PARTY IMPLEMENTATION

A Master's Thesis

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PARTY IMPLEMENTATION

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

PARTY IMPLEMENTATION

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In this study party implementation concept is introduced. Party implementation is an extension of classical implementation allowing different parties to have group-specific choice rules. Members of a party are assumed to act co-operatively according to a common preference. In this context, a choice rule is said to be party implementable if it is robust to co-operative manipulation. In this thesis some necessity and sufficiency results for party implementation are proven. In particular, it is shown that under some restrictions if the societies choice rule is party implementable, an alternative that is chosen by any group should also be chosen by the society. Conversely, it is shown that if the collective choice can be represented by the union of different groups' choice, then the social choice rule should be party implementable.

Keywords: Implementation, Party Implementation

ÖZET

PARTİ YAPISIYLA UYGULANABİLİRLİK

HASAN TAHSİN APAKAN

Yüksek Lisans, Ekonomi Bölümü

Tez Yöneticisi: Yard. Doç. Dr. Farhad Husseinov

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Bu çalışmada parti yapısıyla uygulanabilirlik kavramı ortaya konulmaktadır. Parti yapısıyla uygulanabilirlik klasik uygulanabilirlik kavramının partilerin farklı grup içi seçim kurallarına sahip olmasına izin veren bir uzantısıdır. Bir partinin üyelerinin ortak bir tercih sıralamasına göre işbirliği içerisinde davrandıkları varsayılmıştır. Bu bağlamda, bir seçim kuralı işbirlikçi manipulasyona karşı dayanıklı ise, bunun parti yapısıyla uygulanabilir bir kural olduğu söylenir. Bu tezde, parti yapısıyla uygulanabilirlik ile ilgili bazı gereklilik ve yeterlilik sonuçları ispatlanmıştır. Bazı kısıtlamalar altında, sosyal seçim kuralı parti yapısıyla uygulanabilir ise herhangi bir partinin tercih ettiği seçeneklerin toplum tarafından da seçilmesi gerektiği gösterilmiştir. Bunun yanında, sosyal seçimler farklı grupların tercihlerin birleşimi olarak ifade edilebilirse, sosyal seçim kuralının parti yapısıyla uygulanabilir olduğu gösterilmiştir.

Anahtar Kelimeler: Uygulanabilirlik, Parti Yapısıyla Uygulanabilirlik

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TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

The standard approach in game theory is to model an interactive situation in the form of a game[1] and to find possible outcomes with the help of a reasonable solution concept. By contrast in implementation theory game is something to be designed rather than taken; a planner is assumed to set the rules of the interaction to realize her objectives. With a given solution concept and some set of desired outcomes, one investigates whether there exist a game form that yields the same set of outcomes as equilibria. In order to have a clearer picture, we could describe the implementation framework in some detail.

The implementation problem starts with a society that desires to choose an alternative; so there are individuals who have preferences over a set of alternatives and a procedure called social choice rule that aggregates the individual preferences to choose some subset of alternatives. One might see these alternatives as the desirable ones for the society. A planner then is assumed to design a game form (mechanism) that consists of a strategy space for each agent and an outcome function. The outcome function associates an alternative with the profile of strategies.

[1]A game is a formal description that includes the constraints on the actions that the players can take and the players interest (Osborne and Rubinstein, 1994)

Planner designs the game form such that the equilibria of the game (a game form with realized preferences) are the alternatives chosen by the society. When a game is played, some set of moves are specified by a solution concept which could be seen as the reasonable equilibria of the game. If there is such a game form that the correspondence between the social choice and equilibria takes place for any set of preference profiles then we say the mechanism implements the social choice via the proposed solution concept.

In this thesis implementation via the Nash equilibrium concept will be useful. Maskin (1999)[2] proved some necessity and sufficiency results about Nash implementation. He used two concepts called no-veto power and monotonicity.A choice rule is said to be monotonic if an alternative is selected by a social choice rule for some preference profile, it must also be selected by the rule for any preference profile, where the relative ranking of the alternative weakly improves. A choice rule satisfies no-veto power if an alternative is selected whenever all individuals but possibly one ranks that alternative at the top of the preferences. Maskin showed that monotonicity is necessary while monotonicity and no-veto power condition together are sufficient for Nash implementation.

