WELFARE-BASED EVALUATION OF ALTERNATIVE LOSS FUNCTIONS FOR SMALL OPEN ECONOMIES

A Master's Thesis

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The Institute of Economics and Social Sciences
of
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by

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

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July 2009

This master's thesis compares outcomes of alternative loss functions to optimal monetary policy in small open economies as the degree of openness increases. The small open economy model that is laid out by Gali and Monacelli (2005) is taken as the baseline framework. Based on a second order Taylor approximation to the utility function, the optimal monetary policy is derived. Then, using the optimal policy as a benchmark, four alternative loss functions are evaluated. Among others, minimizing the variance of domestic inflation achieves the minimum loss and is equivalent to optimal policy. Minimization of CPI inflation variance gives higher losses compared to minimization of domestic inflation variance. Lastly, attributing positive weight to dampening exchange rate fluctuations increases welfare losses.

Keywords: Loss function, small open economy, welfare evaluation, optimal monetary policy.

ÖZET

KÜÇÜK AÇIK EKONOMİLERDE REFAH DEĞERLENDİRMESİNE GÖRE ALTERNATİF KAYIP FONKSİYONLARI

ÖZHAN, GALİP KEMAL

Yüksek Lisans, Ekonomi Bölümü

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Bu master tezi küçük açık ekonomilerdeki kayıp fonksiyonlarını optimal para politikasına göre ekonomideki açıklık artarken karşılaştırmaktadır. Gali Monacelli (2005) makalesinde sunulan küçük açık ekonomi modeli temel olarak alınmıştır. Fayda fonksiyonuna ikinci dereceden Taylor yakınlaştırması yapilarak optimal para politikası kuralı bulunmuştur. Daha sonra, optimal para politikasına göre dört farklı alternatif kayıp fonksiyonu değerlendirilmiştir. Diğer kayıp fonksiyonlarına nazaran yerli malı enflasyonundaki sapmayı minimize eden kayıp fonksiyonu optimal para politikasıyla çakışarak minimum kaybı vermiştir. Tüketici enflasyonundaki sapmayı minimize eden refah kaybı fonksiyonu ise yerli malı enflasyonunundaki sapmayı minimize eden kayıp fonksiyonundan daha fazla refah kaybına yol açmıştır. Son olarak, döviz kuru dalgalanmasındaki sapmayı azaltmaya yönelik atılan pozitif ağırlıgın her zaman refah kaybını arttırdığı görülmüştür.

Anahtar Kelimeler: Küçük açık ekonomi, refah değerlendirmesi, kayıp fonksiyonu, optimal para politikası.

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CHAPTER 1

INTRODUCTION

Over the last decade optimal monetary policy in open economy general equilibrium models that feature imperfect competition and nominal rigidities has become a popular area of research. As pointed out by Bernanke (2007), as economies become more open and integrated with the world, prices and wages begin to depend on foreign economic conditions as well as domestic activities, and this linkage may cause globalization, and therefore trade, to affect domestic inflation.

"There is a fair amount of consensus in the academic literature that a desirable monetary policy is the one that achieves a low expected value of a discounted-loss function, where the losses each period are a weighted average of terms quadratic in the deviation of inflation from a target rate and in some measure of output relative to potential" (Woodford, 2003, p. 381). Determination of inflation and output gap in open economies will differ from that of closed economies, leading to different welfare losses from the same policies. In this master's thesis, we pose the following question as our main concern: which alternative loss function can give us the closest loss that is obtained from the fully optimal rule while the degree of openness in the economy is increasing? In order to answer this question we use the small open economy version of the Calvo sticky price model that is laid out in Gali and Monacelli (2005) (henceforth GM).

Openness in this framework represents the import-export activity of the economy being modeled. Figure 1 and Figure 2 report the ratio of import and export activity to the GDP for Turkey and the US. The increase in openness shown in these figures verify our concern about taking degree of openness into account.

This kind of framework was previously used by others. Obstfeld and Rogoff (1999) studied the optimal monetary policy in a model with one-period sticky wages. Clarida, Gali and Gertler (2001) (henceforth CGG (2001)) presented the optimal monetary policy under commitment and discretion. Benigno and Benigno (2003) analyzed the conditions when price stability rises as an equilibrium outcome. Sutherland (2005) analyzed the implications of cost-push shocks, and Zaniboni (2008) studied the dynamics assuming pricing-to-market by firms in this framework.[1]

Following the previous literature, with the assumptions of complete financial markets and price stickiness with a specific type of functional form, we are able to solve the dynamics of the system. After finding the New Keynesian Phillips Curve and the dynamic IS equation, we derive a second order Taylor approximation to the utility function of the home country's household to find the welfare loss function in order to specify our criteria in comparing the alternative policies.

Then, we write four ad-hoc but reasonable and simple alternative loss functions for the monetary authority in case they are not able to follow (or announce) the complicated optimal monetary policy, which is common for central banks. The dynamics of the four alternatives are compared after a technology shock, and losses obtained from our welfare criterion are discussed while the degree of openness in the economy is increasing.

The dynamics of the alternative that considers only minimizing the vari-

[1]Among many others, other examples of this literature include Corsetti and Pesenti (2001), Clarida, Gali and Gertler (2002), and Smets and Wouters (2003).

ance of inflation on domestically produced goods, namely domestic inflation, coincides with optimal monetary policy. Under the assumption of complete markets, a similar result is obtained by Bodenstein, Erceg and Guerrieri (2007) concerning oil price shocks: higher oil prices does not affect the relative wealth of an oil-importing country. It can be interpreted that under the complete markets assumption, if authorities are concerned with only domestic activities, shocks on production influence the dynamics as it does in the closed economy case. Also, this result justifies the view presented by Donald Kohn: independent central banks control their own destinies, although openness affects the parameters of central bank models. In addition, Ferrero, Gertler and Svensson (2008) found that, under an incomplete market assumption, monetary policy regime under current account adjustment scenarios has important consequences for the behavior of domestic variables, but much less for international variables. Their main finding is that the central bank should focus on targeting domestic inflation.

Among the other three alternative loss functions, we see that the aim of output stabilization next to CPI inflation targeting increases the welfare compared to only CPI inflation targeting. Finally, putting weight on dampening nominal exchange rate variance in the policy of the monetary authority leads to higher welfare losses for each degree of openness level.

The rest of the paper is organized as follows: section 2 introduces the small open economy model that is introduced by GM, Section 3 analyzes the optimal monetary policy, derives welfare loss function of the household, introduces alternative policies for the optimal monetary policy and discusses the dynamics and comparisons of the alternatives, and Section 4 concludes.

