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Efficient bipedal locomotion on rough terrain via compliant ankle actuation with energy regulation

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Abstract

Legged locomotion enables robotic platforms to traverse on rough terrain, which is quite challenging for other locomotion types, such as in wheeled and tracked systems. However, this benefit—moving robustly on rough terrain—comes with an inherent drawback due to the higher cost of transport in legged robots. The ultimate need for energy efficiency motivated the utilization of passive dynamics in legged locomotion. Nevertheless, a handicap in passive dynamic walking is the fragile basin of attraction that limits the locomotion capabilities of such systems. There have been various extensions to overcome such limitations by incorporating additional actuators and active control approaches at the expense of compromising the benefits of passivity. Here, we present a novel actuation and control framework, enabling efficient and sustained bipedal locomotion on significantly rough terrain. The proposed approach reinforces the passive dynamics by intermittent active feedback control within a bio-inspired compliant ankle actuation framework. Specifically, we use once-per-step energy regulation to adjust the spring precompression of the compliant ankle based on the liftoff instants—when the toe liftoffs from the ground—of the locomotion. Our results show that the proposed approach achieves highly efficient (with a cost of transport of 0.086) sustained locomotion on rough terrain, withstanding height variations up to 15% of the leg length. We provide theoretical and numerical analysis to demonstrate the performance of our approach, including systematic comparisons with the recent and state-of-the-art techniques in the literature.

1. Introduction

The robotics community has an ongoing pursuit of empowering locomotion of lab-built biped robots in outdoor settings, particularly on rough terrains. Motivated by the idea of expanding the capabilities of biped robots into the areas that are unsafe for humans to venture in, recent studies achieve sustained locomotion on significantly rough terrains utilizing high-dimensional models and physical prototypes. Despite their achievements, these complex bipeds experience high locomotion costs due to complex actuation architectures (Caron *et al* 2019, Hubicki *et al* 2018). In contrast, low-dimensional locomotion approaches have limited performance, but they are scientifically beneficial, allowing systematic analysis and controller design in ways that are often imprac-

tical with complex systems. The concepts developed with low-dimensional models can also be transferred to complex platforms leading to successful locomotion patterns (Uyanik *et al* 2015, Xiong and Ames 2018, Westervelt *et al* 2018).

Passive dynamic walkers, particularly passive compass biped (PCB), have shown to be the simplest and accurate predictor of bipedal locomotion. The PCB model is very straightforward, consisting of only two rigid legs hinged at the hip and driven only by gravity (McGeer 1990, Garcia *et al* 1998). PCB models exhibit actuation-free asymptotically stable walking on downhill sloped terrain while capturing the essential characteristics of bipedal walking. However, the absence of actuators yields a highly fragile basin of attraction, limiting the locomotion capabilities of PCB to slightly inclined downhill terrains. Such pas-

sive models exhibit even more limited performance on rough terrains (Su and Dingwell 2007, Afshar and Ren 2012). Adaptation to such terrains and compensation for perturbations require a large basin of attraction with active control.

Incorporating various actuation schemes enable the compass biped to walk on level ground (Goswami *et al* 1997, Kuo 2002, Srinivasan 2010, Bhounsule *et al* 2014). Hip torque facilitates locomotion of the compass biped on constant downhill slopes (Gritli and Belghith 2018) and downhill rough terrains (Dai and Tedrake 2013) by accurately placing the swing toe. However, locomotion on uphill slopes with compass bipeds requires energy input, which necessitates ankle actuation (Spong *et al* 2007, Spitz *et al* 2015). Ankle actuation also leads to stable locomotion on steep constant downhill slopes (Safa *et al* 2018). Previous works of Byl and Tedrake (2009), Iida and Tedrake (2010), and Lee and Kim (2020) enabled a compass biped model to walk on rough terrain, including downhill and uphill slopes with hip and ankle actuation. Nevertheless, low dimensional approaches have been shown to exhibit limited locomotion performance on rough terrain.

The complex bipeds can achieve locomotion on significantly rough terrains via leveraging the kinematic advantage of multiple limbs and utilizing continuous actuation of the active joints. However, the increased complexity in biped morphology requires sophisticated control and increases the cost of transport. These issues have been addressed to a certain extent via introducing underactuated bipeds (Erez and Smart 2007, Hobbelen and Wisse 2008, Westervelt *et al* 2018, Griffin and Grizzle 2017). Various control methods such as transverse linearization and hybrid zero dynamics have been devised to design joint trajectories for underactuated complex bipeds (Manchester *et al* 2011, Westervelt *et al* 2018, Horn *et al* 2020). Despite reducing the number of actuators in bipeds, these proposed techniques cannot immensely reduce the cost of locomotion due to the inevitable need for continuous trajectory tracking. Moreover, the literature focuses on controlling hip and knee joints of the underactuated bipeds consisting of single support (SS) and instantaneous double-support phases. Unlike humans, limiting the locomotion phases and control actions to the SS phase would also limit the capabilities of the robots on rough terrains. However, there are limited studies of bipeds with finite-time double support (DS) phase (Williams and Martin 2021, Lee and Kim 2020, Shahbazi *et al* 2016).

Another aspect of research in complex bipeds and humanoids is the utilization of all the robot joints to enable locomotion on unstructured environments. In recent years, researchers have developed optimization frameworks and whole-body motion planners to address this issue. Particularly, the authors in (Reher *et al* 2016) exploit hybrid zero

dynamics of a bipedal humanoid robot that utilizes the whole-body dynamics to formulate a nonlinear optimization problem for efficient level terrain locomotion. In (Kuindersma *et al* 2016), the authors introduced the nonlinear trajectory optimization of the whole-body motion of a humanoid that accounts for kinematic constraints and multi-contact motions. The controller developed in (Feng *et al* 2015) solves the whole-body floating base inverse dynamics and inverse kinematics of a humanoid for rough terrain locomotion and ladder climbing via using quadratic programming. The whole-body operational space controllers developed in (Kim *et al* 2016) enable a point foot biped robot to traverse on significant rough terrains. The authors in (Koolen *et al* 2016) developed a whole-body compliant controller which prevents rigid control of a humanoid and enables locomotion on tilted surfaces and stairs. The work of (Hopkins *et al* 2015) presents a quadratic program formulation to solve the whole-body inverse dynamics problem of a humanoid robot for locomotion on uneven terrains. Although these approaches generate very versatile locomotion profiles by exploiting the robot's full capabilities, they require significant energy and processing power that limits their application in real-time scenarios.

The trade-off between robustness on rough terrains and energy efficiency can be circumvented if the underlying passive dynamics can be augmented with an actuation framework. In this regard, ankle actuation offers a significant advantage over hip actuation by rendering energetically efficient, and human-like locomotion behavior (Kuo 2002, Zelik *et al* 2014). In contrast, hip actuation requires the knee or torso to sustain the locomotion on uphill slopes, leading to increased cost and complexity. The compass biped augmented with an impulsive ankle pushoff illustrated the effectiveness of the ankle joint on rough terrain in Byl and Tedrake (2009) and Iida and Tedrake (2010). However, such an actuation scheme has limitations in practical realizations. As an alternative, compliant actuation schemes incorporating springs are shown to be an effective method of actuation for bipeds (Rummel *et al* 2010, Renjewski and Seyfarth 2012, Ernst *et al* 2012) and offer many practical advantages in implementations (Torricelli *et al* 2016). In this respect, the spring-loaded inverted pendulum (SLIP) has proven to be a useful template, enabling control of compliant actuation mechanism (Full and Koditschek 1999, Nguyen *et al* 2017). Particularly, compliant ankle actuation similar to the musculoskeletal mechanism of the human renders efficient locomotion (Ahn and Hogan 2012, Zelik *et al* 2014). However, most studies on compliant ankle actuation are limited to level ground (Ahn and Hogan 2012, Zelik *et al* 2014, Kerimoglu *et al* 2015, Kerimoglu *et al* 2017) and slightly rough terrains (Lee *et al* 2016, Kerimoglu 2017). Recently, Lee and Kim (2020) illustrated the effectiveness of

the compliant ankle actuation on rough terrains; however, the use of linearized dynamics led to limited performance compared to complex nonlinear models.

