ASSET PRICING IN A MULTIPERIOD SECURITIES MARKET WITH NONNEGATIVE WEALTH CONSTRAINTS

A Ph.D Dissertation

by

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July 2007

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The Institute of Economics and Social Sciences
of
Bilkent University

by

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In Partial Fulfilment of the Requirements for the Degree of

DOCTOR OF PHILOSOPHY

in

THE DEPARTMENT OF MANAGEMENT
BILKENT UNIVERSITY
ANKARA

July 2007

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Management.

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ABSTRACT

ASSET PRICING IN A MULTIPERIOD SECURITIES MARKET WITH NONNEGATIVE WEALTH CONSTRAINTS

Yakup Eser Arısoy

Ph.D. Dissertation in Management
Supervisor: Assoc.Prof. Dr. Aslıhan Altay-Salih

July 2007

According to Black-Scholes option pricing model, options are redundant securities, therefore have no importance for the allocation of wealth in the economy. This dissertation shows that options might be nonredundant when two factors are considered - nonnegative wealth and volatility risk. The first part of the dissertation empirically examines whether options are redundant securities or not in the context of volatility risk. It is documented that volatility risk, proxied by zero-beta at-the-money straddles, captures time variation in the stochastic discount factor. In relation to this, alternative explanations to size and value vs. growth anomalies are given. In the second part of the dissertation, a multiperiod securities market is considered, and a model where agents face nonnegative wealth constraints is developed.

Individuals' associated consumption-investment problem is solved under this constraint, and optimal sharing rules for each agent in the economy are derived, subsequently. The optimal consumption for the representative agent leads to a multifactor conditional C-CAPM, which is the main testable hypothesis of the theory. Overall the theory outlined, and the empirical findings documented have implications for asset pricing, portfolio management, and capital markets theories.

Keywords: Nonnegative wealth, option returns, C-CAPM, conditioning variable, volatility risk.

ÖZET

ÇOK PERİYODLU MENKUL KIYMET PİYASALARINDA EKSİ OLMAYAN SERVET KISITLARI İLE VARLIK FİYATLAMASI

Yakup Eser Arısoy

İşletme Doktora Tezi

Tez Yöneticisi: Doç. Dr. Aslıhan Altay-Salih

Temmuz 2007

Black-Scholes opsiyon fiyatlama modeline göre opsiyonlar atıl menkul kıymetlerdir, bu yüzden de ekonomideki servetin dağılımında bir rolleri yoktur. Bu tez eksi olmayan servet kısıtları ve oynaklık riski faktörleri altında opsiyonların atıl olmayabileceğini göstermektedir. Tezin ilk bölümü, opsiyonlarin atıl olup olmadığını oynaklık riski bağlamında ampirik olarak incelemektedir. Sıfır-betalı parada straddle ile temsil edilen oynaklık riskinin stokastik iskonto faktöründe zamansal değişiklikleri yakalayabildiği ortaya konmaktadır. Bununla bağlantılı olarak, firma büyüklüğü ve değer-büyüme anormalliklerine alternatif açıklamalar getirilmektedir. Tezin ikinci bölümünde, çok periyodlu menkul kıymet piyasaları ele alınmakta olup, acentaların eksi olmayan servet kısıtlarıyla karşı karşıya kaldığı bir model geliştirilmektedir. Bireylerin bununla bağlantılı olan tüketim-yatırım problemi çözülmekte, ve sonrasında ekonomideki her acenta için optimal paylaşım kuralları elde edilmektedir. Temsilci acentanın optimal tüketimi, aynı zamanda teorinin temel test edilebilir hipotezi olan çok faktörlü şartlı C-CAPM modeline varmaktadır. Toplamda, ortaya konan teori ve ampirik bulguların varlık fiyatlaması, portföy yönetimi ve sermaye piyasaları teorileri üzerinde etkileri bulunmaktadır.

Anahtar Kelimeler: Eksi olmayan servet, opsiyon getirileri, C-CAPM, şartlı değişken, oynaklık riski.

ACKNOWLEDGEMENTS

I would like to thank to my supervisor Assoc. Prof. Aslıhan Altay-Salih for her patience, guidance, and invaluable comments throughout my doctoral study. She has always been positive and caring when I needed advise, and guided me through difficult paths to a meaningful end. Her enthusiasm and devotion to her students, and academic discipline will always be an inspiration for me throughout my academic life.

I am also thankful to Prof. Joel Vanden, Assoc. Prof. Levent Akdeniz, Asst. Prof. Aydın Yüksel, and Prof. Mustafa Pınar for their valuable comments, and corrections on parts of this thesis.

My family's love and support was with me all the time. Without them, it would have been impossible to complete this thesis. My father Mehmet inspired me as a mathematics professor since my childhood. My mother Hatice was the best teacher I could have in my lifetime. And my sister Özden, hearing her cheerful voice on the phone was worth everything. I feel so lucky for having a family like them. I dedicate this thesis to you.

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CHAPTER 1

INTRODUCTION

This thesis consists of two inter-connected articles that examine option returns, and propose empirical and theoretical explanations for the nonredundancy and allocational role of options in the economy. The first article examines whether volatility risk is priced or not, by using a measure from the options market, i.e. zero-beta at-the-money straddle returns. The empirical results indicate that volatility risk is time varying, and straddle returns are important conditioning variables, i.e agents use straddle returns in forming their expectations about returns of securities. The article also provides alternative explanations to the size and value vs. growth anomalies. The second article proposes to solve individuals' consumption-investment problem with nonnegative wealth constraints in a multiperiod securities market, and subsequently derive optimal sharing rules for each agent in the economy. The derivation of optimal sharing rules in a rational expectations equilibrium yields a multifactor conditional consumption capital asset pricing model (C-CAPM), where the first factor is the change in log aggregate consumption, and the other factors are excess returns on a bundle of options written on the aggregate consumption. Overall, the results have important implications both for asset pricing and for the allocational role of options in the economy.

1.1 RELATED LITERATURE

There are four important lines of literature that sets the motivating ground behind this thesis. These are:

i) the inadequacy of single factor asset pricing models (why do CAPM and C-CAPM fail to explain asset prices although they have sound theoretical backgrounds?)

ii) the notion of market completeness (when do markets become complete and what are the possible frictions causing markets to become incomplete?)

iii) the allocational role of options in the economy (why do we observe so massive trading volumes in the options market if they are redundant and have no allocational role in the economy?)

iv) the implications of nonnegative wealth constraints (what are the equilibrium consequences of nonnegative wealth constraints regarding the agents' consumption-investment problem?).

The following four subsections go over the major articles that have received recognition in their own categories, present their impact on the finance literature, and relate them to this thesis study.

1.1.1 INADEQUACY OF SINGLE FACTOR MODELS

Capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965) and Mossin (1966) have undergone a long way since its celebrated years in mid-sixties and seventies. The power and popularity of CAPM stem from its parsimony and elegance. By determining an asset's return with a single factor, namely its covariance with the market return, the so-called beta, it was theoretically possible to price all traded assets. This simple but powerful model has received its more-than-deserved attention in the academia, and it has been the most popular tool for both theoreticians and practitioners compared to any other tool in the finance literature.

After the publication of Sharpe, Lintner, and Mossin articles, there was a wave of papers seeking to relax the strong assumptions that underpin the original CAPM. The most frequently cited modification is by Black (1972), who shows how the model needs to be adapted when riskless borrowing is not available; which is known as the zero-beta CAPM. Another important variant is by Brennan (1970), who finds that the structure of the original CAPM is retained when taxes are introduced into the equilibrium. Mayers (1972) shows that when the market portfolio includes non-traded assets, the model also remains identical in structure to the original CAPM. The model can also be extended to encompass international investing, as in Solnik (1974) and Black (1974). The theoretical validity of the CAPM has even been shown to be relatively robust if the assumption of homogenous return expectations is relaxed, as in Williams (1977). All these studies have increased the confidence regarding the explanatory power of CAPM, or its versions.

On the empirical side, the situation was similar. Until mid-seventies the cross-sectional tests initiated by Black, Jensen and Scholes (1972), and further tests by Fama and Macbeth (1973), and Blume and Friend (1973) have not rejected zero-beta CAPM (although rejecting the original CAPM due to the significant error term). In contrast to these confirmatory studies, the first important criticism to CAPM was put forward by Roll (1977). Previous tests of the CAPM examine the relationship between equity returns and beta measured relative to an equity market index such as the S&P500. However, Roll demonstrates that the market, as defined in the theoretical CAPM, is not a single equity market, but an index of all wealth. The market index must include bonds, property, foreign assets, human capital and etc., tangible or intangible that adds to the wealth of agents in the economy. Furthermore, Roll shows that tests of CAPM are wrong since we never observe this true market index with certainty. Thus, Roll argues that tests of the CAPM are at best tests of the mean-variance efficiency of the portfolio that is taken as the market proxy. But, since within any sample, there will always be a portfolio that is mean-variance efficient; finding evidence against the efficiency of a given portfolio tells us nothing about whether or not the CAPM is correct.

After this theoretical criticism, came a series of anomalies that have further weakened the ground for CAPM. Now there is a vast amount of empirical evidence that CAPM is unable to explain the cross section of expected returns. Banz (1981) and Reinganum (1981) show that small-sized firms earned higher returns and big-sized firms earned lower returns than the CAPM actually predicts. Rosenberg, Reid, and Lanstein (1984) document that the value portfolios (high book-to-market firms) tend to outperform growth portfolios (low book-to-market firms), which contradicted with CAPM predictions. Basu (1977) find that price-earnings ratios can explain a better proportion of variation in securities return than the beta of a security. Finally, Fama and French (1992) show that size, and book-to market ratios are superior to CAPM's beta in explaining the cross-sectional variation in securities returns.

In the meantime, new research was pouring in from the dynamic asset pricing literature. A key assumption in the original CAPM is that agents make decisions for only one time period. This is an unrealistic assumption since investors can and actually do rebalance their portfolios on a regular basis. The first work that questioned this limitation was the seminal work by

Merton (1973), which is today known as intertemporal CAPM (I-CAPM). One of Merton's key results is that the static CAPM does not in general hold in a dynamic setting. In particular, Merton demonstrates that an agent's welfare at any point in time is not only a function of his own wealth, but also the state of the economy. If the economy is doing well then the agent's welfare will be greater than if it is doing badly, even if the level of wealth is the same. Thus the demand for risky assets will be made up not only of the mean-variance component, as in the static portfolio optimization problem of Markowitz (1952), but also of a demand to hedge adverse shocks to the investment opportunity set. The upshot is that CAPM will still hold at each point in time, but there will be multiple betas, the number of betas being equal to one plus the number of state variables that drive the investment opportunity set through time. Although a major breakthrough, Merton’s analysis was at the same time disconcerting, because it runs counter to the basic intuition of the CAPM that an asset has greater value if its marginal contribution to wealth is greater. The reply to this problem was the consumption CAPM.

Breeden and Litzenberger (1978), and Breeden (1979) reconciled the gap between Merton's I-CAPM, and the classical CAPM by highlighting the dichotomy between wealth and consumption. In an intertemporal setting, Breeden and Litzenberger show that agents’ preferences must be defined over consumption. The implication is that assets are valued by their marginal contribution to the future consumption, not wealth. The model which became known as C-CAPM allows assets to be priced with a single beta as in the traditional CAPM. However, in contrast to the latter, the C-CAPM's beta is measured not with respect to aggregate market wealth, but with respect to an aggregate consumption flow. As Breeden states, *"the higher that an asset's beta with respect to consumption is, the higher its equilibrium expected rate of return"*. C-CAPM has been regarded as superior to the classical CAPM, since an asset's covariance with the marginal utility of consumption as a measure of systematic risk is theoretically more sound than other definitions of risk. Also, CAPM and its extensions can almost always be expressed as either special cases of, or proxies for, the consumption-based model. Moreover, the consumption-based framework is a simple but powerful tool for addressing the criticisms of Merton (1973), that the static-CAPM fails to account for the intertemporal hedging component of asset demand, and Roll (1977), that the market return cannot be adequately proxied by an index of common stocks.

However, empirical tests of C-CAPM have proven to be disappointing. The consumption-based model has been rejected for the U.S. data in its representative agent formulation with time-separable power utility [Hansen and Singleton (1982, 1983)]. Furthermore, it has performed no better and often worse than the simple static-CAPM in explaining the cross section of average asset returns [Mankiw and Shapiro (1986), Breeden, Gibbons, and Litzenberger (1989), Campbell (1996), Cochrane (1996), Hodrick, Ng and Sengmueller (1999)]. However, recently, Lettau and

Ludvigson (2001) use a scaling variable, $c\widehat{a}y_t$, as a proxy of the log of consumption-wealth ratio, and find that conditional versions of C-CAPM with this conditioning variable performs much better than alternative pricing models.

Finally, there is the arbitrage pricing theory (APT), which is an attempt to resolve the inadequacies in single factor models. It is no surprise that Ross (1976) has developed his model almost at the same time with Roll's critique, and the first reported anomalies. While still retaining the core idea of CAPM (covariance of an asset's return with a number of factors are the determinant of the long term average return of that security), the major contribution of the model is its allowance to multiple factors in pricing of securities. Although the choice of appropriate factors still being debated, and there is no clear-cut methodology to which factors should be included, the model's strength comes from the broadness of its assumptions and its testability. Once you define theoretically appropriate factors that affect an asset's return systematically, then APT is relatively superior to classical single factor models.

1.1.2 ARE MARKETS (IN)COMPLETE?

Financial economics deals with agents' decision making (i.e. optimal portfolio choice, optimal consumption-investment choice) under uncertainty.

The general framework of decision making under uncertainty has been established by the seminal works of Arrow (1951, 1953), Debreu (1951), and Arrow and Debreu (1954). These studies have chosen to model uncertainty as the revelation of a state of the world. Individuals in these models face investment and consumption decisions based on payoffs of securities that vary across different states of the world. The basic building blocks of the state-preference theory are the time-event state-contingent claims. A time-event state-contingent claim is a contract that promises to deliver to the holder of that contract a particular commodity when a particular state occurs at a particular time, and delivers nothing at any other state and/or time. Agents maximize their utilities over these time-event state-contingent claims.

The Arrow-Debreu framework has two versions: a state-contingent claims model, and a securities market model. The notion of market completeness refers to having a complete set of state-contingent claim markets in the first version (i.e. each state-contingent claim can be priced in these markets at the beginning of the trade); and number of linearly independent securities being equal to the number of states in the second version. In complete markets, the resulting equilibrium is such that each agent purchases a set of future state-contingent commodities in the initial time period, and then just watches the future states and events unfold.

Since Arrow-Debreu equilibrium conditions are oversimplified versions of the general uncertainty in the economy, following research focused on the relaxation of several assumptions of the Arrow-Debreu economy. The result was a sequential equilibrium framework, where three different versions emerged: temporary equilibrium by Stigum (1969), and Grandmont and Hildenbrand (1974); Radner equilibrium by Radner (1972), and rational expectations equilibrium by Lucas (1972), and Green (1973). In the classical Arrow-Debreu equilibrium, once we have complete markets there is no need for markets to reopen. All trading takes place at the initial period, and then there is no need for trading in subsequent periods. All the three equilibria mentioned above have the same common characteristic, i.e. spot markets and securities markets are open in the sequences following the initial period. The essential distinctions between these theories lie in the form of expectations assumed, i.e. temporary equilibrium does not require perfect foresight, or information-consistency across agents; Radner equilibrium requires perfect foresight, but not consistency; and rational expectations equilibrium require information-consistent expectations. The framework in the second article will be a rational expectations framework with incomplete markets. So it is of importance to further discuss these two concepts and relate them to the setting of this thesis.

A rational expectations equilibrium can be thought of as the special case of the Radner equilibrium when the probabilities assigned by all agents about future states are the same across agents. More specifically, agents are assumed to form "information-consistent" probability assignments. As long as agents have the same information about the future of the economy, then their probability assignments will be the same. In the second article, it is assumed that all agents share a common information structure, $F = \{F_t, t = 0,1,\ldots,T\}$, where each F_t is a partition of the state space Ω. Thus, as information is revealed at each period, each agent knows what state she is in, and forms the same probabilistic assignments with other agents regarding the future possibilities of events.

The basic idea behind incomplete markets is the possibility of having a sequential economy where there are an insufficient number of financial securities. Specifically, markets are said to be incomplete if the number of linearly independent securities is strictly less than the number of possible future states. Although a general Radner equilibrium can still be attained in incomplete markets, the allocation among agents is no more Pareto efficient. The markets in the second article are incomplete given the total number of traded securities, and the consumption patterns of agents. However, once options are introduced, markets are effectively completed and an efficient allocation among agents is achieved. This relates the issue to the allocational role of options in incomplete markets, which is the subject of next section.

1.1.3 ALLOCATIONAL ROLE OF OPTIONS

According to Bank of International Settlements, the size of derivatives markets in 2002 was estimated to exceed $109 trillion in outstanding contracts, and over $400 trillion in trading volume on derivatives exchanges.

Today, the daily trading volumes on currency exchanges are on average $3.5 trillion dollars, much ahead of the spot market transaction volumes. What do these numbers mean about the allocational role of options in the economy? If options are redundant securities as implied by Black-Scholes assumptions, why do we observe huge amounts of options trading in the economy. The answer can not solely relay on hedging purposes or speculation. Today, many researchers question the redundancy of options, and there is a growing amount of literature on the spanning role of options.

The elegant option pricing theory developed by Black and Scholes (1973) relies on a simple rule; the replication of an option's payoff with that of a risky asset and a riskless asset. This no arbitrage condition implies that options are redundant securities, and have no allocational role in the economy. The first study that shows how standard call and put options can be used to complete a securities market goes back to the seminal work of Ross (1976). Ross shows that when the markets are incomplete, one can construct options with prespecified strike prices to span the state space. In the same spirit with this study, Breeden and Litzenberger (1978) show that constructing options whose strike prices coincide with every possible level of aggregate wealth are sufficient to characterize the prices of Arrow-Debreu securities.

The idea of market completion by options has been carried further to a multiperiod setting by Kreps (1982), and Duffie and Huang (1985). They show that markets can be dynamically completed by repeated trading of long-lived securities. This implies that the number of long-lived securities needed to complete markets is far fewer than the total number of states, i.e. it is just equal to the number of branches leaving each node on the event tree representing the information structure. Thus by dynamically trading long-lived securities markets can be completed, and a Pareto optimal allocation can be achieved.

The above theoretical research had significant impact on option pricing literature. Although standard Black-Scholes option pricing model is still widely used in practice, research today has shifted from assuming complete markets to examining the settings of why and how markets become incomplete, focus on the allocational consequences of market incompleteness, and develop alternative option pricing models. The following sub-sections analyze different settings that can cause markets to be incomplete, and summarize recent findings in these settings, correspondingly.

1.1.3.1 Heterogeneous Beliefs

This line of research argues that heterogeneous attitudes towards risk can generate demand for options. For example, Leland (1980) shows that in an economy with terminal consumption only, convex final payoffs such as options will be demanded by more risk-tolerant agents. Grosmann and Zhou (1996) show that if one of the agents, such as a portfolio insurer, is infinitely averse to the risk when his wealth drops below a given threshold, than the demand for options can be an important determinant of the underlying asset price. Bates (2001) considers an economy where crashes can occur and less crash-tolerant investors buy options from more crash-tolerant ones. In his setting, options complete the market by serving as a hedge against crash risk. Buraschi and Jiltsov (2003) consider a symmetric but incomplete information setting; agents agree on the dividend process but differ in their beliefs about the price process unrelated to fundamentals. They find that much of the observed option trading volume can be explained by this heterogeneity in beliefs.

1.1.3.2 Asymmetric Information

Asymmetric information about the dividend process can induce traders with private information to hold options in equilibrium. A number of studies suggest that option may be non-redundant because the price of a traded option can convey some information, which otherwise would be unobservable in the economy. Grossman (1988) argues that an option may appear to be redundant, however it can be nonredundant due to its informational content, thus its removal from the economy would make markets incomplete. Back (1993) shows that the introduction of option trading into a market with asymmetric information may change the stochastic process the underlying asset follows. Hence, options introduced to a complete market may become non-redundant. Also Easley, O'Hara, and Srinivas (1998) suggest that an option market could be a platform for informed trading due to lower transaction costs and greater financial leverage.

1.1.3.3 Stochastic Volatility and Jumps

Presence of stochastic volatility and jumps can severely affect asset prices and thus options that are written on them. The main approach to modeling stock returns is defining a continuous time stochastic volatility diffusion process possibly augmented with an independent jump process in returns. Today, most option pricing models incorporate these two factors in order to account for a more realistic pricing process. It was first Heston (1993) who proposed a stochastic volatility diffusion model, for which one could analytically derive an option pricing formula. Duffie and Kan (1996), and Duffie, Pan, and Singleton (2000) further developed Heston's model to a rich class of affine jump diffusion processes. Several other authors have used stochastic volatility diffusion process augmented by jumps [Bates (1996) Andersen, Benzoni and Lund (2001), Eraker, Johannes and Polson (2001), Pan (2002), Chernov, Gallant, Ghysels and Tauchen (2003)]. Bakshi, Cao, and Chen (1997) compare empirical performances of these alternative option pricing models with respect to three criteria; internal consistency of implied volatility with relevant time series data, out-of-sample pricing errors, and hedging performance. Overall, models that include stochastic volatility and jump processes perform the best.

Besides these theoretical models, recently, a number of empirical papers have demonstrated that options are not redundant. Buraschi and Jackwerth (2001) test whether the pricing kernel of the economy can be spanned by stock and bonds or whether additional securities are required. Their results suggest that option returns do significantly increase the spanning quality of the pricing kernel and that the volatility risk is priced. Coval and Shumway (2001) give preliminary evidence that returns on zero-beta at-the-money straddles can explain a significant amount of S&P 100 index returns, and argue that at-the-money straddles can account for the systematic volatility risk in the securities market. Bakshi and Kapadia (2003) show that delta-hedged option portfolios consistently earn negative returns, indicating that there exists a negative volatility risk premium in option prices, which is consistent with the nonredundancy of options. Liu and Pan (2003) argue that, in the existence of volatility and jump risks, a market consisting of a riskless bond and a risky asset is not enough to replicate the possible payoffs resulting from those risks, thus the markets are strongly incomplete. They show that at-the-money straddles and out-of-money puts can be used to complete the markets and derive optimal demands for those options in a partial equilibrium framework.

1.1.3.4 Market Frictions

The standard asset pricing and option pricing theories assume that markets are frictionless, i.e. no transaction costs, no limitations on short sales, or borrowing. However, real-life practice seldom approves these cases. The presence of transaction costs, and portfolio constraints such as constraints on short selling, or credit constraints such as nonnegative wealth constraints can generate demand for options, and options can have important allocational roles due to those frictions in the economy.

