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Survival of Rationalism Between Hostility and Economic Growth*

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This article examines the interaction of country pairs who have historically been and are potentially hostile. Hostility is described as a function of arms stocks versus bilateral trade. Armament intensifies the current level of hostility whereas trade reduces the possibility of militarized disputes. We argue that welfare-maximizing decisionmakers have to seek methods other than accumulation of arms to increase the security of their nations, and we highlight the strategic nature of trade in overcoming enmity. Rational governments, who consider bilateral trade as a factor that reduces the level of enmity, allocate resources more efficiently between arms imports and consumer goods. The model predicts that understanding the use of trade as a diplomatic tool will lead the economy to grow significantly. The model is designed as a non-cooperative dynamic game and solved numerically using an adaptive learning algorithm called a *genetic algorithm.*

Introduction

There is a large literature (see Brito, 1972; Simaan & Cruz, 1975; Intriligator, 1975; Deger & Sen, 1983; Garfinkel, 1990; van der Ploeg & de Zeeuw, 1990; Levine & Smith, 1997) on the competitive accumulation of weapons between nations dating back (Richardson, 1960) to arms race models. Most of these theoretical analyses employ differential game theory to analyze the intertemporal security/consumption trade-offs inherent in these models. The models proposed are solved by the assumption that countries act as rational agents concerned with both consumption and the public evil of a war. However, there is a paucity of research incorporating the accumulation of capital besides arms to understand the security/growth trade-off in a dynamic game setting.[1] Hence, this article presents a more comprehensive model to examine the long-run growth trajectories of two potentially hostile

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[1] Even though many studies investigate the relationship between defense spending and economic performance, dynamic game structure is yet underplayed in the literature. See Deger & Smith (1983), Faini, Annez & Taylor (1984), Mintz & Huang (1990, 1991), Ward & Davis (1992) and Cappelen, Gleditsch & Bjerkholt (1992) for the trade-off between defense spending and civilian resource use (the guns vs. butter trade-off or the guns vs. investment trade-off). For other perspectives concerning the relationship between military expenditure and economic growth, see the extensive literature listed in Heo (1998).

countries who spend their national income on arms imports, consumption, and investment.

Recently, the international relations literature has challenged both theoretically and empirically the trade/conflict relationship by using game-theoretic or expected utility models (Barbieri, 1996; Reuveny & Kang, 1998; Polachek, Robst & Chang, 1999; Oneal & Russett, 1999). Polachek, Robst & Chang (1999) and Oneal & Russett (1999) draw attention to contiguity, in that conflict between neighboring countries would be greater than observed if it were not for the mitigating effects of trade. A common argument is that international trade prevents conflict because the possible loss of trade reduces the willingness to fight. Accordingly, in this article, we argue that welfare-maximizing decisionmakers have to seek methods other than accumulation of arms to increase the security of their nations, and we highlight the strategic nature of trade in overcoming enmity. The inability of political leaders in conflict-prone countries to capture the benefits of mutual interaction seriously hampers growth.

We measure hostility as the function of arms stocks versus bilateral trade. Armament intensifies the current level of hostility, whereas trade reduces the possibility of militarized disputes. We assume that importers avoid establishing ties with exporters in the adversary country because of the possibility that one government might rupture these ties. Therefore, the decision to accumulate arms influences the existing level of hostility, which has a direct deterrent effect on net bilateral trade and aggregate output.

The welfare of a country depends on the level of consumption and the level of security (which is perceived to be an increasing function of its own weapons stock). Generally, the relative importance of security and consumption on the overall welfare function is described by exogenously determined preference parameters. Here, we endogenize these parameters to explore the indirect effect of hostility on growth. Namely, preferences for security and consumption vary according to the choices made by incumbent governments. If hostility between two nations intensifies, government leaders prefer to invest more in arms since the relative significance of security increases. Thus, we incorporate the rationale for the excessive armament policies of some political leaders whose countries are subject to severe economic constraints.

The interconnection between growth, investment, and military expenditure is necessarily complex. Hence, one of the important innovations of this article is to introduce an adaptive learning algorithm called *genetic algorithm* to study the dynamics of such complicated models under certain plausible parameters. This stochastic, directed search algorithm is a useful representation of trial-and-error learning that has important advantages over existing solution procedures in complex dynamic games.[2] The genetic algorithm helps solve a conundrum that has long bedevilled conventional problem-solving methods: striking a balance between exploration and exploitation. Once one finds a good strategy (policy) it is possible to concentrate on exploiting that strategy. Holland (1975: 69), who is the founder of genetic algorithms, argues that

> the choice carries a hidden cost because exploitation makes the discovery of truly novel strategies unlikely. Improvements come from trying new, risky things. Because many of the risks fail, exploitation involves a degradation of performance. Deciding to what degree the present should be mortgaged for the future is a classic problem for all systems that adapt and learn.

Walt (1999: 22) argues that 'a logically consistent and mathematically rigorous theory is of little value if it does not illuminate some important aspects of the real

[2] See Smoker (1989) for a discussion of artificial intelligence models of arms races.

world'. Our model is devised to make long-run predictions for conflicting countries, so we only provide simulation results using genetic algorithms for supportive evidence. We also present a case study of Russia and Turkey and illustrate how mutual interaction, and thus learning, affect political preferences toward efficiency. The case study aims to justify one of the crucial assumptions of the model on the endogenous preference parameters chosen by the policymakers.

The Model

Consider that there are two potentially hostile countries, i and j. The decisionmaker in each country has preferences described by identical lifetime utility functions:

$$V_i = \max \sum_{t=0}^{\infty} \beta^t u(C_{it}, A_{it}) \qquad (1)$$

where C_{it} is consumption of country i at time t, A_{it} is beginning-of-period arms stock and β is the rate of time preference. Given the initial period arms and capital stocks, A_{i0} and K_{i0} respectively, the government's objective is to choose optimal (maximizing $u(.)$) sequences $\{(C_{it}, N_{it})\}_{t=0}^{\infty}$ where N_{it} is new arms imported at time t.

