HUB LOCATION PROBLEM FOR AIR-GROUND

TRANSPORTATION SYSTEMS WITH TIME RESTRICTIONS

A THESIS

SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL

ENGINEERING

AND THE INSTITUTE OF ENGINEERING AND SCIENCES

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

by

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December, 2006

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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M.S. in Industrial Engineering

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December, 2006

In this thesis, we study the problem of designing a service network for cargo delivery sector. We analyzed the structure of cargo delivery firms in Turkey and identified the features of the network. Generally, in the literature only one type of vehicle is considered when dispatching cargo. However, our analysis showed that in some cases both planes and trucks are used for a better service quality. Therefore, we seek a design in which all cargo between origin and destinations is delivered with minimum cost using trucks or planes within a given time bound. We call the problem "Time Constrained Hierarchical Hub Location Problem (TCHH)" and propose a model for it. The model includes some non-linear constraints. After linearizations, the TCHH is solved with data taken from cargo delivery firms. The computational results are reported and comparison with the current structure of a cargo delivery firm is given.

Keywords: Hub location problem, Time restriction, Cargo delivery, Hierarchical network design.

ÖZET

HAVA ve KARA TAŞIMACILIK SİSTEMİNDE
ZAMAN KISITLI ANA DAĞITIM ÜSSÜ YER SEÇİMİ PROBLEMİ

Seda Elmastaş

Endüstri Mühendisliği Yüksek Lisans

Tez Yöneticileri : Yrd. Doç. Hande Yaman, Yrd. Doç. Bahar Y. Kara

Aralık, 2006

Bu çalışmada, ana dağıtım üssü (ADÜ) yer seçimi problemi kargo sektörü özelinde incelenmiştir. Literatürde geliştirilen modeller ve sezgiseller tek tip araç varsayımı için geçerlidir. Türkiye'deki kargo sektörü incelendiğinde belli servis kaliteleri için uçak kamyon bağlantılarının kullanıldığı ve ağ yapısının hiyerarşik olduğu gözlemlenmiştir. Bu çalışmada, iki tip araç kullanımına olanak sağlayan "Zaman Kısıtlı Hiyerarşik ADÜ Yer Seçimi" problemi tanımlanmış ve modellenmiştir. Doğrusal olmayan bazı kısıtlar doğrusal hale getirilerek, önerilen tamsayılı karar modeli çözülmüştür. Bulunan sonuçlar ile mevcut sistem karşılaştırılmış ve mevcut sistemden çok daha iyi sonuçlar elde edilmiştir. Ayrıca model farklı parametreler ile de denenmiştir.

Anahtar Kelimeler: ADÜ Yer Seçimi Problemi, Zaman Kısıtı, Hiyerarşik Ağ Tasarımı, Kargo Dağıtım Sektörü.

To my family and my departed grandfather, Bekir Elmastaş

ACKOWLEDGEMENT

And at last but not surely least, I want to express my thanks to my departed grandfather, Bekir Elmastaş, for his love, trust and guidance. Although he is not beside me, I always feel his support through my life.

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CHAPTER 1

INTRODUCTION

In this thesis we focus on the cargo delivery sector and we design a service network which is composed of agents, transfer centers and vehicles. Agents are located in the proximity of commercial zones in the cities and customers bring their packages to these agents. After collecting all the packages, agents send these packages to the transfer centers. All outgoing and incoming packages of agents are collected in these transfer centers. Each agent is assigned to a transfer center and transportation of packages is carried out via these transfer centers by vehicles.

In the cargo delivery sector, an important issue is the customer satisfaction. Among many components of customer satisfaction, the cost of the service and delivering the cargo in a timely manner are the key elements of the market.

Cargo delivery firms want to minimize their total cost which includes the cost of operating transfer centers and the transportation costs. In addition to the cost, the delivery time is another factor that must be taken into account by cargo delivery firms. Delivering the cargo within a promised service time, (will be referred to as "time bound"), is important for the quality of the service. It would be a great advantage for the firm if it can deliver the cargo to even most distant locations on time. These two elements, time and cost, are related to each other. To clarify the relationship between these two elements we first give the story of cargo:

A package that originates from an agent first travels to the corresponding transfer center. The transportation between an agent and a transfer center is provided by low capacitated vehicles (will be referred to as "middle trucks"). At the transfer center, vehicles are unloaded and packages are sorted according to their destinations. After sorting operations, packages are loaded to vehicles depending on their destinations. Since the flow between two transfer centers is usually high in volume, transportation is provided by high capacitated vehicles (will be referred to as "main trucks") between the transfer centers. After arriving at the destination transfer center, packages are unloaded from the main trucks and again sorted according to their final destinations. Then packages are transported to the destination agents with middle trucks. All these operations can be called as "The Story of Cargo" as shown in **Figure 1.1.**
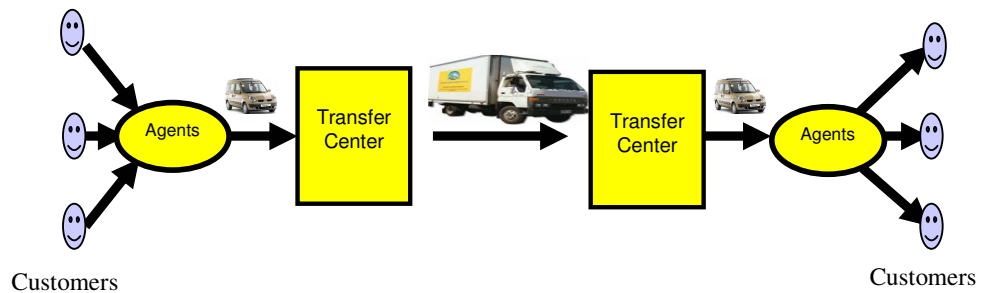
Figure 1.1. The Story of Cargo

Generally, the cargo is transported by trucks. However it may not be possible to travel by trucks when the distance between some origin and destination pairs within the time bound. In this case, some cargo delivery firms use also airway transportation.

Structure of airway transportation is similar to the ground transportation. Packages travel from transfer centers to the airports. Airplanes travel between airports and after arriving at the destination airport, packages are

unloaded from airplanes and again sorted according to their final destinations. Then packages are loaded on main trucks and are transported to the transfer centers. When cargo delivery companies use both ground and airway transportation they have a two layer structure as shown in Figure 1.2.
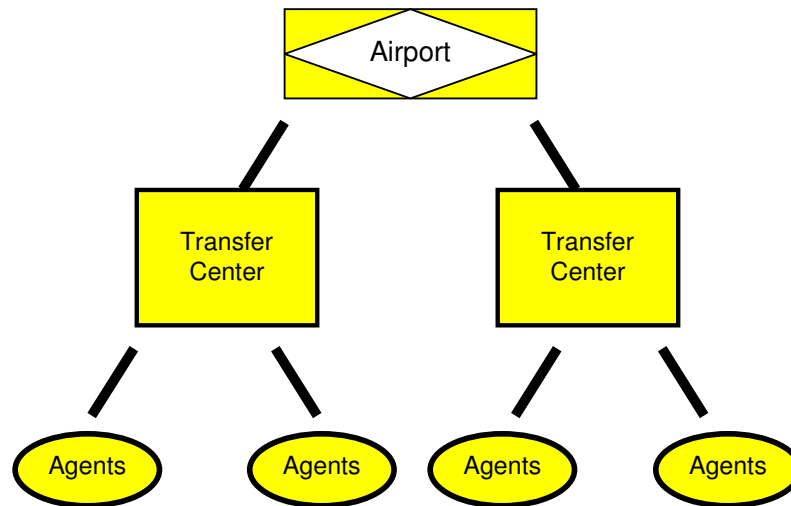
Figure 1.2. The Structure of Ground and Airway Transportation

However, using planes is a costly way. As a result of this fact, the decision of the number of planes and the number of transfer centers/ airports becomes important. On the other hand, it is also important to obey the time bound. Therefore, in this thesis we focus on the problem of designing a cargo delivery network with time restrictions and we develop a mathematical model for this complex problem.

The problem that we study has *two layers*. In the first layer we decide on the number of planes and determine the airports that will be used. In the second layer, we decide on the location of transfer centers. Between first

and second layer the problem is the allocation of transfer centers to the airports. Finally, we decide about the allocation of agents to these transfer centers. This two level structure is shown in Figure 1.3.
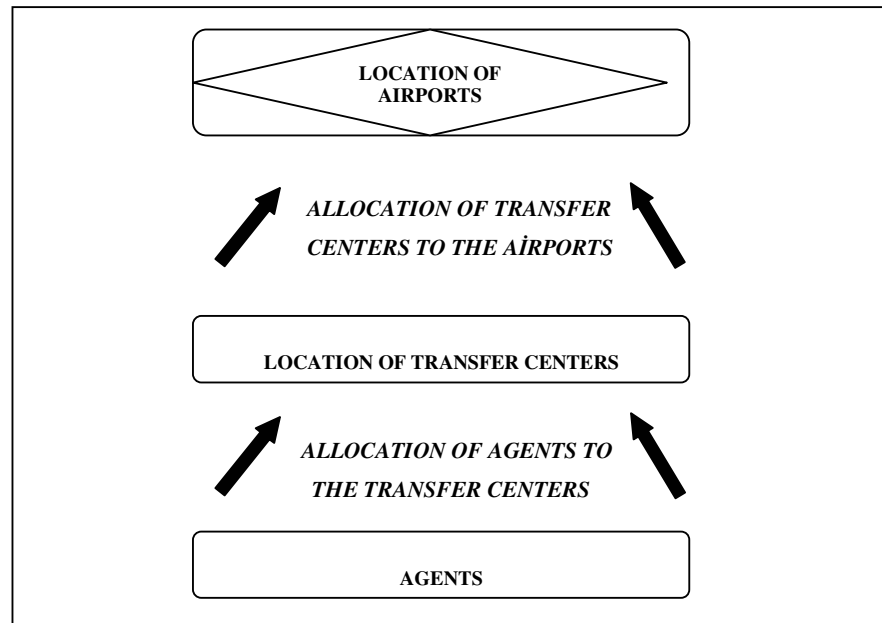
Figure 1.3. The Structure of 2-level problem

In our model, the aim is to minimize total cost. The cost figures that we include are;

- ➢ Transportation cost between airports by planes,
- ➢ Transportation cost between airports and transfer centers,
- ➢ Transportation cost between transfer centers,
- ➢ Transportation cost between transfer centers and agents.

While we are minimizing the total cost we must also obey the time bound. We propose a model to design a network where all packages are sent between origin and destinations using trucks or planes within the time bound with minimum cost. The model includes non-linear

constraints. We first linearize these constraints and then the model is solved by using a commercial mixed linear programming solver.

The problem described briefly is motivated by a real life application. We have interviewed representatives of five different cargo delivery firms to comprehend the basic structure of cargo delivery systems operating in Turkey. In the following chapter, Chapter 2, the detailed descriptions of their cargo delivery process and their service network structure are presented. Among these firms only MNG Cargo uses both ground and airway transportation. Therefore, we focus on MNG Cargo.

The rest of the thesis is organized as follows. A review of the literature is presented in Chapter 3. A detailed description of the problem, a mixed integer program and our proposed solution approach are given in Chapter 4. In chapter 5, we present the results of the proposed model and compare these results with those of the current system of MNG Cargo. The summary of our research and future directions are given in Chapter 6.

CHAPTER 2

THE CARGO DELIVERY FIRMS IN TURKEY

We have interviewed five different cargo delivery firms. Two of them, UPS and FedEx, provide service between Turkey and foreign countries, i.e. either the origin or the destination of the cargo is located in a foreign country. The other three firms provide service within Turkey. This means both the origin and the destination of the cargo are located in Turkey. In the following sections, the companies that provide service only in Turkey will be described first according to their service network structures. We start with Aras Cargo which has the fundamental structure and then continue with Yurtiçi Cargo and MNG Cargo. Next, the cargo delivery firms that provide service between Turkey and the foreign countries, UPS and FedEx, are described. At the last section, we give a synthesis of the cargo delivery sector in Turkey.

2.1 Aras Cargo

ARAS Cargo is a firm that has been providing cargo delivery service since 1989. It has a network in Turkey with 780 agents, 26 transfer centers and 5000 personnel. These agents and transfer centers are managed by 36 region directories. ARAS Cargo uses 2500 trucks, which are firm's own assets, and does not use planes for transportation.

The transfer centers of ARAS Cargo are located in Adana, Afyon, Aksaray, Ankara, Antalya, Balıkesir, Bursa, Denizli, Diyarbakır, Düzce, Elazığ, Erzurum, Eskişehir, Gaziantep, İstanbul(2), İzmir, Kayseri, Kocaeli, Konya, Malatya, Mersin, Merzifon, Samsun, Trabzon and Van.

As mentioned before, customers bring their cargo to the agents. The agents are allocated to transfer centers and dispatch the cargo via these transfer centers. In the service network of Aras Cargo, each agent is allocated to a single transfer center. Hence, all the cargo that originates from the same agent must first travel to the corresponding transfer center.

