

An Adaptive Human Pilot Model for Adaptively Controlled Systems

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Abstract—Despite their success in handling uncertain dynamical systems that are prone to failure, adaptive controllers are observed to have unfavorable interactions with human pilots in certain applications. To alleviate this problem, we need to evaluate the safety and performance of adaptive controllers in the simulation environment using realistic pilot models before conducting flight tests. While many useful human pilot models exist in the literature, models that are adequate for the prediction of adaptive human-adaptive controller interactions are yet to be available. In this letter, we fill this gap by proposing an adaptive human pilot model suited for the prediction of human behavior in the loop with an adaptive controller. The model can serve as a valuable tool guiding the design of adaptive controllers so as to ensure smooth pilot-controller interactions.

Index Terms—Adaptive control, adaptive systems, human-in-the-loop control.

I. INTRODUCTION

IRRESPECTIVE of the numerous advances in the field of control theory, the human operator is still irreplaceable in critical tasks where uncertainty in the controlled elements and unexpected external influences can lead to catastrophic events. And with a long way ahead before reaching highly intelligent controllers that can imitate the versatility and the adaptability of human operators in controlling complex systems, several mathematical human pilot models are developed aiming to enhance the aviation safety and handling quality measures by providing valuable guidance to the control system designer.

One of the most important pilot models is the crossover model proposed by McRuer [1], which is derived based on experimental observations that a human operator controls a system in such a way that results in a well-designed linear feedback system. The crossover model is followed by many extensions forming a large pool a designer can refer to for predicting the human behavior in the loop with a wide range of time-invariant controlled elements [2], [3]. But, when it comes

to modeling systems that are prone to failures, damage, and sudden parametric changes, these fixed pilot models fail to resemble how a human is found to respond adaptively in such critical situations. This shortcoming led to the development of a few adaptive human pilot models.

A prominent adaptive human pilot model is the one proposed in [4] and [5], where the adaptation laws are based on expert knowledge, aiming to make the adaptive pilot model follow the dictates of the crossover model. Inspired by this idea, a promising experimentally-validated adaptive human pilot model is recently developed in [6] and [7], by resorting to model reference adaptive control (MRAC) techniques which allows a rigorous stability analysis using the Lyapunov-Krasovskii stability criteria. These studies are conducted assuming the adaptive human pilot is operating on a linear controller-plant combination. Although there exist studies such as [8], [9], where an adaptive controller is in the loop, the pilot model used is not adaptive.

Developing adaptive pilot models that are suitable for adaptively controlled systems is important because owing to their special nonlinear characteristics, unfavorable interactions of human pilots with the adaptive control systems are observed in several flight tests [10].

The field of flight control reconfiguration emerged to address the issue of actuator failures by replacing redundant physical hardware with control algorithms that utilize the remaining control surfaces to compensate for loss of actuator effectiveness [11]. The consideration of such an uncertainty in the early design stage has become a standard approach in adaptive controller design [8], [12]–[15], and in adaptive control allocation methods [9], [16], [17], after its importance is highlighted in early studies such as [18], [19].

In this letter, we propose an adaptive human pilot model suited to be used in the loop with an adaptive controller, for the first time in the literature. The model serves as a tool for testing adaptive controllers, which can help the designer manipulate the controller parameters in order to ensure smooth pilot-controller interactions. The development of the model is carried out based on MRAC architecture, with a rigorous Lyapunov stability analysis.

A preliminary version of this letter can be found in [20]. This letter distinguishes itself from [20] by introducing additional actuator uncertainty. The presence of this uncertainty adds to the complexity of the adaptive pilot model, necessitating two additional adaptive parameters, and imposing a

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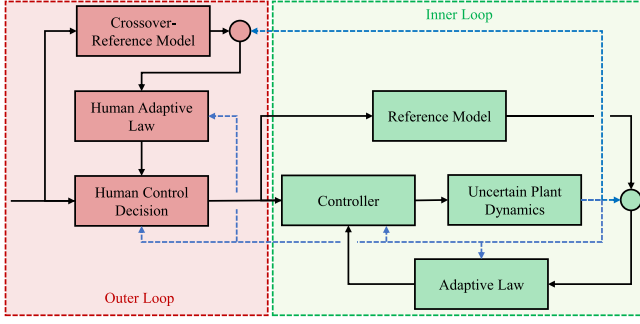


Fig. 1. Block Diagram.

supplemental condition on the adaptive controller. As a result, a time-varying anti-windup design parameter is used to compensate for the additional adaptive parameters, and the design parameter's existence is ensured by guaranteeing the invertibility of the adaptive parameters. In addition, we extend our results to plants with free integrators, where a special treatment is needed to ensure command tracking. The latter allows us to conduct a numerical simulation of a transport aircraft where a human pilot commands all attitude rates using a side-stick aiming to control the vehicle's attitude angles.

The notation used here is standard, where $\mathbb{R}^{p \times q}$ [$\mathbb{S}^{p \times q}$] [$\mathbb{D}^{p \times q}$] denotes the set of real [symmetric real] [diagonal real] p by q matrices, and $\|\cdot\|$ refers to the euclidean norm for vectors ($q = 1$), and the induced-2 norm for matrices. $\|\cdot\|_F$ refers to the Frobenius norm for matrices, $\text{Tr}\{\cdot\}$ refers to the trace operator, and $(\cdot)^T[(\cdot)^{-1}]$ denotes the transpose [inverse] operator. Finally, we write $\lambda_{\min}(A)$ for the minimum eigenvalue of the matrix A and we denote the set of positive definite real matrices by $\mathbb{R}_+^{p \times p}$.

II. PROBLEM STATEMENT

To model the human's adaptive control behavior with an adaptive controller in the loop, we start with a block diagram given in Fig. 1. In the figure, the block diagram is divided into inner and outer loops. The inner-loop consists of an adaptive controller controlling a plant with uncertain dynamics such that the plant states follow those of a reference model by adjusting the control parameters using an adaptive law.

