Time-varying fairness concerns, delay, and disagreement in bargaining

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ARTICLE INFO

Article history:

Received 18 September 2017

Received in revised form

27 November 2017

Accepted 3 January 2018

Available online 9 January 2018

JEL classification:

C72

C78

D03

Keywords:

Bargaining

Deadline effect

Delay

Disagreement

Fairness concerns

Justice sensitivity

ABSTRACT

We study an alternating-offers, bilateral bargaining game where players may derive disutility from accepting shares below what they deem as fair. Moreover, we assume that the values they attach to fairness (i.e., their sensitivity to violations of their fairness judgments) decrease over time, as the deadline approaches. Our results offer a new explanation to delays and disagreements in dynamic negotiations. We show that even mutually compatible fairness judgments do not guarantee an immediate agreement. We partially characterize conditions for delay and disagreement, and study the changes in the length of delay in response to changes in the model parameters.

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1. Introduction

Delays in reaching an agreement and stalemates are ubiquitous features of real life bargaining encounters. Laboratory experiments on bargaining games report similar observations (see Roth et al., 1988; Babcock et al., 1995; Babcock and Loewenstein, 1997; Gächter and Riedl, 2005; Karagözoğlu and Riedl, 2015; Karagözoğlu and Kocher, 2016; among others). Roth et al. (1988), who observed subjects' tendency to reach agreements towards the deadline, labeled this phenomenon as the *deadline effect*.¹ Babcock et al. (1995), Babcock and Loewenstein (1997), Gächter and Riedl (2005), and Karagözoğlu and Riedl (2015) observed that delays in reaching agreements and disagreements are positively correlated with the incompatibility between the bargainers' fairness judgments. Based on these observations, they informally argued that fairmindedness, combined with biased and incompatible fairness judgments, could be one of the reasons behind delays, last-minute agreements, and even disagreements. Standard bargaining models with common knowledge of *rationality*, complete/perfect infor-

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¹ This is also frequently observed outside the lab. For instance, labor unions and employers usually reach agreements just before a strike starts, a phenomenon known as the "eleventh hour deal". Settlements in pre-trial negotiations are usually reached just before the court date. Similarly, in 2011 and 2013, Democrats and Republicans reached an agreement in the negotiations about the U.S. debt ceiling, just before the deadline.

mation, and discounting fail to capture these phenomena. In those models (see [Rubinstein, 1982](#)), the player who makes the first offer uses all the relevant information and makes an offer that the other player cannot reject. This results in an immediate agreement.

Delays and disagreements in negotiations can have important economic implications. For instance, strikes or lockouts due to disagreements between labor and management (e.g., the NBA lockout in the 2011–2012 season, the US Airways union workers' strike in 2014), continuing conflicts between countries (e.g., the Cyprus conflict, the Israeli-Palestinian conflict), postponed decisions due to stalemates in political negotiations (e.g., delays in government formation due to lengthy coalition negotiations in Belgium in 2011 and in the Netherlands in 2017), merger and acquisition negotiations (e.g., the Verizon–Yahoo merger), or litigation negotiations (e.g., the Pennzoil vs. Texaco case) imply great welfare costs not only for the negotiating parties involved but also for the third parties who have interests connected to the negotiation (e.g., fans, tv broadcasting companies, sports brands in the NBA lockout example). Therefore, understanding the factors that cause bargaining delays and disagreements is of utmost importance. Over the last three decades, bargaining scholars offered various theoretical explanations for delay and disagreements.² Asymmetric information or lack of common priors on important bargaining characteristics (e.g., discount factors, outside options), stochastically evolving model parameters (e.g., stake size), or uncertainty regarding player types and reputational concerns are the main reasons for equilibrium delay and disagreement in those papers.

In the current paper, we study an alternating-offers bargaining game with a known deadline between two players where players may derive disutility from accepting shares below what they deem as *fair*. More precisely, each player has some subjective judgment regarding the fair division of the pie, which is common knowledge; and if she receives a share below what she deems as fair, then she derives disutility from that. A natural example addressed in various bargaining experiments is as follows (see [Gantner et al., 2001](#); [Birkeland and Tungodden, 2014](#); [Karagözoğlu and Riedl, 2015](#); [Bolton and Karagözoğlu, 2016](#)): Suppose that two players, *Alan* and *Betty*, exerted efforts to jointly produce the pie they are bargaining over. *Alan's* effort produced 70% of the pie, whereas *Betty's* effort produced 30% of the pie. Now, *Alan* believes that the appropriate justice norm is *equity* and hence the pie should be divided in proportion to their contributions (i.e., 70–30 division); whereas *Betty* believes that the appropriate justice norm is *equality* and hence the pie should be divided equally (i.e., 50–50 division). Each player would experience disutility from receiving shares below their fair shares (70 vs. 50, respectively).

Importantly, we allow the weights the players attach to fairness concerns (i.e., their sensitivity to violations of their fairness judgments) to diminish over time. Accordingly, as the deadline approaches, the players care less and less about fairness and become more and more material-gain-oriented. This modeling choice is inspired by the recent experimental findings, which highlight the primacy of economic concerns over fairness concerns—especially under time/cognitive pressure (see [Moore and Loewenstein, 2004](#); [Knoch et al., 2006](#); [Knoch and Fehr, 2007](#); [Halali et al., 2013](#); [Hochman et al., 2015](#); among others) and the temporal instability of justice sensitivity (see [Fortin et al., 2016](#) for an excellent review). Furthermore, it is also in line with the habituation and costly self-control arguments in the theoretical literature, which would imply that an agent who repetitively receives a stimuli that is cognitively disturbing or costly to handle starts to become less responsive (see [Karagözoğlu, 2014](#) for habituation; and [Fudenberg and Levine, 2006](#); [Dreber et al., 2016](#) for costly self-control).

We, first, analytically show that delay and/or disagreement may exist in the equilibrium of this bargaining game. We provide necessary and sufficient conditions for the existence of delay and disagreement. Our results show that whether players will be able to reach an agreement and if so when depend on multiple factors such as players' fairness judgments, the pace of the decrease in their weights for fairness, the identities of players (i.e., proposer or responder) in different periods, and the horizon length. Furthermore, our simulation results show that our model can produce empirically relevant outcome patterns such as U-shaped or J-shaped distribution of agreement times, which are usually observed in dynamic bargaining experiments (see [Roth et al., 1988](#); [Gächter and Riedl, 2005](#); [Karagözoğlu and Riedl, 2015](#); [Sullivan, 2016](#)). Our comparative static analyses reveal non-trivial interactions between the model parameters and bargaining delay. For instance, more demanding fairness judgments or stronger concerns for fairness may lead to faster agreements. Finally and somewhat surprisingly, we show that mutually compatible fairness judgments do not guarantee an immediate agreement.

It is worthwhile emphasizing that all the relevant information is common knowledge in our model. There is no incomplete or imperfect information. Players are perfectly forward-looking.³ Our model is the first to explain both delay and disagreement in a finite-horizon, alternating offers bargaining framework without resorting to usual incomplete or imperfect information assumptions. Moreover, it is the first model to formally study time-varying fairness concerns. To the best of our knowledge, [Birkeland and Tungodden \(2014\)](#) present the closest study to ours in that they also utilize fairness concerns. These authors work on a static Nash bargaining model, without any time-varying component in fairness judgments. Naturally, their model can explain disagreements, but not delay.