All the transfer centers are connected to the main transfer center in Ankara by trucks. Cargo is collected at this main transfer center and is sorted out according to its destination. Then it is loaded to main trucks and is sent to the destination transfer center. When the cargo arrives at the destination transfer center it is loaded to the middle trucks and it is sent to the destination agents. The service network of Aras Cargo is shown in Figure 2.1.
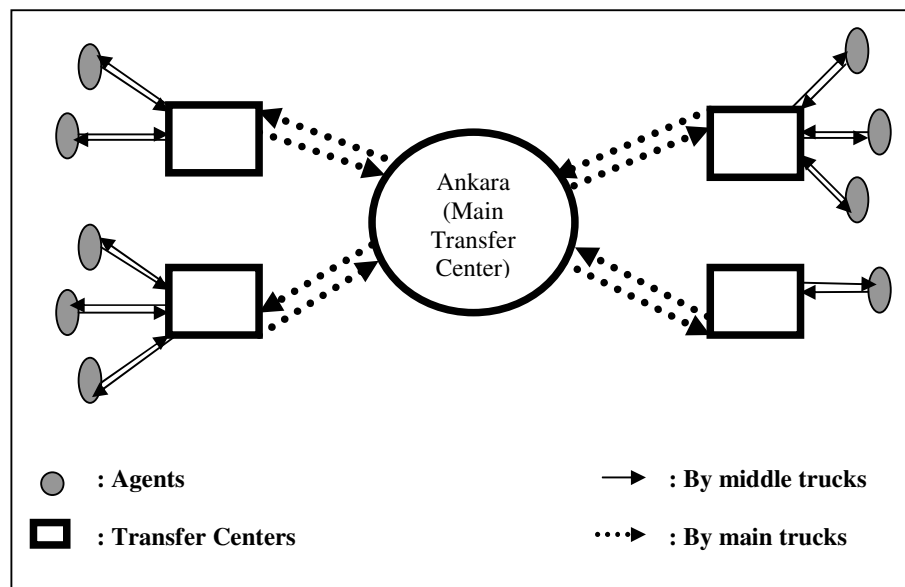
Figure 2.1. The Service Network of ARAS Cargo,

If the distance between the origin and the destination is less than or equal to 600 km then Aras Cargo promises to deliver the cargo in 24 hours. However, if this distance is greater than 600 km then cargo is delivered in two days.

2.2 Yurtiçi Cargo

YURTİÇİ Cargo, established in 1982, is the first private cargo delivery firm in Turkey. The firm performs service via 580 agents and 28 transfer centers. These agents and transfer centers are managed by 14 region directories. YURTİÇİ Cargo uses over 2100 trucks that are firm's own assets.

The service network of Yurtiçi Cargo is similar to that of Aras Cargo. As we mentioned earlier Aras Cargo has a single main transfer center, Ankara. On the other hand, Yurtiçi Cargo has a second main transfer center located in Istanbul. All transfer centers are allocated to these two main transfer centers. Cargo transported from transfer centers is collected at these main transfer centers and it is sorted out according to its destination. After sorting operations the cargo is sent to the destination transfer centers or is sent to the other main transfer center. The service network of Yurtiçi Cargo is shown in Figure 2.2.
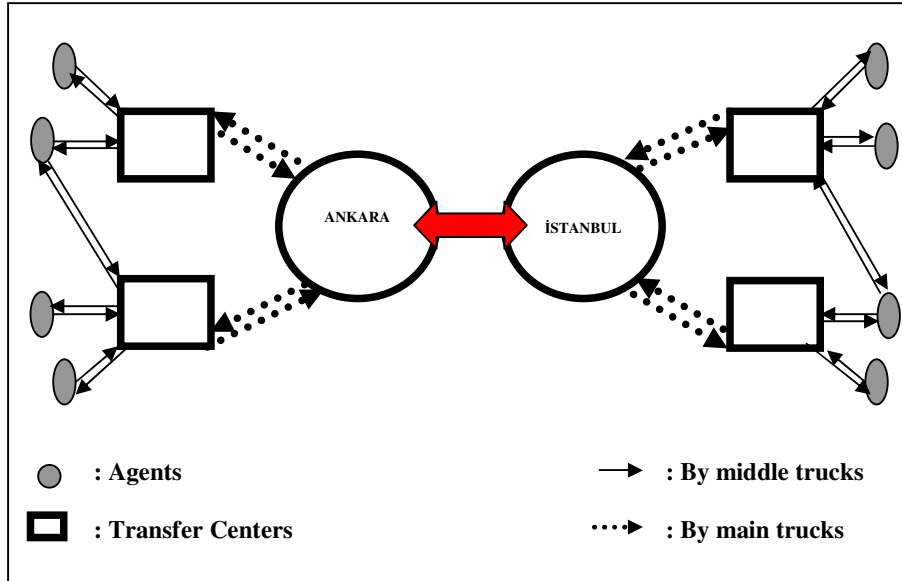
Figure 2.2. The Service Network of Yutiçi Cargo,

In the service network of Yurtiçi Cargo, an agent can be allocated to more than one transfer center. Because of this allocation, cargo that originates from the same agent can be sent to different transfer centers.

Same as Aras Cargo, Yurtiçi Cargo promises to deliver cargo in 24 hours if the distance between the origin-destination pair is less than or equal to 600 km. Otherwise, cargo is delivered within 48 hours.

2.3 MNG Cargo

MNG Cargo has been providing cargo delivery service since 1984 and has a wider service network in Turkey.

The firm performs this service via 22 transfer centers, 12 of which have airports and over 400 agents. These agents and transfer centers are managed by 12 region directories. MNG Cargo has 11 airplanes, and more than 750 trucks.

The story of the cargo is similar for MNG Cargo; agents are allocated to transfer centers and they dispatch the cargo via these transfer centers. Each agent should be allocated to a single transfer center. But different from Aras and Yurtiçi Cargo, MNG Cargo uses both ground and airway transportation. Therefore it has a more complex network structure than the others. In the service network of MNG Cargo, two types of transfer centers are present. First type is the transfer centers with airport and the second type is the transfer centers without airport.

The airway structure of MNG Cargo is similar to the ground structure of Aras Cargo. Each transfer center without an airport can be allocated to a single airport and all airports are connected to the "central airport" that is in Ankara. Hence, all the cargo that originates from the same transfer center must first travel to the corresponding airport and then must fly to the Ankara central airport. If a transfer center is allocated to the central airport, then the cargo will be delivered using main trucks to Ankara. In Ankara, cargo from airports and transfer centers are sorted out according to their destinations and loaded to vehicles or airplanes. The planes that depart from the central airport should wait for all the planes and trucks arriving at this central airport.

MNG Cargo uses airplanes if the distance between the agents is greater than 600 km. Otherwise, MNG Cargo delivers all cargo by trucks.
Cargo has four possible routes and these routes are shown in Figure 2.3;

➤ *Case i* : *Distance between origin destination pairs is less than or equal to 600 km*; transportation by trucks.

In Figure 2.3, the distance between agent *a* and agent *b* is less than 600 km therefore transportation is provided by trucks.

➤ *Case ii: Distance between origin destination pairs is greater than 600 km*; transportation by airplanes and trucks.

o *Case ii-a*: *If both origin and destination transfer centers are allocated to airports other than Ankara*:

First, cargo is delivered to the corresponding origin airport by main truck. Second, it flies to the central airport and then to the final airport. In Figure 2.3, transfer centers *i* and *k* are allocated to the airports other than Ankara and cargo delivery between these two transfer centers is an example of this case.

o *Case ii-b: Origin or destination transfer center is allocated to Ankara and the other one is allocated to any airport other than Ankara.*

- Origin transfer center is allocated to Ankara and destination transfer center is allocated to an airport other than Ankara: Cargo is delivered to Ankara by main truck and it travels from Ankara to the final airport by plane.

- Origin transfer center is allocated to an airport other than Ankara and destination transfer center is allocated to Ankara: Cargo is delivered to the corresponding origin airport by main truck. Then it flies to Ankara and then it is sent from Ankara to the destination transfer center by a main truck.

In Figure 2.3, routes between the transfer centers *m* and *k, m* and *i, j* and *k, j* and *i,* are examples of these last two cases.
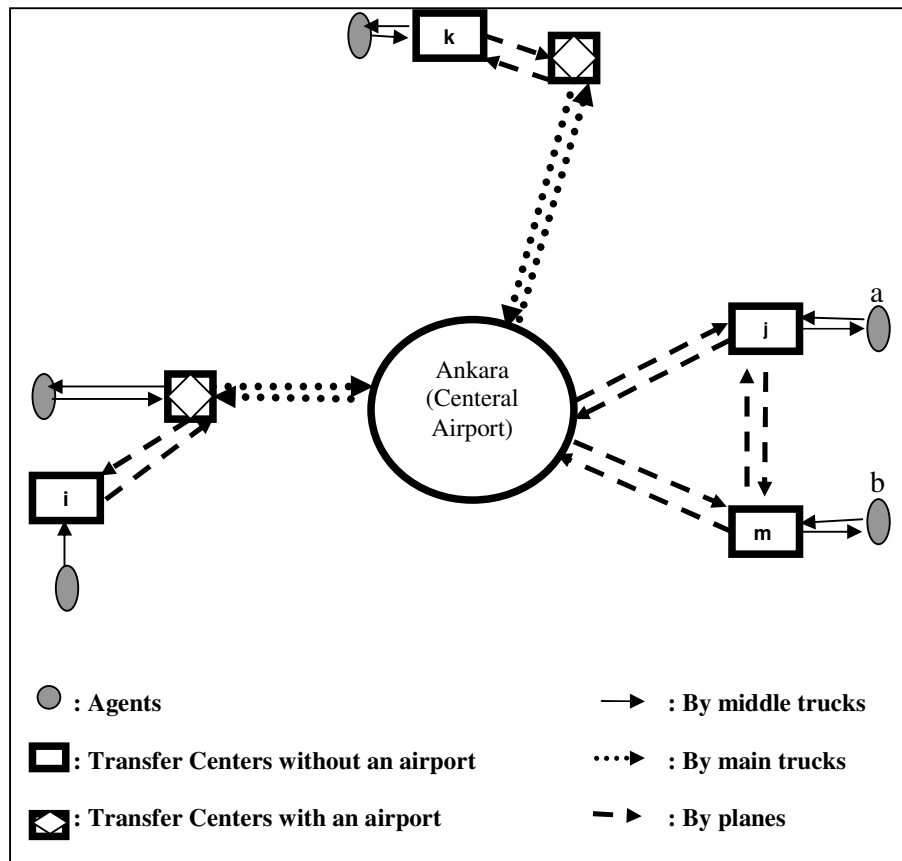
Figure 2.3. The Service Network of MNG Cargo,

Since MNG Cargo uses both ground and airway transportation, firm promises to deliver cargo in 24 hours between all origin-destination pairs.

2.4 UPS

UPS was established in 1907 in USA and serves in 230 countries. Service capacity of the firm is 8 times larger than the most proximate competitor and can deliver approximately 15,000,000 cargo per day. UPS's headquarters are located in Atlanta / USA. UPS-Turkey has its own head office in Istanbul since 1988. UPS provides service between Turkey and foreign countries. In Turkey, service is provided by 3 planes and

approximately 300 vehicles. The cities that are served by UPS in Turkey are Adana, Ankara, Antalya, Balıkesir, Bursa, Denizli, Eskişehir, Gaziantep, İstanbul, İzmir, Kahramanmaraş, Kayseri, Kocaeli, Konya, Manisa, Mersin, Samsun, Nevşehir, Muğla and Tekirdağ. All customs operations are performed only in Istanbul and thus UPS located its only transfer center in this city. All the cargo is collected in the main office Istanbul and then it is sent to the destination points.

2.5 FedEx

FedEx is an international air express company and it provides service between Turkey and foreign countries like UPS. The firm provides service to and from 16 cities in Turkey. These are Adana, Ankara, Antalya, Aydın, Balıkesir, Bursa, Çanakkale, Denizli, Gaziantep, Isparta, İzmir, İzmit, Kayseri, Konya, Kocaeli, Muğla and Tekirdag. The transportation of FedEx within Turkey is performed by a subcontractor firm Express Kargo. Because of the same reasons with UPS, FedEx has its only operation center in Istanbul and all the operations are the same as UPS.

2.6. Synthesis of The Cargo Delivery Sector In Turkey

According to our interviews with cargo delivery firms, we see that the story of the cargo is the similar for all the cargo delivery firms operating in Turkey. Customers bring their cargo to the agents and each agent is allocated to at least one transfer center. All incoming and outgoing cargo are consolidated at these transfer centers and sent to their destinations via these transfer centers. All these operations are completed in a predetermined time bound.

In addition to these common properties, another important point which is common for the cargo delivery firms is the truck departure times. Trucks departing from a transfer center should wait for all other trucks arriving at this transfer center. Otherwise, the cargo that arrives at the transfer center after the departure of the trucks will either require a second truck or wait for another 24 hours for the next day's truck. This property is common for all cargo delivery firms in Turkey and we also use this fact in our mathematical model. Other properties of the firms are summarized in Table 2.1.

Table 2.1. Service network properties of Cargo Delivery Firms in Turkey

Firms	Ground Trans.	Main Transfer Center for Ground Trans.	Airway Trans.	Main Transfer Center for Airway Trans.	Time Bound
Aras Cargo	Yes	Ankara	No	No	24 hours, if distance ≤ 600, 48 hours, if distance>600.
Yurtiçi Cargo	Yes	Ankara & İstanbul	No	No	24 hours, if distance ≤ 600, 48 hours, if distance>600.
MNG Cargo	Yes	No	Yes	Ankara	24 hours, all distances

CHAPTER 3

LITERATURE SURVEY

The critical decision for cargo delivery firms is the location of transfer centers and the allocation of agents to these transfer centers with minimum cost. In the literature this type of problem is called the "**Hub Location Problem**", transfer centers are named as "hubs" and agents are named as "demand points". However, there does not exist a specific term for the transfer centers with airport. Therefore, we call this type of transfer center as "hub airport". In the rest of this thesis, these terms will be used.