The outer-loop consists of the human controlling the inner-loop such that the plant output follows a reference input. The human is assumed to be well trained, i.e., familiar with the nominal plant-controller dynamics. However, he/she is not aware of the uncertainties in the plant dynamics. This motivates modeling the human as an adaptive outer-loop controller, where an adaptive law is utilized to force the plant states to follow the states of the crossover-reference model.

III. INNER LOOP

Consider the following uncertain plant dynamics

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p \Delta u_p(t) + B_p W_p^T \sigma_p(x_p(t)), \\ y_1(t) &= C_1^T x_p(t), \quad y_2(t) = C_2^T x_p(t), \end{aligned} \quad (1)$$

where $x_p(t) \in \mathbb{R}^{n_p}$ is the accessible state vector, $u_p(t) \in \mathbb{R}^m$ is the plant control input, $\sigma_p(x_p(t)) : \mathbb{R}^{n_p} \rightarrow \mathbb{R}^s$ is a known possibly non-linear basis function, $W_p \in \mathbb{R}^{s \times m}$ is an unknown weight matrix, $\Lambda \in \mathbb{R}_+^{m \times m} \cap \mathbb{D}^{m \times m}$ is an unknown control effectiveness matrix with the diagonal elements $\lambda_{i,i} \in (0, 1]$, $A_p \in \mathbb{R}^{n_p \times n_p}$ is an unknown system matrix, $B_p \in \mathbb{R}^{n_p \times m}$ is a known control input matrix, and $C_1 \in \mathbb{R}^{n_p \times m}$ and $C_2 \in \mathbb{R}^{n_p \times r}$ are both known output matrices. The outputs $y_1(t) \in \mathbb{R}^m$ and $y_2(t) \in \mathbb{R}^r$ are the outputs of interest for the inner and outer loops, respectively. Furthermore, it is assumed that the pair (A_p, B_p) is controllable.

Let the nominal plant dynamics be given as

$$\dot{x}_n(t) = A_n x_n(t) + B_p u_n(t), \quad (2)$$

where $u_n(t) \in \mathbb{R}^m$ is a nominal controller given as

$$u_n(t) = -L_x x_n(t) + L_r y_h(t), \quad (3)$$

where $y_h(t) \in \mathbb{R}^m$ is the human command, and $L_x \in \mathbb{R}^{m \times n_p}$ is such that $A_r \triangleq A_n - B_p L_x$ is Hurwitz. It is noted that the human input $y_h(t)$ is bounded due to physical manipulator limits. In the design of the outer loop, given in the next section, human input saturation is considered in the stability analysis. Defining $B_r \triangleq B_p L_r$, the reference model is assigned as

$$\dot{x}_r(t) = A_r x_r(t) + B_r y_h(t). \quad (4)$$

For a constant y_h , at steady state, it is obtained using (4) that $\dot{x}_r(\infty) = 0 = A_r x_r(\infty) + B_r y_h$, and $x_r(\infty) = -A_r^{-1} B_p L_r y_h$. Hence, once $\lim_{t \rightarrow \infty} x_p(t) = x_r(t)$, the plant output $y_1(t)$, given in (1), takes the form $y_1(\infty) = -C_1^T A_r^{-1} B_p L_r y_h$. Selecting

$$L_r = -(C_1^T A_r^{-1} B_p)^{-1}, \quad (5)$$

results in $\lim_{t \rightarrow \infty} y_1(t) = y_h$.

Considering (1), which can be rewritten as

$$\dot{x}_p(t) = A_p x_p(t) + B_p \Lambda (u_p(t) + W^T \sigma_p(x_p(t))), \quad (6)$$

where $W^T \triangleq \Lambda^{-1} W_p^T \in \mathbb{R}^{m \times s}$ is an unknown parameter, we assume that there exist $K_x^* \in \mathbb{R}^{m \times n_p}$ and $K_r^* \in \mathbb{R}^{m \times m}$ such that the matching conditions

$$A_p - B_p \Lambda K_x^* = A_r, \quad B_p \Lambda K_r^* = B_r \triangleq B_p L_r, \quad (7)$$

are satisfied. The second matching condition implies that $K_r^* = \Lambda^{-1} L_r$. We define the plant control law as

$$u_p(t) = -\hat{K}_x(t) x_p(t) + \text{diag}(\hat{\lambda}(t)) L_r y_h(t) - \hat{W}^T(t) \sigma_p(x_p(t)), \quad (8)$$

where $\hat{K}_x(t) \in \mathbb{R}^{m \times n_p}$, $\hat{\lambda}(t) \in \mathbb{R}^m$ and $\hat{W}(t) \in \mathbb{R}^{s \times m}$ are adjustable parameters serving as estimates for the ideal values K_x^* , λ^* and W , respectively. It is noted that $\text{diag}(\lambda^*) = \Lambda^{-1}$ exists since Λ is diagonal positive definite.

Substituting (8) into (6), one can rewrite (6) as

$$\begin{aligned} \dot{x}_p(t) &= A_r x_p(t) + B_r y_h(t) + B_p \Lambda \text{diag}(\tilde{\lambda}(t)) L_r y_h(t) \\ &\quad - B_p \Lambda (\tilde{W}^T(t) \sigma_p(x_p(t)) + \tilde{K}_x(t) x_p(t)), \end{aligned} \quad (9)$$

where $\tilde{K}_x(t) \triangleq \hat{K}_x(t) - K_x^*$, $\tilde{\lambda}(t) \triangleq \hat{\lambda}(t) - \lambda^*$ and $\tilde{W}(t) \triangleq \hat{W}(t) - W$ are the adaptive parameters errors.

Defining $\Theta(t) \triangleq [\hat{W}^T(t), \hat{K}_x^T(t)]^T \in \mathbb{R}^{(s+n_p) \times m}$, with the ideal value $\Theta^* \triangleq [W^T, K_x^*]^T$, (9) can be rewritten as

$$\dot{x}_p(t) = A_r x_p(t) + B_r y_h(t) + B_p \Lambda \text{diag}(\tilde{\lambda}(t)) L_r y_h(t) - B_p \Lambda \tilde{\Theta}^T(t) \sigma(x_p(t)), \quad (10)$$

where $\tilde{\Theta}(t) \triangleq \Theta(t) - \Theta^*$ and $\sigma(x_p(t)) \triangleq [\sigma_p^T(x_p(t)), x_p^T(t)]^T \in \mathbb{R}^{s+n_p}$.