In many settings where individuals interact, co-operation and coordination is possible. Moreover agents care about the others. For example members of a family or a political party may behave according to a common decision and they can co-operate their actions. Using these observation one could introduce an implementation framework and an equilibrium concept. First we will describe the framework and the equilibrium concept. We assume there are separate groups of individuals which form a partition of all individuals. These groups have their different moral-choice agenda, so that each group has a unique preference over alternatives which clearly depend upon its members preferences. A group has a certain procedure for making an aggregate or-

[2]This study circulated as working paper from 1977 to 1998

dering for the alternatives, all the preferences of individuals are known to all the members of the group. We assume the members of a group after learning the aggregate-group preference, act according to it as if they have the group preference as their own. This means the individuals are extremely loyal to their groups aggregate preference. Moreover we assume every group (party) has a social welfare function; one particular form of this (that we use in our model) that groups have social choice rules and welfare functions are induced from them in a natural way.

Besides the extreme-loyalty assumption, we also assume the members of a group can co-operate; that is in a game situation they can deviate their actions together. In this framework party equilibrium is defined naturally: a profile of moves is an equilibrium when no group would like to deviate its actions. In this way a mechanism is said to party implement a choice rule within a group structure (as described above) when party equilibrium of a game (with the induced preferences) are the alternatives chosen by the rule. The equilibrium concept that we use is a specific example of a Strong Nash equilibrium with coalition constraints, but the implementation notion is different from the strong Nash implementation with constraints[3] as we assume individuals use aggregated group preferences.

In Chapter 2 we describe the implementation and party implementation frameworks formally. In Chapter 3 we present our results. In the first part we prove that under some assumptions if a social choice rule is party implementable, then any alternative that is chosen by a group should also be chosen by the society. In the second part we give some sufficiency condition for party implementation. Finally we look at party implementation framework in some specific context. Then in the last chapter we conclude our discussion and propose a possible extension of the framework.

[3] Suh (1996) characterized a variant of Strong Nash Implementation by allowing only some coalitions to form

CHAPTER 2

MODEL

2.1 Implementation of Social Choice Correspondences

In this section we deal with the standard implementation framework. In the next section we will introduce the party implementation framework. A social choice problem is a triplet (N, A, R), where $N = \{1, ..., n\}$ is a finite individual set, A is the alternatives set and $R = \{R_i\}_{i \in N}$ is a profile of preference relations defined on A.

Definition. A **preference relation** R_i on A is a binary relation on A which satisfies the following properties:

- complete: for any $a, b \in A$ with $a \neq b$, we have aR_ib or bR_ia
- trasitive: for any $a, b, c \in A$, aR_ib and bR_ic implies aR_ic
- reflexive: for any $a \in A$, aR_ia.

For each agent (individual) i, let $\Re_i$ denote the set of all possible preference relations of agent i. The set $\Re = \prod_{i \in N} \Re_i$ is the set of all preference profiles. Given any preference relation R_i, we denote its asymmetric part with P_i and call it the strict preference induced by R_i. Given a preference relation R_i we denote its symmetric part with I_i and call it the indifference induced by R_i. For any subset K of the alternative set A, $R_i[K]$ denotes the intersection of the preference relation R_i with the set $K \times K$, that is the restriction of the preference relation R_i on the set K.

A **social choice rule** is a correspondence $F\colon \Re \to A$ which maps every preference profile to a non-empty subset of A. Alternatives in $F(R)$ are interpreted as desirable alternatives for the profile R. In a social problem the planner may not have information about the preferences of agents, so the planner confronts the individuals with a proper mechanism such that the strategic solutions of the mechanism corresponds to the social choice.

Definition. A **mechanism (game form)** is a pair $\Gamma = (\mathbf{M}, g)$ where $M = \prod_{i \in N} \mathbf{M}_i$ is the strategy (action) space and $g : \mathbf{M} \to A$ is the outcome function. Given a mechanism Γ each preference profile R defines a normal form game (Γ, R).

