Eavesdropper Selection Strategies in Wireless Source Localization Networks

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Abstract—We consider a wireless source localization network in which eavesdropper nodes aim to estimate the position of a target node. We formulate the problem of selecting a set of N_E positions out of N possible positions for placing eavesdropper nodes in order to estimate the target node position as accurately as possible. The Cramér-Rao lower bound related to the estimation of the target node position by eavesdropper nodes is derived, and its monotonicity and convexity properties are investigated. Via relaxation of the integer constraints, the eavesdropper selection problem is approximated by a convex optimization problem, which is used to propose two algorithms for eavesdropper selection. Moreover, in the presence of parameter uncertainty, a robust version of the eavesdropper selection problem is investigated. Simulation results are presented to examine performance of the proposed algorithms.

Index Terms—Localization, eavesdropping, estimation.

I. INTRODUCTION

In wireless localization networks, location estimation is performed based on signal exchanges between anchor nodes with known positions and target nodes whose positions are to be estimated [1], [2]. Wireless localization networks are commonly classified as *self localization* and *source (network centric) localization* networks [1]. In the self localization scenario, target nodes estimate their positions via the signals emitted from anchor nodes whereas in the source localization scenario, anchor nodes estimate positions of target nodes based on the signals transmitted by target nodes.

Maintaining high localization accuracy and protecting location secrecy are two major challenges in wireless localization networks [3]. In a wireless localization system, jammer nodes can be placed to degrade the localization accuracy of target nodes by transmitting jamming signals [4]. In addition, eavesdropper nodes can be present in some wireless localization networks, which aim to estimate positions of target nodes by utilizing the broadcast nature of localization networks [3].

In [5], a location secrecy metric (LSM) is developed by considering only the positions of targets nodes and the measurement model of an eavesdropper node. In [3], the definition of the LSM is extended by also taking channel conditions and time offsets into account. For some specific scenarios, [3] specifies the LSMs and proposes algorithms in order to protect location secrecy by diminishing the estimation capability of an eavesdropper node. Based on round-trip-measurements, an adversary eavesdropping model is provided in [6] by using time difference of arrival (TDOA) approaches. Power allocation frameworks for anchor and target nodes are presented in order to degrade the estimation performance of an eavesdropper node while maintaining the localization accuracy of the network [6].

Although the problem of protecting location secrecy against an eavesdropper node has been addressed in [3], [5], [6], there

exist no studies that consider the problem of eavesdropper selection. In the eavesdropper selection problem, the aim is to optimally place a set of eavesdropper nodes to a set of possible positions such that the location secrecy of a target node is reduced as much as possible. From the perspective of eavesdropper nodes, the optimal eavesdropper selection problem can be studied for determining performance limits for eavesdropping. The investigation of eavesdropper selection strategies is important to identify adversarial capabilities of eavesdropper nodes. The eavesdropper selection problem also carries similarities to the anchor placement problem (e.g., [7]–[9]), in which the aim is to determine the optimal positions of anchor (reference) nodes for optimizing performance of target localization.

In this paper, we consider a wireless source localization network with multiple eavesdropper nodes (only one eavesdropper node is considered in [3], [5], [6]), which are synchronized among themselves, and propose and analyze the eavesdropper selection problem. The main contributions of this paper can be summarized as follows: (i) We formulate the eavesdropper selection problem in a wireless source localization network for the first time in the literature. (The studies in [3], [5], [6] aim to degrade the localization performance of a single eavesdropper.) (ii) Considering the Cramér-Rao lower bound (CRLB) as a performance metric for location secrecy, a novel closed-form expression is derived for the CRLB related to estimation of target node position by eavesdropper nodes. (iii) We prove some analytic properties of the CRLB expression (namely, monotonicity and convexity) to express the eavesdropper selection problem as a convex optimization problem after relaxation. (iv) We propose algorithms to solve the eavesdropper selection problem by considering both perfect and imperfect knowledge of system parameters, and develop robust approaches in the presence of imperfect knowledge.

Notation: Throughout the manuscript, $\mathbf{X} \succeq \mathbf{Y}$ denotes that matrix $\mathbf{X} - \mathbf{Y}$ is positive semi-definite, $\mathbf{x} \succeq \mathbf{y}$ means that $x_i \geq y_i$ for all $i = 1, 2, \dots, n$, where $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_n]^T$ and $\mathbf{y} = [y_1 \ y_2 \ \dots \ y_n]^T$, and $\text{tr}\{\cdot\}$ represents the trace of a square matrix. Also, the following definitions are used: (i) Let $f(\cdot)$ be a real-valued function of $\mathbf{z} \in \mathbb{R}^n$. $f(\mathbf{z})$ being non-increasing in $\mathbf{z}$ means that if $\mathbf{z}$ and $\mathbf{w}$ satisfy $\mathbf{z} \succeq \mathbf{w}$, $f(\mathbf{z}) \leq f(\mathbf{w})$ holds. (ii) Let $g(\cdot)$ be a real-valued function of $\mathbf{X} \in S_+^n$, where S_+^n is the set of positive semi-definite matrices in $\mathbb{R}^{n \times n}$. Then, $g(\mathbf{X})$ being non-increasing in $\mathbf{X}$ means that if $\mathbf{X}$ and $\mathbf{Y}$ satisfy $\mathbf{X} \succeq \mathbf{Y}$, $g(\mathbf{X}) \leq g(\mathbf{Y})$ holds.