The organization of the paper is as follows. In Section 2, we introduce our model. In Section 3, we first present our analytical results on the existence of equilibrium involving delay and disagreement. Later, we present simulation results on the relationships between the model parameters and the length of delay. Section 4 concludes.

2. The model

We investigate a two-player finite-horizon alternating offers bargaining game à la Rubinstein (see [Rubinstein, 1982](#)). Let $N = \{1, 2\}$ be the set of players who bargain over a divisible pie (with a normalized size of 1) for a finite number of periods T . In period $t = 1$, *player 1* proposes a division of the pie choosing a strategy (an offer) from $[0, 1]$. Observing *player 1's* offer, *player 2* chooses whether to accept or reject: $\{a, r\}$. If she accepts the offer, then the proposed division is implemented and the game ends. In case she rejects

² See [Rubinstein \(1985\)](#), [Admati and Perry \(1987\)](#), [Gul and Sonnenschein \(1988\)](#), [Cho \(1990\)](#), [Cramton \(1991\)](#), [Fershtman and Seidmann \(1993\)](#), [Ma and Manove \(1993\)](#), [Jehiel and Moldovanu \(1995\)](#), [Merlo and Wilson \(1998\)](#), [Abreu and Gul \(2000\)](#), [Bac \(2000\)](#), [Yildiz \(2004\)](#), [Feinberg and Skrzypacz \(2005\)](#), [Ali \(2006\)](#), [Simsek and Yildiz \(2014\)](#), [Romm \(2016\)](#), [Fanning \(2016\)](#) among others.

³ More precisely, at the beginning of the game, a player knows the complete evolution of the weight she attaches to fairness.

the offer, the game proceeds to the next period. In period $t=2$, it is *player 2's* turn to make an offer, choosing a strategy from $[0, 1]$, after which *player 1* is to decide whether to accept (a) or reject (r) the offer. If *player 1* accepts, then the proposed division is implemented and the game ends; if she rejects, then the game proceeds to period 3 in which everything follows as in the first period. This bargaining procedure continues until one of the players accepts an offer or until period T ends. In case the former happens, each player gets some utility based on the agreed division, whereas if the latter happens, then the game ends with a disagreement which yields a utility of 0 to both players.

Players have fairness concerns. More precisely, each player $i \in N$ believes that it is fair for her to receive a share $\varphi_i \in [0, 1]$ of the pie. If player i gets a share $x_i < \varphi_i$, then she experiences a decrease in the utility derived from the material gain of having $x_i \in [0, 1]$.⁴ This type of preferences is commonly represented with an additively separable utility function in the literature (see Bolton, 1991; Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000). The novel feature of our model is the introduction of *time-varying* fairness concerns. Accordingly, each utility function has different weights for the material gain and the disutility coming from an unfair division, and the weight (relative importance) for fairness concerns weakly decreases through time. We normalize the weight of the material gain in both utility functions to 1, and we let $\alpha_i : \{1, \dots, T\} \rightarrow [0, \infty)$ be a non-increasing function of player i 's relative weights for her fairness concerns at each period. If players agree on an allocation $(x, 1-x)$ at some period $t \in \{1, \dots, T\}$, their utilities from such an agreement are given by:

$$u_1((x, 1-x), t) = x - \alpha_1(t) \cdot \max\{\varphi_1 - x, 0\}$$

$$u_2((x, 1-x), t) = 1 - x - \alpha_2(t) \cdot \max\{\varphi_2 + x - 1, 0\}$$

Players discount future payoffs with $\delta_i \in (0, 1)$. This means that the utilities given above are discounted by a rate of δ_i^{t-1} when evaluated in the first period. All the relevant information (e.g., fairness concerns, the initial values and the evolution of the weights attached to fairness, the discount factors, functional forms, etc.) is common knowledge. We denote this game by Γ^T .

3. The results

In what follows, we first present the analytical results on the existence of equilibrium, delay, and disagreements. Later, we present simulation results on the length of delay.

3.1. Analytical results

Throughout the paper, an equilibrium means (pure strategy) subgame perfect Nash equilibrium. We denote the offer made by *player 1* at some period t by x_t and the offer made by *player 2* at some period t' by $y_{t'}$. As a convention, no matter who the proposer is, an offer always indicates the share of *player 1*. To put it differently, *player 1* proposes the share he wants to receive, whereas *player 2* proposes the share she wants to give. Our first proposition concerns the existence of equilibrium.

Proposition 1. *Let Γ^T be a T -period, bilateral bargaining game with time-varying fairness concerns. An equilibrium of Γ^T exists.*

Proof. Γ^T is a finite-horizon, complete information, extensive form game with continuous utility functions and compact strategy sets at each decision node. The existence result accordingly follows. $\square$

It is worth mentioning that there may exist multiple equilibria and that all equilibria are payoff equivalent: utilizing backward induction, one can see that if a player prefers to make an offer that cannot be rejected, then there exists a unique optimal offer; but if a player prefers to make an offer that will be rejected, then there exists infinitely many optimal offers. The latter would lead to a multiplicity of equilibria, whereas the former guarantees that all equilibria are payoff equivalent. Note that the players may not be able to simply agree on the equilibrium payoffs in the first period since those payoffs may not be feasible in the first period due to higher weights they attach to fairness at the beginning of the game.

Our second proposition concerns the existence of disagreements and delayed agreements on the equilibrium path. For that purpose, it is enough to analyze a two-period bargaining game.

Proposition 2. *Let Γ^2 be a two-period, bilateral bargaining game with time-varying fairness concerns. Then, (i) there exists an equilibrium of Γ^2 with a disagreement and (ii) there exists an equilibrium of Γ^2 with a delayed agreement (i.e., an agreement in the second period).*

Proof. See Appendix A. $\square$

The corollary below shows that the same results follow for any T -period bargaining game.

Corollary 1. *Let Γ^T be a T -period, bilateral bargaining game with time-varying fairness concerns. Then, (i) there exists an equilibrium of Γ^T with a disagreement and (ii) there exists an equilibrium of Γ^T with a delayed agreement (i.e., an agreement some time after the first period).*

Proof. The proof simply follows from Proposition 2 since one can add arbitrary number of periods to the two-period game analyzed in Proposition 2. $\square$

⁴ Note that players' fairness judgments are self-serving in that they do not experience disutility if they receive a share larger than their fair share. Introducing the counterpart for $x_i > \varphi_i$ is neither essential for our model nor changes our results qualitatively. Hence, we opt for simplicity. Satisfaction with advantageous inequity in cognitively demanding situations is supported by experimental research, as well (see Falk and Fischbacher, 2006; Van den Bos et al., 2006).

The proof of Proposition 2 may give the impression that the incompatibility between players' fairness judgments necessitates a delayed agreement, if not a disagreement. By contrast, Proposition 3 shows that even when there is no such conflict between fairness judgments, it is possible to have delayed agreements. The proposition further indicates that disagreement is impossible in that case.

Proposition 3. *Let Γ^2 be a two-period, bilateral bargaining game with time-varying fairness concerns. Let $\varphi_1 + \varphi_2 < 1$, i.e., there exists an allocation which satisfies both players' fairness concerns. Then, (i) there may still exist an equilibrium with a delayed agreement but (ii) there exists no equilibrium with disagreement.*

Proof. See Appendix A. $\square$

Notice that the former statement is about a possibility. When there is no conflict between players' fairness judgments, an immediate agreement is generally reached; however, the opposite might also be true in some cases. For instance, in case the best acceptable offer in the first period is unfair to one of the players⁵ and the weights attached to fairness decrease dramatically in the next period, then that player may prefer to proceed to the next period in which she may find such an unfair offer more acceptable due to having a lower weight for fairness concerns. The latter statement in the proposition is based on the fact that if players' fairness judgments are compatible, either they immediately agree or if they do not, there must be a better alternative in the future, which cannot be a stalemate.

