There have been many attempts to model the tapping-mode operation of atomic force microscope. Some authors tried to solve the problem analytically and others attempted to solve it with computer simulations, both adding to a better insight in understanding and treating the data collected. It is now possible to map local energy dissipation profiles. We hope that a better modeling of the problem will lead to better imaging ability.

Early models neglected attractive forces and treated the tip as a forced oscillator hitting a damped elastic surface. By using boundary conditions on the surface, the differential equation resulting from this model was solved. More involved models take attractive forces into account as well. This was hard to solve analytically, but numerical approaches succeeded in explaining many experimentally observable effects like attractive and repulsive regimes, hysteresis in the frequency sweep, and bistability. However, these models did not take hysteresis in the adhesive forces into account, which was later shown by Tamayo and Garcia that this is the main source of energy dissipation in a tapping mode. Some recent works treat the interaction forces as a double valued function and solve the resulting differential equation numerically. The results of this simulation showed the asymmetry of the necks formed by the sample surface during approach and retraction.

In this letter we present a model, which takes all the previously discussed effects as well as hysteresis. The model treats the tip as a forced oscillator, as usual. The sample is modeled with an equivalent spring constant and a damping effect, which is due to the internal viscosity of the sample. There is a negligible equivalent mass of the sample that we will totally neglect in this work. Interaction forces due to adhesion, surface charges, and some other sources are defined as a single valued function of the distance between the tip and sample surface. The schematic of the model is sketched in Fig. 1. It is important to note that, although the interaction force is defined as a single valued function of the distance between tip and sample, the resulting force–distance relation exhibits hysteresis. Since the attractive forces between tip and sample are proportional to \(1/(x_t - x_s)^n\), there may be more than one stable solution for the sample position \(x_s\). When the tip approaches the sample, the level of the sample surface increases because of the attractive forces. After a certain point, the system becomes unstable and the sample surface jumps to contact with the tip. However, during retraction, the contact is lost at a higher sample surface level. This results in different force–distance relations during approach and retraction.

In order to estimate the spring constant \(k_s\) of the sample, we used the Hertz model applied to deformable bodies. When the model is applied to the tip–sample interaction, the equivalent spring constant is given by the formula:

\[
k_s = \frac{1}{2} E^* \sqrt{R \times z_{def}}.
\]

Here \(R\) is the effective radius of the tip, \(E^*\) is the effective elastic modulus of the system, and \(z_{def}\) is the deformation of the sample. This value changes with force applied to the sample. Therefore, a nominal value of force is needed to estimate the spring constant of the sample. The best choice of this value is the adhesion force, since it is observed when no external force is applied to the tip and reflects the equi-
librium position. Also, the forces observed in tapping modes are around this value. The equivalent spring constant can be written in terms of this force as

\[ k_s = \frac{3}{6}E^* R F_{\text{adh}}. \] (2)

This parameter has a wide range, since elastic modulus varies by several orders of magnitude. The most important approximation we have done up to now is to assume that this spring constant would be applicable also when the surface is pulled up. That is mainly because the forces of adhesion are effective at very short distances and, therefore, the surface is pulled up from a small volume in the vicinity of the tip. This is similar to the repulsion where the force is applied from a small contact area but to the opposite direction. By making this assumption, we can then write the resulting coupled differential equations of motion as

\[ \frac{d^2 x_s}{dt^2} + \frac{m \omega_0}{Q} \frac{dx_s}{dt} + k_s(x_s - a_{\text{sp}}) - F(x_t - x_s) = A \cos(\omega t), \] (3)

\[ \gamma_s \frac{dx_t}{dt} + k_s x_t + F(x_t - x_s) = 0, \] (4)

where \( m, k, \omega_0, \omega, Q, a_{\text{sp}}, \) and \( A \) are the mass, spring constant, free resonance frequency, excitation frequency, quality factor, set point, and driving amplitude of the tip, respect-
The complete analogy between the mechanical and electrical system makes it possible to simulate the mechanical problem with an electrical circuit simulator such as SPICE. This program can make a transient analysis of nonlinear circuits to predict the node voltages and branch currents. We preferred the program can make a transient analysis of nonlinear circuits to predict the node voltages and branch currents. We preferred a nonlinear controlled source. The resulting electrical circuit is depicted in Fig. 2. We can then write the coupled differential equations for the charges \( q_t \) and \( q_s \), of the capacitors as

\[
L \frac{d^2 q_t}{dt^2} + R_c \frac{dq_t}{dt} + q_t - F(q_t - q_s) = A \cos(\omega t) + V_0, \\
R_s \frac{dq_s}{dt} + q_s + F(q_t - q_s) = 0.
\]

The authors thank L. Degertekin of Georgia Institute of Technology and Mesut Meterelliyyoz for helpful comments.

10. HSPICE: Avant! Corporation, 46871 Bayside Parkway, Fremont, CA 94538.
11. HSPICE, version 99.2 does not accept resistor values less than 10 µΩ.

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**Table I.** Correspondence between the mechanical model parameters and electrical circuit parameters. The values given are the ones used in our simulations.

<table>
<thead>
<tr>
<th>Circuit parameter</th>
<th>Mechanical equivalent</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_c )</td>
<td>( \frac{1000 , maw_0}{Q} )</td>
<td>( 400 , \mu \Omega )</td>
</tr>
<tr>
<td>( L )</td>
<td>( 1000 , k_s / \omega_0^2 )</td>
<td>( 1 , \mu H )</td>
</tr>
<tr>
<td>( C_t )</td>
<td>( 1/1000 , k_t )</td>
<td>( 100 , \mu F )</td>
</tr>
<tr>
<td>( R_s )</td>
<td>( 1000 , \gamma_s )</td>
<td>( 100 , \mu \Omega )</td>
</tr>
<tr>
<td>( C_s )</td>
<td>( 1/1000 , k_s )</td>
<td>( 40 , \mu F )</td>
</tr>
<tr>
<td>( F(q_t - q_s) )</td>
<td>( 1000 , F(x_t - x_s) )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>( V_0 )</td>
<td>( 1000 , k_d x_s )</td>
<td>( 0.6 , mV )</td>
</tr>
</tbody>
</table>