Fiscal efficiency, redistribution and welfare

Bilin Neyapti a,*, Zeynep Burcu Bulut-Cevik b

a Bilkent University, Ankara, Turkey
b METU, Ankara, Turkey

A R T I C L E   I N F O

Article history:
Accepted 23 May 2014
Available online 19 June 2014

Keywords:
Redistribution
Fiscal efficiency
Transfer rule

A B S T R A C T

The expanding literature on fiscal decentralization (FD) emphasizes the role of institutional mechanisms for FD’s welfare effects. We analyze the welfare effects of FD in case of a fiscal transfer mechanism that punishes inefficiency in tax collection and compensates for local income deficiency. In addition, a portion of transfers is earmarked for investment. Given a level of FD and these rules, the representative local government chooses its tax collection effort to maximize local utility. The solution of the model reveals that the stricter the redistributive rule, the higher are steady-state fiscal efficiency and welfare. While the effectiveness of the redistributive parameters increases with centralization of the revenue pool, it decreases with the tax rate. Both welfare and income distribution, on the other hand, improve with the degree of revenue centralization and the tax rate. Besides, fiscal efficiency and redistribution decrease with investment-earmarked transfers.

© 2014 Elsevier B.V. All rights reserved.

1. Introduction

Fiscal decentralization (FD), defined as the devolution of fiscal power and responsibilities from the central government toward local governments, has been in practice by varying degrees in a growing number of countries. As a mechanism theorized to promote fiscal efficiency, FD is considered to contribute to social welfare. Following the seminal work of Oates (1972), the literature has discussed the potential advantages and disadvantages of FD widely; the consensus emerging from this literature is that the effectiveness of FD depends on various related institutional and structural factors (see, for example, Oates, 1999; Tanzi, 2000; De Mello, 2000; Bouton et al., 2008; and Neyapti, 2004, 2006, 2010). Among the growing number of studies that investigate the effects of FD, those that are of particular interest for the current study focus mainly on the growth implications of FD, and show mixed evidence at that (see, for example, Davoodi and Zou, 1998; Lin and Liu, 2000; Martinez-Vazquez and McNab, 2006; Thiessen, 2003). More recently, a number of studies focus on the welfare implications of FD and emphasize the significance of the various attributes of redistributive mechanisms for effective implementation of FD (see, for example, Sanguinetti and Tomassi, 2004; Stowhase and Traxler, 2005; Akin et al., 2010).

The current paper contributes to this literature by providing a formal analysis of the welfare implications of a fiscal institutional mechanism defined by a redistributive rule, an investment rule, and FD. By doing this, we address three crucial aspects of fiscal institutional design that complement each other and are considered increasingly in the recent policy reform agendas.1

In view of the large vertical and horizontal gaps in both less developed and developed countries,2 redistribution remains to be a key issue of fiscal policy and redistributive rules are therefore an important aspect of institutional design.3 Favorable macroeconomic effects of fiscal rules have recently been discussed by Alesina and Bayoumi (1996) and by the IMF (2009). In a theoretical paper, Sanguinetti and Tomassi (2004) demonstrate that a redistributive rule is preferred to discretion for local governments to attain fiscal efficiency.4 In a recent study, Akin et al. (2010; ABCN henceforth) investigate the effectiveness of FD considering a transfer mechanism that both punishes inefficiency in tax collection and compensates for the deviation from target income levels.5 The authors show that, under such a redistributive rule, FD increases fiscal efficiency provided

1 Neyapti (2013) provides empirical evidence for the significant impact of fiscal rules on the effectiveness of FD.2 Although developing countries have greater vertical gaps than developed countries on average, even in developed countries that are federal states, such as Canada, Switzerland, US and Germany, central government transfers constitute 50% to 70% of local budgets.3 Boadway and Shah (2007) provide an extensive account of the issues regarding the design of intergovernmental transfers, stressing the importance of assessing local expenditure needs and revenue capacities.4 The authors argue that discretionary redistribution provides insurance against large local shocks, but is not preferable under high degree of FD.5 Ma (1997) points out that among the transfer systems observed in practice, those that take into account both revenue capacities and expenditure needs are the most developed, although also the most demanding, ones.
that budget constraint applies. Neyapti (2013) shows empirically that fiscal rules enhance the fiscal disciplinary effects of FD. These findings are relevant not only for fiscal institutional design in given economy but also for an economic union, for which an exemplary case is the recent economic crises in the EU where fiscal decentralization is not accompanied by well-enforced fiscal rules.6