CHAPTER 2

A SMALL OPEN ECONOMY MODEL

2.1 Households

The model represented here is the standard model that is used in the New Keynesian Open Economy framework which is consisted of a continuum of small open economies represented by the unit interval, and where economies do not have any strategic behavior. Variables without an i index refer to the small open economy being modeled. Variable with $i \in [0,1]$ subscript refer to economy i, and variables with star superscript correspond to the world economy as a whole. The composite consumption index is defined by:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2.1}$$

Consumption of domestic goods, $C_{H,t}$, and consumption of foreign goods, $C_{F,t}$ are given in Dixit-Stiglitz form:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.2}$$

$$C_{F,t} = \left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} \tag{2.3}$$

Here, $C_{i,t}$ refers to consumption of goods imported from country i, and is given by:

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.4}$$

And, ε is the elasticity of substitution between consumption goods produced within any country, η is the elasticity of substitution between domestic and foreign goods, and γ is the elasticity of substitution between goods produced in different foreign countries. Parameter $\alpha \in [0,1]$ represents the openness of the economy as the share of the consumption of goods imported from abroad, and it is the share of the imported goods in the CPI consumption. In other words, the parameter α defines the import-export activity in the model. With this definition, α is inversely related to the degree of home bias in preferences, as in Sutherland (2005). Representative household seeks to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{2.5}$$

s.to

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}C_{i,t}(j)djdi + E_t\{Q_{t,t+1}B_{t+1}\} \leq B_t + W_tN_t + T_t \tag{2.6}$$

where

$$U(C_t, N_t) = \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{(N_t)^{1+\varphi}}{1+\varphi} \tag{2.7}$$

Notation is as follows: B_t is the quantity of one-period nominally riskless discount bonds purchased in t, maturing in $t+1$; $Q_{t,t+1}$ is the stochastic discount factor for one period ahead nominal payoffs; $P_{i,t}(j)$ is the price of the good j imported from country i, expressed in domestic currency; W_t is the nominal wage; T_t expresses the lump-sum taxes; and lastly, N_t is the labor supply. All variables are expressed in domestic currency and households have access to a complete set of contingent claims, traded internationally.

Household's allocation on its domestic consumption expenditures among differentiated goods (resulting from cost minimization) will be in the form:

$$C_{H,t}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} = C_{H,t}(j) \tag{2.8}$$

$$C_{i,t}\left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} = C_{i,t}(j) \tag{2.9}$$

where $P_{H,t} = (\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj)^{\frac{1}{1-\varepsilon}}$ is the index of domestically produced goods, namely domestic price index, and $P_{i,t} = (\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj)^{\frac{1}{1-\varepsilon}}$ is the price index of imported goods from country i in domestic currency. Furthermore, the price index for all imported goods in domestic currency will be denoted as $P_{F,t} = (\int_0^1 P_{i,t}^{1-\gamma} di)^{\frac{1}{1-\gamma}}$. It is convenient to note that $\int_0^1 P_{H,t}(j) C_{H,t}(j) dj = P_{H,t} C_{H,t}$ and $\int_0^1 P_{i,t}(j) C_{i,t}(j) dj = P_{i,t} C_{i,t}$.

Optimal consumption allocation between the domestic and foreign goods implies:

$$\frac{C_{H,t}}{C_{F,t}} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} \tag{2.10}$$

in this case, the consumer price index (CPI) is

$$P_t = \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta}\right]^{\frac{1}{(1-\eta)}}. \tag{2.11}$$

Price indexes for domestic and foreign goods are equal in steady state ($P_F = P_H$), and $C = C^* = Y = Y^*$. Then α corresponds to the share of domestic consumption allocated to imported goods, and combining (2.1) and (2.10) yields $\alpha = \frac{C_F}{Y}$. Since $P_{H,t} C_{H,t} + P_{F,t} C_{F,t} = P_t C_t$, optimality conditions will be:

$$\frac{N_t^{\varphi}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \tag{2.12}$$

$$\beta R_t E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1 \tag{2.13}$$

Equation (2.12) shows the optimal plan for the consumption and labor supply for the household. In addition, it also shows the labor supply schedule in terms of real wages for fixed marginal utility from the consumption. And, the latter is the Euler equation, where $R_t = E_t \left\{ \left(\frac{1}{Q_{t,t+1}} \right) \right\}$ is the gross nominal return on a one-period riskless bond. Equation (2.13) can be interpreted as the ratio of utility loss by buying an Arrow security to discounted utility gain from expected consumption in next period is equal to the gross nominal return.

Now, let's write the optimality conditions in log-linearized form:

$$\varphi n_t + \sigma c_t = w_t - p_t \tag{2.14}$$

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \tag{2.15}$$

where $\pi_t = p_t - p_{t-1}$ is the CPI inflation (with $p_t = logP_t$), $\rho = -log\beta$ is the time discount rate, and $i_t = -logQ_{t,t+1}$ is the nominal interest rate.

In the rest of the world, a representative agent faces the symmetric problem and optimality conditions are symmetric to our small open economy. However, since we assumed the continuum of economies, the rest of the world can be seen as a closed economy because our small economy is negligible according to the rest of the world.

Now, define terms of trade as $S_t = \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$ where $S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$, and in log-linearized form, $s_t = p_{F,t} - p_{H,t}$ (the price of foreign goods in terms of home goods). So, after we log-linearize the CPI formula, we obtain:

$$\begin{aligned} p_t &= (1-\alpha)p_{H,t} + \alpha p_{F,t} \\ &= p_{H,t} + \alpha s_t. \end{aligned} \tag{2.16}$$

Then, we can write the relationship between domestic inflation and CPI-inflation:

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \tag{2.17}$$

Equation (2.17) states that a household in the domestic economy will face with a CPI inflation rate that is influenced by the change of prices in the foreign world, and this influence is augmented as the economy becomes more open.

We do also assume that law-of-one price holds for both import and export prices, which implies $P_{i,t}(j) = \xi_t P^*_{i,t}(j)$ for all $i, j \in [0, 1]$ where $P^*_{i,t}(j)$ denotes the price of good j produced in country i in the currency of country i, and therefore ξ_t is the nominal exchange rate. Here, it is useful to note that the goods that are imported from the rest of the world are not domestically produced.

If we aggregate the previous equation over goods and over all foreign countries, we obtain $P_{F,t} = \xi_t P^*_{F,t}$. Then, combine the log-linearized version of this condition with the terms of trade condition, we obtain the terms of trade condition in terms of nominal exchange rate and foreign good's price in foreign currency:

$$s_t = e_t + p^*_{F,t} - p_{H,t} \tag{2.18}$$

Here $e_t = log \xi_t$. Since our small economy is negligible according to rest of the world, we can write $p^*_{F,t} = p^*_t$.

2.1.1 International Risk Sharing

With the assumption of domestic and international complete financial markets, we can write the Euler equation of a foreign country as a symmetric case of our small open economy's. Then dividing two symmetric equations leads to:

$$C_t = \varkappa C^*_t \Omega_t^{1/\sigma} \tag{2.19}$$

where $\varkappa$ is a constant that depends on initial conditions on relative asset holdings, and $\Omega_t = \frac{\xi_t P_t^*}{P_t}$ is the real exchange rate. By the assumption of zero net foreign asset holding, and identical environments, $\varkappa = 1$.

It is useful to notice here that while the law of one price holds for each individual good, real exchange rate may still fluctuate between different periods due to differences between domestic and world consumption baskets. But, in the symmetric perfect foresight steady state, with the help of symmetry of the productivity levels among all countries, we have $\Omega_t = S_t = 1$, and this implies that purchasing power parity holds at the steady state.

After log-linearizing the equation (2.19) around zero, we obtain the international risk sharing condition which shows us the linkage between the home consumption and consumption in foreign countries and the terms of trade condition via complete financial markets:

$$c_t = c_t^* + \frac{(1-\alpha)}{\sigma} s_t \tag{2.20}$$

Thus, a country whose real exchange rate depreciates will experience faster consumption growth than the rest of the world, and as domestic country becomes more open, the gains from real exchange rate depreciation will be faded.[1]

This equation has its meaning when there exists an import-export activity among the countries. However, as economy becomes fully closed (i.e. $\alpha \rightarrow 0$), due to law of one price assumption and symmetric preferences among countries it again tells us domestic consumption is equal to foreign consumption.

