MULTIMODAL MULTICOMMODITY ROUTING PROBLEM WITH SCHEDULED SERVICES

A THESIS

SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL ENGINEERING

AND THE INSTITUTE OF ENGINEERING AND SCIENCE

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

By

Burak Ayar

November, 2008

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assist. Prof. Dr. Hande Yaman(Advisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Oya Ekin Karaşan

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Haldun Süral

Approved for the Institute of Engineering and Science:

Prof. Dr. Mehmet B. Baray
Director of the Institute

ABSTRACT

MULTIMODAL MULTICOMMODITY ROUTING PROBLEM WITH SCHEDULED SERVICES

Burak Ayar
M.S. in Industrial Engineering
Supervisor: Assist. Prof. Dr. Hande Yaman
November, 2008

We study a multicommodity network flow problem faced by a third party logistics company that has the possibility of using ground and maritime transportation. We are given a set of commodities which should be picked up from their origins at given release times and should be delivered to their destinations no later than their duedates. The commodities may be carried directly from their origins to their destinations on trucks, or they may be carried on trucks to a seaport, may visit several seaports using maritime services, and then to be carried to their destinations on trucks. There is no capacity and time limitation on the use of ground transportation. However, the maritime services are scheduled in advance and the company has limitations on the amounts of volume that it can use on each service. The aim is to determine routes for commodities in order to minimize the sum of transportation cost and stocking costs at seaports, respecting the capacity and time related constraints. We call this problem the *"Multimodal Multicommodity Routing Problem with Scheduled Services (MMR-S)"*. We first prove that the problem is NP-hard. Next, we propose a first mixed integer programming formulation and strengthen it using variable fixing and valid inequalities. We relax the capacity constraints in a Lagrangian manner and show that the relaxed problems decompose into a series of shortest path problems defined on networks augmented by time for each commodity. The corresponding Lagrangian dual yields a lower bound, which may be stronger than that of the linear programming relaxation of our first formulation. Then, we provide an extended formulation whose linear programming relaxation gives the same bound as the Lagrangian dual. Finally, we use the Lagrangian relaxation to devise heuristic methods and report the results of our computational study.

Keywords: Intermodal transportation, multicommodity network flows, time windows, departure times, routing, time-dependent shortest paths.

ÖZET

ÇOK MODLU TARİFELİ SEFERLERE SAHİP TAŞIMA ŞEBEKESİNDE ÇOK ÜRÜNLÜ ROTALAMA PROBLEMİ

Burak Ayar
Endüstri Mühendisliği, Yüksek Lisans
Tez Yöneticisi: Yrd. Doç. Dr. Hande Yaman
Kasım, 2008

Bu tez çalışmasında kara ve deniz taşıma alternatiflerine sahip bir üçüncü şahıs lojistik firmasının çok ürünlü taşıma problemi incelenmiştir. Verilen ürün kümesi bulunduğu noktalardan verilen zaman içinde direkt kamyonla ya da kamyonla alınıp limana ulaştırıldıktan sonra gemiyle ya da tarifeli deniz seferlerini kullanarak son limana taşınmalı ve buradan kamyonla varış noktasına (belirtilen zamandan önce) taşınmalıdır. Kara taşımacılığı ne bir kapasite ne de bir zaman kısıtı içerir. Bununla birlikte deniz taşıma hizmetleri tarifelidir ve firmanın bir seferde kullanabileceği kapasite miktarı belirlenmiştir. Bu çalışmadaki amaç, belirli bir dönemde firmaya ulaşan taleplerin toplam taşıma ve limanlardaki stoklama maliyetini eniyileyen, zaman ve kapasite kısıtlarını sağlayan rotaların bulunmasıdır. Bu probleme *"Çok Modlu Tarifeli Seferlere Sahip Taşıma Şebekesinde Çok Ürünlü Rotalama Problemi "* adı verildi. Öncelikle problemin NP-Zor türü olduğu gösterildi. Daha sonra yeni bir karışık tamsayılı programlama modeli oluşturuldu ve değişken sabitleme ve geçerli eşitsizliklerle model güçlendirildi. Daha sonra doğrusal gevşetmesi ve Lagrangian çifteşi ile aynı sınırı veren genişletilmiş formulasyon verildi. Son olarak, Lagrangian gevşetmesi kullanılarak sezgisel yöntemler geliştirildi ve sayısal çalışmalar rapor edildi.

Anahtar sözcükler: Çok modlu taşımacılık, çok ürünlü şebeke akışları, zaman aralıkları, hareket zamanları, rotalama, zamana bağlı en kısa yollar.

Acknowledgement

I would like to express my deepest and most sincere gratitude to my advisor and mentor, Assist. Prof. Hande Yaman for her invaluable trust, guidance, encouragement and motivation during my graduate study. She has been supervising me with everlasting patience and interest from the beginning to the end.

I would like to express my special thanks to Assoc. Prof. Oya Ekin Karaşan and Assoc. Prof. Haldun Süral for accepting to read and review this thesis and for their suggestions.

I would like to thank my friends Ali Gökay Erön and Sıtkı Gülten for their invaluable camaraderie and helpfulness. I thank them all, for their closest conversations and always being ready to listen me carefully and share their invaluable thoughts.

I would like to thank my friends Zeynep Aydın, Hatice Çalık, Tuğçe Akbaş, G. Didem Batur, Emre Uzun, Safa Onur Bingöl, Adnan Tula, İhsan Yanıkoğlu, Utku Güruşcu, Burak Paç, Ahmet Camcı, Önder Bulut and all other collegaues for providing me such a friendly enviroment to work and also entertaining chats when I want to abstain from doing work. Furthermore, my special thanks to Onur Özkök for his everlasting patience and help, and his keen friendship during my graduate study.

I wish to thank my friends Mustafa Tansu and Ragıp Akkaya for encouraging me to study in industrial engineering after management and sharing their joy with me during my graduate study. Also, I am thankful to Rasim Tansu for being ready and to help me to take a breath any time of any day during the hardest part of my study.

I am also indebted to Volkan Topçu, for his valuable comments on the subject and sparing his valuable time for me.

I would like to thank TÜBİTAK for the financial support they have provided to make this thesis happen.

Last not but least, I would like to express my gratitude to my family for their trust and motivation during my study. I owe them a lot for every success I have in my life. My brother, Fatih Ayar and my sister, Özlem Ayar deserve special mention for their everlasting support and love.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

In today's highly competitive business world, companies' oversea commerce have rapid growth rates. As a result of increased communication and information, a product may have a high demand in a country thousaunds of kilometers away from the production centre. Today, we can see many people who wear the products of the same brand all over the world. It may not be profitable for this brand to produce in all of these countries. Rather, most of the firms perform their operations in different countries where they can carry the least-cost production. Also, it is not required to produce the entire product in one country. In some sectors like automotive, various parts of an automobile are produced in different countries and then assembled in a factory which may be in another country. Then these automobiles are sent to different regions of the world.

Another reason that drives companies to produce outside is the shortage of resources in their home countries. The industrialized countries face resource drain problem, which increases the prices of resources because of low supply and high demand. Companies operating with such scarce resources moved their production to developing countries where industrialization has not been completed and where resources are still plenty [28]. As a result, most of the products produced in developing countries are for developed countries' markets and so output conveyance problem arises.

In order to be competitive, a company must manage its supply chain efficiently. Logistics is a key part of supply chain and its efficient management is a necessity. 40% to 60% of the total cost of an import commodity belongs to transportation and warehousing costs all over the world. Also, the percentage of logistics costs averages 21% of the selling price of a product [30]. Total logistics costs may constitute an important share of the GDP of a country. For the advanced industrialized countries such as the U.S., Japan, and Britain, the logistics cost is about 10%, 11% and 7% of GDP, respectively [27]. In Asia and Pacific countries, logistics costs average about 11.6% of GDP [27]. For example, Thailand's total logistics costs is about 19% of its GDP [27]. These costs make of 17% the GDP of China, which is the 5^{th} biggest and one of the fastest developing economy in the world [27].

Generally, logistics is not in the core competencies of firms and they choose to outsource these services. Today's trend is accepting a logistics company as a partner. This trend is also called as third-party logistics($3PL$). Since this is a partnership, both sides should look for ways to improve their total operating costs and gain competitive advantage.

Multimodal Transportation

Until 1980s, commodities were mostly transported from their origins to destinations with only one type of transportation vehicle [31]. Together with the use of containers in transportation, commodities can be transported without being directly handled and so be transferred from vehicle to vehicle easily. Now, companies seek less costly routes for transportation by considering the possibility of using different types of vehicles at different segments of the routes.

According to European Conference of Ministers of Transport, multimodal transportation is the carriage of goods by at least two different modes of transportation, without any handling of the goods themselves in transshipment between the modes [25]. The carriage of goods without handling is provided by the use of containers. Ground transportation with trucks has several disadvantages. First of all, transportation between countries with no ground connection was not possible. Second, it could be cost inefficient because of low transportation capacities of trucks.

Recently, some countries started to impose limitations on truck transportation because of accidents on roads. For example, the German government started to forbid trucks from its autobahns because of high level of accident risk carried by them [33]. It also decided to encourage rail transportation in inland transportation whose utilization is low, like all over the world [32]. Another restriction we can mention as an example is the use of quotas. Every year, Russian government identifies the number of Turkish licensed trucks that can enter Russia [29].

Additionally, governments encourage the use of alternative transportation modes to avoid high traffic density on roads and ecological disadvantages, and to decrease accident risk and noise pollution. Governments try to balance usage of all transportation options in order to improve the quality of life of their citizens by decreasing the risks and complications. For example, Turkey government provides tax incentives to sea and rail transportation in order to reach to European Union transportation standards and adapt its transportation policy to EU's policy [23].

The logistics operators who experience these enforcements are looking for alternative options in order to provide high quality, fast, reliable and low cost services to their customers. Consequently, these service providers are interested in maritime transportation.

The maritime logistics sector grew rapidly in the last years. The new built vessels' capacities are expanded every year to satisfy the growing maritime transportation demand. The increase in demand for maritime transportation can be explained by its advantages. Firstly, its price. Especially prices for long distance shipping services are less than drayage services. This advantage is a result of high capacities of vessels. Second, shipping services are more enviroment friendly. Also, the risk of accident carried by a ship is less than a truck.

Unless a company is very close to a seaport, it is impossible to directly load or unload goods. To realize maritime transportation, the commodities are first transported by trucks or trains from their origins to sea ports. The reverse process is realized when commodities arrive at the last sea port on their trip. Since commodities are carried by at least two different modes, this type of transportation is called multimodal transportation.

With efficient design of multimodal routes, important amount of gain in costs can be realized. However, the design of a multimodal transportation system is more complex and and more difficult to manage compared to the unimodal one. Also, there exist more parties in the multimodal transportation and hence it becomes difficult to satisfy the needs of all sides.

The use of operations research techniques in multimodal transportation decision process increased in recent years and this attracts the OR researchers and practitioners in this field. The design of multimodal networks and routing of commodities on these networks are the two major subjects in this field.

In our research, we are dealing with operational level decisions of a major logistics company in Turkey that co-operates with the world's largest sea transport service providers. We are interested in the routing of demands of this company on their multimodal network. This company also gives primal and final transportation services called drayage services and direct transportation service from origins to destinations by trucks.

Our aim is to find the minimum total cost routes for a given commodity set of logistics company in a given planning horizon, while satisfying some capacity and time related constraints. A feasible route of a commodity starts when commodity is ready for transportation and ends when it is delivered to its destination no later than its due date. There exist two alternatives in order to transport a commodity. These are multimodal transportation and direct truck transportation options. In multimodal transportation option, a commodity is first transported to a sea terminal with trucks. Then, it is transported with at least one sea service and reaches to last terminal on its trip. The multimodal transportation route of commodity ends with transporting commodity by trucks from last terminal to its destination. The second alternative is direct truck transportation. A commodity can be directly transported by trucks from its origin to destination if it can be delivered before its due date.

In our multimodal transportation system, every service leg between ports has a service time window. The upper bound of this service time window reflects the cutoff times in seaports. In sea transportation, a container is not accepted to be transported with a service if it arrives to port after the cutoff time of this service. After the end of cutoff time, there is an amount of time available for completing all terminal-related activities of service and then vessel departs from the port at its scheduled departure time. On the other hand a truck can depart from a port at any time. We impose no schedules on truck departures and no capacities to drayage services and direct truck transportation services because according to our industrial research, any number of trucks can be available for transportation at any time in any port because of high level of outsourcing alternatives. These cases are also considered in [6] and [15]. Furthermore, we consider the case where storage yard of ports have capacity limits and hence, very early arrivals should be penalized in order to prevent congestion in ports [8].