Inspired by the compliance of the human ankle that enables energy injection during a finite interval, here, we propose a bipedal actuation and control framework. Our approach utilizes the passive dynamics in the context of compliant ankle pushoff to achieve sustained locomotion on rough terrains. We derive the hybrid dynamics and use Poincaré stability analysis to identify the open-loop limit cycles. Then, by applying terrain disturbances to the open-loop limit cycles, we determine the features of these cycles, enabling larger terrain disturbances. Consequently, motivated by the effectiveness of once-per-step control (Kim and Collins 2017) and energy management of biped locomotion (Crews and Travers 2020), we introduce a novel control mechanism that regulates the configuration and speed of the model by adjusting the precompression of the rotary ankle spring via energy feedback. The model is controlled only during the DS phase (once-per-step energy input through the ankle spring), allowing to benefit from the passive dynamics during the single stance phase. Besides, the compliant ankle actuation enables redirection of the center of mass in the finite-time DS phase and eliminates the unnatural instantaneous DS phase. We achieve highly efficient locomotion on stochastically rough terrains, where the model can withstand variations of terrain heights up to 15% of the leg length (15 cm), an unprecedented level of tolerance to terrain variations for such simple walkers. Consequently, relying on passive dynamics and adopting compliant ankle control with energy regulation, the cost of transport of bipedal locomotion can be reduced as low as 0.086, while maintaining robustness on rough terrains. This cost is comparable to the cost of human locomotion on level terrain, which is estimated to be 0.05. To better illustrate the performance of our approach, we have provided a comparison table with the state-of-the-art works.

The practical examples of compliant ankle push-off on physical platforms utilize series elastic actuators (SEAs), a widely adopted actuation scheme in robots (Pratt and Krupp 2004, Chen *et al* 2018). The results presented in this study aim to provide key insights into the design and control of efficient bipedal platforms via SEAs on sloped and rough terrains, targeting robotic platforms, orthoses, and exoskeletons. In this context, compliant ankle actuation and a once-per-step energy feedback control strategy are shown to render highly efficient and robust locomotion behavior with a simple compass biped on significantly rough terrains. The approaches presented here can be extended to high-dimensional models to generate efficient sustained locomotion on rough terrains.

2. Foot-compass biped (FCB) model

This section describes the foot-compass biped (FCB) model and its dynamics, as illustrated in figure 1. The FCB model consists of two rigid massless legs of length L , each having a foot of length l with an infinitesimal mass m located at the heel. The legs are hinged at the frictionless passive hip joint. The torso dynamics are omitted, and its effect is captured with the point hip mass M . A massless rotary spring of stiffness k is attached to each ankle joint. The support leg angle and the interleg angles are represented by θ and ϕ , respectively. The FCB model is constrained to planar walking on inclined terrains with a slope γ , where the positive values correspond to uphill terrains. The proposed FCB model is an extension of Garcia *et al* (1998) by the inclusion of feet, and its assumptions are consistent with the studies in the literature (Garcia *et al* 1998, Zelik *et al* 2014, Ahn and Hogan 2012).

The FCB model has two alternating locomotion phases; SS and DS phases. The transitions between these two phases are triggered by the collision and liftoff events, as depicted in figure 1. Only the support leg is in contact with the ground in the SS phase, while the swing leg rotates freely without any actuation. When the swing leg touches the ground (collision), the model dynamics switch to the DS phase. Here, the trailing heel is assumed to cease contact with the ground, whereas the trailing toe maintains its contact. The ankle spring is triggered just after the collision to release its energy via generating plantarflexion motion. Following the successful extension of the ankle spring, the trailing toe lifts off from the ground and the model transitions to the next SS phase. The superscripts $[-, +, ^1]$ refer to pre-collision, post-collision, and liftoff instants, respectively.

2.1. Single support (SS) phase

The equations of motion for the SS phase are identical to those of the PCB model as described in the literature (Garcia *et al* 1998, Kuo 2002). The ankle springs attached to both feet are disengaged and do not apply any torque. The two-dimensional configuration space for the SS phase is defined using the angles $\mathbf{q}_{ss} := [\theta, \phi]^T$. The FCB model ignores toe scuffing, assuming that toe retraction or knee mechanisms will be used for physical implementations, as commonly used in the literature (Goswami *et al* 1997, Garcia *et al* 1998, Kuo 2002).

2.2. Double support (DS) phase

The DS phase is activated after the collision of the swing heel with the ground. In this phase, the ankle spring of the trailing leg rotates (changing ψ) and generates plantarflexion torque, while the leading toe pivots around its contact point on the ground. The ankle spring is assumed to be precompressed during the SS phase, and its energy is released during the DS phase

by an offset angle $\Delta\psi$ (spring precompression) to propel the hip mass. The DS phase dynamics reduces to single-DOF, parameterized with the vertical angle of the support leg $q_{DS} := \theta$. The joint variables of the DS phase, ψ , L_t and θ_t kinematically depend on θ due to the closed kinematic chain of the legs as shown in figure 2. These dependent variables are derived using the law of cosines as:

$$\psi = \cos^{-1} \left(\frac{l^2 - d_t^2 + 2Ld_t \sin(\theta - \gamma)}{2Ll} \right), \quad (1)$$

$$L_t = \sqrt{l^2 + l^2 - 2Ll \cos \psi}, \quad (2)$$

$$\theta_t = \cos^{-1} \left(\frac{L_t^2 + d_t^2 - L^2}{2L_t d_t} \right), \quad (3)$$

where L_t is the distance between the trailing toe and the hip and θ_t is the angle of L_t to the ground. Here, d_t is the step length starting from the trailing toe, and its value is fixed at the onset of the DS phase as:

$$d_t = 2L \sin \frac{\phi^+}{2} - l. \quad (4)$$

The equations of motion for this phase are obtained using the Lagrange equations with constraints to consider the closed kinematic chain as follows:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \frac{\partial \mathcal{L}}{\partial q_i} - \lambda_1 \frac{\partial f_1}{\partial q_i} - \lambda_2 \frac{\partial f_2}{\partial q_i} = 0, \quad (5)$$

where, $q_i = [\theta, \psi, \theta_t]$, f_1 and f_2 are constraints, λ_1 and λ_2 are the Lagrange multipliers and $\mathcal{L}$ is the Lagrangian. Neglecting the infinitesimal foot masses, the Lagrangian is given below as:

$$\mathcal{L} = \frac{M}{2} (L\dot{\theta})^2 - MgL \cos \theta - \frac{k}{2} (\psi_o - \psi)^2, \quad (6)$$

where $\psi_o = \psi^+ + \Delta\psi$ is the final rotation angle of the ankle spring. Note that in order for the spring to produce plantar torque, the ankle spring offset angle must be greater than zero ($\Delta\psi \geq 0$). The constraints are derived using the closed kinematic chain of the DS phase as:

$$f_1 := L_t \cos(\theta_t + \gamma) + L \sin \theta - d_t \cos \gamma = 0, \quad (7)$$

$$f_2 := L_t \sin(\theta_t + \gamma) - L \cos \theta + d_t \sin \gamma = 0. \quad (8)$$

The Lagrange multipliers are found by solving (5) for the dependent variables ψ and θ_t :

$$\lambda_1 = -\frac{k(\psi_o - \psi)L_t}{Ll \sin \psi} \cos(\theta + \gamma), \quad (9)$$

$$\lambda_2 = -\frac{k(\psi_o - \psi)L_t}{Ll \sin \psi} \sin(\theta + \gamma). \quad (10)$$

Substituting λ_1 and λ_2 in (5) and solving for θ yields the following equation of motion for the DS phase:

$$ML\ddot{\theta} - Mg \sin \theta + \frac{k(\psi_o - \psi)L_t}{Ll \sin \psi} \cos(\theta - \theta_t - \gamma) = 0. \quad (11)$$

The DS phase ends successfully when the ankle spring reaches its final or rest angle ψ_o after injecting its total potential energy into the model. The rest angle ψ_o of the spring is confined as $\psi^+ \leq \psi_o < \pi$ for feasible DS phases. However, not all ψ_o values lead to a feasible DS phase. We consider the DS phase as a failure for ψ_o if $\dot{\psi} < 0$, i.e. when the spring starts compressing without injecting its energy or when the ground reaction forces (GRF) become negative due to enormous spring energy. The physical realization of this phase is possible via an SEA at the ankle, which is triggered at the onset of the DS phase. Note that in the limiting case of too high spring stiffness, this phase reduces to instantaneous duration with a post-collision push-off impulse at the trailing ankle.