2.1.2 Uncovered Interest Parity Condition

What we need to do is to combine the pricing equation of a riskless bond in foreign currency with the domestic bond pricing equation. Namely, $R_t^{-1} =$

[1]This is consistent with the risk sharing condition that is proposed and tested by Backus and Smith (1993).

$E_t\{Q_{t,t+1}\}$ and, $\xi_t R_t^{*-1} = E_t\{Q_{t,t+1}\xi_{t+1}\}$. We obtain:

$$E_t\left\{Q_{t,t+1}\left[R_t - R_t^*\left(\frac{\xi_{t+1}}{\xi_t}\right)\right]\right\} = 0 \tag{2.21}$$

After log-linearizing the last equation around steady state, it yields to:

$$i_t - i_t^* = E_t\{e_{t+1}\} - e_t. \tag{2.22}$$

And, we can combine the uncovered interest parity condition with the previous terms of trade equation to show that variations in the terms of trade are a function of current and anticipated real interest rate differentials:

$$s_t = (i_t^* - E_t\{\pi_{t+1}^*\}) - (i_t - E_t\{\pi_{H,t+1}\}) + E_t\{s_{t+1}\} \tag{2.23}$$

Equation (2.23) shows that a change in terms of trade between two consecutive periods is adjusted by the change in difference of the real interest rates (i.e. a relative increase in prices of imported goods between two consecutive periods will lead to an increase in terms of trade and in foreign inflation next period which will balance each other).

Moreover, since PPP requiring to hold at steady state implies that $lim_{T\to\infty}$ $E_t\{s_T\} = 0$, we can iterate (2.23) to obtain:

$$s_t = E_t\{\sum_{k=0}^{\infty}[(i_{t+k}^* - \pi_{t+k+1}^*) - (i_{t+k} - \pi_{t+k+1})]\} \tag{2.24}$$

Assumption of PPP holding at steady state implies that the difference between foreign and home real interest rates will be zero at steady state.

2.2 Firms

On the supply side of the economy, home economy produces with the following production technology:

$$Y_t(j) = A_t N_t(j) \tag{2.25}$$

where $a_t = logA_t$ follows an AR(1) process $a_t = \rho_a a_{t-1} + \epsilon_t$.

The aggregate domestic output index will be of the form:

$$Y_t = \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.26}$$

In log-linearized terms, we have

$$y_t = a_t + n_t. \tag{2.27}$$

In the rest of the world, firms produce goods with the technology $a_t^* = \rho_a^* a_{t-1}^* + \epsilon_t^*$.

Firms set their prices just like in Calvo(1983) with staggered-price mechanism. We assume that θ is the portion of firms that do not change their prices each period. Optimal price setting strategy for the typical firm resetting its price in period t is approximated by log-linearization:

$$\bar{p}_{H,t} = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{mc_{t+k}\} \tag{2.28}$$

Here $\bar{p}_{H,t}$ is the new set domestic price, $\mu = log(\frac{\varepsilon}{1-\varepsilon})$ is the gross markup in the steady state, equivalently the equilibrium markup of flexible price economy, and mc_t is the real marginal cost. The determination of real marginal cost in terms of domestic output differs from the one in closed economy due to the disparity between consumption and output, and between CPI and domestic prices.

$$
\begin{aligned}
mc_t &= \vartheta + w_t - p_{H,t} - a_t \\
&= \vartheta + w_t - p_t + p_t - p_{H,t} - a_t \\
&= \vartheta + \sigma c_t + \varphi n_t + \alpha s_t - a_t \\
&= \vartheta + \sigma y_t^* + \varphi y_t + s_t - (1+\varphi) a_t
\end{aligned} \tag{2.29}
$$

In this context, $\vartheta = log(1-\tau)$, and τ is the employment subsidy that neutralizes the distortion arisen from firms' market power, and its role will be important while making monetary policy analysis. The effects of domestic output and technology are in the same manner with the closed economy case as they influence the productivity and the employment.[2] However, mc_t is an increasing function in world output and terms of trade, and they are both influencing the real wages through the wealth effect.

The price setting mechanism in the rest of the world is in the same manner as in the small open economy's. For simplicity, we assume $\theta^* = \theta$ which implies the price changing behavior is the same all over the world.

2.3 Equilibrium

2.3.1 Aggregate Demand, Output and Inflation Dynamics

First, by aggregating over all countries, the world market clearing condition will be: $y_t^* = \int_0^1 y_t^i di = \int_0^1 c_t^i di = c_t^*$. Since the preferences of representative household are identical to the one in the small open economy, we can write:

$$
y_t^* = E_t\{y_{t+1}^*\} - \frac{1}{\sigma}(i_t^* - E_t\{\pi_{t+1}^*\} - \rho) \tag{2.30}
$$

[2]See e.g. Clarida, Gali and Gertler (1999).

Goods market clearing in the home country yields to:

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C^i_{H,t}(j)di \tag{2.31}$$

where $C^i_{H,t}(j)$ denotes country i's demand for good j produced in the home economy.

Inserting the optimal consumption demand equations into the latter and using the analogous expression for the rest of the world, we get:

$$\begin{aligned} Y_t(j) &= C_{H,t}(j) + C^*_{H,t}(j) \\ &= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \varkappa Y^*_t \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} (1-\alpha)\Omega_t^{1/\sigma} + \left(\frac{P_{H,t}}{\xi_t P^*_t}\right)\alpha\right] \end{aligned} \tag{2.32}$$

Since $Y_t = \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$, by the use of the previous equation we obtain:

$$Y_t = \varkappa Y^*_t S^\eta_t[(1-\alpha)\Omega_t^{\frac{1}{\sigma}-\eta} + \alpha] \tag{2.33}$$

First order approximation of this expression will give us

$$y_t = y^*_t + \frac{1}{\sigma_\alpha} s_t \tag{2.34}$$

where $\sigma_\alpha = \frac{\sigma}{(1-\alpha)+\alpha\omega}$ and $\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$. Domestic country and foreign country balance their production according to their trade levels. A change in output affects terms of trade which is captured by $\frac{1}{\sigma_\alpha}$. It is also important to note that when $\sigma = \gamma = \eta = 1$, $\omega = 1$, a boom in aggregate output of the home country will follow an increase in foreign output and/or terms of trade due to impact of import-export activity.

In order to see the domestic consumption as a weighted average of domestic and world output for the special case when $\sigma = \gamma = \eta = 1$, we can use the last equation with the international risk sharing equation to get:

$$c_t = (1-\alpha)y_t + \alpha y_t^* \tag{2.35}$$

This implies that CPI consumption demand is divided between goods that are bought from abroad and home product goods depending on the degree of openness of the country.

Finally, the last two equations combined with the Euler gives us the domestic output in terms of domestic real interest rate and world output:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}(i_t - E_t\{\pi_{H,t+1}\} - \rho) + \alpha(\omega - 1)E_t\{\Delta y_{t+1}^*\} \tag{2.36}$$

It is useful to note that any change in domestic real interest rate $(i_t - E_t\{\pi_{H,t+1}\})$ will affect the output, and the sensitivity between these two will be higher as degree of openness of the country changes. For given foreign output, an increase in real rates will lead to a real appreciation which follows an increase in CPI inflation and a decrease in consumption based real rate $(i_t - E_t\{\pi_{t+1}\})$, which offsets the effect of the change.