Regarding the transaction costs, Lee and Yi (2001) test whether greater leverage and lower trading costs make options more attractive to informed traders, and if the relative lack of anonymity in options markets discourages large investors from trading options. They find that the adverse selection component of the bid-ask spread decreases with option delta, implying that options with greater financial leverage attract more informed investors. Kaul, Nimalendran, and Zhang (2002) examine the relation between adverse selection in the underlying stock and spreads on options of different strike prices. Their main finding is that adverse selection costs are highest for at-the-money options. The authors argue that this result is consistent with the trade-off between high leverage and transaction costs. In Basak and Croitoru (2000), a mispricing between a stock and a redundant derivative arises due to portfolio constraints on short selling and investors with heterogeneous beliefs. The degree of mispricing and optimal derivative portfolio holdings becomes non-trivial in their generalized equilibrium framework. Vanden (2004) examine the effect of nonnegative wealth constraints in a single period economy, and in equilibrium agents hold options thus options become nonredundant. The markets are strongly incomplete given the traded options, but options help agents achieve a Pareto efficient allocation, and in equilibrium, options effectively complete the market. Since, in equilibrium, agents agree on the value of all stochastic payoffs, Vanden's findings have important consequences for asset pricing. This is because the payoffs from existing securities (a positive probability of bankruptcy) in addition to the short selling possibilities can lead agents reach negative levels of wealth. Hence, by imposing nonnegative wealth constraints agents are guaranteed to come back to the economy with the ability to repay their debt. The economic intuition and related literature regarding nonnegative wealth is the subject of the next sub-section.

1.1.4 NONNEGATIVE WEALTH CONSTRAINTS

As noted in the previous sub-sections, frictionless markets assumption breaks down in real life practices. There may be some constraints on wealth (or borrowing limits), which can practically affect individuals' optimal consumption-saving choices. For example, constraints like bounded credit by

Dybvig and Huang (1988), or nonnegative wealth by Vanden (2004) might force individuals to alter their unconstrained optimal solutions, which can result in certain payoffs that cannot be replicated by the existing financial instruments.

The analysis of nonnegative wealth constraints and their implications on individual's consumption-investment decision and option pricing goes back to Harrsion and Kreps (1979). In their pioneering work, in a continuous time setting, it is demonstrated that doubling strategies (which refers to one's doubling her bet at a roulette game) can earn arbitrage profits in a finite time interval. Since the core of investment-consumption decision and option pricing rests on the no-arbitrage condition, the existence of doubling strategies, thus arbitrage opportunities, precludes having a solution to the optimal investment-consumption problem, and obviously invalidates the option pricing theory. Harrison and Kreps conjecture that arbitrage possibilities are ruled out if trading strategies are restricted to those having nonnegative wealth at all times. Dybvig and Huang (1988) generalize their work in assuming a lower bound on wealth. This assumption is economically plausible since there are institutional restrictions on the amount of credit an individual can borrow. They show that any lower bound on wealth rules out doubling strategies, and any other strategies that generate a free lunch.

The effect of nonnegative wealth constraint on individual's optimal consumption-investment problem has been studied by Cox and Huang (1989), Grosmann and Vila (1989), and Merton (1990); and recently in an equilibrium framework by Vanden (2004). Although the previous studies examine the problem by considering a single individual's consumption-investment decision framework, the results derived by Vanden assume that all agents simultaneously face nonnegative wealth constraints. The results have important allocational implications regarding the individuals' optimal consumption-investment decisions.

Overall, the above literature can be summarized as follows:

i. Single factor models of asset pricing fail to explain the cross sectional variation in securities returns.

ii. Markets are incomplete due to several real life frictions and options can be used effectively to complete markets, making them non-redundant securities.

iii. An asset pricing model that takes into account theoretical weaknesses in i and ii, is theoretically more sound, and resembles reality better.

This thesis combines the above asset pricing literature, and examines two asset pricing models that are theoretically sound, and empirically testable. The first article proposes a single factor conditional CAPM where straddle returns are used as a conditioning variable, and the second article proposes a multifactor conditional C-CAPM where option returns appear as factors. To the best of our knowledge, the first article is the first study that uses straddle returns in the context of volatility risk, and the second article is the first study to use option returns in a conditional C-CAPM framework.

Overall the tested models provide some supportive evidence for the nonredundancy, and allocational role of options in the economy.

CHAPTER 2

IS VOLATILITY RISK PRICED IN THE SECURITIES MARKET? EVIDENCE FROM S&P 500 INDEX OPTIONS

2.1 INTRODUCTION AND LITERATURE REVIEW

The notion that equity returns exhibit stochastic volatility is well documented in the asset pricing literature.[1] Furthermore, recent evidence indicates the existence of a negative volatility risk premium in the options market [Lamoureux and Lastrapes (1993), Buraschi and Jackwerth (2001), Coval and Shumway, (2001), Bakshi and Kapadia (2003)]. However, the existence of volatility risk in the securities market and its impact on different

[1] See Engle and Ng (1993), Canina and Figlewski (1993), Duffee (1995), Braun, Nelson, and Sunier (1995), Andersen (1996), Bollerslev and Mikkelsen (1999), and Bekaert and Wu (2000).

classes of firms has not been extensively documented. Recently, Coval and Shumway (2001) examines the return characteristics of S&P 100 index straddles and gives preliminary evidence that volatility risk may be a common risk factor in securities markets - a finding that contradicts the classical CAPM.

CAPM suggests that the only common risk factor relevant to the pricing of any asset is its covariance with the market portfolio; thus an asset's beta is the appropriate quantity for measuring the risk of any asset. However, Vanden (2004) shows that when agents face nonnegative wealth constraints, cross sectional variation in securities returns is not explained only by an asset's beta. Instead, excess returns on the traded index options and on the market portfolio explain this variation; implying that options are nonredundant securities. Furthermore, as Detemple and Selden (1991) suggest, if options in the economy are non-redundant securities, then there should be a general interaction between the returns of risky assets and the returns of options. This implies that option returns should help explain security returns.

This article extends the preceding studies and presents evidence that straddle returns are important for asset pricing since they help capture time variation in the stochastic discount factor. The findings suggest that volatility risk is time-varying and that options are nonredundant securities at volatile states of the economy. This has important implications regarding the allocational role of options in the economy. The preliminary time-series regressions, Fama-MacBeth regressions, and GMM-SDF estimations in this article confirm the theory that options are effective tools in pricing securities and allocating wealth among agents as suggested by Vanden (2004). This article also examines the effect of volatility risk in pricing different classes of firms, i.e. small vs. big and value vs. growth, and finds distinct patterns in the returns of these firms, especially at volatile states of the economy.

Asset pricing theories thus far have been unable to provide a satisfactory economic explanation for the size and value vs. growth anomalies.[2] In a rational markets framework, we would expect these abnormal returns to be temporary. Once investors realize arbitrage opportunities, the abnormal profits of small and value stocks are expected to vanish. However, this has not been the case. The persistence of these two anomalies has led to extensive research and has yielded two alternative lines of explanations within the rational markets paradigm.

One line, led by Fama and French (1992, 1993, 1995), argues that a stock's beta is not the only risk factor. This approach suggests that fundamental additional variables such as book-to-market and market value explain equity returns much better, because they are proxies for some unidentified risk factors. However, the weakness of this explanation lies in its failure to address the economic variables underlying these factors. The

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[2] Banz (1981) and Reinganum (1981) document that portfolios formed on small sized firms earn returns higher than the CAPM predicts. Rosenberg, Reid and Leinstein (1985) find that firms with high book-to-market ratios (value firms) earn higher returns than firms with low book-to-market ratios (growth firms). Davis, Fama, and French (2000) report that the value premium in U.S. stocks is robust.

other line of research within the risk-return framework argues that it is the time variation in betas and the market risk premium that cause the static CAPM to fail to explain these anomalies. There is now considerable evidence that conditional versions of CAPM perform much better than their unconditional counterparts.[3]

This article re-examines these two important asset pricing anomalies with an important but somewhat overlooked factor, the volatility risk. There is now a considerable amount of evidence that volatility risk is priced in the options market. First, Jackwerth and Rubinstein (1996) report that at-the-money implied volatilities of call and put options are consistently higher than their realized volatilities, suggesting that a negative volatility premium could be an explanation to this empirical irregularity. Furthermore, Coval and Shumway (2001) report that zero-beta at-the-money straddles on the S&P 100 index earn returns consistently lower than the risk free rate, suggesting the presence of a negative volatility risk premium in the prices of options. As an extension of this study, Driessen and Maenhout (2005) report that volatility risk is also priced in FTSE and Nikkei index options. Finally, Bakshi and Kapadia (2003) show that delta-hedged option portfolios consistently earn negative returns and conclude that there exists a negative volatility risk premium in option prices.

[3] See Ferson (989), Ferson and Harvey (1991), Ferson and Korajczyk (1995), Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Altay-Salih, Akdeniz, and Caner (2003) for the theory behind time-varying beta and conditional CAPM literature.

Although the above evidence indicates that volatility risk is priced in options markets, we are less confident that it is priced in securities markets. Recent studies find that volatility risk can explain the cross-section of expected returns. For example, Moise (2005) uses innovations in the realized stock market volatility, and demonstrate that volatility risk helps explain some of the size anomaly. Furthermore, by using changes in the volatility index (VIX) of Chicago Board Options Exchange (CBOE), Ang, Hodrick, Xing, and Zhang (2006) demonstrate that aggregate volatility is a cross-sectional risk factor. In this study, a measure from the options market, i.e. straddle returns on the S&P 500 index, is used as a proxy for volatility risk. The reason behind using straddle returns is intuitive. As Detemple and Selden (1991) argue, if options are non-redundant securities in the economy, then their returns should appear as factors in explaining the cross section of asset returns. Furthermore, Vanden (2004) reports that returns of call and put options indeed explain a significant amount of variation in securities return, but fail to explain the returns for small and value stocks. The failure of Vanden's model could be due to omitting an important risk factor - the volatility risk. Furthermore, straddles are volatility trades, and they provide insurance against significant downward moves.[4] Thus, overall, straddle returns are ideal for studying the effects of volatility risk in security returns.

[4] This is because increased market volatility coincides with downward market moves, a phenomenon which is reported by French, Schwert, and Stambaugh (1987), and Glosten, Jagannathan, and Runkle (1993). Engle and Ng (1993) show that volatility is more associated with downward market moves due to the leverage effect.

The remainder of this article is organized as follows. First, data and the methodology for calculating straddle returns are presented. Econometric issues in the estimation of the volatility risk premium are discussed in the next section. This is followed by empirical results. The final section offers concluding remarks.

2.2 DATA AND METHODOLOGY

The data consist of two parts - S&P 500 options data and stock return data - covering the period January 1987 through October 1994.[5] Daily S&P 500 options data is obtained from the Chicago Board Options Exchange and consists of daily closing prices of call and put options, the daily closing level of the S&P 500 index, the maturities and strike prices for each option, the dividend yield on the S&P 500 index, and the one-month T-bill rate. For option volatilities, the closing level of CBOE's S&P 500 VIX index is used. For market portfolio, CRSP's value weighted index on all NYSE, AMEX and NASDAQ stocks are used. The return data on size and book-to-market portfolios are obtained from Kenneth French's data library.

The method for calculating daily option returns is as follows. First, options that significantly violate arbitrage-pricing bounds are eliminated. Then, options that expire during the following calendar month are identified.

[5] We are grateful to Ramazan Gencay for providing the data.

This roughly coincides with options that have 14 to 50 days to expiry in our sample. The reason for choosing options that expire the next calendar month is that they are the most liquid data among various maturities.[6] Options that expire within 14 days are excluded from the sample, because they show large deviations in trading volumes, which casts doubt on the reliability of their pricing associated with increased volatility.[7] Next, each option is checked whether it is traded the next trading day or not. If no option is found in the nearest expiry contracts, then options in the second-nearest expiry contracts are used. To calculate the daily return of an option, raw net returns are used. The usage of raw net returns is justified by Coval and Shumway (2001) who argue that log-scaling of option returns can be quite problematic.

Once daily call and put returns are calculated, they are grouped according to their moneyness levels. Although there is no standard procedure for classifying at-the-money options, options with a moneyness level (S-K) between -5 and +5 are classified as at-the-money options. This classification also guarantees that there are at least two options around the spot price. One reason for focusing on zero-beta at-the-money straddles was to capture the effect of volatility risk, as mentioned previously. Another advantage of studying at-the-money options is that they are less prone to pricing errors compared to deep-out-of money options, as cited in option

[6] According to Buraschi and Jackwerth (2001), most of the trading activity in S&P500 options is concentrated in the nearest (0-30 days to expiry) and second nearest (30-60 days to expiry) contracts.
[7] Stoll and Whaley (1987) report abnormal trading volumes for options close to expiry.

pricing literature.[8] Using the above procedure results in 1937 days of return data out of 1980 trading days.

The straddle returns are calculated according to the methodology outlined by Coval and Shumway (2001). In order to capture the effect of volatility risk, zero-beta at-the-money straddle returns on the S&P 500 index are used. The advantage of using S&P 500 index options is that they are highly liquid, thus they are less prone to microstructure and illiquid trading effects. Zero-beta straddles are formed by solving for θ from the following set of equations,

$$r_v = \theta r_c + (1-\theta) r_p \quad (1)$$

$$\theta \beta_c + (1-\theta)\beta_p = 0 \quad (2)$$

where r_v is the straddle return, r_c and r_p are the call and put returns, θ is the fraction of the straddle's value in call options, and β_c and β_p are the market betas of the call and put options, respectively. It is straightforward to calculate returns on call and put options; however, to calculate the return of a straddle, the value of θ is needed, which depends on β_c and β_p. By using the put-call parity theorem, Equation (2) can be reduced into a single unknown, β_c, and the value of θ is derived as follows

[8] Macbeth and Merville (1979) report that the Black-Scholes prices of at-the-money call options are on average less than market prices for in-the-money call options. Also, Gencay and Salih (2001) document that pricing errors are larger in the deeper-out-of-money options compared to at-the-money options.

$$\theta = \frac{-C\beta_c + s}{P\beta_c - C\beta_c + s} \qquad (3)$$

where C is price of the call option, P is price of the put option, and s is the level of the S&P 500 index.

The only parameter that is not directly observable in the above equation is the call option's beta, β_c. We use Black-Scholes' beta, which is defined as

$$\beta_c = \frac{s}{C} N\left[\frac{\ln(s/X) + (r - q + \sigma^2/2)t}{\sigma\sqrt{t}}\right]\beta_s \qquad (4)$$

where $N[.]$ is the cumulative normal distribution, X is the exercise price of call option, r is the risk-free short term interest rate, q is the dividend yield for S&P 500 assets, σ is the standard deviation of S&P 500 returns, and t is the option's time to maturity.

The methodology to calculate zero-beta at-the-money straddle returns is as follows. First, an option's beta is calculated according to Equation (4). Then, θ is derived by incorporating the previously calculated call and put option returns into Equation (3). Finally, straddle returns for each day are calculated according to Equation (1). The daily zero-beta straddle return is then simply the equally-weighted average of at-the money-straddle returns that are found in the final step.

Table 2.1 reports the summary statistics for the daily S&P 500 (SPX) straddle returns. The average daily S&P 500 straddle return is -1.06 % with a minimum return of –87.77% and maximum of 441.79%. The mean and median of the daily zero-beta straddle returns are negative as documented by the earlier literature. Note that call option betas are instantaneous betas, and therefore the straddles are zero-beta at the construction. However, we calculate the zero-beta straddle returns by using daily buy and hold returns. Thus, they are zero-beta instantaneously and their betas might change during the holding period. This might be the possible explanation of negative correlation of -0.54 between the straddle and market returns.[9]

TABLE 2.1
Summary Statistics for Daily Zero-Beta Straddles

Daily Straddle Returns (%)	
Mean	-1.06
Median	-1.58
Minimum	-87.77
Maximum	441.79
Skewness	17.03
Kurtosis	520.03
Correlation	-0.54

Note. This table reports the summary statistics for the returns of daily zero-beta at-the money straddles. The sample covers the period January 1987 to October 1994 (1980 days). After adjusting for moneyness and maturity criteria, we end up with 1937 days of data. Correlation is the correlation of straddle returns with market returns.

2.3 ECONOMETRIC SPECIFICATIONS

In order to test the main hypothesis that volatility risk - proxied by zero-beta at-the-money straddle returns - is priced in securities returns, we

[9] To check the robustness of the results, we set the theoretical position beta in Equation (2) to a constant such that the in-sample straddle beta is exactly zero. Negative mean and median volatility risk premium still persists and furthermore conclusions from time series regressions do not change.

first regress the excess returns of size and book-to-market portfolios on excess straddle returns and on the market factor.[10] The empirical model to be tested is

$$r_{it} - r_{ft} = \alpha_i + \sum_j \beta_{ij} \left(r_{jt} - r_{ft} \right) + \varepsilon_{it} \qquad (5)$$

where r_{it}'s are realized returns of size and book-to-market portfolios, and r_{jt}'s are the returns of factors that are included in the regressions.

The above analysis relies on monthly holding period returns, both because microstructure effects tend to distort daily returns, and to rule out non-synchronous trading effects that could be present in daily data. In order to calculate monthly at-the-money straddle returns, an equally weighted portfolio of at-the-money straddles is formed for each day and then each day's return is cumulated to find monthly holding period returns. This adds up to 94 monthly straddle returns, which are used as an independent variable in the preceding time-series regressions. Although these regressions are not formal tests of whether volatility risk is priced or not, they nevertheless give clues about the potential explanatory power of straddle returns in explaining the cross-section of expected returns.

Next the question of whether volatility risk is a priced risk factor is examined by performing Fama-MacBeth two-pass regressions by using the 25 size and book-to-market portfolios.[11] The model to be tested is

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[10] Vanden (2004) uses a similar model, where he includes call and put option returns and a market factor as explanatory factors.
[11] The returns on 25 portfolios are obtained from Kenneth French's data library.

$$E[r_{it}] = \alpha_i + \beta' \lambda \ . \tag{6}$$

More specifically, in the first pass, portfolio betas are estimated from a single multiple time-series regression via Equation (5). Instead of using the 5-year rolling-window approach, a full sample period is used.[12] In the second pass, a cross-sectional regression is run at each time period, with full-sample betas obtained from the first pass regressions, i.e.

$$E[r_{it}] = \alpha_{it} + \beta_{ij}' \lambda_{jt} \text{, i = 1, 2, …, } N \text{ for each } t. \tag{7}$$

Fama and MacBeth (1973) suggests that we estimate the intercept term and risk premia, α_i and λ_j's, as the average of cross-sectional regression estimates

$$\hat{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{\alpha}_{it} \text{, and } \hat{\lambda}_j = \frac{1}{T}\sum_{t=1}^{T}\hat{\lambda}_{jt} \ .$$

One problem with the Fama-MacBeth procedure is that it ignores the errors-in-variables problem that results from the fact that in the second pass, beta estimates instead of the true betas are used. In order to avoid this problem, a Generalized Method of Moments (GMM) approach within the stochastic discount factor (SDF) representation is employed. The advantage of a GMM approach is that it allows the estimation of model parameters in a single pass, thereby avoiding the errors-in-variables problem. The advantage of the SDF representation relative to the beta representation is that it is

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12 This approach is advocated for data having fewer than 150 time series observations.

extremely general in its assumptions and can be applied to all asset classes, including stocks, bonds, and derivatives. Cochrane (2001) demonstrates that both representations express the same point, but from slightly different viewpoints. However, the SDF view is more general, it encompasses virtually all other commonly known asset pricing models. Ross (1976) and Harrison and Kreps (1979) state that in the absence of arbitrage and when financial markets satisfy the law of one price, there exists a stochastic discount factor, or pricing kernel, m_{t+1}, such that the following equation holds

$$E[R_{it+1}m_{t+1}] = 1, \qquad (8)$$

where R_{it+1} is the gross return (one plus the net return) on any traded asset i, from period t to period $t+1$. We denote this as the unconditional SDF model.

Because considerable evidence exists to suggest that expected excess returns are time-varying, the above unconditional specification may be too restrictive. Thus, to answer the question of whether or not there exists time-variation in the volatility risk premium, both unconditional and conditional models of asset pricing are tested. The conditional SDF model is denoted as

$$E_t[R_{it+1}m_{t+1}] = 1 \qquad (9)$$

where E_t denotes the mathematical expectation operator conditional on the information available at time t.

Following Jagannathan and Wang (1996), we consider a linear factor pricing model with observable factors, f_t. Then, m_{t+1} can be represented as

$$m_{t+1} = a_t + b_t' f_{t+1} \qquad (10)$$

where a_t, and b_t are time-varying parameters. Note that, when a_t, and b_t are constants, we obtain the unconditional version of linear factor models.

The question here is how one can incorporate the information that investors use when they determine expected returns in Equations (9) and (10). Because the investors' true information set is unobservable, one has to find observable variables to proxy for that information set. Cochrane (1996) shows that conditional asset pricing models can be tested via a conditioning time t information variable, z_t. One way of incorporating conditioning variable, z_t, into the model is to scale factor returns, as discussed in Cochrane (2001); and used in Cochrane (1996), Hodrick and Zhang (2001), and Lettau and Ludvigson (2001b). This is done by scaling the factors with z_t, thus modeling the parameters a_t, and b_t as linear functions of z_t as follows

$$a_t = \gamma_0 + \gamma_1 z_t \qquad (11)$$

$$b_t = \eta_0 + \eta_1 z_t \qquad (12)$$

Plugging these equations into Equation (10), and assuming that we have a single factor, we have a scaled multifactor model with constant coefficients taking the form

$$\begin{aligned} m_{t+1} &= (\gamma_o + \gamma_1 z_t) + (\eta_0 + \eta_1 z_t) f_{t+1} \\ &= \gamma_0 + \gamma_1 z_t + \eta_o f_{t+1} + \eta_1 z_t f_{t+1} \qquad (13) \end{aligned}$$

The scaled multifactor model can be tested by rewriting the conditional factor model in Equation (9), as an unconditional factor model with constant coefficients $\gamma_0, \gamma_1, \eta_0$, and η_1 as follows,

$$E\left[R_{it+1}\left(\gamma_0 + \gamma_1 z_t + \eta_o f_{t+1} + \eta_1 z_t f_{t+1}\right)\right] = 1 \qquad (14)$$

In the next section, empirical results of OLS time-series regressions (Equation 5), Fama-MacBeth regressions (Equation 6), and the GMM-SDF estimations (Equation 8) are presented.

2.4 EMPIRICAL FINDINGS

2.4.1 TIME SERIES REGRESSIONS

Coval and Shumway (CS; 2001) argue that zero-beta at-the-money straddles can proxy for volatility risk, which can in turn explain the variation in the cross-section of equity returns. Usually, highly volatile periods are associated with significant downward market moves. Furthermore, index straddles earn positive (negative) returns in times of high (low) volatility, as can be seen by the negative correlation between the straddle and market returns in Table 2.1. CS also argue that volatility risk is a possible explanation for the well-known size anomaly among securities returns. For a preliminary investigation of those two hypotheses, we use a two-factor model, and regress excess returns of CRSP's size deciles on the excess returns of CRSP's value-weighted index on all NYSE, AMEX, and NASDAQ stocks and the excess returns of zero-beta at-the-money straddles. Table 2.2 presents the results of these regressions.