By subtracting (4) from (10), and exploiting the fact that $\Lambda \text{diag}(\tilde{\lambda}(t)) L_r y_h(t) = \text{diag}(L_r y_h(t)) \Lambda \tilde{\lambda}(t)$, we obtain that

$$\dot{e}_1(t) = A_r e_1(t) + B_p \text{diag}(L_r y_h(t)) \Lambda \tilde{\lambda}(t) - B_p \Lambda \tilde{\Theta}^T(t) \sigma(x_p(t)), \quad (11)$$

where $e_1(t) \triangleq x_p(t) - x_r(t)$ is the inner-loop tracking error. We define the inner-loop adaptive laws as

$$\dot{\tilde{\Theta}}(t) = \dot{\Theta}(t) = \gamma_\theta \sigma(x_p(t)) e_1(t)^T P_1 B_p, \quad (12a)$$

$$\dot{\tilde{\lambda}}(t) = \dot{\lambda}(t) = \gamma_\lambda \text{Proj}\left(\hat{\lambda}(t), -\text{diag}(L_r y_h(t)) B_p^T P_1 e_1(t)\right), \quad (12b)$$

where $\text{Proj}(\cdot, \cdot)$ is the projection operator [12], [16] used to bound the adaptive parameter $\hat{\lambda}(t)$ in a compact set with each element $\hat{\lambda}_i(t)$ having positive upper and lower bounds $\hat{\lambda}_{\max_i}$ and $\hat{\lambda}_{\min_i}$. Furthermore, $\gamma_\theta, \gamma_\lambda \in \mathbb{R}_+$ are learning rates, and $P_1 \in \mathbb{R}_+^{n_p \times n_p} \cap \mathbb{S}^{n_p \times n_p}$ is the solution of $A_r^T P_1 + P_1 A_r = -Q_1$, for some $Q_1 \in \mathbb{R}_+^{n_p \times n_p} \cap \mathbb{S}^{n_p \times n_p}$.

Lemma 1: Consider the uncertain dynamical system (1), the reference model (4), and the feedback control law given by (8) and (12). The solution $(e_1(t), \tilde{\Theta}(t), \tilde{\lambda}(t))$ is Lyapunov stable in the large. Furthermore, since the human command $y_h(t)$ is bounded, $\lim_{t \rightarrow \infty} e_1(t) = 0$ and $\tilde{\Theta}(t)$ and $\tilde{\lambda}(t)$ remain bounded along with all the signals in the inner-loop.

Proof: Consider the Lyapunov function candidate

$$V_1 = e_1^T P_1 e_1 + \gamma_\theta^{-1} \text{Tr}\{(\tilde{\Theta} \Lambda^{1/2})^T (\tilde{\Theta} \Lambda^{1/2})\} + \gamma_\lambda^{-1} \tilde{\lambda}^T \Lambda \tilde{\lambda}. \quad (13)$$

Differentiating along the trajectories (11) and (12) yields

$$\begin{aligned} \dot{V}_1 &= \dot{e}_1^T P_1 e_1 + e_1^T P_1 \dot{e}_1 + 2\gamma_\theta^{-1} \text{Tr}\{\tilde{\Theta}^T \dot{\tilde{\Theta}} \Lambda\} + 2\gamma_\lambda^{-1} \tilde{\lambda}^T \Lambda \dot{\tilde{\lambda}} \\ &= -e_1^T Q_1 e_1 + 2\gamma_\theta^{-1} \text{Tr}\{\tilde{\Theta}^T \dot{\tilde{\Theta}} \Lambda\} + 2\gamma_\lambda^{-1} \tilde{\lambda}^T \Lambda \dot{\tilde{\lambda}} \\ &\quad - 2\text{Tr}\{\tilde{\Theta}^T \sigma(x_p) e_1^T P_1 B_p \Lambda\} + 2\tilde{\lambda}^T \Lambda \text{diag}(L_r y_h) B_p^T P_1 e_1 \\ &= -e_1^T Q_1 e_1 + 2\tilde{\lambda}^T \Lambda \left(\text{Proj}(\hat{\lambda}, -\text{diag}(L_r y_h) B_p^T P_1 e_1) \right. \\ &\quad \left. + \text{diag}(L_r y_h) B_p^T P_1 e_1 \right). \end{aligned} \quad (14)$$

Using the property $(\theta_{i,j} - \theta_{i,j}^*)(\text{Proj}(\theta_{i,j}, Y_{i,j}) - Y_{i,j}) \leq 0$ and the fact that Λ is diagonal positive definite, it follows that

$$\dot{V}_1 \leq -e_1^T Q_1 e_1 \leq 0. \quad (15)$$

Hence, the solution $(e_1(t), \tilde{\Theta}(t), \tilde{\lambda}(t))$ is stable in the large.

Equation (4) indicates that the boundedness of $y_h(t)$ implies the boundedness of $x_r(t)$. The latter coupled with the fact that $e_1(t)$ is bounded imply the boundedness of $x_p(t)$, which in turn implies that $(\dot{e}_1(t), \dot{\tilde{\Theta}}(t), \dot{\tilde{\lambda}}(t))$ are bounded. It then follows from Barbalat's lemma that $\lim_{t \rightarrow \infty} e_1(t) = 0$. ■

IV. OUTER LOOP

Using (7), and $\text{diag}(\lambda^*) = \Lambda^{-1}$, (10) can be rewritten as

$$\dot{x}_p(t) = A_r x_p(t) + B_p \Lambda \text{diag}(\hat{\lambda}(t)) L_r y_h(t) - B_p \Lambda \tilde{\Theta}^T(t) \sigma(x_p(t)). \quad (16)$$

Since we assume that the human operator is familiar with the nominal dynamics (2) and (3), the only unknowns in (16) for the outer loop are the parameters Λ , $\hat{\lambda}(t)$ and $\tilde{\Theta}(t)$.