2.3. Collision event

The collision event triggers the transition from the SS to the DS phase. When the swing leg collides with the ground, the model experiences impulsive forces as illustrated in figure 1. The foot contact condition is defined as:

$$\theta - \frac{\phi}{2} - \gamma = 0. \quad (12)$$

The impulsive forces yield a discrete jump in the velocities of the model even though the model configuration is preserved. We assume that the leading heel and trailing toe remain on the ground without slipping, whereas the trailing heel ceases contact with the ground during the inelastic collision event. The collision map is defined as:

$$\theta^+ = -\theta^- + 2\gamma, \quad (13)$$

$$\dot{\theta}^+ = \cos \phi^- \dot{\theta}^-. \quad (14)$$

Note that, even though the foot collision generates a discrete jump in $\dot{\phi}^+$, this change does not affect the model dynamics during the DS phase since these dynamics are only governed by the variable θ . Moreover, the angular velocity of the rear ankle at the post-collision instant, $\dot{\psi}^+$, is given as follows:

$$\dot{\psi}^+ = -\frac{d_t \cos(\gamma - \theta^+(t)) \dot{\theta}^+(t)}{l \sqrt{1 - \frac{(d_t^2 + 2d_t L \sin(\gamma - \theta^+(t)) - l^2)^2}{4l^2 L^2}}}. \quad (15)$$

The components d_t and $\cos(\gamma - \theta^+(t))$ are always positive and $\dot{\theta}^+(t)$ is negative for forward locomotion schemes, all of them varying at each step. Thus, assuming that the denominator yields real values, $\dot{\psi}^+$ is positive, generating propulsion in the forward direction. As usual, the support and swing leg configuration variables are swapped across each successive step to utilize the same form of dynamics.

2.4. Liftoff event

This event triggers the transition from the DS to the SS phase when $\psi = \psi_o$. The liftoff event does not affect the angle and velocity (θ and $\dot{\theta}$) of the leading leg (new support). However, since the DS dynamics depend only on θ , the interleg angle at liftoff ϕ^1 needs to be calculated with respect to θ^1 in preparation for the following SS phase. Hence, ϕ^1 can be calculated using the leg length and angle relations at liftoff instant as:

$$\phi^1 = \cos^{-1} \left(\frac{L^2 + (L_t^1)^2 - l^2}{2LL_t^1} \right) + \frac{\pi}{2} - \theta_t^1 + \theta^l - \gamma, \quad (16)$$

where, L_t^1 , θ_t^1 and θ^l are the variables of the DS phase at the liftoff instant. Consequently, the time derivative of (16) yields the angular velocity of the interleg angle at liftoff instance as:

$$\dot{\phi}^1 = \dot{\theta}^1 \left(\frac{d_t L (l^2 - L^2 + (L_t^1)^2) \cos(\theta^l - \gamma) (L_t^1)^{-2}}{\sqrt{((l - L)^2 - (L_t^1)^2)((L_t^1)^2 - (l + L)^2)}} - \frac{L - d_t \sin(\theta^l - \gamma)}{L^{-1} (L_t^1)^2} + 1 \right). \quad (17)$$

When the trailing leg lifts off the ground, the actuator in the new support ankle should bring the ankle spring to its compressed initial value, $\Delta\psi$, to prepare the model for the subsequent step. This can be achieved via the SEA, which can compress the ankle spring while the swing leg rotates. The effect of this compression event on the model dynamics is neglected for simplicity following the similar assumptions commonly used in the literature (Uyanik *et al* 2011, Ahn and Hogan 2012, Lee and Kim 2020).

2.5. Ground reaction forces

The GRF during the locomotory phases of the model must yield positive values to avoid the flight phase. In this respect, the GRF during the SS phase is straightforward, and it is given below as:

$$Mg \cos \theta - M \frac{(L\dot{\theta})^2}{L} > 0. \quad (18)$$

There are two GRF forces during the DS phase, which are generated from the trailing toe and leading heel. The torque generated by the GRF of the trailing toe F_A should be equal to the plantarflexion ankle torque as:

$$F_A \sin \beta l = k(\psi_o - \psi), \quad (19)$$

where $\beta = \cos^{-1}(\frac{l^2 + L_t^2 - L^2}{2lL_t})$ is the angle between l and L_t . Simplifying the terms yields the GRF of the trailing toe as:

$$F_A = \frac{k(\psi_o - \psi)L_t}{Ll \sin \psi} > 0. \quad (20)$$

Note that for $\psi^+ < \frac{\pi}{2}$ and $\Delta\psi > 0$, F_A remains always positive. The GRF of the leading heel, F_B , is derived using the linear momentum in horizontal direction:

$$F_A \cos(\theta_t + \gamma) - F_B \sin \theta = M(-L\ddot{\theta} \cos \theta + L\dot{\theta}^2 \sin \theta). \quad (21)$$

After substituting for $\ddot{\theta}$ using (11) into (21), F_B is determined as follows:

$$F_B = \frac{k(\psi_o - \psi)L_t}{Ll \sin \psi} \sin(\theta - \theta_t - \gamma) + Mg \cos \theta - ML\dot{\theta}^2 > 0. \quad (22)$$

These forces are utilized as constraints in the numerical simulations in the following sections. Note that there are similar models in the literature that focus solely on energy efficiency on level terrain (Zelik *et al* 2014) and reproducing gait behaviors similar to humans (Ahn and Hogan 2012). However, our motivation to study the proposed model is its ability to leverage ankle compliance and passive dynamics to generate efficient and robust locomotion behavior on rough terrain.

The proposed model asserts various limitations, enabling the analysis in the following sections. The

first limitation is that when the leading heel makes ground contact, the trailing heel lifts off with the trailing toe keeping its contact with the ground. This ensures that there exists 2 points of contact during the DS phase and enables injection of spring potential energy during this phase. However, in reality, an infinite number of contact points affect the energy injection process. The second assumption is that the collision is assumed to be frictionless, which helps to simplify the collision map but limits the applicability of the proposed approach in real-life scenarios. Finally, we assume that there is no knee joint in the DSP phase in contrast to human locomotion. This would also affect the DS phase energy injection, which may lead to different results than that of humans. Although these assumptions would generate distinctive dynamical behaviors compared to more realistic modeling assumptions and real-life scenarios, they enable illustrating the fundamental benefits of utilizing the DS phase and finite-time energy injection.

3. Analyzing the limit cycles: characterizing stability and robustness

In this section, we characterize the limit cycles of the proposed model using the Poincaré stability analysis and investigate the effects of model parameters on the limit cycles for various inclined surfaces. The limit cycles are obtained using open-loop energy injection, i.e. without any feedback mechanisms in the ankle spring. Subsequently, we apply single-step terrain disturbance to these limit cycles under various ankle compliance configurations to determine the locomotion characteristics, enabling higher tolerance to terrain variations.

3.1. Stability of the limit cycles

We utilize Poincaré stability analysis to determine the existence and stability of the open-loop limit cycles of the model. This requires investigating the system response at the discrete hyper surfaces of the state space termed Poincaré sections. We choose the liftoff state during forward locomotion of the SS phase as our Poincaré section, S_Γ . Let $\mathbf{q}_l := [\dot{\theta}, \phi, \dot{\phi}]^T$ be a parametrization of the states on $S_\Gamma = \mathbb{R} \times S^1 \times \mathbb{R}$ and let $\mathbf{q}_l[i]$ denote the i th liftoff state. Then, the Poincaré map takes the form:

$$\mathbf{q}_l[i+1] = \mathbf{G}(\mathbf{q}_l[i]). \quad (23)$$

Consequently, we compute the fixed points on S_Γ , corresponding to the periodic trajectories of the model as:

$$\mathbf{q}_l^* = \mathbf{G}(\mathbf{q}_l^*). \quad (24)$$

The linearization of $\mathbf{G}$ around its fixed point, $\mathbf{q}_l^*$, yields the associated Jacobian matrix $D\mathbf{G}$ as:

$$\mathbf{q}_l[i+1] - \mathbf{q}_l^* \approx D\mathbf{G}|_{(\mathbf{q}_l^*)}(\mathbf{q}_l[i] - \mathbf{q}_l^*). \quad (25)$$

Here, eigenvalues of $D\mathbf{G}|_{(\mathbf{q}_l^*)}$ characterizes the stability of the limit cycles. Any eigenvalue whose absolute value is greater than 1 indicates that the associated limit cycle is unstable. Note that since our study incorporates fully nonlinear, impulsive, and hybrid dynamics of the proposed model, we utilized numerical integration methods to compute the Poincaré map and the Jacobian. Recently, the analytical expression of the Poincaré map of a similar hybrid, the nonlinear dynamic model, has been derived using linearization in Znegui et al (2020). However, the linearized expressions do not fully capture the rich and chaotic dynamical behavior of impulsive, hybrid, nonlinear dynamics of the compass walking models. In this respect, one can rely on numerical computations of the Poincaré map as will be described below. For our purposes, the biped parameters are chosen as follows $M = 1$ kg, $m = 1e-8$ kg, $L = 1$ m and $g = 9.807$ N kg⁻¹ to be consistent with the literature (Wisse et al 2005, Uyanik et al 2016). The equations of motion are evaluated using MATLAB ode45 numerical differential equation solver, and the limit cycles are obtained using the fminsearch unconstrained multivariable optimization function. The optimizer minimizes the norm of the difference between the consecutive liftoff states to numerically determine the limit cycle in (24):

$$\mathbf{q}_l^* = \underset{\mathbf{q}_l}{\operatorname{argmin}} \|\mathbf{G}(\mathbf{q}_l[i]) - \mathbf{q}_l[i]\|^2. \quad (26)$$