Let δ be arms depreciation or obsolescence rate; the next period's arms stock, A_{it+1}, is the summation of net of beginning-of-period arms stock and new arms imports. Then

$$A_{it+1} = (1 - \delta)A_{it} + N_{it} \qquad (2)$$

describes the accumulation of arms stocks for country i at period t. Neither nation has complete freedom of its consumption and weapon expenditures since total expenditures must not exceed the net national output, Y_{it}. Thus the budget constraint is expressed by the equation

$$Y_{it} = C_{it} + I_{it} + pN_{it} + M_{jt} - M_{it} \qquad (3)$$

where p is the price of imported arms relative to the price of consumer goods, M_{it} is the imports of consumer goods from conflicting country j and M_{jt} is exports to country j (or imports of j). I_{it} represents investment or net increase in the stock of physical capital at point in time, K_{it} $I_{it} = K_{it+1} - K_{it}$.

Equation (3) is the national income identity, linking aggregate output to aggregate expenditure. We characterize the aggregate output as $Y_{it} = F(K_{it})$ and define economic growth as the output growth or accumulation of capital over time. By rearranging (3),

$$K_{it+1} - K_{it} = F(K_{it}) - C_{it} - pN_{it} - M_{jt} + M_{it} \qquad (3')$$

we can show that the rate of capital accumulation is affected not only by the home country's arms imports, N_{it}, but also by imports of consumer goods from the opponent country, M_{it} (bilateral trade).

We assumed that both countries are arms importers, so there is no weapons trade between these conflicting parties; but there could be 'ordinary' trade, so we describe this relation as

$$M_{it} = gF(K_{it}) - bZ_{jt} \qquad (4)$$

where Z_{jt} measures country j's hostility against i. In Equation (4), we argue that countries whose incomes are high may trade more, and the parameter $g \geq 0$ denotes the share of increased import demand from the conflicting country. As emphasized by Pollins (1989), the realized imports between nations could be lower than desired due to existing hostility. He showed that importers take account not only of the price and quality of goods and services but also of the place of origin of these products and of the political relationship between the importing and exporting nations. A common parameter $b \geq 0$ captures the worsening effect of hostility on imports in each country. We specify the hostility in each period for country i and j as

$$Z_{it} = \frac{h_i A_{jt}^{\theta}}{M_{it-1}^{\psi_i}} \text{ and } Z_{jt} = \frac{h_j A_{it}^{\theta}}{M_{jt-1}^{\psi_j}} \quad (5)$$
$$\theta, \psi_i, \psi_j, h_i, h_j \geq 0$$

where h_i and h_j denote the inherent constant hostility parameters of conflicting nations against each other respectively. Z grows by the increase in the rival's beginning-of-period arms stock and decreases by the increase in trade links realized at period $t-1$. ψ_i and ψ_j are exogenous parameters chosen by the social planners to weigh the effect of economic links on hostility.[3] If $\psi_i = 0$, this means that the incumbent government in country i ignores trade relations in the calculation of hostility and considers only arms accumulation as the indicator of hostile intentions from the adversary country. If the rulers or politicians have no policy preferences of their own and are conflict averse, then ψ must be different from zero since it is assumed that bilateral trade reduces the tension or hostility between nations, and with that national income (output) increases as well as social welfare. The exponential order of hostility due to arms accumulation is measured by another common parameter, $\theta \geq 0$.

Welfare function is specified as $u(C_{it}, A_{it}) = C_t^{1-\gamma_i} A_t^{\gamma_i}$, where the parameters $1-\gamma_i$ and γ_i denote the government's tastes for consumption and arms stock respectively. Arms stock increases the well-being of the citizens through increased security, but there is a trade-off between consumption and the accumulation of arms. Higher weapon stocks eventually increase the feeling of security (Brito, 1972) and thus welfare, but also mean that there are less resources available for consumption, and therefore welfare diminishes. Nonetheless, the distribution of the aggregate output to consumption and arms stock is determined by the preference term, γ_i, *chosen* by social planners. The taste for armament, γ_i, is specified as follows:

$$\gamma_i = Z_{it} / (\sigma + Z_{it})$$

where $0 \leq \sigma \leq 1$ is any constant parameter to restore diminishing marginal utility, namely, $0 \leq \gamma_i \leq 1$.[4] If the hostility index, Z, is small due to either less armament of the adversary country or to the strong economic link to that country, the incentive $\gamma = Z/(\sigma + Z)$ to accumulate arms decreases. Therefore the income dedicated to arms imports is transferred to consumption. By endogenizing the preference parameters, we are able to incorporate the effect that armament policies not only increase security but (since they also intensify hostility) lead to further armament over time.

Also, nation j solves a similar problem as follows:

$$V_j = \max \sum_{t=0}^{\infty} \beta^t C_{jt}^{1-\gamma_j} A_{jt}^{\gamma_t}, \quad 0 \leq \gamma_j \leq 1 \quad (6)$$

subject to

$$A_{jt+1} = N_{jt} + (1-\delta) A_{jt},$$
$$K_{jt+1} = F(K_{jt}) + K_{jt} - C_{jt} - pN_{jt} - M_{it} + M_{jt},$$
$$M_{jt} = gF(K_{jt}) - bZ_{it}.$$

In the above problem, the initial period values for capital stock, K_{j0}, arms stock, A_{j0}, and home country imports, M_{j-1} are given.[5]

[3] In order to capture the positive effect of trade on hostility, the above formulation of Z necessitates M_{it} and M_{jt} to be greater than 1 for all t.

[4] Most people are subject to diminishing marginal utility, which means that they gain less and less satisfaction per unit as more and more of something is consumed.

