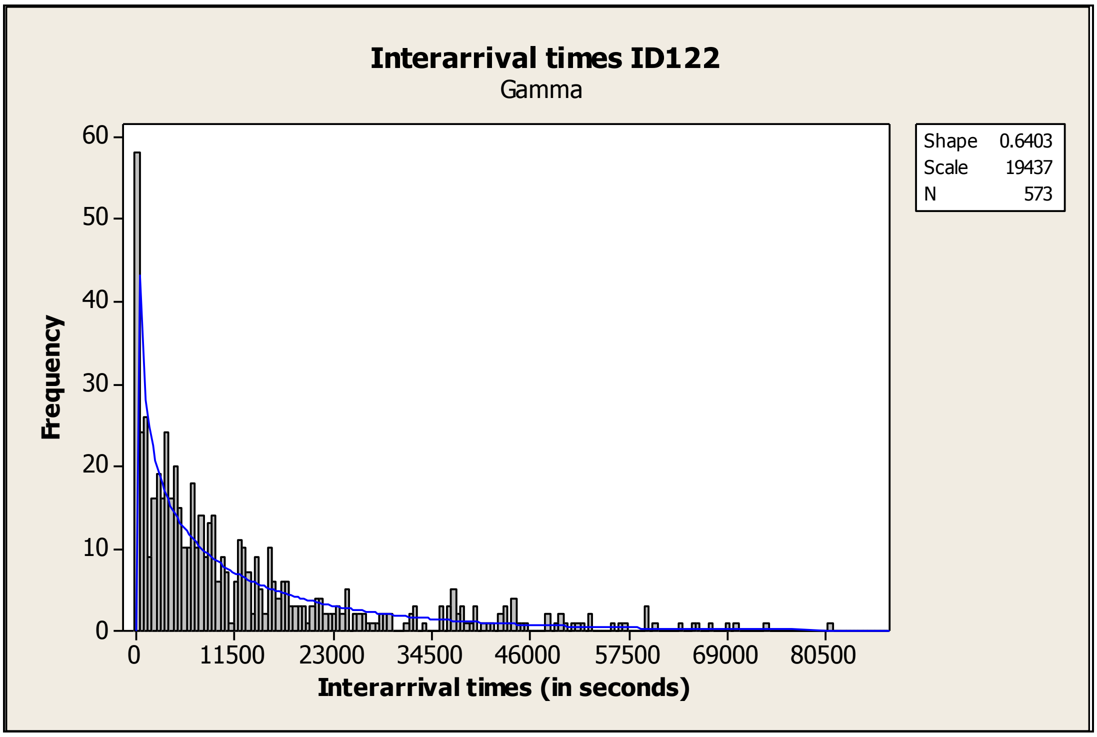

Figure 3.6: Histogram with Gamma fit for interarrival times of item ID122

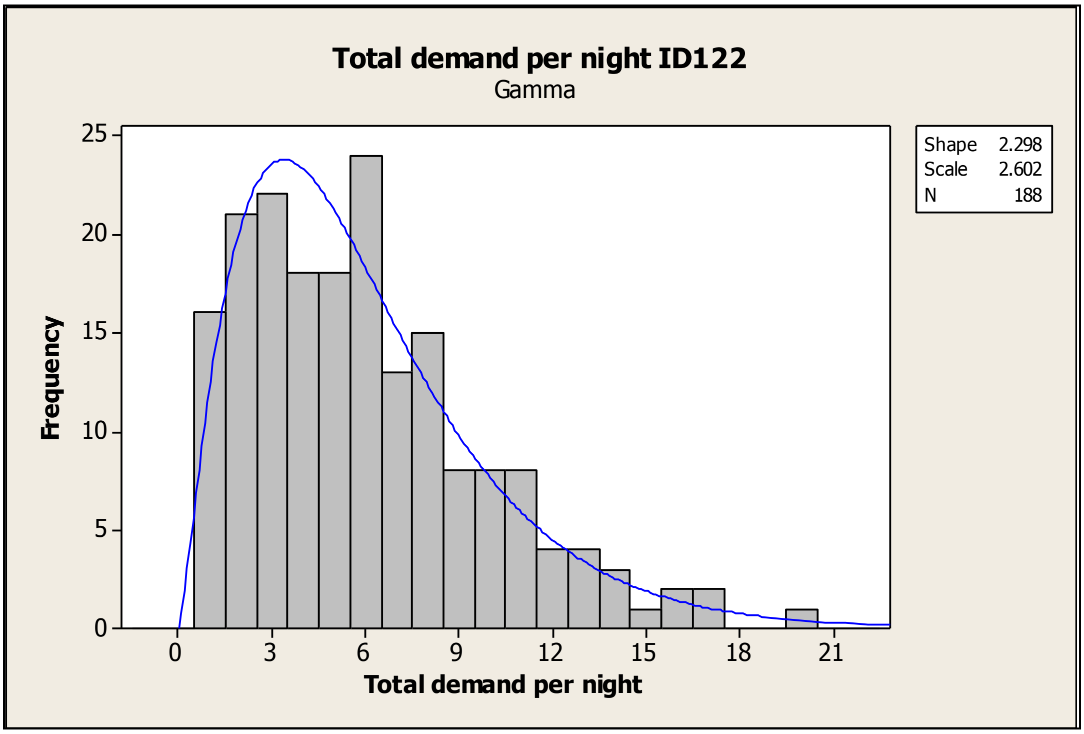

Figure 3.7: Histogram with Gamma fit for item ID122 total demand per night

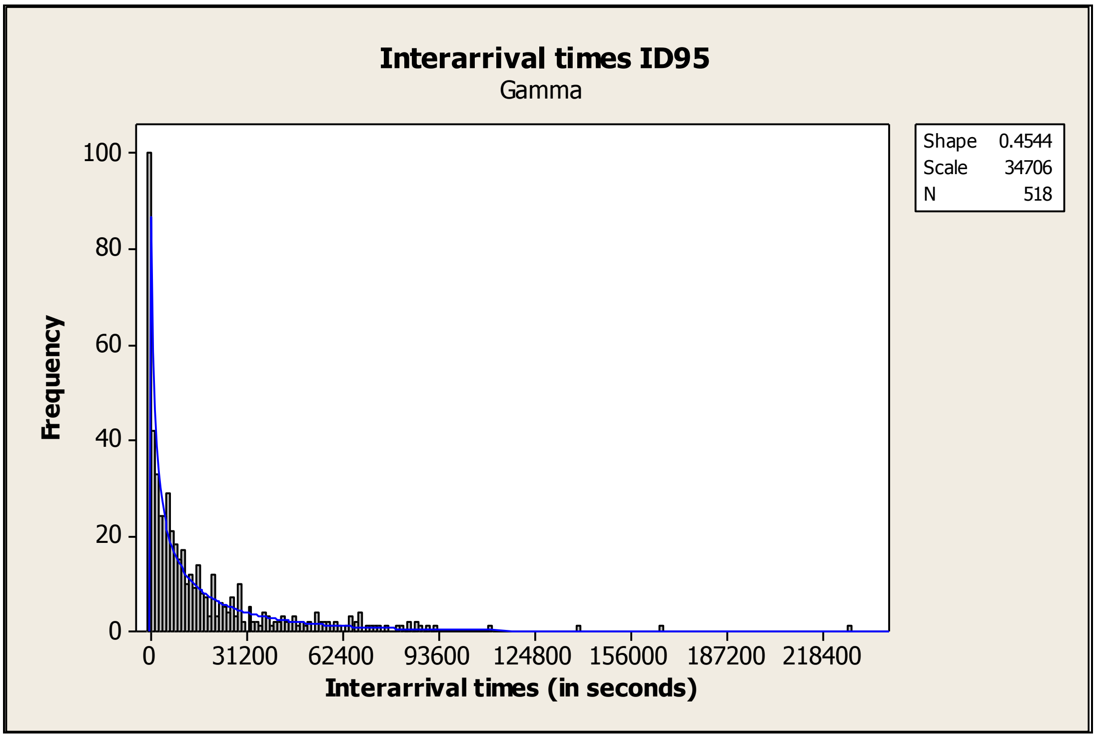

Figure 3.8: Histogram with Gamma fit for interarrival times of item ID95

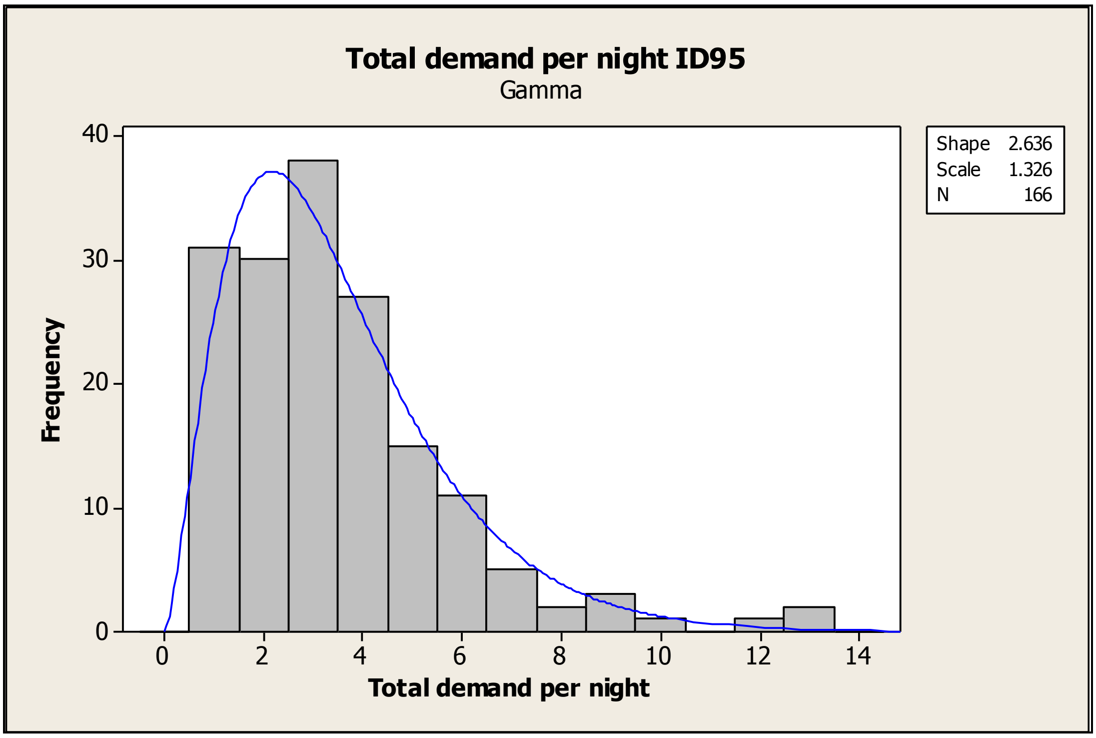

Figure 3.9: Histogram with Gamma fit for item ID95 total demand per night

Among the B items, we analyze the demand structures of the items ID75 and ID112. These items' day-time demand interarrivals are also best fitted to the Gamma distribution with shape parameters 0.8320 and 0.5395, and scale parameters 125077 and 147852 respectively. The related histograms can be seen in Figures 3.10 and 3.11. The demand structures of these items also include compound parts, whose empirical distributions are shown in Table 3.4. The total demands per night distributions for these items are fitted to the empirical distributions as well. In Table 3.5 the probabilities of observing certain amounts of demand at each night are given.

Table 3.4: Distributions of compound parts for B items day-time demands

	Probability Distributions	
Demand per arrival	ID75	ID112
1	0.88	0.93
2	0.10	0.06
3	0.02	-
4	-	0.01