Consider the integral action

$$\dot{x}_c(t) = y_2(t) - r(t) + J(t) \Delta y(t), \quad (17)$$

where $x_c(t) \in \mathbb{R}^r$ is the integrator state, $r(t) \in \mathbb{R}^r$ is a bounded reference input, and $J(t) \in \mathbb{R}^{r \times m}$ is a control parameter defined below. The term $J(t) \Delta y(t)$, which will be explained later, is included to avoid integrator windup, and it only takes action when the human input is saturated [21], [22]. Aggregating (16) and (17) results in the augmented dynamics

$$\begin{aligned} \dot{x}(t) &= A x(t) + B_m r(t) - B_m J(t) \Delta y(t) \\ &\quad + B H^T(t) \sigma(x_p(t)) + B \Lambda_2(t) L_r y_h(t), \end{aligned} \quad (18)$$

where $x(t) \triangleq [x_p^T(t), x_c^T(t)]^T \in \mathbb{R}^n$ is the augmented state vector with $n = n_p + r$, $H^T(t) \triangleq -\Lambda \tilde{\Theta}^T(t)$ and $\Lambda_2(t) \triangleq \Lambda \text{diag}(\hat{\lambda}(t))$ are unknown time-varying parameters, and

$$A \triangleq \begin{bmatrix} A_r & 0_{n_p \times r} \\ C_2^T & 0_{r \times r} \end{bmatrix} \in \mathbb{R}^{n \times n}, \quad (19a)$$

$$B \triangleq \begin{bmatrix} B_p^T & 0_{m \times r} \end{bmatrix}^T \in \mathbb{R}^{n \times m}, \quad (19b)$$

$$B_m \triangleq \begin{bmatrix} 0_{r \times n_p} & -I_{r \times r} \end{bmatrix}^T \in \mathbb{R}^{n \times r}. \quad (19c)$$

The goal of the human is to control the system such that the plant states follow that of a unity feedback reference model with an open loop crossover model transfer function. We refer to the latter as the *crossover-reference model* (Fig. 1). Let the crossover-reference model be given as

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t), \quad (20)$$

where $x_m(t) \in \mathbb{R}^n$ is the crossover-reference model state vector, and $A_m \in \mathbb{R}^{n \times n}$ is Hurwitz.

In an ideal case where the human input is not saturated, and both $H(t)$ and $\Lambda_2(t)$ are known, the following control law achieves the crossover-reference model dynamics

$$\begin{aligned} \mathcal{G}^*(t) &= -\theta_x x(t) - L_r^{-1} H^T(t) \sigma(x_p(t)), \\ y_h^*(t) &= L_r^{-1} \Lambda_2^{-1}(t) L_r \mathcal{G}^*(t), \end{aligned} \quad (21)$$

where we assume that there exists $\theta_x \in \mathbb{R}^{m \times n}$ such that $A_m = A - B L_r \theta_x$. We define the human control input by augmenting the baseline PI-control term $-\theta_x x(t)$ with adaptive components as

$$\mathcal{G}(t) = -\theta_x x(t) - L_r^{-1} \hat{H}^T(t) \sigma(x_p(t)), \quad (22a)$$

$$v(t) = L_r^{-1} \text{diag}(\hat{\lambda}_2(t)) L_r \mathcal{G}(t), \quad (22b)$$

$$y_{h_i}(t) = \begin{cases} v_i(t), & \text{if } |v_i(t)| \leq y_{oi}, \\ y_{oi} \text{sgn}(v_i(t)), & \text{if } |v_i(t)| > y_{oi}, \end{cases} \quad (22c)$$

where $\hat{H}(t)$ and $\hat{\lambda}_2(t)$ are adjustable parameters serving as estimates for the ideal values $H(t)$ and $\lambda_2^*(t)$, respectively. It is

noted that $\text{diag}(\lambda_2^*(t)) = \Lambda_2^{-1}(t)$ exists for all $t \geq 0$ since $\Lambda_2(t)$ is diagonal positive definite at all time instants. Finally, (22c) is an element-wise saturation function where $y_{oi} \in \mathbb{R}_+$ is the saturation limit of $y_{hi}(t)$ (the i^{th} element of $y_h(t)$).

Remark 1: Such a θ_x exists only if A_m is selected such that

$$A_m \triangleq \begin{bmatrix} A_r - B_r \phi & -B_r \psi \\ C_2^T & 0_{r \times r} \end{bmatrix} \in \mathbb{R}^{n \times n}, \quad (23)$$

for some $\phi \in \mathbb{R}^{m \times n_p}$ and $\psi \in \mathbb{R}^{m \times r}$. Then, θ_x can be assigned as

$$\theta_x \triangleq [\phi \quad \psi] \in \mathbb{R}^{m \times n}. \quad (24)$$

Assumption 1: It is assumed that there exists a matrix $M \in \mathbb{R}^{r \times m}$ such that $M\psi \in \mathbb{R}^{r \times r}$ is Hurwitz. This assumption is used in Theorem 1 below.

Substituting (22) into (18), we obtain that

$$\begin{aligned} \dot{x}(t) = & A_m x(t) + B_m r(t) - B_m J(t) \Delta y(t) + B \Lambda_2(t) L_r \Delta y(t) \\ & - B \tilde{H}^T(t) \sigma(x_p(t)) + B \Lambda_2(t) \text{diag}(\tilde{\lambda}_2(t)) L_r \mathcal{G}(t), \end{aligned} \quad (25)$$

where $\tilde{H}(t) \triangleq \hat{H}(t) - H(t)$ and $\tilde{\lambda}_2(t) \triangleq \hat{\lambda}_2(t) - \lambda_2^*(t)$ are outer-loop adaptive parameters errors, and $\Delta y(t) \triangleq y_h(t) - v(t)$ is the control deficiency due to human input saturation.

