to 4 m for the higher-frequency branch of the second mode. For the lower-frequency branch, the single null occurs at the center ($\rho/a = 0$) at about $z_0 = 2.5$ m and the double null occurs at increased values of ρ/a, approaching about 0.56 at $z_0 = 4$ m. Null separation distance in the higher-frequency branch increases as ϕ increases at $z_0 = 2.1$ m but decreases at $z_0 = 3$ m (in the interval from $z_0 = 2.212$ m to 3.738 m). The separation of the double dips is symmetric in ϕ for z_0 larger than the maximum distance where the nulls occur. The maximum distances are found to increase as ϕ increases, as shown in Figure 5.

4. CONCLUSION

Double nulls and double dips that may be useful in identifying an air spherical cavity in the dielectric medium are analyzed in the forward near-field region. When the wavelength is comparable to one half of the cavity radius (second mode), double nulls in the very near region and double dips in the near-field region occur and the null separation in the higher-frequency branch is about 1 to 0.56 times the diameter of the sphere. The null separation changes as ϕ changes, although the dip separation remains unchanged.

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Received 2-16-93

Microwave and Optical Technology Letters, 6/9, 549–552

CCC 0895-2477/93

A GENERALIZED EIGENVALUE METHOD FOR FDTD ANALYSES

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KEY TERMS

Generalized eigenvalue method, finite-difference time-domain method, interpolation and extrapolation of transient time response, generalized pencil-of-function method

ABSTRACT

In this article we apply the generalized eigenvalue method (GEM) to extract the complex exponentials from a truncated time record computed by the finite-difference time-domain (FDTD) code for analyzing microwave integrated circuits. To obtain accurate scattering parameters without further FDTD computations, the truncated FDTD time record is efficiently extended into the future by summing the complex exponentials with complex coefficients determined by the least-squares method. Numerical GEM results with fewer poles are shown to be in good agreement with those obtained by the Prony method with a large number of poles. © 1993 John Wiley & Sons, Inc.

I. INTRODUCTION

In a previous article [1] it has been demonstrated that a FDTD time record long enough for accurate scattering parameter estimation can be efficiently obtained from a relatively short time record by using an extrapolation scheme based upon the Prony method. It has also been found that a relatively large number of complex exponential functions, e.g., 18 for the bandpass filter and 22 for the low-pass filter in [1], is needed to satisfactorily curve-fit the FDTD data. In this article, an alternative curve-fitting scheme based on the generalized eigenvalue method (GEM) [2, 3] is applied to interpolate the FDTD data. It is illustrated in this article that using a small number of complex exponentials derived by GEM can achieve the same degree of accuracy in extrapolating the FDTD data as using a large number of complex exponentials found by the Prony method. The Prony method finds the complex exponentials by seeking the complex zeros of a polynomial. The GEM determines the complex exponentials by finding the generalized eigenvalues. The locations of these two sets of poles may be different, but the interpolations as well as the extrapolations of the FDTD data by both methods are equally accurate in comparison with the direct FDTD generated time response.

II. METHOD OF APPROACH

As a first step in the extrapolation scheme, a curve-fitting procedure is employed to approximate the time record, computed over a relatively short period using the FDTD scheme, in terms of complex exponentials. Thus the FDTD data g_n can be expressed as

$$g_n = \sum_{k=1}^{K} c_k z_k^n, \qquad n = 0, 1, \ldots, N-1, \tag{1}$$

where z_k are complex exponentials and c_k are complex coefficients. Using the FDTD data g_n, $n = 0, 1, \ldots, N-1$, two $(N-M) \times M$, $M = N/2$, matrices G and F are formed in the following manner [3]:

$$G = \begin{bmatrix} g_0 & g_1 & g_2 \cdots g_{M-1} \\ g_1 & g_2 & g_3 \cdots g_M \\ \vdots & \vdots & \ddots \quad \vdots \\ g_{N-1-M} & g_{N-M} & \cdots \quad g_{N-2} \end{bmatrix} \tag{2}$$

and

$$F = \begin{bmatrix} g_1 & g_2 & g_3 \cdots g_M \\ g_2 & g_3 & g_4 \cdots g_{M+1} \\ \vdots & \vdots & \ddots \quad \vdots \\ g_{N-M} & g_{N-M+1} & \cdots \quad g_{N-1} \end{bmatrix}. \tag{3}$$