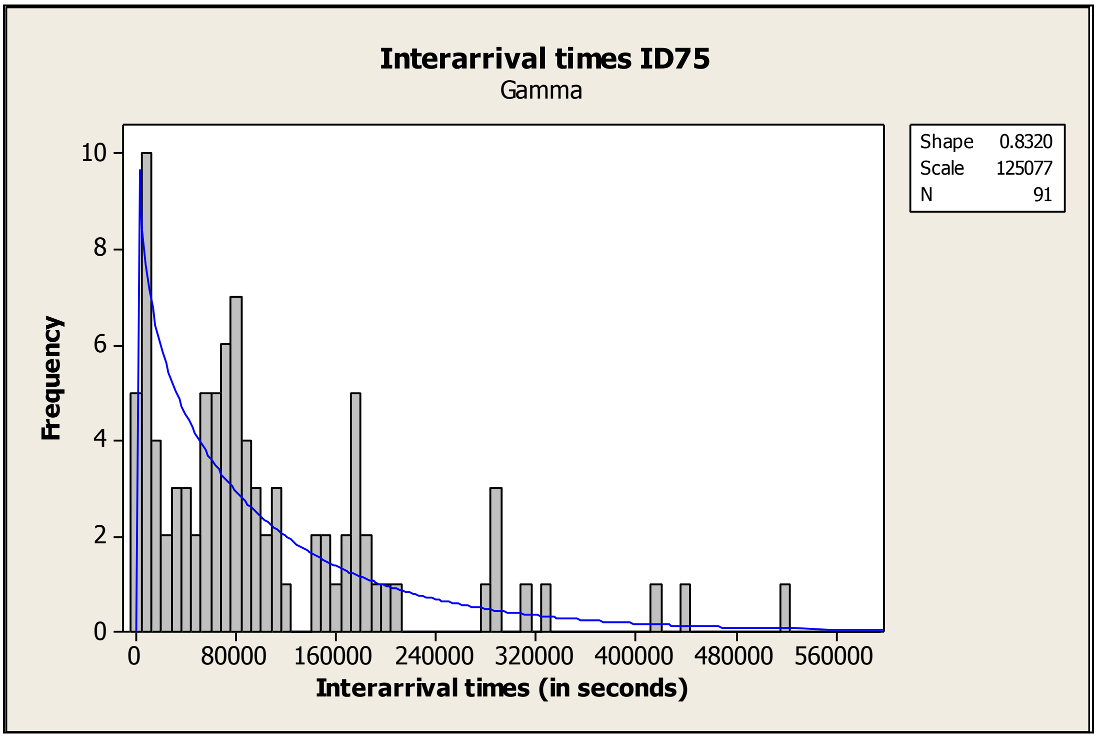

Figure 3.10: Histogram with Gamma fit for interarrival times of item ID75

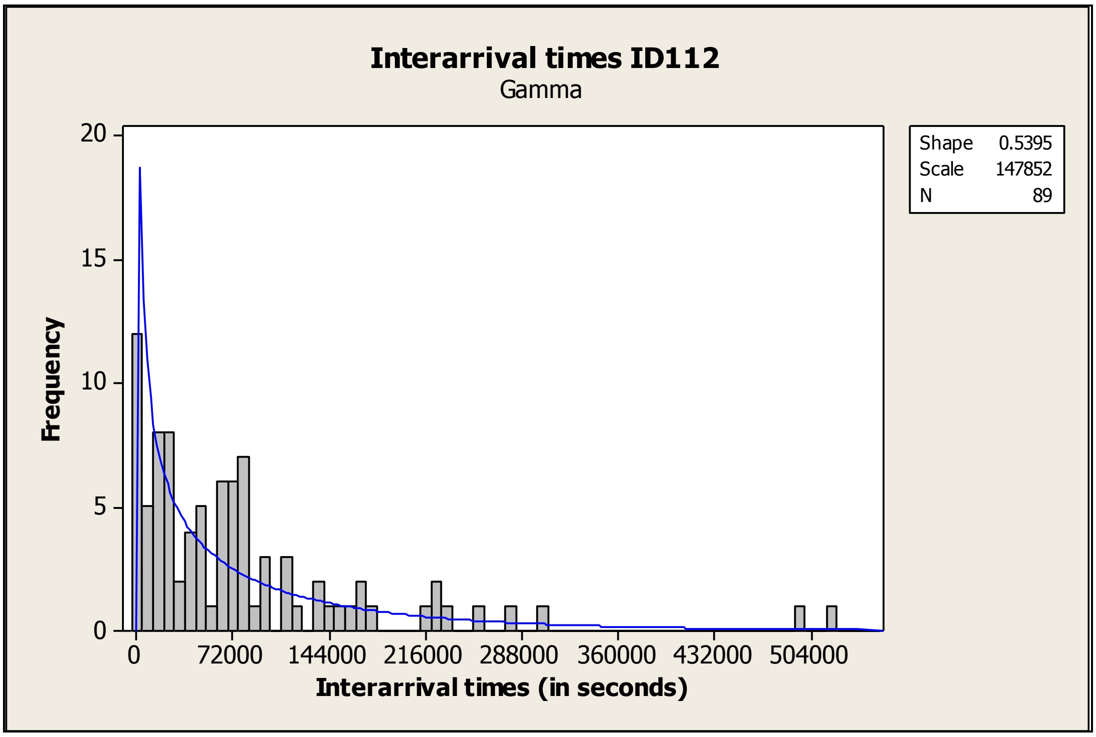

Figure 3.11: Histogram with Gamma fit for interarrival times of item ID112

Table 3.5: Distributions of night-time demands for B items

	Probability Distributions	
Demand per night	ID75	ID112
0	0.71	0.61
1	0.18	0.32
2	0.07	0.06
3	0.01	0.01
4	0.01	-
6	0.01	-
10	0.01	-

To sum up, in this Chapter we analyze the six months transaction data of the nursing station at Block A 3rd Floor. We classify the items into three classes according to total demand proportion of each item within the inventory system. Then, after obtaining the total daily transaction of each item, we find the daily demand distributions, which can be either a known distribution or an empirical distribution, whichever fits the best. Moreover, in order to use in the numerical analysis in Chapter 5, we obtain the best fitted distribution for the interarrival times during the day-time (i.e. time between each transaction during a day). For each item, we find the distribution of total demand per night for using in the numerical analysis. For this analysis, we also find the distribution of the compounding part of demand for each item (i.e. number of items retrieved in each transaction).

Chapter 4

Model and Policies

4.1 Definition of the Inventory Problem

We consider a multi-item two echelon inventory system where the upper echelon is a central warehouse and the lower echelon involves smaller depots which are nursing stations. We assume an ample supply at the central warehouse which is capable of meeting the demand of nursing stations whenever necessary. The central warehouse operates during a certain period of time of any given day, and is closed during the remaining times. Due to such on- and off-periods of the central warehouse, it is possible to replenish the nursing stations during the day-time on a continuous scale but it is not possible to make any replenishment during the night-time.

In our setting, satisfying target service levels (i.e. fill rates) is the prior performance measure rather than the cost measure, while making inventory control decisions. Note that this priority is the natural choice for hospital operations. Nevertheless, we still aim to minimize average inventory levels, because of the capacity of the nursing stations and the inevitable cost considerations. In other words, we want to make sure that the average proportion of demand that cannot be met during day-time and night-time should be less than a specified level with minimum average inventory levels.

During the first time frame (day-time), demand can be observed at anytime and a capacitated cart is used for moving the items from upper echelon (the central warehouse) to the lower echelon (nursing stations) whenever a replenishment occurs. We denote the replenishment lead time by L which starts at the time that an order is given until it is received by the nursing station.

During the second time frame (night-time), demand can still be observed at anytime at the nursing stations. However, the upper echelon is closed during this time frame and for this reason replenishment cannot take place. This particular situation makes it necessary to treat the day-time and night-time demand separately. Since replenishment is possible at anytime during the day-time and achieving high service levels targets is the main concern, controlling inventory at a continuous basis during the day-time would be the most logical choice. However, since replenishment is not possible during the night-time, the inventory control for this time period can only be made at a periodic basis. In particular, a one time decision is made for the whole night every day. Consequently, the day-time and night-time demands should be modeled and handled differently. We assume a compound renewal type demand, i.e. the interarrival times of the demand instances and compounding demand per arrival are both random variables, for the day-time. On the other hand, the whole night-time demand, which is also a random variable, is denoted by a single probability mass distribution.

Consider a 24-hour period starting at some time t_1. This starting time, t_1, also marks the start of the first time frame (day-time). The first time frame ends at time $t_2$which also marks the start of the second time frame (night-time). This second time frame ends at time t_1 of the next 24-hour period. Let X_D be the random variable denoting the compounding part of the demand observed at a demand instance during day-time, Y be the random variable denoting the interarrival times of the demand during day-time, and X_N be the random variable denoting the accumulated total night-time demand between t_2 and t_1.

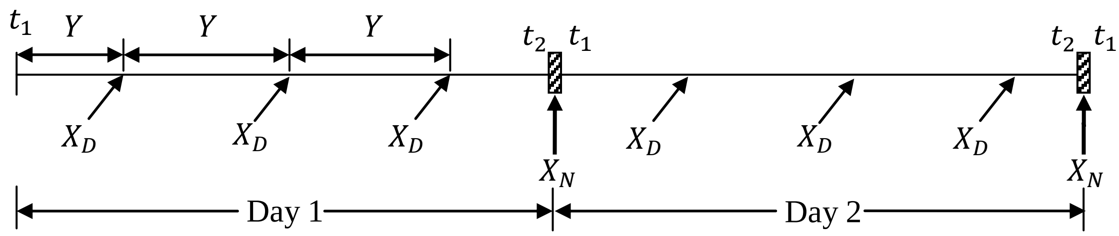

Figure 4.1: Demand structure

Within the scope of our problem, we assume that the medical supplies are independent of each other. Note also that the total demand during night-time is always greater than or equal to the total demand observed during lead time for each medical supply in our particular context. Considering these assumptions and the demand structure illustrated in Figure 4.1, we aim to find an appropriate inventory control policy, which leads high service levels for both day-time and night-time demand, while minimizing the average inventory levels.

4.2 Solution Approaches

Having two structurally different demand streams, as day-time and night-time demands, yields a different inventory control problem than the typical ones. One possible approach for this situation could be to obtain a non-stationary single demand distribution by interlacing these two demand streams and search for the optimal inventory control policies accordingly. By using this aggregated distribution, we may find the optimal inventory control policies for this system; nevertheless, this approach involves difficulties in terms of analyzing the system itself, since the resulting distribution will have a time-dependent non-stationary structure. In this thesis, we aim to find policies which are easy-to-implement and whose parameters are easy to find for each medical supply.

4.2.1 Our Proposed Control Policy

In our system, using a stationary inventory control policy with same parameters for day-time and night-time is not an appropriate way to satisfy the high service levels with low inventory levels. Because, this may cause high inventory levels during the day-time in order to meet the night-time demand or on the contrary it may lead to lower inventory levels but at the same time it may cause failure to satisfy the high service levels during night-time. Thus, separate control policies or same policy with different parameters for day-time and night-time demand can be used in order to make sure that our operational targets are satisfied.

In order to take action before observing stockout at the medical supplies during day-time, continuous review policies would be appropriate. Among the continuous review policies, we search for the ones that are suitable for environments where high service levels are required and a limit on the order size is imposed. For this reason, policies which only consider high service levels but do not take limit on the order size into account, such as $(\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{S})$ and $Q(\boldsymbol{s}, \boldsymbol{S})$ are eliminated (Tanrikulu et. al., 2009). Since $(Q, \boldsymbol{S})$ and $(\boldsymbol{s}, Q)$ policies take the capacity and utilization of that cart into account, we take these policies as the candidate policies. The $(Q, \boldsymbol{S})$ policy employs a fixed order amount Q at each replenishment. Nevertheless, as it is mentioned by Tanrikulu et. al. (2009), it does not take each item's inventory levels into consideration; instead under this policy replenishment is made when the total demand of all items reaches Q. Therefore, this policy cannot respond quickly, if one item's inventory level drops below zero, before the total consumption reaches Q. Another policy, which utilizes a constant order at each replenishment, is the $(\boldsymbol{s}, Q)$ policy. This policy can also satisfy high service levels without carrying excess amount of inventory. Tanrikulu et. al. (2009) points out that this policy outperforms $(Q, \boldsymbol{S})$ policy especially when the high service levels are required and lead times are low. This comparison fits exactly our case when we think backorder costs as service level requirements in our problem.

Consequently, we propose $(\boldsymbol{s}, Q)$ policy to be used for day-time inventory control. Under this policy, inventory positions of all items are reviewed continuously and when one item's inventory position drops to its reorder point s, a fixed order Q is given. In this joint order, each item's order size is determined by a heuristic allocation method, which is also used by Tanrikulu et.al. (2009), with the aim of balancing each item's inventory position in excess of the reorder point. Let IP_i be the inventory position of item i at any time and s_i be the reorder level of item i. By using this allocation method, each item's inventory position (IP_i) is brought above its reorder level (s_i). Moreover, since we use a capacitated cart for the replenishment and we want it to be fully utilized, we allocate the remaining amount of Q in a way that we balance each item's inventory position in excess of its reorder level. We allow for ordering more than one Q if it is necessary at an ordering instance. With this in mind, our proposed allocation method is given below.

for $i = 1{:}\,n$

    if $IP_i < s_i$

        $orderamount(i) = s_i - IP_i$

    else

        $orderamount(i) = 0$

end

$fixedorders = roundup[(\mathrm{sum}_i\, orderamount(i))/Q]$

$remainingQ = fixedorders * Q - (\mathrm{sum}_i\, orderamount(i))$

while $remainingQ \geq 1$

    $\min\{IP_1 - s_1, \ldots, IP_i - s_i\} = \min\{IP_1 - s_1, \ldots, IP_i - s_i\} + 1$

    $remainingQ = remainingQ - 1$

end

In order to satisfy the night-time demand, there is no need to hold inventory in advance throughout the day-time, but giving an order, just one lead time before the night-time starts would be sufficient. Thus, we suggest a periodic review policy for night-time inventory control. In other words if the replenishment lead time L is one hour, then controlling the inventory level and giving order if necessary at $t_2 - L$ will be appropriate for both satisfying night-time service levels and holding less inventory. For the night-time inventory control, we propose $(T, \boldsymbol{S})$ policy, where $T = t_2 - L$. By using this policy for the night-time demand, the necessary amount of order, that is required to bring inventory positions up to each item's S, is given just a lead time before the night-time demand is observed. Thus, without holding inventory for a long time, high service levels can be satisfied for the night-time.

To sum up, we propose a hybrid policy, which involves $(\boldsymbol{s}, Q)$ continuous review policy for the day-time demand and $(T, \boldsymbol{S})$ periodic review policy for the night-time demand. Note that, in the multiple items case, $\boldsymbol{s}$ of the $(\boldsymbol{s}, Q)$ policy and $\boldsymbol{S}$ of the $(T, \boldsymbol{S})$ policy are vectors of numbers corresponding to the reorder and order-up-to points of items respectively. In the remainder, we do not include an index for items for brevity but we note that each expression is valid for each item. In Chapter 5, we will show how we can find the exact or estimated policy parameters for this hybrid policy in order to satisfy the specified service levels.

4.2.2 A Special Case: Single-Item

In this section, we analyze a special case of our system: the single-item case. We explain our approach for an item observing renewal type individual demand but this approach can also be extended to compound demand situations. We develop exact expressions that can be used to calculate different performance measures such as total expected cost, service levels, etc. When there is only one item in the system, then the proposed $(\boldsymbol{s}, Q)$ policy becomes the well known (r, Q) policy. In this special case, we treat the night-time demand in the same way as it is in the multi-item case, hence we suggest (T, S) policy to be used for the night-time inventory control. Summary of the notation used in the remainder is given in Table 4.1.

Table 4.1: Notation

r	:	Reorder point for the day-time replenishment
Q	:	Fixed order amount for the day-time replenishment
T	:	Periodic review instance for the night-time replenishment
S	:	Order up to level for the night-time replenishment
L	:	Replenishment lead time
t_1	:	The start of the day-time
t_2	:	The start of the night-time
X_D^L	:	Random variable denoting the demand during lead time
X_N	:	Random variable denoting the demand during night
$N(t)$	:	Amount of demand observed by time t
$IP(t)$	:	Inventory position at time t
$IL(t)$	:	Inventory level at time t
$IL(t_2)^-$	:	Inventory level at time t_2 after the order arrives for the night and before the night's demand is observed i.e. $IL(t_2)^- = S - X_D^L$
$IL(t_2)^+$	:	Inventory level at time t_2 after the order arrives for the night and night's demand is observed i.e. $IL(t_2)^+ = S - X_D^L - X_N$

As explained in the previous section, at time $T = t_2 - L$, the inventory position of the item is raised up to S. Due to the nature of the system, no orders are placed during $(t_2 - L, t_2)$ and all outstanding orders at time $t_2 - L$ would arrive until time t_2. Therefore, the inventory level at time t_2 before the night-time demand is observed, $IL(t_2)^-$, is given by $S - X_D^L$. Afterwards, the total demand for night-time (X_N) is observed which causes the inventory level at t_1 to drop to the value $IL(t_2)^+ = IL(t_2)^- - X_N$ and next day's starting inventory position to take the value $IP(t_1)$ which is equal to $IL(t_2)^+$ before any order is given in the next day as shown at Figures 4.2a and 4.2b. We analyze two possible scenarios where the actions taken at time t_1 are different. In Figure 4.2a observed night-time demand is so low that the beginning inventory position is greater than the reorder point $(IP(t_1) > r)$ and for this reason no order is given at the beginning of the next day.

