Random Access over Wireless Links: Optimal Rate and Activity Probability Selection

Nurullah Karakoc
School of Electrical, Computer and Energy Engineering
Arizona State University, Tempe, AZ, USA
Email: nkarakoc@asu.edu

Tolga M. Duman
Electrical and Electronics Engineering Department
Bilkent University, 06800 Ankara, Turkey
Email: duman@ee.bilkent.edu.tr

***Abstract*—In this paper, we consider a random access scheme over wireless fading channels based on slotted ALOHA where each user independently decides whether to send a packet or not to a common receiver at any given time slot. To characterize the system throughput, i.e., the expected sum-rate, an information theoretic formulation is developed. We consider two scenarios: classical slotted ALOHA where no multi-user detection (MUD) capability is available and slotted ALOHA with MUD. Our main contribution is that the optimal rates and channel activity probabilities can be characterized as a function of the user distances to the receiver to maximize the system throughput. In addition, we address the issue of fairness among the users and provide solutions, which guarantee a minimum amount of individual throughput.**

I. Introduction

The number of devices in various types of wireless networks is expected to become very large in near future with the introduction of 5G systems necessitating development of new channel access schemes. Due to its distributed and simple nature, random access is a promising solution towards this goal, particularly, in machine-to-machine (M2M) communications and internet of things (IoT) applications [1]. With the proliferation of applications requiring random access over wireless links, the traditional solutions should be modified by taking into account the wireless channel characteristics, and novel approaches should be devised.

In this paper, we consider a slotted ALOHA scheme with probabilistically active users over wireless fading channels. We formulate optimization of transmission rates and user activity probabilities with the objective of maximizing the system throughput while also guaranteeing fairness among different users. To make a realistic analysis with the wireless link considerations, our formulation includes path-loss and small scale (Rayleigh) fading effects. We approach the problem from an information-theoretic perspective at the physical layer without making any changes at the medium access control layer of the standard slotted ALOHA. Also, to preserve the simple nature of ALOHA, no coordination among users is considered, and sophisticated schemes such as rate splitting and superposition coding are avoided.

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In classical slotted ALOHA networks [2], [3], no multi-user detection (MUD) capability is available and collisions result in a loss of all the colliding packets. That is, each successfully received packet is the result of a point-to-point transmission without any interference. In this work, we first consider this setup, and present a closed-form solution to the rate selection problem for each user as a function of its distance to the common destination[1]. We also guarantee a minimum amount of throughput by allowing far away users to send their packets more frequently. We then examine scenarios where the common receiver is endowed with MUD capabilities. We model each collision as a Gaussian multiple access channel (MAC) with fading, and provide methods of obtaining optimal rates and activity probabilities while considering fairness among the users in the system. Noting that there exist practical coding schemes over a MAC with a small number of users (e.g., two users) [4], we focus on cases where at most two users' signals are allowed to collide for successful decoding; however, the proposed approach can be extended to a higher number of colliding packets in a similar manner.

Slotted ALOHA results in low throughputs especially with a large number of users. To handle this problem, there have been extensive efforts, which focus on collision recovery in random access. Towards this goal, [5] investigates network throughput performance by using the capture effect, [6] presents a detailed capture effect analysis with wireless channel considerations, and [7] investigates performance of successive interference cancellation (SIC) in random access networks. As a further example, we also note that multiple replicas of the packets can also be transmitted coupled with suitable iterative decoding algorithms at the receiver side, e.g., through contention resolution diversity slotted ALOHA [8] and irregular repetition slotted ALOHA [9] to increase the system throughput.

We specify the differences between our specific contributions and closely related existing works as follows. Ref. [10] investigates achievable rates for interference-free networks over fading channels. Our work differs from [10] in that we focus on a network with uncoordinated set of users and address the fairness issue explicitly. Ref. [11] provides a method to increase the sum-rate of slotted ALOHA over an additive white

[1] More generally, this can be interpreted as a function of the average signal to noise ratio (which, in our setup, is only a function of the transmitter-receiver separation and the path-loss exponent).