Literature on hub location problems focuses on one type of transportation mode: either by plane or by truck. On the other hand, the competition among the firms has increased and firms use different types of transportation modes to have a competitive advantage. So we also review the literature on "**Intermodal Freight Transportation**". In this chapter, literature on hub location and intermodal freight transportation problem are presented.

3.1. Hub Location Problem and Related Literature

Hubs are central facilities and are commonly used in cargo and postal delivery systems and communication networks. They act as switching points in networks and connect a set of interacting nodes. Generally, hub

location problems involve demand points and demands coming from these points are consolidated in the hubs.

Hub location problems can be classified into two groups according to the connection type of demand points to hubs as single and multi allocation. If each demand point is assigned to exactly one hub, the problem is called the single assignment (allocation) problem. If a demand point can be assigned to more than one hub then the problem is called the multi assignment problem.

The research on hub location problem began with the studies of O'Kelly (1986a, 1986b, 1987). The first description of the hub location problem is given by O'Kelly (1986a). In this paper the two cases are considered; the organization of a single hub network and the organization of systems with two hubs. The author presents real world examples and simple models for these two cases. O'Kelly (1986b) describes the quadratic structure in hub location problem and develops a heuristic for the single assignment problem.

Hub location problems also differ in their objective functions. The most frequently addressed hub location problem has been the p-hub median problem. The p-hub median problem is to locate p hubs in a network and allocate demand points to hubs such that the sum of the costs of transporting flow between all origin destination pairs in the network is minimized (Campbell, 1994). Different from the p-hub median problem, p-hub center problem is a minimax type problem. In other words, p-hub center problem is to locate p hubs in a network and to allocate demand points to hubs such that the maximum travel time (or distance) between any origin-destination pair is minimized (Campbell, 1994a). Another type of hub location problem is the hub covering problem in which the aim is to maximize the covered area by the hubs obeying the maximum time

bound on travel time. Generally, in these hub location problems the fixed cost of opening facilities is ignored. Different from these types, O'Kelly (1992b) introduces the fixed cost of facilities into hub location problems and the number of hubs becomes a decision variable. In the following part, the related literature on these problems is presented in three different subsections. Namely: p-hub median, p-hub center, hub covering and hub location problems with fixed costs.

3.1.1 P-hub Median Problem

As mentioned before the research on hub location began with the work of O'Kelly (1986a, 1986b, 1987). O'Kelly (1987) presents the first mathematical formulation for the single allocation p-hub median problem as a quadratic integer program which minimizes the total network cost. This quadratic integer program is considered as the basic model for hub location problem. The author also presents two heuristic algorithms for this problem. Heuristic 1 assigns the demand points to its nearest hub and Heuristic 2 selects the better of the first and second nearest hub. The heuristics are used to solve the problem with a data based on the airline passenger interactions between 25 U.S. cities in 1970 evaluated by the Civil Aeronautics Board (CAB). Later, this data set has been used by almost all of the hub location researchers and will be referred as the CAB data set.

Kliencewicz (1991) develops exchange heuristics for the single allocation p-hub median problem. These heuristics are compared with a clustering heuristic and heuristics developed in O'Kelly (1987). Among these heuristics the double-exchange heuristics in Kliencewicz (1991) show great promise as a solution technique for p-hub median problems. Skorin-Kapov & Skorin-Kapov (1994), develop a new heuristic method

based on tabu search for the single allocation p-hub median problem. They also compare their results with the heuristics of O'Kelly (1987) with the CAB data set. They get better solutions than other heuristics but the CPU times are higher than the other heuristics. Campbell (1996) presented two heuristics which rely on first solving the multiple assignment problem (via greedy exchange heuristic) and then using the solution of multiple assignment to develop a good network of hubs and allocations for the single assignment problem.

The first linear integer programming formulation for the single allocation p-hub median problem is given by Campbell (1994b). Ernst and Krishnamoorthy (1996) present a new formulation which requires fewer variables and constraints and so it is able to solve larger problems faster. They develop a heuristic algorithm which is based on simulated annealing, and they use the upper bound of the simulation annealing to develop a branch and bound algorithm. They have tested both their heuristic and branch and bound algorithm on the CAB data set and a new data set, consists of 200 nodes that represent postcode districts, along with their coordinates, which is referred as AP (Australian Post) data set.

Sohn and Park (1997) studied the single allocation two-hub median problems. They transform the quadratic 0-1 integer program for single allocation problem in the fixed two hub system into a linear program and solve in polynomial time when the hub locations are fixed.

Up to now we have presented the related literature on single allocation p-hub median problem. Different from single allocation, multi allocation p-hub median problems are also studied in the literature. Campbell (1992) was first to formulate the multiple allocation p-hub median problem as a linear integer program. Although Campbell (1996) studies the single allocation version of the p-median problem, the author obtains solutions

of multiple allocation problem by a greedy-interchange heuristic because the heuristics for solving the single allocation p-hub median problem are based on the solutions of the multiple allocation p-hub median problem.

Skorin-Kapov et al. (1996) develop mixed 0/1 linear formulations with linear programming relaxations. In this paper the authors consider multiple and single allocation *p*-hub median problems. In a subsequent study, Sohn and Park (1998) focused on methods to find optimal solutions for the allocation problems with fixed hub locations. They studied single allocation *p*-hub median problem and they have reduced the number of variables and constraints of the formulation provided by Skorin-Kapov et al. (1996) when the unit flow cost is symmetric. Besides single allocation, they also focus on the multiple allocation problem and they showed that the multiple allocation problem can be solved by the shortest path algorithm when *p* is fixed. O'Kelly et al. (1996) present exact solutions for hub location models and both single and multiple hub allocations are considered. They presented a further reduction in the size of the problem to the Skorin-Kapov et al. (1996) formulation based on the assumption of having a symmetric flow data. An important aspect of O'Kelly et al. (1996) is that it includes the discussion on the sensitivity of the solutions.

Ernst and Krishnamoorthy (1998a) present a new mixed integer linear programming model for the multiple allocation *p*-hub median problem based on the idea that they proposed for the single allocation *p*-hub median problem in 1996. The authors develop exact and heuristic algorithms for the multiple allocation *p*-hub median problem. They outline a heuristic using shortest paths and obtain exact solutions using two methods, namely explicit enumeration and branch and bound. In the paper, computational results with both the CAB and AP data set are presented.

Sasaki et al (1999), consider the 1-stop multiple allocation *p*-hub median problem which is a special case of the problem where they allow using at most one hub by each route in the network and they formulate the model as a *p*-hub median problem. For solving this formulation two algorithms are described in the paper. First one is a branch and bound type algorithm that uses lagrangian relaxation and the second algorithm is a greedy type algorithm. They test the performance of their algorithm on the CAB data set.

3.1.2 *P*-hub Center and Hub Covering Problems

Generally, existing studies in the literature have focused on the *p*-hub median with single and multi allocation. Different from the *p*-hub median problem, O'Kelly and Miller (1991) studied the single facility minimax hub location problem. In this paper, the work of O'Kelly (1986, 1987) is extended to the new problem of siting a hub in order to minimize the maximum cost of interaction in a hub networks system. Several approaches to this problem were reviewed, including: discrete locational evaluation; Helly's Theorem, a graphical approach; linear programming feasibility and Drezner's round trip location algorithm. One of these approaches, Drezner's algorithm, is chosen and applied to a real world example.

Campbell (1994b) extends hub location to center and covering problems by introducing the *p*-hub center and hub covering problems. The author develops integer programming models for these problems considering both single and multiple allocations.

Kara and Tansel (2000) also focus on the minimax criterion and present a new linearization for the single allocation p-hub center problem. They also prove that the single assignment *p*-hub center problem is *NP*-Hard. Campbell et al. (2005) address *p*-hub center problem when hub locations are fixed and they present integer programming formulations for both uncapacitated and capacitated cases.

Kara and Tansel (2003) focus on the hub covering problem. They studied the single allocation hub set covering problem and proved that it is *NP*-Hard. The authors develop a new model and give three linearizations for the old model developed by Campbell (1994b). Computational results show that the new model provides better CPU times than the old model.

Hub location literature discussed above does not consider the transient times spent at hubs for loading-unloading operations. Kara and Tansel, (2001) consider these transient times and identified a new problem that they call the latest arrival hub location problem. In this problem the aim is to minimize the maximum arrival time at destinations. For this model linear and nonlinear IP formulations are given and medium sized problems can effectively be solved using standard optimization tools.

Yaman, Kara and Tansel (2005) propose a mathematical model that allows stopovers for the latest arrival hub location problem. Proposed model is developed as a mixed integer program and it has nonlinear constraints. Linearization techniques are applied to these nonlinear constraints and valid inequalities are developed to strengthen the model. After linearizations and valid inequalities, the final model is tested with the data taken from Turkish cargo delivery firms. Inclusion of the valid inequalities gets the optimal solution in a smaller time.

3.1.3 Hub Location Problem with Fixed Costs

As we mentioned before, in the *p*-hub location problem the fixed cost of opening facilities is disregarded. On the other hand, the simple plant location problem includes fixed facility costs. In 1992, O'Kelly introduces the fixed facility costs into a hub location problem and thereby making the number of hubs a decision variable.

Campbell (1994b) presented the first linear programming formulations for the single and multi allocation hub location problems. Then Abinnoour - Helm (1998) introduced a heuristic to solve the uncapacitated hub location problem which is a hybrid of genetic algorithms and tabu search. He uses genetic algorithms to select the number and the location of hubs and tabu search to assign the demand points to the hubs. Topcuoglu et al. (2005) proposed another genetic algorithm for the uncapacitated hub location problem. They compare their results with the hybrid heuristic of Abdinnour-Helm (1998) on the CAB and AP data sets. Their experimental results show that their heuristic outperforms the heuristic proposed in Abdinnour-Helm (1998) with respect to both solution quality and required computational time. Another heuristic for the uncapacitated single allocation hub location problem is proposed in Chen (in press). He proposes a hybrid heuristic based on simulated annealing method, tabu list and improvement procedures. His computational results demonstrate that the proposed hybrid heuristic outperforms the heuristic presented in Topcuoglu et al. (2005) in terms of runtime and solution quality.

Ernst and Krishnamoorthy (1999) concentrate on the capacitated single allocation hub location problem. They used a modified version of a previous mixed integer linear programming formulation that they

developed in 1996 for the *p*-hub median problems. The authors also develop heuristics for obtaining upper bounds. They obtained optimal solutions by using an LP-based branch and bound method with the initial upper bound provided by the heuristics. They tested their algorithm on the AP data set because CAB data does not include capacities.

J.Ebery et al. (2000) describe a new mixed integer formulation for the capacitated multiple allocation hub location problem. Authors construct an efficient heuristic algorithm based on shortest paths and the upper bound obtained from this heuristic is incorporated in a linear programming based branch and bound solution procedure. Their computational experiments were carried out using the CAB and AP data sets.

Boland et al. (2004) consider formulations and solution approaches for multiple allocation hub location problems. They discuss both the capacitated and the uncapacitated multiple allocation hub location problem. They give the formulations of these problems and they identify the various characteristics of optimal solutions to multiple allocation hub location problems. Then they develop preprocessing procedures and tighten constraints for the existing formulations by using these characteristics. These procedures effectively reduce the computational effort required to obtain optimal solutions.

Marin et al. (2006) studied the uncapacitated multiple allocation hub location problem. They present new formulations of this problem that allow one or two visits to hubs and include more general cost structures that do not need to satisfy the triangle inequality. They checked the strength of these new formulations and compared them with other formulations presented in the literature on the CAB and AP data sets. The results show that formulations are better than the previous studies used

for small and medium problems. Canovas et al. (in press) also deals with the uncapacitated multiple allocation hub location problem. A heuristic method is presented and it is tested with CAB and AP data sets.

A polyhedral study on the multiple allocation uncapacitated hub location problem is presented in Hamacher et al. (2004). The authors determine the dimension and derive some classes of facets for this polyhedron. Labbé and Yaman (2004) studied the single allocation uncapacitated hub location problem. The authors derive a family of facet defining inequalities that can be separated in polynomial time. Labbé et al. (2005) study the capacitated version of the single allocation hub location problem where each hub has a fixed capacity in terms of the traffic that passes through it. They investigated some polyhedral properties of these problems and developed a branch-and-cut algorithm based on these results.

Hub location problems are difficult problems in general. For example the *p*-hub median problem is *NP*-hard. Moreover, even if the locations of the hubs are fixed, the allocation part of the problem remains *NP*-hard (Skorin-Kapov and Skorin-Kapov 1994). The single allocation hub center problem is *NP*-Complete as shown by Kara and Tansel (1999a). Lastly, when we look at the single allocation hub covering problem, we see that this problem is also *NP*-Hard as shown by Kara and Tansel, (2003).