[5] M_{j-1} denotes country j's initial period ($t = -1$) of import demand of consumption goods from country i. We need import demands at time -1 to calculate respective Zs at time 0.

Case Study: Turkish–Russian Relations

In the above model, we made a crucial assumption that eradicating hostility and promoting cooperation is an important step leading to peace and economic growth. One method of diminishing hostility and bringing about cooperation is to increase the cost of hostility between conflicting nations (Polachek, 1980). Policymakers should consider diminution of welfare associated with potential trade losses and learn to expand peaceful flows among them – the widening spread of ideas and knowledge, and flows of goods and people in international trade (Kuznets, 1980). In this respect we analyzed Turkish–Russian relations[6] and presented how centuries-long enmity substantially declined as trade increased. Assumptions that will plausibly turn hostility to healthy competition are as follows:

(1) both countries inherit a strong state tradition, albeit under diverse circumstances. Ad hoc cooperation between echelons of state continue;
(2) although some external circumstances affect mutual misperceptions, confidence-building measures are present;
(3) since the 1998 Russian economic crisis, Turkey has not withheld credit, abrogated agreements, or withdrawn its workforce from Russia;
(4) Turkey and Russia are on a par with each other in their search for political stability, democratization, territorial integrity, and economic growth;
(6) since the demise of the Soviet Union, Turkey's strategic position has become dynamic, freed from NATO's forward defense concept. This may be an asset for Russia and Turkey alike, both of whom are facing a Europe reluctant to receive them as part of the political and security architecture; and
(7) opposite stances that Turkey and Russia took regarding conflicts in the Balkans and Nagorno-Karabakh never became an issue in bilateral relations.

In 1964, trade turnover between the Soviet Union and Turkey was minimal, totalling less than $20 million. By October 1990, they agreed to raise the volume of trade to $4 billion. By 1991, for the first time since Stalin revoked the 1925 Treaty of Friendship and Neutrality in 1945, the two countries referred to each other as friends. This positive development was enhanced by membership in the Black Sea Economic Cooperation Organization (initiated by Turkey in 1990 on the assumption that economic interdependence promotes security), and booming trade, projected to reach a volume of $10 billion by 2000.

Meanwhile, political discord on certain issues such as economic rivalry in the Commonwealth of Independent States (CIS), oil transportation issues, Russian attempts to alter the Conventional Forces in Europe (CEF) agreement, playing the Chechen and Kurdistan Worker's Party (PKK) separatism/terrorism respectively against each other strained bilateral relations. As the initial shock of Soviet disintegration began to erode and Turkey's initial enthusiasm to open up to Central Asia and the Caucasus assumed realistic proportions, a balance seems to have been struck in Russian–Turkish relations. The recent trend appears to have overcome points of discord, and mutual economic benefits have begun to dominate.

Declarations, letters of intent, and even protocols aside, Russia became the second largest trade partner of Turkey within a spectacularly short time. Figures from 1995 point

[6] Even though there is no Turkish–Russian arms race (as they are incompatible in various ways), this relation deserves to be studied from the perspective that although learning may take a long time, similar goals like democratization, commitment to free market economy, and mutual benefit will decrease antagonism.

to stronger Turkish economic ties with Russia than with any other state in the CIS. The trade volume rose to \$3.3 billion, the value of construction work undertaken by Turkish firms reached \$5.7 billion, suitcase trade was \$1 billion, and over one million Russian tourists visited Turkey that year (Babusenko, 1996).

There are three natural gas import projects to/via Turkey: the Azerbaijani pipeline, the Transcaspian pipeline, and the Blue Stream, respectively ranging from the least to the most expensive. The Blue Stream was criticized on various grounds, such as Russia did not have enough natural gas to fill this pipeline and therefore would buy gas cheaply from Turkmenistan and sell it at a higher price to Turkey. The agreement on the Blue Stream project was one gesture from Turkey towards Russia, since there is much at stake for the welfare of both countries not only in economic but also in political terms.

The most tangible cooperation Russia displayed towards Turkey was its refusal to accommodate the PKK leader on its territory after Turkey forced Syria to extradite him in October 1998. Although some factions in the Duma lent support to the Kurdish parliament-in-exile, the Russian government remained true to its pledge in the 1992 Treaty of Friendship and Cooperation to cooperate against terrorism.

Seeking the Optimal Non-cooperative Solution Using Genetic Algorithms

Our model is an infinite-horizon non-cooperative dynamic game between two potentially hostile countries. Traditionally, the optimal strategies or open-loop Nash[7] equilibrium in such a game can be approximated by various optimization techniques under some restrictive assumptions on the functional forms (Başar & Oldser, 1982). However, in this study, we will use a new optimization technique called the *genetic algorithm* (GA) to solve the non-cooperative game between nations. Here, we do not expect decisionmakers to derive first-order conditions for the problem described, but rather allow them to communicate and learn the optimal strategies over time. In the context of our model, we use both the optimization and the learning property of the GA to approximate non-cooperative solutions.

GAs operate on a population of candidate solutions to some well-defined problem. Following each iteration of the algorithm, candidate solutions are evaluated for their performance and are assigned a fitness value. Solutions with relatively high fitness values are more likely to remain in the next *generation* of candidate solutions than are solutions with relatively low fitness values (Grefenstette, 1986). This process captures the notion of survival of the fittest (natural selection). The algorithm then uses the highly fit candidate solutions to breed new candidate solutions, using naturally occurring genetic operations (see Goldberg, 1989; Michalewicz, 1992).