In this thesis, we study the problem faced by a 3PL company operating on an multimodal network. Given the demand over the planning horizon, the aim is to identify cost efficient transportation routes that are compatible with schedules of the selected transportation modes. We call the problem "*Multimodal Multicommodity Routing Problem with Scheduled Services(MMR-S)*" and propose a mathematical model for it. After proposing the model, we first try the exact solution approaches. After this attempt, we look for heuristic approaches. For having an idea about the quality of heuristic solutions, we aim to generate strong lower bounds. For these, we generate some valid inequalities in order to strengthen the LP relaxation of the model and also apply Lagrangian relaxation techniques. Two heuristic solution approaches are implemented. *MMR-S* is both solved on the network of the logistics company with data provided from the same source and solved on randomly generated networks with generated data. The computational results are reported and the best solution approaches are given.

The remainder of the thesis is organized as follows:

In Chapter 2, we provide a review of the literature on multimodal network problems. The studies on the design of the multimodal service network problems are reviewed in the first part of the chapter. Then, single commodity and multicommodity routing on multimodal networks are analyzed.

In Chapter 3, first we define our problem and propose a new mixed integer linear program. Then, we perform variable fixing and add valid inequalities to the proposed model to improve the lower bound and running times for problem instances.

Chapter 4 is organized in two parts. In the first part, we apply Lagrangian relaxation to our model. We describe how we solve the Lagrangian relaxed problem. In the second part, we give an extended formulation.

In Chapter 5, we propose heuristics to get near-optimal solutions for large size networks and higher number of commodities that can not be solved in reasonable running times.

We report the results of our computational study in Chapter 6. First, we give the comparisons between two formulations. We analyze the effects of node and arc sizes and capacities of arcs on the efficiency of the formulations. Also, efficiency of variable fixing and valid inequalities is discussed. Finally, comparison of heuristics and lower bounds are given.

In Chapter 7, we conclude the thesis by giving an overall summary of our contribution to the existing literature and suggesting some possible future research directions.

Chapter 2

Literature Survey

The literature on the use of OR techniques in multimodal transportation is quite limited. However, the number of published papers has been increasing since 2000s. In their review, Macharis and Bontekoning [14] classify the studies in this field using two criteria : type of operator and time horizon of operations. They identify four types of operators : drayage, terminal, network and multimodal, which are based on four main activities in multimodal operations. Three different time horizons according to level of planning are strategic, tactical and operational. According to this classification, our problem is an operational level planning problem which is faced by a intermodal service operator. Also, the authors remark that this category is one of the least studied.

We first review multimodal service network design literature. Then, we analyze the literature on multimodal routing problem. Since multicommodity routing on multimodal network problem is a relatively new topic, both single and multicommodity studies are reviewed in the second part.

2.1 Multimodal Service Network Design Problem Literature

Service network design problems are long-term planning problems which aim at minimizing fixed costs of installing network and long term variable costs of operations. The fixed costs may be associated with installing a new terminal or a new crane, purchasing a new vessel, wagon or a truck.

An early work on multicommodity multimode transportation service network design problems is by Crainic and Rousseau [9]. They work on both strategic and tactical level planning problems when supply level of transportation services and the itineraries of the demands are controlled by the same authority. A general modelling framework is designed to integrate problems in these levels. The objective of this framework is improving the performance of the transportation system by minimizing the total operating and delay costs and maximizing the service quality. They formulate a nonlinear mixed integer programming model and describe an algorithm based on decomposition and column generation principles combined with some heuristic approaches to improve the algorithm's performance. The algorithm is a two step procedure where first, given a fixed level of demand, optimal frequencies of the available services are determined and then the optimal routes are generated. The algorithm is tested on the two data set supplied by Canadian National Railways.

Guelat, Florian and Crainic [11] develop an uncapacitated multimode multiproduct assignment model for strategic planning of freight flows appropriate for national and regional transportation systems. The model is a mixed integer linear model based on path formulation. Modes are defined as types of vehicles used. Various mode alternatives between two terminals are represented using parallel arcs between these terminals. The transfers between modes at a terminal are modeled by adding artificial nodes and arcs to the physical network. They assume that cost of transporting a commodity on an arc depends on the flows of arcs that share same tail and head nodes in both orientations. The structure of their model allows decomposition by commodity and they solve each linear subproblem by an algorithm which is a modification of Djikstra's label setting algorithm. They apply their study on data from Brazilian freight transportation network.

Andersen, Crainic and Christiansen [1] focus on the issue of management of assets, that is vehicle fleets used in operations, while designing the service network. Their model includes adaptation of new and existing services and management of vehicles. The internal and external services are linked at intermodal terminals so intermodal operations take place at these terminals. The objective of their model is minimizing the total time spent of the demand in the system by determining the optimal departure times of integral services with a given level of demand. All demand within the given time period must be transported and services selected for transportation of demand must be covered by vehicles. The application of the model is on Polcorridor study (Polcorridor 2006) and solved by using a mixed integer software with adding some constraints to strengthen the model. Also, they make various scenario analysis related with the integration of different service systems.

Vasiliauskas [17] models the national multimodal freight transportation network of Lithuania. He develops the model by integrating partical modal networks into a general intermodal network through identifying optimal transfer terminals in road, railroad and water networks. Then he determines the demands' optimal routes between origin destination pairs with simple shortest path algorithms. Also, he deals with the design of terminal operations.

As we have seen, in most of multimodal service network design problems, reseracher s initially develop the network according to given level of demand and after that they develop the optimal routes of these demands on new developed service network. Because of this characteristic, the design and routing problems are combined with each other in multimodal concept.

2.2 Multimodal Routing Problem Literature

In the earliest studies in multimodal routing, the researchers either do not relate themselves with time aspects of the problem or they do not directly place them to their models. One of the study in the earliest literature of developing multimodal routes of commodities is by Barnhart and Ratliff [3]. They consider transportation of commodities on an uncapacitated rail-truck combined network. In their study a commodity can be transported directly by truck from its origin to destination or by truck from origin to a rail terminal, then by train over the railway network and lastly from last train terminal to its destination again by truck. This structure resembles to the service offerings of our logistics company. They model the problem upon two types of cost plan on rails which are per trailer rail cost and per flatcar rail cost. Network representations are given for both types of cost plan. They discuss extensions their of model to deal with schedule requirements and flatcar configurations.

Bookbinder and Fox [4] find optimal intermodal routes for Canada-Mexico shipments under NAFTA. The network is composed of cities from Mexico, Canada and U.S. whose trade volume are high and also services between these cities. They analyze the total trade between two countries and offer the most promising transportation alternatives. The water, rail and road links added between the nodes in the network are selected from services of carriers that meet some criteria. They formulate their problem with the objective of minimizing total cost of the route of every shipment.

Nozick and Morlok [16] present a model for medium-term operations planning for a multimodal rail-truck service. The work is motivated from the need of integrating various elements of rail intermodal operations to enhance service quality and to improve operating efficiency and asset utilization. They discuss the schedules of train services, various service classes in railroad segments and available times of goods for movement. They take into account these characteristics of the transportation systems in the modelling process. Also, the needs for empty trailer and flatcar repositioning are considered. They propose an integer programming formulation that aims to find time and equipment feasible routes for demands and that satisfy the desired service level with minimum transportation and repositioning costs. A heuristic initialized with the LP relaxed solution is developed. The tests on data sets provided from several U.S. rail transportation companies are given.

Kim, Barnhart, Ware and Reinhardt [12] work on multimodal express package delivery problem. The objective of their model is minimizing movement costs of packages under tight service windows at hubs and airport terminals and limited capacity of hubs and vehicles. The service time windows at hubs reflect the time periods that a hub can sort packages. Every airport terminal has time limits that denote the latest arrival time of cargo to terminal for using the desired service. Service types are differentiated in terms of speed. They solve the problem using Dantzig- Wolfe decomposition.

Kozan [13] deals with operations on multimodal terminals. He studies the optimization of the total elapsed time for transferring containers between two modes in a terminal. An analytical model is proposed and applied to a seaport terminal in Australia.

Choong, Cole and Kutanoglu [8] discuss the effects of planning period length on empty container management for multimodal transportation networks. Planning period length is an important concept because transportation cost of empty containers by barge is very small and negligible, however, this transportation option is slow. They give an integer program and apply it to case of potential-barge-transportation operations within the Mississipi river.

We also review the literature on finding a cost optimal or a time optimal path for a single commodity on a multimodal, scheduled network because it is a subproblem of our problem. The solution efforts on this subproblem are based on extending classical shortest path solution procedures.

Ziliaskopoulos and Wardell [18] are interested in finding least travel time paths on multimodal transportation networks. The delays realized during the change of modes are considered in the total travel time. One of the example to this type of delays is time spent during the search for a parking place for a truck when this truck arrives at a seaport to transfer containers to a vessel in the port or to the storage area of the port. They also take into account the fixed schedules of available transportation alternatives while designing their solution methodology. They give an optimality condition which is an extension of Bellman optimality condition and construct their algorithm based on this developed optimality principle.

Boussedjra, Bloch and El Moudni [5] develop an exact method to find the least travel time path between an o-d pair in a time-dependent multimodal transportation network. They use a second criterion if there exists a tie between two solutions. The solution with less transshipments is selected. In the solution process, while exploring the graph for the minimum travel time path they use a bidirectional research strategy which are from origin node to forward and from destination node to backward. Every node in the graph has two labels. At each iteration of the method, a label correcting method based on Bellman optimality conditions updates these labels. Also, at each iteration some feasibility conditions for paths are checked in order to eliminate infeasible ones. These feasibility conditions are related with : arrival time of a path to destination node because of the scheduled arrival time to destination node; arrival times to intermediate nodes because of the scheduled departure times of conveying vehicles from terminals; and the total travel time of the path. Their algorithm is an extended version of algorithm developed in [18]. The enhanced method is tested on networks with various sizes and compared with results obtained with branch and bound method. The results show that the developed method is superior to branch and bound method in terms of CPU times.

A latest study on determining the routes of commodities on international multimodal networks is by Chang [7]. He formulates the problem as a multiobjective multimodal multicommodity flow problem with time windows and concave costs. The multiobjectives of the problem are minimizing the total cost and total time of the routes of shipments. The transportation cost on each link is designed as concave piecewise linear function of total flow on that link. He incorporates the transportation mode schedules and delivery times of commodities to the model. A time window associated with each node on the network defines the earliest and latest arrival time to that node in order to use associated services originated from this node. A commodity departs from a node at the end of the service period or leaves the node at a scheduled departure time if exists. Also, transportation capacities for every range of cost curve are established to each link. In the formulation, he allows the splitting of demand between an origin destination pair, that is, demand can be transported from its origin to destination with more than one route. The solution procedure used is combination of several methodologies that are respective implementations of relaxation, separation, decomposition, shortest path and reoptimization. They test the solution approach on a small and large network.

One of the other study that highly correlates with our study is by Moccia, Cordeau, Laporte, Ropke and Valentini [15]. They solve a multicommodity routing problem on a multimodal network. Time windows are included in order to model opening hours of terminals and pickup and delivery time slots for demands. The services are differentiated in terms of departure time and cost function. They give a network representation by exploding the physical network. Column generation algorithms are designed to get lower bounds and they are combined with heuristics to get feasible solutions. They apply solution procedures to data obtained from a freight forwarder operating on Italian market.

Since maritime transportation constitutes a major part of multimodal transportation system in our problem, an overview of it is needed. Maritime transportation is not the major transportation mode worldwide, on the other hand there exists fast growth in the amount of cargo transported with vessels and capacity of ships. Within containerization, shippers prefer to use maritime transportation, especially in long hauls. Various type of vessels are produced in order to satisfy maritime transportation demand of various sides. In chapter 4 of [20], authors give terms used in OR-applications in maritime transportation, problems in strategic, tactical and operational planning, models and solution approaches.

Chapter 3

Model Development

In this chapter, we formalize our problem definition, prove its complexity, give a mixed integer programming formulation and derive some valid inequalities.

In our problem, we are given the commodity set of company over a given horizon where each commodity ahould be taken on their ready times and to be delivered to their destinations no later than their due dates. A commodity can be transported either directly by trucks or combination of truck and maritime services. In this combined route, commodities firstly carried from their origins to seaports, then transported on maritime services and from last sea port, again they are transported with trucks to their destination point. The truck transportation services has no schedules and capacity limitations. On the other hand, maritime services has schedules. There exist a time window for each service which reflects the start time and cutoff time for processing of the related maritime service. A commodity can arrive to port earlier than start time of the service but it is stocked on the port againist a fee. However, it can not use the maritime service if it arrives to port later than cutoff time of time of service. Also, maritime services are capacitated. Our problem is finding the time feasible transportation routes of commodity set that gives us the minimum total transportation and stocking costs.

3.1 Mathematical Model Formulation

We are given a directed network $G = (N, A)$ which possibly contains parallel arcs. Each node in this network corresponds to a seaport and each arc corresponds to a scheduled maritime service.

First, we define the parameters related with the arc set of G. Let $a \in A$. The tail of a is denoted by $s(a)$ and the head of a is denoted by $t(a)$. We denote by c_a the variable cost of transporting and loading and unloading a unit volume with service a. τ_a denotes the travel time and u_a denotes capacity of service a. The last two parameters for arc set A are related with time windows. The time window for arc a is defined by $[e_a, l_a]$ where e_a is the earliest time for the start of service a at node $s(a)$ and l_a is the cutoff time at node $s(a)$ to use service a. A commodity that arrives to port $s(a)$ before e_a should wait in the storage area of $s(a)$ and pay a fee for using the storage area. This fee is increasing linearly with the time a commodity waits at the storage area. On the other hand, arrival before l_a is a strict constraint and means a commodity has no chance to use a if it arrives at port after l_a.

