MAJORITY VOTING RULE AND OLIGARCHIC SOCIAL CHOICE RULES

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ABSTRACT

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In the first part of this study majority voting rule for two alternatives and continuum agents is characterized. As in the finite agent case, symmetry among agents, neutrality between alternatives and positive responsiveness characterize majority voting rule. In the second part, the relation between T-monotonicity and the group which acts as the oligarchy in an oligarchic social choice rule, is analyzed. It is shown that the minimal coalition for which the social choice rule is monotonic constitutes the oligarchy.

Keywords: Social Choice, Majority Voting, Monotonicity, Oligarchy.

ÖZET

OY ÇOKLUĞU SEÇİM KURALI VE OLİGARŞİK SOSYAL SEÇİM KURALLARI

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Bu çalışmanın ilk bölümünde, iki alternatif ve kontinuum temsilci için oy çokluğu seçim kuralı karakterize edilmiştir. Sonlu temsilci durumunda olduğu gibi, temsilcilerde simetriklik, alternatifler arasında tarafsızlık ve pozitif cevaplılık oy çokluğu seçim kuralını karakterize eder. İkinci bölümde, T-monotonluk ve oligarşik bir sosyal seçim kuralında oligarşi olarak davranacak grup arasındaki ilişki analiz edilmiştir. Sosyal seçim kuralının monotonluğu sağladığı en küçük grubun oligarşiyi oluşturduğu gösterilmiştir.

Anahtar Kelimeler: Sosyal Seçim, Oy cokluğu, Monotonluk, Oligarşi.

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Chapter 1

Introduction

Social Choice Theory is the study of systems and institutions for making collective choices. In its most general terms the problem is to define best methods to aggregate individual preferences into social preferences and choices. And, in this paper we will assume that individuals are described exclusively by their preference relations over the alternatives, i.e., the issue of interpersonal comparibility of utilities will not be considered. Another assumption on the individual preferences is that they are part of the data of the problem. An individual might change the social decision in favour of his benefits by misrepresenting his actual preferences. We will not discuss such strategy aspects here and assume that the true preferences of the agents are known. The functional relationship between individual preference orderings and social choice is described by social choice rules. The problem then can be formulated as what restrictions one should impose on the social choice rules. There are many specific social choice rules which are well defined; however, a great many of them fail to satisfy some socially desirable criteria. Hence, instead of considering specific functions, we shall focus on some desirable properties that any social choice rule should satisfy.

The analysis can be made in several cases depending on the number of alternatives. The first case is when there is only one alternative which actually requires no analysis. The results obtained for two alternatives and three or more than three alternatives cases shows the importance of the number of alternatives. For the two alternative case we have some nice results. However, the impossibility results we have, in three or more than three alternatives case make the analysis more difficult.

The central result in two alternatives case is suggested by May (1952). He characterized the majority voting rule for two alternatives and finite agents. The decision mechanism in majority voting rule for finite agents works like this: An alternetive is socially preffered if the number of agents that prefers this alternative is greater than the number of agents that prefer the other alternative. If these two numbers are equal then the society is indifferent between these alternatives. The majority voting rule plays an important role in social choice theory because of some important properties it satisfies: Unanimity, symmetry among agents, neutrality between alternatives and positive responsiveness. Unanimity, which is a very natural restriction, says that if every agent puts the same alternative to the top, i.e., the most preferred alternative for every agent is same then this alternative should also be socially most preferred. Symmetry says that the social choice rule does not depend on the names of the agents. This condition provides the equality among agents. Neutrality means that, smilarly, the social decision does not depend on the labelling of the two alternatives. And the last one, positive responsiveness, says that the social choice rule is sensitive to the individual preferences in the sense that; if an alternative is socially preferred or indifferent to the other alternative and some agents change their preference in favour of this alternative then it becomes socially preferred. As it turns out, symmetry among agents, neutrality between alternatives and positive responsiveness entirely characterize majority voting rule for two alternatives case.