3.2. Intermodal Freight Transportation and Related Literature

The research in OR literature has focused mostly on uni-modal transport problems. Hub location problems are of this type since only one type of vehicle is used such as planes, trucks etc. Since the number of

competitors of the firms has increased, a firm may want to use different modes of transportation. This may be useful in reducing transportation costs or increasing delivery speed. Using different modes of transportation is called intermodal freight transportation. This is a newly emerging research field and therefore there is not a consensus definition and a common conceptual model for the intermodal freight transportation. Intermodal transport is defined by the European Conference of Ministers of Transport (ECMT) as the carriage of goods by at least two different modes of transport in the same loading unit. Another description is given by Arnold et al (2004), intermodal freight transportation is characterized by the combination of the advantages of rail and road, rail for long distances and large quantities, road for collecting and distributing over short or medium distances. Of course, the modes of transportation can be different from rail, such as sea or water. An example of the intermodal freight transportation is seen in Turkish cargo delivery firms as we presented in the second chapter. In Turkey, two modes of transportation are used, planes and trucks.

The intermodal transport system is more complex to model than the unimodal one and the use of OR in intermodal transport research is still limited. Majority of the intermodal literature has been published in the last ten years. In general most of the researchers have focused on the rail truck intermodal chain. The main objective of intermodal rail haul research is to find solutions to the problem of organizing the rail haul in an efficient, profitable and competitive way.

Generally they distinguish three levels of planning and decision making with respect to the organization of the rail haul: strategic planning, tactical planning and operational planning. At the strategic level, the configuration of the service network design is determined. This includes decisions about which rail links to use, which origin and destination

regions to serve, which terminals to use and where to locate new terminals. At the tactical level, the configuration of the train production system is determined. This includes decision about train scheduling, routing and frequency of service. The operational level involves the day to day management decisions about the load order of trains, redistribution of railcars and load units.

Van Duin and Van Ham (1998) focus on to find optimal locations for terminals. They develop an appropriate model for each level: At the strategic level, a linear programming model searches the optimal locations for terminals. This model takes into account the existing terminals in the Netherlands and can then be used in order to find some new good areas. In the next level a concrete location in the interesting area is found by the use of a financial analysis. On the lowest level a discrete event stimulation model of the terminal gives the possibility to stimulate the working of the terminal.

Justice (1996) deals with the problem of a drayage company ensuring sufficient chassis (truck-train) available at terminals in order to meet demand. Reallocation is provided by truck within a region or by train between regions. The objective is to determine when, where, how many and by what means (truck-train) chassis are redistributed and to develop a planning model with minimum cost. The problem is mathematically formulated as a time based (network) transportation problem. The model is applied to aid interconnected terminals across the USA.

Network models for terminal location decisions have been applied by Rutten (1995). Rutten's objective is to find terminal locations that will attract sufficient freight to run daily trains to and from the terminal. He studies the effect of adding terminals to the network on the performance of existing terminals and the overall intermodal network. Woxenius

(1997) focuses upon existing and emerging technologies for European intermodal road-rail transshipment terminals and their impact on urban transport patterns.

Marin et al. (2000) present extensions of the uncapacitated hub location problem with multiple allocations that can be applied to network design problems in intermodal public transportation. They explain the different models of UHL and the relations between these models.

Arnold et al. (2004) deals with the problem of optimally locating multimodal terminals for freight transport. A mixed 0-1 program closely linked to multicommodity fixed charge network design problem is suggested and solved by a heuristic approach. The model is applied to the Iberian Peninsula.

Racunica and Wynter (2005) present an application of locating the optimal configuration of intermodal freight transport hubs. The model that they propose for this application is based on the uncapacitated hub location problem and it allows for nonlinear cost functions. Computational experience on the Alpine freight network is provided. In order to solve this model a linearization procedure and two heuristics is developed.

Çetinkaya et al. (2006) develop an iterative heuristic for the combined hubbing and routing problem in postal delivery systems. In the first stage, hub locations are determined and postal offices are multiply allocated to the hubs. The second stage gives the routes in hub regions. The final stage seeks improvements based on special structures in the routed network. Computational experience is reported for test problems taken from the literature and for a case study using the Turkish postal delivery system data.

In the rest of the thesis we compare the results of the model that we developed for cargo delivery systems with the current structure of MNG Cargo. The proposed model for that problem is presented in Chapter 4.

CHAPTER 4

MODEL DEVELOPMENT

In this chapter we analyze the structure of the problem that is special to cargo delivery firms. The main objective for cargo delivery firms is to minimize the total transportation cost. The p-hub median version of the hub location problem can be used to respond to the cargo delivery firms' objective, but the delivery time is not considered in this type of hub location problems. However, as stated before time is the key element of the customer satisfaction. Therefore, it is important to obey the maximum delivery time between any pair of nodes. Hub covering problem can be used to achieve this objective but in this version of the hub location problem the time that is spent at hubs is not considered. In the literature, transportation times and transient times are considered in the latest arrival hub location problem. On the other hand, in all these hub location problems only one type of vehicle is considered and using different modes of transportation is considered in the intermodal freight transportation problem. Therefore, the structure of our problem is similar to the combination of p-hub median, hub covering, latest arrival hub location and intermodal freight transportation problems which, to the best of our knowledge, has not been studied in the literature.

In addition to these, as mentioned in the first chapter, the problem that we study has two layers: in the first layer we determine the number and the location of hub airports, in the second layer we determine the number and the location of hubs. For these reasons, our problem can be named as a *"Time constrained hierarchical hub location problem (TCHH)"*.

The problem of interest can be stated as follows: Given a transportation network, set of potential nodes for hubs and the set of potential nodes for hub airports (which is a subset of the potential hub set), find the location of hub airports and hubs, allocation of hubs to these hub airports and the allocation of demand points to the hubs so as to minimize the total cost and obey the time bound. The total cost includes;

- ➢ Transportation cost between demand points and hubs with middle trucks,
- ➢ Transportation cost between hubs with main trucks,
- ➢ Transportation cost between hubs and hub airports with main trucks,
- ➢ Transportation cost between airports by planes.

Let us define our problem in more detail:

Cargo is sent to the hubs from demand points and it is transported via these hubs using trucks or planes. Each demand point is allocated to exactly one hub. Different from demand point allocation, each hub will be allocated to at *most* one hub airport.

All hub airports are connected to the central airport. From all the hub airports the planes bring their cargo to the central airport. After loading-unloading, all planes go back to their initial hub airports. In the service network, if it is decided to use a plane then the central airport must be opened. Otherwise, if the cargo is only transported by trucks it is not necessary to open the central airport.

Hubs are allocated to hub airports and they can also be allocated to the central airport. If a hub is allocated to the central airport, then cargo will be delivered using main trucks to the central airport as shown in Figure 4.1. At the central airport, cargo sent from hubs and hub airports is sorted

out according to its destination and is loaded to trucks or planes. The planes that depart from the central airport should wait for all the planes and trucks coming to this central airport and after loading operations planes go back to their initial hub airports. Same property is valid for trucks that depart from hubs to demand centers. All trucks that depart from hubs to demand centers wait for all the trucks and planes that are coming from other hubs. After unloading and loading operations trucks dispatch the cargo to the demand centers. Therefore, the departure time from hubs to demand centers is always greater than the departure time from hubs to other hubs.
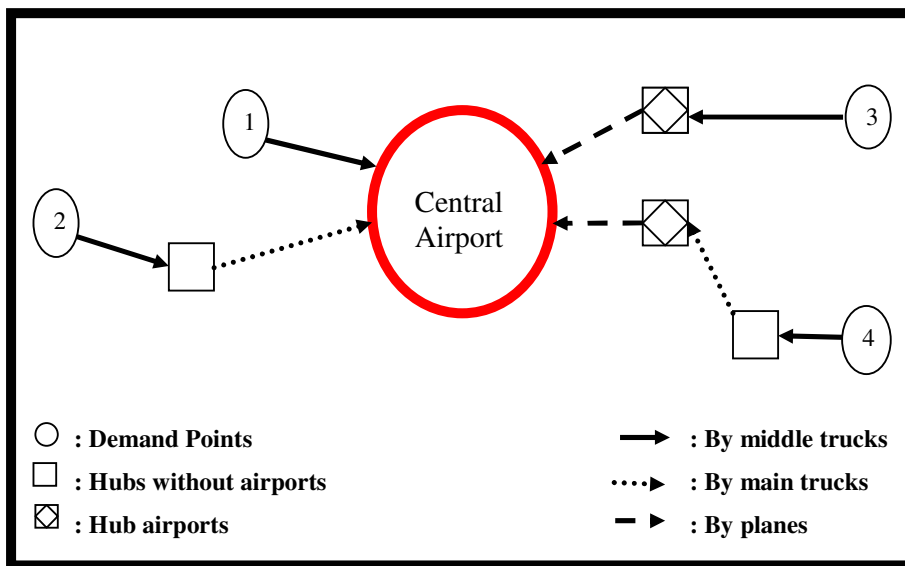
Figure 4.1. The Examples of Allocations,

Numbered cycles symbolize the demand points. First demand point is allocated to the central airport directly and its cargo is sent to the central airport with middle trucks. Second demand point is allocated to a hub and its cargo is sent with middle trucks to the hub and then to the central airport with main trucks. Third demand point is allocated to a hub airport

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so its cargo is transported here with middle trucks and from this hub airport cargo is sent to the central airport by plane. Last demand point is allocated to a hub and its cargo is sent here using middle trucks, then to the hub airport by main trucks and then to the central airport by plane. As we mentioned before it is not necessary to allocate each hub to a hub airport. If there does not exist an airway transportation between any pair of hubs then there must be a ground transportation that connect these two hubs. Otherwise, it is impossible to send the cargo between these two hubs. On the other hand, if there does not exist a ground transportation between any pairs then there exist an airway that connect these two hubs.

The proposed model is formulated as a mixed integer program. The model is subject to assignment constraints, time constraints and connectivity constraints.

As we mentioned before; we have a set of demand points, potential hubs and potential hub airports. D is the set of demand points, H is the set of potential hubs and A is the set of potential hub airports. H is a subset of D which MNG Cargo operates hub. Similarly, A is a subset of H because we select the hub airports from the possible set of hubs which have airports. This means, if node is a hub airport it is also a hub, but not vice versa. Therefore we define the following:

A: set of possible locations for hub airports,

V: set of possible locations for hubs (without airport),

0: central airport. Therefore ;

$$H = A \cup V \cup \{0\}$$

Parameters

In the model p symbolizes the number of hubs to be opened and T symbolizes the maximum time within which cargo should be delivered from any node i to any node j as "time-bound". d_{ij} is the distance between node i and node j, t_{ij} is the time to travel from node i to node j by truck. Since in real life the transportation time with main trucks is smaller than the transportation time with middle trucks, we use α to make this differentiation and α is the discount factor between middle and main trucks. Moreover, t_{i0}^u is the time to travel from hub airport i to the central airport by plane. If a plane is used then its cost equals to $\$u$ per flight. The cost of a truck between a demand point and a hub is equal to $\$f$ per kilometer and the cost of a truck between two hubs equals to $\$f_{hub}$ per kilometer.

In the model we also consider the loading / unloading times at hub airports. The loading / unloading times at any hub airport is min_a and at the central airport is min_0. All parameters are shown in Table 4.1.

Table 4.1. Parameters of the model

p	The required number of hubs
T	time bound
d_{ij}	the distance between node i and node j
t_{ij}	the time to travel between node i and node j by truck
t_{i0}^u	the time to travel from hub airport i to the central airport by plane
u	cost of a plane per flight
f	per kilometer traveling cost of a truck between a demand point and a hub
f_{hub}	per kilometer traveling cost of a truck between two hubs.
min_a	Loading / unloading time at any hub airport
min_0	Loading / unloading time at the central airport,
α	discount factor between middle and main trucks where $0 < \alpha < 1$.

Decision variables

In the model we have two sets of decision variables. The first set is the binary variables and the second set is the continuous variables. In the first set of the decision variables we have x_{ij}, w_{ij}, z_{ij}, u_i, and Y_i which are used for the allocations and operating hubs as described below. These variables are schematically shown in Figure 4.2.

$$x_{ij} : \begin{cases} 1 & \textit{if } \text{demand point } i \in D \text{ is allocated to hub } j \in H \\ 0 & \textit{otherwise} \end{cases}$$

$$w_{ij} : \begin{cases} 1 & \textit{if } \text{hub } i \in V \cup A \text{ is allocated to hub airport } j \in A \cup \{0\} \text{ by truck} \\ 0 & \textit{otherwise} \end{cases}$$

$$z_{ij} : \begin{cases} 1 & \textit{if } \text{there is a truck between hub } i \in H \text{ and hub } j \in H \\ 0 & \textit{otherwise} \end{cases}$$

$$U_i : \begin{cases} 1 & \textit{if } \text{there is a plane from hub airport } i \in A \text{ to central airport} \\ 0 & \textit{otherwise} \end{cases}$$

$$Y_i : \begin{cases} 1 & \textit{if } \text{hub } i \in H \text{ is opened} \\ 0 & \textit{otherwise} \end{cases}$$
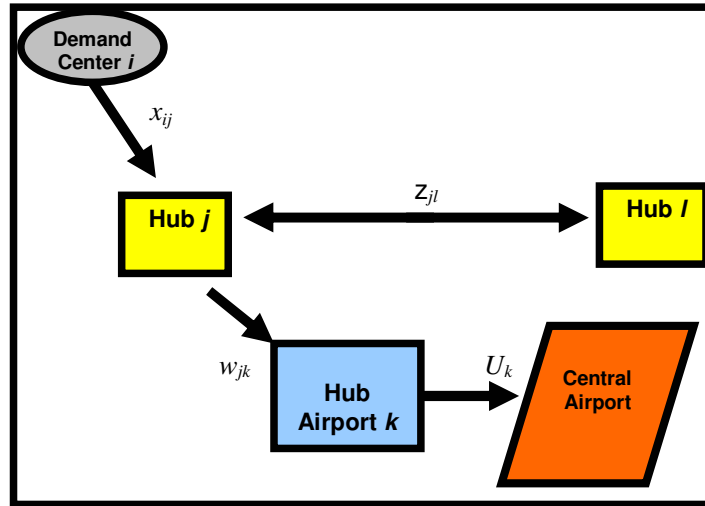
Figure 4.2. The first set of decision variables of the model

Hub j and Hub l are hubs without airport

The decision variables in the second set are used to determine the departure times from hubs to demand centers and from hubs to hubs. These decision variables are defined separately for hubs (without airport) and hub airports.