Definition. A mechanism $\Gamma = (\mathbf{M}, g)$ **implements** a social choice rule F (in Nash equilibria), if for any preference profile R every alternative induced by a Nash equilibrium of the game (Γ, R) is chosen by F for R, and conversely, every alternative chosen by F is induced by a Nash equilibrium: $\forall R \in \Re$, $g(N(\Gamma, R)) = F(R)$.

We now state some necessity and sufficiency results about Nash implementation. Given a preference relation R_i of agent i and an alternative a, **lower contour set** of R_i according to alternative a, $L(R_i, a)$ is the set of alternatives to which a is weakly preferred under R_i, that is $L(R_i, a) = \{b \in A : aR_ib\}$.

Definition. A social choice rule F is said to satisfy **Maskin monotonicity** (or to be **Maskin monotonic**) iff for any $R, R' \in \Re$ with any $a \in F(R)$ and for any $i \in N$: the inclusion $L(R_i, a) \subset L(R'_i, a)$ implies $a \in F(R')$.

Definition. A social choice rule F is said to satisfy **no-veto power** iff for each $R \in \Re$, for each $a \in A$ $|\{i \in N : L(R_i, a) = A\}| \geq |N| - 1$ implies $a \in F(R)$.

Definition. A social choice rule F is said to satisfy **neutrality** iff for any permutation mapping π of the alternative set A and for any preference profile $R \in \Re$, $F(R^\pi) = \pi(F(R))$, where R^π denotes the permuted preference profile.

Definition. A social choice rule F is said to satisfy **anonymity** iff for any permutation mapping σ of the individual set N and for any preference profile $R \in \Re$, $F(R^\sigma) = F(R)$, where R^σ denotes the permuted preference profile.

Maskin (1999) proved the following results:

Theorem 1. *If a social choice rule F is Nash-Implementable, then it is Maskin monotonic.*

Theorem 2. *If a social choice rule F is Maskin monotonic and satisfy no-veto power with the individual set $|N| > 3$, then it is Nash-Implementable.*

2.2 Party Implementation of Social Choice Correspondences

Social choice problem is the same triplet (N, A, R) as before; but the framework is more complicated as there are parties and their group social choice rules. A partition Π_N of N gives the group structure: all members of the partition are separate groups. Each member of the partition has some kind of procedure to choose an alternative, that is for any $S \in \Pi_N$, there exists a sequence of social choice rules defined on any subset of the alternative set. Group-specific social choice rule $(f_S)[K]$ is a correspondence which maps every preference profile of group S on the alternative's set K to a non-empty subset of K. With slight abuse of notation, we will denote this sequence

$\{(f_S)[K]\}_{K\subset A, K\neq\emptyset}$ by f_S. When we say f_S satisfies a property we mean that $(f_S)[K]$ satisfy that property for every non empty set K of A. Any group social choice rule will induce a social welfare function in a natural way. The chosen alternatives in the first round will form the first indifference class. Then the social choice rule is applied to the rest of the alternatives and the newly chosen ones will form the second indifference class. This process goes on till there is no alternative left.

For any $R \in \prod_{i\in N} \Re_i$ let $R_S[A]$ be the projection of preference profile to agent group S on the alternatives subset K. Formally, the method is as follows if $a \in f_S(R_S)$ and $b \notin f_S(R_S)$ then we write $a\ P_S^*\ b$. For the same b if $b \in f_S(R_S[A \setminus f_S(R_S)])$ but $c \notin f_S(R_S)$ and $c \notin f_S(R_S[A \setminus f_S(R_S)])$, then we write $a\ P_S^*\ b\ P_S^*\ c$. Similarly we write $c\ I_S^*\ d$ if $c, d \in f_S(R_S)$. This method is applied to the rest of alternatives, so we get two binary relations I_S^* and P_S^*. Then the union of these binary relations, R_S^*, will be the induced preference of the group S by operation z, so $z(f_S, R_S) = R_S^*$. R_S^* is the group ordering of S under f_S. Individuals in the group act according to this aggregate ordering, they play any game as if they have this group preference as their individual preference's. We assume when $|S| = 1$, f_S choose the top indifference class so that $z(f_S, R_S) = R_S$.