We denote by p_i the cost of stocking a unit volume at seaport $i \in N$.

The company's demands on the given time period is denoted by set K where each item k has an origin $o(k)$, destination $d(k)$, demand $w(k)$, release time $r(k)$ and duedate $q(k)$. The set $K' \subseteq K$ denotes the set of commodities which can wait until their pickup times at their origins at no cost. The remaining commodities should be picked up exactly at their release times.

For $k \in K$, let $c_{o(k)d(k)}$ denote the cost of transporting commodity k from its origin to its destination directly using trucks. For $k \in K$ and $i \in N$, let $c_{o(k)i}$ denote the cost for carrying commodity k from its origin to seaport i, and $c_{id(k)}$ denote the cost for carrying commodity k from seaport i to its destination directly using trucks.

We assume that there does not exist a capacity restriction on drayage services and that all cost factors are nonnegative and $c_{o(k)i} + c_{id(k)} \geq c_{o(k)d(k)}$ for all $k \in K$

and $i \in N$.

Finally, $\tau_{o(k)i}$ denotes the travel time from the origin to seaport i , $\tau_{id(k)}$ denotes the travel time from seaport i to the destination node and $\tau_{o(k)d(k)}$ denotes the travel time of direct transportation from the origin to the destination by truck. These parameters also include the loading times at the origins of the services and the unloading times at the destination of the services. We assume that there does not exist schedules for drayage services.

After the definition of parameters, now we introduce the variables used to formulate the problem. For $k \in K$, y^k is 1 if commodity k is transported directly from its origin to its destination on trucks and 0 otherwise. For $k \in K$ and $i \in N$, $x^k_{o(k)i}$ is 1 if commodity k is carried from its origin to seaport i using trucks and 0 otherwise and $x^k_{id(k)}$ is 1 if commodity k is carried from seaport i to its destination using trucks and 0 otherwise. For $k \in K$ and $a \in A$, x^k_a is 1 if commodity k uses maritime service a and 0 otherwise.

Finally, for $k \in K$ and $i \in N$, v^k_i denotes the arrival time of commodity k at seaport i if this seaport is visited by this commodity and z^k_i is the amount of time that commodity k is stocked at seaport i.

Using the decision variables above, we derive the following mixed integer programming formulation called *ILP-1*.

(ILP-1)

$$min \sum_{k\in K} w(k)\Big[\sum_{j\in N} c_{o(k)j}x^k_{o(k)j} + \sum_{a\in A} c_a x^k_a + \sum_{j\in N} c_{jd(k)}x^k_{jd(k)} + c_{o(k)d(k)}y^k + \sum_{i\in N} p_i z^k_i \Big]$$

s.t.

$$\sum_{j\in N} x^k_{o(k)j} + y^k = 1 \qquad \forall k \in K \qquad (3.1)$$

$$\sum_{a\in A:t(a)=i} x^k_a + x^k_{o(k)i} - \sum_{a\in A:s(a)=i} x^k_a - x^k_{id(k)} = 0 \qquad \forall k \in K, i \in N \qquad (3.2)$$

$$\sum_{j\in N} x^k_{jd(k)} + y^k = 1 \qquad \forall k \in K \qquad (3.3)$$

$$\sum_{a\in A:t(a)=i} x^k_a + x^k_{o(k)i} \leq 1 - y^k \qquad \forall k \in K, i \in N \qquad (3.4)$$

$$\sum_{k\in K} w(k)x^k_a \leq u_a \qquad \forall a \in A \qquad (3.5)$$

$$v^k_i = \sum_{a\in A:t(a)=i} (l_a + \tau_a)x^k_a + (r(k) + \tau_{o(k)i})x^k_{o(k)i} \qquad \forall k \in K, i \in N \qquad (3.6)$$

$$z^k_i \geq \sum_{a\in A:s(a)=i} e_a x^k_a - v^k_i \qquad \forall k \in K \setminus\ K', i \in N \qquad (3.7)$$

$$z^k_i \geq \sum_{a\in A:s(a)=i} e_a x^k_a - v^k_i - Mx_{o(k)i} \qquad \forall k \in K', i \in N \qquad (3.8)$$

$$v^k_i \leq \sum_{a\in A:s(a)=i} l_a x^k_a + (q(k) - \tau_{id(k)})x^k_{id(k)} \qquad \forall k \in K, i \in N \qquad (3.9)$$

$$r(k) + \tau_{o(k)d(k)}y^k \leq q(k) \qquad \forall k \in K \qquad (3.10)$$

$$x^k_{o(k)i} \in \{0, 1\} \qquad \forall k \in K, i \in N \qquad (3.11)$$

$$x^k_{id(k)} \in \{0, 1\} \qquad \forall k \in K, i \in N \qquad (3.12)$$

$$x^k_a \in \{0, 1\} \qquad \forall k \in K, a \in A \qquad (3.13)$$

$$y^k \in \{0, 1\} \qquad \forall k \in K \qquad (3.14)$$

$$v^k_i \geq 0 \qquad \forall k \in K, i \in N \qquad (3.15)$$

$$z^k_i \geq 0 \qquad \forall k \in K, i \in N \qquad (3.16)$$

Constraints (3.1), (3.2) and (3.3) are flow conservation constraints. Constraint (3.1) and (3.3) ensure that the transportation path of a commodity must start at its origin and ends at its destination, respectively. Constraint (3.2) ensures that a commodity can either come to a seaport with a maritime service or with truck from its origin. Also, it either leaves a seaport with a maritime service or with truck to its destination. Constraint (3.4) is designed to eliminate cycles which could arise due to time restrictions(note here that cycles which do not intersect the path can not appear in the optimal solutions when costs are positive). Constraint (3.5) ensures that total volume to be transported on a service can not exceed the capacity of the service.

The arrival times of commodities to sea port terminals are provided by (3.6). If commodity k arrives at seaport i using service a, then it leaves the origin seaport $s(a)$ at time l_a and travels for τ_a time units and hence arrives at i at time $l_a + \tau_a$. If seaport i is the first seaport that commodity k visits, i.e., if commodity k leaves its origin and comes to i using trucks, then $r(k)$ is the time trucks leave the origin node and $\tau_{o(k)i}$ is the trip time. If commodity k does not visit seaport i then its arrival time is taken to be zero.

If a commodity k arrives at seaport i before loading starts for its service, then it is stocked at the seaport. Constraint (3.7) computes the amount of time for which commodity k is stocked at seaport i.

Constraint (3.8) is designed for customers that have depots. If commodity arrives to source seaport of the first maritime service on its transportation path before the start time of the maritime service, then rather than leaving its origin at its release time, it can be stocked at the depot of its origin and leave later than release time. This means early arrival is prevented and the stocking cost at first ports will not be incurred. Here, $M = max\{e_a : a \in A\}$.

Constraints (3.9) ensure that the commodities arrive within the time window of the service they would like to use and that they arrive at their destinations no later than the due dates. Finally, constraints (3.10) avoid direct shipments by trucks if the commodity cannot be on time using this transportation mode. These constraints can be dropped by setting $y^k = 0$ for commodities $k \in K$ such that $r(k) + \tau_{o(k)d(k)} > q(k)$.

The remaining constraints (3.11), (3.12), (3.13), (3.14), (3.15) and (3.16) define the types of decision variables.

3.2 Complexity

Now, we look at the complexity of the problem and give the proof of its complexity.

We define the decision version of the problem *MMR-S* as follows. Given the parameters of the problem and a scalar γ_0, does there exist a feasible solution with cost not more than γ_0? Next, we prove that this problem is NP-complete using a polynomial reduction from the 0-1 knapsack problem.

Proposition 1 *The decision version of MMR-S is NP-complete.*

Proof. Consider the decision version of the 0-1 knapsack problem. Given a set I, nonnegative integers α_i and β_i for each $i \in I$ and two positive scalars α_0 and β_0, does there exist a subset $S \subseteq I$ such that $\sum_{i\in S} \alpha_i \leq \alpha_0$ and $\sum_{i\in S} \beta_i \geq \beta_0$? This problem is NP-complete, see [21] even when $\alpha_i = \beta_i$ for all $i \in I$.

Suppose that there are only two seaports 1 and 2, and that there is a single service from 1 to 2 with capacity α_0. For each item i in set I, define a commodity from node 1 to node 2 with demand volume equal to α_i. The release time is 0 and the duedate is 1 for all commodities. Commodities can be transported at no cost and no time from their origins to seaport 1 and from seaport 2 to their destinations. The maritime service starts at time 0 and ends at time 1. The unit cost of using the service is equal to 1 and the unit cost of delivering a commodity using trucks is 2. Under this specification of parameters, the *MMR-S* reduces to the problem of finding a minimum cost partition of the set of commodities I into two sets S and $I \setminus S$ such that commodities S are transported using the maritime service and commodities $I \setminus S$ are transported using trucks. Such a partition is feasible if $\sum_{i\in S} \alpha_i \leq \alpha_0$. The cost of the associated solution is $\sum_{i\in S} \alpha_i + \sum_{i\in I\setminus S} 2\alpha_i = 2\sum_{i\in I} \alpha_i - \sum_{i\in S} \alpha_i$. Let $\gamma_0 = 2\sum_{i\in I} \alpha_i - \beta_0$.

Then, there exists a solution to the decision version of 0-1 knapsack problem with $\beta_i = \alpha_i$ for all $i \in I$ if and only if there exists a solution to the decision version of the problem *MMR-S*. □

3.3 Variable Fixing

In this section, we present some simple results to reduce the problem size by fixing some of the variables in the model.

Proposition 2 *Let $k \in K$ and $a \in A$. Every feasible solution satisfies $x_a^k = 0$ if $w(k) > u_a$ or $l_a < r(k)$.*

Proof. If the demand volume of a commodity exceeds the capacity of a service or if the ready time of a commodity is later than the cutoff time of a service, then clearly, this commodity cannot use that service. □

The next result is about time characteristic of model. We solve $|K|$ shortest path like problems by applying a modification of Djikstra's label correcting algorithm(see Algorithm 1). We find shortest time paths to all nodes in the graph that gives us the earliest arrival time of a commodity to a node in the graph. We denote this as E_i^k, which means earliest arrival time of commodity k to node i. We do not give the proof of Algorithm 1 because it can be proved same as Djikstra's algorithm.

Proposition 3 *Let $k \in K$ and $a \in A$. Every feasible solution satisfies $x_a^k = 0$ if $E_{s(a)}^k > l_a$.*

Proof. If a commodity's earliest time to reach a terminal is later than the cutoff time of a service originating from this terminal, then we can not transport this commodity on that arc. □

Algorithm 1 Earliest Arrivals-for a given $k \in K$

Initialize two lists, Temporary and Permanent
for all $i \in N \cup \{o(k)\}$
  if $i = o(k)$ **then**
  $E_i^k \leftarrow r(k)$
 else
  $E_i^k \leftarrow \infty$
 endif
Insert all $i \in N \cup \{o(k)\}$ to Temporary list
while Temporary list $\neq \emptyset$ **do**
  Select node i^* from Temporary list such that $E_{i^*}^k = min\left\{E_i^k : i \in Temporarylist\right\}$(break ties arbitrarily)
  Insert i^* to Permanent list, delete from Temporary list
  if $i^* = o(k)$ **then**
    for all $j \in N$ **such that** $\tau_{o(k)j} \neq \infty$
     if $E_j^k > E_{o(k)}^k + \tau_{o(k)j}$
      $E_j^k \leftarrow E_{o(k)}^k + \tau_{o(k)j}$
     endif
  else
    for all $a \in A$ **such that** $s(a) = i^*$
     if $E_{i^*}^k \leq l_a$ and $E_{t(a)}^k > l_a + \tau_a$
      $E_{t(a)}^k \leftarrow l_a + \tau_a$
     endif
  endif
end while

By applying variable fixing, we may reduce the size of our model with the aim of reaching the optimal solutions in shorter times or getting better lower bounds by strengthening the LP relaxation.

3.4 Valid Inequalities

In this section, we try to generate valid inequalities based on time restrictions. Let F denote the set of feasible solutions to model *ILP-1*. Consider commodity $k \in K$ and seaport $i \in N$. If commodity k travels directly from its origin node $o(k)$ to the seaport i, then it arrives there at time $r(k) + \tau_{o(k)i}$. Then it is not possible for this commodity to use any service that starts at node i for which the cutoff time is earlier than $r(k) + \tau_{o(k)i}$. The set $H_i^k = \{a \in A : s(a) = i, r(k) + \tau_{o(k)i} > l_a\}$ is the set of such services.