GAs are powerful general-purpose optimization tools in irregular and complex search spaces. A drawback, however, is the lack of any obvious and generally accepted method of dealing with constraint violations. Given that our model is heavily constrained, this difficulty may seem especially troubling. Nonetheless, we successfully incorporate constraints into 'fitness' or utility functions by way of substitutions. First, we rearrange (3′) and substitute armament Equation (2) to derive C_{it} as

$$C_{it} = F(K_{it}) - \underbrace{(K_{it+1} - K_{it})}_{I_{it}}$$
$$-p\underbrace{(A_{it+1} - (1-\delta)A_{it})}_{N_{it}} - M_{jt} + M_{it}$$

[7] The open-loop Nash equilibrium concept presumes that optimal choices at each point in time are only conditional on the initial state of the model. Open-loop corresponds to the receipt of no information during the play. See Levine & Smith (1997) for further discussions in the context of arms races.

then we insert import demands described by (4) to yield

$$C_{it} = F(K_{it}) - (K_{it+1} - K_{it}) - p(A_{it+1} - (1-\delta)A_{it}) - \underbrace{gF(K_{jt}) + \frac{bh_i A_{jt}^{\theta}}{M_{it-1}^{\psi_i}}}_{M_{jt}} + \underbrace{gF(K_{it}) - \frac{bh_j A_{it}^{\theta}}{M_{jt-1}^{\psi_j}}}_{M_{it}}$$

which is substituted back to Equation (1) and thus, decisionmakers in country i choose time-paths for $\{K_{it+1}\}_{t=0}^{\infty} = K_{i1}, K_{i2}, \ldots$ and $\{A_{it+1}\}_{t=0}^{\infty} = A_{i1}, A_{i2}, \ldots$ to maximize the following fitness function:

$$V_i = \max \sum_{t=0}^{\infty} \beta^t u(K_{it+1}, K_{it}, K_{jt}, A_{it+1}, A_{it}, A_{jt}, M_{it-1}, M_{jt-1}).$$

By the optimal choices of next period's state variables, countries indirectly choose their optimal consumption, C_{it}, and volume of arms imports, N_{it}. This function is the performance measure of the *chosen* strategies given the opponent's policies (K_{jt}, A_{jt}, M_{jt-1}).

For the sake of practicality, the above 'lifelong' fitness function needs to be truncated for some finite period, T, using the methodology described by Mercenier & Michel (1994). This method proposes time aggregation[8] which requires that the finite-horizon model has the same steady state[9] as the infinite-horizon analog. In the numerical experiment, we will use the above function, V_i, to evaluate how chosen strategies meet welfare-maximizing objectives of the governments. Initially, we assume that neither country knows the opponent's strategies. Given some random strategies of country j, $\{(K_{jt}, A_{jt}\}_{t=0}^{T}$, and country i, $\{(K_{it}, A_{it}\}_{t=0}^{T}$, the success of this initial generation of strategies is tested using each country's relevant fitness functions. The learning process starts after initial generation. From the candidate strategies in the initial generation, the best-performing strategy set is publicized by sending this information to what we call an *information exchange center* (Figure 1). Through the game, neither country knows the problems of the opponent or the opponent's preferences in their entirety, but has information about relevant strategies against itself. With the availability of this information, countries update the population of latest strategies and send the *up-to-date best*[10] strategy set (including capital and arms stock) to the information exchange center at the end of each generation. In the convergence state, none of the players alter strategies against each other. Therefore, the game is over for the planning horizon, and the final strategies would be the equilibrium (optimal) time-paths for investment and arms accumulation. This equilibrium is the GA equilibrium of the game.

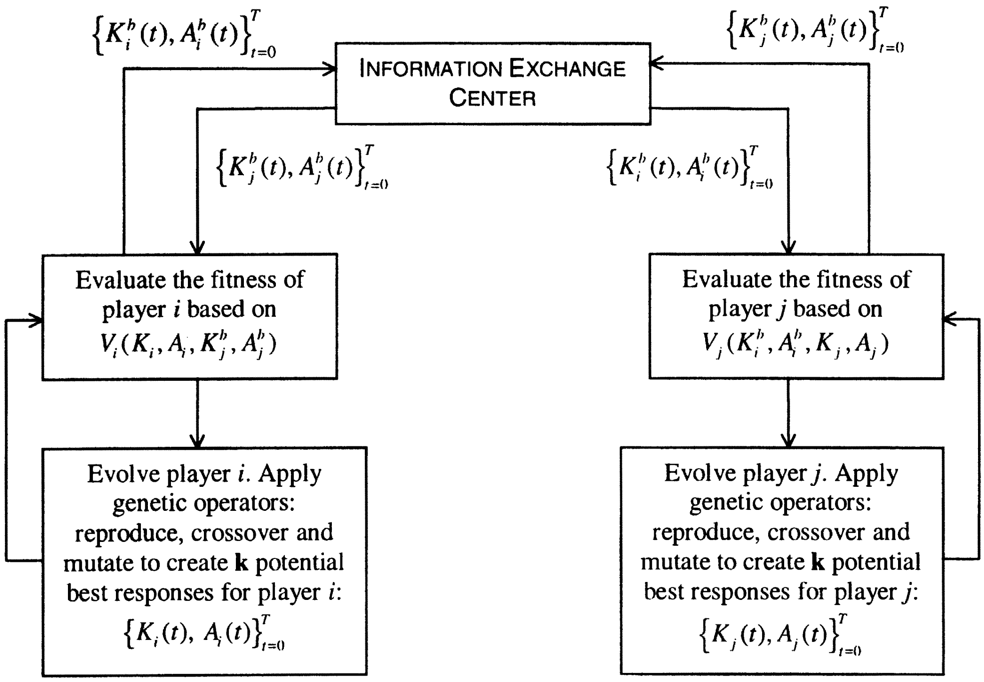
In our model, since there are two nations, we run two separate GAs, each to play one side of the game. Each country has its own utility function and population of possible strategies. The utility functions of each player might have different parameters and functional forms depending on the problem described. Based on the complexity of the fitness function, one player might evaluate the performance of its strategies faster than the other player. However, in order to learn

[8] The transformation of infinite horizon to finite horizon involves various types of decisions. One concern is the length of the finite planning horizon (the transient path). Errors on the optimal trajectory that will result from this approximation may be reduced by increasing the length of the decision horizon with a resulting boost in computational costs. A suitable choice of dynamic aggregation may significantly reduce the dimension of the numerical optimization for a given level of accuracy, hence allowing enrichment of other aspects of the model (see Mercenier & Michel, 1994).