We consider a particular solution to the optimization problem above to be a limit cycle when $\|\mathbf{G}(\mathbf{q}_l[i]) - \mathbf{q}_l[i]\| < 1e-8$. We characterize the limit cycles by illustrating the eigenvalues of their Jacobian matrices for various spring stiffness and foot length parameters as a function of energy input. We also investigate the effect of constant terrain slopes on these limit cycles such as uphill, level, and downhill terrains. In this respect, figure 3 illustrates the maximum absolute value of the eigenvalues of $D\mathbf{G}(\lambda_{\max})$ for various k and l . As the spring precompression, $\Delta\psi$ increases—the energy input increases—the stability of the limit cycles vary depending on the surface inclination and model parameters. Each subfigure of figure 3 depicts the effect of various spring stiffness and terrain slopes on the limit cycle stability. Thus, a lower spring constant and large feet correspond to compliant ankle actuation, whereas a higher spring stiffness and small feet correspond to stiff (impulse-like) ankle actuation. A closer inspection of figure 3 reveals that most of the limit cycles exhibit unstable behavior for all terrain profiles. Even though it is desirable to obtain stable trajectories for control purposes, the unstable periodic orbits also illustrate the model's locomotion capabilities. In particular, the open-loop unstable trajectories given in figure 3 can be stabilized and utilized for sustained locomotion using closed-loop feedback control. Thus, we seek the parameters

Figure 3. The ankle spring energy input (E_{ic}) versus the magnitude of maximum eigenvalues of the limit cycles for various ankle compliance configurations. The purple, blue, red, green and orange curves represent the limit cycles with the spring constants of $k = 0.2$, $k = 0.5$, $k = 1$, $k = 2$, $k = 5$ in Nm rad^{-1} , respectively.

that yield the largest extent of limit cycles as a function of energy input which can be either stable or unstable.

The widest range of limit cycles is achieved on downhill surfaces for all parameters. We observe that a further increase in the energy input leads to the flight phase by violating the inequality (22) for all parameters. One crucial observation is that as the terrain slope changes from downhill to uphill and the ankle actuation becomes stiffer, the extent of limit cycles shrinks. This observation is obvious in uphill limit cycles of figure 3(a), where the model experiences the stiffest ankle actuation with $k = 5 \text{ Nm rad}^{-1}$. In contrast, low spring constant yields larger range of limit cycles for small feet length of $l = 0.05$ m. Increasing the feet length extends the range of limit cycles as illustrated in figure 3(b), enabling more energy injection. However, the model exhibits larger range of limit cycles until $E_{ic} = 1.8$ J for stiffer springs such as $k = 1 \text{ Nm rad}^{-1}$. Further increase in the feet

length ($l = 0.25$ m), figure 3(c), enables the ankle to inject energy up to $E_{ic} = 3$ J, but the model cannot sustain locomotion for the more compliant springs such as $k = 0.2 \text{ Nm rad}^{-1}$ and $k = 0.5 \text{ Nm rad}^{-1}$. This is because the ankle spring becomes too compliant to inject all its energy into the model. Thus the model does not have any limit cycles for such configurations. Finally, for the largest feet length of $l = 0.35$ m, figure 3(d), the model can only exhibit limit cycles for stiffer ankle springs of $k = 2 \text{ Nm rad}^{-1}$ and $k = 5 \text{ Nm rad}^{-1}$. Even though the model exhibits a relatively large range of limit cycles for such springs, very large feet does not support low levels of energy injection, thus limiting the overall range of limit cycles. The reduction of limit cycles with stiff spring and small feet on uphill slopes is because the model comes closer to violating (22) during the DS phase, thus limiting the state space of sustained locomotion. To support this argument, we let $\Delta\psi = \psi_o - \psi$ and multiply the post-collision form of (22) by $\frac{\Delta\psi}{2}$ to

obtain the following condition:

$$E_{lc} \frac{L_t^+ \sin(\theta^+ - \theta_t^+ - \gamma)}{Ll \sin \psi^+} + \frac{\Delta\psi}{2} (Mg \cos \theta^+ - ML(\dot{\theta}^+)^2) > 0. \quad (27)$$

Here, $E_{lc} = k\Delta\psi^2/2$ is the energy supplied to the model by the ankle spring to maintain the forward locomotion. For locomotion on uphill slopes with $\phi^- < \frac{\pi}{2}$, the multiplicative component of E_{lc} is always negative since $\theta^+ < \theta_t$. Furthermore, the multiplicative component of $\frac{\Delta\psi}{2}$ is the GRF constraint (18) which is always positive for sustained locomotion profiles. The locomotion on uphill and level slopes requires external energy to compensate for the kinetic and potential energy losses. In this respect, on uphill slopes, the ankle spring provides the necessary energy for the forward locomotion with $E_{lc} > 0$. The energy input E_{lc} that sustains the forward locomotion can be provided with a larger k by decreasing $\Delta\psi$. Hence, while the tradeoff between k and $\Delta\psi$ does not affect the negative component for a constant E_{lc} in (27), the decrease in $\Delta\psi$ leads to the decreased positive component. Additionally, very small feet and uphill slopes contribute to the increase in the negative multiplicative component in (27). Hence, the model starts experiencing the flight phase earlier on uphill slopes as the ankle actuation becomes stiffer. In contrast, the locomotion profiles on downhill slopes can use the gravitational potential energy to compensate for the collision losses, requiring less energy input and decreasing negative component. In the extreme case, the energy requirement can be zero, and the condition (27) reduces to (18). Moreover, a downhill slope also leads to a decrease in the negative component in (27). As a result, the model exhibits a larger range of limit cycles on downhill slopes without violating (27) compared to uphill slopes. In addition, as the foot length increases the

limit cycles vanishes for low E_{lc} , due to insufficient ankle pushoff. In other words, as the lever arm of the foot increases, the spring stiffness required to rotate the ankle joint also increases. Thus, the model does not exhibit limit cycles for lower E_{lc} values with large feet.

In figure 4, we investigate the effects of l and k on the range of the energy level E_{lc} in detail. Here, each square corresponds to the energy extent of the limit cycles (range of E_{lc}) of the associated $[k, l]$ values for $\gamma = 0^\circ$. The blue and yellow regions indicate the least and most extensive energy levels of the limit cycles. The range of limit cycles enlarges as the feet length increases and the largest extent is attained in the vicinity of $l = 0.2$ – 0.3 m and $k = 1.5$ – 2.5 Nm rad⁻¹. Also, the stiffness leading to the larger E_{lc} increases with the feet length. For very large feet configurations, highly compliant ankle springs generate insufficient push-off force and the limit cycles disappear as seen by the white regions.