One major feature of the majority voting rule is that it satisfies the axiom of the pairwise independence condition which means that society's choice between a pair of alternatives depends only on the preferences for that pair, and so any other characterization of the rule can be simply based on paired comparisons.

In the second chapter we will show that May's result also holds for continuum agent: For two alternatives and continuum agent, symmetry among agents, neutrality between alternatives and positive responsiveness characterizes majority voting social choice rule.

In the three or more than three alternative case the sitiation is more complicated. First of all, the intransitivities that may occur in the social ordering makes it impossible to apply some social choice rules - for example, the majority voting rule. In order to avoid this problem we can impose some restricted domain assumptions on the social choice rule. But if we consider full domain assumption, we end up with the well-known Condercet paradox. To have a better understanding of the problem consider the following example: Let us have three agents and three alternatives x, y, z. Agent one prefers x to y and y to z; agent two prefers z to x and x to y; agent three prefers y to z and z to x. Now, if we apply pairwise majority voting rule to these preferences, x must be socially preferred to y since two agents prefer x to y, and similarly y must be socially preferred to z and z must be socially preferred to x. This cyclic pattern tells us that we can not have transitive social preferences in three or more than three alternative case if we apply the majority voting rule.

The next problem, we come across, is about pairwise independence condition which was first suggested by Arrow. The pairwise independence condition says that the social ordering between two alternatives only depends on the same two alternatives. This is an important restriction which simplifies the problem. The complications that might be created by the existance of irrelavant alternatevis will be ruled out by this assumption. However, many social choice rules does not satisfy this condition. For example the Borda count rule which depends on the placements of the alternatives in the individual orderings does not satisfy the pairwise independence condition.

All these problems mainly summed up in Arrow's central impossibility theorem. The theorem tells that the following conditions are inconsistent: The number of alternatives is at least three; universal domain assumption; social rationality; pairwise independence condition; unanimity and no-dictatorship. So, a social choice rule satisfying the first five conditions is dictatorial in the following sense: There is an agent h such that, for any x, y and any preference profile, we have that x is socially preferred to y whenever agent h prefers x to y. The minimality of the conditions of the theorem makes the result more powerful. However, there are several ways of avoiding this dictatorship conclusion. First way is, as we already mentioned, is restricting the domain of the social choice rule. Another approach which was suggested by Guha (1972) is, relaxing the full rationality condition. If we replace transitivity by quasitransitivity then we get oligarchic social choice rules.

Our main concern in the third chapter will be oligarchic social choice rules. We will focus on the question which group of the members of the society can constitute an oligarchy.

Chapter 2

May's Theorem For Continuum Agents

2.1 Preliminaries

In this chapter we will show that symmetry among agents, neutrality between alternatives and positive responsiveness characterize the majority voting rule for two alternatives and continuum agents. In 1952, May showed that these three properties characterize majority voting rule for two alternatives and finite number of agents.

We assume that there are two alternatives, x and y, and the agent set is the closed interval $[0,1]$. We will denote the Lebesgue measure of a set $A \subset [0,1]$ by $m(A)$ and the set of all Lebesgue measurable subsets of $[0,1]$ by $\mathcal{L}$. A preference profile of the individuals in the society will be described by the function $\theta : [0,1] \rightarrow \{-1,0,1\}$, where θ takes the value 1, 0 or -1 according to whether agent $t \in [0,1]$ prefers alternative x to alternative y, is indifferent between them or prefers alternative y to alternative x, respectively. For example, if we have $\theta(t) = 1$, this means that agent t prefers alternative x to alternative y. We will assume that θ is Lebesque measurable. The set of all measurable preference profiles will be denoted by Θ.