[9] The steady state or long-run equilibrium can be defined as the state or equilibrium where all the variables grow at a constant (possibly zero) rate.

[10] In Figure 1, *up-to-date best* strategies are denoted by superscript *b*.

Figure 1. The Evolution of Learning and Optimization Through GA



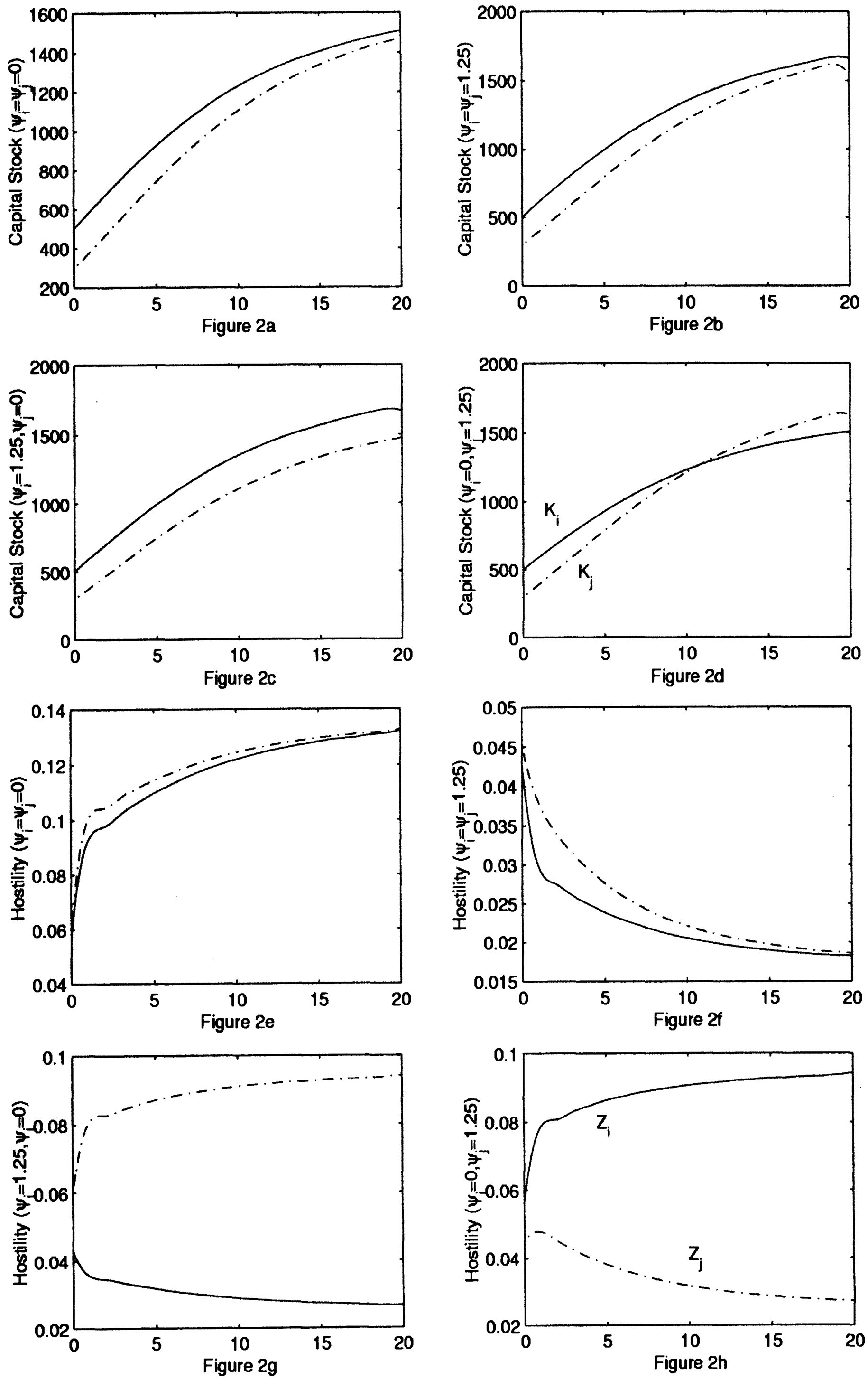
the action of the other player against its strategies, each player waits for the other player's action in each generation. Thus, the game must be played *synchronously* and genetic operators must be applied sequentially to each generation (Özyıldırım, 1997).

Since the algorithm is a search algorithm, in order to find the equilibrium of the problem we do not need to take derivatives (to derive first-order conditions) but only substitute the best values into the evaluation function. No one expects governments to solve mathematical problems, but their objectives are to maximize overall welfare which require optimization. By our alternative procedure, governments merely write down their objective function and search for optimal strategies by trial-and-error learning algorithm. A number of experimental studies (see Goldberg, 1989; Michalewicz, 1992) have shown that GAs exhibit impressive efficiency in practice. While classical gradient search techniques are more efficient for problems which satisfy tight constraints (for instance, continuity, low dimensionality, uni-modility, etc.), GAs consistently outperform both gradient search techniques and various forms of random search on more difficult (and more common) problems such as optimization involving high dimensionality, multi-modality and non-linearity. The model devised in this study is sufficiently high dimensional and non-linear, so we prefer to introduce this random search algorithm which is not confined to searching locally.