3.2. Tolerance to single-step terrain disturbance

After investigating the effects of ankle actuation and constant terrain slopes as a function of energy input on the open-loop limit cycles, we analyze the tolerance of these limit cycles to a range of single-step terrain disturbances. Our primary goal here is to identify the properties of these limit cycles and the model parameters, leading to better tolerance to terrain disturbance. We conjecture that the tolerance to single-step terrain disturbances would shed light on the ability of the model to sustain forward locomotion on rough terrain. We determine a limit cycle's tolerance to a given single-step terrain disturbance by checking its ability to place the swinging heel during the next step on level terrain via adjusting the ankle spring as illustrated in figure 5. We start by applying single-step terrain disturbances to the limit cycles on level terrain in figure 3. First, the locomotion is initiated from

a fixed point $\mathbf{q}_l^*[i]$ on level ground, which is associated with any energy level E_{lc} in figure 3. Then, the model experiences the single-step terrain disturbance of slope γ' , leading to the disturbed post-collision states $\mathbf{q}^+[i]$. Consequently, we set the new spring angle offset as $\Delta\psi'$ that tolerates the disturbed terrain slope by injecting enough energy. The model is considered to successfully tolerate the single-step terrain disturbance when the swinging toe achieves collision on the upcoming level terrain with the post-collision states $\mathbf{q}^+[i+1]$. More formally, we compute the following function for the fixed point $\mathbf{q}_l^*[i]$ and the parameter set $p = [\Delta\psi', \gamma']$:

$$\mathbf{q}^+[i+1] = \mathbf{G}_p'(\mathbf{q}_l^*[i]). \quad (28)$$

The return map $\mathbf{G}_p'$ accepts the fixed point, it evaluates the SS dynamics for the disturbed terrain slope γ' and applies $\Delta\psi'$ during the DS phase. Then it evaluates the SS dynamics once more for level terrain and returns $\mathbf{q}^+[i+1]$ as the output by terminating the locomotion. We consider $\Delta\psi'$ a valid energy input if the model can achieve a feasible post-collision state $\mathbf{q}^+[i+1]$ for the upcoming level terrain by satisfying the foot contact condition in (12). Consequently, we span the parameter space p for valid $\mathbf{q}^+[i+1]$ states via $\mathbf{G}_p'$ to find the maximum terrain disturbance of the fixed points given in figure 3. The curves illustrated in figure 6 corresponds to the maximum single-step terrain disturbances tolerated by the limit cycles as a function of energy input for various model parameters. Each color corresponds to a different spring stiffness configuration. The solid and dashed curves correspond to the maximum step-up and step-down terrain tolerances, respectively.

For small feet, the compliant ankle spring yields larger tolerance to single-step disturbance as illustrated in figure 6(a). The model starts exerting insufficient pushoff forces during step-up disturbances for very compliant spring ($k = 0.2 \text{ Nm rad}^{-1}$) as observed in figure 6(a). The disturbance tolerance increases for feet length of $l = 0.15 \text{ m}$ as seen in figure 6(b), however, springs with relatively low stiffness of $k = 0.5 \text{ Nm rad}^{-1}$ and $k = 1 \text{ Nm rad}^{-1}$ cannot effectively tolerate step-up disturbances. Moreover, very stiff spring also yields limited disturbance tolerance. Further increase in the feet length enables larger disturbance tolerance as depicted in figure 6(c). However, the model fails to tolerate large disturbances for very large feet as illustrated in figure 6(d).

Closer inspection of figure 6 reveals that the largest tolerance to the single-step disturbance for each feet length can be obtained via different ankle compliance configurations. The largest single-step disturbance tolerance across a large range of energy input is attained for $[k, l] = [2 \text{ Nm rad}^{-1}, 0.25 \text{ m}]$. An interesting observation in figure 6 is that the tolerance to step-down terrain disturbances is generally larger than the step-up tolerance for all model parameters. Moreover, there is a trend in figure 6 such that the increase in energy input first leads to an increase in the step-up disturbance tolerance until some energy level, and then it leads to reduced tolerance. We explain these observations in detail by first introducing scenarios that lead to falling behavior of such simple biped walkers under terrain disturbance as:

- The support leg does not have enough kinetic energy to vault over the apex point where the model falls back (Crews and Travers 2020).
- The model violates GRF constraints during locomotion phases.

Figure 6. Single-step terrain disturbance applied to the limit cycles presented in figure 3, where the spring constants are given in Nm rad^{-1} . The solid (dashed) curves with purple, blue, red, green and orange colors represent the step-up (step-down) terrain disturbance with the spring constants of $k = 0.2$, $k = 0.5$, $k = 1$, $k = 2$, $k = 5$, respectively.

- The swing leg is not fast enough to land the swing heel in front of the support heel leading to falling forward.
- The swing leg is too fast relative to the support leg where the model experiences excessive swing leg retraction (SLR). This could either lead to falling back or forward depending on the configuration of the support leg.

The model must avoid the above scenarios for sustained locomotion. Note that the falling behaviors during bipedal locomotion are not limited to the above items, where we exclude the failures that occur due to foot sliding, knee, and torso actuation. Nevertheless, for the sake of discussion, we limit our focus to failure cases of simple models.

The first failure case can be avoided by injecting sufficient energy during the DS phase as long as the spring energy does not lead to the flight phase. The condition for this is given at the post-collision instant as:

$$\frac{k\Delta\psi^2}{2} + \frac{1}{2}M(L\dot{\theta}^+)^2 + MgL \cos \theta^+ > MgL. \quad (29)$$

The second failure case can be prevented by enforcing the conditions (18), (20) and (22). The

third and fourth cases are not trivial for the proposed model since the SS trajectories cannot be directly controlled. However, the passive SS trajectories can be adjusted/shaped to a proper liftoff state during the finite-time DS phase by adjusting the ankle spring energy. As we will show in the following sections, suitable adjustment of the liftoff states enables the model to negotiate significant terrain variations. In this respect, we conjecture that the regulation of the swing leg velocity at the liftoff instant plays a key role in evading third and fourth failure cases. We argue that the passive SS trajectories of the limit cycles with wider step length (interleg angle), i.e. larger step lengths, lead to faster swing leg velocities relative to the support leg velocities and help regulate the swing leg placement on the upcoming terrain. With wider step length, the passive swing leg rotates fast enough to catch up with the support leg and places the swing heel appropriately on the upcoming terrain, which helps avoid failure case three. Also, limit cycles with wider step lengths enable larger ground clearances and eventually make the SS trajectories more robust to terrain disturbance. Consequently, with larger ground clearance, the model can tolerate larger SLR and eventually prevent the last failure case.

Figure 7. Step length versus liftoff velocity ratio of the legs for the limit cycles presented in figure 3.

To support the above arguments, we utilize the open-loop limit cycles and the ratio of the swing leg velocity to support leg velocity at liftoff $\frac{\dot{\phi}^l}{\dot{\theta}^l}$ which can be derived from (16). Since this ratio is not analytically tractable, we numerically evaluate it for the limit cycles introduced in figure 3. Figure 7 illustrates the step length and liftoff velocity ratio of the legs of these limit cycles. Figure 7 indicates that the passive trajectories of the swing leg at liftoff become exponentially faster relative to the support leg as the step length increases due to the increase in energy input. For small feet, the compliant spring boosts this increase across a wide range of step lengths, leading to increased success in landing the swinging heel on the upcoming terrain. For larger feet, even though the ratio of $\frac{\dot{\phi}^l}{\dot{\theta}^l}$ increases with compliant spring, the range of step length remains limited. Similar to the results in figure 6 the largest step length and $\frac{\dot{\phi}^l}{\dot{\theta}^l}$ ratio can be attained for $[k, l] = [2 \text{ Nm rad}^{-1}, 0.25 \text{ m}]$.

Returning to figure 6, as we have mentioned previously, the increase in the step length of the limit cycles enhances the tolerance to step-down terrain disturbance whereas the tolerance to step-up disturbance first increases and then decreases. These observations can be explained using the failure cases and liftoff velocity ratio of the legs. A step-down disturbance to the model causes the late ground collision, introducing extra energy due to the potential difference at the collision. Since the ankle spring cannot reduce the energy, the aforementioned extra energy increases the speed of the support leg. Hence, as illustrated in figure 6, the swing leg trajectories at liftoff, which are relatively slow compared to support leg for the limit cycles with small step length, further lags the support leg during the SS phase and lead to falling forward. However, with a larger step length, the swing leg at liftoff becomes relatively faster and enables the passive SS trajectories to handle larger step-down terrain disturbances. For the step-up disturbance occurring during locomotion with small step lengths, the swing leg falls behind the support leg and fails to place the swing heel on the upcoming terrain. Wider step

length leads to larger step-up disturbance tolerance until the energy input required to traverse the uphill disturbance leads to excessive ankle pushoff force. In other words, the decrease in step-up disturbance tolerance starts when the energy required to satisfy (29) starts violating (22). Hence, the transition of the model to the flight phase becomes easier on uphill slopes with wider step length. Additionally, as the step length further widens, the step-up disturbance tolerance decreases, whereas the step-down disturbance tolerance increases.

Consequently, we have investigated the effects of k and l on the single-step disturbance rejection capability of the model with figure 8. In this figure, each square corresponds to the mean value of the maximum slope tolerances, i.e. mean of the range of $\gamma'_{\max}$ for the associated $[k, l]$ pair. The best tolerance to single-step terrain disturbance can be attained in the neighborhood of $l = 0.25 \text{ m}$ and $k = 1.5 \text{ Nm rad}^{-1}$. Small feet yield low disturbance tolerance, whereas the increase in the feet length leads to increased tolerance. However, very large feet require stiff spring configurations and exhibit reduced disturbance tolerance.