II. SYSTEM MODEL

Consider a two-dimensional wireless source localization network in which a target node (source) transmits signals that

are used by anchors nodes to estimate its location. Also, there exists some prior information about the location of the target node such that it is located at $\mathbf{x}_i \in \mathbb{R}^2$ with probability w_i for $i = 1, 2, \dots, N_T$, where N_T is the number of possible locations for the target node, $w_i \geq 0$ for $i = 1, 2, \dots, N_T$, and $\sum_{i=1}^{N_T} w_i = 1$.

In the wireless localization network, there also exist eavesdropper nodes that listen to the signal transmitted by the target node and aim to estimate its location. The eavesdropper nodes can be placed at certain locations in the network, which are denoted as $\mathbf{p}_k \in \mathbb{R}^2$ for $k = 1, 2, \dots, N$. The set of possible eavesdropper locations is represented by $\mathcal{N} = \{\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_N\}$. Let $\mathcal{N}_L^{(i)}$ and $\mathcal{N}_{NL}^{(i)}$ denote the set of eavesdropper locations that are in line-of-sight (LOS) and non-line-of-sight (NLOS) with respect to the i th target location, respectively. It is evident that for the i th target location, $\mathcal{N} = \mathcal{N}_L^{(i)} \cup \mathcal{N}_{NL}^{(i)}$ for all $i = 1, 2, \dots, N_T$. Also, ϕ_{ik} represents the angle from the i th target location to the k th eavesdropper location, i.e., $\phi_{ik} = \arctan \frac{x_{i2} - p_{k2}}{x_{i1} - p_{k1}}$, where $\mathbf{x}_i \triangleq [x_{i1} \ x_{i2}]^T$ and $\mathbf{p}_k \triangleq [p_{k1} \ p_{k2}]^T$.

It is assumed that at any given time, at most N_E of the N locations can be used for eavesdropping purposes, where $N_E \leq N$. In other words, there exist at most N_E eavesdropper nodes that can be placed at some of the N possible locations. Let $\mathcal{N}_E$ denote the set of locations in $\mathcal{N}$ at which eavesdropper nodes are placed. If there exists an eavesdropper node placed at location $\mathbf{p}_k$, the received signal at that node coming from the target node located at $\mathbf{x}_i$ can be expressed as

$$r_{ik}(t) = \sum_{l=1}^{L_{ik}} \alpha_{ik}^{(l)} s_i(t - \tau_{ik}^{(l)}) + n_{ik}(t) \text{ for } t \in [0, T_{obs}] \quad (1)$$

where $\alpha_{ik}^{(l)}$ and $\tau_{ik}^{(l)}$ denote, respectively, the amplitude and the delay of the l th multipath component between the target node located at $\mathbf{x}_i$ and the eavesdropper node located at $\mathbf{p}_k$. L_{ik} represents the number of paths between the target node located at $\mathbf{x}_i$ and the eavesdropper node located at $\mathbf{p}_k$, $s_i(t)$ is a known signal waveform, $n_{ik}(t)$ is modeled as zero-mean white Gaussian noise with a power spectral density level of σ_k^2 , and $[0, T_{obs}]$ is the observation interval. The delays of the paths are characterized by the following expression:

$$\tau_{ik}^{(l)} = \frac{1}{c} (\|\mathbf{x}_i - \mathbf{p}_k\| + b_{ik}^{(l)} + \Delta_i) \quad (2)$$

where c is the propagation speed, $b_{ik}^{(l)} \geq 0$ is the range bias ($b_{ik}^{(1)} = 0$ for LOS propagation and $b_{ik}^{(1)} > 0$ for NLOS), and Δ_i characterizes the time offset between the clocks of the target node located at $\mathbf{x}_i$ and the eavesdropper nodes. It is assumed that the eavesdropper nodes are perfectly synchronized among themselves. However, there is no synchronization between the target node and the eavesdropper nodes.

III. EAVESDROPPER SELECTION PROBLEM

In the considered wireless localization network, the aim is to choose at most N_E locations from set $\mathcal{N}$ for eavesdropping purposes so that the location of the target node is estimated as accurately as possible in the mean-square sense. We use the CRLB as a performance measure for localization since the mean-squared error of the maximum likelihood (ML) estimator is asymptotically tight to the CRLB in the high SNR regime [10].