Gaussian noise (AWGN) channel with the help of superposition coding and rate splitting, and [12] extends the proposed method to Rayleigh fading channels. While the authors in [12] use average capacity to find the optimal transmission rates, in this paper, we focus on slow (non-ergodic) fading scenarios and use an outage probability formulation. Also, we emphasize that [11] and [12] employ superposition coding, while on the contrary, we adopt a constant rate approach for a given user throughout its transmission. Ref. [13] presents methods for throughput maximization with random arrivals in both coordinated and uncoordinated cases with the same rate assignment to all the users. Our study differs from [13] in that we assume that the users can transmit with different rates, and we focus fairness among them. In [14], the authors suggest adaptation of the encoding rate according to channel traffic to improve the system throughput by allowing MUD without taking into account the wireless channel considerations. Also, while [14] specifically studies the symmetric scenario of equal activity probabilities, here we allow for different (optimized) rates and activity probabilities in an effort to increase the overall system throughput.

The paper is organized as follows. In Section II, we introduce the system model. In Sections III and IV, we present the newly proposed methods for optimal rate and activity probability selection in classical slotted ALOHA and slotted ALOHA with MUD, respectively. In Section V, we provide numerical examples to illustrate our findings and make comparisons with the related approaches in the literature. Finally, we provide our conclusions in Section VI.

II. System Model

We consider slotted ALOHA over a communication medium characterized as a wireless channel with path-loss and small scale fading effects. We assume that there are n users distributed over a ring of inner radius d_{min} and outer radius d_{max}, and there is a common receiver at the center of the ring. User i is active with probability p_i where $0 \leq p_i \leq 1$, and it has a distance d_i to the common receiver (with $d_{min} \leq d_i \leq d_{max}$). While we use a simplified path-loss model to determine the average signal to noise ratio (SNR) for each user, other channel effects such as shadowing can also be taken into account in the same manner. The received power P_i corresponding to the user i's signal is given by

$$P_i = P_t \kappa \left(\frac{d_0}{d_i}\right)^{\gamma} \quad \text{for} \quad i = 1, 2, \ldots, n, \tag{1}$$

where γ is the path-loss exponent, P_t is the transmission power assumed to be the same for all the users, κ and d_0 are constants. To model the small scale fading effects, we consider Rayleigh fading, and assume that the CSI is known at the receiver side only. We also assume that the channel is slowly varying, and the channel gain can be modeled as a constant over each time slot (which is long enough to invoke random coding arguments).

For simplicity of the exposition, we take the channel coefficients as real Rayleigh random variables. Conditioned on the instantaneous channel gain h_i, the point-to-point capacity over a fading channel with AWGN is

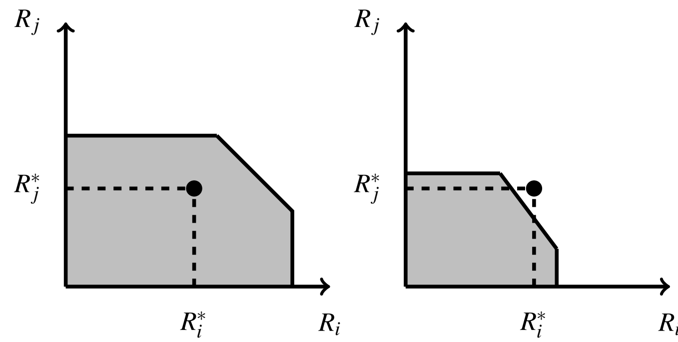

Fig. 1. Two-user Gaussian MAC capacity regions for different fading realizations.

$$C(P_i h_i^2) = \frac{1}{2} \log_2 \left(1 + \frac{P_i h_i^2}{N}\right) \quad \text{bits/channel use} \tag{2}$$

where $C(x) = \frac{1}{2}\log_2(1 + \frac{x}{N})$, and N denotes the additive noise power. Hence, with a channel gain of h_i, a user's coding rate $R_i < C(P_i h_i^2)$ can be supported reliably.