Second, the dynamics of the inflation in the world economy is also characterized by staggered price mechanism Calvo. By combining optimal price setting decision for world economy with evolution of the aggregate price level one can find

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \lambda \tilde{mc_t}^* \tag{2.37}$$

where $\tilde{mc_t}^*$ is the deviation of the marginal cost from its steady state value and $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. The inflation dynamics in the small open economy is analogous to the rest of the world:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda \tilde{mc}_t. \tag{2.38}$$

2.3.2 A Canonical Representation

One can use the marginal cost equation, which is evaluated at the steady state, and equation (2.29) to obtain the natural level of output. Then, we can write the marginal cost in terms of output gap. After using the previous equations, we obtain the relationship between real marginal cost and the output gap in GM.[3]

$$\tilde{mc}_t = (\sigma_\alpha + \varphi)\tilde{y}_t \tag{2.39}$$

Combine this result with inflation dynamics, and we will get the New Keynesian Phillips Curve (NKPC) for the small open economy:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_\alpha \tilde{y}_t \tag{2.40}$$

where $\kappa_\alpha = \lambda(\sigma_\alpha + \varphi)$. And with the help of the Euler, we have the dynamic IS (DIS) equation:

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha}(i_t - E_t\{\pi_{H,t+1}\} - rr_t) \tag{2.41}$$

where the Wicksellian real rate of interest is

$$rr_t = \rho - \frac{\sigma + (1+\varphi)(1-\rho_a)}{\sigma + \varphi\omega} a_t - \varphi \frac{\sigma(1-\omega)}{\sigma + \phi\omega} E_t\{\Delta y^*_{t+1}\} \tag{2.42}$$

GM obtained a NKPC for the open economy very similar to the one in closed economy as in CGG(2001) and CGG(2002). The degree of openness of the country affects the inflation dynamics by influencing the variation in the output gap. In open economy, a change in domestic output has an effect on marginal cost through employment and terms of trade. In addition, DIS shows that the openness influences the output gap sensitivity to interest rate changes, and natural rate depends on expected increase in world output in

[3] Here, $\tilde{y}_t$ denotes the deviation of output from potential.

addition to domestic productivity through openness as previously discussed.

CHAPTER 3

OPTIMAL MONETARY POLICY AND ALTERNATIVE LOSS FUNCTIONS

In this section, we derive the optimal monetary policy and discuss the equilibrium dynamics, first, when both domestic and foreign central banks pursue the optimal monetary policy, and second, when domestic monetary authority aims to minimize different welfare rule based loss functions.

Following Rotemberg and Woodford(1999), fiscal authority in the world economy fully neutralizes the distortions which are caused from firms' market power, with a constant employment subsidy, and the optimal monetary policy replicates the flexible price equilibrium allocation in the world economy (which is considered as a closed economy). Optimal monetary policy in such an environment is the one that fully stabilizes the prices and output gap, that is $\tilde{y}_t^* = \pi_t^* = 0$. The interest rate that supports the optimal allocation is given in equation (2.42). On the other side of the coin, in the open economy, as discussed by Corsetti and Pesenti (2001), there exists another economic distortion that is a direct result of openness. A country may have an incentive to affect its terms of trade by influencing the supply of its products. This statement is a result coming from imperfect elasticity of substitution between domestic and foreign goods. If we set $\sigma = \gamma = \eta = 1$, then there exists a constant employment subsidy that offsets the distortions coming from market power and letting $\gamma = 1$ will yield no distortions due to terms of trade. Then, again the optimal policy becomes the flexible price equilibrium allocation.[1] As in the closed economy, the optimal monetary policy requires stabilizing output gap, $\tilde{y}_t = 0$, then equation (2.40) shows that $\pi_{H,t} = 0$ holds.[2]

3.1 Welfare Analysis and Alternative Loss Functions

After making a second order Taylor approximation to the utility of the representative household in the small open economy, as in GM, we find

$$\hat{W}_t = \frac{(1-\alpha)}{2} \sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon}{\lambda} \pi_{H,t}^2 + (1+\varphi) \tilde{y}_t^{\,2} \right] + t.i.p. + o(||a||^3). \tag{3.1}$$

Here, $t.i.p.$ stands for terms independent of policy, and the derivation of the second order approximation can be seen in Appendix C. Following Rudebusch and Svensson (1999), and Rotemberg and Woodford (1999), with taking unconditional expectations on the latter equation and letting $\beta \to 1$, the expected welfare losses obtained from deviating from strict inflation targeting can be written as variances of domestic inflation and output gap. When $\beta \to 1$, the sum in equation (3.1) becomes unbounded, and it is consisted of two components: one corresponding to the deterministic optimization problem when all shocks are zero, and the other proportional to the variances of the shocks. The latter converges when $\beta \to 1$, because terms approach zero quickly, and the decision problem becomes well defined. The intertemporal loss function approaches the infinite sums of unconditional means of the period loss function, and the scaled loss func-

[1] In a 2-country open economy setting like in Benigno and Benigno (2003), it is enough to set $\sigma = \eta = 1$. However, while considering multiple foreign countries, we must consider setting the substitutability between goods produced in different foreign countries to 1, i.e. $\gamma = 1$.

[2] Derivation of this constant employment subsidy for both economies is shown in Appendix D.

tion, $(1-\beta)\sum_{t=0}^{\infty}\beta^t\left[\frac{1-\alpha}{2}\left(\frac{\varepsilon}{\lambda}\pi_{H,t}^2+(1+\varphi)y_t^2\right)\right]$, approaches the unconditional mean:[3]

$$E[L_t] = \frac{1-\alpha}{2}\left[\frac{\varepsilon}{\lambda}Var[\pi_{H,t}] + (1+\varphi)Var[\tilde{y}_t]\right]. \tag{3.2}$$

So, we can interpret the intertemporal loss function as the unconditional mean of the period loss function, and now we recommend reasonable and simple alternative loss functions to the one derived from the second order approximation. The alternatives are the one which is considering only stabilization of domestic inflation, the one that admires to minimize the loss coming from CPI-inflation and output variations, domestic inflation and exchange rate variability, and lastly, the loss function that wishes to minimize the loss due to variations in CPI inflation and exchange rate variability. Namely,

$$\acute{L}_t^1 = Var[\pi_{H,t}] \tag{3.3}$$

$$\acute{L}_t^2 = Var[\pi_t] + Var[\tilde{y}_t] \tag{3.4}$$

$$\acute{L}_t^3 = Var[\pi_{H,t}] + Var[e_t] \tag{3.5}$$

$$\acute{L}_t^4 = Var[\pi_t] + Var[e_t]. \tag{3.6}$$

Here, one should note that the measure that we use while we are comparing these loss functions is equation (3.2). Monetary policy authority considers the alternative loss functions that are stated in (3.3)-(3.6), and for each policy we calculate the losses according to equation (3.2), which gives us the true welfare losses when alternative policies are applied.

3.2 Calibration and Impulse Responses

We follow the same calibration as in GM. We set $\sigma = \eta = \gamma = 1$, $\varphi = 3$, where the latter implies that elasticity of labor is 1/3. The value of the steady state ratio of the prices to the marginal cost, mark-up, $\mu = 1.2$, and elasticity of

[3]One can check Woodford(2003, Chapter 6) for a more detailed discussion.

substitution between goods, $\varepsilon = 6$. The probability of firms that reset their prices to 0.25, namely $\theta = 0.75$. And, $\beta = 0.99$. Finally, we set $\rho_a^* = \rho_a = 0.9$, and the correlation between the domestic and world shocks is 0.77.