Welfare implications of FD have been investigated in several recent theoretical studies. Lockwood (2002, 2008) employs political economy models to show that the decentralization theorem fails under fairly unrestrictive conditions. Bataglini et al. (2010) propose a dynamic behavioral model to investigate the extent of the free-riding problem, where the central versus decentralized decision making is analyzed with regard to the investment and consumption choices. The authors conclude that the mechanism characterized by the central government decision on investment and redistribution is superior to the decentralized decision in terms of the steady state levels of investment and the public good. The results are supported by an experimental analysis and shed light on the dynamic aspects of public good provision. Chu and Yang (2012) investigate the growth and welfare implications of FD, vis a vis the dynamic aspects of public good provision. Chu and Yang (2012) in terms of the steady state levels of investment and the public good. The rest of the paper is organized as follows. Section 2 describes the model. Section 3 discusses the comparative statistics and reports the results of the simulation analysis. Section 4 concludes.

2. The model

We consider a representative local government $i$ ($i = 1...n$) that maximizes its lifetime total utility derived from the spending of the local private ($Ci$) and government sectors ($Gi$) by choosing the level of efficiency in tax collection: $A_i$:

$$\max_{A_i} \sum_{t=0}^{\infty} \rho^t \left( \alpha \ln C_{i,t} + \beta \ln G_{i,t} \right)$$

where $\rho$ is the discount factor, which is assumed to be constant across the $n$ local governments, $\alpha$ and $\beta$ represent the utility weights of the private and government spending that are given by:

$$C_{i,t} = (1-t_i)y_{i,t}, \text{where } t_i = tA_i$$

$$G_{i,t} = (1-c_i)t_iy_{i,t} + \gamma TR_{i,t}$$

where $y_i$ stands for (per capita) income in the ith locality, $t_i$ is the effective tax rate for local government $i$, $\gamma$ is the tax collection effort (where $0 < \gamma_i \leq 1$). Private consumption is equal to the after-tax income, and there are no private savings. $c_i$ stands for the share of tax revenues accruing to the central government; hence $(1 - c_i)$ represents the degree of fiscal (revenue, to be specific) decentralization, which is assumed to be given exogenously. Given $t_i$, which is determined centrally and is assumed to be uniform across localities. Eq. (3) stands for the budget constraint of local government $i$. The term $\gamma TR_i$ in Eq. (3) implies that $\gamma$ portion of transfers is declared centrally to be used as part of the local governments’ current spending.12 The remaining transfers, $\gamma(1 - \gamma)TR_i$, are invested locally, by the local governments. Hence, the level of investment in region $i$ is given by: $I_{i,t} = (1 - \gamma)TR_{i,t}$. Note that total local spending for locality $i$ ($C_{i,t} + G_{i,t} + I_{i,t}$) is equal to the portion of after-tax income that remains in the locality: $(1 - ct_i)y_{i,t} + \text{the transfers received } (TR_{i,t})$. Hence, total local spending is not necessarily

---

6 A fiscal union for the EU members has been discussed since the 1970 Werner Report.

7 The theorem, due to Gates (1972) states that decentralizing tax collection and public goods provision is welfare-enhancing especially when regions are heterogeneous and the spillovers are small.

8 Petchey and MacDonald (2007) analyze the effects of conditional grants for capital accumulation in South Africa.

9 Lack of coordination between the central and local governments under FD is commonly argued to lead to tax competition and underprovision of the public good. Wilson (1999) argues that in case regions fail to internalize their externalities fully, a corrective subsidy system improves efficiency, similar to the role our transfer rule plays in the current model. Chu and Yang (2012) also suggests combining FD with a central coordination device of fiscal authority.

10 The finding that centralization is associated with higher efficiency is not surprising in the case of lack of major heterogeneity, which only takes the form of income differentials in the current model.

11 One may consider that both central and local governments collect the revenues from a locality where the shares of each are $c$ and $(1 - c)$, which would be equivalent to the current setup with the additional assumption that A’s are identical under the two regimes. Alternatively, Aslim and Neyapti (2013) presents of politico-economic model where $c_i$ is determined endogenously.