D_h : *departure time* from hub h to demand centers ($h \in V$)

$\hat{D}_h$: *departure time* from hub h to other hubs ($h \in V$)

A_h : *departure time* from hub airport h to demand centers ($h \in A \cup \{0\}$)

$\hat{A}_h$: *departure time* from hub airport h to hubs ($h \in A \cup \{0\}$)

These variables are schematically shown in Figure 4.3.
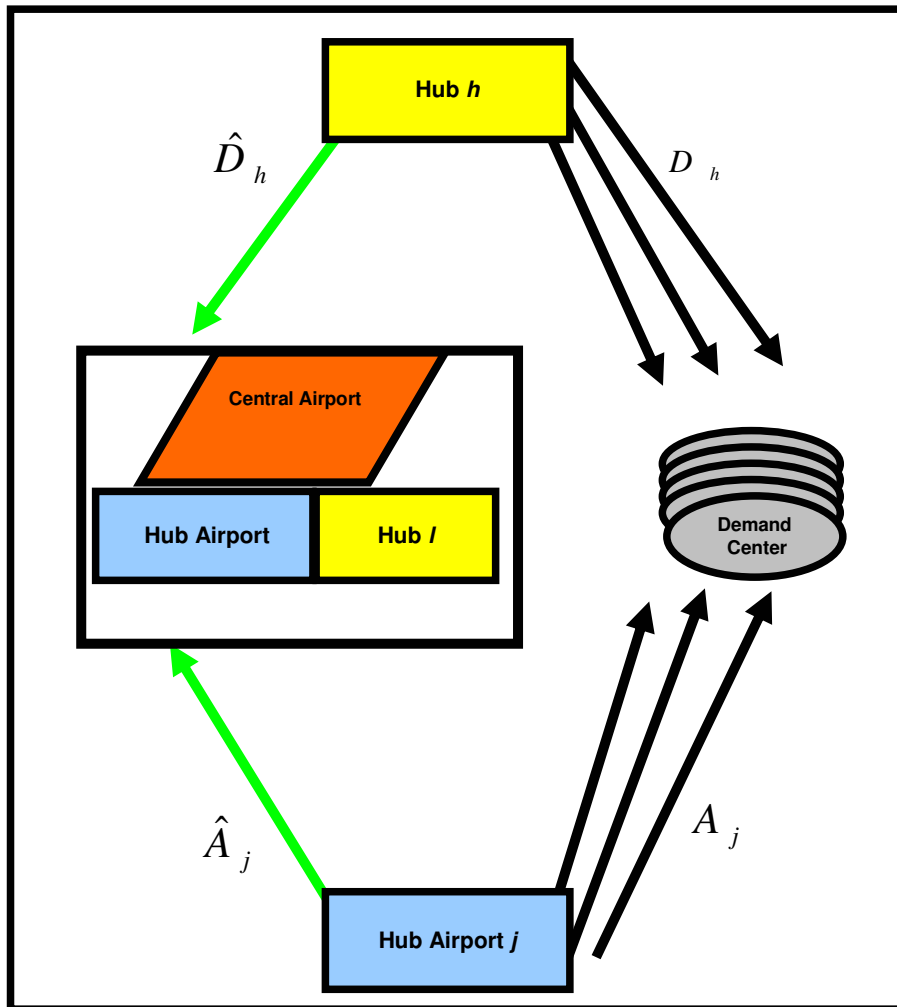
Figure 4.3. The second set of decision variables of the model

Hub h and Hub l are hubs without airport

4.1 Proposed Models

As we mentioned before, hubs can be allocated to at most one airport. Therefore, it is not necessary to use a plane when the cargo is dispatched. This decision is related to the time bound, T. If T is large enough then we do not need to use a plane to deliver cargo. However, if T is small we cannot deliver all cargo using trucks. Therefore, we have two cases. In the first case cargo is delivered by using at least one plane and in the

second case cargo is dispatched only by trucks. We develop two models for these two cases. In the first model, cargo is transported by both planes and trucks and we assume that at least one plane is used (will be referred as "TCHH_Tr.&P"). In the second model, cargo is transported only by trucks (referred as "TCHH_Tr.").

4.1.1 TCHH_Tr.&P

The TCHH_Tr.&P aims to design a network where all packages are sent between origin and destinations with minimum cost by using trucks and planes within the time bound. The model is composed of assignment, connectivity and time constraints. The allocation of demand points to hubs and hubs to hub airports is provided by assignment constraints. The transportation of the cargo between all origin-destination pairs is ensured by the connectivity constraints. And the time constraints provide that cargo is delivered within a time bound.

We use nine sets of decision variables in the model and forty six sets of constraints are developed by using these decision variables. The more detailed explanation of the constraints and the objective function is given in the following four parts.

4.1.1.1 Objective Function;

$$\min \sum_{i \in D} \sum_{j \in H} 2fd_{ij}x_{ij} + \sum_{i \in H} \sum_{j \in H} f_{hub}d_{ij}z_{ij} + \sum_{i \in A} 2u\,U_i$$

Our objective is to minimize the total cost of delivering cargo by trucks and planes. First term is the transportation cost between demand points and hubs. Second term represents the total transportation cost between hubs. The third term is the transportation cost between airports by planes.

4.1.1.2 Assignment and Connectivity Constraints

Constraints (1) to (18) are the assignment and connectivity constraints. The location of hubs and hub airports, allocation of demand points to hubs and the allocation of hubs to hub airports is provided by assignment constraints. After these allocations the connection between these hubs and hub airports is provided by the connectivity constraints. The detailed explanation of each constraint is given below:

$$\sum_{j\in H} x_{ij} = 1 \qquad\qquad \forall i \in D \qquad\qquad (1)$$

$$\sum_{j\in A \cup \{0\}} w_{ij} \leq 1 \qquad\qquad \forall i \in V \qquad\qquad (2)$$

$$\sum_{\substack{j\in A \cup \{0\} \\ j \neq i}} w_{ij} + U_i \leq 1 \qquad\qquad \forall i \in A \qquad\qquad (3)$$

$$\sum_{j\in A} w_{0,j} = 0 \qquad\qquad\qquad\qquad (4)$$

$$\sum_{j\in H} Y_j \leq p \qquad\qquad\qquad\qquad (5)$$

$$U_i \leq Y_i \qquad\qquad \forall i \in A \qquad\qquad (6)$$

$$z_{ij} \leq Y_j \qquad\qquad \forall i, j \in H \qquad\qquad (7)$$

$$z_{ij} \leq Y_i \qquad\qquad \forall i, j \in H \qquad\qquad (8)$$

$$x_{ij} \leq Y_j \qquad\qquad \forall i \in D, j \in H \qquad\qquad (9)$$

$$w_{ij} \leq z_{ij} \qquad\qquad \forall i \in H, j \in (A \cup \{0\}) \qquad (10)$$

$$w_{ij} \leq z_{ji} \qquad\qquad \forall i \in H, j \in (A \cup \{0\}) \qquad (11)$$

$$w_{ij} \leq U_j \qquad\qquad \forall i \in H, j \in A \qquad (12)$$

$$x_{i,i} = Y_i \qquad\qquad \forall i \in H \qquad (13)$$

$$x_{0,0} = 1 \qquad\qquad (14)$$

$$z_{kl} \geq (Y_k - \sum_{r \in A} w_{kr})(Y_l - \sum_{r \in A} w_{lr}) \qquad\qquad \forall k, l \in (V \cup \{0\}) \qquad (15)$$

$$z_{kl} \geq (Y_k - \sum_{r \in A} w_{kr})(Y_l - \sum_{\substack{r \in A \\ r \neq l}} w_{lr})(Y_l - U_l) \qquad\qquad \forall k \in (V \cup \{0\}), l \in A \qquad (16)$$

$$z_{kl} \geq (Y_k - \sum_{\substack{r \in A \\ r \neq k}} w_{kr})(Y_l - \sum_{r \in A} w_{lr})(Y_k - U_k) \qquad\qquad \forall k \in A, l \in (V \cup \{0\}) \qquad (17)$$

$$z_{kl} \geq (Y_k - \sum_{\substack{r \in A \\ r \neq k}} w_{kr})(Y_l - \sum_{\substack{r \in A \\ r \neq l}} w_{lr})(Y_l - U_l)(Y_k - U_k) \quad \forall k, l \in A \qquad (18)$$

First four constraints are the assignment constraints. According to constraints (1) and (9) each demand point is assigned to exactly one hub and an assignment is possible if that hub is opened. Constraints (2) and (12) force each hub $\in$ V, to be assigned to at most one hub airport and an assignment is possible if that hub airport is opened. According to constraint (3) if a hub is from the possible hub airport set and this hub is opened as a hub airport by the model than it cannot be assigned to another hub airport. Central airport is not assigned to other hub airports by constraint (4).

Constraint (5) ensures that the number of hubs is at most "*p*". Constraint (6) forces that if a possible hub is opened as a hub airport then it must also be opened as a hub.

Constraints (7) and (8) allow a truck link between two hubs if the hubs at the endpoints are opened. Constraint (10) and (11) ensure that if a hub is assigned to a hub airport than there must be a truck link between the hub and the hub airport. Constraint (13) ensures that if hub i is opened than demand point i is assigned to that hub. Constraint (14) forces the central airport to be opened.

Constraints (15) to (18) ensure that if two hubs are not assigned to hub airports or do not become hub airports then there must be a truck link between these two nodes. Otherwise, cargo can not be delivered between these two hubs. For instance, constraint (18) provides that if hub k and hub l are not opened as a hub airport and are not assigned to another hub airport than there must be a highway link between these hubs. Moreover, since central airport is neither opened as a hub airport ($\notin A$) and nor assigned to it, there is always a truck link between central airport and a hub, where the hub is not assigned to any hub airport and is not opened as a hub airport.

4.1.1.3 Time Constraints

Constraints between (19) and (37) are the time constraints. These constraints are constructed to keep track of departure times from hubs / hub airports and to provide that all the cargo is delivered to their destination points within time bound T. First we give the constraints and then we present the detailed explanation of the constraints.

$$D_h \geq \left(\hat{D}_r + \alpha\, t_{rh} \right) z_{rh} \qquad\qquad \forall h, r \in V \qquad (19)$$

$$D_h \geq \left(\hat{A}_r + \alpha\, t_{rh} \right) z_{rh} \qquad\qquad \forall h \in V, r \in (A \cup \{0\}) \quad (20)$$

$$D_h \geq \hat{D}_h \qquad\qquad \forall h \in V \qquad (21)$$

$$A_h \geq \left(\hat{A}_0 + t^u_{0,h} \right) U_h \qquad\qquad \forall h \in A \qquad (22)$$

$$A_h \geq \left(\hat{D}_r + \alpha\, t_{rh} \right) z_{rh} \qquad\qquad \forall h \in A, r \in V \qquad (23)$$

$$A_h \geq \left(\hat{A}_r + \alpha\, t_{rh} \right) z_{rh} \qquad\qquad \forall r \in (A \cup \{0\}), h \in A \qquad (24)$$

$$A_h \geq \hat{A}_h + \min_a U_h \qquad\qquad \forall h \in A \qquad (25)$$

$$A_0 \geq \hat{A}_0 + \min_0 \qquad\qquad (26)$$

$$\hat{D}_h \geq t_{ih}\, x_{ih} \qquad\qquad \forall i \in D, h \in V \qquad (27)$$

$$\hat{A}_h \geq t_{ih}\, x_{ih} \qquad\qquad \forall i \in D, h \in A \qquad (28)$$

$$\hat{A}_h \geq \left(\hat{D}_r + \alpha\, t_{rh} \right) w_{rh} \qquad\qquad \forall h \in A, r \in V \qquad (29)$$

$$\hat{A}_h \geq \left(\hat{A}_r + \alpha\, t_{rh} \right) w_{rh} \qquad\qquad \forall r, h \in A \qquad (30)$$

$$\hat{A}_0 \geq t_{i,0}\, x_{i,0} \qquad\qquad \forall i \in D \qquad (31)$$

$$\hat{A}_0 \geq \left(\hat{D}_h + \alpha\, t_{h,0} \right) z_{h,0} \qquad\qquad \forall h \in V \qquad (32)$$

$$\hat{A}_0 \geq \left(\hat{A}_h + \alpha\, t_{h,0} \right) z_{h,0} \qquad\qquad \forall h \in A \qquad (33)$$

$$\hat{A}_0 \geq \left(\hat{A}_h + t^u_{h,0} \right) U_h \qquad\qquad \forall h \in A \qquad (34)$$

$$(D_h + t_{ih})\, x_{ih} \leq T \qquad\qquad \forall i \in D, h \in V \qquad (35)$$

$$(A_h + t_{ih})\, x_{ih} \leq T \qquad\qquad \forall i \in D, h \in A \qquad (36)$$

$$(A_0 + t_{i0})\, x_{i0} \leq T \qquad\qquad \forall i \in D \qquad (37)$$

Constraints (19) to (26) are to keep track of departure times from all hubs (hubs $\in$ H) to demand points. Constraints (19), (20) and (21) are for the hubs in set V. Constraint (19) provides that the departing vehicles at these hubs must wait for the trucks coming from other hubs and constraint (20) ensures that these departing vehicles also wait for the trucks coming from hub airports before delivering the cargo to demand points. Constraint (21) ensures that outgoing cargo must leave the hub before delivering the incoming cargo. Constraints (22) to (24) are written for the hub airports. The departing vehicles at these hub airports must wait for the trucks coming from other hub airports (constraint (24)) and hubs assigned to them (constraint (23)) and also wait for the plane coming from the central airport (constraint (22)) before delivering the cargo. Constraints (25) and (26) are written for the loading and unloading times at hub airports. Constraint (25) is for any hub airport and constraint (26) is for the central airport.

