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In recent years, some managers and researchers have advocated reducing lot sizes by decreasing setup costs, arguing that smaller lot sizes improve quality while reducing inventory levels and associated holding costs. However, smaller lot sizes result in an increased number of shipments which, in turn, exacerbates traffic congestion. This results in longer delivery times and, thereby, higher inventory levels. In this paper we study the relation between lot sizes and traffic congestion by constructing a model with numerous retailers who share a common congested delivery road. Using a numerical example, we illustrate the model's managerial implications with respect to several factors, including lot sizes, traffic congestion, and inventory levels. Our findings suggest that in a physical distribution system, if there are a relatively large number of retailers, no single retailer has an incentive to increase batch sizes because one retailer's effect on reducing traffic congestion will be negligible. If all retailers increase their lotsizes, however, traffic congestion will be reduced and all retailers will experience lower costs.

1. Introduction

In this paper we study the relation between lot sizes and traffic congestion in a physical distribution system. The idea of reducing lot sizes has been popularized in recent years as part of the 'just-in-time' (JIT) philosophy developed in the 1960s by Ohno, and first implemented by Toyota [1]. JIT proponents advocate reducing setup costs to improve flexibility and the ability to adapt quickly to changing demand. It is further argued that smaller lot sizes (ideally a batch size of one) will result in reduced inventory levels requiring less warehouse space and lower costs (see Zipkin [2] for a more comprehensive discussion of JIT philosophy and practice).

Smaller lot sizes, however, lead to more frequent shipments, which increase traffic congestion and result in longer delivery lead times and higher inventory levels. In addition, increased traffic congestion causes environmental degradation and negatively impacts road conditions and social welfare (For example, vehicle carbon dioxide emissions increased from 15% in 1975 to about 20% in 1990 in Japan [3]. Also see NRC [4] for a discussion of related issues in the USA.). These costs are causing managers and government agencies alike to reconsider the theory and practice of small lot ordering.

Negative consequences of lot size reduction have been described in numerous recent articles [5–8]. Referring to the Tokyo metropolitan area, Shiomi *et al.* [3] state bluntly that '...the frequent movements of trucks under JIT create chronic traffic jams in large cities and pollute the environment'. Abrahams [9] quotes the director of basic chemicals at one of Japan's largest plastics manufacturers, who stated that 'Acrylic sheet was infamous for small deliveries, even within the plastics industry. We were sometimes having to make deliveries three times a day. The costs were exorbitant. Not only did you have to cover the cost of freight, but you had to carry larger inventory too.'

In this research we study the impact of small lot ordering on traffic congestion in a supply chain. Specifically, we consider a physical distribution system with a number of retail outlets that share a common road for deliveries (the Interstate-5 corridor in the Seattle metropolitan area is an example of such a situation). We assume that demand at the retail outlets is random, and that fixed order costs are zero. Furthermore we assume that the speed of the delivery vehicles on the common road is a non-increasing function of the number of the vehicles on the road (i.e., the traffic density). This latter assumption implies that order leadtimes are random and depend on the number of vehicles on the road at any point in time.

We initially develop expressions for the operating characteristics of the system. Using these expressions, we show that when JIT ordering creates traffic congestion, the retailers can reduce their operating costs through a coordinated effort by increasing lot sizes (the solution adopted by the Japanese plastics industry). When such a

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coordinated effort does not occur among retailers, there are implications for government agencies that can impose road fees (e.g., toll fees, usage fees, taxes). Such fees would reduce traffic congestion and its associated social, structural and environmental degradations while collecting money for road maintenance.

To the best of our knowledge this is the first work that studies the effect of small lot ordering on traffic congestion, order leadtimes and operating costs in a multi-location physical distribution system. Previous research in multi-echelon distribution system has studied systems where congestion is only a factor in the production process. In these situations, standard queueing models were used for modelling congestion [10–14]. Other related work includes studies between the batch size and production leadtimes in a single-facility production environment by Karmarkar [15] and Karmarkar *et al.* [16]. Assuming that production times are exponentially distributed, the authors modelled the production facility as an $M/M/1$ queue and showed that production leadtimes increase as smaller production batch sizes are used.

The rest of this paper is organized as follows. In Section 2 we describe the physical distribution system and outline the relations which enable us to evaluate the operating measures of such systems. Specifically, we initially compute the steady-state probability distribution of the order leadtimes for our model. Using results from multi-echelon inventory theory, we then find the probability distributions of the number of outstanding orders at each retailer and, subsequently, the average total expected cost rate. In the Section 3 we introduce the functional relation between the speed of the vehicles on the road and the traffic density. In Section 4 we present the results of a numerical experiment that illustrates the impact of traffic congestion on JIT ordering by using the model developed in previous sections. Finally, we summarize our findings and discuss possible extensions of the basic model.