We define a new equilibrium concept for a normal form game which is dependent on the party environment.

Definition. A **party equilibrium** σ of a strategic game $(\mathbf{M}, g, R)$ relative to partition (Π_N) is a profile $x \in M$ of messages with the property that for each $S \in \Pi_N$, and for each $y \in \mathbf{M}$ such that for every $i \in (N \setminus S)$ $x_i = y_i$;for every $j \in S$, $g(x) R_j g(y)$.

$$\begin{tikzcd}
(R, \Pi_N, \{f_S\}_{S\in\Pi_N}) \arrow[r] \arrow[d, "\forall S \in \Pi_N : z(f_S, R_S) = R^*_S"] & F(R) \arrow[d, equal] \\
(\mathbf{M}, g, R^*) \arrow[r, "\sigma(\Pi_N)"] & \sigma(\Pi_N)(\mathbf{M}, g, R^*)
\end{tikzcd}$$

Figure 2.1: Party Implementation

That is any coalition can not be better off by deviating relative to the induced preference. Our next aim is to find a mechanism $(\mathbf{M}, g)$ such that $\sigma(\Pi_N)(\mathbf{M}, g, R^*) = F(R)$ in an environment $(\Pi_N, \{f_S\}_{S\in\Pi_N})$, where R^* is the induced preference as defined above. If there is such a mechanism $(\mathbf{M}, g)$ that the previous equation holds for any R with the corresponding induced preference, then we say F is **party implementable relative to environment** $(\Pi_N, \{f_S\}_{S\in\Pi_N})$. Moreover we propose a generalization of Maskin monotonocity.

Definition. A social choice rule F is said to be **Group-Maskin monotonic** relative to an environment $(\Pi_N, \{f_S\}_{S\in\Pi_N})$ if for any $R, R' \in \Re$ with any $a \in F(R)$ and for any $S \in \Pi_N$ such that $L(R^*_S, a) \subset L(R'^*_S, a)$ we have $\in F(R')$, where $R^*_S = z(f_S, R_S)$ and $R'^*_S = z(f_S, R'_S)$.

CHAPTER 3

RESULTS

3.1 Necessity Conditions

Proposition 1. *Within an environment* $(\Pi_N, \{f_S\}_{S\in\Pi_N})$*, if* F *is party implementable, then* F *should be Group-Maskin monotonic relative to that environment.*

Proof. Since F is party implementable relative to environment $(\Pi_N, \{f_S\}_{S\in\Pi_N})$ and $a \in F(R)$, there is a party equilibrium, where a is chosen in which the groups act according to the induced preference R_S^*. It follows that there is no group which can deviate to any better alternative, as the second induced preference $L({R'_S}^*, a)$ contains $L(R_S^*, a)$. Groups can not deviate as the possible better alternative set got smaller. □

Note: It follows from the previous proposition that if all the group-specific social choice rules are Maskin monotonic and F is party implementable, then F should be Maskin monotonic.

Proposition 2. *In an environment* $(\Pi_N, \{f_S\}_{S\in\Pi_N})$ *assume* $|N| \geq 3$ *and there exists a coalition(a member of the partition)* S *with at least two members which satisfies Maskin monotonicity, neutrality and anonymity. Then if* F *satisfies the same assumptions with no-veto power and* F *is party imple-*

mentable relative to environment $(\Pi_N, \{f_S\}_{S\in\Pi_N})$, *F should choose all the top alternatives. That is* $F(R) \supset \overline{A}$,*where* $\overline{A} = \{a \in A\text{: } \exists i \in N : \forall b \in A,\ aR_ib\ \}$.