Next, we define the sets E_+, E_- and E_0 as follows:
$E_+^\theta = \{t \in [0,1] : \theta(t) = 1\}$
$E_-^\theta = \{t \in [0,1] : \theta(t) = -1\}$
$E_0^\theta = \{t \in [0,1] : \theta(t) = 0\}$

These sets are the sets of agents that indicate the agents who choose alternative x, alternative y and are indifferent between them under the profile θ, respectively. They are pairwise disjoint and $E_+^\theta \cup E_-^\theta \cup E_0^\theta = [0,1]$. Note that these sets are Lebesgue measurable since θ is Lebesgue measurable.

In this chapter we will use the terminology, social welfare functional, for an aggregator of individual preferences into social preferences. Now, we give the formal definition of a social welfare functional.

DEFINITION. A social welfare functional is a rule $F : \Theta \to \{-1, 0, 1\}$ that assigns a social preference to every possible profile of individual preferences.

It should be noted that every measurable profile of individual preferences is included in the domain of F. This is called the universal domain assumption.

Now, we define the majority voting rule for continuum agents by using the notion of Lebesgue measure. In the finite agent case, it simply says that, if the number of agents who prefers an alternative, say x, is strictly greater than the number of agents who prefer the other alternative, y, then x is socially preferred; if these two numbers are equal than society is indifferent between these two alternatives.

DEFINITION. The majority voting rule is the social welfare functional where $F(\theta(\cdot)) = 1$ whenever $m(E_+^\theta) > m(E_-^\theta)$, $F(\theta(\cdot)) = -1$ whenever $m(E_-^\theta) > m(E_+^\theta)$ and $F(\theta(\cdot)) = 0$ whenever $m(E_+^\theta) = m(E_-^\theta)$.

The majority voting rule satisfies three important properties; symmetry among agents, neutrality between alternatives and positive responsiveness. The symmetry among agents, provides equality between the agents. The neutrality between alternatives, suggests that the names or the labelings of the alternatives is not important. And the positive responsiveness means that the social preference is sensitive to the individual preferences in the sense which will be defined in the formal definition.

DEFINITION. A social welfare functional F is symmetric among agents if a permutation of preferences across agents does not change the social preference. Precisely, let $\pi : [0,1] \to [0,1]$ be a one-to-one, measure preserving function. Then for any profile θ, we have $F(\theta(\cdot)) = F(\theta(\pi(\cdot)))$.

DEFINITION. A social welfare functional F is neutral between alternatives if $F(-\theta(\cdot)) = -F(\theta(\cdot))$ for every θ, that is when we reverse the the preferences of all agents the social preference is also reversed.

DEFINITION. A social welfare functional F is positive responsive if, whenever we have two profiles θ and θ' such that $F(\theta(\cdot)) \geqslant 0$ and $\theta'(t) \geqslant \theta(t)$ for all t and there exist $A \in \mathcal{L}$ of positive measure, $m(A) > 0$, such that

$\theta'(t) > \theta(t)$ for all $t \in A$ then we have $F(\theta'(\cdot)) = 1$. That is if x is socially preffered or indifferent to y and some agents raise their consideration of x, then x becomes socially preffered.

In the next section we will show that these three properties; symmetry among agents, neutrality between alternatives and positive responsiveness entirely characterize majority voting rule.

2.2 Result

THEOREM. Suppose there are two alternatives and continuum agents. Then a social welfare functional F is majority voting rule if and only if it is symmetric among agents, neutral between alternatives and positive responsive.