To formulate the eavesdropper selection problem, we introduce a selection vector $\mathbf{z} = [z_1 \ z_2 \ \dots \ z_N]^T$, specified as

$$z_j = \begin{cases} 1, & \text{if } \mathbf{p}_j \in \mathcal{N}_E \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

where $\sum_{j=1}^N z_j \leq N_E$. In addition, for the target position i , $\boldsymbol{\theta}_i$ is defined as $\boldsymbol{\theta}_i \triangleq [\mathbf{x}_i^T \ \Delta_i \ \boldsymbol{\kappa}_{i1}^T \ \boldsymbol{\kappa}_{i2}^T \ \dots \ \boldsymbol{\kappa}_{iN}^T]^T$, where $\boldsymbol{\kappa}_{ik}$ is given by

$$\boldsymbol{\kappa}_{ik} = \begin{cases} [\alpha_{ik}^{(1)} \ b_{ik}^{(2)} \ \alpha_{ik}^{(2)} \ \dots \ b_{ik}^{(L_{ik})} \ \alpha_{ik}^{(L_{ik})}]^T, & \text{if } k \in \mathcal{N}_L^{(i)}, \\ [b_{ik}^{(1)} \ \alpha_{ik}^{(1)} \ b_{ik}^{(2)} \ \alpha_{ik}^{(2)} \ \dots \ b_{ik}^{(L_{ik})} \ \alpha_{ik}^{(L_{ik})}]^T, & \text{if } k \in \mathcal{N}_{NL}^{(i)} \end{cases}$$

for $k = 1, 2, \dots, N$. It is known that the error vector satisfies $\mathbb{E}_{\boldsymbol{\theta}_i} \{(\boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i)(\boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i)^T\} \succeq \mathbf{J}_{\boldsymbol{\theta}_i}^{-1}$ [10], where $\hat{\boldsymbol{\theta}}_i$ is any unbiased estimate of $\boldsymbol{\theta}_i$, and $\mathbf{J}_{\boldsymbol{\theta}_i}$ is the Fisher information matrix (FIM) for the parameter vector $\boldsymbol{\theta}_i$. Hence, the CRLB for estimating the position of the target node located at $\mathbf{x}_i$ is obtained as

$$\mathbb{E}_{\boldsymbol{\theta}_i} \{\|\hat{\mathbf{x}}_i - \mathbf{x}_i\|^2\} \geq \text{tr}\{\mathbf{J}_{\boldsymbol{\theta}_i}^{-1}\}_{2 \times 2} \quad (4)$$

where $\hat{\mathbf{x}}_i$ is any unbiased estimate of $\mathbf{x}_i$. It is noted from (4) that, for the CRLB calculation, we should focus on the equivalent Fisher information matrix (EFIM) for $\mathbf{x}_i$, which is a 2×2 matrix denoted by $\mathbf{J}_e^{(i)}(\mathbf{x}_i)$ such that $[\mathbf{J}_{\boldsymbol{\theta}_i}^{-1}]_{2 \times 2} = (\mathbf{J}_e^{(i)}(\mathbf{x}_i))^{-1}$ [11]. Since $[\mathbf{J}_{\boldsymbol{\theta}_i}]_{2 \times 2}$ is a function of both $\mathbf{x}_i$ and $\mathbf{z}$, it is convenient to write $[\mathbf{J}_{\boldsymbol{\theta}_i}]_{2 \times 2} \triangleq \mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z})$. Hence, we formulate the proposed eavesdropper selection problem as

$$\min_{\mathbf{z}} \sum_{i=1}^{N_T} w_i \text{tr}\{(\mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z}))^{-1}\} \quad (5a)$$

$$\text{subject to} \quad \sum_{j=1}^N z_j \leq N_E, \quad (5b)$$

$$z_j \in \{0, 1\} \text{ for } j = 1, 2, \dots, N. \quad (5c)$$

To simplify the notation, let $f(\mathbf{z})$ be defined as

$$f(\mathbf{z}) \triangleq \sum_{i=1}^{N_T} w_i \text{tr}\{(\mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z}))^{-1}\}. \quad (6)$$

In the rest of this section, we first obtain a closed form expression of $\text{tr}\{(\mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z}))^{-1}\}$ for any target location i , and then analyze monotonicity and convexity properties of $f(\mathbf{z})$ with respect to $\mathbf{z}$.

Proposition 1: For a given eavesdropper selection vector $\mathbf{z}$, the CRLB for estimating the position of the target node located at $\mathbf{x}_i$ is given by

$$\text{tr}\{(\mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z}))^{-1}\} = \quad (7)$$

$$\frac{3 \sum_{k \in \mathcal{N}_L^{(i)}} \sum_{l \in \mathcal{N}_L^{(i)}} z_k z_l \lambda_k^{(i)} \lambda_l^{(i)} p_{k,l}^{(i)}}{4 \sum_{k \in \mathcal{N}_L^{(i)}} \sum_{l \in \mathcal{N}_L^{(i)}} \sum_{m \in \mathcal{N}_L^{(i)}} z_k z_l z_m \lambda_k^{(i)} \lambda_l^{(i)} \lambda_m^{(i)} p_{k,l}^{(i)} p_{l,m}^{(i)} p_{m,k}^{(i)}}$$

where

$$\lambda_k^{(i)} = \frac{8\pi\beta_i^2}{c^2} (1 - \chi_k^{(i)}) \text{SNR}_{ik}^{(1)}, \quad \beta_i^2 = \frac{\int_{-\infty}^{\infty} f^2 |S_i(f)|^2 df}{\int_{-\infty}^{\infty} |S_i(f)|^2 df}, \quad (8)$$

$$\text{SNR}_{ik}^{(1)} = \frac{|\alpha_{ik}^{(1)}|^2 \int_{-\infty}^{\infty} |S_i(f)|^2 df}{2\sigma_k^2}, \quad p_{k,l}^{(i)} = \sin^2 \left(\frac{\phi_{ik} - \phi_{il}}{2} \right) \quad (9)$$

with $S_i(f)$ denoting the Fourier transform of $s_i(t)$, and $\chi_k^{(i)}$ being the path overlap coefficient with $0 \leq \chi_k^{(i)} \leq 1$ [11].