The impulse responses after a positive domestic technology shock while we are applying the fully optimal policy is shown in Figure 3 (for the case $\alpha = 0.40$). Figure 3 also represents the impulse responses when monetary authority considers equation (3.3) in order to minimize only the variation in domestic inflation. As we discussed above, a domestic inflation targeting will yield to $\pi_{H,t} = 0$. This in turn will make $\tilde{y}_t = 0$, through equation (2.40), and optimal policy coincides with first alternative irrespective to the degree of openness parameter. We can say that, for every degree of openness parameter, concerning only domestic inflation will give us the same results with the optimal monetary policy. Besides, we observe that after a technology innovation, domestic nominal interest rate falls in order to support the increase in consumption and output. In addition, uncovered interest parity implies that a fall in the interest rate of the open economy will be adjusted by a relative increase in the next period's terms of trade and domestic currency will depreciate against foreign currency, given the constant world interest rate, which will yield to an increase in CPI inflation.

Figure 4 shows the impulse responses after a technological innovation while the central bank is applying second alternative (equation (3.4)). Here, we fixed the degree of openness parameter, α to 0.40. When the central bank is concerning about stabilizing the summation of variances in CPI inflation and output gap, a positive shock leads to an increase in CPI inflation which is the result of depreciating exchange rates due to an increase in output. In this case, real exchange rate reaction is lessened by considering CPI inflation rather than domestic inflation, which yielded hump shaped graphs for exchange rates.

Impulse response graphs for a positive production shock when the central bank concerns to minimize the variations in exchange rate and domestic inflation are shown in Figure 5. Relative to second alternative, deviations in output gap and domestic inflation increase but they follow a pattern in the same manner. Since output gap is not concerned by the central bank, a higher deviation takes place. And, in order to support the expansion in production, exchange rates depreciate, which in turn yields a decrease in open economy's interest rate (again, for given constant world interest rate).

Lastly, in Figure 6, we obtain similar impulse responses for the fourth alternative according to third alternative. The main difference is the less depreciation of nominal exchange rate with respect to the one for alternative three. The reason is that the central bank is concerned about CPI inflation in alternative four instead of domestic inflation as in alternative three. However, we see a similar pattern for the real exchange rates. Nominal interest rate has a smaller decrease than the one in impulse response functions of alternative three, due to less nominal exchange rate variation.

These impulse response graphs show similar reactions oppositely but symmetrically when a shock comes from foreign world, rather than the home country.

3.3 Welfare Losses

In this part, we calculate the welfare losses of each policy while the degree of openness in the economy is increasing, and make a comparison with the ad-hoc alternative policies. As can be seen from the Table 1, and as we mentioned before, first alternative policy coincides with optimal monetary policy according to welfare losses. Stabilizing the domestic inflation stabilizes the output gap through NKPC irrespective of the degree of openness. Among the other alternatives, it is clearly seen that second alternative policy dominates third and fourth alternative policies. The fall in CPI inflation, with respect to optimal and first alternative case, requires a contractionary monetary policy. In addition, as economy becomes closed, concerning CPI inflation implies concerning domestic inflation. Thus, we see losses close to zero as degree of openness is getting smaller. On the other hand, as economy becomes more open, taking CPI inflation in consideration becomes important.

It is also seen that third alternative policy dominates the fourth policy, while these policies are giving relatively higher losses according to other alternatives. Concerning exchange rate volatility yields to a higher volatility in domestic inflation rather than other alternatives. As economy becomes closed, the optimal loss function penalizes more for including exchange rates in consideration. Lastly, it is observable that not considering output stabilization in the policy gives higher losses for each degree of openness value.

In order to quantify the effects of considering output gap and exchange rate next to CPI inflation in the alternative loss functions two and four, we can modify them as

$$\breve{L}_t^2 = \psi Var[\pi_t] + (1-\psi) Var[\tilde{y}_t] \tag{3.7}$$

$$\breve{L}_t^4 = \varsigma Var[\pi_t] + (1-\varsigma) Var[e_t].$$

where $\varsigma, \psi \in (0,1)$ and determines the weight on each variable. The results obtained by varying ς and ψ are in Table 2 and Table 3 (for constant $\alpha = 0.40$). It is clearly seen that increasing the incentive to stabilize the variation in CPI inflation increases the losses obtained from second alternative, on the contrary, as $\psi \to 0$ losses converge to zero due to the property of the optimal monetary policy. In alternative four, since we did not include the output gap and domestic inflation in the loss function, giving more weight to stabilizing exchange rate variability yields more losses and concerning with only CPI inflation has fewer losses than those obtained from the alternatives three and four.

CHAPTER 4

CONCLUSION

This master's thesis compared ad-hoc but reasonable and simple alternative loss functions as rising alternative monetary policies to the optimal monetary policy while the degree of openness in a country is increasing by using the small open economy model that is presented in GM.

The welfare cost of alternative policies is calculated by using the fully optimal policy as a benchmark. It is found that considering the minimization of domestic inflation variances coincides with the optimal rule. Due to the structure of the NKPC, considering output gap variation in monetary policy leads to fewer losses according to consideration of other variables. It is also observed that taking minimization of variance of nominal exchange rate in consideration yields higher welfare losses for any given policy.

There are several examples from the previous literature that supports this result. In addition to the ones that are discussed in introduction, Kösem Alp(2009) analyzed the relevance of sectoral inflation persistence differences with optimal monetary policy using a two-sector sticky price model, and found that optimal inflation targeting rather than CPI inflation targeting reduces the welfare losses. It can be interpreted as an analysis of the optimal monetary policy under a currency union with heterogeneous regions, and concerning about optimal rule rather than CPI targeting rises as another support to this paper.

A further interesting extension to this paper would be considering incomplete markets to fill the gap in the literature. Considering incomplete markets assumption with monetary policy can yield closer results for emerging countries. Finally, one might quantify the benefits gained from monetary policy coordination under incomplete markets as a further research on this paper.

BIBLIOGRAPHY

Backus, David and Smith, Gregor (1993), "Consumption and Real Exchange Rates in Dynamic Exchange Economies with Nontraded Goods." *Journal of International Economics*, pp. 297-316.

Benigno, Gianluca and Benigno, Pierpaolo (2003), "Price Stability in Open Economies." *Review of Economic Studies*, pp.743-764.

Bernanke, Ben (2007), "Globalization and Monetary Policy." Speech at the Fourth Economic Summit, Stanford Institute for Economic Policy Research.

Bodenstein, Martin; Erceg, Christopher and Guerrieri, Luca (2007), "Oil Shocks and External Adjustment." *International Finance Discussion Papers*, Board of Governors of the Federal Reserve System.

Calvo, Guillermo A. (1983), "Staggered Prices in a Utility Maximizing Framework." *Journal of Monetary Economics*, pp.383-398.

Clarida, Richard; Gali, Jordi and Gertler, Mark (1999), "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature*, pp.1661-1707.

Clarida, Richard; Gali, Jordi and Gertler, Mark (2001), "Optimal Monetary Policy in Open versus Closed Economies: An Integrated Approach." *American Economic Review*, pp.248-252.