2. Specification of the physical distribution model

Consider a physical distribution system consisting of M retailers sharing a common delivery road for their goods. Without loss of generality, we assume that all retailers (with index set $I = \{1, \dots, M\}$) stock one product (but not necessarily the same product) and order units from suppliers in integer multiples, m_i, of a minimum allowable order quantity, $b_i (i \in I)$. (The quantity b_i is determined by packaging or freight considerations.) Inventories at all facilities are reviewed continuously and a (Q, R) ordering policy is used at each location; that is, when inventory position I_i (on-hand + on-order − backorders) reaches R_i, retailer i places another order of size Q_i [17], where $Q_i = m_i b_i$.

Demand for the product occurs at the retailers according to a Poisson process with a mean rate λ_i for retailer i. If the retailer does not have sufficient units on hand to meet demand, a backorder occurs. For ease of exposition, we assume that retailers' orders are filled immediately by the suppliers (i.e., there are no order delays at the supplier). Therefore order leadtime is determined only by the transit time to ship Q_i units from the supplier to retailer i. To simplify the discussion, we will use $\boldsymbol{Q}$ to denote the vector of all order quantities (i.e., $\boldsymbol{Q} = (Q_1, Q_2, \dots, Q_M)$) and $\boldsymbol{R}$ to denote the vector of all reorder points (i.e., $\boldsymbol{R} = (R_1, R_2, \dots, R_M)$).

To study the impact of JIT ordering, we assume that fixed order costs are zero and that retailers order the minimum allowable batch size, b_i (i.e., $m_i = 1 \ \forall\, i \in I$). This assumption is consistent with the goal of JIT ordering [2] and the increased emphasis on EDI in supply chain management.

All orders are shipped on a common road to their respective retailers; the transit time on this road is denoted by the random variable τ. The distance to be travelled by all vehicles on this common road is Y; vehicular speed is v_n when there are n vehicles on the road (i.e., n is a measure of the traffic density). We assume that v_n is a monotonically non-increasing function of n. The common road, however, may only constitute a portion of the total distance from the suppliers to each retailer. Thus we assume that there is an additional distance to each retailer; this latter segment is travelled on a non-congested road and takes a known time, T_i ($\forall\, i \in I$). The total transit time (and, hence, the order leadtime) faced by retailer i, τ_i, is then defined as $\tau_i = \tau + T_i$.

2.1. *Distribution of order leadtimes*

We note that traffic density is determined by the number of vehicles shipping goods to retailers as well as non-commercial traffic that may be using the common road. We will denote the average rate which the non-commercial traffic enters the common road by $\widehat{\lambda}$; therefore the average rate of vehicle entry, λ, to the common road is equal to

$$\lambda = \sum_{i \in I} \frac{\lambda_i}{Q_i} + \widehat{\lambda},$$

given that retailer i orders in batches of Q_i. If the number of retailers in the distribution system is relatively large and/or the non-commercial traffic constitutes a significant portion of the total traffic to the road, the order arrival process to the road can be approximated by a Poisson process (see Feller [18] for a description, Albin [19] for extensive empirical tests that support the approximation, and Zipkin [10] for a general discussion and application of such an approximation). Therefore we approximate the entry of vehicles to the common road by a Poisson process with mean λ.

To derive the probability distribution of the number of vehicles on the road (and, hence, the probability distribution of order leadtimes), we let $x_n(t) = \{x_1(t), \ldots, x_n(t)\}$ denote the stochastic process that describes the status of the vehicles on the common road at time t with n vehicles on the road and $x_i(t)$ denotes the remaining distance to be travelled by the ith vehicle on the road prior to t. Note that vehicles on the road are ordered on a FIFO basis; that is, $0 \le x_1 \le x_2 \le \ldots \le x_n \le Y$. We derive expressions for the steady-state density of $x_n, p_n(x_1, x_2, \ldots, x_n)$, and the steady-state probability of having n vehicles on the road, P_n, in the following proposition.

Proposition 1. *If* $\lim_{n\to\infty} nv_n > \lambda Y$, *then the steady-state distributions for* x_n *exists and*

$$p_n(x_1, \ldots, x_n) = (\lambda^n/[n!\, v_1 \ldots v_n])K \tag{1}$$

and

$$P_n = \left[(\lambda Y)^n \Big/ n! \prod_{i=1}^{n} v_i\right] K, \tag{2}$$

where K is a normalizing constant and is equal to

$$K = \left\{1 + \sum_{n=1}^{\infty} (\lambda Y)^n \Big/ \left[n! \prod_{i=1}^{n} v_i\right]\right\}^{-1}. \tag{3}$$

Proof: The details of the proof are described in Appendix A. ■

It is also shown in Appendix A that the Laplace–Stieltjes transform of the travel time τ of a vehicle on the common road, $L(\cdot)$, can be expressed in terms of the probability-generating function of the number of vehicles on the common road, $M(\cdot)$, as follows:

$$L[\lambda(1-z)] = M(z). \tag{4}$$

For certain forms of v_n (for example, $v_n = \alpha/n$ for $n \ge 1$, where α is a constant), one can obtain closed form expressions for the steady-state probability distribution of the total travel time τ of a vehicle on the common road. Furthermore, from (4), the expected value and variance of τ are defined, respectively, as:

$$E(\tau) = E(N)/\lambda \tag{5}$$

and

$$V(\tau) = [V(N) - E(N)]/\lambda^2, \tag{6}$$

where N is the random variable denoting the number of cars on the common road.