Subtracting (20) from (25), and exploiting the fact that $\Lambda_2(t) \text{diag}(\tilde{\lambda}_2(t)) L_r \mathcal{G}(t) = \text{diag}(L_r \mathcal{G}(t)) \Lambda_2(t) \tilde{\lambda}_2(t)$ results in the outer-loop error dynamics

$$\begin{aligned} \dot{e}_2(t) = & A_m e_2(t) - B_m J(t) \Delta y(t) + B \Lambda_2(t) L_r \Delta y(t) \\ & - B \tilde{H}^T(t) \sigma(x_p(t)) + B \text{diag}(L_r \mathcal{G}(t)) \Lambda_2(t) \tilde{\lambda}_2(t), \end{aligned} \quad (26)$$

where $e_2(t) \triangleq x(t) - x_m(t)$ is the outer-loop tracking error.

We generate an auxiliary signal $e_\Delta(t)$ as in [13], [23]

$$\begin{aligned} \dot{e}_\Delta(t) = & A_m e_\Delta(t) - B_m J(t) \Delta y(t) + B \text{diag}(\hat{\lambda}_3(t)) L_r \Delta y(t), \\ e_\Delta(t_0) = & 0, \end{aligned} \quad (27)$$

where $\hat{\lambda}_3(t)$ is an adjustable parameter serving as an estimate for the ideal value $\lambda_3^*(t)$, and $\text{diag}(\lambda_3^*(t)) = \Lambda_2(t)$. Defining an augmented error signal $e_y(t) \triangleq e_2(t) - e_\Delta(t)$, and using the fact that $\text{diag}(\tilde{\lambda}_3(t)) L_r \Delta y(t) = \text{diag}(L_r \Delta y(t)) \tilde{\lambda}_3(t)$ yields

$$\begin{aligned} \dot{e}_y(t) = & A_m e_y(t) - B \text{diag}(L_r \Delta y(t)) \tilde{\lambda}_3(t) \\ & - B \tilde{H}^T(t) \sigma(x_p(t)) + B \text{diag}(L_r \mathcal{G}(t)) \Lambda_2(t) \tilde{\lambda}_2(t), \end{aligned} \quad (28)$$

where $\tilde{\lambda}_3(t) \triangleq \hat{\lambda}_3(t) - \lambda_3^*(t)$. Equation (28) is in a standard error model form. We propose the adaptive laws

$$\dot{\hat{H}}(t) = \gamma_H \text{Proj}(\hat{H}(t), \sigma(x_p(t)) e_y^T(t) P_2 B), \quad (29a)$$

$$\dot{\hat{\lambda}}_2(t) = \gamma_2 \text{Proj}(\hat{\lambda}_2(t), -\text{diag}(L_r \mathcal{G}(t)) B^T P_2 e_y(t)), \quad (29b)$$

$$\dot{\hat{\lambda}}_3(t) = \gamma_3 \text{Proj}(\hat{\lambda}_3(t), \text{diag}(L_r \Delta y(t)) B^T P_2 e_y(t)), \quad (29c)$$

where $\gamma_H, \gamma_2, \gamma_3 \in \mathbb{R}_+$ are learning rates, and $P_2 \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$ is the solution of $A_m^T P_2 + P_2 A_m = -Q_2$, for some $Q_2 \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$. In addition, we impose bounds on the elements of $\hat{\lambda}_2(t)$ with each element $\hat{\lambda}_{2i}(t)$ having positive upper and lower bounds $\hat{\lambda}_{2max_i}$ and $\hat{\lambda}_{2min_i}$. This ensures that $\text{diag}(\hat{\lambda}_2(t))$ is always invertible.

Finally, regarding the anti-windup term used in (17), and with Assumption 1 satisfied, we define $J(t)$ as

$$J(t) = M L_r^{-1} \text{diag}(\hat{\lambda}_2(t))^{-1} L_r, \quad (30)$$

which is bounded since $\hat{\lambda}_{2min_i} > 0, i = 1, \dots, m$.

Remark 2: It follows from Lemma 1 that $\tilde{\Theta}(t), \hat{\lambda}(t), \dot{\hat{\lambda}}(t)$ and $\hat{\lambda}(t)$ are bounded, which implies the boundedness of $H(t), \Lambda_2(t), \dot{H}(t)$ and $\dot{\Lambda}_2(t)$. Therefore, there exist $h \in \mathbb{R}_+, \dot{h} \in \mathbb{R}_+, \beta_3 \in \mathbb{R}_+$ and $\dot{\beta}_3 \in \mathbb{R}_+$ such that $\|H(t)\| \leq h, \|\dot{H}(t)\| \leq \dot{h}, \|\Lambda_2(t)\|_F \leq \beta_3$ and $\|\dot{\Lambda}_2(t)\|_F \leq \dot{\beta}_3$ for all $t \geq 0$. The latter implies that $\|\lambda_3^*(t)\| \leq \beta_3$ and $\|\dot{\lambda}_3^*(t)\| \leq \dot{\beta}_3$. Moreover, as $\hat{\lambda}_{min_i} > 0$ for $i = 1, \dots, m$, there exists $\beta_2 \in \mathbb{R}_+$ such that $\|\Lambda_2^{-1}(t)\|_F \leq \beta_2$. And since $\frac{d\Lambda_2^{-1}}{dt} = -\Lambda_2^{-1} \dot{\Lambda}_2 \Lambda_2^{-1}$, then there exists $\dot{\beta}_2 \in \mathbb{R}_+$ such that $\left\| \frac{d\Lambda_2^{-1}}{dt} \right\|_F \leq \dot{\beta}_2$. This implies that $\|\lambda_2^*(t)\| \leq \beta_2$ and $\|\dot{\lambda}_2^*(t)\| \leq \dot{\beta}_2$ for all $t \geq 0$.