TABLE 2.2

2-Factor Time Series Regressions

$$r_{it} - r_{ft} = \alpha_i + \beta_{im} (r_{mt} - r_{ft}) + \beta_{iv} (r_{vt} - r_{ft}) + \varepsilon_{it}$$

$r_{it} - r_f$	α_i	t-statistic	β_{im}	t-statistic	β_{iv}	t-statistic	Adj. R^2
Small 10	-0.0024	-0.61	0.7555	6.91***	-0.0109	-4.55***	0.64
Decile 9	-0.0039	-1.23	0.9612	11.37***	-0.0080	-4.29***	0.78
Decile 8	-0.0004	-0.18	1.0106	13.69***	-0.0063	-3.98***	0.84
Decile 7	-0.0017	-0.70	1.0612	14.86***	-0.0052	-3.33***	0.86
Decile 6	0.0009	0.40	1.0553	14.83***	-0.0040	-2.74***	0.88
Decile 5	0.0009	0.51	1.0337	20.91***	-0.0031	-3.02***	0.92
Decile 4	0.0004	0.37	1.0343	27.10***	-0.0024	-2.31**	0.95
Decile 3	0.0007	0.60	1.0917	27.76***	0.0003	0.36	0.96
Decile 2	0.0004	0.55	1.0801	34.26***	0.0019	2.67***	0.98
Big 1	0.0006	0.56	0.9953	32.97***	0.0024	2.99***	0.96

GRS F-Test = 2.3314 (p=0. 0179)

Note. This table reports monthly time-series regression results of excess returns of CRSP's size deciles on market factor and excess straddle returns. The dependent variable is the excess return of CRSP's size-decile portfolio, r_{mt} is the return of CRSP's value-weighted index on all NYSE, AMEX, and NASDAQ stocks, , r_{vt}, is the monthly zero-beta straddle return, and r_f is the 1-month T-bill rate. ***, **, * denote 0.01, 0.05, and 0.10 significance levels, respectively. All t-values are corrected for autocorrelation (with lag=3) and heteroskedasticity as suggested by Newey and West (1987). GRS F-Test reported at the bottom of the table is from Gibbons, Ross, and Shanken (1989).

As can be seen from the table, there exists a statistically significant relationship between straddle returns and securities returns in 9 of the 10 size deciles. Thus, straddle returns and therefore volatility risk could be a significant variable in explaining securities returns. In their recent studies, Moise (2005) and Ang et al. (2006) also document statistically significant negative price of risk for aggregate volatility. In our case, the economic interpretation of this negative volatility risk premium could be that buyers of zero-beta at-the-money straddles are willing to pay a premium for downside market risk. If investors are assumed to be averse to downward market moves, the existence of a negative volatility risk premium would be justified, because downward moves are associated with high volatility periods. Following Vanden's theoretical framework, this would imply that straddles are effective tools in completing the market, because they help investors avoid insolvency and negative wealth levels, during high volatility periods.

A more interesting finding, which also confirms CS's predictions, is the significant pattern observed in the coefficients of straddle returns. The coefficients of straddle returns monotonically increase from the smallest size decile to the largest. This finding, if persistent, can be a potential explanation for the widely known size anomaly. Because stocks with small market capitalizations are the ones that are affected most by highly volatile states of the economy, the volatility coefficients of smaller decile firms are expected to be lower than larger decile firms; i.e., they are associated with more negative volatility risk premia.[13] Moreover, the coefficients of the largest size decile turn out to be significantly positive, suggesting that investors see large firms as hedges against volatility. This finding suggests that, during volatile periods, large firms tend to protect their investors better than small firms.

The explanatory power of the regressions is relatively high with adjusted R^2's ranging from 0.64 to 0.98. Furthermore, none of the intercept terms are significantly different form zero according to the *t*-statistics. However, the Gibson, Ross and Shanken (GRS; 1989) F-test rejects the hypothesis that all the intercepts are jointly equal to zero at the 5% level. Overall, the above results favor the explanation that volatility risk might be a potential priced factor among securities returns.

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[13] This finding is in line with Moise (2005).

Next, the relevance of the volatility risk factor on different classes of firms is examined. To do this, 25 portfolios formed on size and book-to-market are used. One advantage of using this broader portfolio set is to see the robustness of the above results across book-to-market portfolios, as well.

Table 2.3 documents the time-series regression results for the 25 portfolios. As can be seen, straddle returns still explain the variation in the returns of 21 out of 25 portfolios formed according to size and book-to-market. Consistent with the previous results, small size portfolios (the lowest three size quintiles) have statistically significant negative coefficients for most of the book-to-market levels (14 out of 15 portfolios). Although, the intercept term α_i is not statistically significant for 23 of the portfolios, the GRS-*F* test rejects the hypothesis that intercepts are jointly equal to zero. This result is consistent with Vanden (2004) and Coval and Shumway (2001).

Looking across book-to-market portfolios, it is seen that high book-to-market (value) stocks consistently have significant and negative coefficients in the smallest four size quintiles and low book-to-market (growth) stocks have significant and positive coefficients in the biggest size quintile. The positive and significant coefficients for the big-growth portfolios are interesting. This result might indicate that among the big firms, investors see only growth firms as potential hedges against volatile states of the economy. This, in turn, can be a possible explanation for the value vs. growth anomaly.

TABLE 2.3

25 (5x5) Portfolio Regressions

$$r_{it} - r_{ft} = \alpha_i + \beta_{im} (r_{mt} - r_{ft}) + \beta_{iv} (r_{vt} - r_{ft}) + \varepsilon_{it}$$

Size	B/M	α_i	t-statistic	β_{im}	t-statistic	β_{iv}	t-statistic	Adj. R^2
S	L	-0.0115	-2.67^{***}	1.0271	9.51^{***}	-0.0100	-3.89^{***}	0.70
S	2	-0.0018	-0.48	0.9158	9.06^{***}	-0.0098	-4.33^{***}	0.70
S	3	-0.0012	-0.37	0.8589	9.63^{***}	-0.0085	-4.53^{***}	0.76
S	4	0.0011	0.36	0.7602	8.20^{***}	-0.0105	-4.91^{***}	0.72
S	H	0.0018	0.41	0.7808	8.08^{***}	-0.0105	-4.82^{***}	0.65
2	L	-0.0052	-1.67^{*}	1.2560	14.09^{***}	-0.0041	-1.95^{**}	0.81
2	2	-0.0014	-0.47	1.0796	14.50^{***}	-0.0067	-4.16^{***}	0.82
2	3	0.0026	1.05	0.8742	11.08^{***}	-0.0080	-5.19^{***}	0.84
2	4	0.0011	0.49	0.7999	12.43^{***}	-0.0080	-5.70^{***}	0.82
2	H	0.0012	0.35	0.9861	10.79^{***}	-0.0062	-2.90^{***}	0.77
3	L	-0.0013	-0.45	1.2517	18.22^{***}	-0.0014	-0.83	0.83
3	2	0.0010	0.45	1.0854	16.14^{***}	-0.0045	-2.96^{***}	0.88
3	3	-0.0001	-0.07	0.8722	13.27^{***}	-0.0047	-3.03^{***}	0.86
3	4	0.0024	1.07	0.8723	13.77^{***}	-0.0033	-2.27^{**}	0.85
3	H	0.0027	0.98	0.9250	15.26^{***}	-0.0062	-4.08^{***}	0.82
4	L	0.0013	0.73	1.1890	26.99^{***}	0.0013	1.13	0.89
4	2	-0.0006	-0.35	1.0294	25.29^{***}	-0.0050	-4.69^{***}	0.93
4	3	-0.0011	-0.53	1.0834	13.95^{***}	-0.0005	-0.27	0.90
4	4	0.0023	1.35	0.9081	15.45^{***}	0.0022	1.81^{*}	0.89
4	H	0.0027	1.11	0.9264	12.16^{***}	-0.0038	-2.19^{**}	0.82
B	L	0.0012	0.52	1.1202	24.26^{***}	0.0037	3.39^{***}	0.88
B	2	0.0002	0.10	1.1129	24.46^{***}	0.0027	2.55^{**}	0.92
B	3	0.0005	0.25	0.8575	17.83^{***}	-0.0025	-2.54^{**}	0.87
B	4	0.0004	0.26	0.9113	24.29^{***}	0.0043	2.91^{***}	0.83
B	H	0.0027	0.79	0.9354	14.67^{***}	0.0008	0.38	0.70
GRS F-Test = 2.7293 (p=0. 0071)								

Note. This table reports monthly time-series regression results of excess returns of CRSP's 25 size and book-to-market portfolios on market factor and excess straddle returns The returns on 25 portfolios formed on size and book-to-market equity are obtained from Kenneth French's data library. The 25 portfolios constructed at the end of each June, are the intersections of 5 portfolios formed on size (market equity, ME) and 5 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoints for year t are the NYSE market equity quintiles at the end of June of t. BE/ME for June of year t is the book equity for the last fiscal year end in t-1 divided by ME for December of t-1. The BE/ME breakpoints are NYSE quintiles. S and B stands for the smallest and biggest size quintiles; L and H stands for the lowest and highest book-to-market quintiles. r_{it} is the dependent variable which denotes the return on each of the 25 portfolios from January 1987-October 1994. r_{mt} is the return of CRSP's value-weighted index on all NYSE, AMEX, and NASDAQ stocks, r_{vt} is the monthly zero beta straddle return, and r_f is the 1-month T-bill rate obtained from Ibbotson and Associates. ***, **, * denote 0.01, 0.05, and 0.10 significance levels, respectively. All t-values are corrected for autocorrelation (with lag=3) and heteroskedasticity as suggested by Newey and West (1987). GRS F-Test reported at the bottom of the table is from Gibbons, Ross, and Shanken (1989).

To further check the robustness of this explanation, the sample is refined to 6 portfolios based on size and book-to-market. As can be seen from Table 2.4, small-sized firms still have negative and significant coefficients consistent with the previous documented results. Furthermore, among big firm portfolios it is only the growth portfolio, which exhibits a positive and significant volatility risk coefficient. These consistent results indicate that the volatility risk could not only explain the size anomaly but also the value vs. growth anomaly. When formed according to size, it is clearly seen that small firms are more prone to volatility risk, whereas big firms are seen as hedges against this kind of risk. However a detailed analysis reveals that it is actually the growth portfolios among big firms that provide a hedge against volatility risk.

TABLE 2.4
6 (2x3) Portfolio Regressions

$$r_{it} - r_{ft} = \alpha_i + \beta_{im} (r_{mt} - r_{ft}) + \beta_{iv} (r_{vt} - r_{ft}) + \varepsilon_{it}$$

Size	B/M	α_i	t-statistic	β_{im}	t-statistic	β_{iv}	t-statistic	Adj. R^2
S	L	-0.0046	-1.57	1.1557	14.74***	-0.0058	-3.30***	0.83
S	2	0.0056	2.52**	0.8997	13.06***	-0.0066	-4.43***	0.86
S	H	0.0059	2.07**	0.8642	11.58***	-0.0076	-4.52***	0.80
B	L	0.0053	3.24***	1.1287	35.50***	0.0027	3.54***	0.94
B	2	0.0047	4.21***	0.9329	32.55***	0.0010	1.60	0.94
B	H	0.0056	2.97***	0.8659	25.07***	-0.0002	-0.15	0.86
GRS F-Test = 2.3260 (p=0. 0178)								

Note. This table reports monthly time-series regression results of excess returns of CRSP's 6 size and book-to-market portfolios on market factor and excess straddle returns. Portfolios are constructed at the end of each June, which are the intersections of 2 portfolios formed on size (market equity, ME) and 3 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoint for year t is the median NYSE market equity at the end of June of year t. BE/ME for June of year t is the book equity for the last fiscal year end in t-1 divided by ME for December of t-1. The BE/ME breakpoints are the 30th and 70th NYSE percentiles. S and B stands for the smallest and biggest size quintiles; L and H stands for the lowest and highest book-to-market quintiles. r_{it} is the dependent variable which denotes the monthly return on each of the 6 portfolios from January 1987-October 1994. r_{mt} is the monthly return of CRSP's value-weighted index on all NYSE and AMEX stocks, r_{vt} is the monthly zero beta straddle return, and r_f is the 1-month T-bill rate obtained from Ibbotson and Associates. ***, ** , * denote 0.01, 0.05, and 0.10 significance levels, respectively. All t-values are corrected for autocorrelation (with lag=3) and heteroskedasticity as suggested by Newey and West (1987). GRS F-Test reported at the bottom of the table is from Gibbons, Ross, and Shanken (1989).

2.4.2 IS VOLATILTY RISK PRICED?

Up to now, the documented evidence suggests that straddle returns are useful explanatory variables over the sample period studied, but we can not conclude whether volatility risk is priced in security returns or not. In an attempt to answer this question, Fama-MacBeth two-pass regressions are performed and Panel A of Table 2.5 reports the results of these tests for the conditional and unconditional versions of various CAPM specifications. More specifically, risk premia estimated according to Equation (6), their associated Shanken-corrected and uncorrected *t*-statistics, and adjusted R^2 statistics for the cross-sectional regressions are shown.

The first row of Panel A in Table 2.5 presents results for the traditional unconditional CAPM taking the form

$$E[r_{it}] = \alpha_i + \lambda_m \beta_i^m .$$

The statistically insignificant *t*-statistic for the market risk premium shows the inability of the value-weighted market beta to explain the cross-section of average returns. Moreover, the negative sign of the market risk premium contradicts the CAPM theory. These findings are also supported by the very low explanatory power for the model. The results are in line with the Fama and French (1992) findings.

TABLE 2.5

Evaluation of Various CAPM Specifications using 25 Fama-French Portfolios

Panel A: Risk premium estimates using two-pass Fama-MacBeth regressions

ROW	α_i	λ_m	λ_{st}	λ_{SMB}	λ_{HML}	λ_{scaled}	Adj. R^2
1	1.4486	-0.7850					
	(2.17**)	(-0.96)					0.03
	(2.16**)	(-0.95)					
2	1.4274	-0.7254	23.4020				
	(2.16**)	(-0.92)	(0.79)				0.32
	(2.15**)	(-0.91)	(0.78)				
3	0.7525	-0.0643		-0.1794	0.2110		
	(1.81*)	(-0.10)		(-0.68)	(0.83)		0.44
	(1.80*)	(-0.10)		(-0.67)	(0.82)		
4	1.6442	-1.1322	37.8143			-5.6965	
	(2.43**)	(-1.42)	(1.20)			(-2.37**)	0.42
	(2.34**)	(-1.32)	(1.11)			(-2.21**)	
5	1.2121	-0.6912	15.4201	-0.1077	0.2964	-6.0019	
	(3.05***)	(-1.17)	(0.71)	(-0.41)	(1.17)	(-2.37**)	0.52
	(2.94***)	(-1.08)	(0.66)	(-0.38)	(1.08)	(-2.20**)	

Panel B: Stochastic Discount Factor (SDF) estimates using GMM

	δ_0	δ_m	δ_{st}	δ_{SMB}	δ_{HML}	δ_{scaled}	HJ-dist.	HJ-dist. identity
6	0.9179	5.8378					1.0445	0.0121
	(8.55***)	(2.13**)					(0.00)	(0.00)
7	0.9288	5.9155	0.0765				1.0440	0.0116
	(8.59***)	(1.49)	(0.44)				(0.00)	(0.01)
8	0.9108	6.2797		0.6760	-1.0204		1.0438	0.0112
	(8.06***)	(1.92*)		(0.15)	(-0.20)		(0.00)	(0.00)
9	0.9390	6.2940	0.0845			0.3772	1.0155	0.0100
	(8.63***)	(1.38)	(0.42)			(1.82*)	(0.00)	(0.11)
10	0.9435	6.0327	0.0857	0.2176	-0.7585	0.3794	1.0153	0.0096
	(8.35***)	(1.23)	(0.40)	(0.04)	(-0.15)	(2.15**)	(0.00)	(0.13)

Note. This table gives the estimates for the cross-sectional Fama-MacBeth regression model

$$E[r_{it}] = \alpha_i + \lambda_{st}\beta_i^{st} + \lambda_m \beta_i^m + \lambda_{SMB}\beta_i^{SMB} + \lambda_{HML}\beta_i^{HML} + \lambda_{scaled}\beta_i^{scaled}$$

and the model for the moments

$$E\left[(1+r_{it})\left(\delta_0 + \delta_{st} r_t^{st} + \delta_m r_t^m + \delta_{SMB} r_t^{SMB} + \delta_{HML} r_t^{HML} + \delta_{scaled} r_t^{scaled}\right)\right] = 1$$

with either a subset or all of the variables. Panel A reports the individual risk-premium, λ_j, estimates from the second-pass cross-sectional regressions. In the first stage, the time-series betas are computed in one multiple regression of the portfolio of excess returns on the factors. The term r_{it} is the return on 25 Fama-French portfolios (i=1,2,…,25) in month t (January 1987-October 1994). The numbers in parentheses are the two t-statistics for each coefficient estimate. The top statistic uses uncorrected Fama-MacBeth standard errors; the bottom statistic uses Shanken (1992) correction. The term adjusted R^2 denotes the cross-sectional R^2 statistic adjusted for the degrees of freedom. Panel B reports GMM estimates for various SDF representations and their associated t- and p-values. The model for the moments are estimated using the GMM approach with the Hansen-Jagannathan weighting matrix. r_t^{st} is the straddle return, r_t^m is the return on the value-weighted index of all NYSE, AMEX, and NASDAQ stocks, r_t^{SMB}, and r_t^{HML} are the returns on Fama-French mimicking portfolios related to size and book-to-equity ratios, and r_t^{scaled} is the return of the scaled variable, i.e. $r_t^{st} \cdot r_{t+1}^m$. The numbers in parentheses are the t-statistics for each coefficient estimate. ***, **, * denote 0.01, 0.05, and 0.10 significance levels, respectively. The minimized value of the GMM criterion function is the first item under the "HJ-dist.", with the associated p-values immediately below it. The final column reports HJ-dist. using the identity matrix as suggested by Lettau and Ludvigson (2001b).

Next, we test the significance of volatility risk as a priced factor with the following model

$$E[r_{it}] = \alpha_i + \lambda_m \beta_i^m + \lambda_{st} \beta_i^{st}.$$

Row 2 of Panel A shows that adding straddle betas significantly contributes to the explanatory power of the two-factor model. The adjusted R^2 increases dramatically from 3 percent to 32 percent. Although the volatility risk premium is positive, the insignificant *t*-statistic shows that it is not a priced risk factor. This result needs further exploration, as it contradicts the previous findings of significant volatility betas in time-series regressions.

One explanation for this contradiction could be the time variation inherent in the volatility risk premium and the inadequacy of the unconditional models to capture this time variation. The literature on time-varying risk premia argues that conditional versions of factor models better explain this time variation than their unconditional counterparts. Hence, a natural extension is to perform the preceding analysis with conditional factor models.

2.4.1.1 Conditional Factor Models

Cochrane (1996, 2001) argues that conditional factor models can be represented in an unconditional form by using appropriate scaling variables. We posit that investors use time *t* straddle returns when forming their expectations about time *t+1* returns. For the conditional model with one factor (market return) and one scaling variable (straddle return), the scaled market factor would take the form, $r_t^{st} \cdot r_{t+1}^{m}$, and the model would be

$$E[r_{it}] = \alpha_i + \lambda_m \beta_i^m + \lambda_{st} \beta_i^{st} + \lambda_{scaled} \beta_i^{scaled}$$

where β_i^{scaled} is the beta of the scaled market factor. Row 4 of Table 2.5 reports the estimated coefficients of the proposed conditional model. The estimated risk premia for straddle and market returns are still not statistically significant; however, the coefficient of the scaled market beta is negative and statistically significant at the 5% level. The explanatory power of the model also improves from an R^2 of 0.32 to 0.42.

Besides the statistical significance of the scaled factor in the conditional model, we examine the effect of a one standard deviation change in the estimated betas on average returns of various portfolios. This is done to see the sensitivity of average portfolio returns to changes in betas that are estimated in the first-pass. For example, taking the big-growth portfolio, a one standard deviation increase in the beta of the scaled factor causes a 0.19% decrease in the average return of the portfolio. The effect of a one standard deviation increase in the market beta results in a decrease of 0.03% in the average return, whereas a one standard deviation increase in straddle beta increases the average return of the big-growth portfolio by 1.25%. However, one need to be careful while interpreting the risk-premia associated with the scaled returns. Lettau and Ludvigson (2001b) argue that individual risk-premium estimates for the scaled multifactor model should not be interpreted as risk prices as in unconditional models. Cochrane (2001) note that scaled returns act as payoffs to managed portfolios, thus in incomplete market settings state contingencies can be provided through trading strategies using conditioning information. The significance of the scaled market factor in the conditional model indicate that investors use straddle returns in forming their expectations about the future prices of securities. This also supports the non-redundancy of options hypothesis by Vanden (2004). Overall, these results suggest that there exist time variation in the volatility risk premium and that the scaled market return is an important factor for asset pricing.

Lettau and Ludvigson (2001b) show that conditional versions of CAPM perform much better than the unconditional models, using the log consumption-wealth ratio as a conditioning variable. They document that these models perform about as well as the Fama-French three-factor model. In our case, Row 4 of Table 2.5 demonstrates that the conditional CAPM, using straddle returns as a conditioning variable, performs slightly worse than the Fama-French three factor model, where none of the risk premia is statistically significant. Furthermore, we test whether or not the addition of Fama-French factors can explain the cross-section of expected returns not explained by our model. The model to be tested is

$$E[r_{it}] = \alpha_i + \lambda_{st}\beta_i^{st} + \lambda_m\beta_i^m + \lambda_{SMB}\beta_i^{SMB} + \lambda_{HML}\beta_i^{HML} + \lambda_{scaled}\beta_i^{scaled}$$

where scaling is done in a similar manner as in the one factor model. Row 5 of Table 2.5 reports the results of this estimation. Although the explanatory power of the model increases to an R^2 of 52%, the coefficients of the Fama-French factors are still insignificant. The only significant risk premium is that of the scaled market factor. This confirms that the conditional model using straddle returns as a scaling variable is successful in explaining the cross-section of average returns.

2.4.1.2 GMM-SDF Tests

Because the Fama-MacBeth regressions is criticized for having errors-in-variables problem, we also examine whether the volatility risk is priced or not by using a GMM framework in various SDF representations. Panel B of Table 2.5 reports the estimates of SDF coefficients and their associated *t*-statistics, p-values, and Hansen-Jagannathan distances (HJ-dist.).[14] The first model to be tested is the unconditional CAPM, i.e.,

$$E\left[R_{it}\left(\delta_0 + \delta_m r_t^m\right)\right] = 1$$

where R_{it} is the gross return of 25 Fama-French portfolios and r_t^m is the return on the value-weighted index of all NYSE, AMEX, and NASDAQ stocks. Row 6 of Panel B presents the results of this estimation. Contrary to the previous findings, the unconditional CAPM yields a statistically significant coefficient

[14] For a detailed discussion on the calculation of HJ-dist., see Jagannathan and Wang (1996).

for the market factor. However, the estimated HJ-dist. shows that the pricing error is very high, and significantly different from zero, suggesting that this model is a poor SDF representation.