Let δ_i denote demand during leadtime at retailer i. Since demand at each retailer is assumed to be Poisson, then $E[\delta_i]$ is equal to $\lambda_i E[\tau_i]$ on the basis of Little's law. But $\tau_i = \tau + T_i$; therefore

$$E[\delta_i] = (\lambda_i/\lambda)E[N] + \lambda_i T_i \tag{7}$$

by (5). Furthermore we have (see [20]):

$$E[\delta_i(\delta_i - 1)] = \lambda_i E(\tau_i^2),$$

where $E(\tau_i^2)$ is the second moment of τ_i. It follows that the variance of δ_i can be expressed as

$$V(\delta_i) = \left(\frac{\lambda_i}{\lambda}\right)^2 V(N) + \left(\frac{\lambda_i}{\lambda}\right)\left(1 - \frac{\lambda_i}{\lambda}\right)E(N) + \lambda_i T_i. \tag{8}$$

Let B_i and OH_i denote the number of units backordered and the number of units on-hand, respectively, at retailer i. To evaluate the steady-state operating measures at each retailer, we note that

$$E(B_i) = E([\delta_i - I_i]^+), \tag{9}$$

where $(x)^+ = \max(0, x)$.

Since I_i is uniformly distributed in $[R_i + 1, R_i + Q_i]$ [17],

$$E(B_i) = \frac{1}{Q_i} \sum_{j=R_i+1}^{R_i+Q_i} \sum_{k=j}^{\infty} (k-j)\, q_i(k), \tag{10}$$

where $q_i(\cdot)$ denotes the probability distribution of demand for retailer i.

Similarly, the expected on-hand inventory at retailer i can be defined as follows:

$$\begin{aligned} E(OH_i) &= E([I_i - \delta_i]^+) = E(I_i) - E(\delta_i) + E(B_i) \\ &= R_i + \tfrac{1}{2}(Q_i + 1) - \lambda_i[E(N)/\lambda + T_i] + E(B_i). \end{aligned} \tag{11}$$

In the absence of order costs, the expected total cost rate $TC_i(Q_i, R_i)$ at each retailer is given by the sum of average holding and backorder costs

$$\begin{aligned} TC_i(Q_i, R_i) = {} & h_i\{R_i + \tfrac{1}{2}(Q_i + 1) - \lambda_i[E(N)/\lambda + T_i]\} \\ & + (h_i + \pi_i)\, E(B_i), \end{aligned} \tag{12}$$

where h_i is the unit holding cost of an item at retailer i per unit time, and π_i is the unit backorder cost of an item at retailer i per unit time. The expected total cost rate for the system, $TC(\boldsymbol{Q}, \boldsymbol{R})$, is then the sum of the expected costs for all retailers; that is,

$$TC(\boldsymbol{Q}, \boldsymbol{R}) = \sum_{i \in I} TC(Q_i, R_i).$$

If the objective is to minimize $TC(\boldsymbol{Q}, \boldsymbol{R})$, then, for fixed $\boldsymbol{Q}$, the optimal reorder point at retailer i, R_i^*, is the largest value of R_i that satisfies

$$\sum_{j=R_i+1}^{R_i+Q_i} \sum_{k=j+1}^{\infty} q_i(k) \ge \frac{h_i Q_i}{h_i + \pi_i}.$$

3. Vehicular speed on the common (congested) road

The relation between vehicular speed on a road and traffic density can be derived by considering the car-following behavior of single-driver-vehicle systems, or by directly studying the macroscopic behavior of platoons of vehicles on a given road section. As Gazis *et al.* [21] have demonstrated, the two approaches yield similar speed–density relationships. The relations obtained suggest that vehicle speed on a single road at steady state is a monotonic decreasing function of traffic density.

Greenshields [22] was among the first researchers to study the empirical relation between speed and traffic density and proposed an inverse relation approximated by a linear function. In order to better capture the true traffic behavior, especially for high traffic densities, several nonlinear (polynomial, logarithmic, and exponential) relation have since been developed. The particular choice of a relation depends on factors such as the geometry of the road segment and the nature of traffic flow. We refer the interested reader to Gazis [23], Gerlough and Huber [24] and Papageorgiou [25] for a summary of speed–density hypotheses and supporting empirical data.