Similarly, if commodity k travels from seaport i directly to its destination $d(k)$, then it should be ready to depart from i the latest at time $q(k) - \tau_{id(k)}$. Hence any service a which arrives at seaport i later than this time cannot be used by commodity k. $D_i^k = \{a \in A : t(a) = i, l_a + \tau_a + \tau_{id(k)} > q(k)\}$.

Proposition 4 *For $k \in K$ and $i \in N$, the inequalities*

$$x_{o(k)i}^k + \textstyle\sum_{a \in H_i^k} x_a^k \leq 1 - y^k \tag{3.17}$$

and

$$x_{id(k)}^k + \textstyle\sum_{a \in D_i^k} x_a^k \leq 1 - y^k \tag{3.18}$$

are valid for F.

Proof. We give the proof for inequality (3.17). For commodity $k \in K$, if $y^k = 1$, then commodity k is transported from its origin to its destination directly on trucks and hence all associated x variables are zero. Otherwise, if $x_{o_k i}^k = 1$, then by definition of set H_i^k, commodity k cannot use any service in this set and hence $\sum_{a \in H_i^k} x_a^k = 0$. Finally, if $y^k = 0$ and $x_{o_k i}^k = 0$, then $\sum_{a \in H_i^k} x_a^k \leq 1$ as commodity k has to be carried on a simple path. The proof for inequality (3.18) is similar. □

For $a \in A$, define $\Delta_a^+ = \{a' : s(a') = t(a), l_a + \tau_a > l_{a'}\}$ and $\Delta_a^- = \{a' : t(a') = t(a), l_{a'} + \tau_{a'} \geq l_a + \tau_a\}$. Notice that if a commodity k uses service a, then the time it reaches $t(a)$ is $l_a + \tau_a$. If a' is a service that starts at node $t(a)$ and if the latest allowable time for this service is earlier than $l_a + \tau_a$, then the commodity cannot use this service. The set Δ_a^+ is the set of services that start at node $t(a)$ and that cannot be used if service a is used to reach node $t(a)$. The set Δ_a^- is the set of services that arrive at node $t(a)$ not earlier than service a.

Proposition 5 *Let $k \in K$ and $a \in A$.*

If $r(k) + \tau_{o(k)t(a)} \leq l_a + \tau_a$ and $l_a + \tau_a + \tau_{t(a)(dk)} \leq q(k)$, the inequality

$$\textstyle\sum_{a' \in \Delta_a^-} x_{a'}^k + \sum_{a' \in \Delta_a^+} x_{a'}^k \leq 1 - y^k \tag{3.19}$$

is valid for F.

If $r(k) + \tau_{o(k)t(a)} > l_a + \tau_a$ and $l_a + \tau_a + \tau_{t(a)(dk)} \leq q(k)$, the inequality

$$x_{o(k)t(a)}^k + \textstyle\sum_{a' \in \Delta_a^-} x_{a'}^k + \sum_{a' \in \Delta_a^+} x_{a'}^k \leq 1 - y^k \tag{3.20}$$

is valid for F.

If $r(k) + \tau_{o(k)t(a)} \leq l_a + \tau_a$ and $l_a + \tau_a + \tau_{t(a)d(k)} > q(k)$, the inequality

$$\textstyle\sum_{a' \in \Delta_a^-} x_{a'}^k + \sum_{a' \in \Delta_a^+} x_{a'}^k + x_{t(a)d(k)}^k + \leq 1 - y^k \tag{3.21}$$

is valid for F.

If $r(k) + \tau_{o(k)t(a)} > l_a + \tau_a$ and $l_a + \tau_a + \tau_{t(a)d(k)} > q(k)$, the inequality

$$x^k_{o(k)t(a)} + \sum_{a' \in \Delta^-_a} x^k_{a'} + \sum_{a' \in \Delta^+_a} x^k_{a'} + x^k_{t(a)d(k)} + \leq 1 - y^k \tag{3.22}$$

is valid for F.

Proof. We give the proof for inequality (3.20). If $y^k = 1$, then the vector x^k is a zero vector. If $y^k = 0$ and $x^k_a = 1$, then $\sum_{a' \in \Delta^+_a} x^k_{a'} = 0$ since service a arrives at seaport t_a later than the latest allowable time for these services, $x^k_{t_a d_k} = 0$ since it is not possible to meet the duedate as $l_a + \tau_a + \tau_{t_a d_k} > q_k$, $x^k_{o_k t_a} + \sum_{a' \in \Delta^-_a \setminus \{a\}} x^k_{a'} = 0$ since commodity k arrives at seaport t_a using service a and cannot use any other service that ends at the same seaport and cannot be carried to this seaport from its origin. Finally, if $y^k = 0$ and $x^k_a = 0$, then we know that $x^k_{o_k t_a} + \sum_{a' \in \Delta^-_a} x^k_{a'} \leq 1$ and $\sum_{a' \in \Delta^+_a} x^k_{a'} + x^k_{t_a d_k} \leq 1$ since commodity k can arrive at and leave seaport t_a at most once. Moreover, all services in set Δ^-_a arrive at t_a to late to be able to use any service from set Δ^+_a or for the commodity to be delivered to its destination on time. Hence at most one of the sums $x^k_{o_k t_a} + \sum_{a' \in \Delta^-_a} x^k_{a'}$ and $\sum_{a' \in \Delta^+_a} x^k_{a'} + x^k_{t_a d_k}$ can be 1. $\square$

Chapter 4

Lagrangian Relaxation & The Extended Formulation

In the first part of this chapter, we present a Lagrangian relaxation for *MMR-S*. In the second part, we give an extended formulation to our problem which is an integer multicommodity network flow(IMNF) problem defined on a special graph.

4.1 Lagrangian Relaxation of ILP-1

The Lagrangian relaxation is a popular approach among OR practitioners and widely used in the solution process of hard combinatorial optimization problems. The idea behind the Lagrangian relaxation is to remove the complicating constraint/constraints from the constraint set, to add them to the objective function by penalizing their violation and then to solve the easier problem with remaining constraints.

It is important to decide which constraints are complicating before applying the Lagrangian relaxation technique. We relax the capacity constraints (3.5) in our problem as these are the only constraints that link all commodities together.

Let α_a be the Lagrange multiplier associated with constraint (3.5) for service

$a \in A$. For a given vector $\alpha \geq 0$, the relaxed problem disaggregates into $|K|$ problems as follows.

$$\begin{aligned}\mathrm{LR}(\alpha) = & -\textstyle\sum_{a\in A}\alpha_a u_a + \sum_{k\in K: r(k)+\tau_{o(k)d(k)}\leq q(k)} w(k)\min\{LR^k(\alpha), c_{o(k)d(k)}\} \\ & + \textstyle\sum_{k\in K: r(k)+\tau_{o(k)d(k)}>q(k)} w(k) LR^k(\alpha)\end{aligned}$$

where

$$\begin{aligned}\mathrm{LR}^k(\alpha) = \min & \textstyle\sum_{j\in N} c_{o(k)j}x^k_{o(k)j} + \sum_{a\in A}(c_a+\alpha_a)x^k_a + \sum_{j\in N} c_{jd(k)}x^k_{jd(k)} + \\ & \textstyle\sum_{i\in N} p_i z^k_i\end{aligned}$$

s.t.

$$\textstyle\sum_{i\in N} x^k_{o(k)i} = 1 \qquad (4.1)$$

$$\textstyle\sum_{a\in A:t(a)=i} x^k_a + x^k_{o(k)i} - \sum_{a\in A:s(a)=i} x^k_a - x^k_{id(k)} = 0 \qquad \forall i \in N \qquad (4.2)$$

$$\textstyle\sum_{i\in N} x^k_{id(k)} = 1 \qquad (4.3)$$

$$\textstyle\sum_{a\in A:t(a)=i} x^k_a + x^k_{o(k)i} \leq 1 \qquad \forall i \in N \qquad (4.4)$$

$$v^k_i = \textstyle\sum_{a\in A:t(a)=i}(l_a+\tau_a)x^k_a + (r(k)+\tau_{o(k)i})x^k_{o(k)i} \qquad \forall i \in N \qquad (4.5)$$

$$z^k_i \geq \textstyle\sum_{a\in A:s(a)=i} e_a x^k_a - v^k_i \qquad \forall i \in N \qquad (4.6)$$

$$v^k_i \leq \textstyle\sum_{a\in A:s(a)=i} l_a x^k_a + (q(k)-\tau_{id(k)})x^k_{id(k)} \qquad \forall i \in N \qquad (4.7)$$

$$x^k_{o(k)i}, x^k_{id(k)} \in \{0,1\} \qquad \forall i \in N \qquad (4.8)$$

$$x^k_a \in \{0,1\} \qquad \forall a \in A \qquad (4.9)$$

$$z^k_i \geq 0 \qquad \forall i \in N \qquad (4.10)$$

In $LR^k(\alpha)$, direct truck transportation option is not included because it is inserted to $LR(\alpha)$. For commodities which direct truck transportation is feasible, we select the minimum cost route, minimum multimodal(if exists) or direct truck transportation route. For the remaining constraints, direct truck transportation is not time feasible so it is not included.

The aim of the problem is to find a simple path from origin of commodity k to its destination which minimizes the sum of transportation and stocking costs using scheduled services. The cost of using service $a \in A$ is equal to $c_a + \alpha_a$.

We can not solve the subproblems by finding a shortest path in G. The reason is that because of stocking times and cutoff times a shortest path may not satisfy the *Bellman optimality conditions* [19] which is the base of most of the classical shortest path algorithms. Suppose that there exists a shortest path ϕ from an origin node o to a destination node d in our network. Also, suppose that node i is in ϕ. As indicated in *Bellman optimality conditions* [19] , if ϕ is the shortest path from o to d and i is in ϕ, then the path from o to i and i to d must be shortest. But that may not be the case for our problem. In our problem, we see that if there exist a minimum cost path from an origin node to a destination node and any other node different from origin and destination nodes is in this lowest cost path, then the paths from source to that node and from that node to sink node may not be the minimum cost paths. We explain a possible case with an example below.

Suppose there exists only two paths from origin to node i, namely $p1$ and $p2$. Also, suppose there exists only two paths originating from node i, $p3$ and $p4$ which end at the destination of related commodity. Assume cost of $p1$ is 10, $p2$ is 12 , $p3$ is 9 and $p4$ is 14 liras. Also, assume that commodity transported on $p1$ arrives later than $p2$ and if commodity will be transported on $p1$, it can not be transported from node i to the destination on $p3$ due to time conditions. In other words, cutoff time of the first service used on $p3$ is greater than the arrival time of the path $p1$ to the node i. Suppose we apply one of the shortest path algorithms that runs according to *Bellman optimality conditions*. When it labels

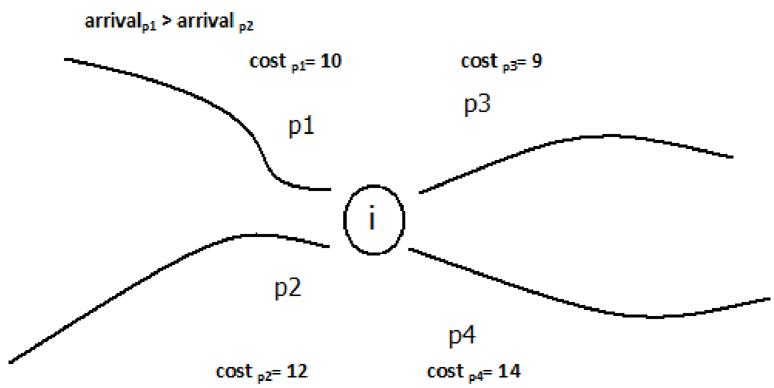

Figure 4.1: A case not satisfies Bellman Optimalitiy Conditions

node i, since $p1$ has less cost than $p2$, i will be labeled as cost of $p1$. On the other hand, after this point, we can not use $p3$ because of time infeasibility, so we have to use $p4$. As a result, we have total cost of transporting commodity from the origin to the destination as $cost_{p1} + cost_{p4}$, that is 24. However, we have a less cost o-d path if we first use $p2$ and then $p3$. In that case, total cost is 21. As we see, node i is in the shortest path but it does not mean from the origin to node i is the shortest one.

The above problem can be solved using a shortest path algorithm on an acyclic graph. We define the auxiliary graph $G_k = (N_k, A_k)$ with procedure given in Algorithm 2. If we assume that all travel times are positive, then the graph G_k is acyclic and can be lexicographically ordered by sorting the nodes $nodeNo$ in non-decreasing order of $timeIndex(nodeNo)$. The maximum node number in graph G_k is $|A|$. Notice that a path from node $o(k)$ to node $d(k)$ in this graph is a simple path which satisfies all time restrictions, i.e., such a path corresponds to a trip that starts at the origin $o(k)$ at time $r(k)$ and ends at the destination $d(k)$ no later than time $q(k)$, and arrives at the origin seaport of each service on its trip before its cutoff time.