Proof. For the most part of the proof we will use only two types of preferences, namely type 1 and type 2. Then we will generalize our results to full domain of preferences using Maskin monotonicity of the social choice rules. Assuming a is in the alternative set, let type 1 be such that: $\forall b \in A \setminus \{a\}$: aPb and type 2: $\forall b \in A \setminus \{a\}$: bPa. Moreover, for both type 1 and type 2; and for each d,c $\in A \setminus \{a\}$ dIc, where P denotes strict preference and I denotes indifference. In matrices types will be shown as:

$$\text{type 1: } \begin{pmatrix} a \\ A \setminus \{a\} \end{pmatrix} \text{ type 2: } \begin{pmatrix} A \setminus \{a\} \\ a \end{pmatrix}$$

Then construct a preference profile R such that only one individual j has type 2 preference and the rest have type 1. Assume $j \in S$, then as F is no veto power we have $a \in F(R)$.Then assume $f_S(R_S) \not\supset \{a\}$, where

$$R^*_S = \begin{pmatrix} A \setminus \{a\} \\ a \end{pmatrix}$$

With the help of the Proposition 1 one could change the preferences of all the individuals in S to type 2. This operation would not change the group preference as f_S is Maskin monotonic. So that F should still choose the alternative a. Then by anonymity of F, one could change the place of a single individual from S with another outside S, then as R^*_S is same with type 2 preference, one could repeat the same operation until all the members of N have type 2 preference. Then by Maskin monotonicity of F we get F is the constant-all rule[1]. We are done.

[1]Constant-all Rule choose all the alternatives for any preference profile.

Now assume $f_S(R_S) \supset \{a\}$. By using an algorithm below we will make everyone's preference outside S of type 2. In the new preference profile a is still chosen. Algorithm is as follows:

Step 1 Change the place of an individual with type 2 preference inside S with another individual of type 1 preference outside S.

Step 2 Change the preference of a single individual within S to type 2.

Step 3 After Step 2 turn back to Step 1, stop the algorithm when Step 1 is not achievable.

This way we eventually will have everyone outside coalition S to have type 2 preference, as a is always top ranked by groups after step 1 and step 2 are executed. By Proposition 1 as the lower contour set did not get smaller, we will have $F(\overline{R}) \supset \{a\}$. Denote this new preference profile $\overline{R}$.

$\overline{R}$ could be written like this, without loss of generality, as F is anonymous:

$$\overline{R} = \begin{pmatrix} a & \cdots & a & A \setminus \{a\} & \cdots & A \setminus \{a\} \\ A \setminus \{a\} & \cdots & A \setminus \{a\} & a & \cdots & a \end{pmatrix}$$

f_S could be the constant-all rule or not. Assume f_S is not constant all rule.

Claim. *Assuming f_S is not constant all rule, there exists a group preference profile R''_S with only type 1 or type 2 preferences such that $f_S(R''_S) \supset \{a\}$ and with the property that if a single individual switches his preference from type 1 to type 2, then alternative a will not be chosen by f_S.*

Proof of Claim: Algorithm: take R_S such that everyone has type 1 preference. In each step change a single individual's preference to *type* 2. Do this till a is not chosen, then the preference in the previous step will have the properties of R''_S in the claim. This process should stop somewhere as f_S is not constant-all, neutral and Maskin monotonic. Claim is proved.

By the help of Proposition 1 change $\overline{R}_S$ to R''_S, where R''_S has the property stated in the claim and $F(\overline{R}_{-S}, R''_S) \supset \{a\}$. Then by anonymity change the places of an individual who has type 1 preference in S with another individual with type 2 preference in the set $N \setminus S$. Then as the stated properties of R''_S show, a is not chosen in the new profile, so group preference of S will be of type 2. Now by first the observation we can make everyone's preference inside S of type 2, without changing the chosen alternative. Now everyone but a single individual has type 2 preference. Then we get F should choose all the top alternatives.