As discussed above, different feet have different stiffness configurations that enlarge the tolerance bandwidth of the model to single-step disturbances. The enhanced disturbance tolerance is achieved by enabling locomotion with larger step lengths and injecting energy via a finite duration DS phase. Larger feet help avoid the failure cases; however, extensively large feet fail to exert pushoff force on step-up disturbances. Thus we conclude that a feet length similar to humans and a relatively compliant spring such as $[k, l] = [1.5\text{--}2 \text{ Nm rad}^{-1}, 0.25\text{--}0.3 \text{ m}]$ exhibits the largest extent of limit cycles and the best disturbance tolerance. In addition to these findings, figure 6 reveals the initial conditions which better tolerate terrain disturbance. In this respect, it is worth noting that the success of the passive SS trajectories of the FCB model heavily depends on the initial conditions during sustained locomotion (McGeer 1990).

This dependence can be reduced by applying hip torque, enabling the control of the swing leg angle and angular velocity, leading to increased locomotion cost and complexity. In summary, the passive SS trajectories augmented by compliant ankle actuation become more robust to step-up and down terrain disturbances with larger step lengths. This is achieved by speeding up the swing leg in the SS phase via higher $\frac{\phi}{\theta}$ and satisfying the GRF constraints in the DS phase via compliant actuation. The following sections will illustrate that the proper initial conditions and feedback control enable the proposed model to negotiate significant terrains without hip actuation or control.

Even though single-step terrain disturbance provides essential information about which gait characteristics would lead to better tolerance to terrain variations, it has limited implications on continuously varying terrains. Moreover, open-loop limit cycles are limited in terms of their ability to adapt to terrain disturbance. This motivates us to test the findings presented in this section with closed-loop limit cycles on stochastically varying rough terrains. The following section will introduce closed-loop limit cycles by designing a feedback controller for the ankle spring.

4. Efficient rough terrain locomotion with energy regulation

This section focuses on the feedback control design for the spring precompression $\Delta\psi$ to achieve sustained locomotion on rough terrain. The ‘rough’ terrain profiles are defined through the vertical height of the terrain computed as a continuous, piecewise linear function of the horizontal position, x , taking the form $y = h_g(x)$. Changes in the terrain slope at each step are randomly drawn from a Gaussian distribution as $\Delta h \sim N(\mu, \sigma)$ where the standard

deviation of the distribution σ represents the roughness of the terrain in degrees and the average terrain slope is adjusted via μ . One hundred such terrains, each consisting of 1000 steps, were generated for different levels of terrain roughness with mean value $\mu = 0^\circ$ and roughness ranging from $\sigma = 0^\circ$ to $\sigma = 2.3^\circ$. The detection of the collision event is performed by monitoring the y -distance of the swing heel and the height function of rough terrain ($h_g(x)$), where x is the distance of the swing heel relative to the support heel coordinates. The collision occurs when the two quantities are equal.

4.1. Energy feedback control of the ankle spring

In this section, we will introduce a feedback control approach that generates sustained locomotion behavior on significantly rough terrains without relying on the perception of the following terrain. We assume that the terrain slope is not available until the leading leg collides with the ground. The proposed feedback control relies on the energy regulation of the model at each collision instant by adjusting the spring precompression $\Delta\psi$ during the DS phase. The idea is to use the liftoff energy level as a target to design $\Delta\psi$ at each post-collision instant.

We start by defining the energy of the model at key instances during the locomotion. Let $E^l[i]$, $E^-[i]$ and $E^+[i]$ be the sum of kinetic and potential energy of the model for the i th step at liftoff, pre-collision and post-collision instances, respectively. Consequently, the following energy relation at the post-collision instant can be derived using the kinematic chain of the DS phase as:

$$E_{\text{ref}}^l[i+1] = E^+[i] + E_s[i], \quad (30)$$

where $E_{\text{ref}}^l[i+1]$ is the reference energy level of the liftoff instance of the following, $(i+1)^{\text{th}}$, step and $E_s[i] = \frac{1}{2}k\Delta\psi[i]^2$ is the energy of the spring to be designed. Consequently, the post-collision energy of

the model, $E^+[i]$, can be expressed in terms of pre-collision energies as:

$$E^+[i] = E^-[i] - K_{\text{imp}}[i] - P_\gamma[i], \quad (31)$$

where $K_{\text{imp}}[i]$ represents the kinetic energy lost due to impulsive collision and $P_\gamma[i]$ denotes the potential energy difference experienced at collision instance due to support leg transfer and they are defined respectively as:

$$K_{\text{imp}}[i] = \frac{1}{2}M \sin^2(2\theta^-)(L\dot{\theta}^-)^2, \quad (32)$$

$$P_\gamma[i] = Mg d_t[i] \sin \gamma[i]. \quad (33)$$

Then, by simply letting $E_{\text{ref}}^l[i+1] = E^l[i]$, i.e. equating the following and the previous liftoff energy and using the fact that $E^l[i] = E^-[i]$ for the conservative SS dynamics, the spring precompression can be obtained as:

$$\Delta\psi[i] := \sqrt{\frac{2}{k}(K_{\text{imp}}[i] + P_\gamma[i])}. \quad (34)$$

With this control law, the model tracks the liftoff energy of the i th step by adjusting $\Delta\psi[i]$ at each step. To illustrate the effect of closed-loop control on the open-loop trajectories we stabilize the unstable orbits in figure 3 via the feedback control law (34). Figure 9 illustrates the magnitude of maximum eigenvalues of the unstable (blue) and feedback-stabilized (red) trajectories for $[k, l] = [0.5 \text{ Nm rad}^{-1}, 0.05 \text{ m}]$ as a function of E_{lc} . The controller effectively stabilizes all the orbits by pushing the maximum absolute value of their eigenvalues below one.

The closed-loop energy feedback adjusts the configuration and speed of the model by modulating the spring energy for the local slope. The controller injects less energy for downhill terrains, whereas it injects more energy for uphill terrains. The above spring energy modulation can be explained in two folds. First, a step-down terrain variation causes late ground collision (larger θ^-); hence the model experiences higher kinetic and lower potential energy at the heel contact compared to the collision instant of the previous step. Consequently, right after the collision, the model regains potential energy due to the height difference between the leading and trailing heels P_γ . Ultimately, the post-collision energy of the biped increases compared to the post-collision energy of the previous step. As discussed in the previous section, the excessive energy gained during step-down terrain disturbances can lead to falling behavior. By setting the target liftoff energy as the previous liftoff energy, the ankle joint compensates for this extra potential energy via injecting less spring energy. If the model faces a large enough step-down terrain, the ankle spring does not inject energy at all, and the model reduces to fully passive trajectories.

Second, a step-up terrain variation causes early ground collision (smaller θ^-) where the model ends

up with relatively lower kinetic and higher potential energy at heel contact. In contrast to the downhill slope, the model loses some of its potential energy during an uphill collision due to the positive height difference. Hence, compared to the collision instant of the previous step, the model has less kinetic and potential energy during an uphill collision. To compensate for this energy loss, the feedback controller injects more spring energy to propel the body during the DS phase. This energy tracking control approach enables the model to negotiate significantly rough terrains without relying on the perception of the upcoming terrain. Note that, the proposed model can also traverse on other rough terrain profiles such as step terrains as long as foot scuffing is ignored and the terrain profile does not include any gaps.

4.2. One-step lookahead energy update

Although the energy regulation in (34) enables the model to sustain locomotion for varying terrain profiles, restricting the target liftoff energy to the previous liftoff energy level would limit the performance. Utilizing the limited knowledge of the upcoming terrain, which can be easily implemented with a light detection and ranging sensor, has shown to increase the rough terrain locomotion performance of such biped model in Byl and Tedrake (2009). In this regard, we augment (34) with a term that updates the target liftoff energy at each step using the information of the upcoming terrain as:

$$\Delta\psi^{\text{la}}[i] := \sqrt{\frac{2}{k}(K_{\text{imp}}[i] + P_\gamma[i] + k_E(P_\gamma[i] - \hat{P}_\gamma[i+1]))}, \quad (35)$$

where k_E is the gain of the update term. Here, $\hat{P}_\gamma[i+1]$ is the estimate of the potential energy difference experienced at the collision instant of the following step:

$$\hat{P}_\gamma[i+1] = Mg d_t[i] \sin \gamma[i+1]. \quad (36)$$

Note that since the step length for the upcoming terrain ($d_t[i+1]$) is not available, the potential energy difference of the following step is estimated using the current step length, $d_t[i]$ and the slope information of the following step $\gamma[i+1]$. The one-step lookahead energy controller $\Delta\psi^{\text{la}}[i]$ incorporates the scaled energy difference between $P_\gamma[i]$ and $\hat{P}_\gamma[i+1]$ into the spring precompression calculation. In doing so, it slightly modulates the spring precompression to help adapt the model to terrain variations of the upcoming step. Specifically, the model better adapts to the upcoming relatively downhill slope by injecting slightly more energy that enables faster swing leg motion during the SS phase, leading to larger ground clearance of the swing toe. Similarly, for the upcoming relatively uphill slope, the model adaptation is improved by injecting slightly less energy. Thus, it leads to slight retraction of the leading leg at the liftoff instant, helps prevent the swing leg from lagging the support leg, and enables larger swing toe clearance.

