Exact Analysis of Offset-Based Service Differentiation in Single-Channel Multi-Class OBS

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***Abstract*—We study a multi-class optical burst switching (OBS) node using the horizon reservation scheme. Multiple traffic classes are differentiated using different offset times per class. Assuming Poisson burst arrivals and phase-type distributed burst lengths, we exactly solve for per-class blocking probabilities using the well-known theory of feedback Markov fluid queues.**

***Index Terms*—OBS, horizon reservation, offset-based service differentiation, Markov fluid queues.**

I. INTRODUCTION

OPTICAL Burst Switching (OBS) is a candidate architecture for the future optical Internet that is based on aggregation of client packets into so-called bursts at the edge of the OBS domain. When a burst is formed, its reservation request is signalled out of band using a burst control packet (BCP). The burst is transmitted after an offset time and is transported in the optical domain. When a BCP arrives at the OBS node, the switch configuration is initiated for the corresponding burst. Different reservation models are proposed for the timing and duration of the reservation. Delayed reservation schemes, such as horizon [1], that do not perform any void filling, are simpler to implement than void filling-based JET (Just Enough Time) schemes [2]. In the horizon scheme, we only keep track of the channel horizon that is the earliest time after which there is no planned use of the channel.

In OBS, service differentiation among traffic classes can be achieved by assigning additional offset values to classes that require preferential treatment. Other methods also exist, see for example [3]. There are analytical models of multi-class OBS systems that rely on the assumption of Poisson arrivals and exponentially distributed burst lengths. In [4], an approximative model is proposed that assumes complete isolation between classes for multi-channel OBS. Per-class loss probabilities for multi-class single-channel JET are approximately calculated in [5] under low offered load assumption and for generally distributed burst lengths. Focusing on horizon-based single-channel OBS networks in this study, we improve upon the existing literature by proposing an exact solution while allowing more general phase-type distributed burst lengths. Our solution is based on the well-known theory of feedback Markov fluid queues (FMFQ) [6].

We provide the stochastic model in Section 2. A brief summary and notation for general FMFQs is given in Section 3. Our proposed solution is presented in Section 4. Numerical examples are given in Section 5 to verify the method. Finally, we conclude.

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II. STOCHASTIC MODEL

To model burst lengths, we use phase-type distributions (PH-type). Consider a Markov process on the states $\{1, 2, \ldots, m, m+1\}$ with initial probability vector $[v, 0]$, $v = [v_1, v_2, \ldots, v_m]$ and infinitesimal generator

$$Q = \begin{bmatrix} S & S^0 \\ 0 & 0 \end{bmatrix},$$

where S is an $m \times m$ nonsingular matrix, S^0 is $m \times 1$, $Se + S^0 = 0$, and e is a column vector of ones of appropriate size. The time till absorption into the absorbing state $m+1$ is a random variable X which is said to have a PH-type distribution with representation (v, S) whose distribution function is written as $F_X(x) = 1 - ve^{Sx}e,\ x \geq 0$.

We focus on a tagged output port of an OBS node comprising a single wavelength channel with the channel capacity normalized to unity. In our system, bursts are assumed to belong to one of the traffic classes in the class-set $\mathcal{I} = \{1, 2, ..., I\}$. We assume that burst (or BCP) arrivals destined to the tagged output port are Poisson with rate λ_i for class-i whose lengths, $1 \leq i \leq I$, are assumed to be modeled by a PH-type distribution characterized with the pair (v_i, S_i) with m_i transient states. Let $S_i^0 = -S_i e = [S_{i,1}^0, S_{i,2}^0, \ldots, S_{i,m_i}^0]^T$ and $v_i = [v_{i,1}, v_{i,2}, \ldots, v_{i,m_i}]$. Also let the $(k,l)^{th}$ entry of S_i be denoted by $\{S_i\}_{k,l}$.

A class-i BCP arrives at the OBS node on behalf of a class-i burst δ_i seconds prior to the arrival of the corresponding burst. The class-i offset δ_i is assumed to be deterministic. If the channel horizon is less than or equal to δ_i at the arrival epoch of a BCP belonging to a class-i burst, then the burst is admitted by reserving the channel for that burst and the channel horizon increases to a value equal to the sum of the burst length (in seconds) and the offset value δ_i. Otherwise, the corresponding burst is blocked (dropped). The channel horizon decreases at a unity rate between burst arrival epochs with a boundary at the origin. Moreover, without loss of generality we assume that inequality $\delta_i < \delta_j$ holds for all $i, j \in \mathcal{I}$ satisfying $i < j$.