12 This type of transfers, where the end use is pre-determined when transfers are disbursed, is referred to as directed- or closed-ended budget transfers.
equal to local income \((y_{i,t})\) because net transfers \((T_{R,t} - c_{t}, y_{i,t})\) can be positive or negative.\(^{13}\)

A representative local government faces a pre-determined rule that redistributes the central pool of revenues \((T_{R} = \Sigma c_{t}, y_{i,t})\) by both punishing the inefficiency in tax collection and compensating for the deviation from the target income level \((y^*_{i,t})\).\(^{14}\):

\[
T_{R,t} = p \times y_{i,t} (A_{i,t} - 1) + m (y^*_{i,t} - y_{i,t}).
\]  

(4)

The rate of punishment for inefficiency in tax collection (that is when \((1 - A_{i}) < 0\)) is \(p\), and the rate of compensation of the deviation of income from its target level is \(m\). Assuming that there is no lack of information or informational asymmetry across the levels of the government, the first part of Eq. (4) implies that a locality that is not fully effective in tax collections \((A_{i} < 1)\) receives less transfers than is implied by the second part of the expression. The production function is of Cobb–Douglas type (Eq. (5)) and exhibits constant returns to scale, where technology is assumed to be fixed (and normalized to 1). The capital stock follows the usual accumulation rule, where capital accumulation occurs only through investment-ended transfers from central to local governments (Eq. (6)):

\[
y_{i,t} = k^0_{i,t} (where \(0 < c < 1\)),
\]

(5)

\[
k_{i,t} = (1 - \delta)k_{i,t-1} + (1 - \gamma)TR_{i,t-1} = (1 - \delta)k_{i,t-1} + L_{i,t-1}.
\]

(6)

where \(\delta\) is the rate of depreciation \((0 < \delta < 1)\) and \(k\) is the per capita level of capital. In brief, local governments maximize (1) subject to (4) and (6), given Eqs. (2), (3) and (5). After substituting Eq. (5) into Eqs. (2) and (3), \(\text{Eq. (4)}\) into Eqs. (3) and (6), and then Eqs. (2) and (3) into Eq. (1), the local government problem can be written as:

\[
\max_{\lambda_i} \sum_{t=1}^{\infty} \left( \alpha \ln \left( 1 - t_1 |k^0_{i} \right) + \beta \ln \left( 1 - c_{t} |k^0_{i} + \gamma m |k^0_{i} + \gamma m |y_{i} - k^0_{i} \right) \right)
\]

(1')

subject to

\[
k_{i,t} = (1 - \delta)k_{i,t-1} + (1 - \gamma) \left( pt_{t} k^0_{i} (A_{i,t} - 1) + m (y^*_{i,t} - y_{i,t}) \right).
\]

(6')

The solution of the above problem yields the following first order condition (see Appendix 1.a):

\[
\rho^t \left[ \frac{\alpha k^0_{i}}{\gamma \eta} + \frac{\beta \eta}{\gamma \eta} \left( 1 - c_{t} |k^0_{i} + \gamma m |k^0_{i} \right) \right] - \lambda_i + \left( \frac{\alpha}{\gamma \eta} \frac{\partial k_{i,t}}{\partial \lambda_i} - \frac{\eta}{\gamma \eta} \frac{\partial k_{i,t}}{\partial \lambda_i} \right) + \left( \frac{\beta}{\gamma \eta} \frac{\partial k_{i,t}}{\partial \lambda_i} + \frac{\partial y_{i,t}}{\partial \lambda_i} \frac{\partial k_{i,t}}{\partial \lambda_i} \right) \right) = 0
\]

(7)

\[
A_{i,t} \leq 1 ; \quad \lambda_i \geq 0 ; \quad \lambda_i (1 - A_{i}) = 0
\]

(8)

where \(\lambda_i\) is the value of Lagrange multiplier at time \(t\). The second order condition for the maximization problem are also satisfied (see Appendix 1.b). Based on the complementarity slackness condition, there are two cases arising, the first one being \(A_{i} < 1\) (and \(\lambda_i = 0\)) and the other is the full effort case \((A_{i} = 1)\). We consider the first case to be the interesting one, although the latter case is also reviewed in the Appendix 1.