As Ross [26] observed, one shortcoming of previously suggested models is that the common speed for the isovelocic flow of traffic is either zero at some 'jam' density or approaches zero asymptotically as the traffic density increases. This assumption results in a non-ergodic queueing system in general and violates the stability condition stated in Proposition 1. Therefore we use a hybrid speed function similar to the one defined by Smulders [27] and supported by Ross [26]. In this case, vehicle speed declines linearly as a function of the number of cars at lower traffic density levels. If the traffic density surpasses a critical density level n_0, vehicle speed is reduced in hyperbolic fashion. The assumed relation between vehicle speed and traffic density can be described mathematically as

$$v_n = \begin{cases} v_{\text{free}} - \alpha n, & 0 < n \le n_0; \\ v_0 n_0/n, & n > n_0; \end{cases} \tag{13}$$

where v_{free} is the free, or equilibrium, vehicular speed as the traffic density, n, approaches zero, and v_0 is the speed at the critical density level n_0. To ensure continuity, we define

$$\alpha = \frac{v_{\text{free}} - v_0}{n_0}.$$

Proposition 1 specifies a stability condition that must hold for a steady-state distribution for x_n to exist; that is,

$$\lim_{n\to\infty} n v_n = v_0 n_0 > \lambda Y.$$

On the basis of this condition, we let ρ denote the ratio $\lambda Y / v_0 n_0$, which provides an effective measure of utilization for the common road. We use this measure in the numerical example described in the following section.

Using (13) and the results obtained in the previous section, we can determine P_n, the probability distribution of the number of vehicles on the road. Let

$$f(n) = \prod_{i=1}^{n} (v_{\text{free}} - \alpha i)$$

and

$$g(n) = \frac{1}{n! f(n)} - \frac{1}{n_0! f(n_0)\,(n_0 v_0)^{n-n_0}}.$$

Then

$$P_n = \begin{cases} \{(\lambda Y)^n / [n! f(n)]\} K, & n \le n_0; \\ \{(\lambda Y)^n / [n_0! f(n_0)(v_0 n_0)^{n-n_0}]\} K, & n > n_0; \end{cases} \tag{14}$$

where

$$K = \left\{ \sum_{n=0}^{n_0} g(n)\,(\lambda Y)^n + \frac{(v_0 n_0)^{n_0+1}}{n_0! f(n_0)\,(v_0 n_0 - \lambda Y)} \right\}^{-1}. \tag{15}$$

The first two moments of the distribution of the number of vehicles on the road can then be expressed as

$$E(N) = K \left\{ \sum_{n=0}^{n_0} n\, g(n)\,(\lambda Y)^n + \frac{\lambda Y (v_0 n_0)^{n_0+1}}{n_0! f(n_0)\,(v_0 n_0 - \lambda Y)^2} \right\} \tag{16}$$

and

$$E(N^2) = K \left\{ \sum_{n=0}^{n_0} n^2 g(n)\,(\lambda Y)^n + \frac{\lambda Y (n_0 v_0 + \lambda Y)(n_0 v_0)^{n_0+1}}{n_0! f(n_0)\,(n_0 v_0 - \lambda Y)^3} \right\}. \tag{17}$$

Using (16) and (17), we can calculate the mean and the variance of the number of orders outstanding at each retailer as defined by (7) and (8). Unfortunately, the specification of the probability distribution of the number of orders outstanding at each retailer i, $q_i(\cdot)$, is extremely complex when $n_0 > 1$. In this case, however, we can approximate the distribution of $q_i(\cdot)$ by using a two-parameter family. It can be easily shown that $E(\delta_i) < V(\delta_i)$. Therefore one alternative will be to approximate the distribution of $q_i(\cdot)$ with a negative binomial distribution with a mean and variance equal to $E(\delta_i)$ and $V(\delta_i)$ and evaluate the operating measures of the retailers using (10) and (11). The negative binomial distribution has been previously used to approximate the steady-state distribution of the leadtime demand at various sites in the multi-echelon inventory literature and has been shown to be effective (see Svoronos and Zipkin [11], Graves [28]).

Furthermore, the average speed on the road is given by

$$\bar{v} = \frac{1}{1 - p_0} \sum_{n=1}^{\infty} v_n p_n,$$

where v_n is the vehicular speed when the traffic density is equal to n. Note that the term $1/(1 - p_0)$ appears because the average vehicular speed is calculated only for the instances when there are vehicles on the road.

The characteristics of the traffic model is illustrated in Fig. 1, which presents the relation between average speed, $\bar{v}$, and the road utilization ratio, ρ. This graph indicates that the average vehicle speed approaches zero asymptotically as the road utilization approaches unity (as ex-

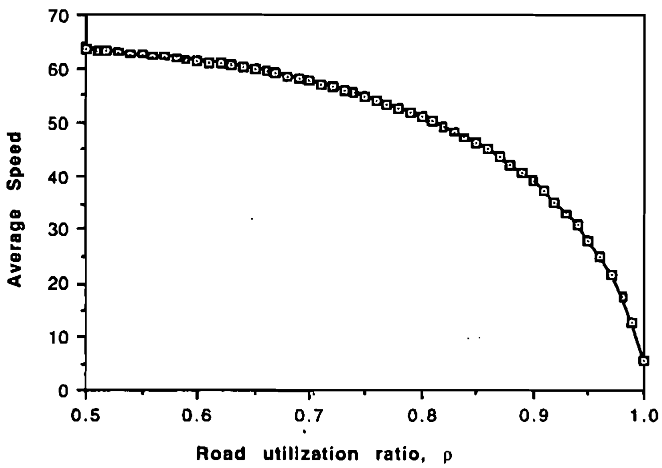

Fig. 1. Average speed, $\bar{v}$, plotted against the road utilization ratio, ρ ($v_{\text{free}} = 70, v_0 = 60, n_0 = 5$, and $Y = 5$).

pected from the stability condition). For the value of $\rho = 0.99$, which we use in the numerical example reported in the following section, Fig. 1 indicates that the average vehicular speed at that level of congestion is approximately 10 km/h.