Proof: We will first show the sufficiency part. Suppose that F is a social welfare functional that satisfies symmetry, neutrality and positive responsiveness. Let $A \subset \mathbb{R}^2$ such that for any $(u, w) \in A$, u, $v \geqslant 0$ and $u + w \leqslant 1$. Symmetry among agents implies that there exists a function $G : A \to -1, 0, 1$ such that $F(\theta(\cdot)) = G(m(E_+^\theta), m(E_-^\theta))$. In deal, let θ' be such that $m(E_+^{\theta'}) = m(E_+^\theta)$ and $m(E_-^{\theta'}) = m(E_-^\theta)$. Let $\pi : [0, 1] \to [0, 1]$ be a one-to-one, measure preserving (i.e., for every $E \in \mathcal{L}$ we have $m(\pi(E)) = m(E)$) function defined as follows: $\pi : E_+^\theta \to E_+^{\theta'}$ is one-to-one and measure preserving, $\pi : E_-^\theta \to E_-^{\theta'}$ is one-to-one and measure preserving, and $\pi : (E_+^\theta \cup E_-^\theta)^C \to (E_+^{\theta'} \cup E_-^{\theta'})^C$ is one-to-one and measure preserving (Royden, 1988). By symmetry, $F(\theta(\pi(\cdot))) = F(\theta(\cdot))$. But, $F(\theta(\pi(\cdot))) = F(\theta'(\cdot))$. So, $F(\theta(\cdot)) = F(\theta'(\cdot))$ which shows that F only depends on $m(E_+^\theta)$ and

$m(E_-^\theta)$.

Now, suppose that θ is a profile such that $m(E_+^\theta) = m(E_-^\theta)$. Then by the observation $m(E_+^{-\theta}) = m(E_-^\theta) = m(E_+^\theta) = m(E_-^{-\theta}$ we have,

$$\begin{aligned} F(\theta(\cdot)) &= G(m(E_+^\theta), m(E_-^\theta)) \\ &= G(m(E_+^{-\theta}), m(E_-^{-\theta})) \\ &= F(-\theta(\cdot)) \\ &= -F(\theta(\cdot)) \end{aligned}$$

Note that the last equality follows from the neutrality between alternatives. So we conclude that if $m(E_+^\theta) = m(E_-^\theta)$ then $F(\theta(\cdot)) = 0$.

Now, let θ be such that $m(E_+^\theta) > m(E_-^\theta)$. Note that there exists $E \in \mathcal{L}$ such that $E \subset E_+^\theta$ and $m(E) = m(E_-^\theta)$ (Royden, 1988). Consider a profile θ' such that $\theta'(t) = 1$ for $t \in E$, $\theta'(t) = -1$ for $t \in E_-^\theta$, and $\theta'(t) = 0$ elsewhere; that is, in the new profile some agents change their individual preferences in favor of y and $m(E_+^{\theta'}) = m(E_-^{\theta'})$. Then by the first part $F(\theta'(\cdot)) = 0$. And since F is positive responsive we have, $F(\theta(\cdot)) = 1$.

Finally suppose θ is such that $m(E_-^\theta) > m(E_+^\theta)$. Then $m(E_+^{-\theta}) > m(E_-^{-\theta})$. Therefore, by neutrality among alternatives $F(\theta(\cdot)) = -F(-\theta(\cdot)) = -1$.

So we conclude that a social welfare functional satisfying these properties is indeed the majority voting rule.

Next we will show that majority voting rule satisfies these properties. Let $\pi : [0,1] \rightarrow [0,1]$ be a one-to-one and measure preserving permutation of the agent set. Then $m(E_+^\theta) = m(E_+^{\theta(\pi(\cdot))})$, $m(E_-^\theta) = m(E_-^{\theta(\pi(\cdot))})$, which implies that $F(\theta(\cdot)) = F(\theta(\pi(\cdot)))$. Therefore F satisfies symmetry among agents.

Now suppose $F(\theta(\cdot)) = 1$ for some $\theta(\cdot)$, i.e., $m(E_+^\theta) > m(E_-^\theta)$. Then $F(-\theta(\cdot)) = -1$ since we have $m(E_+^{-\theta}) = m(E_-^\theta) < m(E_+^\theta) = m(E_-^{-\theta})$, by definition. Hence $F(\theta(\cdot)) = -F(-\theta(\cdot))$. If $F(\theta(\cdot)) = -1$ or 0 then by the same remark we again conclude that $F(\theta(\cdot)) = -F(-\theta(\cdot))$. So F is neutral between alternatives.