Proof: In [11, Thm. 1], the time-of-arrival based EFIM for estimating the location of a single target node is obtained. Even though our network model is quite different from the system model described in Section II of [11], we benefit from the proof of [11, Thm. 1] in the first part of this proof.

In the proof of [11, Thm.1], vector $\mathbf{q}_k$ is defined as $\mathbf{q}_k = [\cos \phi_k \ \sin \phi_k]^\top$. We follow the same steps as in the proof of [11, Thm. 1] by replacing vector $\mathbf{q}_k$ with vector $\mathbf{q}_{ik}$, which is defined as $\mathbf{q}_{ik} = [\cos \phi_{ik} \ \sin \phi_{ik} \ 1]^\top$.¹ Then, we can obtain the EFIM for $[\mathbf{x}_i \ \Delta_i]^\top$, denoted by $\mathbf{J}_e^{(i)}(\mathbf{x}_i, \Delta_i, \mathbf{z})$, as follows:

$$\mathbf{J}_e^{(i)}(\mathbf{x}_i, \Delta_i, \mathbf{z}) = \begin{bmatrix} K_i(\mathbf{z}) & D_i(\mathbf{z}) & C_i(\mathbf{z}) \\ D_i(\mathbf{z}) & E_i(\mathbf{z}) & S_i(\mathbf{z}) \\ C_i(\mathbf{z}) & S_i(\mathbf{z}) & T_i(\mathbf{z}) \end{bmatrix} \quad (10)$$

where

$$\begin{aligned} K_i(\mathbf{z}) &\triangleq \sum_{k \in \mathcal{N}_L^{(i)}} z_k \lambda_k^{(i)} \cos^2 \phi_{ik}, \quad E_i(\mathbf{z}) \triangleq \sum_{k \in \mathcal{N}_L^{(i)}} z_k \lambda_k^{(i)} \sin^2 \phi_{ik} \\ C_i(\mathbf{z}) &\triangleq \sum_{k \in \mathcal{N}_L^{(i)}} z_k \lambda_k^{(i)} \cos \phi_{ik}, \quad S_i(\mathbf{z}) \triangleq \sum_{k \in \mathcal{N}_L^{(i)}} z_k \lambda_k^{(i)} \sin \phi_{ik} \\ D_i(\mathbf{z}) &\triangleq \sum_{k \in \mathcal{N}_L^{(i)}} z_k \lambda_k^{(i)} \sin \phi_{ik} \cos \phi_{ik}, \quad T_i(\mathbf{z}) \triangleq \sum_{k \in \mathcal{N}_L^{(i)}} z_k \lambda_k^{(i)} \end{aligned}$$

By applying the Schur complement formula to (10), the following expression is obtained:

$$\mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z}) = \begin{bmatrix} K_i(\mathbf{z}) & D_i(\mathbf{z}) \\ D_i(\mathbf{z}) & E_i(\mathbf{z}) \end{bmatrix} - \frac{\begin{bmatrix} C_i^2(\mathbf{z}) & C_i(\mathbf{z})S_i(\mathbf{z}) \\ C_i(\mathbf{z})S_i(\mathbf{z}) & S_i^2(\mathbf{z}) \end{bmatrix}}{T_i(\mathbf{z})} \quad (11)$$

After some algebra and using some trigonometric identities, the following expression can be derived from (11):

$$\begin{aligned} \text{tr}\{(\mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z}))^{-1}\} &= \\ &= \frac{3 \sum_{k \in \mathcal{N}_L^{(i)}} \sum_{l \in \mathcal{N}_L^{(i)}} p_{k,l}^{(i)} z_k z_l \lambda_k^{(i)} \lambda_l^{(i)}}{4 \sum_{k \in \mathcal{N}_L^{(i)}} \sum_{l \in \mathcal{N}_L^{(i)}} \sum_{m \in \mathcal{N}_L^{(i)}} p_{k,l}^{(i)} p_{l,m}^{(i)} p_{m,k}^{(i)} z_k z_l z_m \lambda_k^{(i)} \lambda_l^{(i)} \lambda_m^{(i)}} \end{aligned}$$

which is identical to (7). ■

Proposition 1 provides a compact closed-form expression for the CRLB, which can be evaluated in a simple fashion. Therefore, it also facilitates the calculation of the solution of (5) via an exhaustive search over all possible $\mathbf{z}$ vectors when N is sufficiently small. Also, it is noted that the CRLB expression in Proposition 1 depends only on the LOS signals (see (7)), which is in accordance with the results in the literature (e.g., [11, Prop. 1] and [12]).

Remark 1: It is observed from the CRLB expression in (7) that if all $\lambda_k^{(i)}$'s are scaled by the same nonnegative real number ξ , $\text{tr}\{(\mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z}))^{-1}\}$ is scaled by $1/\xi$ for all $i = 1, 2, \dots, N_T$. Therefore, the optimal eavesdropper

¹The reason for using $\mathbf{q}_{ik}$ instead of $\mathbf{q}_k$ stems from the fact that in our system model, the number of the possible target locations is more than one. Also, the additional term 1 in $\mathbf{q}_{ik}$ compared to $\mathbf{q}_k$ is due to the time offset between the target node and the eavesdropper nodes; i.e., due to the Δ_i term.

selection strategy (i.e., the solution of (5)) remains the same in such cases.