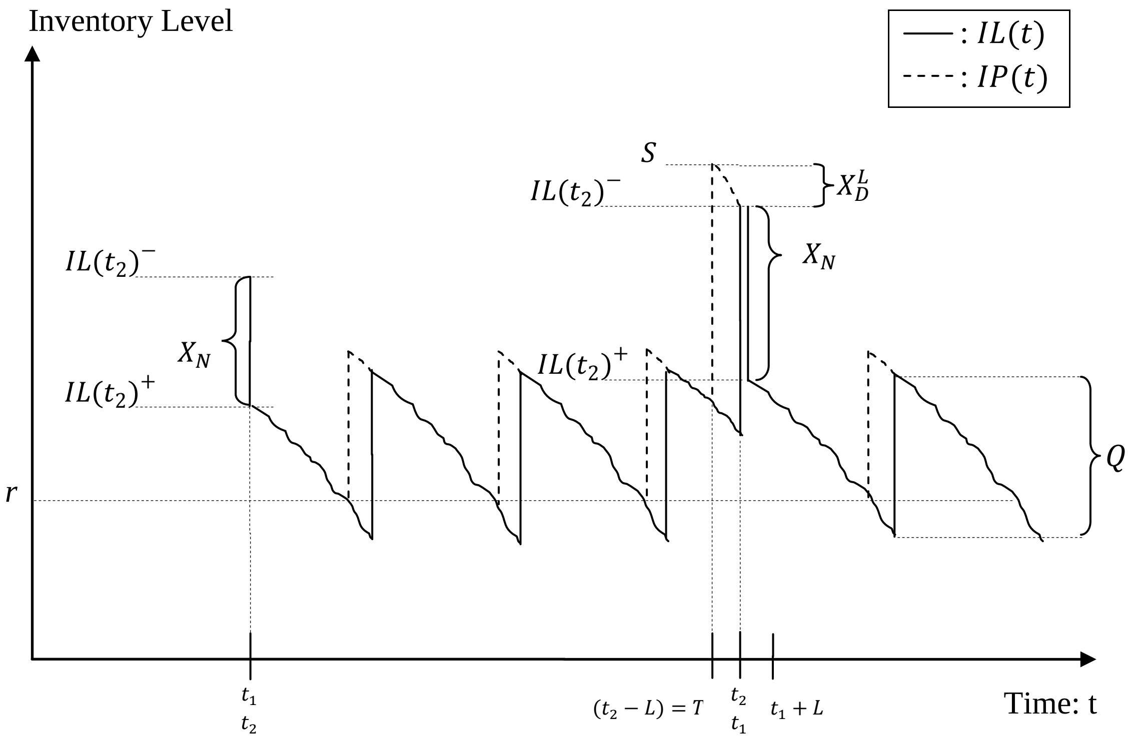

Figure 4.2a: First example for the behavior of inventory level over time

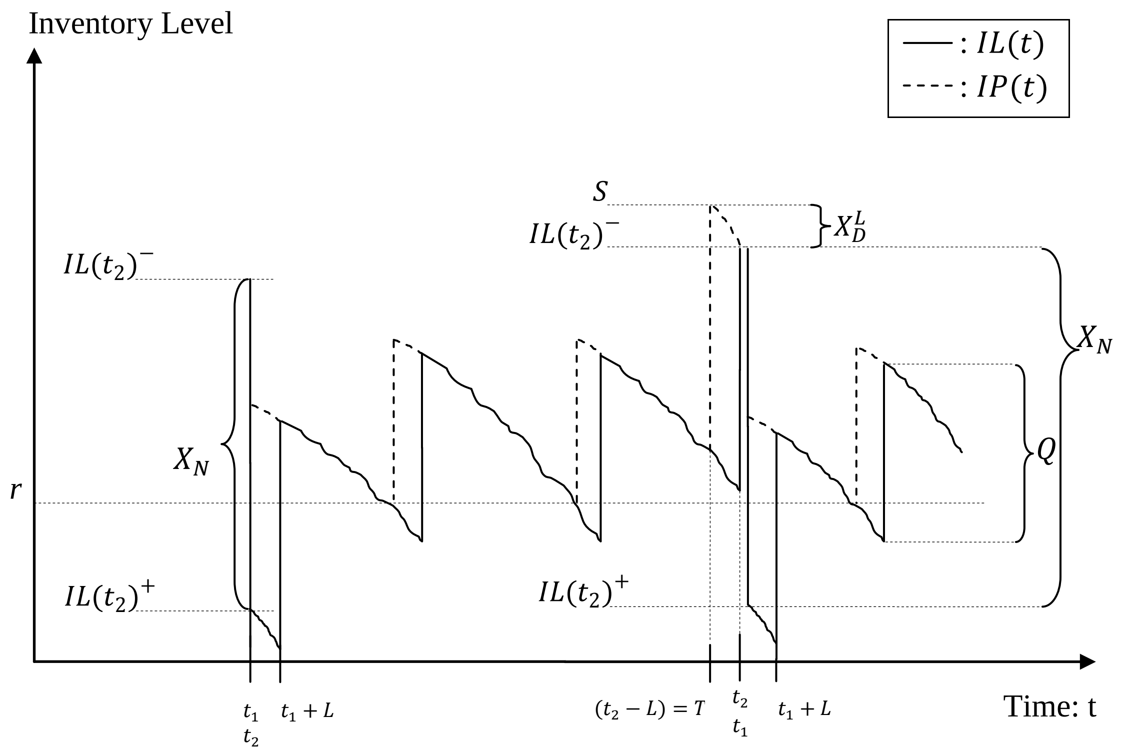

Figure 4.2b: Second example for the behavior of inventory level over time

On the other hand, in Figure 4.2b total demand observed during night-time is high enough that the inventory position at the beginning of the day (t_1) drops below the reorder point ($IP(t_1) < r$) and an order is given at t_1, in order to bring the inventory position to or over r. After the ordering is made at time t_1, the inventory position will take a value between $r+1$ and $r+Q$. After this, regular (r,Q) policy is implemented until time $t_2 - L$ at which the inventory position is raised to S again for the following night-time.

Rather than modeling this particular inventory system as a whole on an infinite horizon continuous time scale, we model the inventory system for every “day-time” (the time range from t_1 to t_2) separately, where the (r,Q) policy is employed on a continuous time scale. Note that each “day-time” is stochastically equivalent to each other. In this model, we also explicitly consider the effect of the (T,S) policy. We first develop exact expressions to estimate the probability distributions of the inventory position at any given time t where $t_1 \leq t \leq t_2$ by using a transient analysis, and then this information is used to find the probability distributions of the inventory levels at any time t.

When the system is controlled by the (r,Q) policy, $IP(t)$ takes a value between $r+1$ and $r+Q$. A typical inventory system operated with the (r,Q) policy can be modeled as continuous time Markov Chain by defining the states of the system as the inventory position. Let $IP(t) = r + k$ for some time t. Whenever a demand arrival occurs after time t, the system will move to the state $r+k-1$ if $1 < k \leq Q$ and to the state $r+Q$ if $k = 1$. Suppose that $IP(\tau_1) = r + k$ at a given time τ_1 and for some k such that $1 \leq k \leq Q$. Then the probability that $IP(\tau_2)$ is equal to $r+l$ at another given time τ_2 and for some l such that $1 \leq l \leq k$, is the probability that there are $k-l$ or $k-l+Q$ or $k-l+2Q$ or ... demand arrivals during the time interval $\tau_2 - \tau_1$. Similarly, the probability that $IP(\tau_2)$ is equal to $r+l$ for some $k \leq l \leq Q$, is the probability that there are $k-l+Q$ or $k-l+2Q$ or ... demand arrivals during the time interval $\tau_2 - \tau_1$.

The case for $1 \leq l \leq k$ is illustrated in Figure 4.3. In this figure, the nodes represent the inventory positions during the time interval $\tau_2 - \tau_1$. After starting at $IP(\tau_1) = r + k$, in order to end at $IP(\tau_2) = r + l$, the first possibility for the total demand arrivals during $\tau_2 - \tau_1$, is $k - l$, and is shown by the arcs, which are marked as “1”. As the second possibility, the total demand may be equal to $k - l + Q$ and this situation can be obtained by starting with arcs “1” and continue with one full cycle by using arcs “2”. The third possibility, which is $k - l + 2Q$, is shown by the arcs “1”+“2”+“3”, in other words this possibility consists of arcs “1” followed by two full cycles.

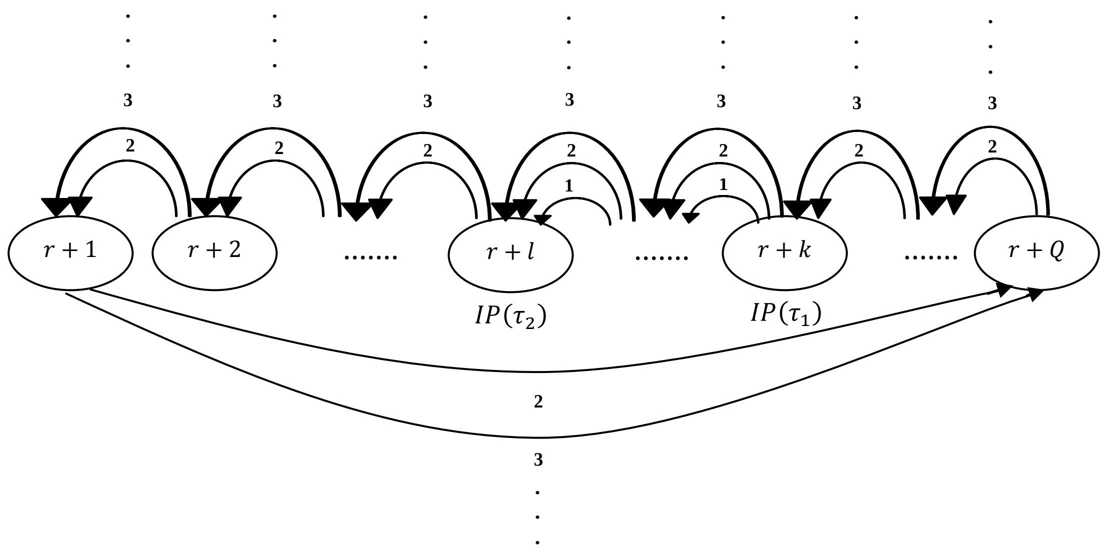

Figure 4.3: Illustration of the demand arrivals within $\tau_2 - \tau_1$ for $1 \leq l \leq k$

The boundary conditions (initial conditions) for this model is given by the probability distribution of the inventory position at time t_1, i.e. $IP(t_1) = IL(t_1)^+$. The starting point of each day-time interval is t_1 and $IP(t_1)$ can be obtained as follows. By conditioning on the smallest number of orders that can bring the inventory position at the beginning of the day to or over the value r, $P\{IP(t_1) = r + Q - i\}$ can be found.

if $IL(t_1)^+ = S - X_D^L - X_N \geq r$ then we do not give any order

$$\begin{aligned} P\{IP(t_1) = r + Q - i\} &= P\{S - X_D^L - X_N = r + Q - i\} \\ &= P\{X_D^L + X_N = S - r - Q + i\} \end{aligned}$$

if $IL(t_1)^+ = S - X_D^L - X_N < r$

if $S - X_D^L - X_N + Q \geq r$ then we order Q

$$P\{IP(t_1) = r + Q - i\} = P\{S - X_D^L - X_N + Q = r + Q - i\}$$
$$= P\{X_D^L + X_N = S - r + i\}$$

if $S - X_D^L - X_N + Q < r \leq S - X_D^L - X_N + 2Q$ then order 2Q

$$P\{IP(t_1) = r + Q - i\} = P\{S - X_D^L - X_N + 2Q = r + Q - i\}$$
$$= P\{X_D^L + X_N = S - r + Q + i\}$$

if $S - X_D^L - X_N + 2Q < r \leq S - X_D^L - X_N + 3Q$ then order 3Q

$$P\{IP(t_1) = r + Q - i\} = P\{S - X_D^L - X_N + 3Q = r + Q - i\}$$
$$= P\{X_D^L + X_N = S - r + 2Q + i\}$$

We can generalize this structure where $k \in \{0,1,..,\infty\}$

$$P\{IP(t_1) = r + Q - i | (S - X_D^L - X_N + (k-1)Q < r \leq S - X_D^L - X_N + kQ)\}$$
$$= P\{IP(t_1) = r + Q - i \mid X_D^L + X_N > S + (k-1)Q - r,\ X_D^L + X_N \leq S + kQ - r\}$$
$$= P\{X_D^L + X_N = S - r + (k-1)Q + i\} \tag{4.1}$$

$$P\{IP(t_1) = r + Q - i\}$$
$$= \sum_{k=0}^{\infty}[P\{IP(t_1) = r + Q - i \mid X_D^L + X_N > S + (k-1)Q - r,\ X_D^L + X_N \leq S + kQ - r\}\, P\{X_D^L + X_N > S + (k-1)Q - r, X_D^L + X_N \leq S + kQ - r\}] \tag{4.2}$$

By substituting Equation (4.1) into (4.2), we obtain:

$$P\{IP(t_1) = r + Q - i\} = \sum_{k=0}^{\infty} P\{X_D^L + X_N = S - r + (k-1)Q + i\}P\{X_D^L + X_N > S + (k-1)Q - r, X_D^L + X_N \leq S + kQ - r\} \tag{4.3}$$

By using this information and the above discussion, we can calculate the inventory position of the item at any given time t as follows.

For the case when $IP(t_1) \leq r + Q$ we drive the equations as follows:

$$P\{IP(t) > r + Q | IP(t_1) \leq r + Q\} = 0. \tag{4.4}$$

Assume $IP(t_1) = r + Q$

$P\{IP(t) = r + Q\} = \sum_{n=0}^{\infty} P\{N(t) = nQ\}$

$P\{IP(t) = r + Q - 1\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + 1\}$

$P\{IP(t) = r + Q - 2\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + 2\}$

...

$P\{IP(t) = r + 2\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + Q - 2\}$

$P\{IP(t) = r + 1\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + Q - 1\}$

Assume $IP(t_1) = r + Q - 1$

$P\{IP(t) = r + Q\} = \sum_{n=0}^{\infty} P\{N(t) = nQ - 1\}$

$P\{IP(t) = r + Q - 1\} = \sum_{n=0}^{\infty} P\{N(t) = nQ\}$

$P\{IP(t) = r + Q - 2\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + 1\}$

...

$P\{IP(t) = r + 2\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + Q - 3\}$

$P\{IP(t) = r + 1\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + Q - 2\}$

We can generalize these expressions for $IP(t_1) \leq r + Q$ where $m \in \{0,1, \dots, Q\}$ and $i \in \{0,1, \dots , Q\}$ as:

$$P\{IP(t) = r + Q - m | IP(t_1) = r + Q - i\} = \sum_{n=0}^{\infty} P\{N(t) = nQ - i + m\}. \quad (4.5)$$

For the case when $IP(t_1) > r + Q$ the structure of $IP(t)$ is different than the previous case.