Next we test whether straddle returns are a part of the stochastic discount factor or not. This gives the following SDF specification

$$E\left[R_{it}\left(\delta_0 + \delta_m r_t^m + \delta_{st} r_t^{st}\right)\right] = 1.$$

Row 7 shows that, including straddle returns in the unconditional model results in slightly lower pricing errors. However, the insignificant coefficient for straddle returns suggests that volatility risk does not play a significant role in constructing a stochastic discount factor in the unconditional form. This result is consistent with the previous Fama-MacBeth results. Next, we test whether the Fama-French factors are significant explanatory variables by the following SDF representation

$$E\left[R_{it}\left(\delta_0 + \delta_m r_t^m + \delta_{SMB} r_t^{SMB} + \delta_{HML} r_t^{HML}\right)\right] = 1.$$

As can be seen in Row 8, the coefficients are still insignificant and the pricing errors are slightly better than that of the traditional CAPM. Row 9 of Panel B presents the results for the conditional CAPM using straddle returns as the conditioning variable. The model to be tested is

$$E\left[R_{it}\left(\delta_0 + \delta_m r_t^m + \delta_{st} r_t^{st} + \delta_{scaled} r_t^{scaled}\right)\right] = 1,$$

where r_t^{scaled} is calculated as before. The statistically significant coefficient for the conditioning variable suggests that this variable plays an important role in constructing a stochastic discount factor. This finding is consistent with our previous results and also confirms that there exists time variation in the volatility risk premium. However, although the pricing error is considerably lower, it is still significantly different from zero. Due to the small-sample problems with GMM estimation, it is not surprising to obtain large HJ-distances that are statistically different from zero. Altonji and Segal (1996), Cochrane (2001), and Lettau and Ludvigson (2001) suggest that using GMM estimates with the identity matrix is far more robust to small-sample problems. The last column of Panel B reports estimates of Hansen-Jagannathan distances using the identity matrix. Note that, HJ-distances estimated with the identity matrix, and therefore pricing errors decrease drastically for all the models. However, only for the conditional models (Rows 9 and 10) are the pricing errors not significantly different from zero. Furthermore, the addition of Fama-French factors to the conditional model does not considerably improve the explanatory power of the model, as reported in Row 10.

Consistent with the earlier findings from Fama-MacBeth regressions, conditional models using straddle returns as a scaling variable perform better than unconditional models examined in this study. Besides this statistical significance, in order to check the economic significance of the results, we examined the impact on the SDF of a one standard deviation change in factor returns. For example, for the conditional model in Row 9 in Table 2.5, a one standard deviation increase in scaled factor returns corresponds to a 0.15 standard deviation increase in the SDF. The effect of a one standard deviation increase in straddle returns is 0.47 standard deviation increase in the SDF, and a one standard deviation increase in market returns cause a 1.22 standard deviation increase in the SDF. As for the economic interpretation of the scaled returns, we can think of them as payoffs to managed portfolios as in Cochrane (2001). For example, an investor who observes high zero-beta straddle returns is expected to decrease her holdings in the market portfolio. Our findings confirm that investors use straddle returns as a conditioning variable when forming their expectations of securities returns. Thus, they are important for asset pricing since they help capture the time variation in the SDF.

2.4.2 EFFECT OF THE 1987 CRASH

The effect of time variation in the volatility risk premium on asset returns can be tested by the threshold regression methodology. We applied the sup-LM test used in Hansen (1996) to explore the question of whether there are statistically significant discrete regime shifts in the risk factors due to certain instrumental variables. VIX Volatility of at-the-money options and the difference between volatilities of at-the-money and out-of-money options are used as instrumental variables, but no significant regime shifts are detected. However, the bootstrap p-values are likely to be poorly estimated in samples of the size encountered here.

Nevertheless, in an attempt to explore the possible effects of a high volatility periods on our results, the sample is divided into two sub-samples, one including the crash period and one excluding it.

TABLE 2.6

10 Size Regressions With and Without 1987 Crash

$$r_{it} - r_{ft} = \alpha_i + \beta_{im}(r_{mt} - r_{ft}) + \beta_{iv}(r_{vt} - r_{ft}) + \varepsilon_{it}$$

$r_{it} - r_f$	α_i	t-statistic	β_{im}	t-statistic	β_{iv}	t-statistic	Adj. R^2
January 1987 - November 1990							
Small 10	-0.0111	-2.54^{**}	0.7806	9.64^{***}	-0.0097	-5.73^{***}	0.85
Decile 9	-0.0099	-2.37^{**}	0.9141	13.64^{***}	-0.0085	-6.23^{***}	0.90
Decile 8	-0.0039	-1.12	0.9848	13.55^{***}	-0.0066	-4.44^{***}	0.91
Decile 7	-0.0062	-1.63	1.0383	13.85^{***}	-0.0055	-3.35^{***}	0.90
Decile 6	-0.0038	-1.24	1.0139	12.72^{***}	-0.0047	-2.93^{***}	0.92
Decile 5	-0.0029	-1.16	1.0052	16.74^{***}	-0.0035	-2.85^{***}	0.94
Decile 4	-0.0003	-0.16	1.0172	24.49^{***}	-0.0029	-2.60^{**}	0.96
Decile 3	-0.0014	-0.79	1.0868	20.06^{***}	0.0004	0.29	0.97
Decile 2	-0.0009	-0.69	1.0770	26.37^{***}	0.0019	2.09^{**}	0.98
Big 1	0.0024	1.51	1.0035	28.42^{***}	0.0025	2.67^{**}	0.97
GRS F-Test = 2.3249 (p=0. 0183)							
December 1990- October 1994							
$r_{it} - r_f$	α_i	t-statistic	β_{im}	t-statistic	β_{iv}	t-statistic	Adj. R^2
Small 10	0.0080	1.58	0.7413	2.31^{**}	-0.0043	-0.25	0.24
Decile 9	0.0030	0.71	1.0906	4.36^{***}	-0.0009	-0.05	0.53
Decile 8	0.0047	1.37	1.1021	6.00^{***}	0.0027	0.22	0.65
Decile 7	0.0058	1.97^{*}	1.1727	8.24^{***}	0.0099	1.06	0.74
Decile 6	0.0087	2.63^{**}	1.2120	11.02^{***}	0.0131	1.04	0.80
Decile 5	0.0062	2.45^{**}	1.1301	15.73^{***}	0.0057	0.54	0.85
Decile 4	0.0033	1.94^{*}	1.1127	15.10^{***}	0.0084	1.63	0.92
Decile 3	0.0034	2.50^{**}	1.1162	34.07^{***}	0.0034	0.90	0.96
Decile 2	0.0028	3.08^{***}	1.1091	40.15^{***}	0.0072	2.29^{**}	0.97
Big 1	-0.0023	-1.78^{*}	0.9564	17.42^{***}	-0.0027	-0.62	0.93
GRS F-Test = 2.8324 (p=0. 0045)							

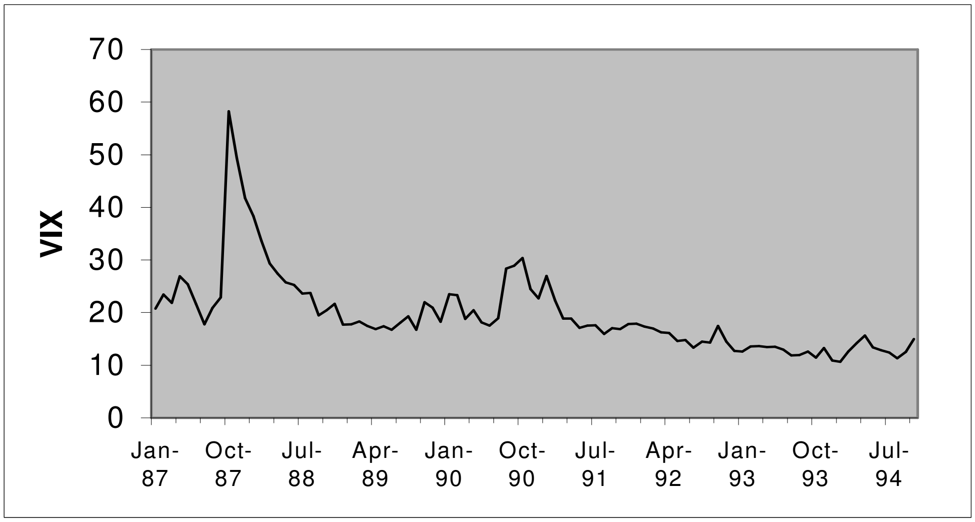
Note. This table reports monthly time-series regression results of excess returns of CRSP's size deciles on market factor and excess straddle returns. The effect of the crash is examined by dividing the sample period into two sub-samples, one from January 1987-November 1990 (47 months), and the other from December 1990-October 1994 (47 months). ***, **, * denote 0.01, 0.05, and 0.10 significance levels, respectively. All t-values are corrected for autocorrelation (with lag=3) and heteroskedasticity as suggested by Newey and West (1987). GRS F-Test reported at the bottom of the table is from Gibbons, Ross, and Shanken (1989).

As can be seen from Table 2.6, when the crash period is excluded from the sample, the significance of the volatility risk factor vanishes for 9 of the 10 size portfolios. This result confirms that there exists time variation in the volatility risk premium and it has several implications regarding the redundancy of options.

According to Vanden (2004), options effectively complete the market when agents face nonnegative wealth constraints. That is options are non-redundant, because they help agents to avoid insolvency while still allowing them to obtain a degree of leverage that is not possible through direct borrowing. Thus, the high explanatory power of the proposed 2-factor model through the crash period makes sense in this manner. Straddles explain asset returns in periods of high volatility, because they allow their investors to hedge volatility risk and help them avoid insolvency in those periods. The failure of straddle returns to explain security returns in periods of low volatility arises because straddles are redundant securities at those times. As the highest volatility period in our sample is around October 1987 (see Figure 2.1), the exclusion of this time period results in less explanatory power for the volatility risk factor. Thus, although volatility risk is priced for all classes of assets at times of high volatility, we cannot assert the same for times of low volatility.

FIGURE 2.1

Monthly Average Implied Volatility of the S&P 500 Index



Note. This figure shows the monthly implied volatilities of the S&P 500 index (VIX) for the period January 1987 through October 1994.. Daily VIX data for the sample period is from the Chicago Board Options Exchange. Monthly implied volatility is the average of daily VIX levels for that month.

Asset return volatility literature documents that high volatility periods tend to coincide with business cycle downturns and recessions. (Turner, Startz, & Nelson (1989), Schwert (1989), Hamilton and Lin (1996), and Perez-Quiros and Timmerman (2001)) Also, Chauvet and Potter (2000) argue that bear markets have higher volatility than bull markets. Our finding of a significant volatility beta in a high volatility period like 1987 is in line with he literature. However, we also report an insignificant volatility beta for the time period of 1991-1992, which is often cited as a period of poor business conditions and high volatility, is at odds with the above literature. We offer two possible explanations for this. First, as can be seen from Figure 2.1, VIX volatility index is much higher in the 1987 crash period compared to the volatility around 1991-1992 downturn. This large difference in the level of volatility, which is captured by straddle returns, might lead the volatility betas to be insignificant for the latter period. One can also argue that it might be the fear of a crash that drives these results. VIX measure is also considered to be a fear indicator among the professionals. High VIX levels are associated with a pessimistic market sentiment and conversely a low level of VIX is considered to be a sign of optimistic market sentiment. The relatively low levels of VIX measure for the second period studied might indicate that investors are optimistic about the market and hence lead the volatility betas to be insignificant for this period. Altogether, these results should be further investigated since the time period studied here covers only one peak and one trough, which makes it hard to reconcile our findings with that of the business cycle literature.

2.5 CONCLUSION

The notion that volatility risk is priced in options markets is now widely documented. However, until recently, very few studies focused on the question of whether volatility risk is priced in the securities market. The answer to this question has important implications for asset pricing, portfolio and risk management, and hedging strategies.

The empirical findings in this article suggest that volatility risk explains a significant amount of variation in securities returns, especially during high volatility periods. In addition, the findings suggest that options are non-redundant securities during those periods. Investors use straddle returns when forming their expectations about securities returns. This implies that straddle returns can be used to price volatility risk.

The findings also indicate different patterns for different classes of firms. For example, during high volatility periods, small firms and value firms are more prone to downside market risk; hence they are associated with negative volatility coefficients. Thus, at times of high volatility, investors see value firms and small firms riskier than their growth and big counterparts and price this risk in their returns via an important factor, volatility risk. Furthermore, investors see big-growth firms as hedges against volatility, regardless of the level of volatility in the market. This could be a potential explanation to why growth firms underperform value firms.

In conclusion, this article presents clear evidence that volatility risk, proxied by straddle returns, is an important factor in asset pricing since it helps capture time variation in the stochastic discount factor. Thus, options play an important role in pricing securities, and allocation of wealth among agents in the economy.

CHAPTER 3

NONNEGATIVE WEALTH, OPTIONS, AND C-CAPM

3.1 INTRODUCTION AND LITERATURE REVIEW

This article proposes to solve individuals' consumption-investment problem with nonnegative wealth constraints in a multi-period securities market framework, and subsequently derive the optimal sharing rules for the agents in an economy where there is the possibility of trading long-lived securities, and time-event contingent claims. The derivation of optimal sharing rules in equilibrium yields a multifactor conditional consumption capital asset pricing model (C-CAPM), where the first factor is the change in log aggregate consumption, and the other factors are excess returns on a bundle of options written on the aggregate consumption. The empirical tests carried for the period 1990-2006 reveals that data supports the predictions of the model. Overall the theory outlined, and the empirical findings documented have implications for asset pricing, portfolio management, and capital markets theories.

There are three important lines of literature that sets the motivating ground for this article. These are:

i) Empirical tests of single factor asset pricing models - why do CAPM and C-CAPM fail to empirically explain asset prices although they have sound theoretical backgrounds?

ii) Market completeness and allocational role of options in the economy - what are the possible frictions that lead to incomplete markets, and do these frictions lead options to play an allocational role in the economy?

iii) Nonnegative wealth constraints - what are the pricing consequences of nonnegative wealth constraints?

Today, there is a vast amount of literature documenting the failure of CAPM and C-CAPM in explaining the cross-section of returns.[15] This might be due to two reasons. Either, multiple factor models are needed as in Merton's I-CAPM (1973) or existing models cannot capture the possible time variation in securities returns. It is documented that conditional and

[15] See Basu (1977), Banz (1981), Reinganum (1981), and Rosenberg, Reid, and Lanstein (1984) for empirical tests of CAPM, and Hansen and Singleton (1982, 1983), Mankiw and Shapiro (1986), Breeden, Gibbons, and Litzenberger (1989), Campbell (1996), Cochrane (1996), Hodrick, Ng and Sengmueller (1999) for tests of C-CAPM.

multifactor models fare better in explaining the cross-section of securities returns than their unconditional and single factor counterparts.[16]

Since the inception of the Black-Scholes option pricing formula [Black and Scholes (1973)], options have been at the heart of many theoretical and empirical research. One line of research that has received extensive attention is on the spanning role of options and market completeness. Although standard Black-Scholes option pricing framework is still widely used in practice, research today has shifted from assuming complete markets to the examination of the possible factors that lead to incomplete markets. The literature identified, heterogeneous believes of agents, asymmetric information, stochastic volatility and jumps, transaction costs, and limitations on short sales or borrowing as possible factors. When these factors are considered, options might become non-redundant, and play an allocational role in the economy.

Research regarding heterogeneous beliefs argues that heterogeneous attitudes towards risk can generate demand for options. Grosmann and Zhou (1996) show that if one of the agents, such as a portfolio insurer, is infinitely averse to the risk when his wealth drops below a given threshold, than the demand for options can be an important determinant of the underlying asset price. Bates (2001) considers an economy where crashes can

[16] See Chen, Roll and Ross (1984), Fama and French (1992, 1993) for tests of multifactor asset pricing models, Jagannathan and Wang (1996) for tests of conditional CAPM, and Lettau and Ludvigson (2001b) for tests of conditional C-CAPM.

occur and less crash-tolerant investors buy options from more crash-tolerant ones. In his setting, options complete the market by serving as a hedge against crash risk. Buraschi and Jiltsov (2003) consider a symmetric but incomplete information setting; agents agree on the dividend process but differ in their beliefs about the price process unrelated to fundamentals. They find that much of the observed option trading volume can be explained by this heterogeneity in beliefs.

A number of studies suggest that options might be non-redundant, because the price of a traded option can convey some information, which otherwise would be unobservable in the economy. For example, Grossman (1988) argues that an option can be non-redundant due to its informational content, thus its removal from the economy would make markets incomplete. Back (1993) shows that, in a market with asymmetric information, the introduction of options might change the stochastic pricing process of the underlying asset. Hence, options introduced to a complete market may be non-redundant. Also Easley, O'Hara, and Srinivas (1998) suggest that an option market could be a platform for informed trading due to lower transaction costs and greater financial leverage.

Furthermore, the presence of stochastic volatility and jumps can severely affect asset price dynamics and thus options that are written on them. The main approach to modeling stock returns is to define a continuous time stochastic volatility diffusion process possibly augmented with an independent jump process in returns. Today, most option pricing models incorporate these two factors in order to account for a more realistic pricing process.[17] Bakshi, Cao, and Chen (1997) compare empirical performances of these alternative option pricing models and conclude that models that include stochastic volatility and jump processes performs better.

Besides these theoretical models, recently, a number of empirical papers have demonstrated that options are non-redundant. Buraschi and Jackwerth (2001) suggest that option returns do significantly increase the spanning quality of the pricing kernel and argue that the volatility risk might be priced in options market. Furthermore, Coval and Shumway (2001) give preliminary evidence that at-the-money straddles can account for the systematic volatility risk in the securities market. Bakshi and Kapadia (2003) show that delta-hedged option portfolios consistently earn negative returns, indicating that there exists a negative volatility risk premium in option prices. Liu and Pan (2003) show that at-the-money straddles, and out-of-money puts can be used to complete the markets when markets are incomplete due to volatility and jump risks. Finally, Arısoy, Salih, and Akdeniz (2007) show that S&P 500 straddle returns play an important role in constructing the stochastic discount factor.

[17] See Heston (1993) who proposed a stochastic volatility diffusion model, for which one could analytically derive an option pricing formula. Duffie and Kan (1996), and Duffie, Pan, and Singleton (2000) further developed Heston's model to a rich class of affine jump diffusion processes. Several other authors have used stochastic volatility diffusion process augmented by jumps [Bates (1996) Andersen, Benzoni and Lund (2001), Eraker, Johannes and Polson (2001), Pan (2002), Chernov, Gallant, Ghysels and Tauchen (2003)].

The standard asset pricing and option pricing theories assume that markets are frictionless. However, the presence of transaction costs, portfolio constraints such as constraints on short selling, or credit constraints such as nonnegative wealth constraints can generate demand for options. For example, Lee and Yi (2001) find that options with lower transaction costs attract more informed investors. Furthermore, Basak and Croitoru (2000), show that a mispricing between a stock and a redundant derivative arises due to portfolio constraints on short selling and investors with heterogeneous beliefs.

There are other imperfections in the market that can cause agents fail to replicate their consumption patterns via existing securities. One such imperfection is the bounded-credit assumption put forward by Dybvig and Huang (1991). Classical asset pricing theories assume no restrictions on borrowing and lending. However, in real life borrowing is limited due to agents' possibility to default. In their model agents can borrow at most the amount equal to their wealth i.e. no guarantees on future income is allowed, allowing agents to come back to the economy with the ability to pay off their debts. This also rules out doubling strategies and arbitrage possibilities posit by Harrison and Kreps (1979).

Recently, Vanden (2004) analyzes the effects of nonnegative wealth constraints and finds that incorporation of such real-life frictions on wealth implies non-redundancy of options in a single period setting. It is argued that options can effectively complete markets via their leverage properties, i.e. by their limited liabilities and theoretically unlimited payoffs and help agents avoid insolvency and thus meet the nonnegative wealth constraint. This difference in the payoff pattern of options help agents to match their consumption patterns better compared to the limited spanning capacity of existing securities in the economy. In Vanden (2004), single period and continuous time equilibrium properties of nonnegative wealth constraints on agents' consumption-investment problem have been derived, however this problem has not yet been analyzed in a multi-period securities market context.

Setting the problem in a multi-period framework is more appealing, since the real-life practice is to trade securities through dynamically managed portfolios. The contribution of this article will be to set up the problem in a multi-period framework with the introduction of short-lived options, i.e. that can be traded in smaller intervals of time (from a day to up to a year), and derive the corresponding equilibrium properties in a multi-factor conditional C-CAPM framework. Thus, unlike Vanden (2004), we assume a dynamically traded portfolio of securities and introduce a multi-period framework where each security can be traded in discrete time intervals. In equilibrium, the pricing agent's optimal consumption incorporates the aggregate consumption and a bundle of short-lived options with different strike prices. This result is similar to Vanden's, yet the asset pricing consequences are different. Since agents dynamically rebalance their portfolios at each period, a multifactor conditional C-CAPM model that is empirically testable. To the best of our knowledge, there have not been any multifactor empirical tests of asset pricing that combines C-CAPM framework with options.

The rest of the article is organized as follows. Section 3.2 introduces the problem and derives the corresponding asset pricing model. Section 3.3 draws the econometric framework for tests of the model. Section 3.4 presents the empirical findings, and Section 3.5 offers concluding remarks.

3.2 THE MODEL

There are *I* agents in the economy indexed by $i = 1,2,\ldots,I$. Agents live in a multiperiod pure exchange economy $(t = 0,1,\ldots,T)$ with reconvening markets, and agree on the possibilities of occurrences of events in the economy. Each event, a_t, is a collection of states, ω. Ω denotes the collection of all possible states of nature, and the true state of the nature is partially revealed to individuals over time.

There is a single perishable consumption good available for consumption at each trading date. Individuals are endowed with time-0 consumption, and time-event contingent claims $\{e_i(0), e_i(a_t), a_t \in F_t; t = 1,\ldots,T\}$, and have the possibility to trade these claims after $t = 0$.[18]

[18] A time-event contingent claim is a security that pays one unit of consumption at a trading date $t \geq 1$ in an event $a_t \in F_t$ and nothing otherwise. The notation and the setting follow the desktop reference Huang and Litzenberger (1988).

Before proceeding with the model we have to note some simplifying assumptions. We assume that i) all agents have the same information structure, F_t, ii) all agents agree on the possible states of nature, iii) all agents are endowed with an initial wealth, iv) all agents have time-additive, and state independent von Neumann-Morgenstern utility functions with identical cautiousness, v) all agents face nonnegative wealth constraints at all periods, and vi) markets are dynamically incomplete. Under these assumptions, the model proceeds as follows.

Individual i has preferences for time-0 consumption, and time-event contingent claims that are increasing, strictly concave, and differentiable, i.e. $u_{i0}\left(z_i\left(o\right)\right)+\sum_{t=1}^{T}\sum_{a_t\in F_t} p_{a_t}u_{i,t}\left(z_i\left(a_t\right)\right)$. p_{a_t} is the homogeneously agreed probability of the occurrence of the event $a_t \in F_t$, $z(0)$ is the time-0 consumption good, and $z(a_t)$ are the payoffs of the time-event contingent claims in the event $a_t \in F_t$ for $t \geq 1$, respectively.