Constraints (27) to (34) are written to keep track of leaving times from hubs to hubs, hub airports and central airport. Constraint (27) ensures that the departing vehicles from the hubs must wait for all the trucks coming from demand points assigned to that hub before sending cargo. Constraints (28) to (30) force that the departing vehicles from hub airports must wait for all the trucks coming from demand points and hubs assigned to that hub airport before sending cargo. Finally, the departing vehicles from central airport must wait for all the trucks coming from demand points and hubs assigned to that hub airport and also must wait

for the planes before sending cargo. These are ensured by constraints (31) to (34).

Constraints (35) to (37) ensure that all the cargo is delivered to their destination points within time bound *T*.

4.1.1.4 Non-negativity and Binary Restrictions

$$x_{ij} \in \{0,1\} \qquad\qquad \forall i \in D, j \in H \qquad (38)$$

$$z_{ij} \in \{0,1\} \qquad\qquad \forall i \in H, j \in H \qquad (39)$$

$$w_{ij} \in \{0,1\} \qquad\qquad \forall i \in H, j \in (A \cup \{0\}) \quad (40)$$

$$U_i \in \{0,1\} \qquad\qquad \forall i \in A \qquad (41)$$

$$Y_i \in \{0,1\} \qquad\qquad \forall i \in H \qquad (42)$$

$$D_h \geq 0 \qquad\qquad \forall h \in V \qquad (43)$$

$$A_h \geq 0 \qquad\qquad \forall h \in (A \cup \{0\}) \qquad (44)$$

$$\hat{D}_h \geq 0 \qquad\qquad \forall h \in V \qquad (45)$$

$$\hat{A}_h \geq 0 \qquad\qquad \forall h \in (A \cup \{0\}) \qquad (46)$$

As mentioned earlier this model is constructed under the assumption of using at least one plane. Constraint (14) and connectivity constraints provide that central airport is opened and at least one plane is used. However, when *T* is large enough it is not necessary to use plane and cargo can be delivered only by trucks. The proposed model for this case is presented in the following section.

4.1.2 TCHH_Tr.

In the TCHH_Tr. the aim is to design a network where all packages are sent between origin and destinations with minimum cost by using only trucks within the time bound.

We use five set of decision variables in the model and 16 constraint sets are developed by using these decision variables. The decision variables that are related with the hub airports and planes are not used in this model. The more detailed explanation of the constraints and the objective function is given below:

$$\min \sum_{i \in D} \sum_{j \in H} 2 f d_{ij} x_{ij} + \sum_{i \in H} \sum_{j \in H} f_{hub} d_{ij} z_{ij}$$

subject to

(1), (5),(7),(8),(9),(13),(38),(39),(42),

$$D_h \geq \left(\hat{D}_r + \alpha t_{rh}\right) z_{rh} \qquad \forall h, r \in H \qquad (19^*)$$

$$D_h \geq \hat{D}_h \qquad \forall h \in H \qquad (21^*)$$

$$\hat{D}_h \geq t_{ih} \, x_{ih} \qquad \forall i \in D, h \in H \qquad (27^*)$$

$$(D_h + t_{ih}) \, x_{ih} \leq T \qquad \forall i \in D, h \in H \qquad (35^*)$$

$$D_h \geq 0 \qquad \forall h \in H \qquad (43^*)$$

$$\hat{D}_h \geq 0 \qquad \forall h \in H \qquad (45^*)$$

$$z_{ij} \geq Y_i + Y_j - 1 \qquad \forall i, j \in H \qquad (47)$$

In this model, the objective function includes the total cost of transportation costs of the trucks. The difference between two objective functions is the transportation cost between airports by planes.

The constraints labeled with (*) are the same as the constraints in the TCHH_Tr.&P model but the set of nodes is different. Since we do not have the hub airports, the set *A* is an empty set and all the hub airports including central airport Ankara are acting as a hub. Therefore, all the hubs are chosen from set *H*. One additional constraint is the constraint (47) which ensures that if two hubs are opened then there must be a link between these two hubs.

These two models include non-linear constraints. In the following part we give the linearization of these non-linear constraints.

4.2 Linearizations:

We have 6 different sets of non-linear constraints. All these nonlinear constraints are linearized and the proofs of these linearizations are given in this part. Since the structure of constraints is similar in each set, we only give the proof of one constraint from each set.

In the first proposition we give the linearization of the first set of nonlinear constraints. This set consists of constraints (15) to (18) which are the connectivity constraints.

Proposition #1 : *Constraints (15) to (18) can be linearized as ;*

$$z_{kl} + \sum_{r \in A} w_{kr} + \sum_{r \in A} w_{lr} \geq Y_k + Y_l - 1 \qquad \forall k,l \in (V \cup \{0\}) \qquad (15')$$

$$z_{kl} + \sum_{r \in A} w_{kr} + \sum_{\substack{r \in A \\ r \neq l}} w_{lr} + U_l \geq Y_k + Y_l - 1 \qquad \forall k \in (V \cup \{0\}), l \in A \quad (16')$$

$$z_{kl} + \sum_{\substack{r \in A \\ r \neq k}} w_{kr} + \sum_{r \in A} w_{lr} + U_k \geq Y_k + Y_l - 1 \qquad \forall k \in A, l \in (V \cup \{0\}) \quad (17')$$

$$z_{kl} + \sum_{\substack{r \in A \\ r \neq k}} w_{kr} + \sum_{\substack{r \in A \\ r \neq l}} w_{lr} + U_l + U_k \geq Y_k + Y_l - 1 \qquad \forall k,l \in A \qquad (18')$$

Proof: Among these constraints the proof of constraint (15) is given.

We have four cases;

1. If $\sum_{r \in A} w_{kr} = 1$ and $\sum_{r \in A} w_{lr} = 1$ then;

$Y_k = 1$ and $Y_l = 1$

Constraint (15) : $z_{kl} \geq 0$

Constraint (15') : $z_{kl} \geq -1$

2. If $\sum_{r \in A} w_{kr} = 1$ and $\sum_{r \in A} w_{lr} = 0$ then we have two cases;

a. $Y_k = 1$ and $Y_l = 0$ **b.** $Y_k = 1$ and $Y_l = 1$

    Constraint (15) : $z_{kl} \geq 0$ Constraint (15) : $z_{kl} \geq 0$

    Constraint (15') : $z_{kl} \geq -1$ Constraint (15') : $z_{kl} \geq 0$

3. If $\sum_{r \in A} w_{kr} = 0$ and $\sum_{r \in A} w_{lr} = 1$ then we have two cases;

a. $Y_k = 0$ and $Y_l = 1$ **b.** $Y_k = 1$ and $Y_l = 1$

    Constraint (15) : $z_{kl} \geq 0$ Constraint (15) : $z_{kl} \geq 0$

    Constraint (15') : $z_{kl} \geq -1$ Constraint (15') : $z_{kl} \geq 0$

4. If $\sum_{r \in A} w_{kr} = 0$ and $\sum_{r \in A} w_{lr} = 0$ then we have four cases;

a. $Y_k = 0$ and $Y_l = 0$ **b.** $Y_k = 1$ and $Y_l = 0$

    Constraint (15) : $z_{kl} \geq 0$ Constraint (15) : $z_{kl} \geq 0$

    Constraint (15') : $z_{kl} \geq -1$ Constraint (15') : $z_{kl} \geq 0$

c. $Y_k = 0$ and $Y_l = 1$ **d.** $Y_k = 1$ and $Y_l = 1$

    Constraint (15) : $z_{kl} \geq 0$ Constraint (15) : $z_{kl} \geq 1$

    Constraint (15') : $z_{kl} \geq 0$ Constraint (15') : $z_{kl} \geq 1$

In each case right hand side of the constraint (15') equals to the one of constraint (15) or it gives a redundant lower bound for the right hand side of constraint (15). However, when constraint (15') gives a redundant lower bound, the nonnegativity constraints ensure that $z_{kl} \geq 0$. So, we can linearize constraint (15) using constraint (15').

As mentioned before the structure of constraints (16) to (18) is similar to the one of constraint (15). These constraints can be linearized in a similar fashion. $\qquad\square$

Second set of nonlinear constraints, constraints (19), (20), (23) and (24), are related with the leaving times from hubs in set V and hubs in set A to demand points. In these constraints the departing vehicles from the hubs wait for the trucks coming from other hubs.

Proposition #2: *Constraints (19), (20), (23) and (24) can be linearized as ;*

$$D_h \geq \hat{D}_r + \alpha\, t_{rh}\, z_{rh} \qquad\qquad \forall h, r \in V \qquad\qquad (19')$$

$$D_h \geq \hat{A}_r + \alpha\, t_{rh}\, z_{rh} \qquad\qquad \forall h \in V, r \in (A \cup \{0\}) \qquad (20')$$

$$A_h \geq \hat{D}_r + \alpha\, t_{rh}\, z_{rh} \qquad\qquad \forall h \in A, r \in V \qquad\qquad (23')$$

$$A_h \geq \hat{A}_r + \alpha\, t_{rh}\, z_{rh} \qquad\qquad \forall r \in (A \cup \{0\}), h \in A \qquad (24')$$

Proof: The proof for constraint (19) is given.
We have two cases:
1. If $z_{rh} = 1$ then both constraint (19) and (19') equal to

$$D_h \geq \hat{D}_r + \alpha\, t_{rh} \; .$$

2. If $z_{rh} = 0$ then

Constraint (19) : $D_h \geq 0$

Constraint (19') : $D_h \geq \hat{D}_r$

When $z_{rh} = 0$ constraint (19') gives a lower bound for D_h.

If $z_{rh} = 0$, then there does not exist a truck between hub r and hub h. From constraints (15) to (19), we know that if there does not exist a truck between any pairs then there exists an airway that connect these two hubs. Otherwise, cargo between hub r and hub h cannot be delivered. Constraints (32) to (34) force that the central airport waits for all the cargo including the one from hub r and constraint (20) forces that hub h also waits for all the cargo coming from the central airport. Therefore hub h also waits for hub r before sending cargo to its demand points. Therefore, constraint (19') is satisfied. So, we can linearize constraint (19) using constraint (19')   □

Third set of constraints are similar to the second set of constraints. They are related with the leaving times from hub airports to demand points but the departing vehicles from hubs wait for the plane coming from hub airports. Constraint (22) is the only element of this set and we give the proof for this constraint.

Proposition #3: *Constraint (22) can be linearized as ;*

$$A_h \geq \hat{A}_0 + t^u_{0,h} U_h \qquad\qquad \forall h \in A \qquad\qquad (22')$$

Proof:

We have two cases:

1. If $U_h = 1$ then constraint (22) and constraint (22') yield the same right hand side.

2.If $U_h = 0$ then

Constraint (22)   : $A_h \geq 0$

Constraint (22')   : $A_h \geq \hat{A}_0$

When $U_h = 0$ constraint (22') gives a lower bound for A_h. U_h equals to zero means there does not exist a plane from hub airport h to central airport. Hub airport h can be allocated to central airport or to another hub airport by truck. From constraint (24) hub airport h also waits for the central airport. Therefore, constraint (22') is satisfied. So, we can linearize constraint (22) as constraint (22')   □

Constraints (29) and (30) are related with the leaving times from hub airports to hubs and they are the fourth set of the nonlinear constraints.

Proposition #4: *Constraints (29) and (30) can be linearized as;*

$$\hat{A}_h \geq (\hat{D}_r + \alpha t_{rh}) - M(1 - w_{rh}) \qquad \forall h \in A, r \in V \qquad (29')$$

$$\hat{A}_h \geq (\hat{A}_r + \alpha t_{rh}) - M(1 - w_{rh}) \qquad \forall r, h \in A \qquad (30')$$

where M is a very big number.

Proof : The proof of constraint (29) is given.

We have two cases;

1. If $w_{rh} = 1$ then constraint (29) and constraint (29') yield the same right hand side.

2. If $w_{rh} = 0$ then;

Constraint (29)   : $\hat{A}_h \geq 0$

Constraint (29')   : $\hat{A}_h \geq (\hat{D}_r + \alpha t_{rh}) - M$

When $w_{rh} = 0$ constraint (29') gives a lower bound for $\hat{A}_h$. Since M is a

big number right hand side of the (29') gives a redundant lower bound on

$\hat{A}_h$. So, we can linearize constraint (29) using constraint (29')   □

Fifth set of constraints, constraints (32) to (34), are related with the leaving times from central airport to hubs $\in H$.