Assume $IP(t_1) = r + Q + 1$

$P\{IP(t) = r + Q + 1\} = P\{N(t) = 0\}$

$P\{IP(t) = r + Q\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + 1\}$

$P\{IP(t) = r + Q - 1\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + 2\}$

...

$P\{IP(t) = r + 2\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + Q - 1\}$

$P\{IP(t) = r + 1\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + Q\}$

Assume $IP(t_1) = r + Q + 2$

$P\{IP(t) = r + Q + 2\} = P\{N(t) = 0\}$

$P\{IP(t) = r + Q + 1\} = P\{N(t) = 1\}$

$P\{IP(t) = r + Q\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + 2\}$

$P\{IP(t) = r + Q - 1\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + 3\}$

...

$P\{IP(t) = r + 2\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + Q\}$

$P\{IP(t) = r + 1\} = \sum_{n=0}^{\infty} P\{N(t) = nQ + Q + 1\}$

We generalize these expressions for $IP(t_1) > r + Q$.

For $m \in \{0,1, \dots, Q\}$ and $i \in \{-\infty, \dots, -1\}$:

$$P\{IP(t) = r + Q - m | IP(t_1) = r + Q - i\} = \sum_{n=0}^{\infty} P\{N(t) = nQ - i + m\}. \quad (4.6)$$

For $m \in \{i, \dots, -1\}$ and $i \in \{-\infty, \dots, -1\}$:

$$P\{IP(t) = r + Q - m | IP(t_1) = r + Q - i\} = P\{N(t) = m - i\}. \quad (4.7)$$

Actually, since $P\{N(t) < 0\} = 0$ for any distribution we can modify the interval for m in Equation (4.7) as $m \in \{-\infty, \dots, -1\}$.

In general, Equations (4.1)-(4.4) can be combined and rewritten as:

$$P\{IP(t) = r + Q - m | IP(t_1) = r + Q - i\} = \sum_{n=0}^{\infty} P\{N(t) = nQ - i + m\}$$

for $m \in \{0,1, \dots, Q\}$ and $i \in \{-\infty, \dots, Q\}$

$$P\{IP(t) = r + Q - m | IP(t_1) = r + Q - i\} = P\{N(t) = m - i\}$$

for $m \in \{-\infty, \dots, -1\}$ and $i \in \{-\infty, \dots, -1\}$

(4.8)

Now we can find $IP(t)$ by using Equation (4.8) conditioning on $IP(t_1)$ as follows:

$$P\{IP(t) = r + Q - m\} = \sum_{i=-\infty}^{Q} [P\{IP(t) = r + Q - m | IP(t_1) = r + Q - i\} \; P\{IP(t_1) = r + Q - i\}]$$

$$P\{IP(t)=r+Q-m\}=$$
$$\begin{cases}\sum_{i=-\infty}^{Q}\sum_{n=0}^{\infty}[P\{N(t)=nQ-i+m\}P\{IP(t_1)=r+Q-i\}] & \text{for } m\in\{0,1,\dots,Q\}\\ \sum_{i=-\infty}^{-1}\sum_{n=0}^{\infty}[P\{N(t)=m-i\}P\{IP(t_1)=r+Q-i\}] & \text{for } m\in\{-\infty,\dots,-1\}\end{cases} \quad (4.9)$$

By inserting Equation (4.3) into (4.9) the unconditional probability distribution of $IP(t)$ can be found.

$$P\{IP(t)=r+Q-m\}=$$
$$\begin{cases}\sum_{i=-\infty}^{Q}\sum_{n=0}^{\infty}[P\{N(t)=nQ-i+m\}P\{IP(t_1)=r+Q-i\}] \text{where } m\in\{0,1,\dots,Q\}\\ \sum_{i=-\infty}^{-1}\sum_{n=0}^{\infty}[P\{N(t)=m-i\}P\{IP(t_1)=r+Q-i\}] \quad \text{where } m\in\{-\infty,\dots,-1\}\end{cases}$$

$$=\begin{cases}\sum_{i=-\infty}^{Q}\sum_{n=0}^{\infty}P\{N(t)=nQ-i+m\}\sum_{k=0}^{\infty}[P\{X_D^L+X_N=S-r+(k-1)Q+i\}\ P\{X_D^L+X_N>S+(k-1)Q-r,X_D^L+X_N\le S+kQ-r\}] \\ \qquad \text{for } m\in\{0,1,\dots,Q\}\\ \sum_{i=-\infty}^{-1}\sum_{n=0}^{\infty}[P\{N(t)=m-i\}\sum_{k=0}^{\infty}[P\{X_D^L+X_N=S-r+(k-1)Q+i\}\ P\{X_D^L+X_N>S+(k-1)Q-r,X_D^L+X_N\le S+kQ-r\}]] \\ \qquad \text{for } m\in\{-\infty,\dots,-1\}\end{cases} \quad (4.10)$$

This analysis is applicable to any renewal demand structure. As an example, let the demand be Poisson distributed, then probability distribution of $N(t)$ can be rewritten as below.

$$P\{N(t)=nQ-i+m\}=\frac{e^{-\lambda t}(\lambda t)^{(nQ-i+m)}}{(nQ-i+m)!} \qquad \text{for } m\in\{0,1,\dots,Q\} \quad (4.11)$$

$$P\{N(t)=m-i\}=\frac{e^{-\lambda t}(\lambda t)^{(m-i)}}{(m-i)!} \qquad \text{for } m\in\{-\infty,\dots,-1\} \quad (4.12)$$

By substituting Equations (4.11) and (4.12) into Equation (4.10) we obtain the Inventory Position distribution for Poisson demand as follows:

$$P\{IP(t)=r+Q-m\}=$$

$$=\begin{cases}\sum_{i=-\infty}^{Q}\sum_{n=0}^{\infty}[\frac{e^{-\lambda t}(\lambda t)^{(nQ-i+m)}}{(nQ-i+m)!}\sum_{k=0}^{\infty}[P\{X_D^L+X_N=S-r+(k-1)Q+i\}\,P\{X_D^L+ \\ X_N>S+(k-1)Q-r,X_D^L+X_N\leq S+kQ-r\}]] \qquad \text{for } m\in\{0,1,\dots,Q\} \\ \\ \sum_{i=-\infty}^{-1}\sum_{n=0}^{\infty}[\frac{e^{-\lambda t}(\lambda t)^{(m-i)}}{(m-i)!}\sum_{k=0}^{\infty}[P\{X_D^L+X_N=S-r+(k-1)Q+i\}\;P\{X_D^L+ \\ X_N>S+(k-1)Q-r,X_D^L+X_N\leq S+kQ-r\}]] \qquad \text{for } m\in\{-\infty,\dots,-1\}\end{cases}$$

Now we can find $IL(t)$ by using $IP(t)$, where $IL(t)$ denotes the inventory level at time t such that $t_1\leq t\leq t_2$

$$P\{IP(t)=r+Q-m\}=P\{IL(t+L)=r+Q-m-X_D^L\}$$

The probability distribution of $IP(t)$ is defined for $t_1\leq t\leq(t_2-L)$ and by using this, the probability distribution of $IL(t)$ can be found at the interval $t_1+L\leq t\leq t_2$ as it is shown below.

$$P\{IL(t+L)=r+Q-m-X_D^L\}=$$

$$=\begin{cases}\sum_{i=0}^{Q}\sum_{n=0}^{\infty}P\{N(t)=nQ-i+m\}\sum_{k=0}^{\infty}P\{X_D^L+X_N=S-r+(k-1)Q+ \\ i\}\,P\{X_D^L+X_N>S+(k-1)Q-r,X_D^L+X_N\leq S+kQ-r\} \\ \qquad\qquad\qquad\qquad \text{for } m\in\{0,1,\dots,Q\} \\ \sum_{i=-\infty}^{-1}\sum_{n=0}^{\infty}[P\{N(t)=m-i\}\;\sum_{k=0}^{\infty}[P\{X_D^L+X_N=S-r+(k-1)Q+ \\ i\}\;P\{X_D^L+X_N>S+(k-1)Q-r,X_D^L+X_N\leq S+kQ-r\}]] \\ \qquad\qquad\qquad\qquad \text{for } m\in\{-\infty,\dots,-1\}\end{cases}$$

Now we need to define $IL(t')$ for $t_1\leq t'<t_1+L$. Let $X_D^{t'}$ denotes the random variable corresponding to demand observed in (t_1,t') interval.

$$IL(t_1)=S-X_D^L-X_N$$
$$IL(t')=S-X_D^L-X_N-X_D^{t'}\quad where\ t_1\leq t'<t_1+L.$$

By using these expressions exact analysis for the service levels can be made and the optimal policy parameters for (r, Q) and (T, S) can be found with specified service level targets.

In order to find optimal policy parameters with the aim of minimizing the expected total cost following equation can be solved where K denotes the fixed ordering cost, p denotes the penalty cost for each unsatisfied demand, and h denotes the inventory holding cost.

$$\min_{r,Q,S} E[Total\ cost] =$$
$$min_{\mathrm{r,Q,S}}\, K.E[Total\ number\ of\ orders\ in\ a\ day] + \int_{t_1}^{t_2} [p.IL(t)^{-} + h.IL(t)^{+}]\, dt \tag{4.13}$$

If the aim is to satisfy a given service level (β), then by using the following expression the optimal policy parameters can be found.

$$min\left\{r, Q, S: \frac{\int_{t_1}^{t_2} IL(t)^{-} dt}{E[Total\ demand\ in\ a\ day]} \leq (1-\beta)\right\} \tag{4.14}$$

Chapter 5

Policy Parameters Estimation

In this Chapter, we aim to estimate the policy parameters in a way that the average inventory level in the system is minimized. We propose methods to estimate the policy parameters close to the optimal values. We test our methods by simulating the inventory system in Arena under different scenarios in terms of demand distributions and item variety, and analyze the impact of the policy parameters on the service levels.

5.1 An Estimation Method to Find Policy Parameters

Recall from Chapter 4 that we propose the hybrid $(\boldsymbol{s}, Q) + (T, \boldsymbol{S})$ policy for the inventory problem under concern. Our main aim is to make sure that the proportion of unsatisfied demand is low enough to meet the required service levels. For this reason, we suggest to use Type 2 service level (β) expressions to find policy parameters.

For the single-item case, we propose (r, Q) policy for the day-time demand and $(T, \boldsymbol{S})$ policy for the night-time demand, as explained in Section 4.2.2. Either with the purpose of obtaining the minimum cost or satisfying certain service levels, the optimal policy parameters for these policies can be found by using the Equations (4.13) and (4.14) respectively, which are obtained by using transient analysis for inventory position as explained in Section 4.3.

For the multiple items case, estimating policy parameters is much more complicated. Tanrikulu et al. (2009) develop a Markov Chain that can be used to model the inventory positions realized under the $(\boldsymbol{s}, Q)$ policy. For n items, this Markov Chain has Q^n states. One might conduct a transient analysis for a multi item case similar to the single item. However, such an approach would require extensive and complex expressions to be solved. Therefore, finding policy parameters by using exact analysis is not straightforward in our problem setting considering the high number of items. By considering the computational difficulties of this analysis, we aim to propose a practical and easy-to-use method to estimate policy parameters for the multi-item case. We then validate our estimation method by comparing the estimated parameters with the optimal parameters found by simulating the inventory control system using Arena.

When the $(\boldsymbol{s}, Q) + (T, \boldsymbol{S})$ policies are implemented as explained, each of the policies affect the inventory and service levels of the two distinct time frames. First of all, Type 2 service level during the interval $(t_2, t_1 + L)$ is dictated by the $(T, \boldsymbol{S})$ policy. Recall that the inventory position of each item is raised to S at time $T = t_2 - L$ and there cannot be any replenishment within $(t_2 - L, t_2)$ time interval. Therefore, at the beginning of the night-time (at time t_2), the inventory level, $IL(t_2)^-$, is equal to $S - X_D^L$ for each item, which is also equal to the inventory position at time t_2. After the night-time demand is observed, an order is placed either immediately at t_1 or at some time later. In any case, the first order placed in day-time arrives at least a lead time later than the beginning of the next day (i.e. at time $t_1 + L$ or later). Thus, we aim to find the S value for each item in a way that we make sure that $IL(t_2)^-$ is high enough to satisfy both the demand during night-time and the demand during first lead time at the beginning of the next day with a certain probability. Type 2 service level for this time frame can be estimated by:

$$\frac{\sum_{i=-\infty}^{S}\sum_{x=i+1}^{\infty}(x-i)P\{X_N+X_D^L=x\}P\{S-X_D^L=i\}}{\sum_{j=-\infty}^{\infty}jP\{X_N+X_D^L=j\}}\leq 1-\beta$$

$$=\frac{\sum_{i=-\infty}^{S}\sum_{x=i+1}^{\infty}\sum_{d=0}^{\infty}(x-i)P\{X_N+X_D^L=x|X_D^L=d\}P\{X_D^L=d\}P\{S-X_D^L=i\}}{\sum_{j=-\infty}^{\infty}jP\{X_N+X_D^L=j\}}\leq 1-\beta$$

$$=\frac{\sum_{i=-\infty}^{S}\sum_{x=i+1}^{\infty}\sum_{d=0}^{x}(x-i)P\{X_N=x-d\}P\{X_D^L=d\}P\{X_D^L=S-i\}}{\sum_{j=-\infty}^{\infty}jP\{X_N+X_D^L=j\}}\leq 1-\beta$$

With this in mind, the following expression can be used to estimate S for each item that will produce the required Type 2 service level:

$$S^*=min\left\{S:\frac{\sum_{i=-\infty}^{S}\sum_{x=i+1}^{\infty}\sum_{d=0}^{x}(x-i)P\{X_N=x-d\}P\{X_D^L=d\}P\{X_D^L=S-i\}}{\sum_{j=-\infty}^{\infty}jP\{X_N+X_D^L=j\}}\leq 1-\beta\right\} \tag{5.1}$$

During the remaining time of the day, i.e. in the (t_1+L,t_2) interval, $(\boldsymbol{s},Q)$ policy affects the Type 2 service level. Recall in $(\boldsymbol{s},Q)$ policy that a joint order is triggered, when an item's inventory position drops below its reorder level (s). At that ordering instance, along with this item, the other items' inventory positions are increased before reaching their s values and the amount of increase depends on the allocation policy employed and the current inventory positions of the items. In this system, it is hard to track each item's inventory position at an ordering instance and know whose inventory position will drop below its reorder point and trigger the order, at the next ordering instance. If we could estimate the inventory levels of all items at an ordering instance, we could find the exact s values that yield the required service levels.