With F and G, we form a pencil $F - sG$, where s is a complex parameter. The eigenvalues of the pencil are [2]

$$s(F, G) = \{s_K | \det(F - s_K G) = 0\}. \tag{4}$$

The eigenvectors q_k of the pencil satisfy

$$Fq_k = s_k G q_k \tag{5}$$

Using singular value decomposition, G can be expressed as [2]

$$G = LDR^H, \tag{6}$$

where L is a unitary matrix of left singular vectors, R is a unitary matrix of right singular vectors, D is a diagonal matrix of singular values, and H means conjugate transpose. Then the generalized inverse G^+ is an $M \times (N - M)$ matrix that can be expressed as [2]

$$G^+ = RD^{-1}L^H \tag{7}$$

where D^{-1}, the inverse of D, is also diagonal. Multiplying (5) by G^+:

$$G^+Fq_k = s_k G^+ G q_k = s_k q_k \tag{8}$$

The eigenvalues s_k's and the eigenvectors q_k's can then be obtained by solving the regular eigenvalue problem in (8) associated with the matrix product of G^+ and F.

Next we show that those s_k's are the same as the z_k's in (1). Substituting (1) in (2) and (3), G and F can be decomposed as [3]

$$G = V_1 C V_2 \tag{9}$$

and

$$F = V_1 C Z V_2. \tag{10}$$

The complex exponentials z_k and the coefficients c_k have been arranged as the diagonal elements of the diagonal matrices Z and C; i.e., $Z = \mathrm{diag}(z_1, z_2, \ldots, z_K)$ and $C = \mathrm{diag}(c_1, c_2, \ldots, c_K)$. The matrices V_1 and V_2 are Vandermonde matrices containing powers of the z_k's [2, 3]. Using (9) and (10), the matrix product G^+F in (8) can now be expressed as

$$G^+F = V_2^+ C^{-1} V_1^+ V_1 C Z V_2 = V_2^+ Z V_2, \tag{11}$$

which says that the eigenvalues s_k's of G^+F are the same as the diagonal elements z_k's of the diagonal matrix Z.

Next we summarize the step-by-step solution procedure as follows:

1. Sample the short FDTD data to generate g_n, $n = 0, 1, \ldots, N - 1$.

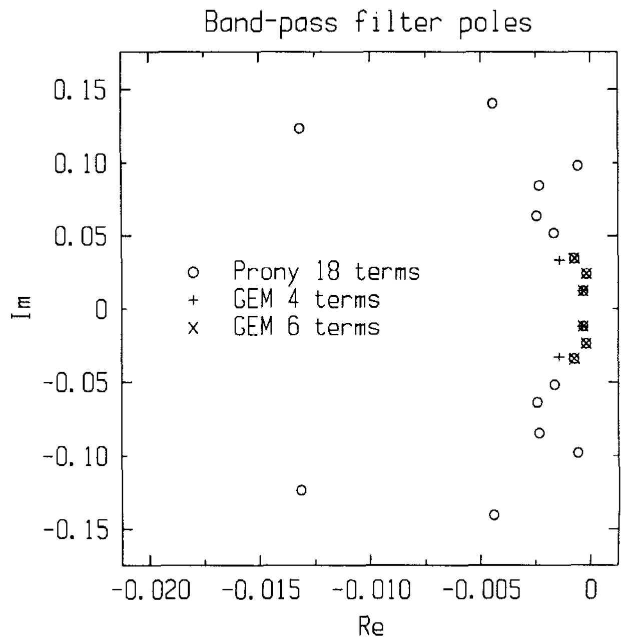

Figure 1 Pole locations

2. Form G and F as shown in (2) and (3).
3. Find G^+ by singular value decomposition of G as shown in (6) and (7).
4. Form the product G^+F and find its eigenvalues s_k's, which are the complex exponentials z_k's in (1).
5. Substitute these z_k's in (1) and solve the resulting least-squares problem to estimate the c_k, $k = 1, 2, \ldots, K$.
6. Check how well (1) can interpolate the FDTD samples. Repeat Steps 1–6 by changing the sampling in Step 1 until a satisfactory interpolation of the sampled data has been obtained.
7. Use (1) to extend the short FDTD data into the future to generate as long a time record as required for accurate scattering parameter estimation, while bypassing the time-consuming FDTD computation altogether.