Figure 9. Maximum absolute value of the eigenvalues of the limit cycles as a function of E_{lc} under open-loop (blue) and closed-loop (red) energy injection for $[k, l] = [0.5 \text{ Nm rad}^{-1}, 0.05 \text{ m}]$.

The input-output relations of the controllers (34) and (35) with the locomotion dynamics are illustrated in figure 10.

4.3. Simulation results

In this section, we characterize the walking performance of the proposed framework under the energy feedback control strategies (34) and (35) as a function of terrain roughness. Each simulation continues until failure or 1000 successful steps on 100 different terrain profiles. Overall, the model is evaluated for 100 000 steps for a particular terrain roughness. We loosely define the stability of the biped on rough terrain by taking continuous steps. In this respect, the total number of successful steps is used to assess the rough terrain locomotion performance of the model. The initial conditions $\mathbf{q}_l[0] = \mathbf{q}_l^*$ for each model configuration are chosen among the limit cycles that equally tolerate single step-up and down disturbance for $\gamma = 0^\circ$. We present the maximum step heights $\Delta h_{\max}$ that the model successfully traverses over uphill and downhill slopes and the average mechanical cost of transport (mCOT). The mCOT for each step is defined as:

$$\text{mCOT} = \frac{\frac{1}{2}k\Delta\psi[i]^2}{Mgd_t[i]}. \quad (37)$$

We start by illustrating the effects of k and l on the rough terrain locomotion performance under the controller (34) with figure 11. Each square corresponds to the number of steps taken before failure for the associated $[k, l]$ pair with a sample terrain roughness of $\sigma = 1.6^\circ$. The regions that the model has high success rate (yellow) are concentrated at $l = 0.2\text{--}0.3 \text{ m}$ and $k = 1.5\text{--}2.5 \text{ Nm rad}^{-1}$, supporting our findings in the previous section. We extend our analysis by investigating the locomotion performance on terrains with increasing roughness for sample ankle actuation configurations with figure 12. As expected, the ankle actuation with small feet, see the top subfigure in figure 12(a), exhibits poor

locomotion performance, degrading quickly beyond $\sigma \approx 0.1^\circ$. The middle plot in figure 12 depicts the maximum step heights $\Delta h_{\max}$ that the model can successfully traverse, where the solid and dashed lines represent uphill and downhill slopes, respectively. This actuation configuration yields a very limited performance with $\Delta h_{\max} = \pm 0.35 \text{ cm}$ for the perfectly sustained locomotion behavior. However, it renders slow but very efficient locomotion on level terrain with $\text{mCOT} \approx 0.003$ at an average walking speed of $v = 0.23 \text{ m s}^{-1}$. As a comparison, human locomotion is reported to have $\text{mCOT} = 0.05$ at approximately 1 m s^{-1} (Collins and Ruina 2005).

Larger feet of $l = 0.15 \text{ m}$ leads to a significant increase in locomotion performance as illustrated in the top plot of figure 12(b). In contrast to the results in figure 12(a), the most compliant spring configuration does not yield the best locomotion performance. Instead, $k = 1 \text{ Nm rad}^{-1}$ yields the best performance where the locomotion is perfectly sustained for 100 000 steps on rough terrains until $\sigma = 1.25^\circ$. Moreover, the maximum step heights for the perfectly sustained locomotion profiles are $\pm 8 \text{ cm}$ with an $\text{mCOT} = 0.072$. However, further increase in the spring stiffness leads to deterioration on the performance. Even though stiffer ankle actuation does not lead to improved tolerance to terrain variations, it can generate more efficient locomotion with $\text{mCOT} \approx 0.02$ at $v \approx 0.4 \text{ m s}^{-1}$. In contrast, more compliant ankle yields less efficient locomotion at $\text{mCOT} \approx 0.07$. This is because the spring with a lower stiffness constant extends more which in return increases (36). Further increase in the feet length ($l = 0.25 \text{ m}$) leads to increase in the locomotion performance as illustrated in figure 12(c). However, the performance deteriorates for very large feet of $l = 0.35 \text{ m}$ as illustrated in figure 12(d). Hence, the comprehensive rough terrain test results in figure 12 complies with our finding in the previous sections that a human-like feet length and a relatively

compliant spring renders the best tolerance to terrains variations.

Based on the above observations, we choose the ankle parameters as $[k, l] = [1.75 \text{ Nm rad}^{-1}, 0.25 \text{ m}]$ which is one of the best performing configurations. With these actuation parameters, we find that the model perfectly walks until $\sigma = 1.7^\circ$ as shown with the circled line in figure 13. Most important of all, this compliance configuration combined with the energy feedback law (34) allows the model to tolerate height differences blindly up to $\pm 12 \text{ cm}$ with $\text{mCOT} = 0.087$ at $v = 0.55 \text{ m s}^{-1}$, a remarkable terrain height tolerance with high energy efficiency for such simple models. For larger σ values, the model cannot sustain perfect locomotion; however, it can withstand terrain heights of approximately 18.5 cm. An important observation is that the mCOT remains mostly unchanged for all compliance configurations across all levels of terrain roughness. Even though larger uphill slopes require relatively more energy to traverse, fully passive SS trajectories enable the model to traverse larger downhill slopes with less energy. Eventually, since $\mu = 0^\circ$ for each terrain profile, mCOT

remains approximately the same as the terrain roughness increases.

In addition, figure 13 illustrates the improvements achieved with the one-step lookahead energy update rule given in (35) under different feedback gains. We observe an important increase in the locomotion performance with $k_E = 0.0015$ where the model sustains locomotion on all terrains until $\sigma = 1.95^\circ$ with $\text{mCOT} = 0.086$. The terrain height tolerance for perfect locomotion can be extended as much as 13 cm and 15.6 cm for uphill and downhill terrains, respectively. An interesting observation is that one-step lookahead feedback also decreases the mCOT slightly. However, a further increase in the feedback gain ($k_E = 0.0025$) leads to reduced locomotion performance. This suggests that there is an optimal k_E , which allows better adaptation to the terrain variations. Due to the highly nonlinear equations of motion, the optimal k_E cannot be attained via analytical tools. Hence, we leave discovering the underlying mechanism of the optimal feedback gain as future work. It is worthwhile noting that the proposed approach enables bipedal locomotion on terrain heights that are frequently

encountered in outdoor settings such as stairs, side-walks, and daily obstacles with a cost less than twice of level human walking. Therefore, we conjecture that the physical bipedal platforms can benefit from the human-ankle-inspired actuation to navigate on rough terrain with longer durations.

4.4. Performance and controller assessment

To assess the proposed model's rough terrain locomotion performance, we performed a comparative analysis with the state-of-the-art planar bipeds in the literature (see table 1). We first compare the proposed model against planar bipeds of similar complexity

with various actuation schemes. The passive compass models can walk on shallow slopes and only tolerate slight variations in terrain roughness on slightly downhill surfaces (Su and Dingwell 2007, Afshar and Ren 2012). Hip actuation significantly improves the rough terrain locomotion performance of compass biped models (Kino *et al* 2019, Dai and Tedrake 2013, Fan and Manchester 2018, Fevre and Schmedeler 2020) and implementation (Manchester *et al* 2011); however hip control cannot directly inject energy to the model, hence limits the biped to only downhill slopes. The works of Byl and Tedrake (2009), Iida and Tedrake (2010) and Lee and Kim (2020) incorporated

ankle actuation that injects energy into the model to enable locomotion on both uphill and downhill rough terrains.