Each agent tries to maximize their expected utilities over their lifetime while facing nonnegative wealth and budget constraints. Now, define $\phi(0)$ as the price of the time-0 consumption good, and $\phi_{a_s}(a_t)$ as the ex-dividend price for the time-event contingent claim paying off at time s in event a_s, conditional on the occurrence of event a_t at time *t*, where

$$\phi_{a_s}(a_t)=\begin{cases}\dfrac{\phi_{a_s}}{\phi_{a_t}} & if\ t<s\ \ and\ \ a_s\subseteq a_t\\ 0 & otherwise\end{cases} \tag{1}$$

Also, for $t<s$, define $p_{a_s}(a_t)$ to be the conditional probability of event a_s given that at time *t* event a_t occurs, so that

$$p_{a_s}(a_t)=\begin{cases}0 & if\ \ a_s\not\subset a_t\\ \dfrac{p_{a_s}}{p_{a_t}} & if\ \ a_s\subseteq a_t\end{cases} \tag{2}$$

With the above assumptions, the problem can be formulated as follows.

$$\underset{\{z_i(0),z_i(a_t)\}}{Max}\ u_{i,0}\left(z_i(0)\right)+\sum_{t=1}^{T}\sum_{\substack{\{a_s\in F_s\\ a_s\subseteq a_t\}}}p_{a_s}(a_t)u_{i,s}\left(z_i(a_s)\right)$$

$$s.t.\ \ \phi_0 z_i(0)+\sum_{t=1}^{T}\sum_{\substack{\{a_s\in F_s\\ a_s\subseteq a_t\}}}\phi_{a_s}(a_t)z_i(a_s)=\phi_0 e_i(0)+\sum_{t=1}^{T}\sum_{\substack{\{a_s\in F_s\\ a_s\not\subset a_t\}}}\phi_{a_s}(a_t)e_i(a_s) \tag{3}$$

$$z_i(0)\geq 0$$

$$z_i(a_s)\geq 0\ \ \ \forall\ a_s\in F_s, a_s\subseteq a_t$$

The solution to the above problem requires the solution of Lagrangian and its associated Kuhn-Tucker conditions. Before proceeding with the solution, we show that when $t=1$, the optimal consumptions are equivalent to Vanden's optimal sharing rules derived in the single period setting, and the marginal rates of substitutions are equal across agents.

Proposition 1. *The solution of the above problem in the single period setting implies that the marginal rates of substitutions of agents are equal for all i=1, …, ,I, and the optimal consumption at* $t=0$, and $t=1$ *for individual* i *is given by* $c_i(0)=\max\left[0,u_{i,0}^{\prime -1}(\gamma_{i,0}\phi_0)\right]$, and $c_i(a_1)=\max\left[0,u_{i,0}^{\prime -1}\left(\frac{\gamma_{i,0}\phi_{a_1}(0)}{p_{a_1}(0)}\right)\right]$.

Proof. The Lagrangian for the single period version can be written as;

$$L\left(z_i(0),z_i(a_1)\right)=u_{i,0}\left(z_i(0)\right)+\sum_{\{a_1\in F_1\}}p_{a_1}(0)u_{i,1}\left(z_i(a_1)\right)$$

$$+\gamma_{i,0}\left(\phi_0\left[e_i(0)-z_i(0)\right]+\sum_{\{a_1\in F_1\}}\phi_{a_1}(0)\left[e_i(a_1)-z_i(a_1)\right]\right)$$

$$+\mu_{i,0}z_i(0)+\mu_{i,a_1}z_i(a_1)$$

The first order conditions (F.O.C.) for the above Lagrangian are evaluated at $c_i(0)$ and $c_i(a_1)$, since the only sources of consumption are the payoffs of time-event contingent claims, $z_i(0)$ and $z_i(a_1)$;

ci(0): $$u'_{i,0}\left(c_i(0)\right)-\gamma_{i,0}\phi_0=0 \tag{4}$$

ci(a1): $$p_{a_1}(0)u'_{i,1}\left(c_i(a_1)\right)-\gamma_{i,0}\phi_{a_1}(0)=0 \tag{5}$$

K-T: $$\mu_{i,0}c_i(0)=0 \tag{6}$$

$$\mu_{i,a_1}c_i(a_1)=0 \tag{7}$$

From Kuhn-Tucker (K-T) conditions (6) and (7), when the nonnegative constraints do not bind, i.e. $c_i(0) > 0$, and $c_i(a_1) > 0$, we have $\mu_{i,0} = \mu_{i,a_1} = 0$. If the nonnegative constraints bind, then wealth at each period and correspondingly consumption at each period is zero. Thus, the above problem has a solution at either zero consumption (when nonnegative constraints bind), or some positive levels of $c_i(0)$, and $c_i(a_1)$ that is evaluated at $\mu_{i,0} = \mu_{i,a_1} = 0$.

Thus, $\frac{p_{a_1}(0)u'_{i,1}(c_i(a_1))}{u'_{i,0}(c_i(0))} = \frac{\phi_{a_1}(0)}{\phi_0}$. In other words, the marginal rate of substitutions of agents in the economy are equal, and independent of the index *i*. Also, the optimal time-0, and time-1 event a_1 consumptions are given by $c_i(0) = \max\left[0, u'^{-1}_{i,0}(\gamma_{i,0}\phi_0)\right]$, and $c_i(a_1) = \max\left[0, u'^{-1}_{i,0}\left(\frac{\gamma_{i,0}\phi_{a_1}(0)}{p_{a_1}(0)}\right)\right]$. This completes the proof. ■

The solution of the original problem follows the same principles. Note that it suffices to solve the problem at any time t, and at any event a_t. The problem is

$$\underset{\substack{\{z_i(a_s), t\geq s\\ a_s\subseteq a_t, a_s\in F_s\}}}{Max} \quad u_{i,t}(z_i(a_t)) + \sum_{s=t+1}^{T} \sum_{\substack{\{a_s\in F_s\\ a_s\subseteq a_t\}}} p_{a_s}(a_t) u_{i,s}(z_i(a_s))$$

$$s.t. \quad z_i(a_t) + \sum_{s=t+1}^{T} \sum_{\substack{\{a_s\in F_s\\ a_s\subseteq a_t\}}} \phi_{a_s}(a_t) z_i(a_s) = c_i(a_t) + \sum_{s=t+1}^{T} \sum_{\substack{\{a_s\in F_s\\ a_s\not\subset a_t\}}} \phi_{a_s}(a_t) c_i(a_s)$$

$$z_i(a_t) \geq 0, \quad z_i(a_s) \geq 0 \quad \forall\, a_s \in F_s, a_s \subseteq a_t$$

The first order conditions (F.O.C.) for the above Lagrangian evaluated at $c_i(a_t)$ and $c_i(a_s)$ are;

ci(at): $u'_{i,t}(c_i(a_t)) - \gamma_{i,a_t} = 0$ (8)

ci(as): $p_{a_s}(a_t) u'_{i,s}(c_i(a_s)) - \gamma_{i,a_t} \phi_{a_s}(a_t) = 0$ (9)

K-T: $\mu_{i,0} c_i(a_t) = 0$ (10)

$\mu_{i,a_s} c_i(a_s) = 0$ (11)

Thus, $\dfrac{p_{a_s}(a_t) u'_{i,s}(c_i(a_s))}{u'_{i,t}(c_i(a_t))} = \phi_{a_s}(a_t)$ (12)

Letting $\gamma_{i,a_t} = \gamma_i \dfrac{\phi_{a_t}}{p_{a_t}}$, and using the definitions of $p_{a_s}(a_t)$ and $\phi_{a_s}(a_t)$, the optimal time-t consumption can be written as

$$c_i(a_t) = \max\left[0, u'^{-1}_{i,t}(\gamma_{i,a_t})\right] = \max\left[0, u'^{-1}_{i,0}\left(\frac{\gamma_i \phi_{a_t}}{p_{a_t}}\right)\right] \quad (13)$$

From Equation (13), we can see that the optimal consumption for the *ith* individual depends on the prices of time-event contingent claim, and the associated probabilities of events in the economy at time *t*, and the Lagrange multiplier of the budget constraint. To derive the corresponding optimal sharing rules, we follow a methodology similar to Vanden.

The aggregate consumption in the economy at time t, and event a_t can be written as;

$$C(a_t) = \sum_{i=1}^{I} c_i(a_t) = \sum_{i=1}^{I} \max\left[0, u_{i,t}'^{-1}\left(\frac{\gamma_i \phi_{a_t}}{p_{a_t}}\right)\right] \qquad (14)$$

Now, define a real-valued function $\Delta(x)$, such that;

$$\Delta(x) = \sum_{i=1}^{I} \max\left[0, u_{i,t}'^{-1}(\gamma_i x)\right] \qquad (15)$$

then $\Delta\left(\frac{\phi_{a_t}}{p_{a_t}}\right) = C(a_t)$, and $\frac{\phi_{a_t}}{p_{a_t}} = \Delta^{-1}\left(C(a_t)\right)$ (16)

and the *ith* agent's optimal sharing rule becomes;

$$c_i(a_t) = \max\left[0, u_{i,t}'^{-1}\left(\gamma_i \Delta^{-1}(C(a_t))\right)\right] \qquad (17)$$

In the above expression, Δ^{-1} is the inverse mapping of the function Δ on the interval where Δ is strictly increasing. The closed form solution of

$\Delta^{-1}(C(a_t))$ for an economy with agents possessing quadratic utility with identical cautiousness is given below.

Assume that the agents' utility function is of the form:

$$u_{i,t}(c_i(a_t)) = c_i(a_t) - \frac{b}{2} c_i^2(a_t) \tag{18}$$

Then the marginal utility and its inverse are given by:

$$u'_{i,t}(c_i(a_t)) = 1 - bc_i(a_t) \tag{19}$$

$$u'^{-1}_{i,t}(c_i(a_t)) = \frac{1}{b}(1 - c_i(a_t)) \tag{20}$$

Now define constants,

$$A_1 = \Delta\left(\frac{1}{\gamma_{2,t}}\right),\ A_2 = \Delta\left(\frac{1}{\gamma_{3,t}}\right),\ \ldots,\ A_{I-1} = \Delta\left(\frac{1}{\gamma_{I,t}}\right) \tag{21}$$

and assume,

$$\gamma_{1,t}^{-1} > \gamma_{2,t}^{-1} > \cdots > \gamma_{I,t}^{-1} \tag{22}$$

Also define $\bar{\gamma}_{k,t} = \sum_{j=1}^{k} \gamma_{j,t}$.

Then, the solution for $\Delta^{-1}(C(a_t))$ is given by,

$$\Delta^{-1}(C(a_t)) = \begin{cases} \dfrac{1 - bC(a_t)}{\gamma_{1,t}} & 0 < C(t) \le A_1 \\ \dfrac{2 - bC(a_t)}{\bar{\gamma}_{2,t}} & A_1 < C(t) \le A_2 \\ \vdots & \\ \dfrac{I - bC(a_t)}{\bar{\gamma}_{I,t}} & A_{I-1} < C(t) \end{cases} \tag{23}$$

Also, one can determine the values for the constants, $A_1, A_2, \ldots, A_{I-1}$ by using definition (21), and assumption (22). For example,

At $A_1 = \Delta\left(\frac{1}{\gamma_{2,t}}\right)$, we have;

$$A_1 = \max\left[0, \frac{1}{b}\left(1 - \frac{\gamma_{1,t}}{\gamma_{2,t}}\right)\right] + \max\left[0, \frac{1}{b}\left(1 - \frac{\gamma_{2,t}}{\gamma_{2,t}}\right)\right] + \max\left[0, \frac{1}{b}\left(1 - \frac{\gamma_{3,t}}{\gamma_{2,t}}\right)\right] + \cdots$$

According to (22), the term inside the first parenthesis is strictly positive, the term inside the second parenthesis is zero, and the terms inside the remaining parentheses are strictly negative. Thus,

$$A_1 = \frac{1}{b}\left(1 - \frac{\gamma_{1,t}}{\gamma_{2,t}}\right) \tag{24}$$

Equation (24) also satisfies $\Delta^{-1}(C(a_t))$ in Equation (23). To see that, note

$$\Delta^{-1}(A_1) = \Delta^{-1}\left(\Delta\left(\frac{1}{\gamma_{2,t}}\right)\right) = \frac{1}{\gamma_{2,t}} = \frac{1-bA_1}{\gamma_{1,t}}.$$

Similarly, at $A_2 = \Delta\left(\frac{1}{\gamma_{3,t}}\right)$, we have;

$$A_2 = \max\left[0, \frac{1}{b}\left(1 - \frac{\gamma_{1,t}}{\gamma_{3,t}}\right)\right] + \max\left[0, \frac{1}{b}\left(1 - \frac{\gamma_{2,t}}{\gamma_{3,t}}\right)\right] + \max\left[0, \frac{1}{b}\left(1 - \frac{\gamma_{3,t}}{\gamma_{3,t}}\right)\right] + \cdots$$

Again by using (22), terms inside the first and second parentheses are strictly positive, the term inside the third parenthesis is zero, and the terms inside the remaining parentheses are strictly negative. Thus,

$$A_2 = \frac{1}{b}\left(2 - \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}}\right) \tag{25}$$

Equation (25) also satisfies Equation (23). To see that, note;

$$\Delta^{-1}(A_2) = \Delta^{-1}\left(\Delta\left(\frac{1}{\gamma_{3,t}}\right)\right) = \frac{1}{\gamma_{3,t}} = \frac{2-bA_2}{\bar{\gamma}_{2,t}}.$$

The rest proceeds similarly, so we can write the predefined constants as;

$$A_1 = \frac{1}{b}\left(1 - \frac{\gamma_{1,t}}{\gamma_{2,t}}\right)\ ,\ A_2 = \frac{1}{b}\left(2 - \frac{\overline{\gamma}_{2,t}}{\gamma_{3,t}}\right)\ ,\ \ldots\ ,\ A_{I-1} = \frac{1}{b}\left(I - \frac{\overline{\gamma}_{I-1,t}}{\gamma_{I,t}}\right) \quad (26)$$

We have shown that the Equation (23) is strictly decreasing in $C(a_t)$ on the intervals for the above defined constants. Now, we can derive the optimal sharing rules for each agent in the economy. To do that, we determine each agent's optimal consumption in each of the possible aggregate consumption intervals.

For $0 < C(a_t) \leq A_1$;

$$c_1(a_t) = \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{1,t}\left(\frac{1 - bC(t)}{\gamma_{1,t}}\right).\right] = C(a_t) \quad (27)$$

$$c_2(t) = \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{2,t}\left(\frac{1 - bC(a_t)}{\gamma_{1,t}}\right).\right]$$

$$= \max\left[0, \frac{1}{b} + \frac{\gamma_{2,t}}{\gamma_{1,t}}\left(C(a_t) - \frac{1}{b}\right).\right]$$

$$= \max\left[0, \frac{\gamma_{2,t}}{\gamma_{1,t}}\left(C(a_t) - \frac{1}{b} + \frac{1}{b}\frac{\gamma_{1,t}}{\gamma_{2,t}}\right).\right]$$

$$c_2(t) = \max\left[0, \frac{\gamma_{2,t}}{\gamma_{1,t}}(C(a_t) - A_1).\right] = 0 \tag{28}$$

The expression in (28) is equal to zero, due to the fact that the term in the parenthesis is less than or equal to zero for $0 < C(a_t) \leq A_1$. Now, by using assumption (22), we can write;

$$c_3(t) = \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{3,t}\left(\frac{1 - bC(a_t)}{\gamma_{1,t}}\right).\right] \leq \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{2,t}\left(\frac{1 - bC(a_t)}{\gamma_{1,t}}\right).\right] = c_2(a_t)$$

because $\gamma_{3,t} > \gamma_{2,t}$.

Similarly, $c_4(a_t), c_5(a_t), \ldots c_I(a_t) = 0$, since $\gamma_{I,t} > \cdots > \gamma_{5,t} > \gamma_{4,t}$ (29)

Next, for $A_1 < C(a_t) \leq A_2$;

$$c_1(a_t) = \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{1,t}\left(\frac{2 - bC(a_t)}{\bar{\gamma}_{2,t}}\right).\right]$$

$$= \max\left[0, \frac{1}{b} + \frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}}\left(C(a_t) - \frac{2}{b}\right).\right]$$

$$= \max\left[0, \frac{1}{b} + \frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}}\left(C(a_t) - \frac{1}{b} + \frac{1}{b}\frac{\gamma_{1,t}}{\gamma_{2,t}} - \frac{1}{b}\left(1 + \frac{\gamma_{1,t}}{\gamma_{2,t}}\right)\right).\right]$$

$$= \max\left[0, \frac{1}{b} - \frac{1}{b}\frac{\gamma_{1,t}}{\gamma_{2,t}} + \frac{\gamma_{1,t}}{\gamma_{2,t}}(C(a_t) - A_1).\right]$$

$$= \max\left[0, A_1 + \frac{\gamma_{1,t}}{\lambda_{2,t}}\left(C(a_t) - A_1\right).\right]$$

$$= A_1 + \frac{\gamma_{1,t}}{\gamma_{2,t}}\left[C(a_t) - A_1\right]$$

$$= A_1 + \left(1 - \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}\right)\left[C(a_t) - A_1\right]$$

$$c_1(a_t) = C(a_t) - \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}\left[C(a_t) - A_1\right] \tag{30}$$

$$c_2(a_t) = \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{2,t}\left(\frac{2 - bC(a_t)}{\bar{\gamma}_{2,t}}\right).\right]$$

$$= \max\left[0, \frac{1}{b} + \frac{\gamma_{2,,t}}{\bar{\gamma}_{2,t}}\left(C(a_t) - \frac{1}{b} + \frac{1}{b}\frac{\gamma_{1,t}}{\gamma_{2,t}} - \frac{1}{b}\left(1 + \frac{\gamma_{1,t}}{\gamma_{2,t}}\right)\right).\right]$$

$$= \max\left[0, \frac{1}{b} - \frac{1}{b}\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}\frac{\bar{\gamma}_{2,t}}{\gamma_{2,t}} + \frac{\gamma_{2,t}}{\gamma_{2,t}}\left(C(a_t) - A_1\right).\right]$$

$$c_2(a_t) = \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}\left[C(a_t) - A_1\right] \tag{31}$$

$$c_3(a_t) = \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{3,t}\left(\frac{2 - bC(a_t)}{\bar{\gamma}_{2,t}}\right).\right]$$

$$= \max\left[0, \frac{\gamma_{3,t}}{\bar{\gamma}_{2,t}}\left(C(a_t) - \frac{1}{b}\left(2 - \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}}\right)\right).\right]$$

$$c_3(a_t) = \max\left[0, \frac{\gamma_{3,t}}{\bar{\gamma}_{2,t}}(C(a_t) - A_2).\right] = 0 \qquad (32)$$

Again, by using assumption (22), we can write;

$$c_4(a_t) = \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{4,t}\left(\frac{2 - bC(a_t)}{\bar{\gamma}_{2,t}}\right).\right] \leq \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{3,t}\left(\frac{2 - bC(a_t)}{\bar{\gamma}_{2,t}}\right).\right] = c_3(a_t)$$

because $\gamma_{4,t} > \gamma_{3,t}$.

Similarly, $c_5(a_t), c_6(a_t), \ldots c_I(a_t) = 0$ since $\gamma_{I,t} > \cdots > \gamma_{6,t} > \gamma_{5,t}$ (33)

Next, for $A_2 < C(a_t) \leq A_3$;

$$c_1(a_t) = \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{1,t}\left(\frac{3 - bC(a_t)}{\bar{\gamma}_{3,t}}\right).\right]$$

$$= \max\left[0, \frac{1}{b} + \frac{\gamma_{1,t}}{\bar{\gamma}_{3,t}}\left(C(a_t) - \frac{3}{b}\right).\right]$$

$$= \max\left[0, \frac{1}{b} + \frac{\gamma_{1,t}}{\bar{\gamma}_{3,t}}\left(C(a_t) - \frac{2}{b} + \frac{1}{b}\frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} - \frac{1}{b}\left(1 + \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}}\right)\right).\right]$$

$$= \max\left[0, \frac{1}{b} - \frac{1}{b}\frac{\gamma_{1,t}}{\gamma_{3,t}} + \frac{\gamma_{1,t}}{\bar{\gamma}_{3,t}}(C(a_t) - A_2).\right]$$

$$= \max\left[0, \frac{1}{b} - \frac{1}{b}\frac{\gamma_{1,t}}{\gamma_{3,t}} + \frac{\gamma_{1,t}}{\bar{\gamma}_{3,t}}(C(a_t) - A_2). - \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}(C(a_t) - A_1) + \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}(C(a_t) - A_1)\right]$$

$$=\max\left[0,\frac{1}{b}-\frac{1}{b}\frac{\gamma_{1,t}}{\gamma_{3,t}}+\frac{\gamma_{1,t}}{\bar{\gamma}_{3,t}}(C(a_t)-A_2).-\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}(C(a_t)-A_1)+\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}\left(C(a_t)-A_2+\frac{1}{b}+\frac{1}{b}\left(-\frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}}+\frac{\gamma_{1,t}}{\gamma_{2,t}}\right)\right)\right]$$

$$=\max\left[0,\frac{1}{b}\left(1-\frac{\gamma_{1,t}}{\gamma_{3,t}}+\frac{\gamma_{2,t}}{\gamma_{2,t}}+\frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}}-\frac{\gamma_{2,t}}{\gamma_{3,t}}\right)+\left(1-\frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}}+\frac{\gamma_{1,t}}{\bar{\gamma}_{3,t}}\right)(C(a_t)-A_2).-\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}(C(a_t)-A_1)\right]$$

After some rearranging,

$$c_1(a_t)=C(a_t)-\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}(C(a_t)-A_1)-\frac{\gamma_{1-1}\gamma_{3,t}}{\bar{\gamma}_{2,t}\bar{\gamma}_{3,t}}(C(a_t)-A_2) \tag{34}$$

$$c_2(a_t)=\max\left[0,\frac{1}{b}-\frac{1}{b}\gamma_{2,t}\left(\frac{3-bC(t)}{\bar{\gamma}_{3,t}}\right).\right]$$

$$=\max\left[0,\frac{1}{b}+\frac{\gamma_{2,t}}{\bar{\gamma}_{3,t}}\left(C(a_t)-\frac{2}{b}+\frac{1}{b}\frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}}-\frac{1}{b}\left(1+\frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}}\right)\right).\right]$$

$$=\max\left[0,\frac{1}{b}-\frac{1}{b}\frac{\gamma_{2,t}}{\gamma_{3,t}}+\frac{\gamma_{2,t}}{\bar{\gamma}_{3,t}}(C(a_t)-A_2).\right]$$

$$=\max\left[0,\frac{1}{b}-\frac{1}{b}\frac{\gamma_{2,t}}{\gamma_{3,t}}+\frac{\gamma_{2,t}}{\bar{\gamma}_{3,t}}(C(a_t)-A_2).-\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}(C(a_t)-A_1)+\frac{\gamma_{2-1}}{\bar{\gamma}_{2,t}}(C(a_t)-A_1)\right]$$

$$=\max\left[0,\frac{1}{b}-\frac{1}{b}\frac{\gamma_{2,t}}{\gamma_{3,t}}+\frac{\gamma_{2,t}}{\bar{\gamma}_{3,t}}(C(a_t)-A_2).-\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}\left(C(a_t)-A_2+\frac{1}{b}+\frac{1}{b}\left(\frac{\gamma_{1,t}}{\gamma_{2,t}}-\frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}}\right)\right)+\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}(C(a_t)-A_1)\right]$$

$$=\max\left[0,\frac{1}{b}\left(1-\frac{\gamma_{2-1}}{\gamma_{3-1}}-\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}-\frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}}+\frac{\gamma_{2,t}}{\gamma_{3,t}}\right)+\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}(C(a_t)-A_1)-\frac{\gamma_{2,t}\gamma_{3,t}}{\bar{\gamma}_{2,t}\bar{\gamma}_{3,t}}(C(a_t)-A_2)\right]$$

$$c_2(a_t)=\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}(C(a_t)-A_1)-\frac{\gamma_{2,t}\gamma_{3,t}}{\bar{\gamma}_{2,t}\bar{\gamma}_{3,t}}(C(a_t)-A_2) \tag{35}$$

$$c_3(a_t)=\max\left[0,\frac{1}{b}-\frac{1}{b}\gamma_{3,t}\left(\frac{3-bC(a_t)}{\bar{\gamma}_{3,t}}\right).\right]$$

$$= \max\left[0, \frac{1}{b} + \frac{\delta_{3,t}}{\bar{\gamma}_{3,t}}\left(C(a_t) - \frac{2}{b} + \frac{1}{b}\frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} - \frac{1}{b}\left(1 + \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}}\right)\right).\right]$$

$$c_3(a_t) = \frac{\gamma_{3,t}}{\bar{\gamma}_{3,t}}\left[C(a_t) - A_2\right] \qquad (36)$$

$$c_4(a_t) = \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{4,t}\left(\frac{3 - bC(a_t)}{\bar{\gamma}_{3,t}}\right).\right]$$

$$= \max\left[0, \frac{\gamma_{4,t}}{\bar{\gamma}_{3,t}}\left(C(a_t) - \frac{1}{b}\left(3 - \frac{\bar{\gamma}_{3,t}}{\gamma_{4,t}}\right)\right).\right]$$

$$c_4(a_t) = \max\left[0, \frac{\gamma_{4,t}}{\bar{\gamma}_{3,t}}\left(C(a_t) - A_3\right).\right] = 0 \qquad (37)$$

By using assumption (22), we can write;

$$c_5(a_t) = \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{5,t}\left(\frac{3 - bC(t)}{\bar{\gamma}_{3,t}}\right).\right] \le \max\left[0, \frac{1}{b} - \frac{1}{b}\gamma_{4,t}\left(\frac{2 - bC(t)}{\bar{\gamma}_{3,t}}\right).\right] = c_4(a_t)$$

because $\gamma_{5,t} > \gamma_{4,t}$.