Theorem 1: Consider the uncertain dynamical system (1), the adaptive controller given by (4), (8) and (12), and the adaptive pilot model given by (17), (20), (22) and (29). Then, the solution $(e_y(t), \tilde{H}(t), \tilde{\lambda}_2(t), \tilde{\lambda}_3(t))$ remains bounded for all $t \geq 0$ and converges to the compact set

$$\begin{aligned} E \triangleq & \left\{ (e_y(t), \tilde{H}(t), \tilde{\lambda}_2(t), \tilde{\lambda}_3(t)) : \|e_y(t)\|^2 \leq \eta, \right. \\ & \left. \|\tilde{H}(t)\| \leq \tilde{h}, \|\tilde{\lambda}_2(t)\| \leq \tilde{\alpha}_2 \text{ and } \|\tilde{\lambda}_3(t)\| \leq \tilde{\alpha}_3 \right\}, \end{aligned} \quad (31)$$

where

$$\eta \triangleq \frac{2\gamma_H^{-1} \tilde{h} \dot{h} + 2\gamma_2^{-1} \tilde{\alpha}_2 \beta_3 \dot{\beta}_2 + \gamma_2^{-1} \tilde{\alpha}_2 \dot{\beta}_3 \tilde{\alpha}_2 + 2\gamma_3^{-1} \tilde{\alpha}_3 \dot{\beta}_3}{\lambda_{min}(Q_2)}, \quad (32)$$

$\tilde{h} \triangleq \|\hat{H}_{max}\| + h, \tilde{\alpha}_2 \triangleq \|\hat{\lambda}_{2max}\| + \beta_2, \tilde{\alpha}_3 \triangleq \|\hat{\lambda}_{3max}\| + \beta_3$, and $h, \dot{h}, \beta_2, \dot{\beta}_2, \beta_3$ and $\dot{\beta}_3$ are defined in Remark 2. Furthermore, all system signals are bounded.

Proof: Consider the Lyapunov function candidate

$$\begin{aligned} V_2(e_y, \tilde{H}, \tilde{\lambda}_2, \tilde{\lambda}_3) = & e_y^T P_2 e_y + \gamma_H^{-1} \text{Tr}\{\tilde{H}^T \tilde{H}\} \\ & + \gamma_2^{-1} \tilde{\lambda}_2^T \Lambda_2 \tilde{\lambda}_2 + \gamma_3^{-1} \tilde{\lambda}_3^T \tilde{\lambda}_3. \end{aligned} \quad (33)$$

Differentiating along the trajectories (28) and (29) yields

$$\begin{aligned} \dot{V}_2 = & \dot{e}_y^T P_2 e_y + e_y^T P_2 \dot{e}_y + 2\gamma_H^{-1} \text{Tr}\{\tilde{H}^T \dot{\tilde{H}}\} \\ & + 2\gamma_2^{-1} \tilde{\lambda}_2^T \Lambda_2 \dot{\tilde{\lambda}}_2 + \gamma_2^{-1} \tilde{\lambda}_2^T \dot{\Lambda}_2 \tilde{\lambda}_2 + 2\gamma_3^{-1} \tilde{\lambda}_3^T \dot{\tilde{\lambda}}_3 \\ = & -e_y^T Q_2 e_y - 2\text{Tr}\{\tilde{H}^T \sigma(x_p) e_y^T P_2 B\} \\ & + 2\tilde{\lambda}_2^T \Lambda_2 \text{diag}(L_r \mathcal{G}) B^T P_2 e_y - 2\tilde{\lambda}_3^T \text{diag}(L_r \Delta y) B^T P_2 e_y \\ & + 2\gamma_H^{-1} \text{Tr}\{\tilde{H}^T \dot{\tilde{H}}\} - 2\gamma_H^{-1} \text{Tr}\{\tilde{H}^T \dot{\tilde{H}}\} \\ & + 2\gamma_2^{-1} \tilde{\lambda}_2^T \Lambda_2 \dot{\tilde{\lambda}}_2 - 2\gamma_2^{-1} \tilde{\lambda}_2^T \dot{\Lambda}_2 \tilde{\lambda}_2 + \gamma_2^{-1} \tilde{\lambda}_2^T \dot{\Lambda}_2 \tilde{\lambda}_2 \\ & + 2\gamma_3^{-1} \tilde{\lambda}_3^T \dot{\tilde{\lambda}}_3 - 2\gamma_3^{-1} \tilde{\lambda}_3^T \dot{\tilde{\lambda}}_3 \\ = & -e_y^T Q_2 e_y + \gamma_2^{-1} \tilde{\lambda}_2^T \dot{\Lambda}_2 \tilde{\lambda}_2 \\ & - 2\gamma_H^{-1} \text{Tr}\{\tilde{H}^T \dot{\tilde{H}}\} - 2\gamma_2^{-1} \tilde{\lambda}_2^T \Lambda_2 \dot{\tilde{\lambda}}_2 - 2\gamma_3^{-1} \tilde{\lambda}_3^T \dot{\tilde{\lambda}}_3 \\ & + 2\text{Tr}\{\tilde{H}^T (\text{Proj}(\hat{H}, Y_H) - Y_H)\} \\ & + 2\tilde{\lambda}_2^T \Lambda_2 (\text{Proj}(\hat{\lambda}_2, Y_2) - Y_2) \\ & + 2\tilde{\lambda}_3^T (\text{Proj}(\hat{\lambda}_3, Y_3) - Y_3), \end{aligned} \quad (34)$$

where $Y_H \triangleq \sigma(x_p)e_y^T P_2 B$, $Y_2 \triangleq -\text{diag}(L_r \mathcal{G}) B^T P_2 e_y$ and $Y_3 \triangleq \text{diag}(L_r \Delta y(t)) B^T P_2 e_y$. Using the projection property $(\theta_{ij} - \theta_{ij}^*)(\text{Proj}(\theta_{ij}, Y_{ij}) - Y_{ij}) \leq 0$, Remark 2, and the fact that $\Lambda_2(t)$ is diagonal positive definite, we obtain

$$\begin{aligned} \dot{V}_2 &\leq -e_y^T Q_2 e_y + \gamma_2^{-1} \tilde{\lambda}_2^T \dot{\Lambda}_2 \tilde{\lambda}_2 \\ &\quad - 2\gamma_H^{-1} \text{Tr}\{\tilde{H}^T \dot{H}\} - 2\gamma_2^{-1} \tilde{\lambda}_2^T \Lambda_2 \dot{\lambda}_2^* - 2\gamma_3^{-1} \tilde{\lambda}_3^T \dot{\lambda}_3^* \\ &\leq -\lambda_{\min}(Q_2) \|e_y\|^2 + \lambda_{\min}(Q_2) \eta. \end{aligned} \quad (35)$$

Hence, $\dot{V}_2 < 0$ outside the compact set (31), which proves the boundedness of $(e_y(t), \hat{H}(t), \hat{\lambda}_2(t), \hat{\lambda}_3(t))$. The boundedness of all the signals in the inner-loop, including $x_p(t)$, follows from Lemma 1. According to (20), since $r(t)$ is bounded, $x_m(t)$ is also bounded, which leaves us only to prove that $x_c(t)$ is bounded to complete the proof.