For a Gaussian MAC with two users, conditioned on the channel gains h_i and h_j, the capacity region is

$$\begin{aligned} R_i &< C(P_i h_i^2) \\ R_j &< C(P_j h_j^2) \\ R_i + R_j &< C(P_i h_i^2 + P_j h_j^2) \end{aligned} \tag{3}$$

where (R_i, R_j) is the users' rate pair. Fig. 1 illustrates two different examples of the capacity region for two different set of channel realizations. All rate pairs that can be supported reliably are in the shaded pentagonal regions represented by (3). A selected rate pair (R_i^*, R_j^*) (marked in the figures) can be supported for the example on the left hand side, while it is outside the capacity region for the one on the right hand side. Namely, for the latter case, the users experience outage due to channel fading.

III. Slotted ALOHA

A. Optimal Rate Selection

In the classical slotted ALOHA with no MUD capabilities at the receiver, collisions result in a loss of all the colliding packets. Therefore, achievable rates are identified with the single user capacity in (2). We denote $R(d_i)$ as the encoding rate of a user with distance d_i to the common receiver. In order to achieve a successful transmission in a given time slot, there should be only one user in that particular slot, and the user's rate should be supported by the specific channel realization. Hence, conditioned on the event that only user i transmits in a given slot we can write

$$T_i = \begin{cases} R(d_i), & \text{if } R(d_i) < C(P_i h_i^2), \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

where T_i shows the throughput of user i in a given slot for the channel gain h_i.

Denoting the expected throughput in a slot by T, from the law of total expectation, we obtain

$$\begin{aligned} T &= \sum_{i=1}^{n} \mathbf{E}_{h_i}(T_i)\mathbf{P}(\text{only } i \text{ transmits}) \\ &= \sum_{i=1}^{n} R(d_i)\mathbf{P}(R(d_i) < C(P_i h_i^2))\mathbf{P}(\text{only } i \text{ transmits}) \end{aligned} \tag{5}$$

where $\mathbf{P}(.)$ denotes the probability of an event. The expectation $\mathbf{E}_{h_i}(.)$ is taken over the random channel gain resulting in the average throughput of user i given that only this user transmits in this particular slot. Since transmissions are modeled as independent Bernoulli trials,

$$\mathbf{P}(\text{only } i \text{ transmits}) = p_i \prod_{\substack{j=1 \\ j\neq i}}^{n} (1-p_j). \tag{6}$$

Noting that h_i^2 is an exponential random variable, the probability of non-outage is given by

$$\begin{aligned} \mathbf{P}(R(d_i) < C(P_i h_i^2)) &= \mathbf{P}_i\left(R(d_i) < \frac{1}{2}\log_2\left(1+\frac{P_i h_i^2}{N}\right)\right) \\ &= e^{-(2^{2R(d_i)}-1)\frac{N d_i^{\gamma}}{P_0}} \end{aligned} \tag{7}$$

with $P_0 = P_t \kappa d_0^{\gamma}$.

By combining (5), (6) and (7), the optimization problem becomes

$$\max_{R(d_i)} \quad \sum_{i=1}^{n} R(d_i) e^{-(2^{2R(d_i)}-1)\frac{N d_i^{\gamma}}{P_0}} p_i \prod_{\substack{j=1 \\ j\neq i}}^{n} (1-p_j), \tag{8}$$

and the optimal rate of user i, $R^*(d_i)$, can then be characterized as a function of its distance d_i with the help of calculus of variations, specifically, with the Euler-Lagrange equation. It is given by

$$R^*(d_i) = \frac{W(\frac{P_0}{N d_i^{\gamma}})}{\ln(4)} \tag{9}$$

where $W(.)$ is Lambert-W function[2].