Algorithm 2 Graph Generation for commodity $k \in K$

```
nodeNo = 0;
arcNo = 0;
List1 := ∅
label origin node as node 0, timeIndex(0) := r(k), real(0) := o(k)
nodeNo := nodeNo + 1
for i = 1 : |N|
   if τ_{o(k)i} ≠ ∞
    timeIndex(nodeNo) := r(k) + τ_{o(k)i}, real(nodeNo) := i
    head(arcNo) := 0, tail(arcNo) := nodeNo,
    cost(arcNo) := c_{o(k)i}
    Insert nodeNo to List1
    nodeNo := nodeNo + 1, arcNo := arcNo + 1
whileList1 ≠ ∅
 select first node in List1, currentNode
 for all arcs a in A such that s(a) := real(currentNode)
   if l_a ≥ timeIndex(currentNode)
    if e_a > timeIndex(currentNode)
       timeIndex(nodeNo) := l_a + τ_a if it has not been initialized
       arcNo, head(arcNo) := currentNode, tail(arcNo) := nodeNo, cost(arcNo)
:= p_{real(currentnode)} * (e_a − timeIndex(currentNode)) + c_a + α_a
       Insert nodeNo to List1
       nodeNo := nodeNo + 1, arcNo := arcNo + 1
    else e_a ≤ timeIndex(currentNode)
       Initialize a new node numbered as nodeNo and timeIndex(nodeNo) :=
l_a + τ_a if it has not been initialized
       Initialize a new arc numbered as arcNo, head(arcNo) := currentNode,
tail(arcNo) := nodeNo, cost(arcNo) := c_a + α_a
       nodeNo := nodeNo + 1, arcNo := arcNo + 1
       Insert nodeNo to List1
  delete currentNode from List1
endwhile
for i = 1 : |N|
  if τ_{id(k)} ≠ ∞
    for all node j in new graph such that real(j) := i
     if timeIndex(j) + τ_{id(k)} ≤ q(k)
     timeIndex(nodeNo) = timeIndex(j) + τ_{id(k)}, real(nodeNo) := d(k)
     head(arcNo) := j, tail(arcNo) := nodeNo,
     cost(arcNo) := c_{id(k)}
     nodeNo := nodeNo + 1; arcNo := arcNo + 1
if r(k) + τ_{o(k)d(k)} ≤ q(k)
  timeIndex(nodeNo) := r(k) + τ_{o(k)d(k)}, real(nodeNo) := d(k)
  head(arcNo) := 0, tail(arcNo) := nodeNo,
  cost(arcNo) := c_{o(k)d(k)}
```

We explain Algorithm 2 with a simple example briefly. Suppose in the original graph there exists 3 ports, 1,2 and 3 and 3 services, 1, s(1) = 1, t(1) = 2, 2, s(2)=1, t(2)=3 and 3, s(3) = 2 and t(3)=3. In order to generate G_k, we first start with adding origin node $o(k)$. Suppose we can only reach from origin of commodity k to seaport 1 and 2. Then, we generate two nodes, $(1, r(k) + \tau_{o(k)1})$ and $(2, r(k) + \tau_{o(k)2})$. The unit volume transportation cost on these arcs are $c_{o(k)1}$ and $c_{o(k)2}$ respectively.

Next, we develop maritime services. Consider, $r(k) + \tau_{o(k)1} < e_1$. Then we develop node $(2, l_1 + \tau_1)$ and an arc from $(1, r(k) + \tau_{o(k)1})$ to node $(2, l_1 + \tau_1)$ with cost $c_1 + p_1(e_1 - r(k) - \tau_{o(k)1}) + \alpha_1$. Suppose $r(k) + \tau_{o(k)1} > l_2$. Then we do not add an arc for service 2. Consider $l_1 + \tau_1 \geq e_3$ and $l_1 + \tau_1 \leq l_3$. Then, first we develop node $(3, l_3 + \tau_3)$ and add an arc from node $(2, l_1 + \tau_1)$ to $(3, l_3 + \tau_3)$ with cost $c_3 + \alpha_3$. Also, assume $r(k) + \tau_{o(k)2} < e_3$. So, we add an arc from node $(2, r(k) + \tau_{o(k2)})$ to node $(3, l_3 + \tau_3)$ with cost $c_3 + p_2(e_3 - r(k) - \tau_{o(k)2}) + \alpha_3$. Now, we add arcs from sea ports to destination. Suppose we can only reach destination from node 3. Also, suppose that $l_3 + \tau_3 + \tau_{3d(k)} \leq q(k)$. Then we add an arc from $(3, l_3 + \tau_3)$ to destination node with cost $c_{3d(k)}$. Assume that direct truck transportation is infeasible for commodity k so we do not add any arc from origin to destination corresponds to direct truck transportation option. Now, our graph for commodity k is ready.

Now $LR^k(\alpha)$ is equal to the length of a shortest path from the origin $o(k)$ to the destination $d(k)$ in graph G_k. Hence $LR(\alpha)$ can be computed efficiently.

For getting the solution of $LR^k(\alpha)$, we apply the shortest path algorithm designed for acyclic digraphs which runs in O(nlogn) [19] where n = $|N|$.

The other requirement is to find the best Lagrangian multipliers for the relaxed constraints. There are several techniques to find the best Lagrangian multipliers. Among these, we use the subgradient approach for solving the Lagrangian dual problem. The algorithm is given in Algorithm 3.

We start with assigning 0 to all Lagrangian multipliers. Then at each Lagrangian step we solve the Lagrangian relaxed problem and check whether it is

Algorithm 3 Subgradient

$\sigma \leftarrow 2$, $\alpha \leftarrow 0$, $noimp \leftarrow 0$
$lb \leftarrow 0$ and $ub \leftarrow \infty$
while $\sigma > 10^{-2}$ and $\frac{ub-lb}{lb} < 0,01$ **do**
  Compute $LR(\alpha)$ and let x be the optimal solution
  if $\sum_{k\in K} w(k)x_a^k \leq u_a$ and $\alpha_a(\sum_{k\in K} w(k)x_a^k - u_a) = 0$ for all $a \in A$ **then**
    STOP, (x, y, z, v) is optimal for *MMR-S*
  else
    if $LR(\alpha) > lb$ **then**
      $lb \leftarrow LR(\alpha)$
      $noimp \leftarrow 0$
    else
      increment $noimp$
    end if
    $x^f \leftarrow Heuristic(x)$
    if $Cost(x^f) < ub$ **then**
      $ub \leftarrow Cost(x^f)$
    end if
    if $noimp > 15$ **then**
      $noimp \leftarrow 0$
      $\sigma \leftarrow \sigma/2$
    end if
  end if
  $s \leftarrow \frac{\sigma(ub-LR(\alpha))}{\sum_{a\in A}(\sum_{k\in K} w(k)x_a^k - u_a)^2}$
  $\alpha_a \leftarrow max\{0, \alpha_a + s(\sum_{k\in K} w(k)x_a^k - u_a)\ \}$ for all $a \in A$
end while

feasible to our original problem and satisfies complementary slackness conditions or not . If it is feasible and satisfies complementary slackness conditions, we stop otherwise we update the lower bound if the value of the Lagrangian relaxed problem is better than current lower bound. After, we apply some heuristic techniques in order to get a feasible solution from the solution of the Lagrangian relaxed problem. We will update upper bound if we get a better feasible solution from the current best. The step size-s- is calculated and finally Lagrangian multipliers are updated.

4.2 The Extended Formulation

The Lagrangian dual bound is $LD = \max_{\alpha \geq 0} LR(\alpha)$. This bound is at least as good as the linear programming bound of model *ILP-1*. As the subproblems are shortest path problems, we can derive an extended formulation which yields the same bound as the Lagrangian dual.

In this section, we give the mathematical formulation of the extended model. Then, we give a proof about the equality of lower bounds obtained from the Lagrangian dual and the linear relaxation of the extended formulation.

For each commodity $k \in K$, we define a graph where a simple path from origin $o(k)$ to $d(k)$ defines a trip which starts at $o(k)$, ends at $d(k)$, does not visit any node more than once, and respects all time restrictions. We define the costs of arcs on the new graph in such a way that the sum of costs of arcs on a path is equal to the transportation and stocking cost for the corresponding trip. Hence the problem *MMR-S* then is equivalent to the problem of finding a path for each commodity k such that the capacity constraints are satisfied.

For each $k \in K$, we define the following graph $G_k^{'} = (N_k, A_k^{'})$. The set $A_k^{'}$ consists of arcs of O_k and D_k and a set of service arcs $S_k^{'}$ defined as follows. For each service $a \in A$ and nodes (i_1, j_1) and (i_2, j_2) in N_k such that $s(a) = i_1$, $t(a) = i_2$, $j_1 \leq l_a$ and $l_a + \tau_a = j_2$, we add an arc ω from node (i_1, j_1) to node (i_2, j_2) with cost $\sigma_\omega = c_a + p_{i_1}(e_a - j_1)^+$ and capacity $\upsilon_\omega = u_a$. We define

$service(\omega) = a$.

Let $k \in K$. For arc $(o(k),(i,r(k)+\tau_{o(k)i})) \in O_k$, let $f^k_{o(k),(i,r(k)+\tau_{o(k)i})}$ be 1 if commodity k travels directly from its origin to seaport i and arrives there at time $r(k)+\tau_{o(k)i}$ and 0 otherwise. For arc $((i,j),d(k)) \in D_k$, let $f^k_{(i,j),d(k)}$ be 1 if commodity k arrives at seaport i at time j and travels from i directly to its destination and 0 otherwise. Finally, for arc $\omega \in S^{'}_k$ from node (i_1,j_1) to (i_2,j_2), we define f^k_ω to be 1 if commodity k arrives at seaport i_1 at time j_1 and uses service $service(\omega)$ that starts at sepaort i_1 and arrives at seaport i_2 at time j_2 and 0 otherwise.

Let $\delta^-(k,i1,j1)$ and $\delta^+(k,i1,j1)$ be the sets of incoming and outgoing arcs of node $(i_1,j_1) \in N_k$ in graph $G^{'}_k$. The extended formulation, *ILP-2* is as follows.

(ILP-2)

$$\min \sum_{k\in K} w(k)\Big[\sum_{(o(k),(i,r(k)+\tau_{o(k)i}))\in O_k} c_{o(k)i} f^k_{o(k),(i,r(k)+\tau_{o(k)i})} + \sum_{\omega\in S^{'}_k} \sigma_\omega f^k_\omega + \sum_{((i,j),d(k))\in D_k} c_{id(k)} f^k_{(i,j),d(k)} + c_{o(k)d(k)} y^k\Big]$$

s.t.

$$\sum_{(o(k),(i,r(k)+\tau_{o(k)i}))\in O_k} f^k_{o(k),(i,r(k)+\tau_{o(k)i})} + y^k = 1 \qquad \forall k \in K \tag{4.11}$$

$$\sum_{\omega\in\delta^+(k,i1,j1)} f^k_\omega - \sum_{\omega\in\delta^-(k,i1,j1)} f^k_\omega = 0 \qquad \forall k \in K, (i_1,j_1) \in N_k \tag{4.12}$$

$$\sum_{((i,j),d(k))\in D_k} f^k_{(i,j),d(k)} + y^k = 1 \qquad \forall k \in K \tag{4.13}$$

$$\sum_{k\in K} w(k) \sum_{\omega\in S^{'}_k:service(\omega)=a} f^k_\omega \le u_a \qquad \forall a \in A \tag{4.14}$$

$$r(k) + \tau_{o(k)d(k)} y^k \le q(k) \qquad \forall k \in K \tag{4.15}$$

$$f^k_{o(k),(i,r(k)+\tau_{o(k)i})} \in \{0,1\} \qquad \forall k \in K, (o(k),(i,r(k)+\tau_{o(k)i})) \in O_k \tag{4.16}$$

$$f^k_{(i,j),d(k)} \in \{0,1\} \qquad \forall k \in K, ((i,j),d(k)) \in D_k \tag{4.17}$$

$$f^k_\omega \in \{0,1\} \qquad \forall k \in K, \omega \in S'_k \tag{4.18}$$

$$y^k \in \{0,1\} \qquad \forall k \in K \tag{4.19}$$

Proposition 6 *The linear programming bound of ILP-2 is equal to the Lagrangian dual bound LD and is at least as good as the linear programming bound of ILP-1.*

Proof. Clearly, the Lagrangian dual bound LD is at least as good as the linear programming bound of *ILP-1*. In the sequel, we prove that it is equal to the linear programming bound of *ILP-2*.