13 For the aggregate, however, \(\Sigma_{i} y_{i,t} - c_{t}, y_{i,t} = \Sigma_{i} c_{i} y_{i,t}\) for all \(t\).

14 as has been formulated originally by ABCN.
indicating that the second region is more capital-intensive than the first. The following general government budget constraint applies:

\[
TR_i = (TR_{x,t} + TR_{z,t}) \leq c\left(A\left(y_{1,t} + y_{z,t}\right)\right),
\]

(12)

which states that total transfers cannot exceed total tax revenues of the central government; hence an individual locality may receive positive or negative transfers. The left side of the inequality also satisfies Eq. (4).

Simulations of the steady state solutions yield in 6285 data points that meet the admissibility constraints.20 Based on these data, the responses of the optimal steady state values \(A_t; R_t; y_t; U_t\) to the fiscal parameters \(p, m, \gamma, c, \theta\) are analyzed and are reported in the following table:

A crucial finding reported in Table 2 is that increasing both the punishment rate, \(p\), and the income compensation rate, \(m\), increases the steady-state level of fiscal effort, or the tax collection efficiency, unambiguously. This observation supports the efficiency enhancing effect of the suggested redistributive rule. The positive effect of \(p\) is as expected, and that of \(m\) appears counterintuitive at first but can be explained as follows. When the income compensation rate is high, the steady-state level of transfers is also high, which gives incentive to the local governments to increase their effort to get a larger share of the increasing pool. This can be interpreted as the substitution effect of the income-compensation parameter.

**Result 1.** The parameters of the suggested redistribution rule all have an unambiguous positive effect on fiscal efficiency: \(\partial A/\partial p > 0; \partial A/\partial m > 0;\) and \(\partial A/\partial (1 - \gamma) > 0\) (see Appendix 2).

The positive effects of the redistributive parameters on efficiency (total effort of the two regions) are diminishing in the respective parameters, although they increase in the degree of centralization, \(c\). This points at the significant role of fiscal rules in alleviating the public good problem; in the presence of a fiscal rule, increased common pool of revenues incentivize local governments to increase their efficiency in order to obtain a greater share of that common pool. By contrast, the efficiency effects of the fiscal rule parameters decrease with the tax rate as the local governments try to counterbalance the negative effect of increasing the tax rate on local disposable incomes and thus utility derived from private consumption.

**Remark 1.** The more centralized the fiscal revenues, the more effective is the proposed fiscal rule in increasing fiscal efficiency.

Besides its indirect (second order) effects on fiscal efficiency, the tax rate also has a direct negative effect on the tax collection effort, for the reason just explained. This negative relationship increases in \(c\), but decreases in \(t\); the reason for the first is that as the centralized pool of revenues increases, ceteris paribus, reducing fiscal effort compensates for the negative effect of increasing taxes on local incomes without reducing the level of transfers that may now be received in through income compensation given the hard budget constraint (Eq. (12)). The observation of the diminishing negative effect of the tax rate on tax collection effort points at the decreasing role of income compensation in overcoming the increase in the disutility from punishment due to the reduction in the effort in tax collection; tax effort therefore falls at a decreasing rate as the tax rate rises.

**Result 2.** The negative effect of taxes on fiscal efficiency increases with the centralization of revenues but decreases in the tax rate. (see Appendix 3).

The rate at which transfers are earmarked for investment \((1 - \gamma)\) has a negative effect on both tax collection effort and transfers.21 While the reason for this may not be obvious, it reflects the long-term positive effect of increasing investment, ceteris paribus, on income (remember that investment ended transfers is the only way for capital accumulation), which in turn allows for lower steady-state effort to attain the same level of utility, which can be viewed as the wealth effect.22 Since local governments receive utility both from their own (current and investment) spending and from the private sector spending, an increase in the long-term capital stock and thus disposable income of the private sector, local governments can afford to exert lower tax effort to attain the same level of utility as before. The same argument can be used to explain the negative effect of the initial capital level on the steady-state level of tax effort.

**Result 3.** Investment-ended transfers are associated with long-term efficiency in tax collection (see Appendix 4).

A testable implication arising from the above results is that fiscal rules are likely to be more effective in countries that centralize fiscal revenues.23 Given a level of decentralization and fiscal rule, increasing the rates of tax and investment-ended transfers, however, may not contribute to efficiency.