We prove the boundedness of $x_c(t)$ (and hence $x(t)$) by considering the cases where a) $\Delta y(t) = 0$ and b) $\Delta y(t) \neq 0$.

Case a) $\Delta y(t) = 0$: It follows from (27) that $e_\Delta(t)$ is bounded which by the fact that $e_y(t)$ is bounded, shows the boundedness of $e_2(t)$ and $x(t)$.

Case b) $\Delta y(t) \neq 0$: Using $\Delta y(t) = y_h(t) - v(t)$, (17) can be rewritten as

$$\dot{x}_c(t) = C_2^T x_p(t) - r(t) + J(t)y_h(t) - J(t)v(t). \quad (36)$$

Substituting (22b) and (22a), one can rewrite (36) as

$$\begin{aligned} \dot{x}_c(t) &= C_2^T x_p(t) - r(t) + J(t)y_h(t) \\ &\quad + J(t)L_r^{-1} \text{diag}(\hat{\lambda}_2(t)) L_r \theta_x x(t) \\ &\quad + J(t)L_r^{-1} \text{diag}(\hat{\lambda}_2(t)) \hat{H}^T(t) \sigma(x_p(t)). \end{aligned} \quad (37)$$

Using (24), it is obtained by substituting $\theta_x x(t) = \phi x_p(t) + \psi x_c(t)$ and (30) into (37) that

$$\begin{aligned} \dot{x}_c(t) &= J(t)L_r^{-1} \text{diag}(\hat{\lambda}_2(t)) L_r \psi x_c(t) + w(t), \\ &= M \psi x_c(t) + w(t), \\ w(t) &\triangleq (C_2^T + M \phi) x_p(t) - r(t) + J(t)y_h(t) \\ &\quad + M L_r^{-1} \hat{H}^T(t) \sigma(x_p(t)). \end{aligned} \quad (38)$$

As $r(t)$, $\hat{H}(t)$, $y_h(t)$, $J(t)$ and $x_p(t)$ are bounded, then so is $w(t)$. Therefore, it follows from Assumption 1 and (38) that $x_c(t)$ is bounded which implies the boundedness of $x(t)$.

The boundedness of $e_2(t)$ follows from the boundedness of $x(t)$ as $x_m(t)$ is bounded. Since $e_y(t) = e_2(t) - e_\Delta(t)$ is bounded, then $e_\Delta(t)$ is also bounded, which implies by (27) that $\Delta y(t)$ is bounded and completes the proof. ■

V. SPECIAL CASE: SYSTEMS WITH FREE INTEGRATORS

The assignment of L_r given in (5) is invalid when $C_1^T A_r^{-1} B_p$ is singular. This may happen, for example, when the pilot commands attitude rates to the inner-loop controller aiming to achieve desired attitude angles. In such a case, (1) can be modified by separating the states as

$$\begin{aligned} \begin{bmatrix} \dot{x}_s \\ \dot{x}_\theta \end{bmatrix} &= \begin{bmatrix} A_s & 0 \\ C_{1s}^T & 0 \end{bmatrix} \begin{bmatrix} x_s \\ x_\theta \end{bmatrix} + \begin{bmatrix} B_s \\ 0 \end{bmatrix} \Lambda(u_p(t) + W^T \sigma_p(x_s(t))), \\ y_1(t) = \dot{x}_\theta(t) &= C_{1s}^T x_s(t), \quad y_2(t) = x_\theta(t) = C_2^T x_p(t), \end{aligned} \quad (39)$$

where $x_s(t) \in \mathbb{R}^{n_s}$, $x_\theta(t) \in \mathbb{R}^m$, and $x_p(t) \triangleq [x_s^T, x_\theta^T]^T \in \mathbb{R}^{n_p}$. To ensure the existence of L_r , the adaptive controller is

TABLE I
NUMERICAL DATA USED IN THE SIMULATIONS

A_{ns}	[-0.6582 0.9705 0 0 0; -3.3105 -1.4741 0 0 0; 0 0 -0.1706 -0.0075 -1; 0 0 -2.4792 1.3585 0.5897; 0 0 0.8050 0.0559 -0.5584]
A_s	[-0.8489 1.1494 -0.0356 -0.0014 -0.0087; -12.8627 7.4865 -1.7827 -0.0688 -0.4341; -0.1071 -0.0004 -0.6684 -0.0140 -1.6727; -0.0807 -0.0474 -0.6318 3.0151 -0.3975; 1.0380 0.0093 5.3771 -0.0697 6.0156]
A_{rs}	[-0.6582 0.9705 0 0 0; -3.3105 -1.4741 0 0 0; 0 0 -0.1161 0.0321 -0.9471; 0 0 0 -1.5 0; 0 0 0 0 -1]
B_s	[-0.0734 0 0; -3.6764 0 0; 0 0 0.0256; 0 1.0924 0.1240; 0 -0.1240 -0.2600]
W_p	[0.5 0.5 0.5; 0.2 0.2 0.2; 0.1 0.1 0.1; 0.5 0.5 0.5]

made responsible only for the control of $x_s(t)$ dynamics while making sure that the human input $y_h(t)$ is being tracked by the plant. This yields inner-loop dynamics similar to (10) as

$$\begin{aligned} \dot{x}_s(t) &= A_{rs} x_s(t) + B_{rs} y_h(t) + B_s \Lambda \text{diag}(\tilde{\lambda}(t)) L_r y_h(t) \\ &\quad - B_s \Lambda \tilde{\Theta}^T(t) \sigma(x_s(t)), \end{aligned} \quad (40)$$

where the pair (A_{rs}, B_{rs}) describe the modified reference model dynamics with the nominal dynamics of $x_s(t)$ represented by (A_{ns}, B_s) . Augmenting (40) with $x_\theta(t)$ dynamics results in a closed loop system similar to (16) which directly extends our previous results to the case in hand.