First observe that $LR(\alpha) = -\sum_{a\in A}\alpha_a u_a + \sum_{k\in K} w(k)\overline{LR}^k(\alpha)$ where

$$\overline{LR}^k(\alpha) = \min \sum_{(o(k),(i,r(k)+\tau_{o(k)i}))\in O_k} c_{o(k)i} f^k_{o(k),(i,r(k)+\tau_{o(k)i})} + \sum_{\omega\in S'_k}(\sigma_\omega + \alpha_{service(\omega)}) f^k_\omega + \sum_{((i,j),d(k))\in D_k} c_{id(k)} f^k_{(i,j),d(k)} + c_{o(k)d(k)} y^k$$

s.t.

$$\sum_{(o(k),(i,r(k)+\tau_{o(k)i}))\in O_k} f^k_{o(k),(i,r(k)+\tau_{o(k)i})} + y^k = 1 \tag{4.10}$$

$$\sum_{\omega\in\delta^+(k,i1,j1)} f^k_\omega - \sum_{\omega\in\delta^-(k,i1,j1)} f^k_\omega = 0 \qquad \forall (i_1,j_1)\in N_k \tag{4.11}$$

$$\sum_{((i,j),d(k))\in D_k} f^k_{(i,j),d(k)} + y^k = 1 \tag{4.12}$$

$$f^k_{o(k),(i,r(k)+\tau_{o(k)i})} \in \{0,1\} \qquad \forall (o(k),(i,r(k)+\tau_{o(k)i}))\in O_k$$

$$f^k_{(i,j),d(k)} \in \{0,1\} \qquad \forall ((i,j),d(k))\in D_k$$

$$f^k_\omega \in \{0,1\} \qquad \forall \omega \in S'_k$$

$$y^k \in \{0,1\}$$

As the above problem is a shortest path problem, it has the linearity property and the convex hull of its feasible solutions is described by constraints (4.10)-(4.12) and nonnegativity constraints on variables. Hence the Lagrangian dual bound LD is equal to the linear programming bound of *ILP-2*. □

Chapter 5

Heuristics

We solve the Lagrangian dual problem using the subgradient algorithm given in Algorithm 3. At each iteration of the algorithm where we solve a relaxed problem, we check whether the optimal solution is feasible and has cost less than the current upper bound. If the optimal solution is not feasible, then we call a heuristic algorithm which tries to generate a feasible solution starting with the optimal solution of the relaxed problem.

We propose two heuristic approaches. The first heuristic works as follows. Suppose we solve the $|K|$ shortest path problems and we have an infeasible solution.

Let x be the optimal solution of the relaxed problem and $\Pi = \{a \in A : \sum_{k \in K \,:\, x_a^k = 1} w(k) > u_a\}$. So, Π is the set of overcapacitated arcs. Also, for $a \in \Pi$, $\Upsilon_a = \{k \in K : x_a^k = 1\}$ is the set of commodities that use arc a.

We need to decide which commodities will be forced out from $a \in \Pi$ and will be assigned to new paths. We try four different commodity selection procedures. In all alternatives, for each $a \in \Pi$ we define a set called Ω_a which keeps the commodities that will be banned from a.

In our first commodity selection procedure, for each $a \in \Pi$ we first select commodity k^* such that $w(k^*) = \min\{w(k) : k \in \Upsilon_a\}$, then add it to Ω_a. The motivation in order to start selection from the commodity with the minimum demand is the expectation of less waiting costs at terminals if the commodity will be assigned to a new intermodal path and less direct truck transportation cost if the commodity will be assigned to direct truck transportation.

After selecting the commodity k^*, we perform a feasibility check operation. If $\sum_{k \in \Upsilon_a} w(k) - w(k^*) \leq u_a$ then we proceed to next $a \in \Pi$. If not, we select the commodity k^{**}, which has the second minimum demand and repeat the feasibility check procedure above. We proceed by selecting third, fourth, .. minimum demand commodity until total flow on a does not exceed its capacity.

The second criterion we adapt to select the commodities is the number of infeasible arcs in a commodity's current path. For $a \in \Pi$, we start with commodity $k^* \in \Upsilon_a$, whose number of infeasible arcs on its path in the solution x is largest. If there is a tie between commodities, then we select the one with smaller demand. Then, we add selected commodity to Ω_a. We apply the same feasibility check as described above.

The third criterion used is the decrease in the number of infeasible arcs if we reroute a commodity. We calculate these values for each $k \in K$ and then for each $a \in \Pi$, we start by the commodity $k^* \in \Upsilon_a$ with largest value.

The last criterion taken in commodity selection process is as follows. For $k \in K$, we calculate how much volume we will gain if we reroute k. For $a \in \Pi$ such that $k^* \in \Upsilon_a$, if we reroute k^*, we gain $min\{\sum_{k \in \Upsilon_a} w(k) - u_a, w(k^*)\}$ on a. After calculating total gains for all commodities, for $a \in \Pi$ we start with $k^* \in \Upsilon_a$ which has largest gain, add it to Ω_a and proceed as before until a becomes feasible.

After developing Ω_a for $a \in \Pi$ with one of the alternative ways, first we calculate the residual capacities of the arcs by the following. For each $k \in K$, if $k \notin \Omega_a$, for all $a \in \Pi$, then we assign k to its path in x. We decrease the capacities of arcs on the path of k by $w(k)$. If there exists $a \in \Pi$ such that $k \in \Omega_a$, this means k wil be rerouted and no capacity adjustment is done. The residual capacities of arcs are named as ru_a.

Let K^* be the set of commodities which will be rerouted, $K^* = \cup_{a \in \Pi} \Omega_a$. We adapt the graphs for commodities $k \in K^*$. This adaptation is done by extracting arc a from graph $k \in K^*$ if $k \in \Omega_a$ or if $ru_a < w(k)$. Then we solve shortest path problems for all commodities $k \in K^*$. This two-step procedure is called as rerouting.

Our procedure may not end because of the fact that there may exist new overcapacitated arcs after rerouting. We perform rerouting procedure 2 or 3 times according to capacity factor used in the associated network.

After the last rerouting, we determine the sets Π and Υ_a for $a \in \Pi$ and solve an optimization problem in order to decide which commodities will be assigned to direct truck transportation. We define $K^{**} = \cup_{a \in \Pi} \Upsilon_a$.

Let μ^k be 1 if commodity $k \in K^{**}$ will be assigned to direct transportation option and 0 otherwise. Let $optcost^k$ denote the cost of the last assigned path of commodity k and $used_a$ be the total volume assigned to service $a \in A$ at the end of the last rerouting procedure.

We solve the following problem.

$$min \sum_{k \in K^{**}} w(k)(c_{o(k)d(k)} - optcost^k)\mu^k$$

s.t.

$$\sum_{k \in \Upsilon_a} w(k)\mu^k \geq used_a - ru_a \qquad \forall a \in \Pi \qquad (5.1)$$

$$\mu^k \in \{0, 1\} \qquad \forall k \in K^{**} \qquad (5.2)$$

If, in the optimal solution, we have $\mu^k = 1$ for commodity k, then commodity k is carried directly from its origin to its destination using trucks.

The resulting solution is a feasible solution to our original problem.

The other heuristic works as follows. First, we develop Π. Let us define two sets B_1 and B_2. For $k \in K$, if there exists $a \in \Pi$ such that $x_a^k = 1$ then insert k to B_1. Otherwise, insert it to B_2. Now, we adapt the capacities of arcs in the problem. For $a \in A$, $u_a = u_a - \sum_{k \in K \,:\, x_a^k = 1,\ k \in B_2} w(k)$. Then, we formulate the original problem with all $k \in B_1$ and new capacities and solve optimality. The resulting solution is a feasible solution.

Chapter 6

Computational Results

In this chapter, we first describe the test data and then we present the results obtained by solving the integer models. Finally, we report the lower bounds obtained from the Lagrangian relaxation and the results of the heuristics.

6.1 Input Data and Solution Methodology

We use two service networks in our computational study. The first one is the company's network which is composed of 34 important sea ports in Europe, Russia, China, Hong Kong, Singapur and Malaysia and 167 services of the world's major sea transportation operators in the world like MSC, Hanjin Shipping, Emes, Grimaldi and Hapag-Lloyd (see figure 6.1). The schedule of services is obtained from the company. This schedule is for services provided in March 2008. Also, capacities allocated to our company are given.

The other parameters we obtained from the company are sea transportation prices, truck transportation prices from various cities to nearest sea terminals(i.e. from Ankara to Mersin and İstanbul), direct truck transportation prices, loading and unloading (both to truck and to vessel) costs at terminals and also stocking costs at seaports. The company's sea transportation price index is the index which is valid in March 2008 and the truck transportation price index is valid between January 1, 2008 to June 30, 2008. Stocking costs may change from terminal to terminal, we take an average value of $p_i = 8.5$ liras per day for all terminals.

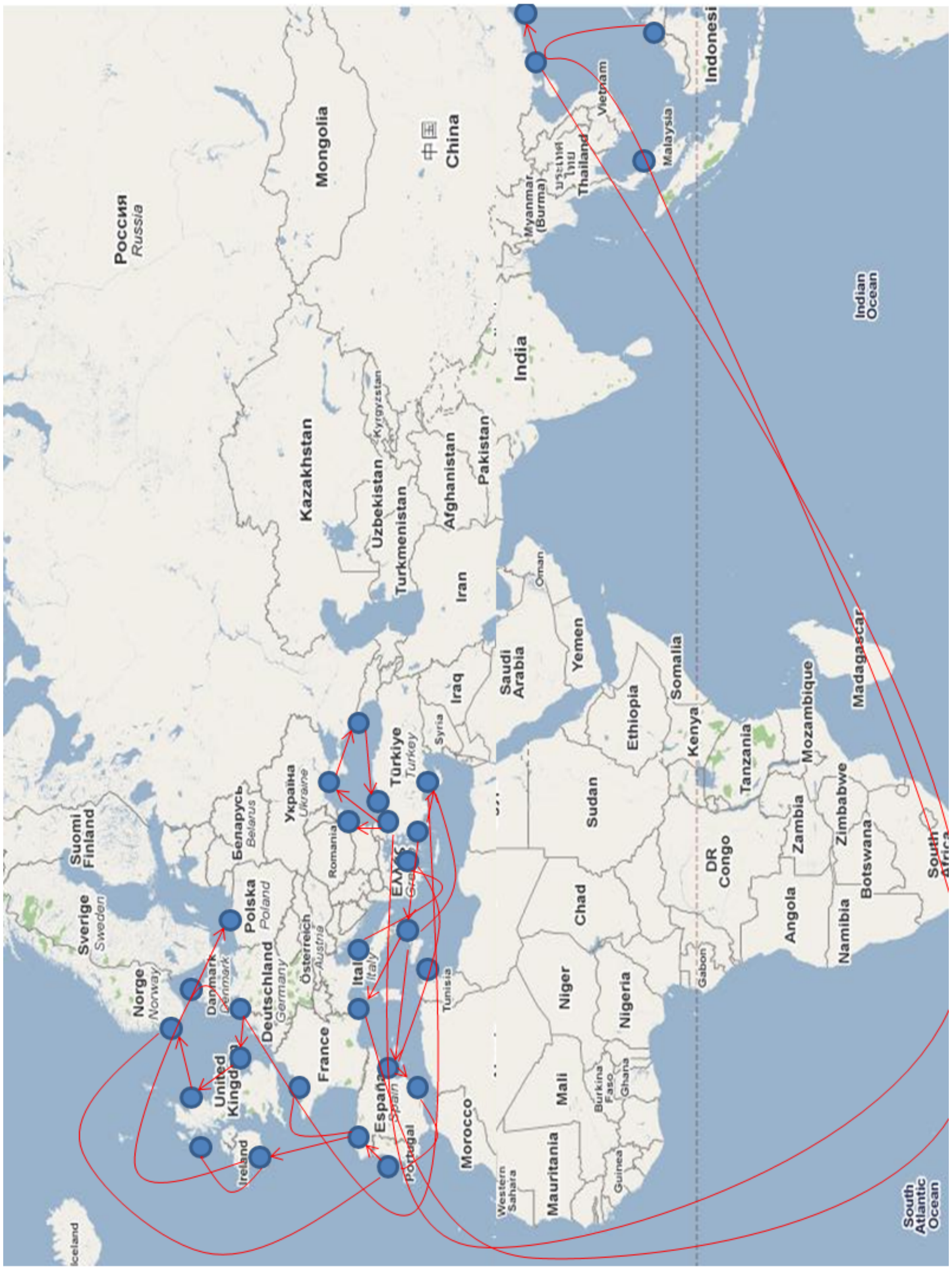

Figure 6.1: Service network of the logistics company

We named company's graph as O1 and random graph as O2.

For the instances on network O1, we randomly generate the demands. For the other network instances, we generate all data randomly.

For every graph, we generate 4 problem core instances in which $|K|$ equals to 400, 600, 800 and 1000. $w(k)$ is uniformly distributed over the interval $[50, 250]$, $c_{o(k)d(k)}$ is uniformly distributed over the interval $[1200, 3500]$, $c_{o(k)i}$ and $c_{id(k)}$ are uniformly distributed over the interval $[100, 600]$, $\tau_{o(k)i}$ and $\tau_{id(k)}$ are uniformly distributed over the interval $[1/10, 5/2]$, $\tau_{o(k)d(k)}$ is uniformly distributed over the interval $[7, 25]$, $r(k)$ and $q(k)$ are uniformly distributed over the intervals $[1, 10]$ and $[20, 35]$ respectively. This parameters are obtained from company for instances in O1 network.