Another important implication of the redistributive mechanism studied is in regard to income distribution (measured here as \(Y_1 / Y_2\)). Fairness in income distribution increases in the punishment rate; with regard to the rate pervasiveness (rates of taxes and their centralization), however, the relationship is negative.24 Similarly, welfare (sum of regional utilities) decreases in the pervasiveness of the central government. In addition, income distribution is observed to worsen in \(c\). This is because the volume of transfers arising from punishment also increases in \(c\), which is the moral hazard effect of increased income. It is worth noting that

20 The number increases with the reduced intervals chosen for the parameter values in the simulation. Admissibility criteria consist, in addition to the parameter constraints listed in Table 1, of the non-negativity of capital, consumption, and government spending, in addition to (0.1) bound for the tax collection effort.

21 This result may not hold when a consumer optimization problem is added to the current framework, which is beyond the scope of the current paper however.

22 The generality of this result, however, may be questioned on the grounds that the current model assumes away heterogeneity in preferences.

23 Neyapti (2013) shows that the effectiveness of FD increases with fiscal rules. As better fiscal rules \((FR)\) may be implemented in more decentralized countries, the findings of this paper suggest that the same quality of FR may perform better the more centralized is the fiscal policy.

24 The finding that decentralization leads to greater equality than the case of centralization (compare Block 1 to Column 3) is inconsistent with the finding of ABCN (2011); using the same transfer rule in a static setup, ABCN shows that centralization leads to higher equality whereas decentralization leads to greater efficiency than the other. The difference in the findings points at the importance of taking the long-term perspective in designing institutions to achieve fiscal efficiency.
when \( \theta \) is less than or equal to 0.3 (the US figure) the admissible optimal steady state values of the model variables are associated with the rate of punishment of only 1.

**Remark 2.** The less pervasive the central government, the greater the welfare and the distributional effects of the proposed redistributive rule (see Appendix 5).

**Remark 3.** Under the proposed redistributive rule, conditions for improving welfare and income distribution are the same, whereas they are opposite of those for efficiency: increasing tax rate reduces both fiscal efficiency and welfare and worsens income distribution. FD, on the other hand reduces efficiency but improves welfare and income distribution.

Simulations also reveal that transfers fall in the degree at which transfers are directed to investments \((1 - \gamma)\). Transfers also decrease in \( p \) but increase in \( m \), as consistent with the redistributive rule (Eq. (4)).

### 3.1. Some policy exercises

Table 3 reports the simulated values of the aggregate (summed across the regions) tax-collection effort; income and welfare corresponding to some benchmark parameter values. To get a better sense of these parameters one can note that the average tax rate is at most 0.5 or 0.6 in the most developed countries, and much less in developing countries (such as 0.2 in Mexico). The rate of revenue decentralization in even the most decentralized developed countries, say in Switzerland, is less than 0.5.\(^{25}\) Hence, a pervasive central government amounts to either highly centralized revenue collection, with or without a high average tax rate, or high average tax rate plus reasonably high revenue decentralization. In that regard, Denmark’s (where \( c = 0.3, r = 0.5 \)) central government is not pervasive, whereas China’s is (where \( c = 0.95 \)).

In Table 3, the calibration results for \( A, Y \) and \( U \) are the sum over the two regions. Hence, one can interpret the Table as the following. Line i, representing relatively low \( c \) and \( t \) values, may stand for a country like Mexico in case it applies a full measure of the proposed redistributive rule (where \( p = m = 1 \)). This can be contrasted with lines ii and v, where income compensation is much more in cases of low \( (\text{ii}) \) and high \( (\text{iv}) \) income shares of capital. Of these, highest effort obtains in the first, and lowest welfare in the last case. Line vi may be viewed to stand for the case of China (with high revenue centralization but low tax rates), if China adopted high punishment rate but low income compensation. This case, which only differs in \( c \) when compared to the line above, yields higher steady-state effort as well as higher welfare.

Comparing lines ii and vi (or lines vii and viii) reveals that increasing centralization, ceteris paribus, increases fiscal efficiency but not welfare. Comparing lines ii to iv shows the effect of increasing the tax rate, which is negative on both efficiency and welfare. Finally, comparing lines ii and iii shows that decreasing the portion investment-earmarked transfers (increasing \( \gamma \)) leads to higher tax effort. These observations are all in lines with the general results reported earlier.