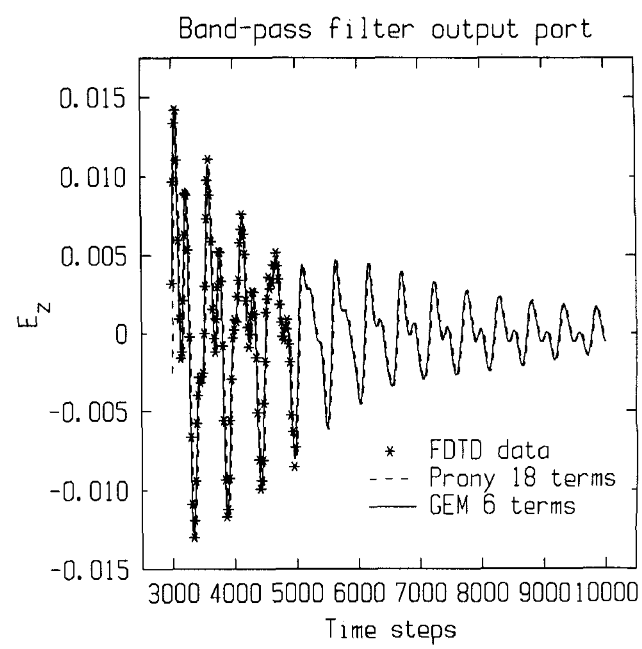

Figure 2 Interpolation 3000–5000; extrapolation 5000–10,000

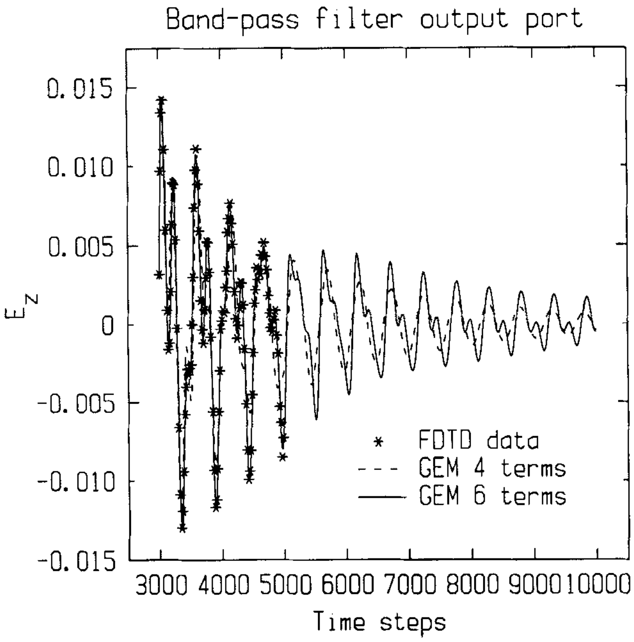

Figure 3 Interpolation 3000–5000; extrapolation 5000–10,000

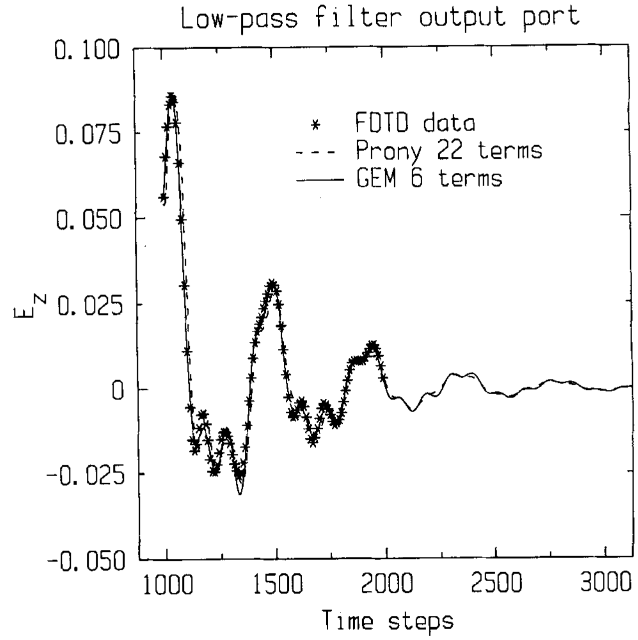

Figure 5 Interpolation 1000–2000; extrapolation 2000–3000

III. RESULTS

The bandpass filter and the low-pass filter in [1] are used to illustrate the GEM approach. For the bandpass filter, a truncated FDTD time record from the 3000th to the 5000th time steps is sampled at every 20 time steps to obtain 100 data points from which complex exponentials are found by GEM or the Prony method. The poles found by GEM are plotted in Figure 1, together with the poles found by Prony's method. In both the GEM and the Prony method, the coefficients of the complex exponentials are determined by a least-squares approach. The GEM series of six complex exponentials and the Prony series of 18 complex exponentials are used in interpolating as well as extrapolating the FDTD data, as shown in Figure 2. For comparison, the direct FDTD data are also plotted in the extrapolation region from 5000 to 10,000 time steps. Figure 3 shows the inadequacy of using less than 6 GEM poles in the extrapolation. Similar results are obtained for the low-pass filter in [1] and illustrated in Figures 4 and 5. In this case, the interpolation region is from 1000 to 2000 time steps and the extrapolation region is from 2000 to 3000 time steps.

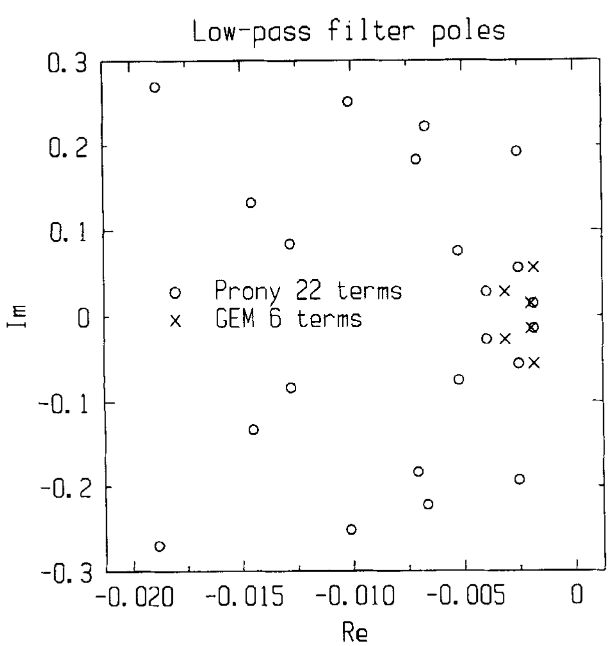

Figure 4 Pole locations

IV. CONCLUSIONS

As shown by comparison with the direct FDTD generated results, the GEM using fewer poles and the Prony method using more poles achieve the same degree of accuracy in extrapolating the short FDTD time record to generate a long enough time response for accurate scattering parameter estimation. Like the Prony method, the GEM can be used for other potential applications that require the computation of a long time response for Fourier transforming to extract frequency-domain parameters.

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Received 2-21-93

Microwave and Optical Technology Letters, 6/9, 552–554

CCC 0895-2477/93