Proposition #5: *Constraints (32) to (34) can be linearized as;*

$$\hat{A}_0 \geq \hat{D}_h + \alpha \, t_{h,0} \, z_{h,0} \qquad\qquad \forall h \in V \qquad (32')$$

$$\hat{A}_0 \geq \hat{A}_h + \alpha \, t_{h,0} \, z_{h,0} \qquad\qquad \forall h \in A \qquad (33')$$

$$\hat{A}_0 \geq \hat{A}_h + t_{h,0}^u \, U_h \qquad\qquad \forall h \in A \qquad (34')$$

Proof: The proof of constraint (32) is given.

We have two cases:

1. If $z_{h,0} = 1$ then constraint (32) and constraint (32') yield the same right hand side.

2. If $z_{h,0} = 0$ then

Constraint (32)   : $\hat{A}_0 \geq 0$

Constraint (32')   : $\hat{A}_0 \geq \hat{D}_h$

When $z_{h,0} = 0$ constraint (32') gives a lower bound for $\hat{A}_0$.

If $z_{h,0} = 0$, then there does not exist a highway connection between hub h and central airport. From constraints (15) to (19), we know that if there does not exist a highway connection between any pairs then there exist an airway that connect these two points. Otherwise, cargo between hub h and central airport cannot be delivered. Hub h can be allocated to any

other hub airport. By constraint (34), central airport waits for all cargo coming from the hub airports. Therefore, constraint (32) is satisfied. $\square$

The last set is the constraints (35) to (37) which are constructed for the time bound.

Proposition #6: _Constraints (35) to (37) can be linearized as;_

$$D_h + t_{ih} x_{ih} \leq T \qquad\qquad \forall i \in D, h \in V \qquad (35')$$

$$A_h + t_{ih} x_{ih} \leq T \qquad\qquad \forall i \in D, h \in A \qquad (36')$$

$$A_0 + t_{i0} x_{i0} \leq T \qquad\qquad \forall i \in D \qquad\qquad (37')$$

Proof : The proof of constraint (35) is given.

We have two cases:

1. If x_{ih} = 1 then constraint (35) and constraint (35') yield the same right hand side.

2. If x_{ih} = 0 then

Constraint (35)   : $0 \leq T$

Constraint (35') : $D_h \leq T$

When x_{ih} = 0 constraint (35') gives an upper bound for D_h. D_h must be smaller than T because we must deliver all cargo within T-hours. Therefore, constraint (35) is satisfied. $\square$

After these linerarizations, we examine the number of decision variables and constraints for the two linear models. The number of decision variables in the TCHH_Tr.& P equals to

$$|D||H| + |V \cup A||A \cup \{0\}| + |H|^2 + |A| + |H| + 2|V| + 2|A \cup \{0\}| =$$

$|A+1|\{2 + |V \cup A|\} + |H|\{|D| + |H| + 1\} + 2|V| + |A|$ and in the TCHH_Tr. the number of decision variables equals to $|D||H| + |H|^2 + 3|H|$.

When we evaluate the number of constraints we see that in the TCHH_Tr.& P, the number of constraints equals to

$$2|H|^2 + 3|A|^2 + 2|V|^2 + 3|H| + 6|V| + 3|D| + 9|A| + 3|A\|H| + |D\|H| + 2|D\|V|$$
$$+ 2|D\|A| + 5|A\|V| + 5$$

In the TCHH_Tr. the number of constraints equals to

$$3|D\|H| + 4|H|^2 + 2|H| + |D| + 1.$$

In the worst case the number of demand points, hubs and hub airports equals to n. In other words, all demand points are possible hub airports and they are also possible hubs. In such a situation the number of decision variables and constraints are given in Table 4.2 for two models.

Table 4.2. The number of decision variables and constraints in the worst case for linear models

	TCHH_Tr.& P	**TCHH_Tr.**
# of decision variables	$3n^2 + 6n$	$2n^2 + 3n$
# of constraints	$20n^2 + 21n + 5$	$7n^2 + 2n + 1$

In the next chapter, we present the results of the proposed model and compare the current system of MNG Cargo with our results.

CHAPTER 5

COMPUTATIONAL RESULTS

In this section we describe the current system of MNG Cargo and explain how the input data is processed. Then we present the results obtained by solving the mixed integer programs given in Chapter 4 and we compare the current system of MNG Cargo with our results.

5.1 Current System of MNG Cargo

In the current system, MNG Cargo provides its service with 22 hubs, 12 of which have hub-airports and over 400 agents. The hubs and hub airports are listed in Table 5.1.

Table 5.1. Hubs and the hub airports of MNG Cargo.

Hubs	Adana	Bursa	Erzurum	İzmir	Samsun	Düzce
	Afyon	Denizli	Eskişehir	Kayseri	Trabzon	Merzifon
	Ankara	Diyarbakır	Gaziantep	Konya	Van	
	Antalya	Elazığ	İstanbul	Malatya	Aksaray	
Hub Airports	Adana	Antalya	Erzurum	Malatya	Samsun	Van
	Ankara	İstanbul	Diyarbakır	İzmir	Trabzon	Gaziantep

Generally, the demand points are allocated to the nearest hub and hubs are allocated to the nearest hub airport including the central airport. For the current implementation, the allocation of hubs to hub airports is given in Figure 5.1 and it is represented by red lines. As shown in the figure,

each hub is allocated to only one hub airport and each hub airport is assigned to the central airport as shown by black lines.
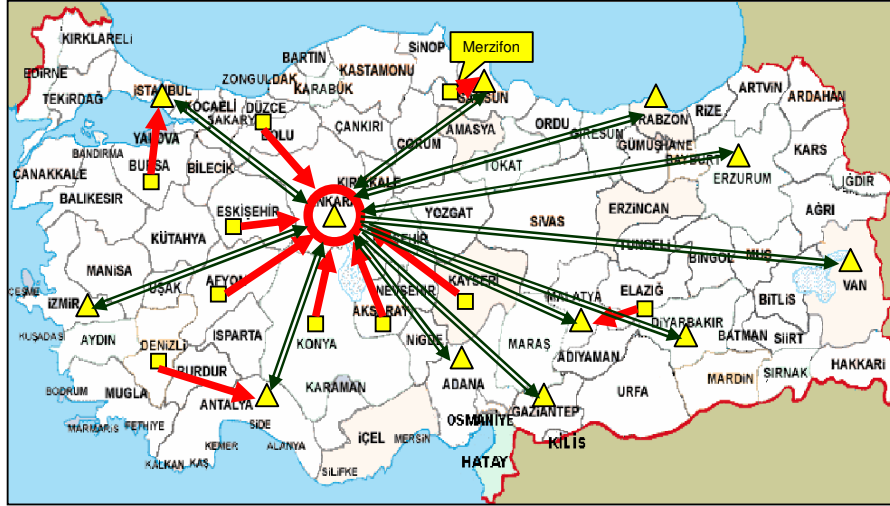
Figure 5.1. The allocation of hubs (☐) to hub airports (△) of MNG Cargo.

5.2 Input Data Processing

Since the demand locations are the points scattered throughout Turkey, we have grouped all of these points into clusters. We take all major cities of Turkey as clusters which makes 81 points and we also count Merzifon which is a hub in the current system. Therefore, we have 82 different demand points. The firm requires that each cargo will be sent in $T = 24$ hours between these demand points.

Since we do not have a fixed cost of operating a hub or hub airport we take these values parametric and we fixed the number of hubs to p. In the current system $p = 22$.

During the interviews, we get the cost of using plane per flight and from the trucking industry we get the unit distance cost for vehicles. The unit distance cost for each middle truck is taken as $f = 0,8$ YTL per kilometer

and the unit distance cost for each main truck is taken as $f_{hub} = 1$ *YTL* per kilometer. The transportation cost between airports by plane is taken as *u=3.500 YTL* for each flight.

When we test our models, we take the loading/unloading times min_a =30 minutes at any airport for a Є A and min_0 =120 minutes at central airport. The time to travel from hub airport i to the central airport by plane, t_{i0}^u , is taken to be 90 minutes for all i Є A.

Using these parameters we solve the model for the current system. Moreover, to see the effects of changes in parameters, we also solve the model for p = {22,20,15,10,5}, T={40,35,32,31,…,20} as given in Table 5.2.

Table 5.2. Different T and p values for fixed cost ratio.

p	T							
	40	35	32	31	30	29	28	27
22	26	25	24	23	22	21	20	
20	40	35	32	31	30	29	28	27
	26	25	24	23	22	21	20	
15	40	35	32	31	30	29	28	27
	26	25	24	23	22	21	20	
10	40	35	32	31	30	29	28	27
	26	25	24	23	22	21	20	
5	40	35	32	31	30	29	28	27
	26	25	24	23	22	21	20	

The solutions are presented in the following part.

5.3 Solution of the Model

Using CPLEX 9.1, the TCHH Tr.&P is solved to optimality in 1674.39 CPU seconds for p = 22 and T = 24. The optimal solution has a total cost

of *44,354 YTL*, using two hub airports in Diyarbakır and İzmir in addition to the central airport and 22 hubs. Compared to the current implementation, the reduction in the number of planes is 81.8 %.

In the above solution, to compare the current implementation and our results, we fixed the number of hubs, *p*, to 22 and our model provides a better solution. We also solve the model for $p \leq 22$ and the mixed integer linear program is solved to optimality in 11.627 CPU seconds. The optimal solution came up with a total cost of 37,910 YTL, using one hub airport in Diyarbakır in addition to the central airport and eleven hubs. Namely: Afyon, Ankara, Diyarbakır, Düzce, Elazığ, Erzurum, Gaziantep, Kayseri, Merzifon, Trabzon, Van. The number of hubs and the number of planes are given in the below table for our model and for the current implementation. As it can be seen from the table, the number of hubs decreases by 50% and the number of planes decreases by 90.9 %. The optimal solution is depicted in Figure 5.2.

Table 5.3. Comparison of the optimal solution and the current implementation

	# of Hubs	# of Planes
Current Implementation	22	11
Optimal Solution	11	1

In Figure 5.2., blue lines symbolize the allocation of demand points to hubs, red lines symbolize the allocation of hubs to hub airports and black line symbolizes the allocation of hub airport to the central airport, Ankara.
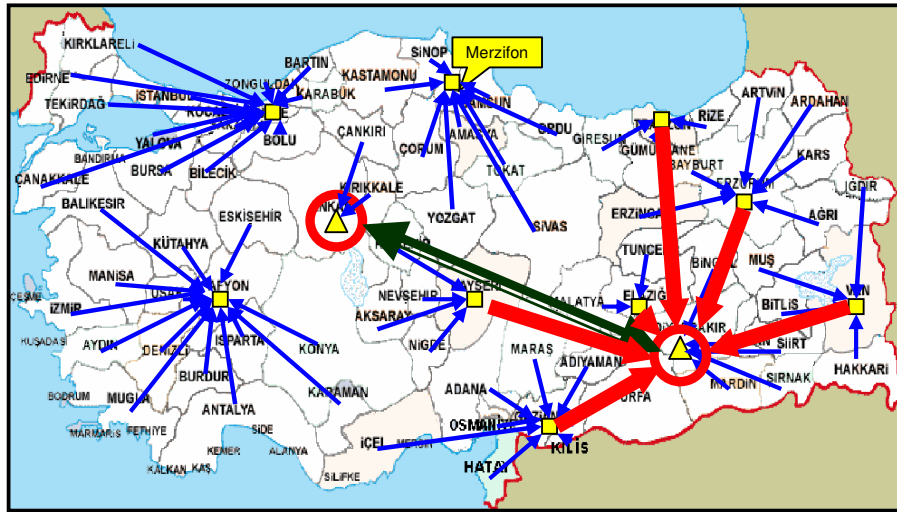
Figure 5.2. The optimal solution for $p \leq 22$

As it is mentioned in Chapter 4, we developed two models: TCHH_Tr.&P is developed for the case where we use both planes and trucks and the TCHH_Tr. is developed for the case where we use only trucks. Let us explain the necessity of these two models:

In Turkey, the furthest two demand points are Çanakkale and Iğdır and the time to travel between these two demand points is 1950 minutes (= 32.5 hours) by truck. This means, a cargo between these two points cannot be delivered in less than 32.5 hours by truck. In view of that, if we want to deliver cargo between any two demand points in less than 32.5 hours we have to use a plane. Therefore, we developed two models as presented in Chapter 4. The first model, TCHH_Tr.&P, is used for $T <$ 32.5 and the second model, TCHH_Tr., is used when $T \geq 32.5$.

As it can be seen in Figure 5.2, the optimal solution is obtained from the TCHH_Tr.&P. We test our model for different T values using these two models. In the following tables, we first present the solutions of the TCHH_Tr.&P and then, we will present the solutions of TCHH_Tr in Table 5.8.

In Table 5.4, for $p \leq 22$, optimal number of hubs, optimal number of hub airports, optimal values and CPU times in seconds for different T values are given. The first column denotes the time-bound, the second and third columns indicate the number of hub airports and locations, respectively.

Table 5.4. Results for $p \leq 22$ and different T values.