The foregoing observations enable important policy recommendations regarding fiscal institutional design: the proposed transfer rule yields higher welfare under decentralization than centralization, although it yields higher fiscal efficiency under a centralized system than under decentralization.

### 4. Concluding remarks

This paper presents a dynamic model to explore the efficiency of a redistributive rule that punishes the lack of tax collection efficiency of local governments and compensates for the deviation of local incomes from a target level. The redistributive rule is coupled by a policy of directed transfers that allocates an exogenously specified (by the central government) portion of the transfers (determined according to the aforementioned rule) to local capital accumulation.

The findings based on the comparative statics of the model’s solution for the steady state reveal that both punishment of fiscal inefficiency and compensation of regional income deficiency are essential for increasing fiscal discipline and welfare. However, while centralization of revenues improves fiscal efficiency, it deteriorates both welfare and income distribution. Given the proposed redistributive rule, increasing the tax rate and investment-ended transfers, especially in cases of highly centralized revenues, do not contribute positively to efficiency or welfare. The findings shed light to the contradiction between the original arguments of the decentralization literature and the recent literature that formally demonstrates the failure of the decentralization theorem, by demonstrating the role of the fiscal rules for welfare implications of fiscal decentralization.

The findings bear interesting implications for fiscal discipline in economic unions as well as in a single economy with large vertical and horizontal imbalances that necessitate an effective transfer mechanism. In the case of the European Union, for example, where, akin to different local decision making units of a fiscally decentralized country, the union faces a single central bank but independent fiscal authorities, the importance of imposing a fiscal rule for redistribution across the states has become even more evident with the recent crises. As future potential research areas, the observations based of the proposed model can be tested empirically. Further extensions could involve modifications of the current framework to incorporate the political economy aspects, such as modeling the game between the central and the local governments, and the optimal choice of FD given the degrees of political fragmentation and spillover effects, which are among the ongoing projects of the authors of the current study.

### Appendix 1. Optimality checks

#### A1. First order analysis

The constraint qualification is obviously satisfied since the constraints are linear. Also note that objective function is concave. So we can apply Kuhn Tucker’s theorem. The Lagrangean expression is obtained after substituting Eq. (4) into Eq. (3) and substituting Eqs. (2) and (3) into Eq. (1):

\[
L = \sum_{t=1}^{\infty} \left( \alpha \ln \left( y_{1t} - \tau A_{1t} y_{1t} \right) + \beta \ln \left( (1-c)\tau A_{1t} y_{1t} + \gamma (\rho Y_{1t} (A_{1t} - 1) \right) + m(y_{1t} - y_{1t}^*) \right) + \lambda_t (1-A_{1t}) \right).
\]

<table>
<thead>
<tr>
<th>( p )</th>
<th>( m )</th>
<th>( c )</th>
<th>( t )</th>
<th>( \gamma )</th>
<th>( \theta )</th>
<th>( A )</th>
<th>( Y )</th>
<th>( U )</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>1</td>
<td>1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>1.47</td>
<td>0.2256</td>
</tr>
<tr>
<td>ii</td>
<td>1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>0.90</td>
<td>0.2256</td>
</tr>
<tr>
<td>iii</td>
<td>1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>1.08</td>
<td>0.2256</td>
</tr>
<tr>
<td>iv</td>
<td>1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.7</td>
<td>0.5</td>
<td>0.1</td>
<td>0.86</td>
<td>0.2256</td>
</tr>
<tr>
<td>v</td>
<td>1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.5</td>
<td>0.3</td>
<td>0.90</td>
<td>0.0029</td>
</tr>
<tr>
<td>vi</td>
<td>1</td>
<td>0.1</td>
<td>0.9</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>1.79</td>
<td>0.2256</td>
</tr>
<tr>
<td>vii</td>
<td>1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
<td>0.9</td>
<td>0.3</td>
<td>1.23</td>
<td>0.0029</td>
</tr>
<tr>
<td>viii</td>
<td>1</td>
<td>0.2</td>
<td>0.9</td>
<td>0.5</td>
<td>0.9</td>
<td>0.3</td>
<td>1.97</td>
<td>0.0029</td>
</tr>
<tr>
<td>ix</td>
<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.5</td>
<td>0.14</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Maximizing this expression yields the following first order conditions with respect to \( A_t \):