To illustrate the operational policy for a two class OBS horizon system, we depict a sample path of the channel horizon process in Fig 1. We take $\delta_1 = 2$ and $\delta_2 = 4$. Burst lengths for class-1 and class-2 are 1 and 2, respectively. Class-1 BCP arrivals occur at $t = 1, 3, 8$ whereas class-2 arrivals at $t = 3.5, 5.5$, and 7. The BCPs arriving at $t = 7, 8$ are blocked since the channel horizon (denoted by $H(t)$) is strictly larger than the corresponding offset times at the arrival instants. Note

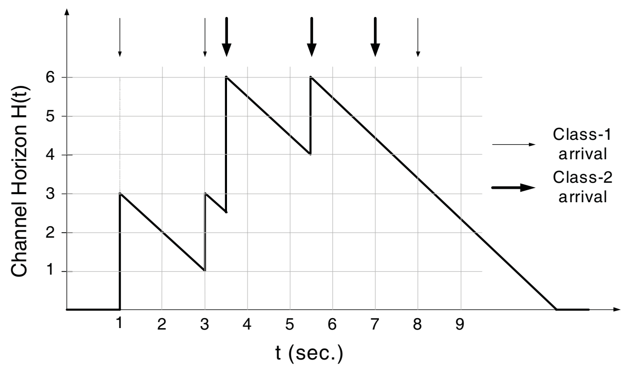


Fig. 1. Evolution of the channel horizon $H(t)$ for an example of a two-class OBS node.

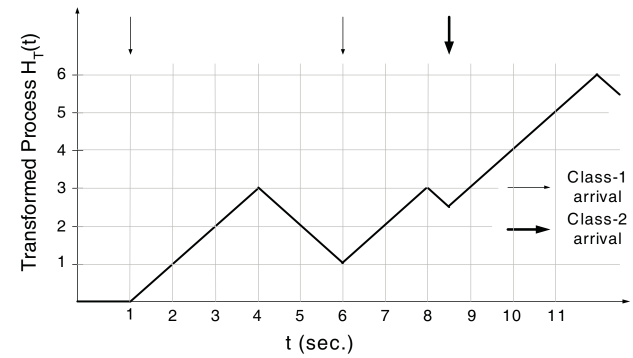


Fig. 2. Sample path for the transformed process $H_T(t)$ for the example of Fig. 1.

that an accepted burst leads to an immediate jump in $H(t)$ whereas a blocked burst does not have any effect.

III. Multi-regime Feedback Fluid Queues

We briefly describe the FMFQ model based on [6]. We let $C(t)$ denote the fluid level in the queue and $M(t)$ denote the state of the background process at time t. Thresholds for the FMFQ are $0 = T^{(0)} < T^{(1)} < \ldots < T^{(K)} = \infty$. The fluid queue is said to be in regime k (at threshold $T^{(k)}$) if $T^{(k-1)} < C(t) < T^{(k)}$ ($C(t) = T^{(k)}$). We assume that the background process $\{M(t); t \geq 0\}$ has a finite state space $\{1, 2, \ldots, M\}$. When the system is in regime k (at threshold $T^{(k)}$) then the background process $M(t)$ behaves according to a Markov process with generator $Q^{(k)}$ ($\tilde{Q}^{(k)}$). The drift (net rate of change of the queue) while at state m, $1 \leq m \leq M$, in regime k (at threshold $T^{(k)}$) is denoted by $r_m^{(k)}$ ($\tilde{r}_m^{(k)}$). We let $R^{(k)}$ ($\tilde{R}^{(k)}$) to be the diagonal matrix of drifts in regime k (at threshold $T^{(k)}$). The dynamics of the buffer content for the FMFQ is given by:

$$\frac{d\,C(t)}{dt} = \begin{cases} \max(0, \tilde{r}^{(0)}_{M(t)}) & \text{if } C(t) = 0, \\ r^{(k)}_{M(t)} & \text{if } T^{(k-1)} < C(t) < T^{(k)}, \\ \tilde{r}^{(k)}_{M(t)} & \text{if } C(t) = T^{(k)}. \end{cases} \tag{1}$$

Let $F_m(x, t)$ denote the joint transient probability distribution function $F_m(x, t) = Pr\{C(t) \leq x, M(t) = m\}$ for $1 \leq m \leq M$. The steady-state joint distribution function can then be defined by taking the limit $F_m(x) = \lim_{t \to \infty} F_m(x, t)$. A spectral solution to the steady-state behavior, i.e. $F_m(\cdot)$, $1 \leq m \leq M$, of the FMFQ is given in [6]. This method requires the solution of K eigenvalue problems for matrices of size M and the solution of a matrix equation of size at most KM. In this method, all eigenvalues for a given regime other than the ones at zero are assumed to be distinct [6].