Clarida, Richard; Gali, Jordi and Gertler, Mark (2002) "A Simple Framework for International Monetary Policy Analysis." *Journal of Monetary Economics*, pp.879-904.

Corsetti, Giancarlo and Pesenti, Paolo (2001), "Welfare and Macroeconomic Interdependence." *Quarterly Journal of Economics*, pp.281-305.

Ferrero, Andrea; Gertler, Mark and Svensson, Lars E.O. (2008), "Current Account Dynamics and Monetary Policy." NBER Working Paper No. 13906.

Juillard, Michel (2001), "DYNARE: A Program for the Simulation of Rational Expectation Models." *Computing in Economics and Finance*, Society for Computational Economics.

Kohn, Donald L. (2006), "The Effects of Globalization on Inflation and Their Implications for Monetary Policy." Remarks at the Federal Reserve Bank of Boston's 51st Economic Conference, Chatham, Massachusetts.

Kösem Alp, Sevim (2009), "Heterogeneity in Inflation Persistence and Optimal Monetary Policy." mimeo, Bilkent University.

Obstfeld, Maurice and Rogoff, Kenneth (1999), "New Directions for Stochastic Open Economy Models." *Journal of International Economics*, pp.117-153.

Rotemberg, Julio and Woodford, Michael (1999), " Interest Rate Rules in an Estimated Sticky Price Model." In J.B. Taylor (ed.) Monetary Policy Rules, University of Chicago Press.

Rudebusch, Glenn and Svensson, Lars E.O. (1999) "Policy Rules for Inflation Targeting." In J.B. Taylor (ed.) *Monetary Policy Rules*, University of Chicago Press.

Smets, Frank and Wouters, Raf. (2002), "Openness, Imperfect Exchange Rate Pass-Through and Monetary Policy." *Journal of Monetary Economics*, pp. 947-981.

Sutherland, Alan (2005), "Cost-Push Shocks and Monetary Policy in Open Economies." *Oxford Economic Papers*.

Woodford, Michael (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton, NJ: Princeton University Press.

Zaniboni, Nicola (2008), "Globalization and the Phillips Curve." mimeo, Princeton University.

APPENDICES

APPENDIX A

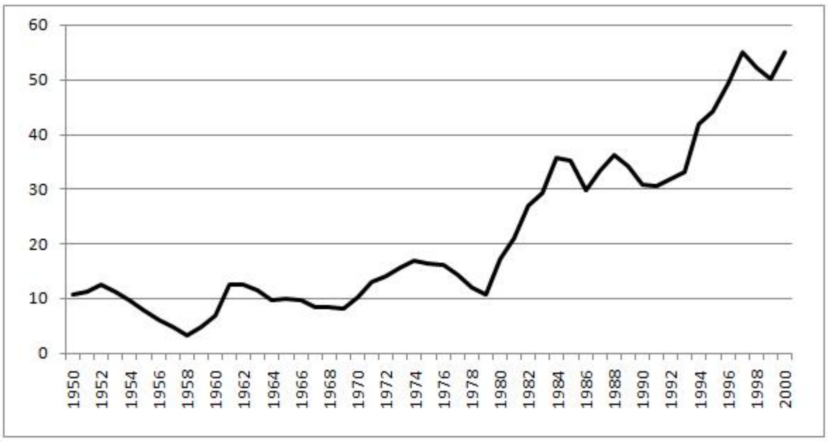

Figure 4.1: Ratio of Imports plus Exports to GDP (Turkey)

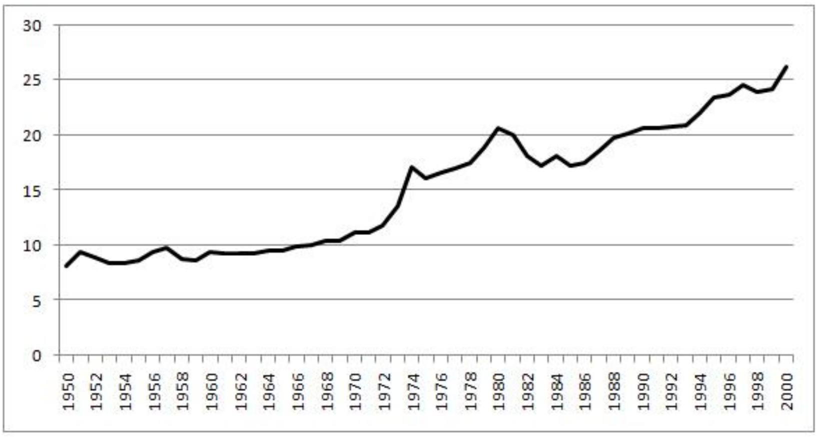

Figure 4.2: Ratio of Imports plus Exports to GDP (USA)

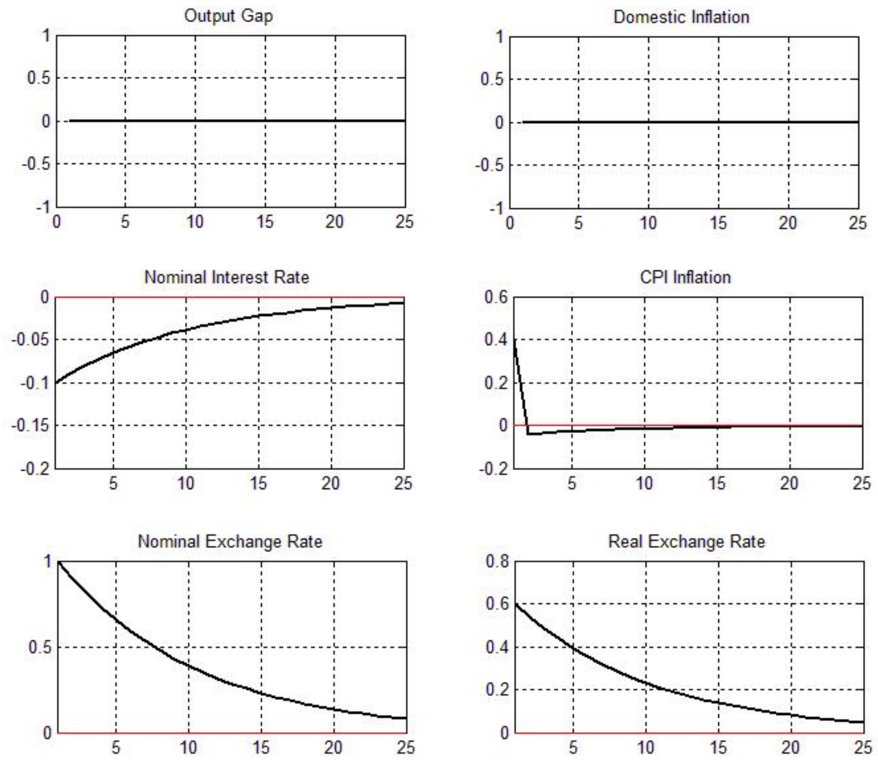

Figure 4.3: Impulse Responses to a Domestic Productivity Shock under the Optimal Policy and Alternative 1