$p \leq 22$					
T	# of Hub Airports (except central airport)	Location of hub airports	# of Hubs	Opt. Value (*YTL*)	CPU Times (*sec.*)
32 – 25	1	Malatya	12	35,658	--
24	1	Diyarbakır	11	37,910	11627.4
23	1	Diyarbakır	10	38,101	13649.88
22	1	Erzurum	9	40,991	83908.06
21	2	Diyarbakır / Trabzon	10	42,281	847239.49
20	3	Diyarbakır / İzmir / Trabzon	13	45,626	228331.23

Since the model gives the same solutions for case $T= \{32, 31,…. , 25\}$, we give these solutions together in the first row in Table 5.4. In these cases, the CPU time does not differ a lot where the average CPU time is 1597.86 seconds.

When we decrease the time bound, the number of hub airports increases to deliver all cargo within the given time-bound. The number of hubs also increases but in case $T=24$, the number of hubs decreases because to deliver all cargo within 24 hours with minimum cost, the location of the hub airport changes. With this new location, the number of hubs, allocation of these hubs and cost also change and by operating eleven hubs and one hub airport all cargo can be delivered within 24 hours. Same situation is also valid for $T=23$ and $T=22$. It goes without argument that the proposed model provides a better solution than the current

implementation to deliver cargo within 24 hours. Moreover, it also provides a solution that delivers all cargo in Turkey by using 3 hub airports and 13 hubs within 20 hours.

When we analyze the CPU times, we see that it differs a lot from instance to instance. However, except the last five instances, the model can be solved in at most 45 minutes and for the cases $T=24$ and $T=23$, the model can be solved within 4 hours which is reasonable for such a data set size.

As we mentioned before, we test the model for different p values. As presented in Table 5.5 for $p \leq 22$, $p \leq 20$ and $p \leq 15$, our model opens the same number of hubs, hub airports and obtains the same optimal values because in each case the model can deliver all cargo within given time-bound at most using 13 hubs.

Table 5.5. Results for different p and T values.

T	Number of Hub Airports (except central airport)	Number of Hubs	Opt. Value (YTL)	CPU Times (sec.) $p \leq 22$	CPU Times (sec.) $p \leq 20$	CPU Times (sec.) $p \leq 15$
32	1	12	35,658	2554.12	712.75	1718.55
31	1	12	35,658	2486.94	913.63	1145.29
30	1	12	35,658	1480.46	2232.37	2389.81
29	1	12	35,658	695.90	8526.37	1135.28
28	1	12	35,658	1276.55	1537.36	1510.48
27	1	12	35,658	1805.69	3266.27	1037.59
26	1	12	35,658	1638.52	2507.20	742.97
25	1	12	35,746	844.73	751.04	1936.16
24	1	11	37,910	11627.4	46142.77	9209.34
23	1	10	38,101	13649.88	10628.06	3522.98
22	1	9	40,991	83908.06	85462.29	39142.28
21	2	10	42,281	847239.49	40969.59	97087.63
20	3	13	45,626	228331.23	154553.25	112872.45

Since the model solves the problem for $p \leq 15$, we reduce the number of hubs and we test our model that allows at most 10 hubs. The results are presented in the Table5.6. Since the model gives the same solutions for

case $T= \{32, 31,\ldots, 26\}$, we give this solution together in the first row in Table 5.6. In these cases, the CPU time does not differ a lot where the average CPU time is 2151.62 seconds.

In each case, except $T=22$, the number of operating hubs are the same in Table 5.6. In case $T = 22$, to deliver all cargo in the given time bound the location of the hub airport changes and with this new location, the number of hubs and allocation of these hubs also change and with operating nine hubs and one hub airport all cargo can be delivered within 22 hours. When we compare Table 5.5 and Table 5.6 for $T=23$, $T=22$ and $T=21$ we obtain the same solutions because the number of hubs is at most 10. We also test the model for $T<20$ and we get feasible solutions up to $T=13$. In other words, it is impossible to deliver all cargo within 13 hours (or less) by operating at most 10 hubs.

Table 5.6. Results for $p \le 10$

$p \le 10$					
T	Number of Hub Airports (except central airport)	Location of Hub Airports	Number of Hubs	Opt. Value (YTL)	CPU Times (sec.)
32-26	1	Malatya	10	35,872	--
25	1	Malatya	10	35,961	1012.65
24	1	Diyarbakır	10	38,013	40103.53
23	1	Diyarbakır	10	38,101	4264.09
22	1	Erzurum	9	40,991	84544.44
21	2	Diyarbakır / Trabzon	10	42,281	62842.26
20	2	Diyarbakır / Erzurum	10	46,865	223462.78

We continue to decrease the number of hubs and we solve the model for $p \le 5$. When we analyze Table 5.7., we see that, the number of operating hubs is fixed to 5 in each case and the number of hub airports increases. Different from Table 5.6, the second hub airport is opened at $T=23$ and

the third one is opened at *T*=20 when the model allows at most 5 hubs. We also test the model for *T*<20 but we cannot obtain feasible solutions. It is impossible to deliver all cargo within 19 hours (or less) by operating at most 5 hubs.

Table 5.7. Results for $p \leq 5$

$p \leq 5$					
T	# of Hub Airports (except central airport)	Location of Hub Airports	# of Hubs	Opt. Value (*YTL*)	CPU Times (*sec.*)
32	1	Diyarbakır	5	41,.208	686.35
31	1	Diyarbakır	5	41,208	299.52
30	1	Diyarbakır	5	41,208	394.49
29	1	Diyarbakır	5	41,208	350.77
28	1	Diyarbakır	5	41,340	836.10
27	1	Diyarbakır	5	41,344	644.62
26	1	Diyarbakır	5	41,633	1159.85
25	1	Diyarbakır	5	42,152	1057.82
24	1	Diyarbakır	5	42,434	677.72
23	2	Diyarbakır / Erzurum	5	48,351	3496.83
22	2	Diyarbakır / Erzurum	5	48,490	763.77
21	2	Diyarbakır / Erzurum	5	48,490	261.39
20	3	Diyarbakır / Erzurum / İzmir	5	54,230	31.69

Comparing the current implementation with the optimal solution in terms of hub airports and hubs, the total number of planes decrease from 11 to 1 and the total number of hubs decrease from 22 to 5 for *T*=24. Moreover, using 3 planes and 5 hubs, all cargo can be delivered within 20 hours.

Up to now, we present the solutions of the TCHH_Tr.&P. In Table5.8 we present the solutions of TCHH_Tr. for different *p* values.

Table 5.8. Results of the TCHH_Tr. for different p values

p	T	Opt. Value for TCHH_Tr.	Number of hubs	CPU Times (*sec.*)
$p \leq 22$	40	43,553	4	200.49
	35	43,553	4	194.87
$p \leq 20$	40	43,553	4	141.98
	35	43,553	4	159.71
$p \leq 15$	40	43,553	4	246.55
	35	43,553	4	258.6
$p \leq 10$	40	43,553	4	299.37
	35	43,553	4	295.95
$p \leq 5$	40	43,553	4	1445.36
	35	43,553	4	1601.42

We observe that in Table 5.8 the optimal values of the TCHH_Tr. are always greater than the optimal values of the TCHH_Tr.&P that are given in the previous tables. In view of that we also solve the models for $T \geq 32.5$ with the TCHH_Tr.&P and we get the following table.

Table 5.9. Results for $T \geq 32.5$ with the TCHH_Tr.&P

p	T	Opt. Value for TCHH_Tr.& P	Number of hub airport	Number of hubs	CPU Times (*sec.*)
$P \leq 22$	40	35,656	1	13	4054.67
	35	35,658	1	12	3004.61
$P \leq 20$	40	35,656	1	13	1840.81
	35	35,658	1	12	1179.59
$p \leq 15$	40	35,656	1	13	956.31
	35	35,658	1	12	3371.49
$p \leq 10$	40	35,872	1	10	1664.05
	35	35,872	1	10	3115.70
$p \leq 5$	40	41,208	1	5	562.37
	35	41,208	1	5	717.66

When we compare Table 5.8 and Table 5.9, we see that the number of hub in Table 5.9 is always greater than in Table 5.8, but this augmentation does not increase our optimal value because we do not have a fixed cost of operating hubs. We note that dispatching cargo by planes and trucks is always cheaper than dispatching only by trucks for $T \geq 32.5$.

Up to now we examined the number of hub airports, hubs and the optimal values by changing the maximum number of hubs, p and the time-bound, T. However we do not make any differentiation with cost parameters. To see the effects of changes in cost parameters we change the ratio of the cost parameters and we solve the system for fixed $T = 24$, $p = 22$ and different cost ratios as given in Table 5.10.

Table 5.10. Different cost ratios for fixed p and T value.

p	T	u / f_{hub}				
		3500	3250	3000	2750	2500
22	24	2250	2000	1750	1500	1400
		1250	1000	750	600	500

The results are presented in Table 5.11.

Table 5.11. Results for T=24, p$\leq$22 and different cost ratios

$T = 24$ & $p \leq 22$				
u / f_{hub}	# of Hub Airports (except central airport)	# of Hubs	Opt. Value (*YTL*)	CPU Times (*sec.*)
3500	1	11	37,910	11.627.4
3250	1	11	37,410	5349.86
3000	1	11	36,910	13552.91
2750	1	11	36,410	7682.22
2500	2	14	35,742	14527.77
2250	2	14	34,742	7191.55
2000	2	14	33,742	3059.87
1750	2	14	32,742	3134.24
1500	2	14	31,742	1098.29
1400	2	14	31,342	1598.54
1250	3	15	30,531	625.64
1000	3	15	29,031	281.77
750	4	15	27,370	110.53
600	4	15	26,170	107.27
500	4	15	25,370	38.47

In Table 5.11, the first column denotes the ratio of the cost of a plane for each flight to the cost of unit distance cost for each main truck, u / f_{hub}.

As we mentioned before, in the current system, the cost of unit distance cost for each main truck is taken as $f_{hub} = 1$ *YTL* per kilometer and the transportation cost of a plane is $u = 3,500$ *YTL* for each flight. We start with the current implementation ratio presented as in the first row and when we continue to decrease the cost ratio, the number of hubs and the number of hub airports increase. On the other hand, concurrently with this decrease the objective values also decrease. Table 5.11 also shows that the critical ratios for the change of the number of hub airports and gives the interval for the same number of hub airports. For instance, when the cost ratio is between 2500 and 1400, the number of hub airport equals to 2 and the solutions are the same, but when we increase the ratio to 2750 then the number of hub airport decreases from 2 to 1 or when we decrease the ratio to 1250 the number of hub airport increases from 2 to 3.

CHAPTER 6

CONCLUSION AND FUTURE REMARKS

In this thesis we focus on the cargo delivery companies in Turkey. We first analyze the structure of these cargo delivery firms. During this analysis, we identified a problem which is not satisfactorily modeled in the hub location literature. We analyze the problem and propose a model for it, *"Time constrained hierarchical hub location problem (TCHH)"*. TCHH is similar to the combination of p-hub median, hub covering, latest arrival hub location and intermodal freight transportation problems which cannot be seen in the literature.

In TCHH, two different types of transportation modes are considered, planes and trucks. Generally, Turkish companies use trucks but one of these companies, MNG Cargo, uses both trucks and planes and promises to deliver cargo in 24 hours between all origin-destination pairs. We focus on its structure and propose a mixed integer programming formulation to design a network where all packages are sent between origin and destinations with minimum cost using trucks or planes within the time bound. We give the linearizations of the non-linear constraints and after linearizations, the final linear model is tested with different parameters.

The mixed integer linear programs are solved to optimality with CPLEX 9.1. When we compare the current implementation with the optimal solution obtained for the TCHH in terms of hub airports, the total number of hub airports decreases from 12 to 3 for $p = 22$ and $T=24$. Subsequently, we test the model for $p \leq 22$ and the number of hubs and hub airports decrease to 11 and 1, respectively. We also test our model for different T and p values. According to our results cargo can be delivered using 10 hubs and 8 hub airports within 14 hours which is a significant improvement compared to the current implementation. Additionally, for $p \leq 5$ and $T=20$, cargo can be dispatched by operating 5 hubs and 3 hub airports. In a word, our results show that one can surely benefit from a better plan offered through mixed integer programming. We also test our model for different cost ratios and we examine the relations between the number of hub airports and cost ratios.

As a future research direction some additional constraints may be added into the model. For example, one may wish to consider the flow between origin destination pairs. In such a case, it is needed to add the capacity constraints. Concurrently with adding capacity constraints, it is needed to take into account the demand of the hubs and the hub airports. Since our model is uncapacitated model, we do not take into consideration the number of planes between airports and we assume that there exists only one plane between hub airports. However, when someone studies with the capacitated formulation then the number of planes between hub airports becomes important and there can be more than one plane between hub airports according to their demands. On the other hand, similar scenario is also valid for trucks. The number of trucks can be different between hubs according to the demand of these hubs. Of course, this may also require defining new decision variables and new upper bounds such as the number of trucks and the number of planes. On the

other hand, adding new constraints and new decision variables makes problem harder and this may require new modeling techniques.

Also, in our model we do not take into consideration the fixed cost of operating hub and we fixed the number of hubs to p. However, one may wish to consider the fixed cost of hubs. In such a case, it is needed to add a new term to the objective function and some constraints will be omitted from the model. By considering the cost of operating hubs, the number of hubs becomes important and the changes in the hub numbers affect optimal value.

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