Numerical Experiments

We simulate the above model under four different cases of political leaders' preferences on the importance of bilateral trade as foundation of peace: (i) $\psi_i = \psi_j = 0$, (ii) $\psi_i = \psi_j = 1.25$, (iii) $\psi_i = 1.25$, $\psi_j = 0$, and (iv) $\psi_i = 0$, $\psi_j = 1.25$. The production function is

Figure 2. Benchmark Case



assumed as Cobb–Douglas for both countries: $F(K) = aK^{\alpha}$ and the other parameters[11] are: $a = 1$, $b = 1$, $\alpha = 0.65$, $h_i = h_j = 0.02$, $g = 0.02$, $\sigma = 0.4$, $\delta = 0.2$, $\beta = 0.95$, $p = 1$, $\theta = 0.25$, $K_{i0} = 500$, $K_{j0} = 300$, $A_{i0} = 5$, $A_{j0} = 4$, and $M_{i-1} = M_{j-1} = 1.25$. In the time aggregated model we assume 20 periods with a dense equally spaced gridding of the time horizon, $T = 200$, which is sufficient to capture convergence.

With this parametrization, Figure 2 presents the simulated trajectories of capital stock and hostility for our numerical experiments. Although our model is deterministic, the non-linear structure of the model raises the possibility of multiple equilibria. However, under these parameters, optimal trajectories derived by GA are unique and stationary. GA seeks *only* the global optimum as the evolutionary equilibrium. The implicit parallelism of GAs ensures that the search is efficient. The central idea behind the parallelism of GAs is that each of the formula elements defines hyperplanes, i.e. subregions of the search space where GA traverses all these subregions to find the best solution.

First, we summarize the main findings in the long run:

- Countries grow more in the long run if they are governed by political leaders who consider trade as a tool to reduce hostility. This result is more evident when the cases $\psi_i = \psi_j = 1.25$ and $\psi_i = \psi_j = 0$ are compared. In the former case, steady state national products are $Y_i^* = 124.03$, $Y_j^* = 122$ (see Figure 2b), whereas in the latter, respective outputs are $Y_i^* = 116.44$, $Y_j^* = 114.65$ (see Figure 2a).
- Countries initially small in terms of physical capital or national product, but ruled by governments that consider trade's positive effect on hostility, grow more and become wealthier than their adversaries (for $\psi_i = 0$, $\psi_j = 1.25$, initial outputs, $Y_{j0} = 40.75$, $Y_{i0} = 56.80$ become $Y_j^* = 122.57$, $Y_i^* = 116.44$ in the long run). From Figure 2d, we observe that in country j, which initially has less capital stock, K_{j0} reaches the same level as in i around the tenth aggregated time-period, and grows further thereafter.
- If a country has initially higher national product than its opponent and is ruled by a government that recognizes the importance of bilateral trade in the construction of mutual concord ($\psi_i = 1.25$, $\psi_j = 0$), the output gap ($Y_i^* - Y_j^*$) between these two countries increases further in the long run (see Figure 2c: initial $Y_{i0} = 56.80$ and $Y_{j0} = 40.75$ turn into $Y_i^* = 124.39$, $Y_j^* = 114.65$ at the steady state).
- Long-run welfare compositions drastically change in favor of arms accumulation among conflicting countries that are ruled by ignorant governments. When $\psi_i = \psi_j = 0$, initial composition of $C_{i0} > A_{i0}$ ($C_{j0} > A_{j0}$) changes to $C_i^* < A_i^*$ ($C_j^* > A_j^*$) in the long run.

We also present evidence that hostility declines over time if either country has $\psi = 0$ (see Figures 2g and 2h) or both countries have ψs other than zero (see Figure 2f). In the model, we described the preference parameter on security as $\gamma = Z/(Z + \sigma)$; hence, declining patterns of hostility suggest that preference for armament declines over time. From the citizen's point of view, the diminishing desire of governments for armament means that more resources would be allocated to consumer goods (see Table I). Thus, not only more is invested but also the composition of total resources invested favors the consumer goods sector as compared with the military sector. In all cases other than the benchmark case $\psi_i = \psi_j = 0$, arms stock is lower than consumption over the transition paths. In the benchmark case,

[11] Some parameter values are used by Levine & Smith (1997) to simulate the arms race between Greece and Turkey.

the volume of arms stock increases so fast that after the fourth aggregated time-period, military stocks in each period become higher than consumption. Concomitant with higher capital accumulation and output (Figure 2a), more resources are devoted to the military sector at the expense of necessary goods.[12]

Finally, from Table II, total respective discounted welfare and changes in the defense burden (share of arms imports in national output) over time may be compared.

We observe substantial welfare gains, even when one of the governments in this arms race has less hostile intentions and understands the importance of trade in this process. Also, when we compare the burden of military expenditures at period 0 and steady state, it is apparent that in cases where governments are 'rational' ($\psi_i \neq 0$ and/or $\psi_j \neq 0$), they keep the burden at almost the same level or reduce it further. However, governments that ignore trade in their decisions would endure an increasing burden of military expenditures up to 15–20% of the national output at the steady state.

Sensitivity Analysis

In order to further assess the robustness of the results reported in the previous section, the basic model has been tested for changes in the focus parameters: exponential impact of trade on hostility, ψ, and exponential impact of the arms stock of the adversary on hostility, θ. We have so far confined attention to situations in which either one country totally ignores trade's importance on hostility or both countries give the same weight to the trade effect. Thus, in order to discern the relative sensitivity of the equilibrium paths to the strengths of these effects, we resolve our model for different unequal values of ψs and θs. Specifically, we run 12 different simulations for $\psi = \{0, 0.25, 0.75, 1.25\}$ and $\theta = \{0.25, 0.35\}$. The optimal trajectories are reported in Tables III and IV (see Appendix). First, we observed that as long as ψ increases, hostility decreases. Hence, by the decline in the motivation of arms accumulation, available resources are distributed more to the physical capital (see Figures 3a and 3e). Thus, countries with more capital grow more in the long run. Second, in the experiments[13] under different θs, all else being equal, if arms accumulation is weighted more as the indication of enmity, rivals' armament increases together with growing hostility (see Figures 3d and 3h). Hence, countries having higher θ grow less in the steady state (see Figures 3c and 3g). Finally, in Table III, we show that even for unequal values of ψ, the result that excessive growth of an initially capital-poor (but $\psi_j > \psi_i$) country over an initially capital-rich one is robust. Over time, $K_{j0} < K_{i0}$ turns to $K^*_j > K^*_i$ at the equilibrium.