Proposition 4 presents a necessary and sufficient condition for the existence of a disagreement in equilibrium. In fact, it boils down to generalizing the condition (*) given in the proof of Proposition 2.

Proposition 4. *Let Γ^T be a T -period, bilateral bargaining game with time-varying fairness concerns. An equilibrium of Γ^T involves disagreement if and only if*

$$\frac{1 + \alpha_2(T) \cdot (1 - \varphi_2)}{1 + \alpha_2(T)} < \frac{\alpha_1(T) \cdot \varphi_1}{1 + \alpha_1(T)}.$$

Proof. See Appendix A. $\square$

The left-hand-side of the inequality in the proposition is the share of the pie which yields a utility of 0 to *player 2* in period T , thereby making *player 2* indifferent between agreement and disagreement, whereas the right-hand-side is the share of the pie which yields a utility of 0 to *player 1* in the same period. Accordingly, the left-hand-side represents the highest offer *player 2* is willing to accept in period T and the right-hand-side represents the lowest offer *player 1* is willing to make in the same period. In case the former is less than the latter, there exists no offer that is acceptable to both players. This implies a disagreement. Note that if such a condition is satisfied in period T , similar conditions would be satisfied in any period $t < T$.

3.1.1. The Length of Delay: A Partial Characterization

A natural question at this point is: Can we identify the length of delay (or equivalently, the agreement period)? It turns out that writing the length of delay as a function of the model parameters is far from trivial; and hence, is left for future research. Here we take an intermediate step. In order to partially characterize the agreement period, we make use of the following interpretation: Consider a finite horizon, alternating offers bargaining game with no fairness concerns (i.e., with standard utility functions). In the first period, players bargain over a set with a linear frontier passing from the utility pairs $(0, 1)$ and $(1, 0)$.⁶ As a matter of fact, a similar interpretation follows for each period with the difference that the bargaining set in period t is always larger than that in period $t + 1$ for every $t \in \mathbb{N}$ due to the discount factor $\delta_t \in (0, 1)$ (see Fig. 1). This is the reason why there is an immediate agreement in the standard model: suppose that players agree in some period $t > 1$ and get discounted utilities of $\bar{u}_1$ and $\bar{u}_2$, we can easily see that *player 1* could offer $\bar{u}_2$ to *player 2* in the first period and receive a higher utility than $\bar{u}_1$; a contradiction.

In our model with time-varying fairness concerns, there is a number of possibilities regarding how the above-described bargaining sets change across periods. For instance, if there is a conflict between players' fairness concerns and if $\alpha_i(t)$ is sufficiently high for each player, then it may be that the bargaining set in period t does not include a utility pair greater than or equal to $(0, 0)$, in vector terms (see S^1 in Fig. 2). In such a period, we know for sure that there exists no agreement. On the other hand, when $\alpha_i(t)$ is sufficiently low for each player, then there exist allocations yielding positive utilities to both players (see S^2 in Fig. 2). Note that if this is the case in some period t , then for every period $t' > t$, there always exist allocations yielding positive utilities to both players. Furthermore, note that due to time-varying fairness concerns, we cannot simply say that the bargaining set in period t' is always smaller or larger than the bargaining set in period t'' for some periods $t', t'' > t$. The reason is that although the discount factor shrinks the bargaining set in the next period, non-increasing fairness concerns expand the bargaining set. And these parameters may be such that the bargaining sets in period t' and t'' do not contain one another (see S^3, S^4 , and S^5 in Fig. 2).

Because of these different forms each bargaining set may take, it is a very difficult task to identify the agreement period in closed form. This is the reason why we turn to a partial characterization. We start with the following remark.

Remark 1. Let S^t denote the bargaining set in period t . If S^1 does not include a utility pair greater than or equal to $(0, 0)$, i.e., if for every $x \in [0, 1]$,

$$x - \alpha_1(1) \cdot (\varphi_1 - x) < 0 \quad \text{and} \quad 1 - x - \alpha_2(1) \cdot (\varphi_2 + x - 1) < 0,$$

⁵ Although there exist allocations which both parties deem as fair, an 'optimal' offer might be unfair to one of the parties due to the asymmetry created by the identity of the proposer in the final period.

⁶ Although the model restricts players to choose a utility pair on the frontier, we can also assume that players are able to choose from the whole bargaining set. Since none of the interior points will be chosen at the equilibrium, this would be a distinction without a difference.

Fig. 1. A possible collection of bargaining sets in the standard model.

then there exists delay or disagreement.

From now on, we avoid trivial cases and only consider the ones in which S^1 includes at least one utility pair greater than $(0,0)$, in vector terms. In the following, we define two concepts about the corresponding bargaining sets in each period and use them to provide additional conditions for having a delayed agreement. First, we define *domination in bargaining sets* and describe the conditions for $S^{t'} \subset S^t$.

Definition 1. Let Γ^T be a T -period, bilateral bargaining game with time-varying fairness concerns. A period t dominates period $t' \neq t$ in bargaining sets if $S^{t'} \subset S^t$.

The following observation summarizes the conditions for the bargaining set in period t to dominate the bargaining set in period $t' \neq t$.

Observation 1. Let Γ^T be a T -period, bilateral bargaining game with time-varying fairness concerns. Consider two periods t and t' satisfying

$$\begin{aligned} u_1 \left(\left(1 - \frac{\alpha_2(t) \cdot \varphi_2}{1 + \alpha_2(t)}, \frac{\alpha_2(t) \cdot \varphi_2}{1 + \alpha_2(t)} \right), t \right) &> \delta_1^{t'-t} \cdot u_1 \left(\left(1 - \frac{\alpha_2(t') \cdot \varphi_2}{1 + \alpha_2(t')}, \frac{\alpha_2(t') \cdot \varphi_2}{1 + \alpha_2(t')} \right), t' \right) \\ u_2 \left(\left(\frac{\alpha_1(t) \cdot \varphi_1}{1 + \alpha_1(t)}, 1 - \frac{\alpha_1(t) \cdot \varphi_1}{1 + \alpha_1(t)} \right), t \right) &> \delta_2^{t'-t} \cdot u_2 \left(\left(\frac{\alpha_1(t') \cdot \varphi_1}{1 + \alpha_1(t')}, 1 - \frac{\alpha_1(t') \cdot \varphi_1}{1 + \alpha_1(t')} \right), t' \right). \end{aligned}$$

If

$$1 - \frac{\alpha_2(t') \cdot \varphi_2}{1 + \alpha_2(t')} < \varphi_1 \text{ and } 1 - \frac{\alpha_1(t') \cdot \varphi_1}{1 + \alpha_1(t')} > \varphi_2,$$

then period t dominates period t' in bargaining sets; and if

$$1 - \frac{\alpha_2(t') \cdot \varphi_2}{1 + \alpha_2(t')} > \varphi_1 \text{ or } 1 - \frac{\alpha_1(t') \cdot \varphi_1}{1 + \alpha_1(t')} > \varphi_2,$$

then period t dominates period t' in bargaining sets only when $t < t'$.