B. System Design with Fairness

Optimal rate function in (9) states that the closer users to the destination have higher optimal rates, and hence for the case of identical activity probabilities, i.e., $p_1 = p_2 = \ldots = p_n$, they enjoy higher individual throughputs compared to the far away users. In order to deal with this fairness issue among different users, we now propose a method that adjusts the activity probabilities and guarantees a minimum amount of individual throughput to each user by allowing the far away users to send their packets more frequently.

For simplicity of calculations, the users are divided into k groups in terms of their distances, and we assume that the number of active users in each group in a given slot is modeled as a Poisson random variable with parameter $\lambda_j = n_j p_j$ where $j = 1, 2, \ldots, k$ denotes the group index, d_j denotes the distance of group j users to the receiver, and n_j and p_j are number of users and the activity probability of the users in group j, respectively. λ_j is defined as the j^{th} group's load, and $\sum_{j=1}^{k}\lambda_j$ is the channel load.

The probability that there is only one active user in the group j and there are no other active users in the system is $\lambda_j \exp(-\sum_{i=1}^{k}\lambda_i)$. Then, by using (8) and (9), the optimization problem can be written as

$$\begin{aligned} \max_{\lambda_1,\lambda_2,\ldots,\lambda_k} \quad & (e^{-\sum_{i=1}^{k}\lambda_i}) \sum_{j=1}^{k} \lambda_j r_j \\ \text{s.t.} \quad & (e^{-\sum_{i=1}^{k}\lambda_i})\lambda_1 r_1 \geq K \\ & (e^{-\sum_{i=1}^{k}\lambda_i})\lambda_2 r_2 \geq K \\ & \vdots \\ & (e^{-\sum_{i=1}^{k}\lambda_i})\lambda_k r_k \geq K \\ & \lambda_1, \lambda_2, \ldots, \lambda_k \geq 0 \end{aligned} \tag{10}$$

where K is the minimum throughput required for each group, and r_j is the effective rate of group j given by

$$r_j = R^*(d_j) e^{-(2^{2R^*(d_j)}-1)\frac{N d_j^{\gamma}}{P_0}}. \tag{11}$$

Notice that r_j is independent of the group load λ_j, and it can be taken as a constant in the optimization problem. The Lagrangian L can be written as

$$L = \sum_{j=1}^{k} -(e^{-\sum_{i=1}^{k}\lambda_i})\lambda_j r_j - \mu_j \lambda_j - \nu_j\left(\lambda_j r_j (e^{-\sum_{i=1}^{k}\lambda_i}) - K\right) \tag{12}$$

where $\mu_1, \mu_2, \ldots, \mu_k \geq 0$, and $\nu_1, \nu_2, \ldots, \nu_k \geq 0$. We have

$$\begin{aligned} \nu_j\left(\lambda_j r_j (e^{-\sum_{i=1}^{k}\lambda_i}) - K\right) &= 0, \quad j = 1, 2, \ldots, k \\ \frac{\partial L}{\partial \lambda_j} &= 0, \quad j = 1, 2, \ldots, k \\ \mu_j \lambda_j &= 0, \quad j = 1, 2, \ldots, k. \end{aligned} \tag{13}$$

Hence, for $r_1 \geq r_2, \ldots, r_k$, the optimal group loads are found as

$$\lambda_1 = 1 - \sum_{j=2}^{k} \frac{Ke}{r_j} \quad \text{and} \quad \lambda_j = \frac{Ke}{r_j}, \quad j = 2, 3, \ldots, k, \tag{14}$$

and the optimal activity probabilities of the users in group j are calculated as $p_j = \lambda_j / n_j$ for $j = 1, 2, \ldots, k$.[3]

We observe that independent of the value of K, the optimal channel load is the same all the time, i.e., $\lambda_1 + \lambda_2 + \ldots + \lambda_k = 1$. In other words, all the groups have just enough load for guaranteeing a throughput of K, and then the remaining load

[2]The details of this calculation are omitted due to space constraints, and can be found in [15, Chp. 3] where global optimality of (9) is also verified. We also note that this result is also derived in [10] by using a different approach.