IV. Proposed Solution

The basic idea is as follows. We like to use FMFQs to model multi-class OBS nodes. However, the channel horizon $H(t)$ can not be modeled directly by an FMFQ due to the jumps involved at arrival epochs. However, consider the transformed process $H_T(t)$ obtained through $H(t)$ by simply replacing the jumps in the sample path by a linear increase (say with unity rate) which makes it possible for us to use an FMFQ to describe the evolution of $H_T(t)$; see Fig. 2 for an evolution of $H_T(t)$ for the same example given in Fig. 1. Note that we do not allow new arrivals when $H_T(t)$ is increasing. We will then show that the steady-state behavior of the original process can be derived from that of the transformed process.

For this purpose, we first define $I+1$ regimes by defining $I+2$ thresholds $0 = T^{(0)} < T^{(1)} = \delta_1 < T^{(2)} = \delta_2 < ... < T^{(I)} = \delta_I < T^{(I+1)} = \infty$. We denote the state corresponding to a decrease in $H_T(t)$ by P. Moreover, for each class-i, we define m_i dummy states which we denote by D_i^j, for $1 \leq j \leq m_i$. We inherit the notation we used for FMFQs in Section 2 including the queue content $C(t)$, the background process $M(t)$, and the matrices $Q^{(k)}$, $\tilde{Q}^{(k)}$, $R^{(k)}$, and $\tilde{R}^{(k)}$. Moreover, we assume that the states P and D_i^j are the p^{th} and $d_i^{j\,th}$ states of the background process $M(t)$, respectively. Let $q_{i,j}^{(k)}$ and $\tilde{q}_{i,j}^{(k)}$ denote the $(i,j)^{th}$ entry of $Q^{(k)}$ and $\tilde{Q}^{(k)}$. Also note that the cardinality of the state space of this FMFQ is $M = 1 + \sum_{i=1}^{I} m_i$.

We now find the parameters of the FMFQ corresponding to the process $H_T(t)$. Note that $C(t)$ is to track the same path as $H_T(t)$ and therefore when the background process is in state P, we define the drift at that state as

$$r_p^{(k)} = -1,\ 1 \leq k \leq I+1, \tag{2}$$

$$\tilde{r}_p^{(k)} = \begin{cases} -1 & \text{if } k > 0, \\ 0 & \text{if } k = 0. \end{cases} \tag{3}$$

When a new class-i burst is admitted, then a transition occurs from state P to D_i^j with probability $v_{i,j}$. However, for a class-i burst to be admitted, the queue content should be less than δ_i. Therefore, for $1 \leq k \leq I$

$$\tilde{q}_{p,l}^{(k)} = q_{p,l}^{(k)} = \begin{cases} -\sum\limits_{i=k}^{I} \lambda_i & \text{if } l = p, \\ \lambda_i v_{i,j} & \text{if } l = d_i^j \text{ and } i \geq k, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Moreover,

$$q_{p,l}^{(I+1)} = 0,\ 1 \leq l \leq M, \tag{5}$$

$$\tilde{q}_{p,l}^{(0)} = q_{p,l}^{(1)},\ 1 \leq l \leq M. \tag{6}$$

If the background process is at state D_i^j, $C(t)$ increases with unity rate up to δ_i without any state transitions. Then, $C(t)$

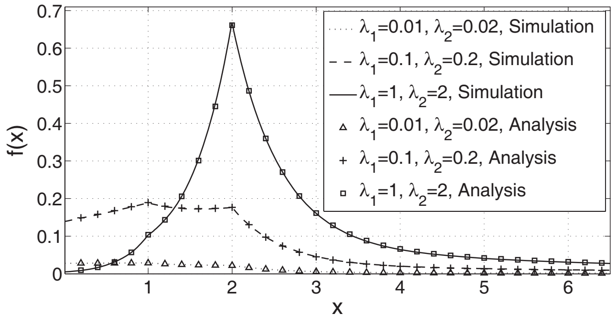
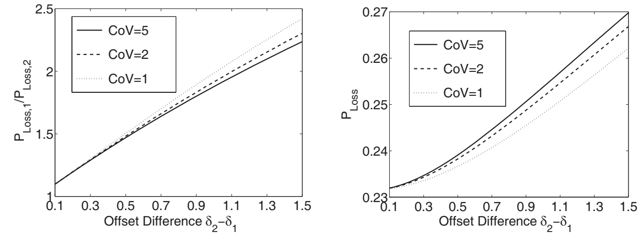