4. Numerical results

In order to better illustrate the tradeoffs and managerial implications between lotsizing decisions and traffic congestion, we present the results of a numerical experiment where we analyzed 1000 scenarios. In each scenario we assumed that $M = 30$ and that the retailers, having identical parameters, used equal lotsizes. For all retailers $i \in I$, we fixed $h_i = h = 1$ and set the ratio $\pi_i/(\pi_i + h_i) = \pi/(\pi + h)$ to values $(0.90, 0.95, 0.98, 0.99)$. Furthermore we varied the constant portion of the delivery leadtimes and set $T_i = T = (0.1, 0.25, 0.5, 0.75, 1.0)$. To allow for differences in product attributes, we varied the number of items in a minimum shipment, and set all $b_i = b = (1, 5, 10, 25, 50)$.

With respect to the common road, we set $v_{\text{free}} = 70$, $v_0 = 60$, $n_0 = 5$, and $Y = 5$. To investigate the impact of non-commercial traffic on road congestion, we varied the fraction of non-commercial traffic, f_{NC}, on the common road from 0.50 to 0.95 in increments of 0.05. Finally, the values of λ_i (average demand at the ith retailer) were set for all retailers at a value such that the road utilization ratio $\rho = \lambda Y/v_0 n_0$ was 0.99 when JIT ordering was used (i.e., when $Q_i = b_i$). This implies that

$$\hat{\lambda} = 0.99(n_0 v_0) f_{\text{NC}}/Y$$

and

$$\lambda_i = 0.99\, b n_0 v_0[1 - f_{\text{NC}}]/M Y \quad \forall\, i \in I.$$

This value of ρ was chosen to highlight the traffic congestion effects as indicated in Fig. 1. It should be noted, however, that we varied ρ in other numerical experiments; our results in all cases were similar to those reported here.

Using these parameters, we examined a number of performance measures under various conditions; results are presented in Figs 2 and 3. In Fig. 2 we compare the expected total cost $TC(Q, R)$ for JIT ordering (i.e., minimum batch sizes when $Q_i = b_i$) with optimal batch sizes for varying values of f_{NC}. (In this case we set $T = 0.75$, $b = 10$, and $\pi/(\pi + h) = 0.98$; however, similar results were found for other values of these parameters.) As the fraction of non-commercial traffic increases (i.e., as the contribution of the orders from the retailer group to the overall vehicle traffic decreases), the cost differential between JIT ordering (denoted in Fig. 2 by TC_{jit}) and optimal lotsizing (denoted in Fig. 2 by TC_{opt}) diminishes. This is as expected; as the fraction of non-commercial traffic increases, the impact of increasing lotsizes on leadtimes is reduced. It should also be noted that, although expected total cost decreases as a function of f_{NC}, we are solving different inventory problems for each value of f_{NC}. Therefore the significance of Fig. 2 lies in the relative difference between the expected costs of JIT and optimal ordering, and not the absolute values of the total costs.

Fig. 3 illustrates the impact of lotsizing as a function of the size of the constant portion of the delivery leadtime, T, when $f_{\text{NC}} = 0.70$, $b = 10$, and $\pi/(\pi + h) = 0.98$. (Again, similar results were found for other values of these parameters.) As T increases, the impact of lotsizing gets smaller because traffic congestion and resulting travel time play a less significant role in the overall delivery delays. This has location policy implications for both retailers and suppliers; the supplier–retailer pairs that are closer to each other experience the biggest impact of the congestion on the road. However, as the distance between suppliers and retailers increases, the time on the con-

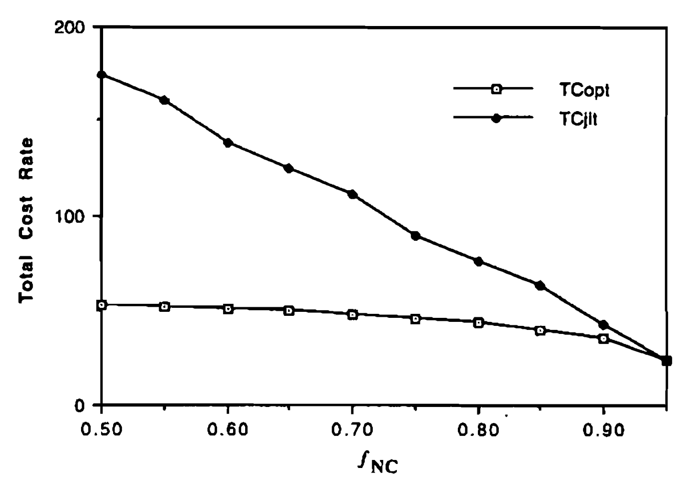

Fig. 2. Expected total cost for JIT ordering and optimal batch ordering ($T = 0.75, b = 10, \pi/(\pi + h) = 0.98$).