[3]Global optimality of the solution is verified in [15, Chp. 3] by using the fact that objective function and constraints in (10) are log-concave functions of λ_i's.

is assigned to the group with the highest effective rate r_j, namely, the closest one to the receiver (i.e., the one with the highest average SNR).

We can also consider a fully fair system in which all the groups have equal throughputs. Namely, we can solve

$$
\begin{aligned}
\max_{\lambda_1,\lambda_2,\ldots,\lambda_k} \quad & K \\
\text{s.t.} \quad & (e^{-\sum_{i=1}^{k}\lambda_i})\lambda_1 r_1 = K \\
& (e^{-\sum_{i=1}^{k}\lambda_i})\lambda_2 r_2 = K \\
& \quad \vdots \\
& (e^{-\sum_{i=1}^{k}\lambda_i})\lambda_k r_k = K \\
& \lambda_1,\lambda_2,\ldots,\lambda_k \geq 0.
\end{aligned}
\tag{15}
$$

Following similar steps as in the previous optimization procedure, the optimal threshold K^* can be found as

$$
K^* = \frac{1}{e}\left(\sum_{i=1}^{k}\frac{1}{r_i}\right)^{-1}, \tag{16}
$$

while the optimal load λ_j^* for each group is given by

$$
\lambda_j = \frac{1}{r_j}\left(\sum_{i=1}^{k}\frac{1}{r_i}\right)^{-1}, \tag{17}
$$

with $j = 1,2,\ldots,k$. These results imply that a fully fair system can be achieved by only using users' effective rate (i.e., their average SNR) information.

IV. Slotted ALOHA with Multi-User Detection

A. Optimal Rate Selection

In this section, we extend our approach in the previous section to systems with a (common) receiver with MUD capabilities. We assume that the MUD capability is limited to the collisions of two packets. We model each collision as a Gaussian MAC with fading, and denote the rate of user i by R_i, $i = 1,2,\ldots,n$. For the expected throughput formulation, we modify (5) with the addition of decodable two-user collisions. Therefore, in the formulation, there is a summation of n terms for single level decoding which occurs if there is only one active user in a slot, and there is an additional summation of $\binom{n}{2}$ terms for two-level decoding (which means that the decoder may be successful when at most two packets collide). For each of these $\binom{n}{2}$ terms, there are three inequalities specified in (3) for the two-user Gaussian MAC capacity determining whether the decoding result is successful or there is a decoder failure[4].

[4]Similarly, for m-level decoding, there would be $\binom{n}{m}$ additional terms, and for each of them (2^m-1) inequalities would be needed compared to the $(m-1)$-level decoding. Hence, the formulation here can be easily extended, however, the optimization procedure will be more cumbersome.

The expected throughput in (5) can then be written as[5]

$$
\begin{aligned}
T = & \sum_{i=1}^{n} R_i \mathbf{P}(R_i < C(P_i h_i^2)) p_i \prod_{\substack{j=1\\ j\neq i}}^{n}(1-p_j) \\
& + \sum_{i=1}^{n} p_i \sum_{j=i+1}^{n} p_j \prod_{\substack{k=1\\ k\neq i,j}}^{n}(1-p_k)(R_i+R_j) \\
& \mathbf{P}\Big(R_i < C(P_i h_i^2), R_j < C(P_j h_j^2), \\
& \qquad (R_i+R_j) < C(P_i h_i^2 + P_j h_j^2)\Big).
\end{aligned}
\tag{18}
$$