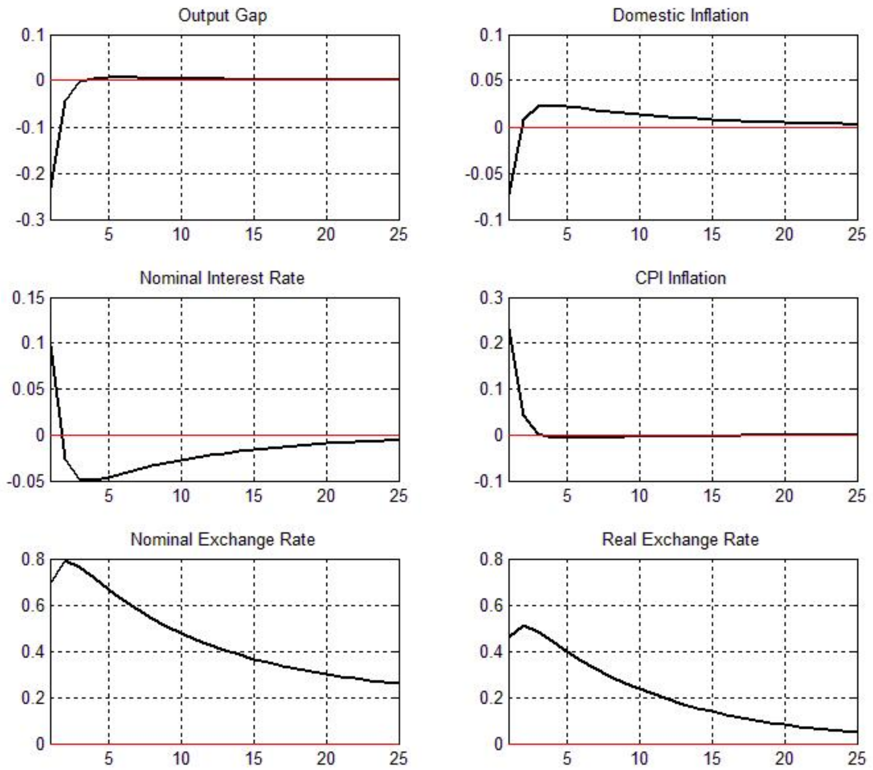

Figure 4.4: Impulse Responses to a Domestic Productivity Shock under the Alternative Policy 2

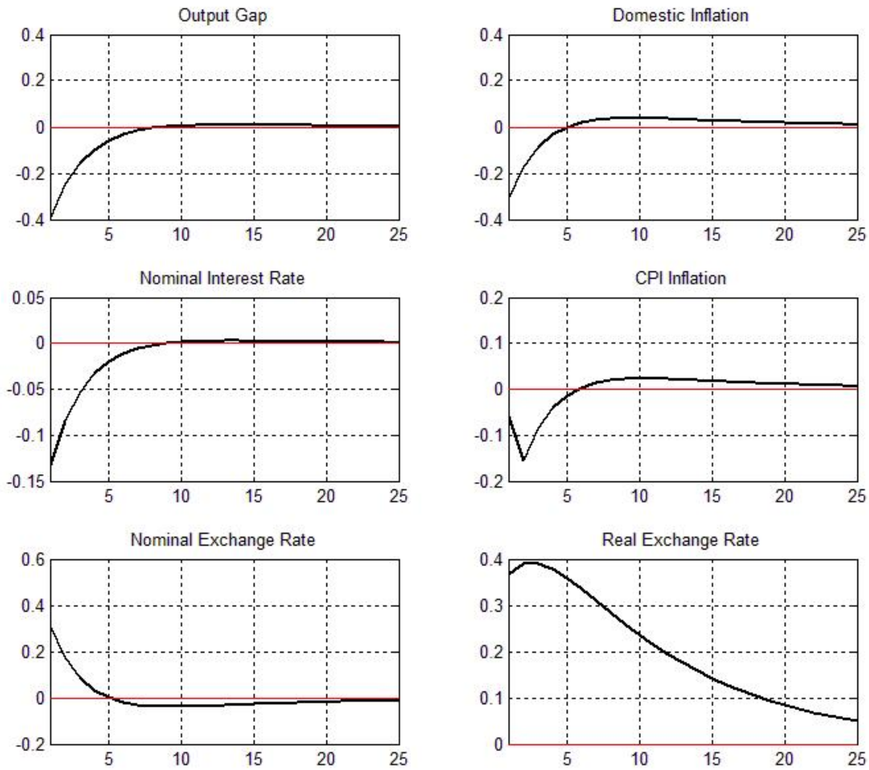

Figure 4.5: Impulse Responses to a Domestic Productivity Shock under the Alternative Policy 3

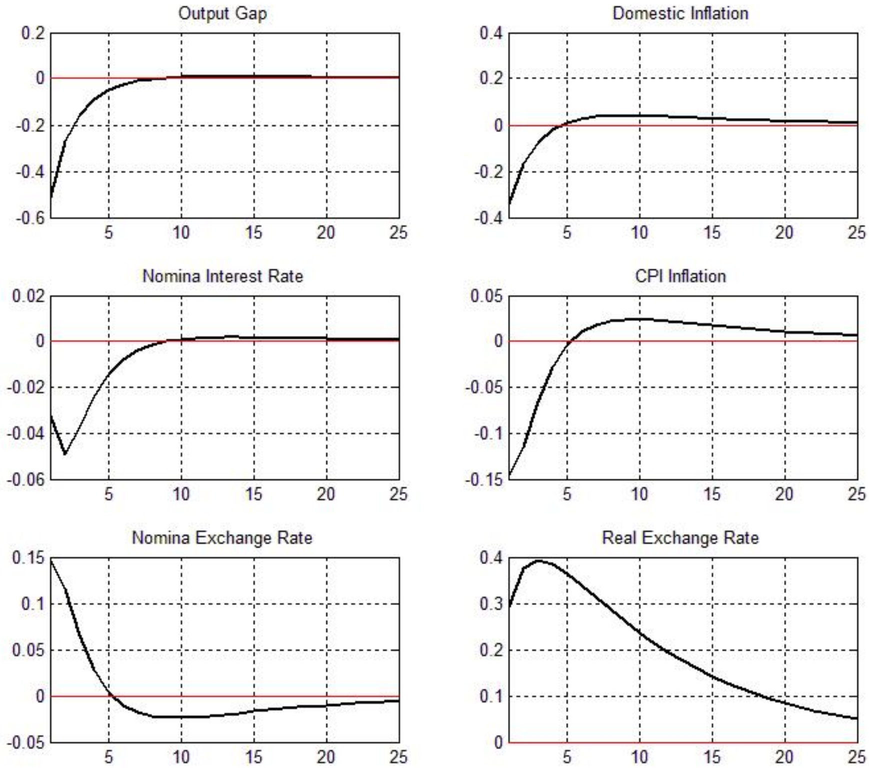

Figure 4.6: Impulse Responses to a Domestic Productivity Shock under the Alternative Policy 4

APPENDIX B

Table 4.1: Welfare Losses According to Each Policy

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Optimal Policy	0	0	0	0	0	0	0	0	0
Alternative 1	0	0	0	0	0	0	0	0	0
Alternative 2	0.03	0.09	0.18	0.25	0.31	0.34	0.33	0.28	0.17
Alternative 3	5.06	4.50	3.94	3.37	2.81	2.25	1.68	1.12	0.56
Alternative 4	5.27	4.88	4.45	3.97	3.44	2.85	2.22	1.53	0.79

Table 4.2: Welfare Losses While Changing the Weight on CPI Inflation and Output Gap

ψ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Alternative 2	0.01	0.04	0.09	0.16	0.25	0.39	0.57	0.82	1.12

Table 4.3: Welfare Losses While Changing the Weight on CPI Inflation and Nominal Exchange Rate

ς	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Alternative 4	4.81	4.65	4.47	4.24	3.97	3.62	3.19	2.65	2.02

APPENDIX C

As discussed in the main text, the derivation of the welfare loss function is done for the case where $\gamma = \eta = \sigma = 1$.

$$\frac{Y_t - Y}{Y} = y_t + \frac{1}{2}y_t^2 + o(||a||^3) \tag{4.1}$$

where $o(||a||^3)$ represents the terms that are in higher order than 3^{rd}. Now the utility consumption,

$$\log C_t = \bar{c_t} + \tilde{c_t} = \bar{c_t} + (1-\alpha)\tilde{y_t} \tag{4.2}$$

where $\bar{c_t}$ is the potential consumption level, and the latter is written by using equation (2.34).