The following lemma characterizes the monotonicity of $f(\mathbf{z})$ in (6) (i.e., the objective function in (5)) with respect to $\mathbf{z}$, which is also utilized in the analysis in Section IV (Lemma 2).

Lemma 1: $f(\mathbf{z})$ is non-increasing in $\mathbf{z}$.

This result is actually quite intuitive as one expects improved performance for estimating the location of a target node as the number of eavesdropper nodes is increased. The proof of this fact is not presented due to the space limitation.

Next, we prove the convexity of the objective function in (5) with respect to $\mathbf{z}$.

Proposition 2: $f(\mathbf{z})$ in (6) is a convex function of $\mathbf{z}$.

Proof: As $w_i \geq 0$ for $i = 1, 2, \dots, N_T$ in (6), it is sufficient to prove that $\text{tr}\{(\mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z}))^{-1}\}$ is a convex function of $\mathbf{z}$. It is known that $\text{tr}\{\mathbf{X}^{-1}\}$ is a convex function of $\mathbf{X}$ for any positive semi-definite $\mathbf{X}$ [13]. Also, $\text{tr}\{\mathbf{X}^{-1}\}$ is non-increasing in $\mathbf{X}$. Therefore, it is sufficient to prove that $\mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z})$ is a concave function of $\mathbf{z}$.

In order to prove that $\mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z})$ is a concave function of $\mathbf{z}$, we should show that for any $\gamma \in [0, 1]$ and $\mathbf{z}, \mathbf{w} \in \mathbb{R}^N$, the following relation is true:

$$\mathbf{J}_e^{(i)}(\mathbf{x}_i, \gamma \mathbf{z} + (1 - \gamma) \mathbf{w}) \succeq \gamma \mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z}) + (1 - \gamma) \mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{w}). \quad (12)$$

Towards that aim, let $J_1^{(i)}(\mathbf{x}_i, \mathbf{z})$ and $J_2^{(i)}(\mathbf{x}_i, \mathbf{z})$ be defined as the first and second terms in (11). Based on the relation in (11), the inequality in (12) can be reduced to the following:

$$\gamma J_2^{(i)}(\mathbf{x}_i, \mathbf{z}) + (1 - \gamma) J_2^{(i)}(\mathbf{x}_i, \mathbf{w}) \succeq J_2^{(i)}(\mathbf{x}_i, \gamma \mathbf{z} + (1 - \gamma) \mathbf{w}) \quad (13)$$

since $J_1^{(i)}(\mathbf{x}_i, \mathbf{z})$ is linear in $\mathbf{z}$.

It is deduced from (11) that for any $\mathbf{y} = [y_1 \ y_2]^\top \in \mathbb{R}^2$, $\mathbf{y}^\top J_2^{(i)}(\mathbf{x}_i, \mathbf{z}) \mathbf{y} = \frac{(y_1 C_i(\mathbf{z}) + y_2 S_i(\mathbf{z}))^2}{T_i(\mathbf{z})}$. Therefore, in order to prove (13), it is sufficient to show that

$$\begin{aligned} &\gamma \frac{(y_1 C_i(\mathbf{z}) + y_2 S_i(\mathbf{z}))^2}{T_i(\mathbf{z})} + (1 - \gamma) \frac{(y_1 C_i(\mathbf{w}) + y_2 S_i(\mathbf{w}))^2}{T_i(\mathbf{w})} \\ &\geq \frac{(y_1 C_i(\mathbf{s}) + y_2 S_i(\mathbf{s}))^2}{T_i(\mathbf{s})} \end{aligned} \quad (14)$$

where $\mathbf{s} = \gamma \mathbf{z} + (1 - \gamma) \mathbf{w}$. By applying the Cauchy-Schwarz inequality to the left-hand-side of (14), and using the fact that $C_i(\cdot)$, $S_i(\cdot)$, and $T_i(\cdot)$ are linear in their arguments, we reach the desired conclusion. ■

As a consequence of Proposition 2, the optimization problem in (5) becomes a convex optimization problem by relaxing the last constraint in (5c). Furthermore, it is deduced from Lemma 1 that if $\mathbf{z}^* = [z_1^* \ z_2^* \ \dots \ z_N^*]^\top$ is a solution of (5), then (5b) must be satisfied with equality, i.e., $\sum_{j=1}^N z_j^* = N_E$ must hold. Therefore, the relaxed version of (5) can be formulated as follows:

$$\min_{\mathbf{z}} \sum_{i=1}^{N_T} w_i \text{tr}\{(\mathbf{J}_e^{(i)}(\mathbf{x}_i, \mathbf{z}))^{-1}\} \quad (15a)$$

$$\text{subject to} \quad \sum_{j=1}^N z_j = N_E, \quad (15b)$$

$$0 \leq z_j \leq 1 \text{ for } j = 1, 2, \dots, N. \quad (15c)$$

As (15) is a convex problem, its solution can be obtained via convex optimization tools [13]. After finding the solution of (15), we propose the following two algorithms to obtain a solution of the original problem in (5). First, we can simply set the largest N_E components of the solution of (15) to one, and the others to zero (called the *largest- N_E algorithm* in Section V). Second, starting from this solution, we can use the swap algorithm discussed in the *Local Optimization* section in [14] and obtain the solution of (15) (called the *swap algorithm* in Section V). In each run of the swap algorithm, one checks whether there is a decrease in the objective function by simply swapping one of the N_E selected positions with one of the $N - N_E$ positions that are not selected.