Fig. 3. The steady-state horizon pdf $f(x)$ of the two class OBS example



(a) $\frac{P_{Loss,1}}{P_{Loss,2}}$ as a function of $\delta_2 - \delta_1$ (b) P_{Loss} as a function of $\delta_2 - \delta_1$

Fig. 4. Performance of a two-class OBS system with respect to offset difference

holds on to its unity rate increase until the corresponding phase type distribution reaches its absorbing state. Therefore,

$$\tilde{r}^{(k-1)}_{d_i^j} = r^{(k)}_{d_i^j} = 1,\ 1 \le k \le I+1, \tag{7}$$

$$\tilde{q}^{(k-1)}_{d_i^j,l} = q^{(k)}_{d_i^j,l} = 0,\ 1 \le l \le M,\ 1 \le k \le i, \tag{8}$$

$$\tilde{q}^{(k-1)}_{d_i^j,l} = q^{(k)}_{d_i^j,l} = \begin{cases} S^0_{i,j} & \text{if } l = p, \\ \{S_i\}_{j,n} & \text{if } l = d_i^n,\ 1 \le n \le m_i \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

This concludes the characterization of the FMFQ describing the process $H_T(t)$. We can then solve for the steady-state joint distribution function $F_p(x)$ using the method of [6]. If we condition only on the P state then the steady-state distribution of the original process can be found. This is due to the observation that if we delete the intervals in Fig. 2 during which the queue is increasing, then we obtain the sample path given in Fig. 1. Mathematically,

$$F(x) = \lim_{t\to\infty} Pr\{H(t) \le x\} = F_p(x)/F_p(\infty). \tag{10}$$

Let $f(x)$ denote the corresponding density function such that $F(x) = \int_{-\infty}^{x} f(y)dy$. Moreover, due to Poisson arrivals and from the PASTA property, a class-i burst reservation request is blocked with probability $P_{Loss,i} = 1 - F(\delta_i)$.

V. Numerical Examples

Let m_X and c_X denote the mean and the coefficient of variation (CoV) of a random variable X. For given $m_X > 0$ and $c_X > 1$ based on empirical data, one can find a 2-phase PH-type distribution, i.e., hyper-exponential distribution, whose mean and CoV match to m_X and c_X, respectively [7]. Note that, for an exponential random variable X, $c_X = 1$. We use the fitting procedure of [7] in the numerical examples to follow. We first assume a two-class OBS system with parameters $\delta_1 = 1, \delta_2 = 2$. Moreover, burst lengths for both classes have a mean of 1 and CoV of 2. In Fig. 3, we compare the steady-state horizon pdf obtained using the method of this letter and simulations for light, moderate, and heavy loaded systems. We show that the results exactly match irrespective of the load. In the next example, we study the effect of the second order statistics of the distribution of burst lengths on loss probabilities. The system under study has $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, and $\delta_1 = 1$. Burst lengths of both classes possess the same distribution with unity mean but varying CoV =1, 2, and 5. In Fig. 4(a), we plot the loss probability ratio $P_{Loss,1}/P_{Loss,2}$ as a function of $\delta_2 - \delta_1$. We observe that the degree of loss differentiation increases with increased offset difference as expected but decreases as a function of the CoV of the burst lengths. In Fig. 4(b), we plot the overall loss probability P_{Loss} as a function of $\delta_2 - \delta_1$. It is clear that P_{Loss} increases with the offset difference due to the lack of a void filling mechanism. However, the increase in P_{Loss} is much higher for larger burst length CoV. We therefore conclude that second order statistics of the burst length is crucial in multi-class OBS systems. This result is important because several approximate methods in the literature use the Erlang-B loss model as a basis which is insensitive to the higher order statistics of the burst lengths and in particular the CoV [8],[9],[10].

VI. Conclusions

We exactly derive the steady-state distribution for the channel horizon and per-class blocking probabilities for a multi-class single-channel OBS node with offset-based service differentiation among traffic classes. For this purpose, we use the theory of feedback Markov fluid queues and the algorithmic solution given in [6]. We validate the proposed method by numerical examples.

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