This observation is based on the fact that domination in bargaining sets can be identified using the endpoints of the corresponding bargaining frontiers. The latter part of the observation indicates that when there exists no *strong* conflict (i.e., when there exists a utility pair which satisfies the fairness concern of one player and yields a positive utility for the other player), only an earlier period can dominate a particular period in bargaining sets.⁷ Utilizing this observation, we present the following necessary condition for a delayed agreement.

Remark 2. If for every $i \in \{1, 2\}$ and every $t \in \{1, \dots, T\}$,

$$\frac{\alpha_i(t)}{\alpha_i(1)} > \delta_i^{t-1},$$

⁷ Remember the result in Proposition 3: even when there is no conflict in players' fairness concerns, $t = 1$ does not necessarily dominate the following period(s) in bargaining sets.

Fig. 2. A possible collection of bargaining sets in our model.

then $t=1$ dominates all the other periods in bargaining sets. Accordingly, there is an immediate agreement in any equilibrium of Γ^T . In order to have a delayed agreement in equilibrium, the inequality above must be violated for some $i \in \{1, 2\}$ and some $t \in \{1, \dots, T\}$.

In other words, if the inequality in Remark 2 is satisfied for both players and for any period, then the discount factors are sufficiently low such that discounting shrinks the bargaining sets in the following periods more than non-increasing fairness concerns expands the bargaining set in the respective period. This implies that the first period dominates all the other periods in bargaining sets, so that players would be better off agreeing in the first period.

Below, we provide the definition of *domination through the union of bargaining sets*, which will be useful in (partially) identifying the agreement period.

Definition 2. Let Γ^T be a T -period, bilateral bargaining game with time-varying fairness concerns. A collection of periods $t^1, \dots, t^k$ dominates another period t through the union of bargaining sets if

$$S^t \subset \bigcup_{i \in \{1, \dots, k\}} S^{t^i}.$$

Furthermore, a period t is undominated, if no collection of periods dominates period t through the union of bargaining sets.

Now, we are ready to describe a property of the agreement period and another sufficient condition for delay.

Remark 3. In a T -period bargaining game, the agreement period t^* is undominated.⁸ Thus, if the first period is dominated by some collection of periods, then there exists delay or disagreement; and if there exists a unique period $\hat{t}$ which is undominated, then $\hat{t}$ is the agreement period.

As mentioned above, there may be multiple undominated periods in the presence of time-varying fairness concerns. In that case, in which period players will agree depends on multiple factors such as the identity of the proposers in the undominated periods, the shape of bargaining sets in the undominated periods, the endpoints of bargaining frontiers in the undominated periods, etc. For such situations, we argue that solving for the equilibrium is the only way to determine the agreement period.

3.1.2. The Length of Delay: An Illustrative Example

We now provide a three-period example in which we characterize the subgame perfect Nash equilibrium. This is to understand how the equilibrium conditions in our model appear and how the values of model parameters determine whether there will be an agreement and if so when. Note that (i) any T -period game can be analyzed in a similar manner for any odd $T \in \mathbb{N}$ and (ii) the equilibrium analysis below can be extended to the cases where T is even.

Example. Consider the bargaining game Γ^3 and assume that the disagreement condition presented in Proposition 4 is not satisfied. Although $T=3$, we stick to general notation in the following analysis.

Consider period T in which *player 1* is the proposer. The analysis for her optimal offer has already been made in the proof of Proposition 4. Given the assumption of “no disagreement” specified above, *player 1* offers

$$x_T^* = \frac{1 + \alpha_2(T) \cdot (1 - \varphi_2)}{1 + \alpha_2(T)}$$

⁸ For instance, in Fig. 2, the agreement period is either period 3 or period 4.

which will be accepted by *player 2*.

Consider period $T-1$ in which *player 2* is the proposer. She anticipates that if her offer is rejected, then she will receive a utility of 0 in period T , whereas *player 1* will receive a utility of $\delta_1 \cdot u_1((x_T^*, 1 - x_T^*), T)$. Accordingly, *player 2* makes an offer so as to make *player 1* indifferent between a and r , if such an offer yields *player 2* a utility not less than 0. We divide our equilibrium analysis into six cases:

For these cases, we define

$$m_{T-1} = \frac{\delta_1 x_T^* + \alpha_1(T-1) \cdot \varphi_1}{1 + \alpha_1(T-1)} \text{ and}$$

$$n_{T-1} = \frac{\delta_1 \cdot [x_T^* - \alpha_1(T) \cdot (\varphi_1 - x_T^*)] + \alpha_1(T-1) \cdot \varphi_1}{1 + \alpha_1(T-1)}.$$

Case A [$\varphi_1 \leq \delta_1 x_T^* \leq x_T^*$ and $u_2((\delta_1 x_T^*, 1 - \delta_1 x_T^*), T-1) \geq 0$]:

In this case, *player 1*'s offer in $t=T$ is fair for her. Such an offer yields her a utility of x_T^* . *Player 2* makes the following offer

$$y_{T-1}^* = \delta_1 x_T^*.$$

Case B [$\varphi_1 \leq \delta_1 x_T^* \leq x_T^*$ and $u_2((\delta_1 x_T^*, 1 - \delta_1 x_T^*), T-1) < 0$]:

It turns out that an offer that will be accepted by *player 1* yields *player 2* a negative utility. Hence, *player 2* can make any offer less than $\delta_1 x_T^*$, and any such offer will be rejected by *player 1*.

Case C [$\delta_1 x_T^* \leq \varphi_1 \leq x_T^*$ and $u_2((m_{T-1}, 1 - m_{T-1}), T-1) \geq 0$]:

In this case, *player 1*'s offer in $t=T$ is fair for her. Such an offer yields her a utility of x_T^* . *Player 2* makes the following offer

$$y_{T-1}^* - \alpha_1(T-1) \cdot (\varphi_1 - y_{T-1}^*) = \delta_1 x_T^*$$

$$y_{T-1}^* = m_{T-1}.$$

Case D [$\delta_1 x_T^* \leq \varphi_1 \leq x_T^*$ and $u_2((m_{T-1}, 1 - m_{T-1}), T-1) < 0$]:

It turns out that an offer that will be accepted by *player 1* yields *player 2* a negative utility. Hence, *player 2* can make any offer less than m_{T-1} , and any such offer will be rejected by *player 1*.

Case E [$\varphi_1 \geq x_T^*$ and $u_2((n_{T-1}, 1 - n_{T-1}), T-1) \geq 0$]:

In this case, *player 1*'s offer in $t=T$ is unfair for her. Such an offer yields her a utility of $x_T^* - \alpha_1(T) \cdot (\varphi_1 - x_T^*)$. *Player 2* makes the following offer

$$y_{T-1}^* - \alpha_1(T-1) \cdot (\varphi_1 - y_{T-1}^*) = \delta_1 \cdot [x_T^* - \alpha_1(T) \cdot (\varphi_1 - x_T^*)]$$

$$y_{T-1}^* = n_{T-1}.$$

Case F [$\varphi_1 \geq x_T^*$ and $u_2((n_{T-1}, 1 - n_{T-1}), T-1) < 0$]:

It turns out that an offer that will be accepted by *player 1* yields *player 2* a negative utility. Hence, *player 2* can make any offer less than n_{T-1} , and any such offer will be rejected by *player 1*.