IV. ROBUST EAVESDROPPER SELECTION PROBLEM

In the previous section, it is assumed that the eavesdropper nodes have the perfect knowledge of $\{\lambda_k^{(i)}\}_{i=1, k=1}^{N_T, N}$ (see (7) and (8)). In this section, we propose a robust eavesdropper selection problem in the presence of imperfect knowledge about the system parameters by introducing some uncertainty in $\{\lambda_k^{(i)}\}_{i=1, k=1}^{N_T, N}$. Towards this aim, we define Λ as follows:

$$\Lambda \triangleq [\lambda^{(1)} \lambda^{(2)} \dots \lambda^{(N_T)}], \lambda^{(i)} \triangleq [\lambda_1^{(i)} \lambda_2^{(i)} \dots \lambda_N^{(i)}]^T. \quad (16)$$

We also introduce the estimated versions of $\lambda^{(i)}$ as $\hat{\lambda}^{(i)}$ for $i = 1, 2, \dots, N_T$, where $\hat{\lambda}^{(i)} \triangleq [\hat{\lambda}_1^{(i)} \hat{\lambda}_2^{(i)} \dots \hat{\lambda}_N^{(i)}]^T$, with $\hat{\lambda}_k^{(i)}$ denoting the estimate of $\lambda_k^{(i)}$ for $k = 1, \dots, N$. These estimated values represent the imperfect knowledge of the $\lambda_k^{(i)}$ parameters at the eavesdropper nodes. Let $\Delta\lambda^{(i)}$ denote the error vector that generates the uncertainty; that is,

$\hat{\lambda}^{(i)} = \lambda^{(i)} + \Delta\lambda^{(i)}$ with $\Delta\lambda^{(i)} \triangleq [\Delta\lambda_1^{(i)} \Delta\lambda_2^{(i)} \dots \Delta\lambda_N^{(i)}]^T$ for $i = 1, 2, \dots, N_T$. Also, let $\Delta\Lambda$ and $\hat{\Lambda}$ be the matrices containing the error vectors and the estimation vectors, respectively, as follows:

$$\Delta\Lambda \triangleq [\Delta\lambda^{(1)} \Delta\lambda^{(2)} \dots \Delta\lambda^{(N_T)}], \hat{\Lambda} \triangleq [\hat{\lambda}^{(1)} \hat{\lambda}^{(2)} \dots \hat{\lambda}^{(N_T)}]$$

In this scenario, the notation for the objective function $f(\mathbf{z})$ is modified as $f(\mathbf{z}, \Lambda)$ to emphasize the dependence on Λ (since $\Delta\Lambda$ becomes another parameter of interest in the presence of uncertainty). As in [15]–[17], we employ a bounded error model for the uncertainty. In particular, for the eavesdropper selection problem in the presence of parameter uncertainty, the following model is assumed for the error matrix $\Delta\Lambda$:

$$\Delta\Lambda \in \mathcal{E} \triangleq \{\Delta\lambda^{(i)} \in \mathbb{R}^N : |\Delta\lambda_k^{(i)}| \leq \delta_k^{(i)}, \forall i = 1, 2, \dots, N_T \text{ and } \forall k = 1, 2, \dots, N\} \quad (17)$$

where $\{\delta_k^{(i)}\}_{i=1, k=1}^{N_T, N}$ determine the size of the uncertainty region $\mathcal{E}$ with $\delta_k^{(i)} \geq 0$ for all $i = 1, 2, \dots, N_T$ and $k = 1, 2, \dots, N$.

The aim is to minimize the worst-case CRLB as in [4] and [17]. Therefore, under this setup, the proposed optimization problem can be formulated as

$$\begin{aligned} \min_{\mathbf{z}} \max_{\Delta\Lambda \in \mathcal{E}} \quad & f(\mathbf{z}, \Lambda) \\ \text{subject to} \quad & \sum_{j=1}^N z_j = N_E, \quad \Lambda = \hat{\Lambda} - \Delta\Lambda, \\ & 0 \leq z_j \leq 1 \text{ for } j = 1, 2, \dots, N. \end{aligned} \quad (18)$$

To solve the optimization problem in (18), the following lemma is utilized.

Lemma 2: $f(\mathbf{z}, \Lambda)$ is non-increasing in $\lambda^{(i)}$ for all $i = 1, 2, \dots, N_T$.

The proof of this intuitive result is not presented due to the space limitation.