Similarly, $c_6(a_t), c_7(a_t), \ldots c_I(a_t) = 0$ since $\gamma_{I,t} > \cdots > \gamma_{7,t} > \gamma_{6,t}$ (38)

The rest proceeds in a similar manner. In general, we observe the following. A switch to a higher consumption interval (i.e. an increase in aggregate consumption from the $i-1st$ bracket to the ith bracket) results in one more agent joining the economy. This last agent's optimal consumption is given by $c_i(a_t) = \frac{\gamma_{i,t}}{\bar{\gamma}_{i,t}}\left(C(a_t) - A_i\right)$. Furthermore, the remaining agents' optimal sharing rules are adjusted in such a way to compensate the inclusion of this last agent to the economy. That is, they will consume in a similar pattern when this agent was not present (i.e. as in the $i-1st$ bracket) minus they will make an adjustment to compensate for the joining agent's consumption, $c_i(a_t)$, that is proportional to their shadow prices, $\gamma_{j,t}$'s, where $j=1,\ldots i-1$. Thus, we observe the optimal sharing rules for various levels of aggregate consumption as in Table 3.1.

$c_2(a_t), c_3(a_t), \ldots c_I(a_t)$ are the optimal consumptions for the 2^{nd}, 3^{rd}, and I^{th} agents, when they are first included in the economy, i.e. $c_i(a_t) = \frac{\gamma_{i,t}}{\bar{\gamma}_{i,t}}(C(a_t) - A_{i-1})$. We observe that the equilibrium condition $\sum_{i=1}^{I} c_i(a_t) = C(a_t)$ is satisfied for all possible levels of aggregate consumption. Also note that in this economy, the agent indexed with $i=1$ is the representative agent, since its nonnegative wealth constraint never binds in any of the states. We can write the optimal consumption pattern of this representative agent as:

$$c_1(a_t) = C(a_t) - \sum_{j=1}^{I-1} \frac{\gamma_{1,t}\gamma_{j+1,t}}{\bar{\gamma}_{j,t}\bar{\gamma}_{j+1,t}} \max\left[0, C(a_t) - A_j\right] \qquad (39)$$

Table 3.1
Optimal Sharing Rules

	$i=1$	$i=2$	$i=3$	$\ldots$	i	$\ldots$	$i=I$
$0<C(a_t)\leq A_1$	C(at)	0	0	$\ldots$	0	$\ldots$	0
$A_1<C(a_t)\leq A_2$	$C(a_t)-c_2(a_t)$	$\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}(C(a_t)-A_1)$	0	$\ldots$	0	$\ldots$	0
$A_2<C(a_t)\leq A_3$	$C(a_t)-c_2(a_t)$ $-\frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}}c_3(a_t)$	$c_2(a_t)-\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}c_3(a_t)$	$\frac{\gamma_{3,t}}{\bar{\gamma}_{3,t}}(C(a_t)-A_2)$	$\ldots$	0	$\ldots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$A_{i-1}<C(a_t)\leq A_i$	$C(a_t)-c_2(a_t)-\frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}}c_3(a_t)$ $-\cdots-\frac{\gamma_{1,t}}{\bar{\gamma}_{i-1,t}}c_i(a_t)$	$c_2(t)-\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}c_3(t)$ $\cdots-\frac{\gamma_{2,t}}{\bar{\gamma}_{i-1,t}}c_i(t)$	$c_3(a_t)-\frac{\gamma_{3,t}}{\bar{\gamma}_{4,t}}c_4(a_t)$ $\cdots-\frac{\gamma_{3,t}}{\bar{\gamma}_{i-1,t}}c_i(a_t)$	$\ldots$	$\frac{\gamma_{i,t}}{\bar{\gamma}_{i,t}}(C(a_t)-A_{i-1})$	$\ldots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$A_{I-1}<C(a_t)$	$C(a_t)-c_2(a_t)-\frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}}c_3(a_t)$ $\cdots-\frac{\gamma_{1,t}}{\bar{\gamma}_{I-1,t}}c_I(a_t)$	$c_2(a_t)-\frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}}c_3(a_t)$ $\cdots-\frac{\gamma_{2,t}}{\bar{\gamma}_{I-1,t}}c_I(a_t)$	$c_3(a_t)-\frac{\gamma_{3,t}}{\bar{\gamma}_{4,t}}c_4(a_t)$ $\cdots-\frac{\gamma_{3,t}}{\bar{\gamma}_{I-1,t}}c_I(a_t)$	$\ldots$	$c_i(a_t)-\cdots$ $-\frac{\gamma_{i,t}}{\bar{\gamma}_{I,t}}c_I(a_t)$	$\ldots$	$\frac{\gamma_{I,t}}{\bar{\gamma}_{I,t}}(C(a(t))-A_{I-1})$

Thus the representative agent holds the aggregate consumption and $I-1$ call options written on the aggregate consumption with strike prices:

$$A_j = \frac{1}{b}\left(j - \frac{\bar{\gamma}_{j,t}}{\gamma_{j+1,t}} \right), \; j = 1,2,\ldots, I-1 \tag{40}$$

Equation (39) can also be written in a put option format noting the following relation

$$\max\left[0, C(a_t) - A_j\right] = \max\left[0, A_j - C(a_t)\right] + C(a_t) - A_j \tag{41}$$

Plugging (41) into (40) we have the put option representation;

$$c_1(a_t) = C(a_t) - \sum_{j=1}^{I-1} \frac{\gamma_{1,t}\gamma_{j+1,t}}{\bar{\gamma}_{j,t}\bar{\gamma}_{j+1,t}} \max\left[0, A_j - C(a_t)\right] - \sum_{j=1}^{I-1} \frac{\gamma_{1,t}\gamma_{j+1,t}}{\bar{\gamma}_{j,t}\bar{\gamma}_{j+1,t}} \left(C(a_t) - A_j\right)$$

$$\begin{aligned} c_1(a_t) &= C(a_t)\left[1 - \sum_{j=1}^{I-1} \frac{\gamma_{1,t}\gamma_{j+1,t}}{\bar{\gamma}_{j,t}\bar{\gamma}_{j+1,t}}\right] - \sum_{j=1}^{I-1} \frac{\gamma_{1,t}\gamma_{j+1,t}}{\bar{\gamma}_{j,t}\bar{\gamma}_{j+1,t}} \max\left[0, A_j - C(a_t)\right] \\ &+ \sum_{j=1}^{I-1} \frac{\gamma_{1,t}\gamma_{j+1,t}}{\bar{\gamma}_{j,t}\bar{\gamma}_{j+1,t}} A_j \end{aligned} \tag{42}$$

Thus, the representative agent holds a fraction of the aggregate consumption plus a portfolio of put options written on the aggregate consumption. The last term is irrelevant since it is a combination of constants.

The fact that the representative agent holds the optimal portfolio as in (39) or (42) in markets reconvening at each period has important pricing consequences. The setting of a multiperiod securities market with rational expectations equilibrium results in a C-CAPM, whereas the nonnegative wealth constraints in our multiperiod setting with dynamically incomplete markets results in a multibeta C-CAPM. The first beta is the covariance of the return of a risky asset with an asset that is highly correlated with the aggregate consumption, and the remaining $I-1$ betas are the covariances of the return of the asset with the returns of options with strike prices given in (40). This can be formalized with the following proposition.

Proposition 2. *The optimal portfolio in (39) held by the representative agent indexed with $i=1$, and having a quadratic utility, results in a multifactor conditional C-CAPM, where the first factor is the change in log aggregate consumption and the remaining $I-1$ factors are option returns with strike prices given in (40).*

Proof. We will give the proof for the case of call options. The case for put options is similar. Let the sequence $\{F_t, t=0,1,\ldots,T\}$ be an information structure, such that the possible realizations of F_t from time 0 to time t generate a state space Ω. Assume that the representative agent is endowed with this information structure and has a quadratic utility given by

$u_{1t}(c_1(t)) = c_1(t) - \frac{b}{2}c_1^2(t)$. The utility function of the representative agent is strictly concave and differentiable. Also assume that F_0 is just $\{\Omega\}$.

The price of a long-lived security in this economy is given by

$$S_j(a_t,t) = \sum_{s=t+1}^{T} \sum_{\left\{\substack{a_s \in F_s \\ a_s \subseteq a_t}\right\}} \phi_{a_s}(a_t) X_j(a_s) \tag{43}$$

where $X_j(a_s)$ is the dividend paid by security j in event a_s.[19]

By using Equation (12), we can rewrite the price of a long-lived security as

$$S_j(a_t,t) = \sum_{s=t+1}^{T} \sum_{\left\{\substack{a_s \in F_s \\ a_s \subseteq a_t}\right\}} \frac{p_{a_s}(a_t) u'_{1,s}(c_1(a_s))}{u'_{1,t}(c_1(a_t))} X_j(a_s) \tag{44}$$

By using the definition of $S_j(a_{t-1}, t-1)$ from Equation (43), one can write

$$S_j(a_{t-1},t-1) = \sum_{\left\{\substack{a_t \in F_t \\ a_t \subseteq a_{t-1}}\right\}} \frac{p_{a_t}(a_{t-1}) u'_{1,t}(c_1(a_t))}{u'_{1,t-1}(c_1(a_{t-1}))} \left[X_j(a_t) + S_j(a_t,t) \right] \tag{45}$$

Also by using the definitions of expected value, the random ex-dividend price of a long-lived security is given as

$$S_j(t-1) = E\left[\sum_{s=t}^{T} \frac{u'_{1,s}(c_1(s))}{u'_{1,t-1}(c_1(t-1))} X_j(s) \middle| F_{t-1} \right] \tag{46}$$

[19] A long-lived security is a complex security that is available for trading at all periods, and is composed of time-0 consumption good and a bundle of time-event contingent claims, and is represented by $X = \{X_0, X_{a_t}; a_t \in F_t, t = 1,\ldots;T\}$, where X_0 and X_{a_t} are the dividends paid at time 0 and at time t in event a_t, respectively, in units of consumption good.

$$S_j(t-1) = E\left[\frac{u'_{1,t}(c_1(t))}{u'_{1,t-1}(c_1(t-1))} \left[X_j(t) + S_j(t) \right] \middle| F_{t-1} \right] \tag{47}$$

$$1 = E\left[\frac{u'_{1,t}(c_1(t))}{u'_{1,t-1}(c_1(t-1))} \left[1 + \tilde{R}_j(t) \right] \middle| F_{t-1} \right] \tag{48}$$

is the expected return process for a long-lived security j.

From the definition of the covariance, equation (48) can be rewritten as;

$$1 = Cov\left[\frac{u'_{1,t}(c_1(t))}{u'_{1,t-1}(c_1(t-1))}, 1 + \tilde{R}_j(t) \middle| F_{t-1} \right] + E\left[\frac{u'_{1,t}(c_1(t))}{u'_{1,t-1}(c_1(t-1))} \middle| F_{t-1} \right] E\left[1 + \tilde{R}_j(t) \middle| F_{t-1} \right] \tag{49}$$

From Equation (48), the existence of a risk-free asset implies that

$$\frac{1}{1 + R_f(t)} = E\left[\frac{u'_{1,t}(c_1(t))}{u'_{1,t-1}(c_1(t-1))} \middle| F_{t-1} \right] \tag{50}$$

Substituting (50) into (49), we have;

$$E\left[\tilde{R}_j(t) \middle| F_{t-1} \right] - R_f(t) = -\left(1 + R_f(t)\right) Cov\left[\tilde{R}_j(t), \frac{u'_{1,t}(c_1(t))}{u'_{1,t-1}(c_1(t-1))} \middle| F_{t-1} \right] \tag{51}$$

$$E\left[\tilde{R}_j(t) \middle| F_{t-1} \right] - R_f(t) = -\left(1 + R_f(t)\right) Cov\left[\tilde{R}_j(t), \frac{bc_1(t)}{bc_1(t-1)} \middle| F_{t-1} \right] \tag{52}$$

$$E\left[\tilde{R}_j(t) \middle| F_{t-1} \right] - R_f(t) = -\left(1 + R_f(t)\right) Cov\left[\tilde{R}_j(t), \tilde{R}_{c_1}(t) \middle| F_{t-1} \right] \tag{53}$$

Since this equation holds for any traded asset, or a portfolio of traded assets, it should also hold for the representative agent's portfolio, $\tilde{R}_{c_1}(t) = \left(\tilde{R}_C(t), \tilde{R}_{O_2}(t), \ldots \tilde{R}_{O_{I-1}}(t) \right)^T$, where $\tilde{R}_C(t)$ is the change in aggregate consumption, and $\tilde{R}_{O_j}(t)$ is the return of the j^{th} option at time t. Thus,

$$E\left[\tilde{R}_{c_1}(t)\middle|F_{t-1}\right]-R_f(t)=-(1+R_f(t))Cov\left[\tilde{R}^T_{c_1}(t),\tilde{R}_{c_1}(t)\middle|F_{t-1}\right] \tag{54}$$

Substituting (54) into (53) gives

$$E\left[\tilde{R}_j(t)\middle|F_{t-1}\right]-R_f(t)=\frac{Cov\left(\tilde{R}_j(t),\tilde{R}_{c_1}(t)\middle|F_{t-1}\right)}{Cov_t\left(\tilde{R}^T_{c_1}(t),\tilde{R}_{c_1}(t)\middle|F_{t-1}\right)}\left(E\left[\tilde{R}_{c_1}(t)\middle|F_{t-1}\right]-R_f(t)\right) \tag{55}$$

In general for a vector of N risky assets, $\tilde{R}(t)=\left(\tilde{R}_1(t),\tilde{R}_2(t),\ldots\tilde{R}_N(t)\right)^T$;

$$E\left[\tilde{R}(t)\middle|F_{t-1}\right]-R_f(t)1=\frac{Cov\left(\tilde{R}(t),\tilde{R}_{c_1}(t)\middle|F_{t-1}\right)}{Cov\left(\tilde{R}^T_{c_1}(t),\tilde{R}_{c_1}(t)\middle|F_{t-1}\right)}\left(E\left[\tilde{R}_{c_1}(t)\middle|F_{t-1}\right]-R_f(t)1\right) \tag{56}$$

where $Cov\left(\tilde{R}(t),\tilde{R}_{c_1}(t)\middle|F_{t-1}\right)Cov\left(\tilde{R}_{c_1}(t),\tilde{R}_{c_1}(t)\middle|F_{t-1}\right)^{-1}$ is an NxI matrix of conditional betas for N risky assets with the return's of the representative agent's portfolio, and $\tilde{R}_{c_1}(t)=\left(\tilde{R}_C(t),\tilde{R}_{O_2}(t),\ldots\tilde{R}_{O_{I-1}}(t)\right)^T$ is the $Ix1$ vector of returns for the representative agent's portfolio.

Equation (56) can be written in a multibeta representation as,

$$E\left[\tilde{R}(t)\middle|F_{t-1}\right]-R_f(t)1=\beta_{Nc_1}\beta^{-1}_{c_1c_1}\left\{E\left[\tilde{R}_{c_1}(t)\middle|F_{t-1}\right]-R_f(t)1\right\} \tag{57}$$

where

$$\beta_{Nc_1} = \begin{bmatrix} Cov_{t-1}\left(\tilde{R}_1(t), \tilde{R}_C(t)\right) & Cov_{t-1}\left(\tilde{R}_1(t), \tilde{R}_{O_1}(t)\right) & \cdots & Cov_{t-1}\left(\tilde{R}_1(t), \tilde{R}_{O_{l-1}}(t)\right) \\ Cov_{t-1}\left(\tilde{R}_2(t), \tilde{R}_C(t)\right) & Cov_{t-1}\left(\tilde{R}_2(t), \tilde{R}_{O_1}(t)\right) & \cdots & Cov_{t-1}\left(\tilde{R}_2(t), \tilde{R}_{O_{l-1}}(t)\right) \\ \vdots & \vdots & \ddots & \vdots \\ Cov_{t-1}\left(\tilde{R}_N(t), \tilde{R}_C(t)\right) & Cov_{t-1}\left(\tilde{R}_N(t), \tilde{R}_{O_1}(t)\right) & \cdots & Cov_{t-1}\left(\tilde{R}_N(t), \tilde{R}_{O_{l-1}}(t)\right) \end{bmatrix}$$

and

$$\beta_{c_1c_1} = \begin{bmatrix} Var_{t-1}\left(\tilde{R}_C(t)\right) & Cov_{t-1}\left(\tilde{R}_C(t), \tilde{R}_{O_1}(t)\right) & \cdots & Cov_{t-1}\left(\tilde{R}_C(t), \tilde{R}_{O_{l-1}}(t)\right) \\ Cov_{t-1}\left(\tilde{R}_{O_1}(t), \tilde{R}_C(t)\right) & Var_{t-1}\left(\tilde{R}_{O_1}(t)\right) & \cdots & Cov_{t-1}\left(\tilde{R}_{O_1}(t), \tilde{R}_{O_{l-1}}(t)\right) \\ \vdots & \vdots & \ddots & \vdots \\ Cov_{t-1}\left(\tilde{R}_{O_{l-1}}, \tilde{R}_C(t)\right) & Cov_{t-1}\left(\tilde{R}_{O_{l-1}}, \tilde{R}_{O_1}(t)\right) & \cdots & Var_{t-1}\left(\tilde{R}_{O_{l-1}}(t)\right) \end{bmatrix}$$

This completes the proof. ■

Equation (57) is the main testable result of the outlined theory. It suggests that a multifactor conditional C-CAPM model with option returns as the factors, should explain the cross-sectional variation in securities returns. The following sections outline the econometric framework for the tests of (57), and present the empirical findings associated with these tests.

3.3 ECONOMETRIC SPECIFICATIONS

According to Equation (57) an asset's return at time t, conditional on the information at $t-1$, should be explained by the changes in the aggregate consumption and returns of options written on the aggregate consumption. To test this, we first specify the general versions of conditional (and unconditional) C-CAPM models used for testing the theory, and then present the data used to test them in this section. The following section presents the empirical results of these tests.

3.3.1 CONDITIONAL MODEL

We start by the stochastic discount factor framework outlined by Harrison and Kreps (1979). Their existence theorem states that, in the absence of arbitrage, there exists a stochastic discount factor, m_{t+1}, which satisfies

$$E_t\left[\left(1+\tilde{R}_{j,t+1}\right)m_{t+1}\middle|F_t\right]=1 \tag{58}$$

where E_t denotes the mathematical expectation operator conditional on the information available at time t, and $\tilde{R}_{j,t+1}$ is the net return of any traded asset j. The conditional form of the SDF is be represented by

$$m_{t+1} = a_t + b_t \tilde{R}_{e,t+1} \tag{59}$$

where $\tilde{R}_{e,t+1}$ is the net return on an unobservable mean-variance efficient frontier.

The above conditional form implies a conditional beta representation given by

$$E_t\left[\tilde{R}_{j,t+1}\right] = R_{o,t} - b_t R_{0,t} Var_t\left(\tilde{R}_{e,t+1}\right)\beta_{j,t} \tag{60}$$

where $R_{o,t}$ is the net return on a zero-beta portfolio that is uncorrelated with m_{t+1} ,

$$b_t = -\frac{E_t\left[\tilde{R}_{e-t+1}\right] - R_{o,t}}{R_{o,t} Var_t\left(\tilde{R}_{e,t+1}\right)} \tag{61}$$

and

$$\beta_{j,t} = \frac{Cov_t\left(\tilde{R}_{j,t+1}, \tilde{R}_{e,t+1}\right)}{Var_t\left(\tilde{R}_{e,t+1}\right)} \tag{62}$$

The question here is how one can incorporate the information that investors use when they determine expected returns in Equation (58). Because the investors' true information set is unobservable, one has to find observable variables to proxy for that information set. Cochrane (1996) shows that conditional asset pricing models can be tested via a conditioning time t information variable, z_t. One way of incorporating conditioning variable, z_t, into the model is to scale factor returns, as discussed in Cochrane (2001); and used in Cochrane (1996), Hodrick and Zhang (2001), and Lettau and Ludvigson (2001b). This is done by scaling the factors with z_t, thus modeling the parameters a_t, and b_t as linear functions of z_t, such that $a_t = \gamma_0 + \gamma_1 z_t$, and $b_t = \eta_0 + \eta_1 z_t$.

Plugging these equations into Equation (59), we have a scaled multifactor model with constant coefficients taking the form

$$
\begin{aligned}
m_{t+1} &= (\gamma_o + \gamma_1 z_t) + (\eta_0 + \eta_1 z_t)\tilde{R}_{e,t+1} \\
&= \gamma_0 + \gamma_1 z_t + \eta_o \tilde{R}_{e,t+1} + \eta_1 z_t \tilde{R}_{e,t+1} \qquad (63)
\end{aligned}
$$

The scaled multifactor model can be tested by rewriting the conditional factor model in Equation (58), as an unconditional factor model with constant coefficients $\gamma_0, \gamma_1, \eta_0$, and η_1 as follows,

$$
E\left[\left(1+\tilde{R}_{j,t+1}\right)\left(\gamma_0 + \gamma_1 z_t + \eta_o \tilde{R}_{e,t+1} + \eta_1 z_t \tilde{R}_{e,t+1}\right)\right] = 1 \qquad (64)
$$

In order to be able to test the theory's main predictions, we have to put more structure on the SDF, m_{t+1}, and on the unobservable mean-variance efficient frontier, $\tilde{R}_{e,t+1}$. Following Cochrane (1996), we consider a linear factor pricing model with a vector of observable factors, f_t. For example, in the classical conditional CAPM tests, f_t is the return of the value-weighted market portfolio. The only requirement for the components of f_t is that the factors should be observable and relevant to the model.