To sum up, *player 2*'s best offer is described by the following piecewise set-valued function:⁹

$$y_{T-1}^* = \begin{cases} \delta_1 x_T^* & \text{if Case A} \\ m_{T-1} & \text{if Case C} \\ n_{T-1} & \text{if Case E} \\ [0, \delta_1 x_T^*) & \text{if Case B} \\ [0, m_{T-1}) & \text{if Case D} \\ [0, n_{T-1}) & \text{if Case F} \end{cases}$$

Consider period $T-2$ in which *player 1* is the proposer. In cases A, C, and E above: *player 1* anticipates that if her offer is rejected, then she will receive a utility of

$$U_{T-1} \equiv \delta_1 \cdot u_1((y_{T-1}^*, 1 - y_{T-1}^*), T-1),$$

whereas *player 2* will receive a utility of

$$\delta_2 \cdot u_2((y_{T-1}^*, 1 - y_{T-1}^*), T-1).$$

Accordingly, *player 1* makes an offer so as to make *player 2* indifferent between a and r , if such an offer yields *player 1* a utility not less than U_{T-1} . We divide our equilibrium analysis into six cases:

⁹ By an abuse of notation, a singleton set is represented by the single element it includes.

For these cases, we define

$$k_{T-2} = \delta_2 \cdot (1 - y_{T-1}^*),$$

$$m_{T-2} = \frac{1 + \alpha_2(T-2) \cdot (1 - \varphi_2) - \delta_2 \cdot (1 - y_{T-1}^*)}{1 + \alpha_2(T-2)} \text{ and}$$

$$n_{T-2} = \frac{1 + \alpha_2(T-2) \cdot (1 - \varphi_2) - \delta_2 \cdot [1 - y_{T-1}^* - \alpha_2(T-1) \cdot (\varphi_2 + y_{T-1}^* - 1)]}{1 + \alpha_2(T-2)}.$$

Case 1A [$\varphi_2 \leq k_{T-2} \leq 1 - y_{T-1}^*$ and $u_1((k_{T-2}, 1 - k_{T-2}), T-2) \geq U_{T-1}$]:

In this case, *player 2*'s offer in $t = T-1$ is fair for her. Such an offer yields her a utility of $1 - y_{T-1}^*$. *Player 1* makes the following offer

$$x_{T-2}^* = k_{T-2}.$$

Case 1B [$\varphi_2 \leq k_{T-2} \leq 1 - y_{T-1}^*$ and $u_1((k_{T-2}, 1 - k_{T-2}), T-2) < U_{T-1}$]:

It turns out that an offer that will be accepted by *player 2* yields *player 1* an undesirable utility. Hence, *player 1* can make any offer higher than k_{T-2} , and any such offer will be rejected by *player 2*.

Case 1C [$k_{T-2} \leq \varphi_2 \leq 1 - y_{T-1}^*$ and $u_1((m_{T-2}, 1 - m_{T-2}), T-2) \geq U_{T-1}$]:

In this case, *player 2*'s offer in $t = T-1$ is fair for her. Such an offer yields her a utility of $1 - y_{T-1}^*$. *Player 1* makes the following offer

$$1 - x_{T-2}^* - \alpha_2(T-2) \cdot (\varphi_2 + x_{T-2}^* - 1) = \delta_2 \cdot (1 - y_{T-1}^*)$$

$$x_{T-2}^* = m_{T-2}.$$

Case 1D [$k_{T-2} \leq \varphi_2 \leq 1 - y_{T-1}^*$ and $u_1((m_{T-2}, 1 - m_{T-2}), T-2) < U_{T-1}$]:

It turns out that an offer that will be accepted by *player 2* yields *player 1* an undesirable utility. Hence, *player 1* can make any offer higher than m_{T-2} , and any such offer will be rejected by *player 2*.

Case 1E [$\varphi_2 \geq 1 - y_{T-1}^*$ and $u_1((n_{T-2}, 1 - n_{T-2}), T-2) \geq U_{T-1}$]:

In this case, *player 2*'s offer in $t = T-1$ is unfair for her. Such an offer yields her a utility of $1 - y_{T-1}^* - \alpha_2(T-1) \cdot (\varphi_2 + y_{T-1}^* - 1)$. *Player 1* makes the following offer

$$1 - x_{T-2}^* - \alpha_2(T-2) \cdot (\varphi_2 + x_{T-2}^* - 1) = \delta_2 \cdot [1 - y_{T-1}^* - \alpha_2(T-1) \cdot (\varphi_2 + y_{T-1}^* - 1)]$$

$$x_{T-2}^* = n_{T-2}.$$

Case 1F [$\varphi_2 \geq 1 - y_{T-1}^*$ and $u_1((n_{T-2}, 1 - n_{T-2}), T-2) < U_{T-1}$]:

It turns out that an offer that will be accepted by *player 2* yields *player 1* an undesirable utility. Hence, *player 1* can make any offer higher than n_{T-2} , and any such offer will be rejected by *player 2*.

In cases B, D, and F above: *player 1* anticipates that if her offer is rejected, then she will receive a utility of

$$V_{T-1} \equiv \delta_1^2 \cdot u_1((x_T^*, 1 - x_T^*), T),$$

whereas *player 2* will receive a utility of 0. Accordingly, *player 1* makes an offer so as to make *player 2* indifferent between a and r , if such an offer yields *player 1* a utility not less than V_{T-1} . We divide our equilibrium analysis into two cases:

For these cases, we define

$$p_{T-2} = \frac{1 + \alpha_2(T-2) \cdot (1 - \varphi_2)}{1 + \alpha_2(T-2)}.$$

Case 2A [$u_1((p_{T-2}, 1 - p_{T-2}), T-2) \geq V_{T-1}$]:

In this case, *player 2*'s offer in $t = T-1$ is irrelevant and *player 1*'s offer in period T yields *player 2* a utility of 0. *Player 1* makes the following offer

$$x_{T-2}^* = p_{T-2}.$$

Case 2B [$u_1((p_{T-2}, 1 - p_{T-2}), T-2) < V_{T-1}$]:

It turns out that an offer that will be accepted by *player 2* yields *player 1* an undesirable utility. Hence, *player 1* can make any offer higher than p_{T-2} , and any such offer will be rejected by *player 2*.

To sum up, *player 1*'s best offer is described by the following piecewise set-valued function:

$$x_{T-2}^* = \begin{cases} k_{T-2} & \text{if Case 1A} \\ m_{T-2} & \text{if Case 1C} \\ n_{T-2} & \text{if Case 1E} \\ p_{T-2} & \text{if Case 2A} \\ (k_{T-2}, 1] & \text{if Case 1B} \\ (m_{T-2}, 1] & \text{if Case 1D} \\ (n_{T-2}, 1] & \text{if Case 1F} \\ (p_{T-2}, 1] & \text{if Case 2B} \end{cases}$$

This completes our analysis of equilibrium in closed form. Now, we consider different sets of parameter values in order to illustrate that agreement in any period is possible. We first fix

$$\varphi_1 = \varphi_2 = 0.8 \quad \text{and} \quad \delta_1 = \delta_2 = 0.8$$

If $\alpha_1(T) = \alpha_2(T) = 2$, then it follows by Proposition 4 that we have a disagreement. Instead, assuming that

$$\begin{aligned} \alpha_1(T) &= \alpha_2(T) = 1 \\ \alpha_1(T-1) &= \alpha_2(T-1) = 2 \end{aligned}$$

we have a delayed agreement in period $T=3$. Assuming that

$$\begin{aligned} \alpha_1(T) &= \alpha_2(T) = 2/3 \\ \alpha_1(T-1) &= \alpha_2(T-1) = 3/4 \\ \alpha_1(T-2) &= \alpha_2(T-2) = 2 \end{aligned}$$

we have a delayed agreement in period $T-1=2$. Finally, considering standard preferences with no fairness concerns, we would have an immediate agreement in period $T-2=1$. $\diamond$