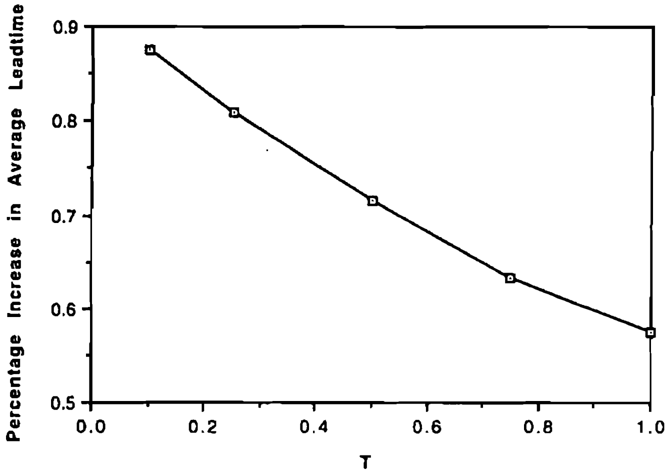

Fig. 3. Expected delivery leadtime, $E(\tau)$, plotted against the constant leadtime, T ($f_{\mathrm{NC}} = 0.70, b = 10, \pi/(\pi + h) = 0.98$).

gested road becomes a smaller fraction of the total leadtime, thereby having a smaller impact on batching decisions.

One of the cited benefits of JIT is that it reduces inventory and associated holding costs. To investigate the impact of congestion under JIT practice, we examined the on-hand inventory levels. In this case, $T = 0.75$, $b = 10$, and $\pi/(\pi + h) = 0.98$. Fig. 4 illustrates the ratio of the on-hand inventory levels under JIT practice and optimal ordering as a function of the fraction of non-commercial traffic on the congested road, f_{NC}. We note that the on-hand inventory levels are significantly lower when a retailer facing congestion deviates from JIT and orders in batch sizes larger than the minimum allowable shipment size.

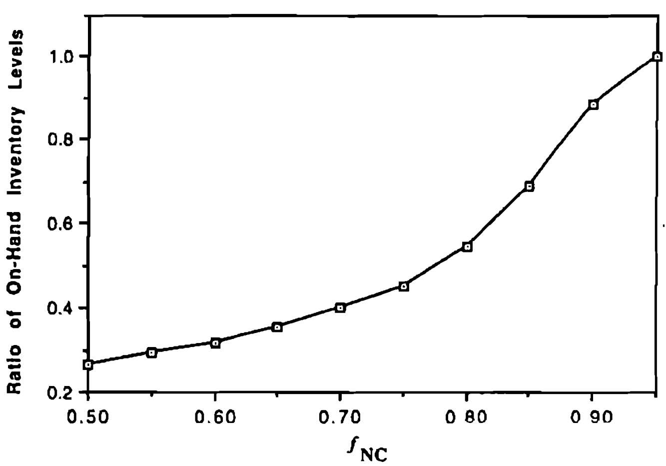

Fig. 4. Ratio of on-hand inventory levels under JIT practice and optimal ordering levels ($T = 0.75, b = 10, \pi/(\pi + h) = 0.98$).

5. Conclusions and extensions

In this paper we studied a physical distribution system where a number of retailers share a common congested road. Assuming that order costs are negligible, we first developed a model for the traffic flow on the congested road. Using results from multi-echelon inventory theory, we derived the probability distributions of the number of outstanding orders at each retailer and, subsequently, the average total expected cost rate.

Our results indicate that ordering minimum batch sizes, even when there is no fixed cost associated with placing an order, may not be optimal in terms of minimizing total expect cost when road congestion is present. Aside from the operating cost perspective of the retailers, traffic congestion has public policy implications as well. As the traffic intensity increases due to commercial use, the travel time of all vehicles on the road increases. Other negative externalities occur such as degradation in road conditions and environmental pollution.

It is interesting to note that, if there are a relatively large number of retailers, no single retailer has an incentive to increase batch sizes because one retailer's effect on reducing traffic congestion is negligible. If all retailers increase their lotsizes, however, traffic congestion is reduced and all retailers experience lower costs. Given the environmental benefits and other externalities, this may be justification for imposing tolls on certain congested roads. The model presented in this paper could be used as a starting point for determining the magnitude of such tolls.