Finally, to see that majority voting satisfies positive responsiveness, first consider two profiles θ and θ' such that $F(\theta(\cdot)) \geqslant 0$ and $\theta'(t) \geqslant \theta(t)$ for all t and there exist $A \in \mathcal{L}$ such that $\theta'(t) > \theta(t)$ for all $t \in A$. Then, by definition of majority voting rule, $F(\theta'(\cdot)) = 1$. Hence F is positive responsive.

So, we showed that these three properties characterizes the majority voting rule for two alternatives and continuum agents case. $\square$

Chapter 3

Oligarchic Social Choice Rules

3.1 Preliminaries

In this chapter we will consider the case where there are at least three alternatives. Our general setting will be as follows: We will denote the set of alternatives by A and set of agents by N and will assume that both A and N are finite and nonempty. Every agent has a rational, i.e., complete and transitive preference relation on A. The set of all possible rational preference relations on A and the set of all possible strict preference relations on A, will be denoted by $\mathcal{R}$ and $\mathcal{P}$, respectively. $\mathcal{R}^N$ and $\mathcal{P}^N$, will stand for all preference profiles on A and strict preference profiles on A, respectively. A preference profile in $\mathcal{R}^N$ and $\mathcal{P}^N$ will be denoted by R and P, and individual orderings in these profiles by R_i and P_i, respectively. We then define a social choice rule as a function that assigns a subset of the alternative set to preference profiles of individuals, i.e., $F : S \to 2^A$ where $S \subset \mathcal{R}^N$.

The natural questions that arise at this point are these: What restrictions should one impose on the social choice rule? And what will be the results of imposing such restrictions? The central result of this issue is Arrow's impossibility theorem: Suppose that the number of alternatives is at least three and that the domain of admissible individual profiles, is either $\mathcal{R}^N$ or $\mathcal{P}^N$. Then every social choice rule that is unanimious and satisfies the pairwise independence condition is dictatorial.

Now, we will look more closely to the conditions of Arrow's theorem. First, it should be noted that the number three plays an important role since intransitivities can only occur on three or more alternatives; that is, if agents have rational preferences in many cases we end up with intransitive social orderings which together with universal domain assumption rules out an important class of social choice rules. For example, the majority voting rule for the case of two alternatives which we analysed in chapter two for two alternatives case can not be applied for social decision in the case of three or more than three alternatives because of this intransitivities. This problem in general is known as the Condorcet paradox. By restricting the domain of the social choice rule we can avoid this problem but we will confine our attention to the social choice rules defined for all possible profiles of individual preferences.

The first restriction of the theorem on the social choice rule is unanimity which says that if every agent puts the same alternative to the top then this alternative should be chosen. The next restriction first suggested by Arrow, is the pairwise independence condition. It means that the social decision between two alternatives does not depend on the other alternatives. The justification for this assumption is simply practicality. By this assumption, the focus is on the relevant alternatives and it also makes task of making social decision easier since it helps to seperate problems. Another issue, on the justification of this assumption which will not be discussed in details here, is the intimate relation between pairwise independence condition and the providing of the right inducements for the truthful revealation of individual preferences. A well known example of social choice rules that does not satisfy this condition is the Borda count rule. The Borda count rule depends on the ordering of an alternative among others, hence, even if the comparison between two alternatives does not change in two different preference profiles, the social ordering on these two alternatives may change because of the orderings of the other alternatives. However, by applying an aggregation rule that uses only the information about the ordering of these two alternatives in individual preferences, we may avoid this problem - but then, as we already mentioned we have the Condorcet paradox problem.

As these explanations of the conditions of Arrow's theorem suggest, what makes Arrow's result disturbing is the minimality and reasonableness of these conditions. However we can still avoid dictatorship conclusion by relaxing some of these conditions. One way is to define the social choice rule on restricted domains. The most important restricted domain condition is single-peakedness. A rational preference relation is single-peaked if there exists an alternative that represents a peak of satisfaction where satisfaction increases as we approach this peak. Whenever the preferences of all agents are single-peaked with respect to the same linear order pairwise majority voting rule can be applied for social decision and a Condorcet winner is obtained.