3.2. Simulations

In this subsection, we report simulation results identifying the agreement period and analyze how it responds to changes in the model parameters.¹⁰ For the sake of tractability, we consider the following functional form for $\alpha_i(\cdot)$:

$$\alpha_i(t) = \kappa_i a_i^t$$

for every $i \in \{1, 2\}$, and use the following values for the model parameters

$$\begin{aligned} T &\in \{20, 21\} \\ \delta_i &\in \{0.95\} \\ \varphi_i &\in \{0.1, 0.2, \dots, 0.9, 1\} \\ \kappa_i &\in \{6, 24\} \\ a_i &\in \{0.1, 0.2, \dots, 0.8, 0.9\} \end{aligned}$$

for every $i \in \{1, 2\}$, which corresponds to a total of 64,800 numerical results.¹¹

The number of periods T is chosen in order to capture sufficient amount of variability in the agreement periods. The functional form for α_i and the corresponding parameter values cover a fairly wide range of interesting situations. For instance, if $a_i=0.1$ and $\kappa_i=6$, then *player i* always values material gain more than she values fairness; if $a_i=0.9$ and $\kappa_i=24$, then *player i* cares about fairness at the beginning of the game so much that her material gain never becomes relatively more important throughout the game; and in all of the remaining cases, fairness is relatively more valuable for *player i* at the beginning, but it becomes relatively less valuable somewhere during the game as the deadline approaches. The switching period significantly depends on the actual values of these parameters: a_i governs the speed of decrease in *player i*'s weight for fairness concerns and κ_i determines whether such a speed is enough to make the material gain relatively more important before the end.

¹⁰ The simulations are conducted in *Visual Studio* using C++ as the programming language.

¹¹ Due to space limitations, we report a small subset of these results in Figs. 3 and 5. All equilibrium agreement periods can be found in the online appendix provided in the corresponding author's web page.

$\kappa_1 = 6, \kappa_2 = 6$				$\kappa_1 = 24, \kappa_2 = 6$			
.4	.6	.8		.4	.6	.8	
.4	1	2	2	.4	1	2	2
.6	1	3	4	.6	2	2	3
.8	3	4	4	.8	11	11	11
$a_1 = 0.2, a_2 = 0.2$				$a_1 = 0.2, a_2 = 0.2$			
.4	.6	.8		.4	.6	.8	
.4	1	2	4	.4	1	2	4
.6	1	4	4	.6	2	4	4
.8	3	4	4	.8	11	11	11
$a_1 = 0.2, a_2 = 0.8$				$a_1 = 0.8, a_2 = 0.8$			
.4	.6	.8		.4	.6	.8	
.4	1	2	4	.4	1	2	4
.6	1	4	4	.6	1	4	4
.8	4	4	4	.8	15	15	15
$a_1 = 0.8, a_2 = 0.2$				$a_1 = 0.8, a_2 = 0.8$			
.4	.6	.8		.4	.6	.8	
.4	1	1	6	.4	1	1	6
.6	1	3	6	.6	1	4	6
.8	3	4	6	.8	4	4	6
$a_1 = 0.2, a_2 = 0.2$				$a_1 = 0.8, a_2 = 0.2$			
.4	.6	.8		.4	.6	.8	
.4	1	1	4	.4	1	1	6
.6	1	3	4	.6	1	4	6
.8	3	4	4	.8	4	4	6
$a_1 = 0.2, a_2 = 0.8$				$a_1 = 0.8, a_2 = 0.8$			
.4	.6	.8		.4	.6	.8	
.4	1	1	8	.4	1	1	10
.6	1	9	13	.6	1	13	15
.8	13	13	17	.8	17	19	19
$a_1 = 0.2, a_2 = 0.2$				$a_1 = 0.8, a_2 = 0.2$			
.4	.6	.8		.4	.6	.8	
.4	1	1	12	.4	1	1	19
.6	1	10	19	.6	1	19	19
.8	13	12	19	.8	19	19	19
$a_1 = 0.2, a_2 = 0.8$				$a_1 = 0.8, a_2 = 0.8$			

Fig. 3. The agreement periods when $T=20$. (Boldface numbers in rows and columns are the values for φ_1 and φ_2 , respectively.)

To provide a clear understanding, here we restrict the set of parameter values and report the results in Fig. 3.¹² As it can be seen in this figure, we observe a wide range of agreement periods; with an immediate agreement when players' belief about their fair shares are sufficiently low, and with the agreement period generally increasing as φ_i , a_i , or κ_i increase reaching a maximum value of 19 when $\varphi_1 = \varphi_2 = 0.8$, $a_1 = a_2 = 0.8$, and $\kappa_1 = \kappa_2 = 24$.

Below, we present two observations from our simulation results. The first observation is about a commonly reported finding in the experimental/empirical literature on bargaining: the distribution of agreement periods is U- or J-shaped, meaning that players mostly agree either close to the beginning or close to the deadline, with latter being observed more frequently, but they rarely agree in the middle of the game (see Roth et al., 1988; Kessler, 1996; Gächter and Riedl, 2005; Karagözoğlu and Riedl, 2015; Sullivan, 2016; Vasserman and Yildiz, 2016, among others). This observation can be supported by our theoretical model for some values of model parameters. For example, consider $a_1 = a_2 = 0.9$ and $\kappa_1 = \kappa_2 = 6$ when $T=20$ and $\delta_1 = \delta_2 = 0.95$. Setting periods 1–7 as the beginning part and periods 15–20 as the end part of the game, and assuming that each player's belief about her fair share is uniformly distributed over $\{0.2, \dots, 0.9\}$,¹³ we can argue that the distribution of agreement periods is U-shaped (see the chart on the left in Fig. 4). As for another example, consider $a_1 = a_2 = 0.8$, $\kappa_1 = \kappa_2 = 24$, and $\varphi_1, \varphi_2 \in \{0.3, \dots, 0.9\}$ keeping the other parameter values unchanged. Now, the distribution of agreement periods becomes J-shaped (see the chart on the right in Fig. 4).

The second observation is that the agreement period is not necessarily monotonic in a player's belief about her fair share φ_i or in her fairness weight parameter a_i . That is, as one of these parameters increases for a player, it may turn out that players reach an agreement more quickly. For example, fixing $a_1 = a_2 = 0.8$ and $\kappa_1 = \kappa_2 = 6$, players agree in the 15th period when $\varphi_1 = 0.8$ and $\varphi_2 = 0.5$; whereas they agree in the 13th period when φ_1 remains to be 0.8 and φ_2 increases to 0.6. Although such cases are quite uncommon in the domain covered by the parameter values we used, they do exist, preventing us to reach a general monotonicity result.