The O2 network which includes 66 nodes and 1200 arcs. The characteristics of arcs are generated as follows. e_a is uniformly distributed over the interval $[1, 26]$, l_a is the sum of e_a and a random variable which is uniformly distributed over the interval $[1, 2]$, u_a is uniformly distributed over the interval $[100, 350]$, τ_a is uniformly distributed over the interval $[2, 12]$, $c_a = 100\tau_a$ and p_i is uniformly distributed over the interval $[5, 10]$.

We also derive 2 more test instances from each core instance by multiplying the capacities of the arcs in the network with factors 1.5 and 3. As a result, we have 24 problem instances for testing our various solution approaches. A problem is named as name of the graph-capacity multiplier-number of commodities.

We coded our models using JAVA programming language in ILOG Concert Technology in order to solve models by CPLEX 11.0. Also, algorithms and heuristic approaches are coded in JAVA programming language. All runs are taken on a 2.67 Ghz 2$\times$Quadcore Xeon Processor with 8 GB Ram.

6.2 Comparison of Models

In this section, we will give the comparison between our models in terms of LP relaxation gap, CPU times(in seconds) and number of branch and cut nodes. Also, the effects of variable fixing and valid inequalities are analyzed. We try to analyze which formulation is better in which cases.

We note that in model ILP-2, we apply variable fixing as follows. For $k \in K$ and $a \in A$, if $w(k) > u_a$ then commodity k can not use service a.

We impose a time limit of one hour. For problems that are not solved optimality in one hour, we report the remaining percentage gap in paranthesis in tables reporting CPU times.

In the sequel, "ILP-1 + v" refers the model valid inequlities added to ILP-1, "ILP-1+p" represents the model ILP-1 with variable fixing, and "ILP-1.2" corresponds to model ILP-1 with valid inequalities and variable fixing.

We start with instances on company's network(O1).

CPU seconds for O1 instances

Instance No	ILP-1	ILP-1 + v	ILP-1 + p	ILP-1.2	ILP-2
O1-1-400	480.6	38.7	441.5	37.9	4.4
O1-1-600	1h*(0.18)	58.0	1h*(0.16)	57.9	42.8
O1-1-800	1h*(0.44)	76.0	1h*(0.44)	67.4	197.9
O1-1-1000	1h*(0.39)	109.3	1h*(0.38)	108.6	1h*(0.02)
O1-1.5-400	812.4	47.5	779.3	38.7	18.1
O1-1.5-600	1h*(0.58)	80.1	1h*(0.58)	64.6	334.2
O1-1.5-800	1h*(18.13)	108.7	1h*(18.13)	105.6	1h*(0.05)
O1-1.5-1000	1h*(21.12)	155.8	1h*(21.10)	194.4	1h*(0.11)
O1-3-400	1131.4	47.4	1022.2	46.2	31.5
O1-3-600	1h*(1.74)	94.7	1h*(1.74)	75.5	1h*(0.13)
O1-3-800	1h*(2.44)	137.4	1h*(2.44)	138.3	1h*(0.21)
O1-3-1000	1h*(4.28)	303.3	1h*(4.28)	240.9	1h*(0.24)

*could not reach to optimal

Table 6.1:

	LP gaps(%) for O1 instances				
Instance No	ILP-1	ILP-1 + v	ILP-1 + p	ILP-1.2	ILP-2
O1-1-400	22.52	18.12	5.64	1.72	0.53
O1-1-600	16.35	13.63	4.92	1.76	0.40
O1-1-800	13.40	11.23	5.11	2.41	1.00
O1-1-1000	11.39	9.96	4.79	2.74	1.74
average	15.91	13.24	5.12	2.16	0.67
O1-1.5-400	19.56	12.69	12.38	2.59	1.71
O1-1.5-600	14.73	10.27	10.45	3.90	1.68
O1-1.5-800	12.36	8.73	9.53	4.77	1.71
O1-1.5-1000	10.44	7.81	8.18	4.82	1.73
average	14.28	9.72	10.11	3.75	1.61
O1-3-400	16.91	4.08	16.87	4.06	1.04
O1-3-600	13.52	4.24	13.50	4.22	0.75
O1-3-800	12.59	5.10	12.55	5.08	0.81
O1-3-1000	11.08	5.35	11.02	5.30	0.86
average	13.53	4.69	13.48	4.68	0.87
overall average	14.56	9.32	9.58	3.53	1.05

Table 6.2:

One of the most important comparison between the performance of two models is CPU times spend for reaching exact solutions. For this, we need to analyze Table 6.1. Regardless of the value of the capacity factor, we can not reach optimality with ILP-1 in less than one hour on instances with high number of commodities. On the other hand, with ILP-1.2 which includes variable fixing and valid inequlities, we can reach optimality for all instances in O1 network. The longest solution time is about 4 minutes. Comparing the results of ILP-1 with ILP-1+v and ILP-1+p, we see that rather than variable fixing, valid inequalities help us to reach exact solutions quickly.

In the last column of Table 6.1, we see the solution times with ILP-2. We see that as the capacity factor increases, the solution times of ILP-2 increase. Also, the number of instances for which ILP-2 can not reach optimality increases with increasing capacity factor. The instances for which it can not reach optimality, the remaining gaps increase as the capacity factor increases.

In the comparison between ILP-1.2 and ILP-2, results show that ILP-2 is

Number of nodes for O1 instances

Instance No	ILP-1	ILP-1 + v	ILP-1 + p	ILP-1.2	ILP-2
O1-1-400	830	0	786	0	0
O1-1-600	-*	0	-*	0	1252
O1-1-800	-*	0	-*	0	5235
O1-1-1000	-*	67	-*	67	-*
O1-1.5-400	1054	0	992	0	1520
O1-1.5-600	-*	1	-*	1	10195
O1-1.5-800	-*	1	-*	1	-*
O1-1.5-1000	-*	409	-*	409	-*
O1-3-400	2887	10	2887	10	2729
O1-3-600	-*	96	-*	96	-*
O1-3-800	-*	221	-*	201	-*
O1-3-1000	-*	1259	-*	1252	-*

*could not reach to optimal

Table 6.3:

faster than ILP-1.2 on instances with 400 and 600 commodities with capacity factor 1, and it is faster with 400 commodities with 1.5 and 3 capacity factors. But the differences are not too large. On the other hand, in instances with larger number of commodities, ILP-2 can not reach optimality but ILP-1.2 does at most within 4 minutes.

Another criterion we use in our comparison is LP gaps of the models. We start with analyzing the effects of variable fixing and valid inequalities and then proceed with comparison of ILP-1.2 and ILP-2. We calculate these gaps by 100*(optimal-relaxed)/optimal.

From Table 6.2, we see that LP relaxation gap average is about 15% for model ILP-1. The gap varies between 10% and 23% in instances on O1. Variable fixing performs better than valid inequalities in O1-1 network instances because of the tight capacities. On average, it improves LP gap by about 11%, on the other hand valid inequalities improve by about 3%. But when we increase the capacity of services in O1, then valid inequalities become more efficient than variable fixing. The improvement provided by variable fixing is about 0.1% with capacity factor 3.

The effect of variable fixing decreases with increasing capacities. This is expected since the fixing, $x_a^k = 0$ if $w(k) > u_a$, loses its efficiency.

We analyze the LP gaps of ILP-1.2, and we see that we improve the gap about 11% on the average. With LP relaxation of ILP-1.2, we are now at most 5.3% far from the optimal value. The gaps obtained from the LP relaxation of ILP-2 is 1% on the average. The maximum deviation with ILP-2 is 1.74%.

We compare the LP gaps of ILP-1.2 and ILP-2 and we see that in all instances, ILP-2 gives better LP relaxation results than ILP-1.2.

From Table 6.3, we see that number of nodes reduces to 0 with variable fixing and valid inequalities for instances O1-1-400, O1-1-600, 01-1-800 and O1-1.5-400. Also, it reduces with variable fixing and valid inequalities for other instances.

Now, we will give and analyze the results obtained from solving instances on the randomly generated network O2. This network contains about 7 times more arcs than the original graph.

CPU seconds for O2 instances

Instance No	ILP-1	ILP-1 + v	ILP-1 + p	ILP-1.2	ILP-2
O2-1-400	240.9	111.5	232.8	101.9	272.1
O2-1-600	1h*(0.78)	166.4	1h*(0.78)	183.7	298.2
O2-1-800	1h*(2.58)	231.1	1h*(2.58)	247.1	503.9
O2-1-1000	1h*(4.04)	335.0	1h*(4.04)	346.2	552.4
O2-1.5-400	209.1	110.9	198.1	99.3	269.1
O2-1.5-600	3211.9	151.2	3054.3	136.4	378.5
O2-1.5-800	1h*(1.12)	227.0	1h*(1.12)	219.2	468.2
O2-1.5-1000	1h*(2.74)	240.6	1h*(2.74)	231.5	548.1
O2-3-400	232.3	103.6	229.1	101.2	254.0
O2-3-600	667.1	162.4	663.0	160.8	362.7
O2-3-800	1h*(0.60)	188.2	1h*	185.3	476.7
O2-3-1000	1h*(0.84)	418.7	1h*	414.2	572.3

*could not reach to optimal

Table 6.4:

With model ILP-1, we can not reach optimality in less than one hour in many

LP gaps(%) for O2 instances

Instance No	ILP-1	ILP-1 + v	ILP-1 + p	ILP-1.2	ILP-2
O2-1-400	43.48	40.47	8.78	4.08	0.84
O2-1-600	40.22	37.27	9.72	5.07	1.28
O2-1-800	35.55	32.02	10.31	8.26	1.82
O2-1-1000	33.61	28.66	10.79	9.20	2.36
average	38.26	34.89	9.63	6.42	1.57
O2-1.5-400	15.97	10.69	14.85	10.38	0.26
O2-1.5-600	14.91	7.59	14.22	7.41	0.31
O2-1.5-800	14.86	8.67	14.37	8.53	0.53
O2-1.5-1000	14.85	13.01	14.48	12.91	0.50
average	15.19	9.59	14.41	9.78	0.4
O2-3-400	15.84	10.55	14.59	10.13	0.03
O2-3-600	14.54	7.17	13.77	6.91	0.01
O2-3-800	14.35	7.80	13.79	7.61	0.01
O2-3-1000	14.68	8.79	14.26	8.65	0.03
average	14.84	8.65	13.56	8.46	0.02
overall average	22.76	17.71	12.53	8.22	0.67

Table 6.5:

of the instances in O2. The number of opened branch-and-cut nodes increases as we increase the number of commodities and this results increase in of solution times. With variable fixing and valid inequalities, the number of opened nodes decreases and this results in improvements of solution times.

According to results in O2 network, ILP-1.2 again performs better than ILP-2 in terms of CPU times. Although in some instances opened nodes are more in ILP-1.2, it is faster than ILP-2.

An interesting point is the LP relaxation gaps of ILP-1 with tight capacities. From Table 6.5, it is seen that average gap of instances with capacity factor 1 is about 39%. With efforts, we decrease this gap to 6.5%.

In O2, LP gaps with ILP-1 decrease when we increase the capacities of the services in the network. This also holds in O1. The LP gaps and capacity factors move reversely. This is not the case for ILP-1.2.

	Number of nodes for O2 instances				
Instance No	ILP-1	ILP-1 + v	ILP-1 + p	ILP-1.2	ILP-2
O2-1-400	1640	0	1589	0	0
O2-1-600	-*	0	-*	0	0
O2-1-800	-*	0	-*	0	0
O2-1-1000	-*	0	-*	0	0
O2-1.5-400	69	0	69	0	0
O2-1.5-600	495	0	487	0	0
O2-1.5-800	-*	33	-*	33	0
O2-1.5-1000	-*	228	-*	228	0
O2-3-400	0	0	0	0	0
O2-3-600	10	0	10	0	0
O2-3-800	-*	28	-*	28	0
O2-3-1000	-*	111	-*	111	0

*could not reach to optimal

Table 6.6:

We analyzed solutions of some instances and we have seen that when capacities are tight, many commodities could not be transported on their shortest paths in IP solutions because of insufficient capacities. On the other hand, some proportion of commodities are transported on shortest paths in LP relaxation solutions. When we increase capacities, the number of commodities transported on their shortest paths increase because now most of the arcs in the system have sufficient capacities. Also, these commodities are fully transported on their shortest paths in LP relaxation solution. So, gaps are decreasing with the increase in the number of commodities transported on their optimal paths.

We can conclude that ILP-1.2 performs better than extended formulation ILP-2 in terms of solution times. The generated valid inequalities work well and improve the solution times.

The variable fixing efforts work well with tight capacities. On the other hand, their benefit decreases with the increase in capacities. We think that efficiency of variable fixing is directly related with the characteristics of network and commodities.

The LP gaps obtained with ILP-2 are better than gaps obtained with ILP-1.2 in spite of our strengthening efforts.

6.3 Lower Bounds and Heuristic Results

In this section, we will give the lower and upper bounds obtained with different heuristic procedures. The results are deviations from the optimal values and given as percentages. Also, the number of iterations performed with each heuristic is given. We run all heuristics for 15 minutes in all instances.