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Appendix A

Proof of Proposition 1

Let $p_n(t,x_1,\ldots,x_n)$ denote the probability density of the road status's being x_n at time t. We derive the system of partial differential equations and their boundary conditions that describe the status of the vehicles of the road. Our approach is similar to that employed by Gnedenko and Kovalenko [29], Schmidt and Nahmias [30], Moinzadeh [31] and Moinzadeh and Schmidt [32].

Case 1: $n=0$. Let $\Delta>0$ be a small number. In order to have no vehicles on the road at time $t+\Delta$, either there were no vehicles on the road at t and no new vehicle entered the road during $(t,t+\Delta)$ or there was one vehicle on the road at time t travelling at a speed v_1 with a remaining distance to be travelled less than $v_1\Delta$. This means that by time $t+\Delta$, the vehicle has reached its destination and therefore left the road. Thus we have:

$$p_0(t+\Delta,\cdot)=(1-\lambda\Delta)p_0(t,\cdot)+(1-\lambda\Delta)\int_0^{\Delta v_1}p_1(t,\zeta)\mathrm{d}\zeta+o(\Delta). \quad \text{(A1)}$$

Using the integral mean value theorem ([33], pp. 139–143), we can write (A1) as:

$$[p_0(t+\Delta,\cdot)-p_0(t,\cdot)]/\Delta=-\lambda p_0(t,\cdot)+v_1p_1(t,\varepsilon)+o(\Delta),$$

where $0\le\varepsilon\le\Delta$.
Letting $\Delta\to 0$ and assuming that a steady-state solution exists, we get:

$$\lambda p_0(\cdot)=v_1p_1(0). \quad \text{(A2)}$$

Case 2: $n>0$, $x_1>0$ and $x_n<Y$. To be at x_n at time $t+\Delta$, one of the following should happen:

(1) There were n vehicles on the road at time t with remaining distances to be travelled equal to $(x_1+\Delta v_n,\ldots,x_n+\Delta v_n)$ and there were no new entries to the road during $(t,t+\Delta)$.
(2) There were $(n+1)$ vehicles on the road at time t with remaining distances to be travelled equal to $(\zeta,x_1+\Delta v_{n+1},\ldots,x_n+\Delta v_{n+1})$, where $0\le\zeta\le\Delta v_{n+1}$, and there were no new entries to the road during $(t,t+\Delta)$. This means that the first vehicle on the road reaches its destination and leaves the road by $t+\Delta$.

Thus, we have:

$$p_n(t+\Delta,x_1,\ldots,x_n) = (1-\lambda\Delta)p_n(t,x_1+\Delta v_n,\ldots,x_n+\Delta v_n) + (1-\lambda\Delta) \times \int_0^{\Delta v_{n+1}} p_{n+1}(t,\zeta,x_1+\Delta v_{n+1},\ldots,x_n+\Delta v_{n+1})\mathrm{d}\zeta + o(\Delta).$$

Using the integral mean value theorem, adding and subtracting successive terms and letting $\Delta \to 0$ and $t \to \infty$, we get:

$$\lambda p_n(x_1,\ldots,x_n) - \sum_{i=1}^{n} \frac{\partial p_n}{\partial x_i} = v_{n+1}p_{n+1}(0,x_1,\ldots,x_n). \quad \text{(A3)}$$

Next we derive the boundary conditions for the above system of partial differential equations. Consider $[n,\zeta_1(t),\ldots,\zeta_n(t)]$ to be the position of a particle at time t located in the region $0 \le \zeta_1(t) \le \ldots \le \zeta_n(t) \le Y$. The motion of the particle is discontinuous when a new delivery vehicle enters the road (a new order is placed). The boundary conditions are derived by considering such discontinuities. Define:

$$Q_n(t,x_1,\ldots,x_{n-1},Y) = \int_{x_1-\delta}^{x_1}\cdots\int_{x_{n-1}-\delta}^{x_{n-1}}\int_{Y-\delta}^{Y} p_n(t,\zeta_1,\ldots,\zeta_n)\mathrm{d}\zeta_n\ldots\mathrm{d}\zeta_1, \quad \text{(A4)}$$

where $\delta \ge 0$ is a small number. $Q_n(t,x_1,\ldots,x_{n-1},Y)$ represents the probability mass in a neighborhood of the boundary $(x_1,\ldots,x_{n-1},Y)$.

Applying the mean value theorem, for some $0 \le \varepsilon_i \le \delta$ $(i=1,\ldots,n)$, we get:

$$Q_n(t,x_1,\ldots,x_{n-1},Y) = \delta^n p_n(t,x_1-\varepsilon_1,\ldots,Y-\varepsilon_n) + o(\delta^n). \quad \text{(A5)}$$

Furthermore,

$$Q_n(t+\delta/v_n,x_1,\ldots,x_{n-1},Y) = \lambda\frac{\delta}{v_n}\int_{x_1-\delta+\delta(v_{n-1}/v_n)}^{x_1+\delta(v_{n-1}/v_n)}\cdots\int_{x_{n-1}-\delta+\delta(v_{n-1}/v_n)}^{x_{n-1}+\delta(v_{n-1}/v_n)} p_{n-1}(t,\zeta_1,\ldots,\zeta_{n-1})\mathrm{d}\zeta_{n-1}\ldots\mathrm{d}\zeta_1.$$