\[
\rho \left[ \frac{\partial (c_t)}{\partial A_t} \right] = \lambda - \rho \left[ \frac{\partial (c_t)}{\partial A_t} \right] + \rho \left[ \frac{\partial (c_t)}{\partial A_t} \right] \lambda
\]

\[
\rho \left[ \frac{\partial (c_t)}{\partial A_t} \right] = \lambda - \rho \left[ \frac{\partial (c_t)}{\partial A_t} \right] + \rho \left[ \frac{\partial (c_t)}{\partial A_t} \right] \lambda
\]

with respect to \( \lambda \): \( A_t \leq 1 \), \( \lambda_t \geq 0 \), \( \lambda_t (1 - A_t) = 0 \).

Case 1. If \( \lambda_t = 0 \), then \( A_t \leq 1 \) (given the constraint \( A_t \leq 1 \) in Eq. (8)). Substituting (2), (3), (4) and Eq. (17) into the above Lagrangean function yields the optimum level of \( A_t \), an explicit solution of which cannot be reached, however. Instead, Matlab is used to obtain symbolic solutions. 26

Case 2. Taking \( \lambda_t > 0 \) implies that \( A_t = 1 \), based on which Eq. (4) becomes:

\[ TR_t = \frac{m}{(1 + \gamma)}k^{\beta}_{t-1} - k^{\beta}_{t}. \]

At the steady state, \( \bar{A} = 1 \); and \( \bar{\lambda} = (1 - \gamma) \frac{1}{\bar{\lambda}} \).

In the full effort case, steady state capital decreases in \( \gamma, \delta, \) and \( \theta \) (because the term in parentheses is less than one) and increases in \( m \) unambiguously, however, it is not plausible to take Case 2 as optimal in any given period due to the incentives local governments usually face to spend less than full tax collection effort.

A2. Second order analysis

Because the constraint qualification and first order condition are satisfied, then, in a generic form:

\[ D^2L(x, \lambda) = D^2f(x) + \sum_{i=1}^{j} \lambda_i D^2g_i(x) \]

where \( \lambda \) is the optimal choice variable, \( f \) is the objective function and \( j \) is the number of constraints.

Since the model’s inequality constraint is linear, its second order derivative is zero, that is, the second part of the above equation disappears.

So,

\[
\rho^2 \left[ \frac{\partial^2 (c_t)}{\partial A_t^2} \right] = \frac{\alpha}{\gamma} \left[ -\delta_t k_t^\beta + \frac{\beta}{\gamma} \left( (1 - c_t) k_t^\beta + \gamma t k_t^\beta \right) \right] - \frac{\rho}{\gamma} \left[ \frac{\partial (c_t)}{\partial A_t} \right] \lambda
\]

\[
\rho^2 \left[ \frac{\partial^2 (c_t)}{\partial A_t^2} \right] = \frac{\alpha}{\gamma} \left[ -\delta_t k_t^\beta + \frac{\beta}{\gamma} \left( (1 - c_t) k_t^\beta + \gamma t k_t^\beta \right) \right] - \frac{\rho}{\gamma} \left[ \frac{\partial (c_t)}{\partial A_t} \right] \lambda
\]

Since \( \frac{\partial^2 (c_t)}{\partial A_t^2} = 0 \), \( \alpha > 0 \), \( \beta > 0 \) and the other terms are in square forms then \( D^2L(x, \lambda) < 0 \). Hence, the second order condition is satisfied unconditionally.

---

26 The above Lagrangean equation is nonlinear; MATLAB solves for a nonlinear function if there is a symbolic solution. For this specific problem, the solution for the optimal tax effort exists, which is too long to report here (available upon request).

27 The capital accumulation rule yields the steady state value of capital; but since no explicit can be obtained, it is calculated numerically.
Appendix 3. Pervasiveness and the response of tax effort to the tax rate

Appendix 4. Investment-ended transfers \((1 - \gamma)\) (the vertical axis should be read as negative)

Appendix 5. Income distribution, welfare and state pervasiveness

References