Considering the difficulties of finding actual Type 2 service levels for the daytime demand, we propose to use a cycle approach. Under the one-order-outstanding assumption, the following generic formulation can be used to estimate the Type 2 service level for the policies with a reorder point and a fixed order size, such as $(\boldsymbol{s}, Q)$ policy:

$$\frac{Expected\ shortages\ per\ replenishment\ cycle\ (ESPRC)}{Expected\ demand\ per\ replenishment\ cycle\ (EDPRC)}$$

We know that during the day-time, a stockout occasion can only be observed within the replenishment lead time, since we are using a policy with reorder levels. For this reason, finding $ESPRC$ is equal to finding the expected number of shortages in lead time ($ESLT$). For a single item case, we have $ESLT = \sum_{x=s+1}^{\infty}(x-s)P\{X_D^L = x\}$ and $EDPRC = Q$. However in the multi-item case, it is not clear from which reference point to estimate $ESLT$ value. For the item that triggers the order, $\sum_{x=s+1}^{\infty}(x-s)P\{X_D^L = x\}$ is valid but it would be an overestimation for the expected shortages for the other items. Similarly, $EDPRC$ for the multi item case is also not clear, as Q is not equal to the expected demand for each item in a given cycle or the cycle length. Moreover, since by using $(T, \boldsymbol{S})$ policy for the night-time an order is given a lead time ahead of the end of the day-time, the final replenishment cycle before the end of the day and the first replenishment cycle at the beginning of the may not be complete cycles. Moreover, due to impact of $(T, \boldsymbol{S})$ policy the system cannot reach to steady state and a steady state analysis cannot be conducted for this system.

Due to these difficulties, we come up with the following simple expression which yields upper bound estimates for the s value of each item.

$$s^u = min\left\{ s: \frac{\sum_{x=s+1}^{\infty}(x-s)P\{X_D^L = x\}}{\sum_{j=-\infty}^{\infty} jP\{X_D^L = j\}} \leq 1-\beta \right\} \tag{5.2}$$

Under the assumption of one order outstanding in a lead time, by using Equation (5.2) the estimated s value, which satisfies the required service level during the lead time, can be obtained. This estimated value can be considered as an upper bound for the optimal value of s. In other words, since the replenishment lead time is less than the length of the daytime and a stockout occasion can only be observed during the lead time, the s value, which satisfies the required service level during the lead time, also satisfies the service level requirement of the daytime.

In the above expressions, one should estimate the probability distribution of demand during lead time for each item. First we present the method of estimating this distribution by using the probability distribution of the compounding part of the demand and that of the number of demand instances during a lead time period (Axsater, 2006).

Let

f_j	=	Probability of demand size j ($j = 1, 2, \dots$)
f_j^k	=	Probability that k customers give the total demand j
X_D^L	=	Stochastic demand in the time interval L
$N(L)$	=	Number of demand occurrences during the time interval L
$P\{N(L) = k\}$	=	The probability that k customers arrive in the time interval L

$f_0^0 = 1$ and $f_j^1 = f_j$, then by using the following recursive formulation we can obtain f_j^k values.

$$f_j^k = \sum_{i=k-1}^{j-1} f_i^{k-1} f_{j-i}, \quad k = 2, 3, 4, \dots$$

By using these f_j^k values, we can find stochastic demand in the time interval L as follows:

$$P(X_D^L = j) = \sum_{k=0}^{\infty} P\{N(L) = k\} f_j^k$$

For items whose interarrival times are exponentially distributed, $P\{N(L) = k\}$ is given by the Poisson distribution. For cases, with general interarrival distributions, this probability can be estimated from the data by fitting a distribution to the number of demand occurrences observed in each day. After finding the best fitted distribution for the number of arrivals during lead time, we can insert that distribution as $P\{N(L) = k\}$ in the formulation above and obtain stochastic demand in the time interval L.

To sum up, by using Equation (5.2) the overestimated s value can be obtained for a given day-time service level requirement and by using Equation (5.1) the exact value of S can be obtained for a given night-time service level requirement for each item. Note that, this method is applicable to any renewal type of distribution.

In the next section, we find the optimal s values through simulation in Arena and gather insights on how to set the s values in identical and non-identical multi item situations, as well as insights on the proximity of the upper bounds to the optimal s values.

5.2 The Simulation Results for the Inventory System

We use Arena 11 for simulating the inventory system under the proposed $(\boldsymbol{s}, Q) + (T, \boldsymbol{S})$ inventory control policies. We construct an allocation method in our simulation so that the joint order is allocated to the items by balancing each item's inventory position in excess of its reorder point, as it is explained in Chapter 4. We obtain and analyze the results for the identical and non-identical multi-item systems, where we use Poisson and Gamma distributions for the day-time and night-time demands. While analyzing the results, we focus on the effects of joint replenishment (i.e. the interaction between the items), the effects of $(T, \boldsymbol{S})$ policy on $(\boldsymbol{s}, Q)$ policy, and the effects of the fixed order size (Q).

In order to find optimal policy parameters by simulating the system, we run the simulation model for a sufficiently long time (92340 hours) by excluding a warm up period (4860 hours), which we decide by using the Output Analyzer of Arena. For all items, we use the exact values for parameters S, which are found by using Equation (5.1), and the upper bound values for parameters s, which are obtained by using the estimation expression given by Equation (5.2), as the initial values of our optimality search and continue to search by decreasing the s value until we find the optimal one.

First of all, we introduce the scenarios that we use for the analysis of the identical multi-item case. Note that in all of the scenarios, we set the replenishment leadtime (L) to 1 hour. As the first scenario, we take a system, which has identical four items with Poisson distributed day-time and night-time demands. In the second scenario, we test the system for identical four items with Poisson distributed demands with lower demand rates. Then, we use another set of scenarios, in which different set of identical items with Poisson distributed demands are used, in order to show the effect of $(T, \boldsymbol{S})$ policy on the $(\boldsymbol{s}, Q)$ policy, if there is any. While constructing that set of scenarios (Scenario 3, 4 and 5), we variate them in terms of the day-time and night-time demand ratios. In the sixth and seventh scenarios we have identical four items with Gamma distributed demands with high and low demanded items separately.

Scenario 1: *Identical four-items with Poisson distribution (high demanded)*

Item 1, 2, 3 and 4 lead time demand rate $(\lambda_{D1}^{L}, \lambda_{D2}^{L}, \lambda_{D3}^{L}, \lambda_{D4}^{L})$	:	2
Item 1, 2, 3 and 4 night-time demand rate $(\lambda_{N1}, \lambda_{N2}, \lambda_{N3}, \lambda_{N4})$	:	18
Fixed order quantity (Q)	:	24

Scenario 2: *Identical four-items with Poisson distribution (low demanded)*

Item 1, 2, 3 and 4 lead time demand rate $(\lambda_{D1}^{L}, \lambda_{D2}^{L}, \lambda_{D3}^{L}, \lambda_{D4}^{L})$	:	1
Item 1, 2, 3 and 4 night-time demand rate $(\lambda_{N1}, \lambda_{N2}, \lambda_{N3}, \lambda_{N4})$	:	9
Fixed order quantity (Q)	:	12

Scenarios 3-5: *Identical four-items with Poisson distribution (different night-time demands)*

	Scenario 3	Scenario 4	Scenario 5
$\lambda^{L}_{D1}, \lambda^{L}_{D2}, \lambda^{L}_{D3}, \lambda^{L}_{D4}$:	6	6	6
$\lambda_{N1}, \lambda_{N2}, \lambda_{N3}, \lambda_{N4}$:	0	8	24

Scenario 6: *Identical four-items with Gamma distribution (high demanded, ID51 in our Data)*

Interarrival times of item 1, 2, 3, 4	:	Gamma (0.4874, 10168)
Compound parts at each arrival item 1, 2, 3, 4	:	Empirical Distribution: 1, 0.51, 2, 0.26, 3, 0.09, 4, 0.05, 5, 0.06, 6, 0.02, 8, 0.01
Total demand per night 1, 2, 3, 4	:	Gamma (3.171, 5.950)

Scenario 7: *Identical four-items with Gamma distribution (low demanded, ID52 in our Data)*

Interarrival times of item 1, 2, 3, 4	:	Gamma (0.5462, 23176)
Compound parts at each arrival item 1, 2, 3, 4	:	Empirical Distribution: 1, 0.52, 2, 0.28, 3, 0.13, 4, 0.03, 5, 0.04
Total demand per night 1, 2, 3, 4	:	Gamma (2.523,4.142)

We first calculate the S and s values for each item by Equations (5.1) and (5.2) and for Scenario 1 we find that $S = 27$ and $s = 5$ for each item in order to satisfy 98% service levels. Then starting from these values, we simulate the system and calculate the realized Type 2 service levels. We calculate the realized Type 2 service levels by dividing total amount of stockout observed, by the total amount of demand realized within a given simulation run-time. This gives us the proportion of unsatisfied demand and we find the proportion of satisfied demand by subtracting this number from one. The results that we obtain for this scenario are given in Table 5.1. In the first row of the table, it can be seen that 99% service level for the related s values (given in the second main column), and 98% service level for the S values are estimated, but by using those values 100% and 98% service levels are realized respectively. This shows that the estimated values that we calculate for S is exact and s is an upper bound. Then, for each item by decreasing s one by one, we find the optimal s value that satisfies the target service level while keeping minimum level of inventory. Observe that in the second row the required service levels of 98% for the day-time and night-time can be satisfied by using a lower value for s and the same value for S, with a lower level of inventory. The same approach is undertaken and rows 3 and 4 are obtained for 99% service levels. Note that while finding the optimal values for the multi-item case, we look for the minimum average inventory level per item.

Next, we test our estimation method under Scenario 2, in which there are four identical items with Poisson distributed day-time and night-time demands which have lower rates than the previous scenario. The results are given in Table 5.2. Again, observe that the optimal s values are lower than the initial s values and the optimal S values are exactly the same with what we find by using our method.

Afterwards, in order to analyze the effect of $(T, \boldsymbol{S})$ policy on $(\boldsymbol{s}, Q)$ policy, we run our simulation model for the Scenarios 3-5, in which there are also four identical items, but with different night-time demands at each scenario. The results are given in Table 5.3. From this table, we can observe that $(T, \boldsymbol{S})$ policy along with the night-time demand have an effect up to the fourth digit on the observed day-time service levels. As we increase the night-time demand as well as the related S value, the day-time service levels are increased by a very small amount, which cannot yield a different value of s compared with each other.

For another four identical items scenario (Scenario 6), we use item ID51's demand structure, which we obtain in Chapter 3. The results that we find by using our method and simulation are given in Table 5.4. In Table 5.4 observe that the initial values of parameters $\boldsymbol{s}$ are equal to 6, while in the optimal solution parameters s are equal to 5 for satisfying 98% service level. Moreover, both estimated and optimal S values are equal to 43. We also observe the system for a lower demanded item (item ID52) compared to ID51. The related estimated and optimal results of this Scenario 7 are presented in Table 5.5.

Afterwards, we test this inventory control system for non-identical four items case, in which we use a variety of Poisson demanded items, whose properties are given in the Scenario 8. For this non-identical items case we set different demand rates for these four items, so that they can represent the item classification explained in Chapter 3 in terms of the rate of demand. For representing group A item 1 and item 2 are used, while item 3 and item 4 are used for representing group B.

Scenario 8: *Non-identical four-items with Poisson distribution*

Item 1 lead time demand rate (λ_{D1}^{L})	:	2
Item 2 lead time demand rate (λ_{D2}^{L})	:	1
Item 3 lead time demand rate (λ_{D3}^{L})	:	0.1
Item 4 lead time demand rate (λ_{D4}^{L})	:	0.06
Item 1 night-time demand rate (λ_{N1})	:	18
Item 2 night-time demand rate (λ_{N2})	:	9
Item 3 night-time demand rate (λ_{N3})	:	1
Item 4 night-time demand rate (λ_{N4})	:	0.54
Fixed order quantity (Q)	:	10

Observe that in Table 5.6 for 98% service level the optimal s value of item 1, whose specifications are also used in the Scenario 1, is greater than the optimal s value given in Table 5.1. In other words, even the same demand rates are used for the item 1 in the non-identical case and the items in the identical case, the optimal s value differs for this highest demanded item. However, for item 2, which has a lower demand rate during day-time, we obtain a smaller value for the parameter s for satisfying 98% service level. If this item is put in a non-identical system along with item 1, which has higher demand rate, the optimal s value for the item 2 decreases to 1. By combining the two arguments that we come up with, we can conclude that the items with different demand rates have an interaction between each other. This is in a way that the item with highest demand rate dominates the less demanded items at each joint ordering instance. This is because the faster moving item reaches its reorder level before the others, and these items' inventory positions are also raised up to a certain amount due to allocation method. For this reason, in such a system, the optimal s values for the relatively low demanded items tend to be less.

Next, in Scenarios 9-11 we use non-identical four items with Gamma distributions with different fixed order sizes (Q), in order to observe the effect of Q on our system. In these scenarios, we apply our estimation method to a non-identical multi-item system, in which four representative items are used from the Guven Hospital's inventory system. We choose items ID51 and ID52 from group A, items ID75 and ID112 from group B, whose distributions are given in Chapter 3.

The results of the Scenario 9 are given in Table 5.7. Note that, among these items the one with the highest demand rate is item 1 (ID51). Observe that, for 99% service level target the dominant item ID51 has the same reorder level in both identical and non-identical cases, whose results are shown in Tables 5.4 and 5.7, respectively. On the other hand, for satisfying the service level of 99% for the item 2 (ID52) it is enough to set s value to 1 in the non-identical case, while it is 2 in the identical case, as it can be seen in Tables 5.7 and 5.5 respectively. This is due to dominating effect of ID51 at the ordering instances, as it is also explained for the items with Poisson day-time and night-time demands. This scenario also validates our argument that there is an interaction between the non-identical items with different demand rates.

Our final analysis is for observing the effect of the fixed order quantity Q on the service levels. For this analysis, we change the Q values of Scenario 9. In Table 5.7 the results for $Q = 10$ are shown. We observe that the actual number of replenishments in Scenario 9, where $Q = 10$ is equal to one. For this reason, we decrease the value of Q to 5 and 3 in Scenarios 10 and 11, so that we increase the number of replenishments, and obtain the results for the service level of 99%, which are shown in Table 5.8, under these scenarios. Observe that, as we decrease the fixed order size, the reorder level of each item tend to increase in order to satisfy certain service level requirements.