The intuition behind this somewhat counterintuitive result is as follows. Consider the numerical example above. We can see via backward induction that players would agree in the 17th period yielding an expected utility of 0.3399 to player 2 in both cases. From that agreement player 1's expected utility is equal to 0.2727 in case (i) $\varphi_2 = 0.5$ and to 0.2667 in case (ii) $\varphi_2 = 0.6$. Simply, in the latter case, player 1 has to offer a higher share to player 2 in order to make her indifferent between accepting and rejecting the offer, so that player 1's own expected utility turns out to be lower. Now, in case (i), player 1 has a chance to offer an amount yielding 0.3399 to player 2 in the 15th period increasing her own expected utility to 0.2910. Thus, players agree in the 15th period. On the other hand, in case (ii), player 1 does not wish to offer an expected utility of 0.3399 to player 2 in the 15th period. The reason is that player 2's fair amount is now higher so that an unfair division would give her a higher disutility (in comparison to the former case), so that making player 2 indifferent between accepting and rejecting turns out not to be beneficial for player 1. Thus, they do not agree in the 15th period. Finally, in the 13th period of both cases (i) and (ii), player 1 is to offer an expected utility of 0.3399 to player 2 in order to make her indifferent between accepting and rejecting the offer. However, in case (i), player 1 would make the offer only if it yields an expected utility no less than 0.2910 to herself; whereas in case (ii), she would make the offer only if it yields an expected utility

¹² For $T=21$, for which the last proposer is player 1, the results are relegated to the Appendix A.

¹³ Arguably, cases in which $\varphi_i = 0.1$ or $\varphi_i = 1$ are unrealistic, although these values of φ_i are included in our extended computation results.

Fig. 4. U- or J-shaped agreements.

no less than 0.2667 to herself. Given that the offer that makes *player 2* indifferent yields an expected utility of 0.2854 to *player 1*, the offer is made in case (ii) leading to an agreement in the 13th period, but not made in case (i). A similar result as well as a similar interpretation are valid for a_i .¹⁴

4. Conclusion

We study a finite horizon bargaining game between two players who value *not only* the absolute amount they receive *but also* the relative share. The novelty of our model is that the weights players attach to fairness decrease over time as the deadline approaches. Thus, as the push comes to shove (i.e., as the deadline approaches and losing the money on the table becomes more salient), players become more focused on material gains and less focused on fairness. This modeling assumption is inspired by the recent experimental studies on hot vs. cold psychological states, the relative primacy of economic vs. social concerns, time pressure, and the temporal instability of justice sensitivity.

Our results shed light on the influence of fairness judgments on delays, last-minute agreements, and disagreements in dynamic bargaining situations. We, first, analytically show that delay or disagreement may exist in the equilibrium path. Then, we provide necessary and sufficient conditions for disagreement and delay. We also find that mutually compatible fairness judgments do not guarantee an immediate agreement. Our analytical results show that whether players will be able to reach an agreement and if so when depends on multiple factors such as players' fairness judgments, the pace of the decrease in their weights for fairness, the identities of players (i.e., proposer or responder) in different periods, and the horizon length. Our model produces empirically relevant outcome patterns such as U-shaped or J-shaped distribution of agreement times, which are frequently observed in dynamic bargaining experiments. Finally, our simulation results reveal some interesting comparative static results. For instance, more demanding fairness judgments or stronger concerns for fairness may lead to faster agreements.

Here we presented an empirically inspired model of bargaining, which successfully matches multiple regularities in the experimental/empirical data. To the best of our knowledge, this is the first finite-horizon alternating-offers bargaining model that produces delay and disagreement in equilibrium without resorting to usual incomplete or imperfect information assumptions.¹⁵ Obviously, to better understand the relationship between time-varying fairness concerns and delay/disagreement in bargaining, experiments particularly designed to test the predictions of our model should be conducted.

This is a first step in incorporating time-varying fairness judgments into bargaining models. A further step would be to model the micro-foundations of time-varying fairness judgments. To achieve this goal, we believe that experiments investigating the changes in brain activity in the regions that react to unfairness—as a proxy for changes in the value people attach to fairness—during negotiations with fMRI studies or alternative methods such as skin conductance would be natural complements.

Acknowledgments

We would like to thank the editor, two anonymous reviewers, and Gary Charness for useful comments and suggestions. We are grateful to Hamide Turan for her help in writing the code used for the simulation results. We also thank audiences at the “Bargaining: Theory and Experiments” workshop at Bilkent University and the 2nd Turkish Experimental and Behavioral Economics Workshop at İstanbul Bilgi University. Emin Karagözoğlu thanks TÜBİTAK (The Scientific and Technological Research Council of Turkey) for the post-doctoral research fellowship, and Massachusetts Institute of Technology, Department of Economics for their hospitality. The usual disclaimers apply.

Appendix A.

Proof of Proposition 2. Consider $t=2$ in which *player 2* is the proposer. She makes an offer, which makes *player 1* indifferent between playing a and r . Noting that playing r would yield *player 1* a utility of 0, we find *player 2*'s optimal offer as follows:

¹⁴ As a matter of fact, one can find numerical examples for the non-monotonicity in a_i in the restricted set of examples we have reported in Figs. 3 and 5.

¹⁵ Incorporating uncertainty into the evolution of players' fairness preferences over time may be a reasonable extension. Nevertheless, it would unnecessarily complicate the model without qualitatively changing our results. Hence, we opt for a complete and perfect information model.

$$y_2^* - \alpha_1(2) \cdot (\varphi_1 - y_2^*) = 0$$

$$y_2^* = \frac{\alpha_1(2) \cdot \varphi_1}{1 + \alpha_1(2)}$$

Notice that *player 2* would not prefer to make this offer, if the remaining pie amount yields her a negative utility; i.e., if¹⁶

$$1 - y_2^* - \alpha_2(2) \cdot (\varphi_2 + y_2^* - 1) < 0. \quad (*)$$

If this is the case, *player 2* makes some offer less than y_2^* , which will then be rejected by *player 1*. So, there will not be any agreement in $t=2$. If this is not the case, there will be an agreement on the allocation $(y_2^*, 1 - y_2^*)$.

Now, to complete the proof, we must show that the bargaining proceeds to $t=2$. Consider $t=1$. Note that *player 1* is the proposer and that she anticipates the outcome of $t=2$. She makes an offer that makes *player 2* indifferent between playing a and r . Noting that playing r would yield *player 2* a utility of

$$u_2(y_2^*, 1 - y_2^*, 2)$$

in $t=2$, we divide the problem into two cases:

$$(i) \quad \varphi_2 \leq 1 - y_2^* \quad \text{and} \quad (ii) \quad \varphi_2 > 1 - y_2^*.$$

In case (i), *player 2*'s utility is $1 - y_2^*$. Divide this case into two subcases:

$$(i.a) \quad \varphi_2 \leq \delta_2 \cdot (1 - y_2^*) \quad \text{and} \quad (i.b) \quad \varphi_2 > \delta_2 \cdot (1 - y_2^*).$$

In case (i.a), *player 1*'s optimal offer is:

$$x_1^* = 1 - \delta_2 \cdot (1 - y_2^*)$$

In case (i.b), *player 1*'s optimal offer can be found by the following:

$$1 - x_1^* - \alpha_2(1) \cdot (\varphi_2 + x_1^* - 1) = \delta_2 \cdot (1 - y_2^*)$$

$$x_1^* = \frac{1 + \alpha_2(1) \cdot (1 - \varphi_2) - \delta_2 \cdot (1 - y_2^*)}{1 + \alpha_2(1)}$$