VI. SIMULATIONS

Consider a large transport aircraft model [14] linearized at Mach 3 and altitude 1000m in the form of (39), where

$$x_s(t) = [\alpha \quad q \quad \beta \quad p \quad r]^T, \quad \dot{x}_\theta(t) = [p \quad q \quad r]^T, \quad (41)$$

α (rad) is the angle of attack, β (rad) is the side-slip angle, and p, q and r (rad/s) are the roll, pitch and yaw rates, respectively. The input vector is defined as

$$u_p(t) = [\delta_e \quad \delta_a \quad \delta_r]^T, \quad (42)$$

consisting of the deflections (in rad) of 2 ganged elevators δ_e , 2 ganged ailerons δ_a and 2 ganged rudders δ_r . The numerical data used in the simulations are given in Table I.

The matched system uncertainty $W_p^T \sigma_p(x_s(t))$ is given by $\sigma_p(x_s(t)) \triangleq [1, q, p, r]^T$. A pilot is controlling the aircraft to achieve desired attitude angles by feeding attitude rate commands to the inner-loop controller using a side-stick.

To design the crossover-reference model (20), the LQR method is used with $Q_{LQR} = \text{diag}([0_{1 \times 8}, 5, 5, 5])$ and $R_{LQR} = 5I_{3 \times 3}$ which yields θ_x in (24) with the sub-matrix $\psi = I_{3 \times 3}$. As all eigen values of ψ have positive real parts, we assume $M = -100I_{3 \times 3}$ which ensures that $M\psi$ is Hurwitz. In addition, the Lyapunov matrices are taken as $Q_1 = I_{5 \times 5}$ and $Q_2 = 2I_{11 \times 11}$, and the human pilot's learning rates are assumed to be $\gamma_H = \gamma_2 = \gamma_3 = 20$. The human's command is saturated as in (22c) with $y_{oi} = 20$ deg/s for $i = 1, \dots, 3$, $\Lambda = I_{3 \times 3}$ for $t < 40$ sec, and we introduce a failure into the system by making $\Lambda = 0.3I_{3 \times 3}$ for $t \geq 40$ sec.

Figs. 2 and 3 show the aircraft attitude angles and the evolution of the pilot commands, respectively for different inner-loop learning rates. Good tracking performance is observed

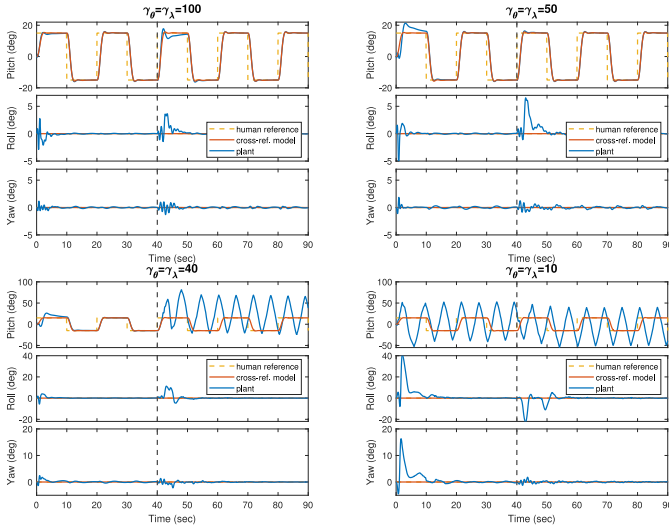


Fig. 2. Attitude angles for different inner-loop learning rates.

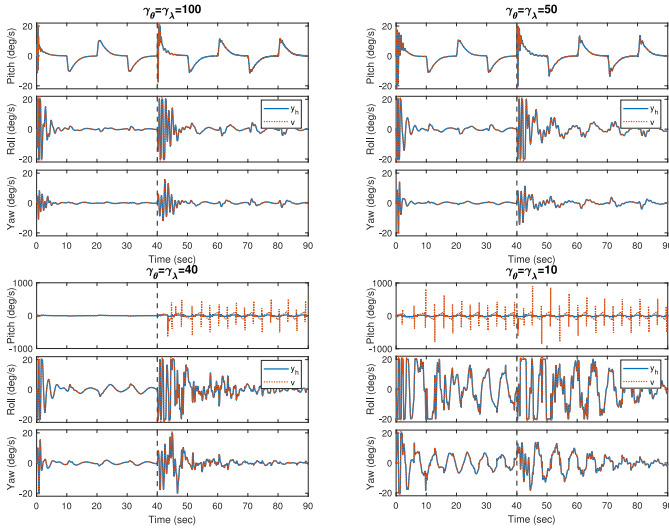


Fig. 3. Pilot commands for different inner-loop learning rates.

for the cases where $\gamma_\theta = \gamma_\lambda = 100$ and $\gamma_\theta = \gamma_\lambda = 50$ while the pilot commands show an oscillatory behavior when an uncertainty is introduced to the system. However, for the cases where $\gamma_\theta = \gamma_\lambda = 40$ and $\gamma_\theta = \gamma_\lambda = 10$, despite the boundedness of all the signals, the tracking performance significantly deteriorates resulting in consistent high amplitude oscillations. Furthermore, pilot commands saturate in the pitch direction since the pilot spends more effort to maintain a satisfactory performance due to a slower adaptive controller.

The simulations in Figs. 2 and 3 are not aimed towards providing high-performance controller implementation examples, but rather to demonstrate a use case for the presented

modeling approach where the effect of the adaptive controller learning rate on the behavior of the pilot is investigated.

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