Even though their performance has improved over the years, compass bipeds still have limited locomotion capabilities on both uphill and downhill rough terrains. These limitations are circumvented by introducing multiple limbs and advanced control approaches. Moreover, such complex models are used to represent and control the state-of-the-art physical platforms. However, due to multiple limbs and continuous trajectory tracking, they have the disadvantage of increased complexity and cost of transport. Recently, the planar model of ATRIAS biped in Nguyen *et al* (2017) is reported to tolerate 30 cm of step height via utilizing passive dynamics and compliant hip joints. The mCOT of the ATRIAS model on rough terrain is not reported in Nguyen *et al* (2017); however, its 3D implementation is reported to have an mCOT = 1 with the maximum ground height variation of 15 cm (Hubicki *et al* 2018). Similarly, the 3D model of MARLO biped is reported to withstand uphill and downhill terrain variations of

8 cm and 12 cm with an mCOT ≈ 0.24 (Griffin and Grizzle 2017), respectively. These successful implementations achieve efficient sustained locomotion on significantly rough terrains via utilizing the SLIP model to control their legs. Nevertheless, they use passive ankle joints and continuous trajectory tracking for hip joints which reduces their efficiency.

As a remedy to commonly faced robustness and energy efficiency issues in the literature, this paper presents an actuation framework that injects energy via the finite-time DS phase. The proposed ankle actuation and control scheme empowers a simple biped model to tolerate substantial terrain variations, yielding the best rough terrain performance among compass bipeds. Moreover, the proposed approach yields similar (and even better in some cases) locomotion performance compared to complex planar models, incorporating torso (Chen and Byl 2012, Fevre *et al* 2019) along with knees (Saglam and Byl 2014, Saglam and Byl 2018, Nguyen *et al* 2017) without compromising energy efficiency. Ankle compliance and energy feedback enable the FCB model to achieve sustained locomotion on rough terrain up to $\sigma =$

Table 1. Sustained locomotion performance comparison of planar bipeds^c.

Compass biped/actuation	Terrain roughness	Largest height	Largest slope	Terrain knowledge	mCOT
FCB/A—proposed approach	$\sigma: 1.7^\circ$ ($\sigma: 2.8$ cm)	12	8.5°	Unknown	0.087
	$\sigma: 1.95^\circ$ ($\sigma: 3.1$ cm)	15.6	9.2°	One-step	0.086
Su and Dingwell (2007)/P	$0^\circ < \Delta\gamma < -1^\circ$	—	-1°	—	0
Afshar and Ren (2012)/P	$-2.4 < \Delta h < -2.6$ cm	-2.6	—	—	0
Kino <i>et al</i> (2019)/H	$-0.7^\circ < \Delta\gamma < -2.5^\circ$	—	-2.5°	Unknown	—
Dai and Tedrake (2013)/H	$-8^\circ < \Delta\gamma < -2^\circ$	—	-8°	Unknown	—
Fan and Manchester (2018)/H	—	—	-5° ^a	Unknown	—
Fevre and Schmiedeler (2020)/H	$\sigma: 0.06$ cm	—	—	Unknown	—
Lee and Kim (2020)/A	$-3^\circ < \Delta\gamma < 3^\circ$	4.3	—	One-step	—
Byl and Tedrake (2009)/H + A	$\sigma: 0.8$ cm	—	—	One-step	—
Iida and Tedrake (2010) ^b /H + A	$-2.6^\circ < \Delta\gamma < 3.75^\circ$	3	—	Unknown	—
Manchester <i>et al</i> (2011) ^b /H	—	-2 ^a	—	Three-steps	—
Complex biped/actuation					
Dai and Tedrake (2013)/H + K	$-2^\circ < \Delta\gamma < 2^\circ$	—	2°	Unknown	—
Chen and Byl (2012)/H	$\sigma: 2.7$ cm	10% l_0	—	Unknown	—
Saglam and Byl (2014)/H + K	$\sigma: 1.5^\circ$	—	—	One-step	0.5
Saglam and Byl (2018)/H + K	$\sigma: 1.8^\circ$	—	—	Unknown	—
Fevre <i>et al</i> (2019)/H	$\sigma: 0.85^\circ$ (HZD)	—	—	Unknown	0.15
	$\sigma: 1.55^\circ$ (Vel Dec)	—	—	Unknown	—
	$\sigma: 1.8^\circ$ (TLFC)	—	—	Unknown	—
Nguyen <i>et al</i> (2017)/H + K	$-30 < \Delta h < 30$ cm	30	—	One-step	—

^aSingle step-down disturbance.^bExperimental compass biped.^cLargest height values are given in cm and l_0 stands for leg length. Actuation methods: P = passive, H = hip, K = knee, A = ankle. σ : standard deviation of terrain roughness. Δh : step height change, $\Delta\gamma$: step slope change. HZD: hybrid zero dynamics, Vel Dec: velocity decomposition, TLFC: transverse linearization feedback control.

1.95° and tolerate terrain heights up to 15 cm with a very low mCOT = 0.086.

Although the proposed approach draws inspiration from the compliance of human ankle pushoff, it incorporates key differences. Specifically, the ankle-foot structure in humans stores energy during the early SS phase and releases this energy during late SS and DS phases (Hansen *et al* 2004, Torricelli *et al* 2016). Additionally, the stiffness of human ankle joint is dynamically modulated during SS phase in preparation for ankle pushoff (Lee *et al* 2016). However, realizing all the characteristics of the ankle joint in the same way as humans utilize during locomotion may not be practical on a physical platform. This is because the human muscle-tendon structure is very compact and efficient compared to the electromechanical components used in robotic platforms. Thus, motivated by the energy storage-release and finite-time energy injection mechanisms of the human ankle, we aim to realize these features on physical platforms by using appropriate tools instead of perfectly replicating human functionality. Hence, we implement the compliant actuation scheme of the human ankle by using a rotary ankle spring of constant stiffness, which is only active during the DS phase. Furthermore, we rely on the passive dynamics during the SS phase. We choose to mimic the human ankle compliance by adjusting the final rotation angle of the spring instead of adjusting its stiffness. This is because the physical realization of spring compression (via

an SEA) would be more practical than realizing a variable-stiffness actuator.

Another aspect of our motivation in developing the proposed approach is that the human ankle angle exhibits large variations during locomotion on uneven terrain compared to level walking (Shultz and Goldfarb 2018, Voloshina *et al* 2013). Hence, by allowing angle variations in the ankle joint via energy regulation, the proposed controller enables locomotion on significant rough terrain. As a result, instead of fully replicating the human ankle features, we aim to exploit the design parameters of compliant ankle actuation and realize its advantages for efficient and effective rough terrain locomotion. As a final remark, the energy regulation of ankle compliance could lead to a decrease in the number of actively controlled joints when it is extended to high-dimensional models and physical platforms. Thus, it offers efficient locomotion by reducing complexity while maintaining robustness to terrain variations.

5. Conclusions

Energy efficiency and robustness in bipedal locomotion on rough terrain play a critical role in having legged robots outside the lab environment to perform daily tasks for humans. However, there is a trade-off between energy efficiency and robustness considering the available models and controllers for

legged locomotion. On the one hand, robustness can be easily achieved with active control of all locomotion phases, increasing the energy need. On the other hand, reliance on passive dynamics reduces the cost of transport at the expense of compromised robustness.

In this paper, driven by the idea of ankle compliance of humans that enables finite-time energy injection, we presented a compliant ankle actuation and control framework for biped walkers that balances the trade-off between efficient and robust locomotion on rough terrain. Using open-loop limit cycles, we first showed that the tolerance to terrain disturbances increases as the feet length increases, and the spring stiffness yielding the best tolerance also increases with the feet length. This trend continues until the feet reaches to a length similar to humans. Very large feet lead to lower disturbance tolerance since the ankle only provides limited pushoff force on step-up disturbances. Thus, there is an optimal interval of feet length and spring stiffness which achieves the best locomotion performance on rough terrains. Then, we developed once-per-step energy feedback control strategy that modulates the ankle spring precompression using the energy relations of key instants of locomotion. Finally, we augmented the energy feedback using the terrain information of the upcoming step to improve the locomotion performance. We find that a foot length of $l = 0.2\text{--}0.3$ m, similar to humans, and a relatively compliant ankle stiffness of $k = 1.5\text{--}2.5$ Nm rad⁻¹ can better evade the failure cases of bipedal locomotion.

Our results showed that efficient and robust locomotion on significantly rough terrain is made possible by preserving the passive dynamics and tracking the energy level of the liftoff instance along with a one-step lookahead energy update strategy. It is reasonable to expect that additional feedback and tighter regulation of the passive dynamics could further improve the locomotion performance. From the perspective of robotics, we believe that this study can be used as a potential design tool to develop new hardware leading to energetically efficient bipedal locomotion on rough terrain. The next step in our research is to augment the energy regulation with the lookahead feedback by exploiting the optimization of model dynamics. Then, we seek to have a physical implementation of the model and test the proposed control principles on a biped platform.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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