Another way to escape dictatorship conclusion is to weaken the rationality of the social welfare ordering. If we replace the transitivity requirement by quasitransitivity, then we end up with oligarchic social choice rules. This approach was first suggested by Guha. We will analyze oligarchic social choice rules in detail in the next section.

3.2 Oligarchies and T - Monotonicity

In this section we will focus on oligarchic social choice rules, more precisely on oligarchies and how they are related to the concept T - monotonicity. We will consider three different definitions of oligarchic social choice rules. Basically, the social decision will be formed by a group of members of society, so called oligarchy and other members of society will not have word on this decision. Differences occur due to the decision mechanism within the oligarchy. The first rule we will consider is a function that chooses an alternative socially if it is unanimously choosen by the members of oligarchy. Guha(1972) used this definition and obtained some results. Another definition of an oligarchic social choice rule is as follows: At every possible profile of individuals, F selects only those alternatives that are Pareto undominated with respect to the oligarchy. This rule is characterized by Kaya (1999): A social choice rule is oligarchic in the sense defined above if and only if it satisfies a monotonicity conditon called oligarchic monotonicity (which is stronger than Maskin monotonicity) and is unanimous. Finally, we will consider the social choice rule that only chooses the top ranked alternatives of the oligarchy.

Guha (1972) showed that under a social choice rule satisfying universal domain assumption, pairwise independence condition and Pareto condition, there is one and only one oligarchy , in the sense he defined. This obviously applies to the other definitions of oligarchic social choice rules that we just mentioned. The question next arises is which group of the society constitutes the oligarchy. Kaya suggested for her definition of oligarchic social choice rules that the minimal T - monotonic set is the oligarchy. We will analyse this claim and conclude that it is also true for the other cases as well.

We will subsequently follow the general setting described in the previous section. The domain of the social choice rules that we consider in this section will be $\mathcal{P}^N$. First we will define a monotonicity concept for a social choice rule -or, more prcisely the concept of Maskin monotonicity, after Maskin, who first suggested this concept. The lower contour set of an alternative a for agent i in profile P is defined as $\{x \in A : aP_ix\}$ and is denoted by $L_i(a, P)$.

DEFINITION. A social choice rule F is Maskin monotonic if and only if for every P, $P' \in \mathcal{P}^N$ and $a \in F(P)$, the inclusions $L_i(a, P) \subset L_i(a, P')$ for all $i \in N$ imply $a \in F(P')$.

It is convenient to give an alternative definition of Maskin monotonicity which is equivalent to the original definition. We will use this definition which was suggested by Kaya, to prove some results presented in this section.

DEFINITION. A social choice rule is Maskin monotonic if and only if for every P, $P' \in \mathcal{P}^N$ and $a \in F(P') \backslash F(P)$ there exists an alternative $b \in L_i(a, P')$ and an agent $i \in N$ such that bP_ia.

Next we define T - monotonicity which was again suggested by Kaya:

DEFINITION. A social choice rule F is T - monotonic where $T \subset N$ if and only if for every P, $P' \in \mathcal{P}^N$ and $a \in F(P)$, the inclusions $L_i(a, P) \subset L_i(a, P')$ for all $i \in T$ imply $a \in F(P)$.

Now, we will confine our attention to the oligarchic social choice rule with oligarchy $S \subset N$ which chooses only those alternatives that are Pareto undominated in S. Kaya (1999), claimed that if a social choice rule F is oligarchic with some $S \subset N$, then S is the minimal coalition for which F is monotonic. However, some claims she made in order to prove this result are not correct. She suggests that if F is unanimous and T - and S - monotonic for some S, $T \subset N$ then F is $S \cap T$ - monotonic. Following example shows that this observation is not true: Suppose there are four agents, $\{1, 2, 3, 4\}$, and three alternatives, $\{a, b, c\}$. Let $S = \{1, 2, 3\}$ and $T = \{2, 3, 4\}$. Let F be unanimous and T - and S - monotonic. Consider a profile P such that aP_ibP_ic for $i = 1, 4$ and bP_iaP_ic for $i = 2, 3$. Let $a \in F(P)$ (since a is Pareto undominated for S and T we can safely make this assumption). Now, let P' be a profile such that $bP_i'aP_i'c$ for $i = 1, 2, 3, 4$. By unanimity, the only alternative chosen in this profile is b. Even though, a preserved its position in P' for $i = 2, 3$, it is not chosen in P'. Hence F is not $S \cap T$ - monotonic.