Conclusion

This article offers insights into the connection between bilateral trade and politics and the learning process in a dynamic game setting.[14] It constructs a model in which political leaders are assumed to be utility maximizers who seek to satisfy security as well as economic welfare. Parameters that summarize the relative importance of security and consumption are endogenously determined. The optimal choice of arms import affects the existing level of hostility, which we argue is a crucial decisive factor. The level of enmity is assumed to be positively related to the rival's armament, and inversely related to trade with the rival country.

[12] This result can be observed in various countries whose citizens have suffered for long periods. A recent case is North Korea. This country is one of the world's poorest and maintains the world's fifth largest army.

[13] We ran six experiments for $\theta = 0.30$, but do not report the (similar) findings due to space limitations.

[14] In this study, we ignored the cooperative game since optimal strategies would not be credible in such complex international problems.

Table I. Optimal Time Paths of Arms Stock and Consumption

	$\psi_i = 0$		$\psi_i = 1.25$		$\psi_i = 1.25$		$\psi_i = 0$	
t	A_i	C_i	A_i	C_i	A_i	C_i	A_i	C_i
0	5.00	43.29	5.00	43.60	5.00	43.64	5.00	43.34
1	39.13	45.51	14.03	51.49	16.56	50.74	34.26	46.94
2	46.32	50.45	15.01	58.37	18.12	57.52	39.71	52.21
3	53.52	55.10	15.98	64.89	19.67	63.80	45.16	57.23
4	60.52	59.47	16.95	71.07	21.04	69.95	50.22	61.97
5	67.14	63.27	17.53	76.60	22.40	75.57	55.08	66.12
6	73.17	66.92	18.12	82.02	23.57	80.74	59.36	70.11
7	79.00	70.20	18.70	86.78	24.54	85.49	63.44	73.73
8	84.26	73.02	19.09	91.15	25.32	90.00	67.14	77.01
9	89.12	75.77	19.48	95.13	26.09	93.88	70.45	80.00
10	93.59	78.22	19.87	98.76	26.87	97.43	73.36	82.52
11	97.68	80.27	20.06	101.89	27.46	100.51	76.09	84.84
12	101.18	82.14	20.45	104.91	27.84	103.48	78.42	86.95
13	104.49	83.84	20.65	107.48	28.43	106.00	80.56	88.70
14	107.21	85.18	20.84	109.81	28.62	108.15	82.51	90.47
15	110.13	86.44	21.04	111.91	29.01	110.12	84.26	91.90
16	112.07	87.48	21.04	113.50	29.40	111.95	85.62	93.20
17	114.02	88.59	21.23	115.01	29.60	113.65	87.17	94.37
18	115.96	89.43	21.23	116.41	29.79	115.02	88.15	95.28
19	117.52	90.32	21.62	120.94	30.18	119.66	89.31	95.96
20	121.22	92.16	21.81	119.62	30.76	118.08	91.84	98.15

	$\psi_j = 0$		$\psi_j = 1.25$		$\psi_j = 1.25$		$\psi_j = 0$	
t	A_j	C_j	A_j	C_j	A_j	C_j	A_j	C_j
0	4.00	29.03	4.00	29.34	4.00	29.15	4.00	29.40
1	28.43	32.86	12.09	36.95	23.95	34.19	14.81	36.10
2	35.43	38.42	13.26	44.08	29.21	39.90	16.56	43.11
3	42.63	43.50	14.42	50.98	34.65	45.62	18.51	49.90
4	49.83	48.60	15.59	57.67	40.10	51.04	20.06	56.47
5	56.63	53.09	16.37	64.12	45.35	56.02	21.62	62.81
6	63.25	57.76	17.15	70.28	50.22	60.79	22.79	68.89
7	69.86	61.73	17.92	76.11	54.88	65.14	23.95	74.48
8	75.89	65.56	18.51	81.27	59.16	69.28	24.93	79.65
9	81.34	68.83	18.90	86.22	63.25	72.83	25.70	84.61
10	86.40	71.81	19.29	90.70	66.94	76.25	26.48	89.11
11	91.07	74.54	19.67	94.58	70.25	79.16	27.07	93.03
12	95.35	77.05	20.06	98.33	73.17	81.81	27.65	96.80
13	98.85	79.34	20.26	101.54	75.89	84.06	28.23	100.02
14	102.35	81.35	20.45	104.30	78.03	86.17	28.62	102.98
15	105.65	82.97	20.65	107.03	80.17	88.24	29.01	105.68
16	108.57	84.58	20.84	109.31	82.31	89.93	29.21	107.77
17	110.13	85.84	21.04	111.43	83.87	91.50	29.60	109.93
18	113.05	87.27	21.04	113.36	85.42	92.73	29.79	111.74
19	115.19	87.96	21.43	124.74	86.79	93.85	30.18	116.89
20	119.66	90.76	21.81	114.24	90.29	96.75	30.96	116.31

These transition paths are derived after averaging results of ten experiments which are run with different initial random starting points. In a typical run we use a population size of 50, crossover rate of 0.60 and mutation rate of 0.01.