Denote the vector $F_{t+1} = \left(1, z_t, f_{t+1}^T, f_{t+1}^T z_t\right)^T$, or in a more compact form $F_{t+1} = \left(1, \bar{f}_{t+1}^T\right)^T$, where $\bar{f}_{t+1} = \left(z_t, f_{t+1}, f_{t+1} z_t\right)$. The stochastic discount factor in equation (63) can be represented by

$$m_{t+1} = \delta^T F_{t+1} \qquad (65)$$

where $\delta = \left(\gamma_o, b^T\right)$ is a constant vector, and $b = \left(\gamma_1, \eta_0^T, \eta_1^T\right)$ is the vector of constant coefficients on the variable factors, $\bar{f}_{t+1}$. Equation (65) implies an unconditional multifactor beta representation for asset j,

$$E\left[\tilde{R}_{j,t+1}\right] = E\left[\tilde{R}_{0,t}\right] + \beta^T \lambda \qquad (66)$$

where $\beta = \dfrac{Cov\left(\tilde{R}_{j,t+1}, \bar{f}\right)}{Cov\left(\bar{f}^T, \bar{f}\right)}$ is a vector of regression coefficients from a multiple regression of returns on the variable factors.

3.3.2 CONDITIONING VARIABLE

The choice of the conditioning variable is important because it summarizes the information that investors use while forming their expectations about securities returns. Due to its role in constructing the SDF one has to find a relevant and theoretically sound variable. Regarding its success in explaining the cross-section of expected returns, we have decided to use the log consumption-wealth ratio that is advocated by Lettau and Ludvigson (2001a, 2001b). First of all it has a significant explanatory power in the conditional versions of C-CAPM, and on the theoretical side it summarizes agents' expectations of future returns on the market portfolio. Second, Cochrane (1996) shows that when the log consumption-wealth ratio is used as a conditioning variable, one can derive CAPM as special cases of C-CAPM. However, one problem with the log consumption-wealth ratio is that it is unobservable. In order to overcome this, we follow the methodology outlined by Lettau and Ludvigson (2001a, 2001b), and choose cay_t as an estimate of the log consumption-wealth ratio. Lettau and Ludvigson (2001a) argue that the log aggregate consumption, c_t, log asset wealth, a_t, and log labor earnings, y_t, are cointegrated, and they share a common trend. They define the trend term as cay_t, which is the cointegrating residual between c_t, a_t, and y_t. Then, cay_t is defined to be $cay_t = c_t - \omega a_t - (1-\omega)y_t$, where ω denotes the share of nonhuman (asset) wealth, A_t, in total wealth, W_t.

The empirical work with consumption data has used expenditures on nondurables and services, $c_{n,t}$, as a measure of the aggregate consumption, and assumed that aggregate consumption is a constant multiple of nondurables and services consumption, i.e. $c_t = \kappa c_{n,t}$, where $\kappa > 1$. Thus, $cay_t = c_{n,t} - \beta_a a_t - \beta_y y_t$, where $\beta_a = (1/\kappa)\omega$, and $\beta_y = (1/\kappa)(1-\omega)$. β_a and β_y are estimated using the following multivariate regression via OLS:

$$c_{n,t} = \alpha + \beta_a a_t + \beta_y y_t + \sum_{i=-k}^{k} b_{a,i}\Delta a_{t-i} + \sum_{i=-k}^{k} b_{y,i}\Delta y_{t-i} + \varepsilon_i \qquad (67)$$

where Δ is the first difference operator. Then the estimated trend deviation is given by

$$c\widehat{a}y_t \equiv c_{n,t} - \hat{\beta}_a a_t - \hat{\beta}_y y_t \qquad (68)$$

where hats denote the estimated parameters.

3.3.3 FUNDAMENTAL FACTORS

Since consumption growth and option returns appear as factors in the asset pricing model of Equation (57), we assume that the SDF can be approximated as a linear function of consumption growth and returns on options written on the aggregate consumption. However, there do not exist any traded options written on the aggregate consumption, therefore we assume that there exists a function $g(.;F_t)$ such that $\tilde{C}_t = g(\tilde{A}_t;F_t)$, where $\tilde{A}_t$ denotes the aggregate wealth at time t. We also assume that the S&P 500 index at time t, resembles a fairly well representation of, and is highly correlated with, $\tilde{A}_t$. Thus, instead of options written on the aggregate consumption, we use observable proxies, i.e. options written on the S&P 500 index to test our model.

More specifically, the vector of observable factors is chosen to be $f_t = (\Delta c_t, \tilde{R}_O)$, (or any subset of it), where Δc_t is the change in log consumption, and $\tilde{R}_O$ is an $I-1$ vector of option returns written on the S&P 500 index. Thus the general conditional (or unconditional) form of the stochastic factor would be

$$m_{t+1} = \gamma_0 + \gamma_1 cay_t + \eta_0^T f_{t+1} + \eta_1^T cay_t f_{t+1} \qquad (69)$$

or subsets of it.

3.3.4 DATA AND METHODOLOGY

For all the econometric analyses, we use quarterly data. This is due to the announcement of the gross domestic product (GDP) data by U.S. Bureau and Economic Analysis (BEA) quarterly. The data covers the period 1990

Q1:2006Q1 for a total of 65 quarters (195 months). In estimating cay_t, the data for log consumption, c_t, log asset wealth, a_t, and log labor income, y_t, are downloaded from Martin Lettau's website.[20] The empirical tests use 10 portfolios sorted according to their market capitalizations, and 25 portfolios sorted according to size and book-to-market value ratios. Monthly return data for the portfolios and the risk-free rate are downloaded from Kenneth French's website.[21] The data for S&P 500 (SPX) call and put options are from the Chicago Board Options Exchange's (CBOE) Market Data Express (MDX). Finally, for the market portfolio, CRSP's value weighted index on all NYSE, AMEX and NASDAQ stocks are used.

For cay_t estimation, we have used a variety of leads and lags $k = 1,\ldots,8$ and report here the results for $k = 8$. The results for other lags are similar. The estimated value for cay_t for the above test period is found to be $c\hat{a}y_t \equiv c_t - 0.18a_t - 0.61y_t - 1.83$. This estimated value is used as the conditioning variable in the empirical tests of conditional C-CAPM models.

The method for calculating daily option returns is as follows. First, we choose daily closing prices for SPX call and puts for a variety of strike prices and maturities and for the class of non-leap options. Second, options that expire during the following calendar month are identified. The reason for choosing options that expire the next calendar month is that they are the

[20] http://pages.stern.nyu.edu/~mlettau/data_cay.html
[21] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

most liquid data among various maturities.[22] Then, options that expire within 14 days are excluded from the sample, because they show large deviations in trading volumes, which casts doubt on the reliability of their pricing associated with increased volatility. Then, we group the call and put options according to their moneyness levels. More specifically, for the vector of observable factors, $f_t = \left(\Delta c_t, \tilde{R}_O\right)$, we choose 3 option classes according to their moneyness levels. Thus, $O = (c_{at}, c_{in}, c_{out})$, or $O = (p_{at}, p_{in}, p_{out})$ where c_{at}, c_{in}, and c_{out} stand for at-the-money, in-the-money, and out-of-money call options; and p_{at}, p_{in}, and p_{out} stand for at-the-money, in-the-money, and out-of-money put options, respectively. We have used the following criteria for moneyness classification.

c_{at}, p_{at}	c_{in}, p_{out}	c_{out}, p_{in}
$-5 \le S - K \le 5$	$1.03 \le S / K \le 1.06$	$0.94 \le S / K \le 0.97$

Classifying at-the-money options with moneyness level (S-K) between -5 and +5 follows Coval and Shumway (2001), and is chosen in order to guarantee that there are at least two and at most three options around the spot price. The classification for in-the-money and out-of-money options follows Bakshi, Cao, and Chen (1997). Returns for options for the above six categories are then calculated.

[22] According to Buraschi and Jackwerth (2001), most of the trading activity in S&P500 options is concentrated in the nearest (0-30 days to expiry) and second nearest (30-60 days to expiry) contracts.

We use raw returns, because Coval and Shumway (2001) report that using log returns could be quite problematic. The daily average option return, is then, the equally weighted average of the returns of options that belong to that category. These returns are then cumulated to quarterly returns ending up with 65 quarterly (195 monthly return data for the six categories of options. Table 3.2 reports the summary statistics for daily call and put option returns for different moneyness levels.

TABLE 3.2

Summary Statistics for SPX options

	c_{at}	c_{in}	c_{out}	p_{at}	p_{in}	p_{out}
Mean	**0,35**	**0,17**	**0.84**	**-1.27**	**-0.71**	**-1.76**
Median	**-0,58**	**-0,09**	**-4.80**	**-3.25**	**-1.69**	**-5.99**
Minimum	**-74.73**	**-66.23**	**-84.52**	**-69.02**	**-55.16**	**-75.95**
Maximum	**131.40**	**78.53**	**299.05**	**221.85**	**130.12**	**407.55**
Skewness	**0.74**	**0.33**	**1.65**	**1.1591**	**0.61**	**2.31**
Kurtosis	**2.04**	**1.67**	**5.03**	**4.6268**	**2.55**	**15.84**

Note. This table reports the summary statistics for the returns of daily call and put option returns on the S&P 500 index. The sample covers the period January 1990 to March 2006. The return figures are in percentages.

Daily returns for at-the-money call and put options are consistent with what has been documented in the literature [Coval and Shumway (2001), Vanden (2004)]. Daily average returns for call (put) options are positive (negative) regardless of their moneyness levels. Furthermore returns increase in absolute values as one goes from in-the-money options to out-of-money options. It is clear that out-of-money calls (puts) are the biggest earners (losers) among the three given moneyness levels.

The next section presents findings from the time-series, cross-sectional and GMM-SDF estimations for a variety of conditional and unconditional C-CAPM models.

3.4 EMPIRICAL RESULTS

In this section we test four versions of C-CAPM.

i) The unconditional C-CAPM

$$f_t = (\Delta c_t)$$

ii) Unconditional C-CAPM with call and put options

$$f_t = (\Delta c_t, R_{c_{at},t}, R_{c_{in},t}, R_{c_{out},t})$$

$$f_t = (\Delta c_t, R_{p_{at},t}, R_{p_{in},t}, R_{p_{out},t})$$

iii) Conditional C-CAPM

$$f_t = (c\widehat{a}y_t, \Delta c_t, c\widehat{a}y_t \Delta c_{t+1})$$

iv) Conditional C-CAPM with call and put options

$$f_t = (c\widehat{a}y_t, \Delta c_t, R_{c_{at},t}, R_{c_{in},t}, R_{c_{out},t}, c\widehat{a}y_t \Delta c_{t+1}, c\widehat{a}y_t R_{c_{at},t+1}, c\widehat{a}y_t R_{c_{in},t+1}, c\widehat{a}y_t R_{c_{out},t+1})$$

$$f_t = (c\widehat{a}y_t, \Delta c_t, R_{p_{at},t}, R_{p_{in},t}, R_{p_{out},t}, c\widehat{a}y_t \Delta c_{t+1}, c\widehat{a}y_t R_{p_{at},t+1}, c\widehat{a}y_t R_{p_{in},t+1}, c\widehat{a}y_t R_{p_{out},t+1})$$

3.4.1 TIME SERIES REGRESSIONS

First, we test whether option returns explain the factor loadings of different portfolios. To do this, we take the unconditional C-CAPM with options (ii) as our base model. Thus, the empirical model to be tested is

$$R_{it} = \alpha_i + \beta_i f_t^T + \varepsilon_{it} \qquad (70)$$

where R_{it}'s are realized quarterly excess returns of 10 size, and 25 size and book-to-market (BV/MV) portfolios, β_i is a row vector of betas for the *ith* portfolio, and f_t is as given in ii. Tables 3.3 and 3.4 report the time series regression results.

As can be seen from Tables 3.3 and 3.4, option returns help explain the variation in returns of the chosen portfolios. Panel A of Table 3.3 report the regression results of excess portfolio returns formed according to size on the aggregate consumption and excess call returns. It is seen that change in aggregate consumption, together with the excess returns of call options help explain the variation in the returns of size portfolios. The adjusted R^2's range from 0.20 (for smallest size portfolio) to 0.58 (for biggest size portfolio), and the model tends to explain the returns of bigger size portfolios better. Furthermore, although the intercepts terms are individually significant, a

TABLE 3.3
10 size regressions

PANEL A: 10 size regressions using excess call returns

	α_i	t-stat	$\beta_{i,\Delta c}$	t-stat	$\beta_{i,c_{at}}$	t-stat	$\beta_{i,c_{in}}$	t-stat	$\beta_{i,c_{out}}$	t-stat	Adj. R^2
Small1	0.0301	2.71***	-0.0123	-1.60	-0.0152	-1.69*	0.0877	2.55**	-0.0180	-1.84*	0.20
Decile2	0.0259	2.68***	-0.0193	-2.54**	-0.0196	-2.45**	0.0988	3.03***	-0.0182	-1.84*	0.31
Decile3	0.0236	3.00***	-0.0190	-2.72***	-0.0170	-2.22**	0.0922	3.57***	-0.0169	-2.08**	0.35
Decile4	0.0194	2.76***	-0.0167	-2.47**	-0.0192	-2.80***	0.0963	3.75***	-0.0168	-2.18**	0.38
Decile5	0.0219	3.05***	-0.0210	-3.35***	-0.0197	-3.05***	0.0964	4.00***	-0.0171	-2.44**	0.43
Decile6	0.0188	2.93***	-0.0184	-3.08***	-0.0145	-2.70***	0.0849	3.95***	-0.0153	-2.33**	0.45
Decile7	0.0240	3.61***	-0.0189	-2.92***	-0.0135	-2.04**	0.0812	3.33***	-0.0149	-1.83*	0.44
Decile8	0.0213	2.99***	-0.0178	-3.18***	-0.0142	-2.48**	0.0784	3.26***	-0.0135	-1.82*	0.41
Decile9	0.0209	3.90***	-0.0155	-3.10***	-0.0083	-1.88*	0.0653	3.40***	-0.0114	-1.87*	0.46
Big10	0.0138	2.04**	-0.0147	-3.46***	-0.0055	-1.30	0.0646	3.95***	-0.0107	-1.83*	0.58

GRS (10,51) = 0.6517 (0.763)

PANEL B: 10 size regressions using excess put returns

	α_i	t-stat	$\beta_{i,\Delta c}$	t-stat	$\beta_{i,p_{at}}$	t-stat	$\beta_{i,p_{in}}$	t-stat	$\beta_{i,p_{out}}$	t-stat	Adj. R^2
Small1	0.0044	0.22	0.0033	0.56	-0.0246	-1.11	-0.0601	-1.91*	0.0141	0.45	0.16
Decile2	0.0049	0.24	-0.0042	-0.45	-0.0205	-0.98	-0.0747	-2.80***	0.0245	0.89	0.25
Decile3	-0.0045	-0.02	-0.0045	-0.52	-0.0185	-0.97	-0.0715	-2.67***	0.0176	0.66	0.30
Decile4	-0.0036	-0.21	-0.0016	-0.19	-0.0202	-1.18	-0.0755	-2.93***	0.0224	0.89	0.33
Decile5	-0.0034	-0.23	-0.0053	-0.69	-0.0205	-1.73*	-0.0743	-3.21***	0.0191	0.87	0.39
Decile6	-0.0065	-0.51	-0.0026	-0.39	-0.0256	-2.48**	-0.0698	-3.58***	0.0210	1.11	0.44
Decile7	-0.0010	-0.08	-0.0047	-0.64	-0.0227	-2.24**	-0.0623	-3.92***	0.0152	0.92	0.41
Decile8	-0.0010	-0.07	-0.0038	-0.56	-0.0188	-1.53	-0.0706	-3.53***	0.0204	1.02	0.42
Decile9	-0.0005	-0.04	-0.0035	-0.49	-0.0170	-1.48	-0.0644	-3.90***	0.0171	1.00	0.49
Big10	-0.0067	-0.39	-0.0050	-0.66	-0.0125	-0.76	-0.0653	-5.19***	0.0150	1.03	0.53

GRS (10,51) = 0.4451 (0.917)

Note. This table reports quarterly time-series regression results of excess returns of CRSP's size deciles on change in log consumption and excess call and put returns. ,i.e. $f_t = \left(\Delta c_t, R_{c_{at},t}, R_{c_{in},t}, R_{c_{out},t}\right)$, and $f_t = \left(\Delta c_t, R_{p_{at},t}, R_{p_{in},t}, R_{p_{out},t}\right)$for Panels A, and B, respectively. ***, **, * denote 0.01, 0.05, and 0.10 significance levels, respectively. All t-values are corrected for autocorrelation (with lag=3) and heteroskedasticity as suggested by Newey and West (1987). GRS F-Test reported at the bottom of the table is from Gibbons, Ross, and Shanken (1989).

low GRS F-statistic with a p-value of 0.763 rejects the joint test of significance of intercept terms, i.e they are not jointly significantly different from zero.

Looking at Panel B of Table 3.3, it can be seen that in-the-money-put returns are significant in among all size portfolios. The adjusted R^2's are slightly lower than the model with excess call returns, and range from 0.16 (smallest size portfolio) to 0.53 (biggest size portfolio), and the tendency to explain the returns of bigger size portfolios remain. The GRS F-statistic is slightly lower with a p-value of 0.917, and clearly rejects the joint test of significance of intercept terms.

Next, we test whether option returns help explain the returns of 25 portfolios formed according to size and book-to-market. Panel A of Table 3.4 reports the results for the model with excess call returns. We see that although the significance of the beta coefficients for out-of-money calls drop slightly, the results are quite consistent with the previous findings. The factor loadings for at-the-money call returns are significant for 22 of 25 portfolios, and moreover in-the-money call returns are significant across all portfolios. The adjusted R^2's range from 0.20 to 0.51, and a high GRS F-test statistic with a p-value of 0.162 cannot reject the joint significance of the intercept terms.

TABLE 3.4
25 size and book-to-market regressions

PANEL A: 25 size and book-to-market regressions using excess call returns

	α_i	**t-stat**	$\beta_{i,\Delta c}$	**t-stat**	$\beta_{i,c_{at}}$	**t-stat**	$\beta_{i,c_{in}}$	**t-stat**	$\beta_{i,c_{out}}$	**t-stat**	**Adj. R^2**
SL	0.0027	0.17	-0.0304	-3.47***	-0.0232	-1.94*	0.1307	2.59**	-0.0295	-2.01**	0.29
S2	0.0288	2.52**	-0.0208	-2.76***	-0.0189	-1.97*	0.1036	2.81***	-0.0218	-1.95*	0.27
S3	0.0321	3.53***	-0.0123	-1.94*	-0.0183	-2.62**	0.0824	3.28***	-0.0137	-1.70*	0.24
S4	0.0397	4.14***	-0.0129	-1.77*	-0.0141	-2.38**	0.0691	3.19***	-0.0120	-1.69*	0.20
SH	0.0401	3.99***	-0.0089	-1.32	-0.0180	-1.87*	0.0854	2.72***	-0.0133	-1.40	0.21
2L	0.0114	1.08	-0.0274	-3.34***	-0.0181	-1.98*	0.1131	3.15***	-0.0231	-2.21**	0.36
22	0.0182	2.32**	-0.0166	-2.34**	-0.0176	-2.58**	0.0902	3.78***	-0.0156	-2.29**	0.33
23	0.0298	3.60***	-0.0144	-2.23**	-0.0180	-2.40**	0.0787	3.70***	-0.0135	-1.72*	0.30
24	0.0312	3.62***	-0.0105	-1.28	-0.0161	-2.41**	0.0735	3.71***	-0.0109	-1.68*	0.25
2H	0.0290	3.01***	-0.0077	-1.35	-0.0198	-2.15**	0.0880	3.13***	-0.0131	-1.54	0.25
3L	0.0125	1.26	-0.0306	-3.18***	-0.0170	-2.09**	0.1105	3.44***	-0.0236	-2.65**	0.42
32	0.0230	3.16***	-0.0180	-2.65**	-0.0194	-3.91***	0.0825	4.47***	-0.0112	-2.19**	0.39
33	0.0249	3.20***	-0.0126	-2.26**	-0.0117	-2.07**	0.0634	3.88***	-0.0098	-1.50	0.31
34	0.0234	2.51**	-0.0089	-1.46	-0.0147	-2.49**	0.0721	3.33***	-0.0115	-1.69*	0.27
3H	0.0337	3.29***	-0.0080	-1.41	-0.0161	-2.45**	0.0753	3.13***	-0.0109	-1.51	0.23
4L	0.0214	1.94*	-0.0327	-3.41***	-0.0155	-1.90*	0.0939	2.89***	-0.0189	-2.04**	0.42
42	0.0226	3.53***	-0.0137	-2.46**	-0.0099	-2.02**	0.0630	3.41***	-0.0093	-1.53	0.35
43	0.0261	3.43***	-0.0099	-1.63	-0.0101	-1.88*	0.0649	3.35***	-0.0111	-1.57	0.29
44	0.0270	3.25***	-0.0080	-1.61	-0.0124	-2.54**	0.0665	3.35***	-0.0099	-1.53	0.30
4H	0.0235	3.06***	-0.0028	-0.70	-0.0112	-1.74*	0.0649	2.65**	-0.0093	-1.06	0.22
5L	0.0159	1.87*	-0.0193	-3.86***	-0.0035	-0.65	0.0642	3.46***	-0.0122	-2.07**	0.51
52	0.0196	3.94***	-0.0115	-2.50**	-0.0066	-1.80*	0.0637	4.44***	-0.0096	-1.82*	0.51
53	0.0195	3.10***	-0.0067	-1.15	-0.0079	-2.09**	0.0546	4.05***	-0.0062	-1.12	0.38
54	0.0173	2.18**	-0.0039	-0.77	-0.0038	-0.96	0.0514	3.41***	-0.0079	-1.32	0.31
5H	0.0166	1.59	-0.0078	-1.32	-0.0065	-1.14	0.0549	2.41**	-0.0086	-1.14	0.27

GRS (25,36) = 1.4273 (0.162)