She next defines the minimal coalition for which F is monotonic as follows: $M(F) = \cap\{S \in 2^N \backslash \varnothing : F \text{ is } T - monotonic\}$. And claims that F is $M(F)$ - monotonic. By the above arguement this is also not true. However, the main claim is still true since it supposes that F is oligarchic. Here, to prove this claim we will proceed in the following way: We will first show that if F is oligarchic with oligarchy $S \subset N$, then F is S - monotonic. Next, we will show that then there does not exist a $T \subsetneqq S$ such that F is T - monotonic.

PROPOSITION. Suppose F is oligarchic with oligarchy $S \subset N$. Then F is S - monotonic.

Proof: We will show that F satisfies the alternative definition of monotonicity for $S \subset N$. Let P, $P' \in \mathcal{P}^N$ and $a \in F(P')\backslash F(P)$. Since $a \in F(P')$ a is Pareto undominated in S. So for all $b \in A\backslash\{a\}$ there exists $i \in S$ such that $aP_i'b$. Since $a \notin F(P)$ a is Pareto dominated in the restriction of P to S. That means, there exists at least one $b \in A\backslash a$ such that bP_ia for all i, which in P' for at least one i, $aP_i'b$. So there exists an alternative $b \in L_i(a, P')$ and an agent $i \in N$ such that bP_ia. Hence F is S - monotonic. □

PROPOSITION. Suppose F is oligarchic with oligarchy $S \subset N$. Then there does not exist a $T \subsetneqq S$ such that F is T - monotonic.

Proof: Suppose there exists $T \subset S$ such that F is T - monotonic. Consider a profile P such that aP_ib for all $i \in S\backslash T$ and bP_ia for all $i \in T$. since a is Pareto undominated in S, $a \in F(P)$. Now consider a profile P' such that bP_ia for all $i \in S$. Since F is T - monotonic we have $a \in F(P')$. On the other hand since a is Pareto dominated by b in S, $a \notin F(P')$. So, we reach a contradiction. Hence, there does not exist a $T \in S$ such that F is T - monotonic. □

These results apply to the other two definitions of oligarchic social choice rules as well. The proofs are almost the same so we will not give the proofs here. Thereby we conclude that for three slightly diffrent definitions of oligarchic social choice rules the smallest group of members of society for which F is monotonic, constitutes the oligarchy.

Chapter 4

Conclusion

In this paper, we analysed some social choice rules for two alternatives and three or more than three alternatives cases. In the first part, we considered the two alternatives case. May characterized the majority voting rule for two alternatives and finite agent by some desirable conditions, namely, by symmetry among agents, neutrality between alternatives and positive responsiveness. We showed that the conditions he suggested, characterizes majority voting rule for two alternatives and continuum agents. The idea of the proof is very much like the finite case. Differences are basically technical. We defined the sets E_+, E_- and E_0 and their outer measure and reformulated the definitions accordingly. Then we concluded that majority voting rule is characterized by these reformulated conditions.

In the second part, we analysed the three or more than three alternatives case. Despite the Arrow's impossibility theorem, it is possible to obtain so-called oligarchic social choice rules, by relaxing the transitivity assumption of the social ordering into quasitransitivity. We investigated the relation between T - monotonicity concept, which was suggested by Kaya, and oligarchies. We considered three different definitions of oligarchic social choice rules and showed that the minimal set of agents for which the social choice rule is monotonic constitutes the oligarchy.

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