Using the Taylor expansion together with the mean value theorem, $Q_n(t+\delta/v_n,x_1,\ldots,x_{n-1},Y)$ can be also written as

$$Q_n(t+\delta/v_n,x_1,\ldots,x_{n-1},Y) = (\lambda\delta^n/v_n)p_{n-1}(x_1-\varepsilon_1,\ldots,x_{n-1}-\varepsilon_{n-1}) + o(\delta^n). \quad \text{(A6)}$$

Equating (A5) and (A6), dividing by δ^n and letting $t \to \infty$, we get:

$$v_n p_n(x_1,\ldots,x_{n-1},Y) = \lambda p_{n-1}(x_1,\ldots,x_{n-1}). \quad \text{(A7)}$$

Assuming that a steady-state solution exists, a solution to the above differential equations and their boundary conditions is:

$$p_n(x_1,\ldots,x_n) = \frac{\lambda^n}{n!v_1\ldots v_n}K,$$

where K is the normalizing constant.

The steady-state probability of having n vehicles on the road is now obtained as

$$P_n = \int_0^Y\int_{x_1}^Y\cdots\int_{x_{n-1}}^Y p_n(x_1,\ldots,x_n)\,\mathrm{d}x_n\ldots\mathrm{d}x_2\mathrm{d}x_1 = \left[(\lambda Y)^n\Big/\left(n!\prod_{i=1}^n v_i\right)\right]K. \quad \text{(A8)}$$

Finally, the normalizing constant, K, is found by summing the probabilities to unity, which yields (3). For the steady state solution to exist, K must be finite. This in turn means that

$$\lim_{n\to\infty}(P_{n+1}/P_n) < 1. \quad \text{(A9)}$$

Substituting (A7) into (3) $\lim_{n\to\infty} nv_n > \lambda Y$ as the condition for the existence of the steady-state solution. ■

6. Derivation of (4)

To prove that (4) holds, we first focus on the times when a vehicle departs the road. Let:

$P_n^{(D)}$ = steady-state probability of having n vehicles on the road as a vehicle departs the road

Then

$$P_n^{(D)} = \frac{\Pr\{\text{a departure is about to occur and there are } n+1 \text{ vehicles on the road}\}}{\Pr\{\text{a departure is about to occur}\}} = \frac{v_{n+1}\int_0^Y\int_{x_1}^Y\cdots\int_{x_n}^Y p_{n+1}(0,x_1,\ldots,x_n)\,\mathrm{d}x_n\ldots\mathrm{d}x_2\mathrm{d}x_1}{\sum_{i=0}^{\infty} v_{i+1}\int_0^Y\int_{x_1}^Y\cdots\int_{x_i}^Y p_{i+1}(0,x_1,\ldots,x_i)\,\mathrm{d}x_i\ldots\mathrm{d}x_2\mathrm{d}x_1}$$

Applying the results from Proposition 1, we get:

$$P_n^{(D)} = \frac{\lambda v_{n+1}\left[(\lambda Y)^n\Big/\left(n!\prod_{j=1}^{n+1} v_j\right)\right]K}{\lambda\sum_{i=0}^{\infty} v_{i+1}\left[(\lambda Y)^i\Big/\left(i!\prod_{j=1}^{i+1} v_j\right)\right]K} = \frac{(\lambda Y)^n}{n!\prod_{j=1}^n v_j}K = P_n. \quad \text{(A10)}$$

Therefore we conclude that the steady-state probability of having n vehicles on the road at a departure point is equal to the steady-state probability of having n vehicles on the road at an arbitrary point in time.

Because the vehicles leave the road on a FIFO basis, arrivals to the road are Poisson and, as shown in Proposition 1, $P_n(x_1, \ldots, x_n)$ is independent of $(x_1, \ldots, x_n)$; then, from the law of total probability, we can express $P_n^{(D)}$ (and thus P_n) as:

$$P_n = P_n^{(D)} = \int_0^\infty P_{n|t}^{(D)} dF(t), \quad \text{(A11)}$$

where $P_{n|t}^{(D)}$ denotes the steady-state probability that a departing vehicle leaves n vehicles on the road given that its travel time on the road was t time units, and $F(\cdot)$ is the cumulative distribution function of time spent on the road, τ, by a vehicle.

Furthermore, assuming that the process x_n, as defined before, is ergodic (that is, $\lim_{n\to\infty} nv_n > \lambda Y$), as shown in Stidham [34], Little's law must hold. Thus we conclude (Cox [35], pp. 45–46) that

$$P_{n|t}^{D} = \frac{(\lambda t)^n}{n!} \exp(-\lambda t)$$

and

$$P_n = \int_0^\infty \frac{(\lambda t)^n}{n!} \exp(-\lambda t)\, dF(t),$$

which, following Gross and Harris ([36], pp. 272–273), results in (4).

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