In the sequel, by "Heuristic1-a", "Heuristic1-b", "Heuristic1-c" and "Heuristic1-d", we mean the application of first, second, third and fourth alternatives given in the first heuristic.

We start with solutions obtained on company's network.

	Heuristic1-a		Heuristic1-b		Heuristic1-c		Heuristic1-d		Heuristic2	
Instance No.	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
O1-1-400	0.65	2.17	0.84	2.33	0.58	0.95	0.63	1.35	1.32	0.70
O1-1-600	0.74	1.66	0.52	2.27	0.71	1.68	0.71	1.56	0.85	0.45
O1-1-800	2.13	1.61	2.03	2.24	1.99	1.86	1.97	1.63	2.09	0.88
O1-1-1000	1.46	2.04	1.85	1.95	1.06	1.56	1.55	1.67	1.77	1.01
O1-1.5-400	2.31	1.19	2.00	1.31	2.08	1.02	1.85	1.14	2.38	0.59
O1-1.5-600	2.29	1.51	2.23	1.45	2.16	1.56	2.03	1.35	2.29	0.75
O1-1.5-800	2.31	1.89	2.35	1.81	2.29	1.54	2.23	1.36	2.51	0.74
O1-1.5-1000	2.69	2.13	2.64	2.01	2.52	1.86	2.42	1.92	2.91	1.12
O1-3-400	1.75	1.08	1.71	0.97	1.68	1.14	1.52	1.12	1.83	0.83
O1-3-600	1.95	1.26	1.90	1.09	1.73	1.13	1.67	1.18	1.80	0.89
O1-3-800	2.07	1.39	1.97	1.50	1.95	1.44	1.88	1.26	2.25	1.14
O1-3-1000	2.08	1.89	2.02	1.78	1.96	1.85	1.99	1.77	2.03	1.15

Table 6.7: Deviation of upper and lower bounds from the optimal solution values for O1 instances

In the instances on the company's graph, it is seen that the best upper bound obtained is at most 1.15% far from the optimal value.

Number of iterations for O1 instances					
Instance No.	Heur1-a	Heur1-b	Heur1-c	Heur1-d	Heur2
O1-1-400	426	418	415	419	328
O1-1-600	284	279	277	279	219
O1-1-800	213	209	208	210	164
O1-1-1000	170	167	166	168	131
O1-1.5-400	498	485	476	480	341
O1-1.5-600	332	323	317	320	227
O1-1.5-800	249	243	238	240	171
O1-1.5-1000	199	194	190	192	136
O1-3-400	509	492	485	488	359
O1-3-600	339	328	323	325	239
O1-3-800	255	246	243	244	180
01-3-1000	204	197	194	195	141

Table 6.8:

In all instances, Heuristic 2 gives the best upper bound. The reason behind this is that we solve the remaining problem to optimality on generated residual network for the commodities using overcapacitated arcs in heuristic 2. On the other hand, generally lower bounds obtained from Lagrangian relaxation with Heuristic 2 are worse probably because the number of iterations done with Heuristic 2 in 15 minutes is less than Heuristic 1. Solving remaining problem to optimality consumes more time.

The results obtained for the randomly generated network are similar to the results obtained for the O1 network. In all networks, generally, heuristics performance decreases with increase in capacities. We think that the number of commodities transported on multimodal paths increases with the increase in capacities. We perform 2 or 3 rerouting procedure but it may not be sufficient to reach better solutions. Also, it is difficult to find a common criterion for deciding which commodities will be extracted from overcapacitated arcs. The good criterion may change from one arc to another.

The gap of the lower bounds we obtain by Lagrangian relaxation is between 1.5% and 3%. We conclude that this is a result of slow convergence of subgradient approach.

The iterations taken with subgradient algorithm decreases with the increase in the network size and number of commodities. On the other hand, iteration number increases with the increase of capacities. When we increase capacities, the number of infeasible arcs decreases. So, the time consumed during the heuristic process decrease.

	Heuristic1-a		Heuristic1-b		Heuristic1-c		Heuristic1-d		Heuristic2	
Instance No.	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
O2-1-400	1.28	1.94	0.97	1.67	1.18	1.90	1.19	1.93	1.72	0.87
O2-1-600	2.88	1.81	2.37	1.62	2.44	1.26	1.36	1.94	2.16	0.90
O2-1-800	1.99	2.02	1.96	1.76	1.96	1.87	1.97	2.25	2.45	1.45
O2-1-1000	2.67	2.17	2.76	2.26	2.64	2.38	2.77	2.24	3.14	1.74
O2-1.5-400	1.56	1.45	1.58	1.27	0.97	1.58	0.93	1.87	1.80	1.22
O2-1.5-600	1.53	1.69	1.05	1.93	1.04	1.99	1.03	1.82	1.74	1.76
O2-1.5-800	1.38	2.24	1.04	2.30	1.02	2.05	1.03	2.21	1.41	2.03
O2-1.5-1000	1.16	1.94	1.36	2.03	1.52	1.96	1.30	2.21	1.31	1.66
O2-3-400	1.25	2.03	1.23	1.84	1.09	1.78	0.74	2.20	1.31	1.36
O2-3-600	2.17	2.02	2.01	2.11	1.60	2.22	1.37	1.91	1.79	1.64
O2-3-800	1.58	2.38	1.50	2.10	1.46	2.13	1.44	2.22	1.60	2.13
O2-3-1000	1.31	2.10	1.49	1.94	1.49	2.24	1.24	2.20	1.37	1.65

Table 6.9: Deviation of upper and lower bounds from the optimal solution values for O2 instances

Number of iterations for O2 instances

Instance No.	Heur1-a	Heur1-b	Heur1-c	Heur1-d	Heur2
O2-1-400	108	99	97	97	46
O2-1-600	72	66	65	66	31
O2-1-800	54	50	49	50	23
O2-1-100	43	40	39	39	18
O2-1.5-400	137	126	123	128	65
O2-1.5-600	91	84	82	85	43
O2-1.5-800	69	63	62	64	33
O2-1.5-100	55	50	49	52	26
O2-3-400	141	129	125	124	67
O2-3-600	94	86	83	83	45
O2-3-800	71	65	63	62	34
O2-3-100	56	52	50	50	27

Table 6.10:

Chapter 7

Conclusion and Future Research

In this thesis, we consider a multimodal transportation network with scheduled services. The problem is minimizing the total cost of routing a given set of demands of a logistics company over a planning horizon by satisfying capacity and time-related constraints on this network. The truck-vessel combination and direct truck transportation routes are allowed on the transportation network.

First, we developed an integer linear model ILP-1, in order to solve our problem exactly. ILP-1 could not reach optimality for problems with large number of commodities and also networks with large number of arcs in reasonable times. We performed variable fixing and then add valid inequalities to ILP-1 to get optimal solutions in shorter times. The resulting model is called ILP-1.2. We reached optimality with the improved model ILP-1.2 in less than 10 minutes for all instances in our computational study.

We also applied Lagrangian relaxation to obtain tighter lower bounds and devise heuristic algorithms. We relaxed the capacity constraints in our problem and we get $|K|$ subproblems in order to solve the Lagrangian relaxed problem. To solve each subproblem, we developed a technique that starts with a graph generation procedure for each commodity and continues with application of a shortest path algorithm designed for acyclic graphs.

Then, we developed an extended formulation, ILP-2 to our problem. This model is formulated on new graphs generated from the original graph. All time related characteristics are included in the new graph generation procedure. Then, we formulate our problem as an integer multicommodity network flow problem.

Finally, we propose two heuristics based on Lagrangian relaxation.

We compared the performance of our models in terms of CPU times, LP lower bounds. Also, we compare the performances of heuristics and lower bounds obtained from Lagrangian relaxation.

We conclude that our first model strengthened with variable fixing and valid inequalities beats our extended model in terms of CPU times in our instances.

The heuristics performances varied according to the capacity levels of services. Generally, the cost of the best feasible solution is at most 2% far away from the cost of optimal. In all instances we solved, heuristic 2 gives the best feasible solution.

We believe that more work can be done on deriving valid inequalities to strength both formulations. Also, different heuristic approaches may be interesting to develop.

Further research can incorporate various mode alternatives to the model. Rail and air transportation services can be added to transportation alternatives portfolio.

Another future research direction is related with the cost structure of the problem. Rather than linear cost functions, piecewise linear cost functions may be used to model transportation costs.

Our problem is related with operational level decisions in the logistics company and can be combined with tactical level decisions. With routing, the most required services can be identified and using the information gathered from routing, company can identify for which services they demand further capacity from shippers.

Bibliography

[1] Andersen J, Crainic TD, Christiansen M. Service Network Design with Management and Coordination of Multiple Fleets. *European Journal of Operational Research* 2009, 193 : 377-389

[2] Barnhart C, Hane CA, Vance PH. Using Branch-and-Price-and-Cut to Solve Origin-Destination Integer Multicommodity Problems. *Operations Research* 2000, Vol.48 No.2 : 318-326

[3] Barnhart C, Ratliff HD. Modeling Intermodal Routing. *Journal of Business Logistics* 1993, Vol.14 No.1 : 205-223

[4] Bookbinder JH, Fox NF. Intermodal Routing of Canada-Mexico Shipments Under NAFTA. *Transportation Research* 1998, Vol.34 No.4 : 289-303

[5] Boussedjra M, Bloch C, El Moudni A. An Exact Method to Find the Intermodal Shortest Path. *Proceedings of the IEEE* 2004, 1075-1080

[6] Caramia M, Guerriero F. A Heuristic Approach to Long-Haul Freight Transportation with Multiple Objective Functions *The International Journal of Management Science* Article in Press.

[7] Chang T.-S. Best Routes Selection in International Intermodal Networks. *Computers and Operations Research* 2007, doi: 10.1016/j.cor.2006.12.025

[8] Choong SK, Cole MH, Kutanoglu E. Empty Container Management for Intermodal Transportation Networks. *Transportation Research* 2002, 38 : 423-438

[9] Crainic TG, Rousseau JM. Multicommodity Multimode Freight Transportation: A General Modeling and Algorithmic Framework for the Service Network Design Problem. *Transportation Research* 1986, Vol.20B No.3 : 225-242

[10] Desrochers M, Soumis F. A Generalized Permanent Labelling Algorithm for the Shortest Path Problem with Time Windows *INFOR*, 1988, Vol 26. No.3 : 191:211

[11] Guelat J, Florian M, Crainic TG. A Multimode Multiproduct Network Assignment Model for Strategic Planning of Freight Flows. *Transportation Science* 1990, Vol.24 No.1 : 25-39

[12] Kim D, Barnhart C, Ware K, Reinhardt G. Multimodal Express Package Delivery: A Service Network Design Application. *Transportation Science* 1999, Vol.33, No.4 :391-407

[13] Kozan E. Optimising Container Transfers at Multimodal Terminals. *Mathematical and Computer Modelling* 2000, 31 : 235-243

[14] Macharis B, Bontekoning YM. Opportunities for OR in Intermodal Freight Transportation Research: A Review. *European Journal of Operational Research* 2004, 153 : 400-416

[15] Moccia L, Cordeau JF, Laporte G, Ropke S, Valentini MP. Modeling and Solving a Multimodal Routing Problem with Timetables and Time Windows

[16] Nozick LK, Morlok EK. A Model for Medium-Term Operations Planning in an Intermodal Rail-Truck Service. *Transportation Research* 1997, Vol.31 No.2 : 91-107

[17] Vasiliauskas AV. Modelling of Intermodal Freight Transportation Network. *Transport* 2002, Vol.17 No.3 : 117-121

[18] Ziliaskopoulos A, Wardell W. An Intermodal Optimum Path Algorithm for Multimodal Networks with Dynamic Arc Travel Times and Switching Delays. *European Journal of Operational Research* 2000, 125 : 486-502

[19] Ahuja RK, Magnanti TL, Orlin JB. Network Flows, *Prentice Hall*, 1993.

[20] Maritime Transportation In Barnhart C, Laporte G.(Editors), Transportation Vol.14 *Elsevier*, 2007.

[21] Garey MR, Johnson WS. Computers and Intractability, A Guide to the Theory of NP-Completeness. *Freeman*, 1979.

[22] Wolsey LA. Integer Programming, *Wiley*, 1998.

[23] Çelikkaya A. Dünyada ve Türkiyede Deniz Taşımacılığına Sağlanan Vergi Teşvikleri, *Beta*, 2008.

[24] European Union of Road Transportation, 2008 Road Statistics.

[25] European Conference of Ministers of Transport website, www.cemt.org

[26] Hurda Araçlara Vergi Düzenlemesi, www.transport.com

[27] Tarihi İpekyolunun Canlandırılması ve Uluslararası Lojistik, logisticsclub.com

[28] The Borderless Workforce, 2008. Supply/Demand.

[29] Türk-Rus Kara Ulaştırması Karma Komisyon Toplantısı Protokolü, 2006.

[30] Worldbank Reports, www.info.worldbank.org

[31] www.en.wikipedia.org, source: Association of American Railroads.

[32] www.lojistikhaber.com

[33] www.nomegatrucks.eu