We define the two-user non-outage probability as $A(R_i,R_j)$. Since h_i^2 and h_j^2 are independent and identically distributed (i.i.d.) exponential random variables, we can write

$$
\begin{aligned}
& A(R_i,R_j) \\
& = \mathbf{P}\Big(R_i < C(P_i h_i^2), R_j < C(P_j h_j^2), R_i+R_j < C(P_i h_i^2 + P_j h_j^2)\Big) \\
& = \mathbf{P}(\gamma_i \geq 2^{2R_i}-1, \gamma_j \geq 2^{2R_j}-1, \gamma_i+\gamma_j \geq 2^{2(R_i+R_j)}-1) \\
& = \int_{2^{2R_i}-1}^{\infty} f_{\gamma_i}(x)\mathbf{P}(\gamma_j \geq \max(2^{2R_j}-1, 2^{2(R_i+R_j)}-1-x))\,dx
\end{aligned}
\tag{19}
$$

where $\gamma_i = P_i h_i^2/N$, and $f_{\gamma_i}(x)$ denotes the probability density function of γ_i. For $\alpha_i \neq \alpha_j$, Eq. (19) can be simplified to

$$
\begin{aligned}
A(R_i,R_j) = & \frac{\alpha_i}{\alpha_j-\alpha_i}\exp(-\alpha_j(2^{2(R_i+R_j)}-1)) \\
& \Big(\exp((\alpha_j-\alpha_i)(2^{2R_i}-1)2^{2R_j}) \\
& \quad - \exp((\alpha_j-\alpha_i)(2^{2R_i}-1))\Big) \\
& + \exp(-\alpha_j(2^{2R_j}-1))\exp(-\alpha_i(2^{2R_i}-1)2^{2R_j})
\end{aligned}
\tag{20}
$$

where $\alpha_i = N d_i^{\gamma}/P_0$. The details of these calculations and the symmetric case with $\alpha_i = \alpha_j$ are omitted due to space constraints, and can be found in [15, Chp. 4].

By combining (7), (18) and (20), the optimization problem becomes

$$
\begin{aligned}
\max_{R_1,R_2,\ldots,R_n} & \sum_{i=1}^{n} p_i \prod_{\substack{j=1\\ j\neq i}}^{n}(1-p_j) R_i \exp(-\alpha_i(2^{2R_i}-1)) \\
& + \sum_{i=1}^{n} p_i \sum_{j=i+1}^{n} p_j \prod_{\substack{k=1\\ k\neq i,j}}^{n}(1-p_k)(R_i+R_j)A(R_i,R_j).
\end{aligned}
\tag{21}
$$

Given the activity probabilities p_i's and the user distances d_i's, the optimal rates R_i^*'s can be found via numerical tools such as gradient descent or interior-point methods[6]. We also note that some optimal rates can be zero as the sole objective is the overall throughput maximization.

[5]We exclude the cases where one of the two users can be decoded by treating the other as noise (i.e., we assume the users are either both decodable or both undecodable in a collision).

[6]The details of these techniques are not provided in this paper, and can be found in [16].

B. System Design with Fairness

We observe that in highly asymmetric scenarios where some users are much closer to the destination than the others, individual throughputs of far away users become very low. With this motivation, we now consider the extension of the proposed scheme for the case with MUD by also taking into account fairness among the users. Specifically, we impose a minimum individual throughput constraint, and propose a method to compute the optimal rates and activity probabilities.

The optimization problem in this case can be formulated as

$$\begin{aligned}
\max_{\substack{p_1,p_2,\ldots,p_n\\ R_1,R_2,\ldots,R_n}} & \sum_{i=1}^{n} p_i \prod_{\substack{j=1\\ j\neq i}}^{n}(1-p_j)R_i \exp(-\alpha_i(2^{2R_i}-1)) \\
& + \sum_{i=1}^{n} p_i \sum_{j=i+1}^{n} p_j \prod_{\substack{k=1\\ k\neq i,j}}^{n}(1-p_k)(R_i+R_j)A(R_i,R_j) \\
\text{s.t.} \quad & R_i\Bigg(p_i \prod_{\substack{j=1\\ j\neq i}}^{n}(1-p_j)\exp(-\alpha_i(2^{2R_i}-1)) \\
& \quad + p_i \sum_{\substack{j=1\\ j\neq i}}^{n} p_j \prod_{\substack{k=1\\ k\neq i,j}}^{n}(1-p_k)A(R_i,R_j)\Bigg) \geq K \\
& 0 \leq p_i \leq 1, \quad \text{for} \quad i=1,2,\ldots,n.
\end{aligned} \tag{22}$$