In case (ii), *player 2*'s utility is $1 - y_2^* - \alpha_2(2) \cdot (\varphi_2 + y_2^* - 1)$. *Player 1*'s optimal offer –which should be unfair for *player 2*– can be found by the following:

$$1 - x_1^* - \alpha_2(1) \cdot (\varphi_2 + x_1^* - 1) = \delta_2 \cdot [1 - y_2^* - \alpha_2(2) \cdot (\varphi_2 + y_2^* - 1)]$$

$$x_1^* = \frac{1 + \alpha_2(1) \cdot (1 - \varphi_2) - \delta_2 \cdot [1 - y_2^* - \alpha_2(2) \cdot (\varphi_2 + y_2^* - 1)]}{1 + \alpha_2(1)}$$

To sum up, if x_1^* yields a negative utility to *player 1* in $t=1$ for one of the above cases, then for that particular case, there is no agreement in the first period. Note that this is possible, if $\alpha_1(1)$ is sufficiently high. If this happens in case (i), then there is an agreement in $t=2$. If this happens in case (ii), then there is an agreement in $t=2$ if the equation (*) is not satisfied, and there is disagreement if the equation (*) is satisfied. $\square$

Proof of Proposition 3. For the former, we provide an example in which there exists delay. Consider a two-period model with

$$\delta_1 = \delta_2 = 0.9,$$

$$\varphi_1 = \varphi_2 = 0.4,$$

$$\alpha_1(2) = \alpha_2(2) = 0.25, \text{ and}$$

$$\alpha_1(1) = \alpha_2(1) = 1.$$

Consider $t=2$ in which *player 2* is the proposer. She makes an offer so as to make *player 1* indifferent between a and r . This offer is $y_2^* = 0.08$, which yields a utility of 0 to *player 1* and a discounted utility of 0.828 to *player 2*. Now, consider $t=1$. Anticipating the outcome of $t=2$, *player 1*'s offer could be to take 0.172 for herself, and thereby making *player 2* indifferent between playing a and r . However, such an offer would yield a utility of $0.172 - 0.228 = -0.056 < 0$. Then, *player 1* would prefer making an offer which *player 2* rejects. Hence, in equilibrium, agreement is reached with delay.

For the latter, suppose that there exists disagreement. This also means that players did not agree until period T (i.e., they reached the final period). Consider period T . Without loss of generality, assume that *player 1* is the proposer. In this period there exists an offer for *player 1* which yields a positive utility to herself and a utility of 0 to *player 2*, since $\varphi_1 + \varphi_2 < 1$ by assumption. Since disagreement

¹⁶ This is not necessarily *player 2*'s utility, because if $1 - y_2^*$ is fair for her, then her utility would be $1 - y_2^*$. In such a case, *player 2* gets a positive utility for sure. This is the reason why 'the negative utility condition' reduces to the equation (*).

yields a utility of 0 to both players, *player 1* would deviate to such an offer and *player 2* would accept. This is a contradiction. Hence, there is always an agreement. $\square$

Proof of Proposition 4. Assume that T is odd (for even T , similar arguments follow). Consider period T in which *player 1* is the proposer. She makes an offer that makes *player 2* indifferent between playing a and r . Noting that playing r would yield *player 2* a utility of 0, we find *player 1*'s optimal offer as follows:

$$1 - x_T^* - \alpha_2(T) \cdot (\varphi_2 + x_T^* - 1) = 0$$

$$x_T^* = \frac{1 + \alpha_2(T) \cdot (1 - \varphi_2)}{1 + \alpha_2(T)}$$

Now, if such an offer is not beneficial for *player 1*, i.e., if

$$x_T^* - \alpha_1(T) \cdot (\varphi_1 - x_T^*) < 0$$

$$x_T^* < \frac{\alpha_1(T) \cdot \varphi_1}{1 + \alpha_1(T)}$$

then *player 1* would offer something higher than x_T^* and *player 2* would reject.

To sum up, if

$$\frac{1 + \alpha_2(T) \cdot (1 - \varphi_2)}{1 + \alpha_2(T)} < \frac{\alpha_1(T) \cdot \varphi_1}{1 + \alpha_1(T)},$$

there is no agreement in the final period. Notice that if this is the case, then there would not be an agreement in the previous periods either. To be more precise, if there is no allocation that yields a utility pair greater than $(0, 0)$ in period T , then there is no allocation that yields such a utility pair in the previous periods. This is because $\alpha_i(t) \geq \alpha_i(T)$ for every $i \in \{1, 2\}$ and $t < T$. This completes the proof. $\square$

Simulation results when $T=21$:

$\kappa_1 = 6, \kappa_2 = 6$

	.4	.6	.8
.4	1	2	2
.6	1	3	4
.8	3	4	4

$a_1 = 0.2, a_2 = 0.2$

	.4	.6	.8
.4	1	2	2
.6	1	3	4
.8	4	4	4

$a_1 = 0.8, a_2 = 0.2$

$\kappa_1 = 24, \kappa_2 = 6$

	.4	.6	.8
.4	1	2	2
.6	2	2	3
.8	7	7	7

$a_1 = 0.2, a_2 = 0.2$

	.4	.6	.8
.4	1	2	2
.6	2	2	4
.8	5	5	5

$a_1 = 0.8, a_2 = 0.2$

$\kappa_1 = 6, \kappa_2 = 0.8$

	.4	.6	.8
.4	1	2	4
.6	1	4	4
.8	3	4	4

$a_1 = 0.2, a_2 = 0.8$

	.4	.6	.8
.4	1	2	4
.6	1	4	4
.8	4	4	4

$a_1 = 0.8, a_2 = 0.8$

$\kappa_1 = 24, \kappa_2 = 0.8$

	.4	.6	.8
.4	1	2	4
.6	2	4	4
.8	7	7	7

$a_1 = 0.2, a_2 = 0.8$

	.4	.6	.8
.4	1	2	4
.6	1	4	4
.8	5	5	5

$a_1 = 0.8, a_2 = 0.8$

$\kappa_1 = 6, \kappa_2 = 24$

	.4	.6	.8
.4	1	1	8
.6	1	3	8
.8	3	4	8

$a_1 = 0.2, a_2 = 0.2$

	.4	.6	.8
.4	1	1	6
.6	1	4	8
.8	4	4	8

$a_1 = 0.8, a_2 = 0.2$

$\kappa_1 = 24, \kappa_2 = 24$

	.4	.6	.8
.4	1	1	8
.6	1	8	12
.8	8	12	14

$a_1 = 0.2, a_2 = 0.2$

	.4	.6	.8
.4	1	1	10
.6	1	11	14
.8	14	16	18

$a_1 = 0.8, a_2 = 0.2$

$\kappa_1 = 6, \kappa_2 = 0.8$

	.4	.6	.8
.4	1	1	12
.6	1	3	12
.8	3	3	12

$a_1 = 0.2, a_2 = 0.8$

	.4	.6	.8
.4	1	1	12
.6	1	4	12
.8	4	4	12

$a_1 = 0.8, a_2 = 0.8$

$\kappa_1 = 24, \kappa_2 = 0.8$

	.4	.6	.8
.4	1	1	14
.6	1	12	16
.8	8	14	16

$a_1 = 0.2, a_2 = 0.8$

	.4	.6	.8
.4	1	1	14
.6	1	14	18
.8	14	18	20

$a_1 = 0.8, a_2 = 0.8$

Fig. 5. The agreement periods when $T=21$. (Boldface numbers in rows and columns are the values for φ_1 and φ_2 , respectively.)

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