If f_S is constant-all rule, then F should be constant-all rule too. This is similar to the first part of the proof. Make everyone in S bottom rank alternative a and change the places with individuals outside S. Since group preference do not change, eventually we will have everyone bottom rank a. □

Corollary. *Assume all assumptions of the Proposition 2 are satisfied. Assume moreover that there exists a coalition S in the partition such that f_S does not choose a top alternative in some R' and all the group-specific social choice rules are Maskin monotonic. If F is party implementable then it should be constant-all rule.*

Proof. Take a preference profile R' such that agent j in the coalition S top ranks a but $f_S(R') \not\supset \{a\}$. Then take another preference profile R such that only player j has type 1 preference and all the other players have type 2 preferences. Alternative a should be chosen by Proposition 1.

As f_S is Maskin monotonic, $f_S(R) \not\supset \{a\}$, that is $z(f_S, R_S)$ is a type 2 preference, so make everyone bottom rank a in S. By Proposition 1 alternative a should still be chosen. By Maskin monotonicity F is constant-all rule. □

Theorem 3. *Assume all the conditions of the Corollary 1 for a social choice rule F and for an environment $(\Pi_N, \{f_S\}_{S \in \Pi_N})$ are satisfied. If F is party implementable relative to that environment, then the expression $\bigcup_{S \in \Pi_N} f_S(R_S) \subset F(R)$ must hold for any R.*

Proof. By Corollary 1 we know that this expression holds if some $f_S(R_S)$ does not choose a top alternative. So we can assume they do choose all the top alternatives. Moreover, assume the expression above does not hold. Then there exists a preference profile R such that $a \notin F(R)$ but $a \in f_T(R_T)$ for some $T \in \Pi_N$. By Proposition 2 alternative a should not be top-ranked by any individual. Take $j \in T$ and make j top ranks alternative a without changing the preference relations of any other individual. Call this new preference profile R'_T. We know $a \in f_T(R'_T)$, so $L(R^*_T, a) = L(R'^*_T, a)$ but we have $a \in F(R_{-T}, R'_T)$. This implies $a \in F(R)$ by Group-Maskin monotonicity of F. Thus we obtain contradiction.

□

Remarks

- When there is no S such that $|S| \geq 2$, we will have only singletons as groups. Moreover we preserve our assumption $z(f_S, R_S) = R^*_S = R_S$ for singleton coalitions. As a result our implementation notion boils down to Nash-implementation.

- If on the other edge $\Pi_N = \{N\}$ with $f_N = F$ the rule will be party-implementable as a result of a further observation.

  Claim. *If $f_N \neq F$, that is if there exists R with $f_N(R) \neq F(R)$, F will not be party implementable.*

  Proof. Assume F is party-implementable.

  It is clear that $f_N \supset F$ must hold since otherwise some coalition would deviate to any other alternative $a \in f_N(R)$.

The other direction is also clear: assume the opposite. Then there must be a message profile $M \in \prod_{i \in N} \mathbf{M}_i$ in which $g(M) = \{a\}$ where $a \in f_N(R)$ but $a \notin F(R)$, where a is clearly chosen by a party equilibrium, Contradiction. $\square$

- When $|N| > |A|$ there is a social choice rule with the properties stated in the Proposition 2 which does not choose every top alternative. Let's define this rule as F choose alternatives that at least two agents top rank. So by pigeon-hole principle this rule is non-empty, and other properties are easy to show. As we see from this observation, there are Maskin monotonic, neutral, anonymous and no veto power social choice rules that are not party implementable relative to any environment $(\Pi_N, \{f_S\}_{S \in \Pi_N})$ with f_S having the properties stated in the Prop. 2.

3.2 A Sufficiency Condition

Proposition 3. *In an environment* $(\Pi_N, \{f_S\}_{S \in \Pi_N})$, *where* $N \geq 3$,
if the equation $\bigcup_{S \in \Pi_N} f_S(R_S) = F(R)$ *holds, then* F *is party implementable.*