Here K is the minimum throughput required for each user. The optimal rates R_i^* and activity probabilities p_i^* are then determined via the following steps:

1) Set the initial values for R_i's.
2) Fix R_i's and find the optimal activity probabilities p_i^*'s via an interior-point method, and update p_i's with the newly found values.
3) Fix p_i's and find the optimal rates R_i^*'s via an interior-point method, and update R_i's with the newly found values. Calculate T_{fair}^* with the current values of p_i's and R_i's.
4) Examine whether $T_{fair}^* - T_{fair}$ is higher than some small tolerance ε. If so, return to step 2 and update T_{fair} with T_{fair}^*. If not, stop and set as the optimal rates and activity probabilities R_i and p_i, respectively.

This method is guaranteed to converge, however, the solution may be locally optimal one. This is because the objective function increases after each iteration, and it is bounded from above. It can also be noted that, the optimal rates and activity probabilities can be found jointly in one step, however, the proposed step by step approach helps reduce the computational complexity and is more practical.

V. Numerical Examples

A. Slotted ALOHA

A comparison among the throughput performances of various rate selections is given in Fig. 2. We set $\gamma = 3$, $n = 40000$ and $p_i = 1/40000$ for all the users, and assume that they are distributed uniformly on a ring of inner radius $d_{min} = 200m$ and outer radius $d_{max} = 1000m$. We name the scheme obtained by (9) as the optimal rate method (ORM), and we refer to the approach that uses the rates $R(d_i) = \frac{1}{2}\log_2\left(1+\frac{P_i}{N}\right)$ where P_i is the average received power of user i as the sub-optimal rate method (S-ORM), and the one using the same rate for all the users (equal to the average channel capacity) as the fixed rate method (FRM). The results in Fig. 2 clearly show that the proposed solution (ORM) is highly superior in terms of the expected throughput, especially for high average SNRs.

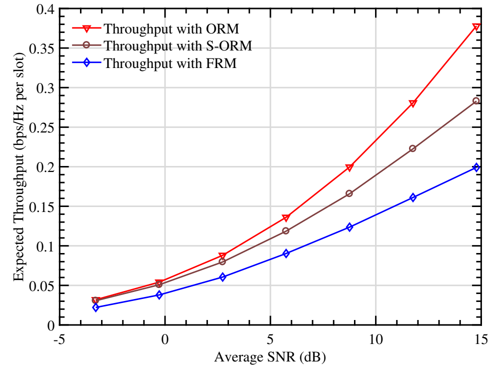

Fig. 2. Performance comparison for different rate selections (with $\gamma = 3$, $n = 40000$ and $p_i = 1/40000$ for all the users that are distributed uniformly on a ring with $d_{min} = 200m$ and $d_{max} = 1000m$).

To illustrate how the fairness issue is addressed, we provide an example for $k = 4$ distinct group of users in Fig. 3. We set the group load λ_j as in (14), and the minimum group throughput as $K = 0.06$. The plot on the left hand side shows the throughput of users in each region while the one on the right hand side shows the activity probabilities. As shown, far away users should send their packets more frequently than the closer ones.

Jain's fairness index [17] can be used as a quantitative measure of the fairness among different users. It takes values between $\frac{1}{n}$ and 1. In this setup, this fairness index can be calculated as 0.771. If an application requires a stronger fairness, one can increase the value of K, or even use the solutions in (16) and (17) for which the maximum achievable