PANEL B: 10 size regressions using excess put returns

	α_i	**t-stat**	$\beta_{i,\Delta c}$	**t-stat**	$\beta_{i,p_{at}}$	**t-stat**	$\beta_{i,p_{in}}$	**t-stat**	$\beta_{i,p_{out}}$	**t-stat**	**Adj. R^2**
SL	-0.0313	-1.27	-0.0079	-0.77	-0.0132	-0.55	-0.1126	-3.56***	0.0204	0.71	0.29
S2	-0.0014	-0.07	-0.0038	-0.48	-0.0105	-0.52	-0.0848	-2.81***	0.0097	0.32	0.27
S3	0.0121	0.68	0.0003	0.06	-0.0203	-1.06	-0.0560	-2.28**	0.0166	0.64	0.18
S4	0.0190	1.02	-0.0011	-0.16	-0.0242	-1.18	-0.0412	-1.42	0.0115	0.40	0.16
SH	0.0190	0.96	0.0033	0.55	-0.0248	-1.21	-0.0529	-1.49	0.0170	0.50	0.13
2L	-0.0152	-0.70	-0.0082	-0.79	-0.0128	-0.58	-0.1121	-4.09***	0.0303	1.19	0.39
22	-0.0041	-0.22	-0.0052	-0.54	-0.0114	-0.57	-0.0648	-2.16**	0.0108	0.37	0.25
23	0.0067	0.41	-0.0006	-0.09	-0.0245	-1.65	-0.0502	-1.68*	0.0130	0.49	0.26
24	0.0055	0.32	0.0025	0.30	-0.0302	-2.07**	-0.0412	-1.34	0.0098	0.34	0.20
2H	0.0013	0.07	0.0037	0.62	-0.0185	-1.10	-0.0492	-1.43	0.0015	0.05	0.15
3L	-0.0088	-0.45	-0.0111	-0.96	-0.0181	-0.91	-0.1093	-5.19***	0.0398	1.99*	0.43
32	-0.0025	-0.20	-0.0045	-0.52	-0.0236	-2.50**	-0.0590	-2.47**	0.0141	0.64	0.35
33	-0.0023	-0.19	-0.0005	-0.11	-0.0241	-2.75***	-0.0421	-1.60	0.0037	0.16	0.31
34	-0.0073	-0.58	0.0041	0.86	-0.0333	-5.36***	-0.0293	-0.96	-0.0008	-0.03	0.21
3H	0.0002	0.01	0.0019	0.53	-0.0127	-1.11	-0.0400	-1.12	-0.0149	-0.48	0.19
4L	0.0018	0.09	-0.0165	-1.44	-0.0154	-0.79	-0.0910	-4.52***	0.0309	1.48	0.43
42	0.0001	0.01	-0.0032	-0.52	-0.0214	-2.33**	-0.0471	-1.87*	0.0102	0.45	0.30
43	-0.0039	-0.33	0.0028	0.55	-0.0253	-3.51***	-0.0422	-1.47	0.0008	0.03	0.19
44	-0.0011	-0.08	0.0029	0.52	-0.0194	-2.56**	-0.0428	-1.66*	-0.0010	-0.04	0.27
4H	0.0032	0.20	0.0045	0.90	-0.0112	-1.00	-0.0429	-1.42	0.0025	0.10	0.13
5L	-0.0007	-0.03	-0.0099	-1.32	-0.0073	-0.36	-0.0735	-4.83***	0.0202	1.21	0.48
52	0.0010	0.08	-0.0038	-0.52	-0.0144	-1.54	-0.0515	-2.98***	0.0118	0.68	0.36
53	-0.0049	-0.46	0.0012	0.18	-0.0157	-1.47	-0.0430	-2.61**	0.0030	0.16	0.31
54	-0.0097	-0.72	0.0063	1.00	-0.0247	-2.32**	-0.0397	-1.82*	0.0046	0.24	0.28
5H	-0.0205	-1.65	-0.0065	-2.12	0.0056	0.56	-0.0342	-1.32	-0.0365	-1.75*	0.33
GRS (25,36) = 3.3280 (0.001)											

Note. This table reports quarterly time-series regression results of excess returns of CRSP's size deciles on change in log consumption and excess call and put returns. ,i.e. $f_t = \left(\Delta c_t, R_{c_{at},t}, R_{c_{in},t}, R_{c_{out},t}\right)$, and $f_t = \left(\Delta c_t, R_{p_{at},t}, R_{p_{in},t}, R_{p_{out},t}\right)$ for Panels A, and B, respectively. ***, **, * denote 0.01, 0.05, and 0.10 significance levels, respectively. All t-values are corrected for autocorrelation (with lag=3) and heteroskedasticity as suggested by Newey and West (1987). GRS F-Test reported at the bottom of the table is from Gibbons, Ross, and Shanken (1989).

The results in Panel B of Table 3.4 are similar with that of Table 3.3. The returns of in-the-money puts are significant in 15 portfolios, and the model with put options again fares well in the biggest size quintile in terms of explanatory power, with adjusted R^2's ranging from 0.13 to 0.48. The joint significance of the intercept terms cannot be rejected due to the high GRS F-statistic with a p-value of 0.001.

Overall the above results indicate that the call and put returns explain a significant amount of variation in securities returns. Thus, the results favor the hypothesis that option returns are useful tools in explaining securities returns. In the following two subsections, we formalize these findings by testing whether the risk premiums for option returns are priced or not.

3.4.2 FAMA-MACBETH ESTIMATIONS

In order to have a formal comparison between the proposed 4 models, and examine the power of various beta representations to explain the cross section of expected returns, we perform Fama-MacBeth regressions and estimate the associated risk premia for each model. The model to be tested is

$$E\left[\tilde{R}_{j,t+1}\right] = \alpha_i + \beta_j^T \lambda \qquad (71)$$

where $\beta_j = \dfrac{Cov\left(\tilde{R}_{j,t+1}, f_t\right)}{Cov\left(f_t^T, f_t\right)}$ is a vector of regression coefficients from a multiple regression of quarterly returns on the variable factors, given in i, ii, iii, iv, and $R_{j,t+1}$ are the quarterly returns of 25 portfolios sorted according to size and book-to-market.

The procedure to estimate λ is as follows. In the first pass, portfolio betas are estimated from a single multiple time-series regression via Equation (70). Due to having a data set for 65 quarters, instead of using the 5-year rolling-window approach, we use a full sample period. In the second pass, a cross-sectional regression is run at each time period, with full-sample betas obtained from the first pass regressions. We estimate the intercept term and risk premia, α_i and λ_j's, as the average of these cross-sectional regression estimates, as outlined by Fama and MacBeth (1973).

Table 3.5 gives the results of Fama-MacBeth estimations. As can be seen from Row 1, the unconditional C-CAPM is very poor in explaining the cross section of expected securities returns. The results are consistent with the existing C-CAPM literature. The risk premium for consumption is insignificant, and the adjusted R^2 indicates that only 14 percent of cross sectional variation of securities returns is explained by the unconditional C-CAPM. Row 2 confirms the findings of Lettau and Ludvigson (2001b) that the conditional C-CAPM (using $c\widehat{a}y_t$ as the conditioning variable) performs superior to its unconditional counterpart. The scaled factor is significant and the model explains 34 percent of the cross-sectional variation in securities returns.

TABLE 3.5
Fama-MacBeth Regressions

	Fundamental Factors									Scaled Factors							
	α_i	λ_{cay}	$\lambda_{o,\Delta c}$	$\lambda_{o,c_{at}}$	$\lambda_{o,c_{in}}$	$\lambda_{o,c_{out}}$	$\lambda_{o,p_{at}}$	$\lambda_{o,p_{in}}$	$\lambda_{o,p_{out}}$	$\lambda_{1,\Delta c}$	$\lambda_{1,c_{at}}$	$\lambda_{1,c_{in}}$	$\lambda_{1,c_{out}}$	$\lambda_{1,p_{at}}$	$\lambda_{1,p_{in}}$	$\lambda_{1,p_{out}}$	**Adj. R^2**
Row1	2.63		0.17														0.14
	(2.45**)		(0.75)														
	(2.41**)		(0.74)														
Row2	3.73	-0.11	0.03							0.04							0.34
	(2.63**)	(-0.23)	(0.19)							(2.19**)							
	(2.46**)	(-0,22)	(0.18)							(2.07**)							
Row3	5.05		0.16	-2.22	-0.76	-1.42											0.47
	(4.13***)		(0.40)	(-2.64**)	(-1.83*)	(-1.19)											
	(3.60***)		(0.35)	(-2.32**)	(-1.59)	(-1.04)											
Row4	3.60		0.26				-0.07	0.32	0.44								0.37
	(3.32***)		(0.66)				(-0.24)	(1.41)	(2.40**)								
	(2.87***)		(0.57)				(-0.21)	(1.22)	(2.09**)								
Row5	4.85	-0.05	0.03	-1.81	-0.84	-0.38				0.01	0.04	0.02	0.01				0.61
	(3.81***)	(-1.04)	(0.20)	(-2.69**)	(-2.01**)	(-0.38)				(2.32**)	(2.29**)	(1.98*)	(0.37)				
	(3.16***)	(-0.67)	(0.16)	(-2.10**)	(-1.57)	(-0.30)				(1.81*)	(1.78*)	(1.54)	(0.29)				
Row6	3.84	-0.06	-0.04				0.23	0.29	0.48	0.02				0.02	-0.04	-0.10	0.53
	(2.71***)	(-1.55)	(-0.26)				(0.97)	(1.56)	(2.63**)	(2.78***)				(0.24)	(-0.84)	(-1.89*)	
	(2.31**)	(-1.17)	(-0.18)				(0.80)	(1.28)	(2.15**)	(2.27**)				(0.20)	(-0.68)	(-1.54)	

Note. This table gives the estimates for the cross-sectional Fama-MacBeth regression model $E\left[\tilde{R}_{j,t+1}\right] = \alpha_i + \beta_j^T \lambda$ where β_j 's are estimated by a single time-series regression via Equation 62 using a full sample period, and using f_t given by i, ii, iii, iv. The estimated coefficients from the second-pass are $\lambda = \left(\lambda_{cay}, \lambda_o, \lambda_1\right)$, where λ_0, and λ_1 denote the vector of coefficients for fundamental factors and scaled factors. $\lambda_o = \left(\lambda_{0,\Delta c}\right)$ for Rows 1 and 2, $\lambda_o = \left(\lambda_{0,\Delta c}, \lambda_{0,c_{at}}, \lambda_{0,c_{in}}, \lambda_{0,c_{out}}\right)$ for Rows 3 and 5, and $\lambda_o = \left(\lambda_{0,\Delta c}, \lambda_{o,p_{at}}, \lambda_{o,p_{in}}, \lambda_{o,p_{out}}\right)$ for Rows 4 and 6. Subsequently, $\lambda_1 = \left(\lambda_{1,\Delta c}\right)$ for Row 2, $\lambda_1 = \left(\lambda_{1,\Delta c}, \lambda_{1,c_{at}}, \lambda_{1,c_{in}}, \lambda_{1,c_{out}}\right)$ for Row 5, and $\lambda_1 = \left(\lambda_{1,\Delta c}, \lambda_{1,p_{at}}, \lambda_{1,p_{in}}, \lambda_{1,p_{out}}\right)$ for Row 6. The term $R_{j,t+1}$ is the return on 25 Fama-French portfolios (j=1,2,…,25) in quarter t (1990Q1:2006Q1). The numbers in parentheses are the two t-statistics for each coefficient estimate. The top statistic uses uncorrected Fama-MacBeth standard errors; the bottom statistic uses Shanken (1992) correction. The term adjusted R^2 denotes the cross-sectional R^2 statistic adjusted for the degrees of freedom.

One important point to be noted here is that the estimated coefficients of scaled variables, i.e. $\lambda_{1,\Delta c}$, should not be taken as the classical risk premia in unconditional models. As discussed by Lettau and Ludvigson (2001b) we should take into account the fact that each scaled unconditional multifactor model has an associated conditional model from which it is derived. Thus, the true risk prices for period t should actually be λ_t. Howeverthe scaled multifactor model uses the unconditional covariance matrix $Cov\left(f_{t+1,} f_{t+1}^T\right)$, instead of the conditional covariance matrix $Cov_t\left(f_{t+1,} f_{t+1}^T\right)$, which is needed to estimate the true period t risk premia. Thus there is no simple relationship between the actual period t risk premia, λ_t, and the estimated scaled λ's.

Row 3 presents the results for the unconditional version of C-CAPM when call returns are included. Looking at the Shanken corrected t-statistics, it is seen that the returns of at-the-money-call options have a negative and significant risk premium. Furthermore, when call option returns are included there is a significant improvement in the explanatory power of the unconditional model, from 14 percent to 47 percent. Similarly, Row 4 presents the results of unconditional C-CAPM with put returns included. The risk premium for out-of-money put returns is significant and positive. Although the adjusted R^2 is lower than the model using call returns, it is nevertheless higher than the unconditional C-CAPM. The two results confirm that the inclusion of option returns help increase the explanatory power of C-CAPM.

Finally, Rows 5 and 6 present the results for the model that is predicted by the theory outlined here. The conditional version of C-CAPM, which uses $c\hat{a}y_t$ as the conditioning variable, and option returns as fundamental factors perform significantly better than all the three previously tested models. Looking at Row 5, we can see that the previous significant risk premium of at-the-money call returns, and the significant coefficient of scaled consumption remain, and in addition, the scaled at-the-money calls also have a significant coefficient at 10 percent level. Furthermore, the overall explanatory power of the model rises to 61 percent. Similarly, Row 6 preserves the significant coefficients of the previous out-of-money put returns, and scaled consumption. Although none of the scaled put returns are significant, the overall explanatory power of the model is 53 percent, which is higher than its unconditional counterpart in Row 4.

Thus overall, the empirical results confirm the theory that option returns help explain securities returns, and imply that investors see options as instruments for hedging against nonnegative wealth levels.

3.4.3 GMM-SDF Estimations

To further check the robustness of the previous findings, we also performed Generalized Method of Moments (GMM) estimations within the Stochastic Discount Factor (SDF) framework outlined in Section 3.3.1. The advantage of a GMM approach is that it allows the estimation of model parameters in a single pass, thereby avoiding the error-in-variables problem in Fama-MacBeth kind of two-pass regressions. Another advantage of GMM, is that it is extremely general in its assumptions and can be applied to all classes of assets.

The set of equations for the method of moments for unconditional versions of C-CAPM is $E\left[\left(1+R_{j,t+1}\right)m_{t+1}\right]=1$, where $R_{j,t+1}$ is the net return for a risky asset j, and $m_{t+1}=\gamma_0+\gamma_1 c\widehat{a}y_t+\eta_o^T f_{t+1}$. The vector of coefficients for fundamental factors is subsets of $\eta_o=\left(\eta_{0,\Delta c},\eta_{0,c_{at}},\eta_{0,c_{in}},\eta_{0,c_{out}},\eta_{o,p_{at}},\eta_{o,p_{in}},\eta_{o,p_{out}}\right)$, and f_t is as given in i, or ii.

The model for the method of moments for conditional versions of C-CAPM is $E_t\left[\left(1+R_{j,t+1}\right)m_{t+1}\right]=1$, where $m_{t+1}=\gamma_0+\gamma_1 cay_t+\eta_0^T f_{t+1}+\eta_1^T cay_t f_{t+1}$. The vector of coefficients for fundamental factors is the subset of $\eta_o=\left(\eta_{0,\Delta c},\eta_{0,c_{at}},\eta_{0,c_{in}},\eta_{0,c_{out}},\eta_{o,p_{at}},\eta_{o,p_{in}},\eta_{o,p_{out}}\right)$, and the vector of coefficients for scaled factors is subsets of $\eta_1=\left(\eta_{1,\Delta c},\eta_{1,c_{at}},\eta_{1,c_{in}},\eta_{1,c_{out}},\eta_{1,p_{at}},\eta_{1,p_{in}},\eta_{1,p_{out}}\right)$. f_t is as given in iii, or iv.

Table 3.6 presents results of the above GMM-SDF estimations for the 4 models tested. Looking at Row 1, we see that the change in log-consumption does not play a role in constructing the SDF. It has an insignificant coefficient, and furthermore, the pricing errors with this model are significantly different from zero. Row 2 presents the results for the conditional C-CAPM. Consistent with the results of Fama-MacBeth estimations, we can see that the the coefficient of log aggregate consumption scaled with cay_t is significant, thus it is an important variable in the construction of the SDF. However, although the pricing errors are lower than its unconditional counterpart, they are still significantly different from zero.

Rows 3 and 4 present the SDF coefficient estimates for the unconditional C-CAPM when call and put returns are included, respectively. Consistent with Fama-MacBeth regressions, we see significant coefficients for the returns of at-the-money calls, and out-of money puts. The pricing errors of the model using call returns are lower than its counterpart using put returns. The errors using the HJ weighting matrix are still far from zero, but when the identity matrix is used, pricing errors are within 20 percent limit of not rejecting that they are equal to zero.

Next, we check the explanatory power of models which are relevant to the theory outlined here. The results in Rows 5 and 6 present the estimated SDF coefficients for the conditional model using call and put returns, respectively. To summarize, in both versions of the model, at-the-money call, out-of-money puts, scaled consumption, and scaled at-the-money calls appear to be significant factors in constructing the SDF. Furthermore, the pricing errors estimated by using the identity matrix are significantly lower than all the other models presented, thus doing a better job in pricing.

TABLE 3.6
GMM-SDF Estimations

			Fundamental Factors							Scaled Factors								
	γ_0	γ_1	$\eta_{o,\Delta c}$	$\eta_{o,c_{at}}$	$\eta_{o,c_{in}}$	$\eta_{o,c_{out}}$	$\eta_{o,p_{at}}$	$\eta_{o,p_{in}}$	$\eta_{o,p_{out}}$	$\eta_{1,\Delta c}$	$\eta_{1,c_{at}}$	$\eta_{1,c_{in}}$	$\eta_{1,c_{out}}$	$\eta_{1,p_{at}}$	$\eta_{1,p_{in}}$	$\eta_{1,p_{out}}$	**HJ dist.**	**HJ dist. (id)**
Row1	1.32		-81.21														1.1131	0.0378
	(29.68)		(-0.59)														(0.00)	(0.72)
	(0.00)		(0.56)															
Row2	1.29	1.90	-17.30							-258.62							1.1093	0.0349
	(14.82)	(1.08)	(-0.31)							(-3.39)							(0.010)	(0.75)
	(0.00)	(0.28)	(0.76)							(0.00)								
Row3	1.50		-142.39	0.71	0.23	0.24											0.9680	0.0248
	(3.66)		(-1.44)	(2.35)	(0.82)	(1.10)											(0.17)	(0.80)
	(0.00)		(0.16)	(0.02)	(0.42)	(0.28)												
Row4	2.22		-358.30				0.70	-0.56	-0.57								1.0913	0.0230
	(4.69)		(-1.38)				(1.41)	(-1.36)	(-1.81)								(0.11)	(0.82)
	(0.00)		(0.17)				(0.16)	(0.18)	(0.08)									
Row5	3.00	8.31	-67.45	1.17	0.33	0.51				-86.71	-52.00	55.37	80.53				0.7839	0.0205
	(3.04)	(1.39)	(-0.42)	(2.11)	(1.11)	(1.33)				(-2.64)	(-1.87)	(0.66)	(1.32)				(0.24)	(0.85)
	(0.00)	(0.17)	(0.68)	(0.04)	(0.27)	(0.19)				(0.01)	(0.07)	(0.51)	(0.19)					
Row6	2.80	13.22	-298.75				-0.06	1.75	-3.23	-248.74				-15.17	58.43	-71.13	0.8459	0.0214
	(2.02)	(1.24)	(-1.25)				(-0.04)	(1.05)	(-2.42)	(-2.56)				(-0.33)	(1.39)	(-1.51)	(0.22)	(0.83)
	(0.05)	(0.22)	(0.22)				(0.97)	(0.30)	(0.02)	(0.01)				(0.74)	(0.17)	(0.14)		

Note. This table gives the estimates for the models of moments $E\left[\left(1+R_{j,t+1}\right)m_{t+1}\right]=1$, and $E_t\left[\left(1+R_{j,t+1}\right)m_{t+1}\right]=1$ for the unconditional and conditional versions of C-CAPM, respectively. $R_{j,t+1}$ is the net return for Fama-French's 25 size and book-to-market portfolios, and the data period is 1990:Q1-2006:Q1. For unconditional models, $m_{t+1}=\gamma_0+\eta_0^T f_{t+1}$, where $\eta_o=\left(\eta_{0,\Delta c},\eta_{0,c_{at}},\eta_{0,c_{in}},\eta_{0,c_{out}},\eta_{o,p_{at}},\eta_{o,p_{in}},\eta_{o,p_{out}}\right)$, or subsets of it, and f_t is given by i, or ii. For conditional models, $m_{t+1}=\gamma_0+\gamma_1 cay_t+\eta_0^T f_{t+1}+\eta_1^T cay_t f_{t+1}$, where $\eta_o=\left(\eta_{0,\Delta c},\eta_{0,c_{at}},\eta_{0,c_{in}},\eta_{0,c_{out}},\eta_{o,p_{at}},\eta_{o,p_{in}},\eta_{o,p_{out}}\right)$, and $\eta_1=\left(\eta_{1,\Delta c},\eta_{1,c_{at}},\eta_{1,c_{in}},\eta_{1,c_{out}},\eta_{1,p_{at}},\eta_{1,p_{in}},\eta_{1,p_{out}}\right)$, or subsets of them, and f_t is given by iii, iv. The model for the moments are estimated using the GMM approach with the Hansen-Jagannathan weighting matrix. The numbers in parentheses are the t-statistics and their associated p-values respectively. The minimized value of the GMM criterion function is the first item under the "HJ-dist.", with the associated p-values immediately below it. The final column reports HJ-dist. using the identity matrix as suggested by Lettau and Ludvigson (2001).

Overall, the conditional C-CAPM using option returns outperform all its conditional and unconditional counterparts with or without options, and presents confirmatory evidence regarding the predictions of the theory outlined.

3.5 CONCLUSION

In a multiperiod securities markets, where agents are able to trade risky securities at each period in time, we show that options are non-redundant securities due to the nonnegative wealth constraints that agents face in solving their optimal consumption-investment problem. The results are similar to that of Vanden's such that the representative agent holds the aggregate consumption plus options written on the aggregate consumption. However, the contributions are twofold. First on the theoretical side, due to the characteristic of multiperiod reconvening markets, the pricing agent's optimal portfolio leads to a multifactor conditional C-CAPM with option returns as factors. Second, on the empirical side, there have been no tests of asset pricing in the framework of conditional C-CAPM that includes option returns as explanatory variables. The model tested performs better than its conditional and unconditional counterparts, confirming the theory that option returns should turn up as explanatory variables in securities returns.

Merton stresses that "*the core of financial economic theory is the study of individual behavior of households in the allocation of their resources in an* *environment of uncertainty and of the role of economic organizations in facilitating these allocations*". Thus, the theory outlined and the findings presented here have important implications both for improving the allocational efficiency of resources in the economy, for asset pricing, and for capital markets theories.

CHAPTER 4

CONCLUSION

This thesis explores the nonredundancy and allocational role of options in the context of volatility risk, and nonnegative wealth constraints. Options have been at the heart of many theoretical, and empirical research. We believe that a better understanding of the conditions that lead options to become nonredundant securities have important consequences for asset pricing, portfolio management, and capital markets theories.

The empirical results and theory outlined can be summarized as follows:

i) Zero-beta at-the money straddles are good proxies for volatility risk.

ii) Straddle returns are important conditioning variables in constructing the SDF.

iii) Volatility risk is time varying. Firms with low market capitalizations (small firms) have negative volatility betas, whereas firms with high market capitalizations and low market-to-book values (big-growth firms) have positive volatility betas.

iv) When agents face nonnegative wealth constraints in a multiperiod securities market, options are held optimally in agents' portfolios.

v) The optimal portfolio held by the representative agent leads to a multifactor conditional C-CAPM, where option returns appear as factors.

vi) A conditional C-CAPM with option returns as factors perform superior compared to its conditional and unconditional counterparts.

vii) At-the-money calls and out-of-money puts are priced risk factors.

Overall the model developed and empirical findings support that options are nonredundant securities, and have an allocational role in the economy.

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