Proof. Assume $\overline{F}$ chooses only alternatives that at least one agent top ranks. Then $\overline{F}$ is Maskin monotonic and no-veto power. By Theorem 2 (Maskin) $\overline{F}$ should be Nash implementable if $N \geq 3$. So using this observation take a single individual from each group and name this set N_1. Then with a slight abuse of notation we have $\bigcup_{S \in \Pi_N} f_S(R_S) = F(R) = \overline{F}(R^*_{N_1}) = \overline{F}(R)$, where $R^*_{N_1}$ is the projection of the induced R^* onto set N_1. Then take the Nash mechanism of $\overline{F}$ for the set N_1, and add non effective moves for $N \setminus N_1$, so it is a game for N. That is if the Nash mechanism is $(\prod_{i \in N_1}(\mathbf{M}_i), g)$ take $(\prod_{i \in N}(\mathbf{M}_i), g')$ as a new mechanism, where for each $m \in \prod_{i \in N \setminus N_1}(\mathbf{M}_i)$, $g(m) = g(m, a)$. Then Nash equilibrium of this game clearly corresponds with the party equilibrium relative to the group structure. $\square$

3.3 Party Implementation with veto power

It is easy to observe that Proposition 2 needs the assumption called the no-veto power; take the veto-free correspondence for instance that is $F(R) = \{a \in A : \text{ there is no } i \in N : \forall b \in A, bP_i a\}$. This correspondence is non-empty when $|N| < |A|$ and it has the other properties stated in Prop. 1. Take any rule which is contained in the veto-free correspondence; then if it is party implementable in a $(\Pi_N, \{f_S\}_{S\in\Pi_N})$ environment then any f_S such that $S \in \Pi_N$, where f_S is monotonic will satisfy the veto power property. This is easy to see, take the preference profile in the beginning of Proposition 2, where a is not chosen by F, as the individual j vetos alternative a. On the other hand, as f_S satisfies the no-veto power assumption alternative a should be chosen. Change everyone's preference to type 1, so that a should be chosen in the new preference profile. But as F is Group-Maskin monotonic, we have a contradiction.

3.4 Party Implementation of Constant-All Correspondence

In an environment $(\Pi_N, \{f_S\}_{S\in\Pi_N})$ if there exists $S, T, K \in \Pi_N$ all distinct, then mechanism with the message space $M_i = (A \times \mathbf{N})$ for each individual where $\mathbf{N}$ is the natural numbers set, party implements F. In the mechanism if an alternative is played by all groups except one, corresponding alternative will be chosen irrespective of the integers chosen. In other instances the alternative is chosen by the individual who says the highest integer. There is an unimportant tie-breaking rule. This mechanism party-implements F (the constant-all rule) relative to environment $(\Pi_N, \{f_S\}_{S\in\Pi_N})$, as for any alternative a there is a party-equilibrium, where all groups choose that alternative.

3.5 Existence of a Dictatorial Social Choice Rule

When there is a dictatorial choice rule f_S for a set $S \in \Pi_N$ with the property $|S| \geq 2$, if F is a social choice rule with the properties in the proposition 2, F should choose all the top alternatives

Proof. A shorter version of the Proposition 2's proof. $\square$

CHAPTER 4

CONCLUSION

In this thesis, we introduced a new setup for implementation. We assumed there are groups which have distinct social choice rules. These groups act cooperatively according to a common aggregated preference. On this framework a new implementation notion called party-implementation is introduced. In brief a social choice rule is said to be party implementable if there is a mechanism which is robust to any group-specific manipulation. Our most important finding is that under some restrictions if the societies choice rule is party implementable, an alternative that is chosen by any group should also be chosen by the society. Secondly, a sufficient condition for party implementation is given; it is shown that if the collective choice can be represented by the union of different groups' choice, then the social choice rule should be implementable. Within various environments some distinct conditions are given for party-implementation. To sum up we can say that, in many plausible environments, where parties and societies have very distinct procedures for choosing an alternative, party implementability is hard to achieve.

On the other hand, some further research is possible; one could generalize the party environment with a suitable network structure where the links between individuals show whether individuals care about the others. Moreover, one could relax the assumption that all the members of some coalition act according to the same common preference, they may have distinct ways of caring about the other's preferences. The equilibrium notion can be generalized allowing different coalitions to act cooperatively. In such a setup new robustness conditions for implementability can be proposed.

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