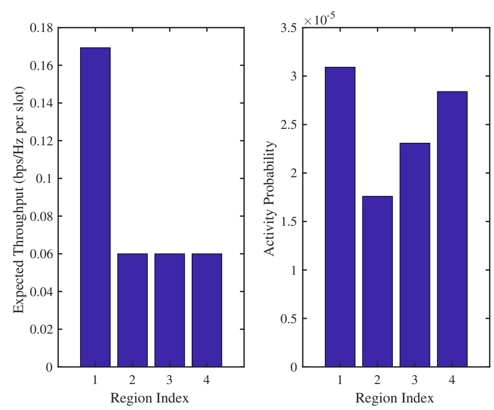

Fig. 3. A simulation of a fair system with equally populated four groups of users (with $\gamma = 3$, $d_1 = 468$m, $d_2 = 688$m, $d_3 = 832$m, $d_4 = 948$m, $n = 40000$, average SNR $= 8.7$ dB).

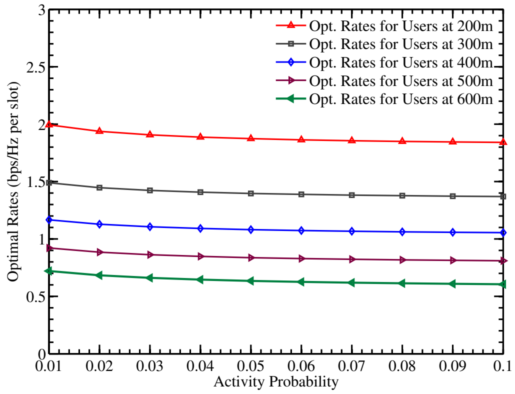

Fig. 4. Optimal rates for 5 groups of users with MUD.

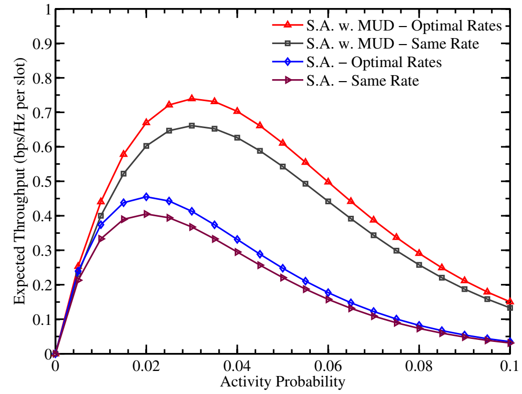

Fig. 5. Expected throughput for different setups with $n = 50$. The user distances are distributed uniformly between $d_{min} = 200m$ and $d_{max} = 600m$.

Jain's index of 1 will be attained.

B. Slotted ALOHA with MUD

To exemplify the optimal rate selection in slotted ALOHA with MUD, we consider 5 groups of users placed at distances of 200m, 300m, 400m, 500m and 600m. We assume that each group has 20 users, and we set the path-loss exponent $\gamma = 3$, and the average SNRs of groups 1-5 as 22.6dB, 17.3dB, 13.6dB, 10.7dB, 8.4dB, respectively. Fig. 4 shows the resulting optimal rates in the symmetric scenario of equal activity probabilities. Optimal rates in asymmetric scenarios can also be found similarly as the framework adopted in (21) is general.

Comparisons among the throughput performance of our model and the one with the same rate assignment to all the users are presented in Fig. 5 for slotted ALOHA with and without MUD. We observe that the newly proposed method outperforms the results of the same rate assumption by 12% for both scenarios. We also note that, if the users are more spatially separated, the gains will increase further.

Our results about fairness are very similar to the case with no MUD, hence we do not provide any specific examples here, and we refer the reader to [15, Chp. 4] for details. Basically, as in the case of single user detection, the proposed solution prioritizes the closer users after guaranteeing a minimum amount of individual throughput for each one.

VI. Conclusions

In this paper, we have proposed methods for obtaining optimal rates and activity probabilities over wireless fading channels for random access networks. We have obtained closed-form solutions for the classical slotted ALOHA framework, while we have resorted to numerical optimization approaches to find the optimal operating parameters in the case with MUD. Our results indicate that with the proposed optimization framework, which allows for unequal transmission rates while providing fairness, the expected throughputs can be improved significantly compared to the existing solutions.

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