Scenarios 9-11: *Non-identical four-items with Gamma distribution (ID51, ID52, ID75, ID112 in our Data) Q =10 in Scenario 9, Q =5 in Scenario 10, Q =3 in Scenario 11*

Interarrival times of item 1	:	Gamma (0.4874, 10168)
Interarrival times of item 2	:	Gamma (0.5462, 23176)
Interarrival times of item 3	:	Gamma (0.8320, 125077)
Interarrival times of item 4	:	Gamma (0.5395, 147852)
Compound parts at each arrival item 1	:	Empirical Distribution: 1, 0.51, 2, 0.26, 3, 0.09, 4, 0.05, 5, 0.06, 6, 0.02, 8, 0.01
Compound parts at each arrival item 2	:	Empirical Distribution: 1, 0.52, 2, 0.28, 3, 0.13, 4, 0.03, 5, 0.04
Compound parts at each arrival item 3	:	Empirical Distribution: 1, 0.88, 2, 0.10, 3, 0.02
Compound parts at each arrival item 4	:	Empirical Distribution: 1, 0.93, 2, 0.06, 4, 0.01
Total demand per night 1	:	Gamma (3.171, 5.950)
Total demand per night 2	:	Gamma (2.523, 4.142)
Total demand per night 3	:	Empirical Distribution: 0, 0.71, 1, 0.18, 2, 0.07, 3, 0.01, 4, 0.01, 6, 0.01, 10, 0.01
Total demand per night 4	:	Empirical Distribution: 0, 0.61, 1, 0.32, 2, 0.06, 3, 0.01

Table 5.1: Results for the Scenario 1

	Calculated Service Levels for s values				s values				Calculated Service Level for S values				S values				Day-time Realized Service Level				Night-time Realized Service Level				Avg. Inventory Level per item	# of Replenish-ments during day-time
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4	1	2	3	4	1	2	3	4		
Initial for 98%	0.99	0.99	0.99	0.99	5	5	5	5	0.98	0.98	0.98	0.98	27	27	27	27	1.00	1.00	1.00	1.00	0.98	0.98	0.98	0.98	8.793	3.324
Optimal for 98%	0.89	0.89	0.89	0.89	3	3	3	3	0.98	0.98	0.98	0.98	27	27	27	27	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	7.280	3.00
Initial for 99%	0.99	0.99	0.99	0.99	5	5	5	5	0.99	0.99	0.99	0.99	28	28	28	28	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	9.019	3.167
Optimal for 99%	0.96	0.96	0.96	0.96	4	4	4	4	0.99	0.99	0.99	0.99	28	28	28	28	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	7.529	2.835

Table 5.2: Results for the Scenario 2

	Calculated Service Levels for s values				s values				Calculated Service Level for S values				S values				Day-time Realized Service Level				Night-time Realized Service Level				Avg. Inventory Level per item	# of Replenish-ments during day-time
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4	1	2	3	4	1	2	3	4		
Initial for 98%	0.99	0.99	0.99	0.99	3	3	3	3	0.98	0.98	0.98	0.98	15	15	15	15	1.00	1.00	1.00	1.00	0.98	0.98	0.98	0.98	5.792	3.253
Optimal for 98%	0.90	0.90	0.90	0.90	2	2	2	2	0.98	0.98	0.98	0.98	15	15	15	15	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.98	5.063	2.937
Initial for 99%	0.99	0.99	0.99	0.99	3	3	3	3	0.99	0.99	0.99	0.99	16	16	16	16	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	6.054	2.937
Optimal for 99%	0.90	0.90	0.90	0.90	2	2	2	2	0.99	0.99	0.99	0.99	16	16	16	16	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	5.373	2.622

Table 5.3: Results for Scenarios 3-5

	Calculated S.L. for s	s values	Calculated S.L. for S	S values	Day-time Realized Service Level				Night-time Realized Service Level				Avg. Inventory Level per item
	s_i	s_i	S_i	S_i	1	2	3	4	1	2	3	4	
Scenario 3 for 99%	0.84	6	0.99	18	0.988743	0.988194	0.987140	0.986496	0.988932	0.989073	0.988739	0.989123	7.397
Scenario 4 for 99%	0.84	6	0.99	27	0.989271	0.988886	0.987807	0.986984	0.989506	0.989493	0.989542	0.990368	7.567
Scenario 5 for 99%	0.84	6	0.99	43	0.990753	0.990063	0.988798	0.987976	0.987509	0.987547	0.987779	0.988370	7.681

Table 5.4: Results for the Scenario 6

	Calculated Service Levels for s values				s values				Calculated Service Level for S values				S values				Day-time Realized Service Level				Night-time Realized Service Level				Avg. Inventory Level per item	# of Replenish-ments during day-time
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4	1	2	3	4	1	2	3	4		
Initial for 98%	0.98	0.98	0.98	0.98	6	6	6	6	0.98	0.98	0.98	0.98	43	43	43	43	0.99	0.99	0.98	0.99	0.98	0.98	0.98	0.98	19.069	1.987
Optimal for 98%	0.96	0.96	0.96	0.96	5	5	5	5	0.98	0.98	0.98	0.98	43	43	43	43	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	18.842	1.825
Initial for 99%	0.99	0.99	0.99	0.99	7	7	7	7	0.99	0.99	0.99	0.99	48	48	48	48	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	23.237	1.387
Optimal for 99%	0.96	0.96	0.96	0.96	5	5	5	5	0.99	0.99	0.99	0.99	48	48	48	48	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	22.929	1.144

Table 5.5: Results for the Scenario 7

	Calculated Service Levels for s values				s values				Calculated Service Level for S values				S values				Day-time Realized Service Level				Night-time Realized Service Level				Avg. Inventory Level per item	# of Replenish-ments during day-time
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4	1	2	3	4	1	2	3	4		
Initial for 98%	0.98	0.98	0.98	0.98	4	4	4	4	0.98	0.98	0.98	0.98	26	26	26	26	1.00	0.99	1.00	1.00	0.98	0.98	0.98	0.98	14.393	0.809
Optimal for 98%	0.56	0.56	0.56	0.56	1	1	1	1	0.98	0.98	0.98	0.98	26	26	26	26	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	13.924	0.501
Initial for 99%	1.00	1.00	1.00	1.00	5	5	5	5	0.99	0.99	0.99	0.99	28	28	28	28	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	16.214	0.699
Optimal for 99%	0.83	0.83	0.83	0.83	2	2	2	2	0.99	0.99	0.99	0.99	28	28	28	28	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	15.799	0.422

Table 5.6: Results for the Scenario 8

	Calculated Service Levels for s values				s values				Calculated Service Level for S values				S values				Day-time Realized Service Level				Night-time Realized Service Level				Avg. Inventory Level per item	# of Replenish-ments during day-time
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4	1	2	3	4	1	2	3	4		
Initial for 98%	0.99	0.99	0.99	0.99	5	3	2	2	0.98	0.98	0.99	0.99	27	15	4	3	1.00	1.00	1.00	1.00	0.99	0.98	1.00	1.00	7.029	3.134
Optimal for 98%	0.96	0.63	0.00	0.00	4	1	0	0	0.98	0.98	0.99	0.99	27	15	4	3	0.99	0.98	1.00	1.00	0.99	0.98	1.00	1.00	5.489	2.845
Initial for 99%	0.99	0.99	0.99	0.99	5	3	2	2	0.99	0.99	0.99	1.00	28	16	4	3	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00	7.138	2.941
Optimal for 99%	0.96	0.90	0.00	0.00	4	2	0	0	0.99	0.99	0.99	1.00	28	16	4	3	0.99	0.99	1.00	1.00	0.99	0.99	1.00	1.00	5.769	2.743

Table 5.7: Results for the Scenario 9

	Calculated Service Levels for s values				s values				Calculated Service Level for S values				S values				Day-time Realized Service Level				Night-time Realized Service Level				Avg. Inventory Level per item	# of Replenish-ments during day-time
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4	1	2	3	4	1	2	3	4		
Initial for 98%	0.98	0.98	1.00	1.00	6	4	1	1	0.98	0.98	0.98	0.98	43	26	9	3	0.98	1.00	1.00	1.00	0.98	0.98	0.98	1.00	12.068	0.795
Optimal for 98%	0.96	0.56	0.00	0.00	5	1	0	0	0.98	0.98	0.98	0.98	43	26	9	3	0.98	0.98	1.00	0.99	0.98	0.98	0.98	1.00	11.604	0.661
Initial for 99%	0.99	1.00	1.00	1.00	7	5	1	1	0.99	0.99	1.00	0.99	48	28	10	4	0.99	0.99	1.00	1.00	0.99	0.99	1.00	1.00	13.724	0.594
Optimal for 99%	0.96	0.56	0.00	0.00	5	1	0	0	0.99	0.99	1.00	0.99	48	28	10	4	0.99	0.99	1.00	0.99	0.99	0.99	1.00	1.00	13.253	0.436

Table 5.8: Results for Scenarios 10 and 11 where $Q = 5$ and $Q = 3$

	Calculated Service Levels for s values				s values				Calculated Service Level for S values				S values				Day-time Realized Service Level				Night-time Realized Service Level				Avg. Inventory Level per item	# of Replenish-ments during day-time
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4	1	2	3	4	1	2	3	4		
Optimal for 99% $Q = 5$	0.98	0.83	0.00	0.00	6	2	0	0	0.99	0.99	1.00	0.99	48	28	10	4	0.99	0.99	1.00	0.99	0.99	0.99	1.00	1.00	12.767	0.721
Optimal for 99% $Q = 3$	0.99	0.83	0.00	1.00	7	2	0	1	0.99	0.99	1.00	0.99	48	28	10	4	0.99	0.99	1.00	1.00	0.99	0.99	1.00	1.00	12.765	1.195

To sum up, in this chapter we analyze our proposed inventory control policy $(\boldsymbol{s}, Q) + (T, \boldsymbol{S})$ under different scenarios by simulating the system in Arena. Firstly, we show that the S values that we find by using the Equation (5.1) are the exact values, while the s values that we find by using the Equation (5.2) are the overestimated values under a specified service level target. We can also conclude that there is actually an interaction between the items, which have different demand rates. The item which has highest demand rate dominates the other items, and the orders are triggered by that item in most of the cases. For this reason, this item's reorder level is closer to what we initially find by our calculations, while the other items' reorder levels tend to be much smaller. With this in mind, the search for optimal s values can be facilitated by using the following search method: For the dominant item, start from the upper bound and decrease the s value one by one, and for the other less demanded items' start from a much lower value than the upper bound (even from zero) and increase the s value within the range until the required service levels are obtained. Additionally, it can be concluded that there is a negligible effect of $(T, \boldsymbol{S})$ policy on $(\boldsymbol{s}, Q)$ policy, so it can be assumed that the optimal s value is not affected by the night-time demand and its control policy. Finally, it is shown that the fixed order quantity has an effect on the reorder levels in a way that, as we decrease the fixed order quantity, the reorder levels increase and become closer to our initially calculated s values.

Chapter 6

Conclusion

In this thesis, we study the inventory control problem of medical supplies that arises in a particular hospital environment. A patient's health is the main concern in hospitals. Therefore, rather than minimizing the cost, we mainly focus on satisfying the target service levels. Type 2 service level, which is the proportion of unsatisfied demand, is taken as the performance indicator for this system. Specifically, we consider a nursing station from which the nurses retrieve items for the patient needs. The nursing stations can be replenished by the central warehouse at anytime during day-time by using capacitated porter carts, but there cannot be any replenishment during night-time since the central warehouse is closed at that period of time. Hence, there are two distinct time frames (day-time and night-time) with different replenishment characteristics. For the day-time inventory control, we can propose a continuous review policy while for the night-time inventory control, we can only propose a periodic review policy. With this in mind, we suggest a hybrid $(s, Q) + (T, S)$ policy to be used, in order to satisfy the required service levels for the day-time and night-time.

Before analyzing the system under these policies, we make a detailed data analysis of the six-months data obtained from the nursing station. First of all, we divide the items into three classes A, B and C according to total demand proportion of each item within the inventory system. Then, in order to use in the numerical analysis we obtain the components of demand distribution of each item. Demand distribution of each item is composed of the distribution of the interarrival times, the distribution of the compounding parts at each transaction, the distribution of the total number of arrivals within lead time and the distribution of total demand per night. Interarrival times are obtained by finding time between each consecutive transaction and the best fitted distribution is chosen. The compounding parts at each transaction are also gathered from the data and a distribution is fitted to this as well. In order to handle the compounded distributions while finding the demand during lead time, we obtain total number of arrivals within lead time by dividing the total daily demand to the length of the day-time and fitting the best distribution to this data. Finally, for the total demand per night distribution we sum up all transactions occurred during night and fit the best distribution.

Our data analysis points out that 8% of the items constitute almost 83% of all items under concern. Hence, even though there are 195 items in total, only 16 of them are the most critical. Special attention should be paid to these items. According to our classification, there are 45 B items and 134 C items. For controlling the most important items, $(s,Q)+(T,S)$ policy can be used; but for the C items individual control can be adapted.

As a special case, for the single-item system we derive exact expressions, which yield us the optimal policy parameters. However, since deriving such expressions for the multi-item case is computationally hard, we simulate the system under different scenarios and analyze the effect of the policy parameters on the service level targets. By using the demand distribution components mentioned in the previous paragraph, we can find the initially estimated values of s and S by considering required service levels with different probability distributions. We use Arena for simulating the system, and we search for optimal values by starting from the initial parameter values. Our results show that the estimation method for the parameter S gives the optimal value and overestimated values for the parameter s. Note that, we set parameter T to one lead time ahead of the end of the day and parameter Q to the capacity of the porter cart.

According to simulation results, it can also be concluded that (T, S) policy has an effect up to the fourth digit on the observed daytime service levels. It is also shown that the fixed order quantity, Q, has an effect on the reorder levels in such a way that, as the value of Q is decreased, s values increase and become closer to our initially estimated s values. Moreover, simulation results indicate that there is an interaction between the items in a system, which consists of items with various demand rates. This interaction shows us the item with the highest demand rate dominates the less demanded items at each joint ordering instance. For this reason, in this type of system, the optimal s values for the relatively low demanded items tend to be less, while for items with high demand the optimal s values tend to be close to our initially estimated values.

By using the discussion above, we propose the following approach to set the policy parameters for the problem under concern:

- Classify the items as A, B and C and find the most important items. (Chapter 3)
- For each selected item, obtain the interarrival times, compounding parts at each transaction, total number of arrivals within lead time and the total demand per night. Then fit the best distribution to each of these data. (Chapter 4)
- Estimate S parameter for each item by using Equation 5.1. (Chapter 5)
- Estimate s^u for each item by using Equation 5.2. (Chapter 5)
- In order to facilitate the searching process for the optimal parameter values; search for the optimal values of s, for the dominating items (in terms of average demand) starting from the upper bound and for the other items starting from zero. (Chapter 5)

This study can be extended by implementing the $(\boldsymbol{s}, Q) + (T, \boldsymbol{S})$ policy to all of the nursing stations and the items at all stations can also be replenished jointly. The effects of jointly replenishing all stations need to be examined. Moreover, the effect of this hybrid policy on the central warehouse can also be analyzed as a future research.