Let the value of $\Delta\Lambda$ that maximizes $f(\mathbf{z}, \Lambda)$ over set $\mathcal{E}$ be denoted as $\Delta\Lambda^*$ and let $\{\Delta\lambda_k^{(i),*}\}_{i,k}$ represent the elements of $\Delta\Lambda^*$. Based on Lemma 2, it is obtained that

$$\Delta\lambda_k^{(i),*} = \delta_k^{(i)}. \quad (19)$$

Therefore, solving (18) is equivalent to solving the following optimization problem:

$$\min_{\mathbf{z}} \quad f(\mathbf{z}, \hat{\Lambda} - \Delta\Lambda^*) \quad (20a)$$

$$\text{subject to} \quad \sum_{j=1}^N z_j = N_E, \quad (20b)$$

$$0 \leq z_j \leq 1 \text{ for } j = 1, 2, \dots, N. \quad (20c)$$

It should be noted that (20) is in the form of (15). Thus, the solution approaches discussed in Section III can also be applied to this problem.

V. SIMULATION RESULTS AND CONCLUSIONS

Consider a wireless source localization network, where the target node is located at one of the 81 possible positions with equal probabilities (i.e., $1/81$). In particular, for $i = 1, \dots, 81$, the possible positions of the target node are given by (all in meters)

$$\mathbf{x}_i = \begin{cases} \lceil [i/9] + 1 \rceil \text{ (mod } 9), & \text{if } (i \text{ mod } 9) \neq 0 \\ \lceil [i/9] \rceil, & \text{otherwise} \end{cases} \quad (21)$$

In addition, there exists 10 possible positions for the eavesdropper nodes denoted as $\{\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_{10}\} = \{[15 \ 5], [18 \ 5], [-5 \ 8], [-10 \ 5], [13 \ 13], [5 \ -4], [5 \ 12], [10 \ -5], [-5 \ -5], [-2 \ 9]\}$ meters.

A. Simulation Results with Perfect Knowledge of Parameters

In this part, the eavesdropper selection problem in (5), which is proposed in Section III, is considered and the performance of the following algorithms is compared:

- *Relaxation:* This approach corresponds to the relaxed version (see (15)) of the original problem in (5), and the solution of (15) is obtained via CVX [18]. As (15) is a relaxed version of (5), its solution provides a lower bound for (5), which is generally not achievable.
- *Largest- N_E :* In this algorithm, after obtaining the solution of the relaxed problem in (15), we set the largest N_E components of the solution to one, and the others to zero in order to obtain a feasible solution of (5).
- *Swap:* In this algorithm, starting from the solution of the largest- N_E algorithm, the swap operation is performed as described at the end of Section III.
- *Exhaustive:* This approach corresponds to searching exhaustively over all feasible $\mathbf{z}$ vectors in (5) to find the optimal solution. (Of course, this approach has the highest complexity and not applicable for large N .)

In the simulations, the same parameters as in [19] are used. It is assumed that $\sigma_k^2 = \sigma^2$ for all $k = 1, 2, \dots, N$. Moreover, $|\alpha_{ik}^{(l)}|^2$ and $\chi_k^{(i)}$ are modeled as $|\alpha_{ik}^{(l)}|^2 = \|\mathbf{x}_i - \mathbf{p}_k\|^{-2}$ and $\chi_k^{(i)} = 0$. Hence, $\lambda_k^{(i)}$ is expressed as $\lambda_k^{(i)} = 4\pi\beta_i^2 E_i / (c^2 \|\mathbf{x}_i - \mathbf{p}_k\|^2 \sigma^2)$, where $E_i = \int_{-\infty}^{\infty} |S_i(f)|^2 df$ is the energy of the signal $s_i(t)$. Then, the signal parameters are selected such that $\lambda_k^{(i)}$ is given by $\lambda_k^{(i)} = 200 / (\|\mathbf{x}_i - \mathbf{p}_k\|^2 \sigma^2)$ [19].

In Fig. 1, the average CRLB performance of each algorithm is plotted versus N_E for two different noise levels: $\sigma^2 = 1/100$ and $\sigma^2 = 1$. For the same setting, Fig. 2 presents the average CRLB performance of each algorithm versus $1/\sigma^2$ for $N_E = 4$ and $N_E = 6$. From Figs. 1 and 2, it is observed that the solution of the relaxed problem provides a performance lower bound, as expected, and the other algorithms perform very similarly in this scenario.

As $\sigma_k^2 = \sigma^2$ for all $k = 1, 2, \dots, N$, it is noted that by changing σ^2 , we in fact scale all $\lambda_k^{(i)}$'s with the same factor. Therefore, by Remark 1, it is concluded that the objective function is also scaled as can be observed from Fig. 1. In particular, in Fig. 1, the curves for $\sigma^2 = 1$ are just the scaled versions of the curves for $\sigma^2 = 1/100$, where the scaling factor is 100. Moreover, from Remark 1, it is known that the solution of a given algorithm is the same for all σ^2 's when N_E is fixed. For instance, when $N_E = 5$, the selection strategies for the largest- N_E algorithm for both $\sigma^2 = 1$ and $\sigma^2 = 1/100$ are given by $z_{\text{largest-}N_E} = [1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1]^T$.