Now, approximate the disutility gained from labor,

$$\frac{N_t^{1+\varphi}}{1+\varphi} = \frac{\bar{N_t}^{1+\varphi}}{1+\varphi} + \bar{N_t}^{1+\varphi}\left[\tilde{n_t} + \frac{1}{2}(1+\varphi)\tilde{n_t}^2\right] + o(||a||^3) \tag{4.3}$$

Since, $N_t = (\frac{Y_t}{A_t})\int_0^1 (\frac{P_{H,t}(j)}{P_{H,t}})^{-\varepsilon} dj$ we have,

$$\tilde{n_t} = \tilde{y_t} + \log \int_0^1 (\frac{P_{H,t}(j)}{P_{H,t}})^{-\varepsilon} dj \tag{4.4}$$

Now, define $\bar{p}_{H,t}(j) = p_{H,t}(j) - p_{H,t}$. We also have $\int_0^1 (\frac{P_{H,t}(j)}{P_{H,t}})^{1-\varepsilon} dj = 1$.

Now,

$$(\frac{P_{H,t}(j)}{P_{H,t}})^{1-\varepsilon} = \exp[(1-\varepsilon)\bar{p}_{H,t}(j)] \tag{4.5}$$
$$= 1 + (1-\varepsilon)\bar{p}_{H,t}(j) + \frac{(1-\varepsilon)^2}{2}\bar{p}_{H,t}(j)^2 + o(||a||^3).$$

A second approximation to $(\frac{P_{H,t}(j)}{P_{H,t}})^{-\varepsilon}$ yields,

$$(\frac{P_{H,t}(j)}{P_{H,t}})^{-\varepsilon} = 1 - \varepsilon\bar{p}_{H,t}(j) + \frac{\varepsilon^2}{2}\bar{p}_{H,t}(j)^2 + o(||a||^3) \tag{4.6}$$

Here, combining the results give us,

$$\int_0^1 (\frac{P_{H,t}(j)}{P_{H,t}})^{-\varepsilon} dj = 1 + \frac{\varepsilon}{2}E\{\bar{p}_{H,t}(j)^2\} \tag{4.7}$$
$$= 1 + \frac{\varepsilon}{2}Var\{\bar{p}_{H,t}(j)\}$$

Then, it follows that,

$$\log \int_0^1 (\frac{P_{H,t}(j)}{P_{H,t}})^{-\varepsilon} dj = \frac{\varepsilon}{2}Var\{\bar{p}_{H,t}(j)\} + o(||a||^3) \tag{4.8}$$

Now, we can write the disutility from labor as,

$$\frac{N_t^{1+\varphi}}{1+\varphi} = \frac{\bar{N}_t^{1+\varphi}}{1+\varphi} + \bar{N}_t^{1+\varphi}[\tilde{y}_t + \frac{\varepsilon}{2}Var\{\bar{p}_{H,t}(j)\} + \frac{1}{2}\bar{(1+\varphi)}\tilde{y}_t{}^2] + o(||a||^3). \tag{4.9}$$

Insert the optimality condition that $\bar{N_t}^{1+\varphi} = (1-\alpha)$ to convert the utility function into,

$$-(1-\alpha)[\frac{\varepsilon}{2}Var\{\bar{p}_{H,t}(j)\} + \frac{1}{2}(1+\varphi)\tilde{y}_t{}^2] + o(||a||^3). \tag{4.10}$$

Lemma 1. $\sum_{t=0}^{\infty}\beta^t Var\{\bar{p}_{H,t}(j)\} = \frac{1}{\lambda}\sum_{t=0}^{\infty}\beta^t \pi_{H,t}^2$, *where* $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

Proof. $Woodford$ (2003, $Chapter$ 6). □

Combining the above lemma with the previous result gives us the second order approximation to the small open economy's household's utility function:

$$\hat{W}_t = -\frac{(1-\alpha)}{2}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\lambda}\pi_{H,t}^2 + (1+\varphi)\tilde{y}_t^2\right] + t.i.p. + o(||a||^3). \qquad (4.11)$$

APPENDIX D

In order to find the employment subsidy that is considered by the world economy, first, in order to find optimal allocation, one should maximize $U(C_t^*, N_t^*) = \frac{(C_t^*)^{1-\sigma}}{1-\sigma} - \frac{(N_t^*)^{1+\varphi}}{1+\varphi}$ subject to $C_t^* = A_t^* N_t^*$, for all t. First order condition is given by, $-N_t^{*\varphi} = C_t^{*-\sigma} A_t^*$.

Whereas, the flexible price equilibrium satisfies

$$\begin{aligned} 1 - \frac{1}{\varepsilon} &= \bar{MC}_t^* \\ &= \frac{(1-\tau^*)}{A_t^*} \frac{-N_t^{*\varphi}}{C_t^{*-\sigma}}. \end{aligned} \tag{4.12}$$

Then, setting $\tau^* = \frac{1}{\varepsilon}$ will tell us that world policymaker fully stabilizes the distortions that are risen by market power. Thus, optimal monetary policy in such an environment is the one that obtains $\tilde{y}_t^* = \pi_t^* = 0$.

On the other hand, in the small open economy maximization of $U(C_t, N_t) = \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{(N_t)^{1+\varphi}}{1+\varphi}$ will be subject to two more constraints. Namely, the risk sharing condition in equation (2.19) and implicitly defined relationship between terms of trade, and domestic and foreign output in equation (2.34). In order to solve this problem, we set $\eta = \sigma = 1$, and we obtain the consumption demand of home country as $C_t = Y_t^{1-\alpha} Y_t^{*\alpha}$. Then first order condition to maximization problem yields $-N_t^{1+\varphi} = (1-\alpha) C_t^{1-\sigma}$.

Whereas, the flexible equilibrium satisfies

$$1 - \frac{1}{\varepsilon} = \bar{MC}_t \tag{4.13}$$

$$
\begin{aligned}
&= \frac{(1-\tau)}{A_t}\left(\frac{Y_t}{Y_t^*}\right)^{-\alpha}\frac{-N_t^{*\varphi}}{C_t^{*-\sigma}} \\
&= (1-\tau)(1-\alpha).
\end{aligned}
$$

Hence, setting $(1-\tau)(1-\alpha) = 1-\frac{1}{\varepsilon}$, will lead to the small open economy's optimality of flexible price equilibrium. Which in turn, the optimal monetary policy in small open economy is the one that requires fully stabilizing the output gap and domestic inflation (i.e. $\tilde{y}_t = \pi_{H,t} = 0$).