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Appendix A

The Detailed Information of the Items

APPENDIX A. THE DETAILED INFORMATION OF THE ITEMS

Table A.1: Six months data for 195 medical items

ID	Item name	Total demand (6 months)	Cumulative percent of total demand	Cumulative percent of item
A ITEMS				
41	ELDİVEN NON STERİL LARGE	8223	19.070	0.5128
51	ENJEKTÖR 5ML	6937	35.157	1.0256
42	ELDİVEN NON STERİL MEDIUM	5145	47.088	1.5385
52	ENJEKTÖR 10ML	2941	53.909	2.0513
122	SET SERUM	1868	58.241	2.5641
146	STRİP GLUCOSE İÇİN	1762	62.327	3.0769
20	BANT ENJEKSİYON İÇİN YUVARLAK	1729	66.337	3.5897
95	MUSLUK ÜÇ YOLLU	1153	69.010	4.1026
44	ELDİVEN PET	982	71.288	4.6154
74	İNTRAKET 20G PEMBE	880	73.329	5.1282
90	LANSET ŞEKER ÖLÇMEK İÇİN	803	75.191	5.6410
91	MASKE 3 KATLI BAĞCIKLI	700	76.814	6.1538
27	BEZ YETİŞKİN İÇİN LARGE	662	78.349	6.6667
22	BANT KOL İÇİN YETİŞKİN	651	79.859	7.1795
38	DERECE	629	81.318	7.6923
31	BONE	619	82.753	8.2051
B ITEMS				
60	FLASTER HYPAFIX 5CMX10M 1ADET:1/10	468	83.839	8.7179
53	ENJEKTÖR 20ML	460	84.905	9.2308
56	ENJEKTÖR İNSÜLİN 1ML	423	85.886	9.7436
163	TÜP KAN ALMA VAKUMLU BOŞ 4ML	388	86.786	10.2564
49	ELEKTROD EKG YETİŞKİN F55	378	87.663	10.7692
65	İDRAR KABI STERİL 100ML	294	88.344	11.2821
61	FLASTER HYPAFIX 10CMX10M 1ADET:1/10	284	89.003	11.7949
165	TÜP KAN ALMA VAKUMLU EDTALI 3ML	281	89.655	12.3077
118	SET POMPA İNFÜZYON -BODYGUARD-	209	90.139	12.8205
104	SARGI YARA PRİMAPORE 8.3CMX6CM	198	90.599	13.3333
75	İNTRAKET 22G MAVİ	197	91.055	13.8462
112	SET DAMLA AYAR	192	91.501	14.3590
69	İĞNE NOVOFINE 30G 0.3/8MM	191	91.944	14.8718
2	ADAPTÖR LUER VENOJECT	188	92.380	15.3846
54	ENJEKTÖR 50ML	167	92.767	15.8974
115	SET KAN	154	93.124	16.4103
71	İĞNE VENOJECT 21G YEŞİL	148	93.467	16.9231
120	SET POMPA LIFECARE 5000 I.V. G868	140	93.792	17.4359
153	TORBA İDRAR İÇİN MUSLUKLU	115	94.059	17.9487
166	TÜP KAN ALMA VAKUMLU HEPARİNLİ 4ML	114	94.323	18.4615

APPENDIX A. THE DETAILED INFORMATION OF THE ITEMS

24	BEZ SARGI ELASTİK 5CMX4M	113	94.585	18.9744
94	MASKE OKSİJEN NAZAL	91	94.796	19.4872
105	SARGI YARA PRİMAPORE 10CMX8CM	89	95.002	20.0000
50	ENJEKTÖR 2ML	82	95.193	20.5128
29	BİSTÜRİ NO:11	78	95.373	21.0256
21	BANT KOL İÇİN ANNE-BEBEK	77	95.552	21.5385
167	TÜP KAN ALMA VAKUMLU SİTRATLI 2.9ML SEDİM	75	95.726	22.0513
168	TÜP KAN ALMA VAKUMLU SİTRATLI 2ML	74	95.898	22.5641
73	İNTRAKET 18G YEŞİL	69	96.058	23.0769
119	SET POMPA İNFÜZYON BODYGUARD NITROGLISERINLİ	69	96.218	23.5897
34	ÇORAP ANTİEMBOLİZM DİZ ÜSTÜ MEDIUM	67	96.373	24.1026
89	KOVA TIBBİ ATIK İÇİN MİNİ	67	96.528	24.6154
6	ALEZ HASTA İÇİN 60CMX90CM	63	96.674	25.1282
55	ENJEKTÖR 50MLCATHETER	63	96.821	25.6410
58	FLASTER BETAFİX 5CMX10M 1ADET:1/10	57	96.953	26.1538
1	ABESLANG	56	97.083	26.6667
145	SPREY OPSITE 100ML 1KT:1/10	48	97.194	27.1795
46	ELDİVEN STERİL NO:7,5	47	97.303	27.6923
25	BEZ SARGI ELASTİK 10CMX4M	44	97.405	28.2051
88	KOVA TIBBİ ATIK İÇİN BÜYÜK	37	97.491	28.7179
59	FLASTER BEZ 5CMX5M 1ADET:1/5	36	97.574	29.2308
70	İĞNE PORT 19G-20G-21G-22G LUERLOCK	34	97.653	29.7436
33	ÇORAP ANTİEMBOLİZM DİZ ÜSTÜ LARGE	33	97.730	30.2564
111	SET ATOMİZER MASKELİ	33	97.806	30.7692
47	ELDİVEN STERİL NO:8	31	97.878	31.2821
C ITEMS				
7	ASKI İDRAR TORBASI İÇİN	30	97.948	31.7949
43	ELDİVEN NON STERİL SMALL	30	98.017	32.3077
57	FLASTER BETAFİX 10CMX10M 1ADET:1/10	30	98.087	32.8205
106	SARGI YARA PRİMAPORE 15CMX8CM	28	98.152	33.3333
125	SONDA ASPİRASYON NO: 6	28	98.217	33.8462
124	SOLUNUM EGZERSİZ ALETİ TRIFLO	27	98.279	34.3590
129	SONDA ASPİRASYON NO:16	27	98.342	34.8718
67	İĞNE ENJEKTÖR İÇİN 21G YEŞİL	26	98.402	35.3846
76	İNTRAKET 24G SARI	26	98.462	35.8974
62	FLASTER İPEK 5CMX5M 1ADET:1/5	25	98.520	36.4103
97	PAMUK ALÇI İÇİN 10CM	25	98.578	36.9231
100	PROXI-STRIP 12,7MMX100MM 104K	24	98.634	37.4359
96	ÖRTÜ YARA COMFEEL 10*10	23	98.687	37.9487
23	BEZ BEBEK İÇİN 3-6KG	21	98.736	38.4615
37	ÇUBUK EKİVYON STERİL TÜPLÜ	20	98.782	38.9744
109	SARGI YARA PRİMAPORE 30CMX10CM	20	98.829	39.4872

170	UZATMA BASINCA DAYANIKLI 145CM MF	20	98.875	40.0000
63	GAİTA KABI PLASTİK	18	98.917	40.5128
107	SARGI YARA PRİMAPORE 20CMX10CM	16	98.954	41.0256
64	İDRAR KABI 24 SAATLİK	15	98.989	41.5385
68	İĞNE ENJEKTÖR İÇİN 22G SİYAH	15	99.024	42.0513
108	SARGI YARA PRİMAPORE 25CMX10CM	15	99.058	42.5641
17	BANDAJ ELASTİK 10CM	13	99.089	43.0769
26	BEZ SARGI ELASTİK 15CMX4M	13	99.119	43.5897
45	ELDİVEN STERİL NO:7	13	99.149	44.1026
48	ELDİVEN STERİL NO:8,5	13	99.179	44.6154
93	MASKE OKSİJEN İNHALASYONU İÇİN	13	99.209	45.1282
150	SÜTYEN SPORCU DESTEKLİ	13	99.239	45.6410
35	ÇORAP ANTİEMBOLİZM DİZ ÜSTÜ SMALL	12	99.267	46.1538
39	DRAPE OPSITE 15CMX28CM	11	99.293	46.6667
102	SARGI BACTIGRAS 10CMX10CM	11	99.318	47.1795
116	SET LAVMAN	11	99.344	47.6923
128	SONDA ASPİRASYON NO:14	11	99.369	48.2051
133	SONDA FOLEY 2 YOLLU NO:16 SİLİKONLU	11	99.395	48.7179
123	SET SÜT SAĞMAK İÇİN (PUMPSET ECO)	10	99.418	49.2308
92	MASKE KORUYUCU N95	9	99.439	49.7436
121	SET POMPA LIFECARE 5000 I.V. NITROGLISERIN L218	9	99.460	50.2564
143	SONDA PREZERVATİF LARGE	9	99.481	50.7692
15	BANDAJ ELASTİK 6CM	8	99.499	51.2821
18	BANDAJ ELASTİK 12CM	8	99.518	51.7949
19	BANDAJ ELASTİK 15CM	8	99.536	52.3077
98	PAMUK ALÇI İÇİN 15CM	8	99.555	52.8205
147	SÜRGÜ PLASTİK	8	99.573	53.3333
72	İLAÇ KADEHİ PLASTİK	7	99.590	53.8462
110	SET ATOMİZER AĞIZLIKLI	7	99.606	54.3590
127	SONDA ASPİRASYON NO:12	7	99.622	54.8718
132	SONDA FOLEY 2 YOLLU NO:16	7	99.638	55.3846
144	SONDA PREZERVATİF MEDIUM	7	99.654	55.8974
83	KOLTUK DEĞNEĞİ	6	99.668	56.4103
103	SARGI BACTIGRAS 15CMX20CM	6	99.682	56.9231
117	SET POMPA FLEXIFLO COMPANION G938	6	99.696	57.4359
157	TURNİKE	6	99.710	57.9487
169	UZATMA 145CM MF	6	99.724	58.4615
84	KOLTUK DEĞNEĞİ KANEDYEN	5	99.736	58.9744
101	PROXI-STRIP 6,4MMX100MM 103K	5	99.747	59.4872
130	SONDA FOLEY 2 YOLLU NO:14	5	99.759	60.0000
135	SONDA FOLEY 2 YOLLU NO:18 SİLİKONLU	5	99.770	60.5128
85	KONNEKTÖR T	4	99.780	61.0256

APPENDIX A. THE DETAILED INFORMATION OF THE ITEMS

131	SONDA FOLEY 2 YOLLU NO:14 SİLİKONLU	4	99.789	61.5385
149	SÜTÜR İPEK S5255 NO:2/0 25MM 75CM KESKİN İĞNE	4	99.798	62.0513
152	TERMOFOR	4	99.808	62.5641
171	ÜROFİX (SAATLİK İDRAR ÖLÇÜMÜ İÇİN)	4	99.817	63.0769
4	AIRWAY YETİŞKİN NO:4	3	99.824	63.5897
9	ASKI KOL İÇİN MEDIUM	3	99.831	64.1026
12	AYAKKABI ALÇI İÇİN LARGE	3	99.838	64.6154
30	BİSTÜRİ NO:20	3	99.845	65.1282
154	TORBA İDRAR İÇİN ERKEK	3	99.852	65.6410
159	TÜP EDTALI 1 MM PEDİATRİK	3	99.859	66.1538
164	TÜP KAN ALMA VAKUMLU BOŞ 9ML	3	99.865	66.6667
3	AIRWAY YETİŞKİN NO:3	2	99.870	67.1795
8	ASKI KOL İÇİN LARGE	2	99.875	67.6923
10	ASKI KOL İÇİN SMALL	2	99.879	68.2051
11	ASKI KOL İÇİN XSMALL	2	99.884	68.7179
14	AYAKKABI ALÇI İÇİN SMALL	2	99.889	69.2308
77	JEL EKG 1LT	2	99.893	69.7436
80	KATETER SECALON T 16G GRİ	2	99.898	70.2564
86	KORSE KARIN İÇİN ABDOMİNOCARE KOD:2510	2	99.903	70.7692
99	PAMUK ALÇI İÇİN 20CM	2	99.907	71.2821
113	SET EPİDURAL SOFT 701	2	99.912	71.7949
126	SONDA ASPİRASYON NO: 8	2	99.917	72.3077
134	SONDA FOLEY 2 YOLLU NO:18	2	99.921	72.8205
136	SONDA FOLEY 2 YOLLU NO:20 SİLİKONLU	2	99.926	73.3333
139	SONDA NAZOGASTRİK NO:16	2	99.930	73.8462
148	SÜTÜR İPEK S4255 NO:3/0 25MM 75CM KESKİN İĞNE	2	99.935	74.3590
156	TORBA SICAK-SOĞUK KOMPRES İÇİN BÜYÜK	2	99.940	74.8718
158	TÜP BESLENME FLEXIFLO NO:12	2	99.944	75.3846
5	AIRWAY YETİŞKİN NO:5	1	99.947	75.8974
13	AYAKKABI ALÇI İÇİN MEDIUM	1	99.949	76.4103
16	BANDAJ ELASTİK 8CM	1	99.951	76.9231
28	BİSTÜRİ NO:10	1	99.954	77.4359
32	ÇAM AĞACI PLASTİK ÇİFT YÖNLÜ	1	99.956	77.9487
36	ÇORAP VARİS İÇİN DİZÜSTÜ MEDİVEN ELEGANCE 169	1	99.958	78.4615
40	DRAPE OPSITE 30CMX28CM	1	99.961	78.9744
66	İĞNE ENJEKTÖR İÇİN 18G PEMBE	1	99.963	79.4872
78	KATETER SANTRAL VENÖZ CAVAFİX 16G 45CM	1	99.965	80.0000
79	KATETER SANTRAL VENÖZ ÇİFT LÜMENLİ 7F-9F	1	99.968	80.5128
81	KATETER SECALON T 18G YEŞİL	1	99.970	81.0256
82	KLEMP EPİDURAL KATETER NO:16/18G	1	99.972	81.5385
87	KORSE LUMBOCARE HIGH ÇELİK BALENLİ KOD:2540	1	99.974	82.0513
114	SET INTRODUCER 8F	1	99.977	82.5641