In the scenario for Figs. 1 and 2, the SNR terms are calculated based on the path-loss (distance) formula only and $\lambda_k^{(i)}$'s are defined as $\lambda_k^{(i)} = 200 / (\|\mathbf{x}_i - \mathbf{p}_k\|^2 \sigma^2)$ (see (8) and (9)). However, in practice, fading occurs and SNRs vary randomly around the values predicted by the path-loss formula, which is called shadow fading [20]. In order to perform simulations considering the shadowing effect, $\lambda_k^{(i)}$'s are multiplied by independent log-normal random variables with mean parameter 0 and variance parameter 2. For this purpose, 810 realizations are generated in a matrix form with dimensions 81×10 by using MATLAB (the seed is equal to 1), where the (i, k) th element of this matrix corresponds to the log-normal random variable for the channel between the i th target position and the k th eavesdropper position. Figs. 3 and 4 illustrate the average CRLB performance of each algorithm versus N_E and $1/\sigma^2$, respectively, in the presence of shadowing. It is observed that shadowing can significantly degrade the estimation accuracy, as can be noted from the increase of the average CRLB values in Figs. 3 and 4 compared to those in Figs. 1 and 2. It is also noted that the swap algorithm achieves very close performance to the exhaustive search, and outperforms the largest- N_E algorithm for some values of N_E in the considered scenario with shadow fading.

B. Simulations Results in Presence of Parameter Uncertainty

In this part, the eavesdropper selection problem is considered in the presence of parameter uncertainty as discussed in Section IV. The worst-case CRLB performance of different algorithms is compared for both robust and non-robust approaches. In the robust approach, the optimization problem in

Fig. 1. Average CRLB versus N_E when $\sigma^2 = 1/100$ and 1.

Fig. 2. Average CRLB versus $1/\sigma^2$ when $N_E = 4$ and 6.

(20) is solved. However, in the non-robust case, the following optimization problem is considered:

$$\min_{\mathbf{z}} \quad f(\mathbf{z}, \hat{\mathbf{\Lambda}}) \quad (22a)$$

$$\text{subject to} \quad \sum_{j=1}^N z_j = N_E, \quad (22b)$$

$$0 \leq z_j \leq 1 \text{ for } j = 1, 2, \dots, N \quad (22c)$$

Fig. 3. Average CRLB versus N_E in the presence of log-normal shadowing when $\sigma^2 = 1/100$ and 1.

Fig. 4. Average CRLB versus $1/\sigma^2$ in the presence of log-normal shadowing when $N_E = 4$.

where $\hat{\Lambda}$ is a known matrix representing the available (imperfect) information at the eavesdropper nodes.

In the simulations, both the robust and non-robust approaches are considered, and two different selection vectors denoted as $\mathbf{z}_R$ and $\mathbf{z}_{NR}$ (corresponding to robust and non-robust, respectively) are obtained for each algorithm. Then, for $\mathbf{z}_R$ and $\mathbf{z}_{NR}$, the corresponding worst-case CRLBs are computed, which are given by $f(\mathbf{z}_R, \hat{\Lambda} - \Delta\Lambda^*)$ and $f(\mathbf{z}_{NR}, \hat{\Lambda} - \Delta\Lambda^*)$, respectively.

For the uncertainty region $\mathcal{E}$, each $\lambda_k^{(i)}$ is modeled as $\lambda_k^{(i)} \in [(1 - \epsilon_k^{(i)})\hat{\lambda}_k^{(i)}, (1 + \epsilon_k^{(i)})\hat{\lambda}_k^{(i)}]$ for some $\epsilon_k^{(i)} \in [0, 1]$. Therefore, the eavesdropper selection is based on $(1 - \epsilon_k^{(i)})\hat{\lambda}_k^{(i)}$'s for the robust approach whereas $\hat{\lambda}_k^{(i)}$'s are used for the non-robust approach. It is noted that $\delta_k^{(i)}$ in (17) can be expressed as $\delta_k^{(i)} = \epsilon_k^{(i)}\hat{\lambda}_k^{(i)}$. It is important to observe that if $\epsilon_k^{(i)}$'s are the same for all i and k , from Remark 1, the selection strategies for the robust and the non-robust approaches become exactly the same. However, if all $\epsilon_k^{(i)}$'s are not identical (which is commonly the case in practice), we expect performance difference between the robust and non-robust approaches. To that aim, we generate $N \times N_T = 810$ realizations of independent uniform random variables distributed in $[0, 1]$ for $\epsilon_k^{(i)}$'s by using MATLAB (the seed is equal to 1). These realizations are again generated in a matrix form with dimensions 81×10 , where the (i, k) element of the matrix corresponds to $\epsilon_k^{(i)}$.

In Fig. 5, the worst-case CRLB performance of the considered algorithms is presented for both the robust and non-robust approaches. As expected, the robust approach yields lower worst-case CRLBs than the non-robust approach. An interesting observation is that, in the non-robust approach, the exhaustive algorithm can yield lower worst-case CRLBs than the relaxation algorithm. This is due to the fact that during the calculation of the algorithms for the non-robust approach, the imperfect (estimated) values, $\hat{\Lambda}$, are employed. However, for calculating the worst-case CRLB, $\hat{\Lambda} - \Delta\Lambda^*$ is used. Therefore, for the non-robust approach, it is not guaranteed that either the relaxation algorithm provides a lower bound for the other algorithms or the swap algorithm yields lower objective values than the largest- N_E algorithm.

Fig. 5. Worst-case CRLB versus N_E for both robust and non-robust approaches, where $\sigma^2 = 1$.

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