137	SONDA FOLEY 3 YOLLU NO:20	1	99.979	83.0769
138	SONDA NAZOGASTRİK NO:14	1	99.981	83.5897
140	SONDA NAZOGASTRİK NO:18	1	99.984	84.1026
141	SONDA NELATON NO:14	1	99.986	84.6154
142	SONDA NELATON NO:16	1	99.988	85.1282
151	TABLET EZİCİ	1	99.991	85.6410
155	TORBA İDRAR İÇİN KIZ	1	99.993	86.1538
160	TÜP ENTÜBASYON NO:4,5 KAFSIZ	1	99.995	86.6667
161	TÜP ENTÜBASYON NO:7,5 KAFLI	1	99.998	87.1795
162	TÜP ENTÜBASYON NO:8,5 KAFLI	1	100.000	87.6923
172	SET KELEBEK 21G MAVİ	0	100.000	88.2051
173	İNTRAKET 14	0	100.000	88.7179
174	İNTRAKET 16	0	100.000	89.2308
175	STYLE ENTÜBASYON S	0	100.000	89.7436
176	STYLE ENTÜBASYON M	0	100.000	90.2564
177	STYLE ENTÜBASYON L	0	100.000	90.7692
178	TÜP ENTÜBASYON NO:2,5	0	100.000	91.2821
179	TÜP ENTÜBASYON NO:3	0	100.000	91.7949
180	TÜP ENTÜBASYON NO:3,5	0	100.000	92.3077
181	TÜP ENTÜBASYON NO:4	0	100.000	92.8205
182	TÜP ENTÜBASYON NO:5	0	100.000	93.3333
183	TÜP ENTÜBASYON NO:5,5	0	100.000	93.8462
184	TÜP ENTÜBASYON NO:6	0	100.000	94.3590
185	TÜP ENTÜBASYON NO:6,5	0	100.000	94.8718
186	TÜP ENTÜBASYON NO:7	0	100.000	95.3846
187	TÜP ENTÜBASYON NO:8	0	100.000	95.8974
188	TÜP ENTÜBASYON NO:9	0	100.000	96.4103
189	AIRWAY YETİŞKİN NO:0	0	100.000	96.9231
190	AIRWAY YETİŞKİN NO:1	0	100.000	97.4359
191	AIRWAY YETİŞKİN NO:2	0	100.000	97.9487
192	HOLDER ENDOTRAKEAL TÜPÜ İÇİN ADULT	0	100.000	98.4615
193	HOLDER ENDOTRAKEAL TÜPÜ İÇİN PEDİATRİ	0	100.000	98.9744
194	BİSTÜRİ NO:21	0	100.000	99.4872
195	SONDA ASPİRASYON NO:10	0	100.000	100.0000

Table A.2: Descriptive statistics of some items

ID	Mean	Standard Deviation	Variance	Coefficient of Variance	Median	Skewness
41	42.39	32.88	1080.86	77.56	34.5	2.21
51	35.94	20.6	424.16	57.3	31	1.6
42	30.26	24.22	586.44	80.02	25	2.01
52	15.73	8.646	74.759	54.98	14	0.76
122	9.629	5.503	30.286	57.15	9	0.79
146	10.37	7.858	61.748	75.81	8	1.06
20	9.005	4.933	24.33	54.77	8	0.87
95	6.199	3.756	14.106	60.59	5	1.41
44	11.16	13.59	184.78	121.81	7.5	4.2
74	5	3.576	12.789	71.52	4	1.37
90	4.751	3.047	9.283	64.12	4	0.84
91	7.61	11.76	138.2	154.5	3	2.89
27	5.213	4.027	16.216	77.25	4	1.23
22	4.043	2.871	8.242	71	4	2.08
38	3.722	2.249	5.059	60.43	3	0.78
31	3.893	2.863	8.197	73.54	3	1.8
60	3.467	3.325	11.057	95.92	2	2.38
53	3.046	1.964	3.858	64.47	3	1.39
56	3.881	3.011	9.069	77.6	3	1.94
163	2.504	1.9	3.61	75.89	2	1.93
49	5.25	3.201	10.246	60.97	4	2.58
65	1.534	1.068	1.141	69.63	1	2.44
61	2.757	1.86	3.46	67.46	2	1.51
165	2.178	1.518	2.304	69.68	2	1.69
118	1.833	0.9769	0.9543	53.28	2	1.04
104	2.676	1.851	3.428	69.19	2	0.87
75	1.791	1.382	1.91	77.17	1	3.51
112	1.67	0.915	0.8372	54.8	1	1.48
69	2.513	1.447	2.093	57.57	2	1.34
2	2	1.503	2.258	75.13	1	1.9
54	1.856	1.204	1.451	64.91	1	2.14
115	1.812	1.239	1.536	68.4	1	2.41
71	1.701	1.101	1.212	64.71	1	2.12
120	1.296	0.568	0.3226	43.82	1	1.79
153	1.797	1.287	1.656	71.63	1	2.33
166	1.81	1.105	1.221	61.07	1	1.95
24	1.592	0.935	0.874	58.73	1	1.99
94	1.358	0.6675	0.4455	49.14	1	1.96

105	1.816	1.364	1.861	75.11	1	1.99
50	2.929	3.066	9.402	104.7	2	2.93
29	1.66	0.841	0.708	50.69	1	1.64
21	1.833	2.284	5.215	124.57	1	4.95

Appendix B

Daily Demand Distributions of Some Items

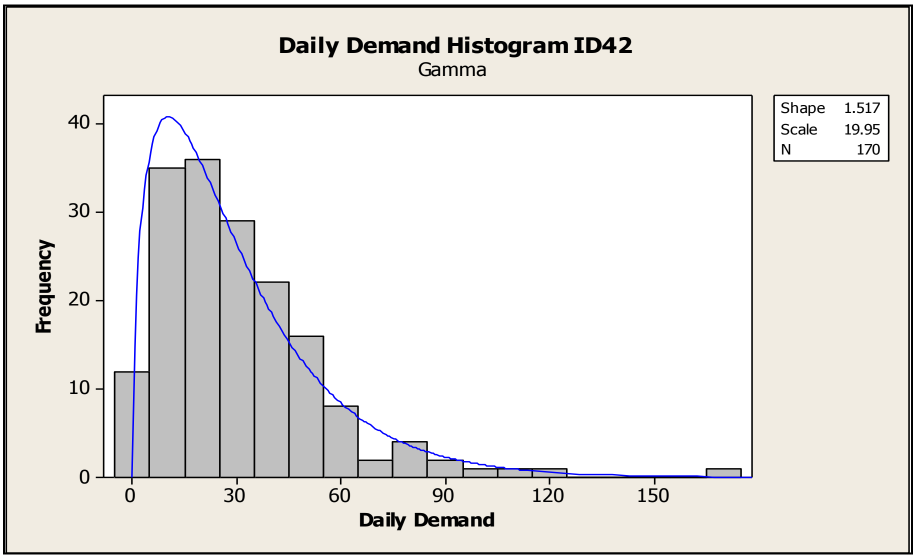

Figure B.1: Daily Demand Distributions for the item ID42

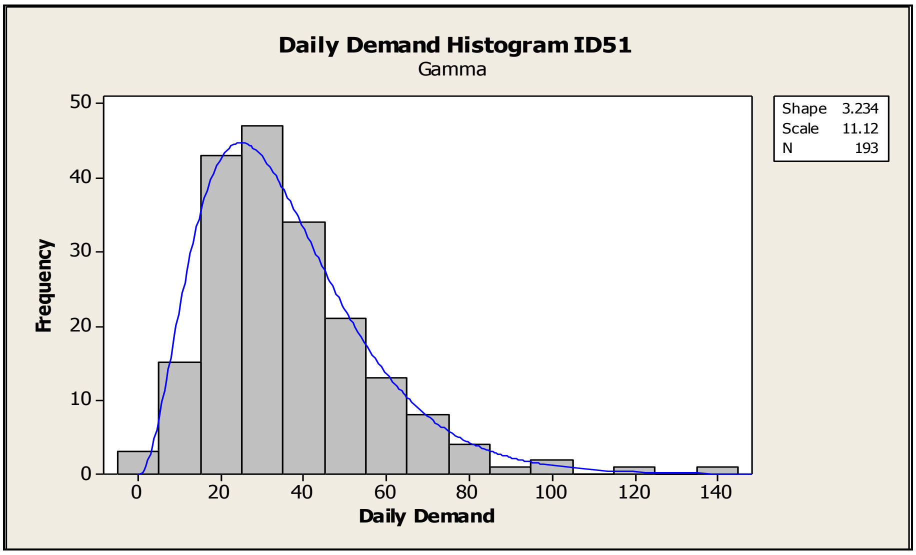

Figure B.2: Daily Demand Distributions for the item ID51

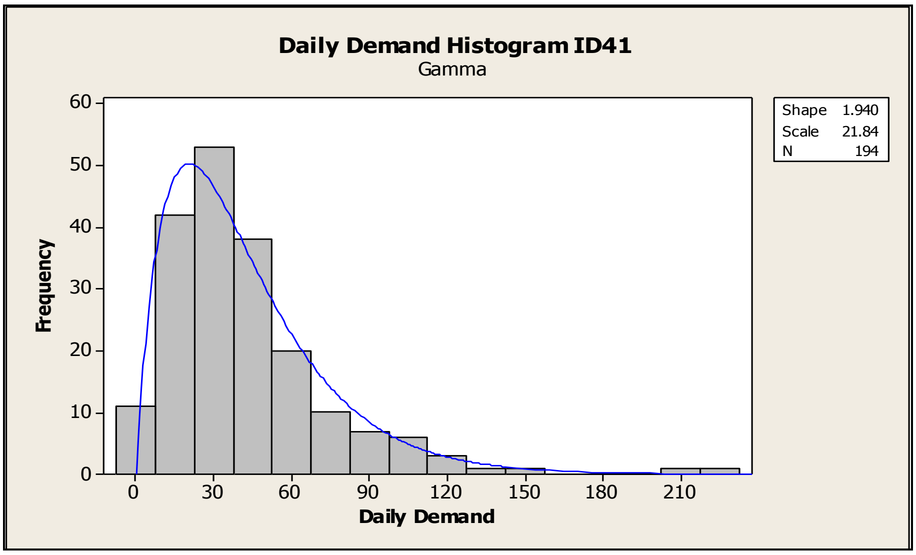

Figure B.3: Daily Demand Distributions for the item ID41

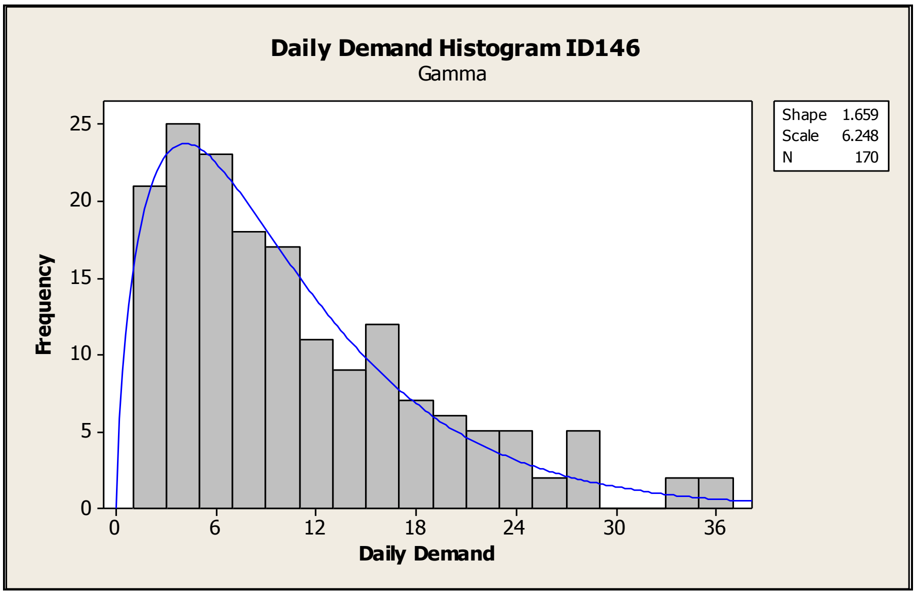

Figure B.4: Daily Demand Distributions for the item ID146

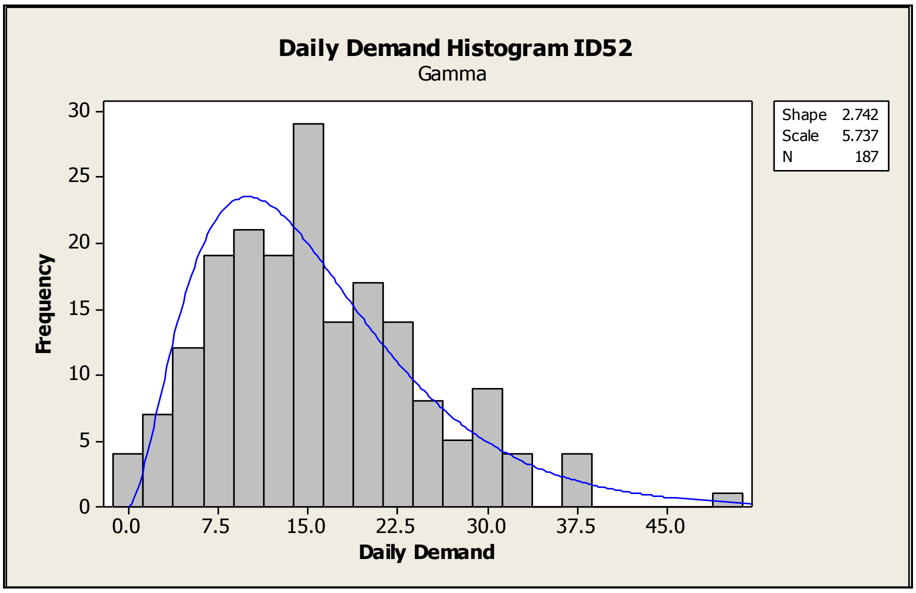

Figure B.5: Daily Demand Distributions for the item ID52

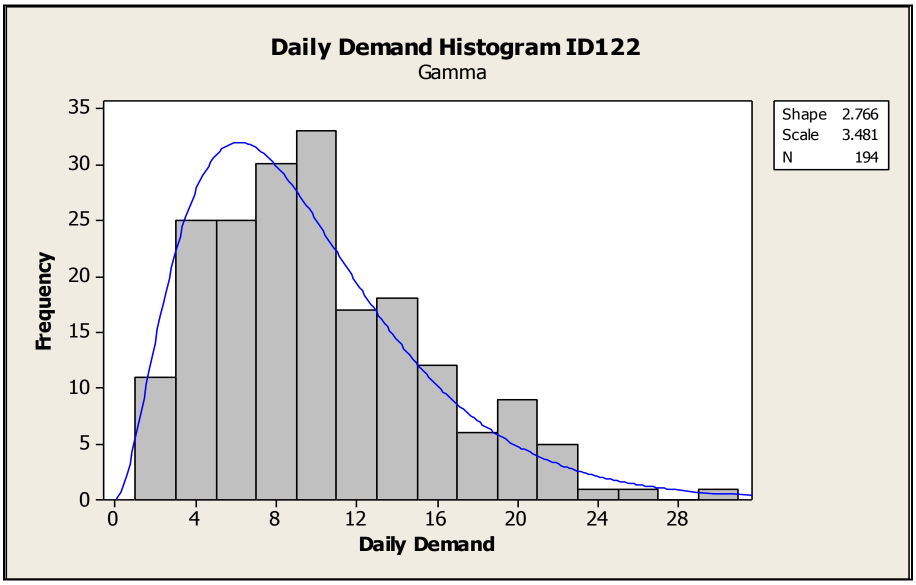

Figure B.6: Daily Demand Distributions for the item ID122