Table II. Total Discounted Welfare and Defense Burden

		Country i			Country j		
ψ_i	ψ_j	V_i	$pN_i(0)/Y_i(0)$	pN_i^* / Y_i^*	Vj	$pNj(0) / Yj(0)$	pN_i^*/Y_i^*
0	0	4.9839	0.0777	0.2082	4.5970	0.0796	0.2087
1.25	1.25	5.0814	0.0335	0.0352	4.6835	0.0395	0.0368
1.25	0	5.0651	0.0380	0.0495	4.6012	0.0686	0.1575
0	1.25	4.9873	0.0691	0.1578	4.6636	0.0462	0.0505

Steady-state values are denoted by an asterisk. V_i and V_j denote respective discounted welfare.

We introduce an adaptive learning algorithm to study the dynamics of such complicated models under plausible parameters. Our analysis concentrates on cases in which, during a hostile peace period, rational governments that recognize the importance of bilateral trade in the construction of mutual concord would surpass their rivals. We show that divergence in initial capital stocks and attitudes of governments to hostility makes substantial differences in the steady-state growth figures. Countries with especially poor resources will benefit more if they are ruled by governments that are 'rational' in the sense that trade is viewed as a diplomatic tool.

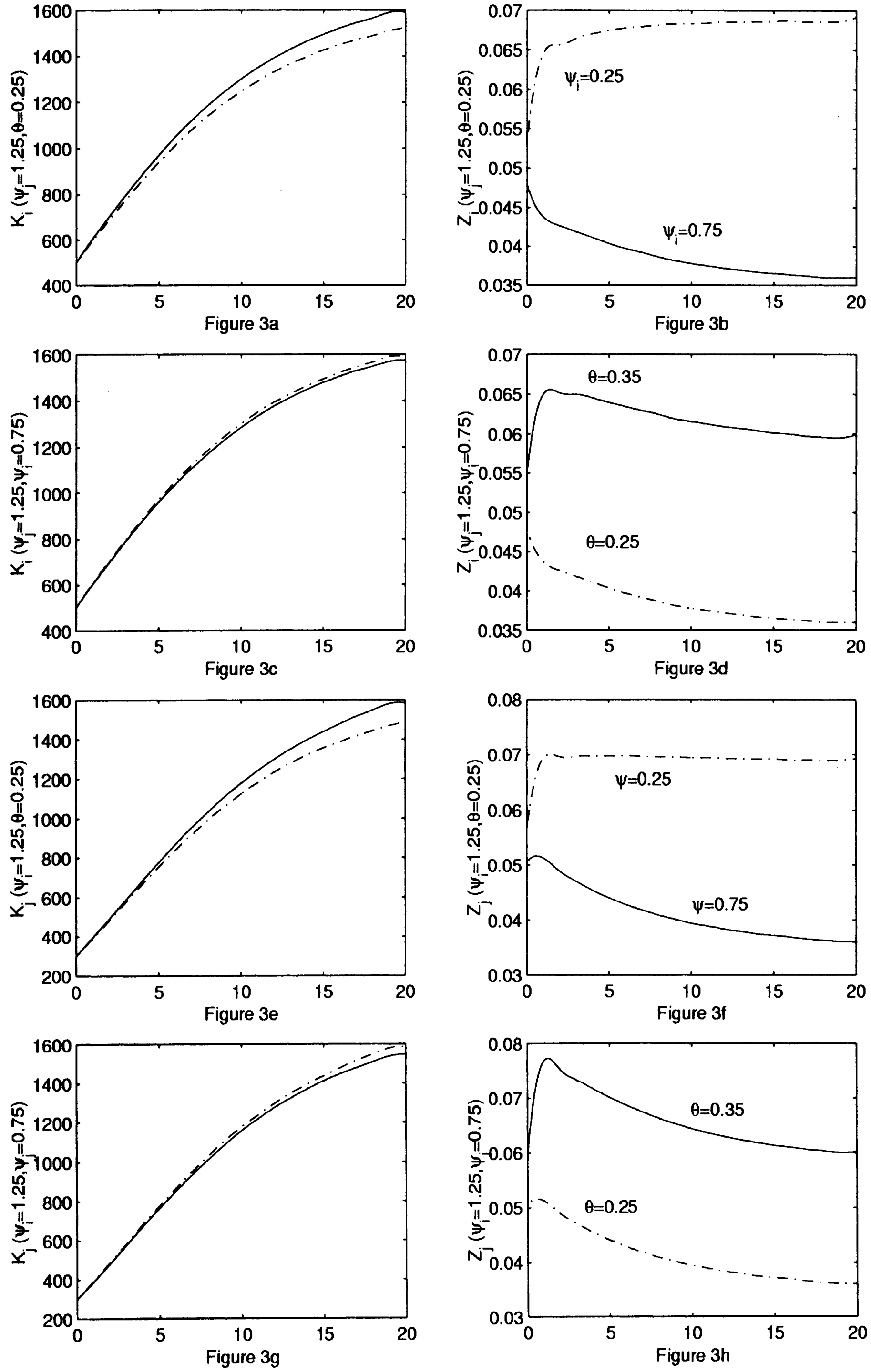
The case study of Turkey and Russia explains the importance of trade as a diplomatic tool in constructing peace in regional conflicts. In the existing literature on trade and conflict, various studies (see Hirschman, 1980; Gasiorowski & Polachek, 1982; Liberman, 1996) illustrate that the manipulation of issues such as trade to gain cooperation from other conflicting players are mostly initiated by the policymakers of developed and democratic countries. In this sense, our example is unique since it illustrates the diplomatic property of interdependence between two 'emerging' economies that have had a long state tradition as well as historical enmity.

The model and the numerical results are useful in understanding the importance of policymakers' rationality in the construction of peace. Conflicting country pairings like Eritrea–Ethiopia, Greece–Turkey, India–Pakistan, Iran–Iraq, and North–South Korea are mostly subject to severe economic constraints but are still spending substantial amounts on arms imports (Levine & Smith, 1997). Our model suggests to the policymakers in those conflicting countries that international conflict can be eased considerably by engaging a hostile nation in trade.

Obviously, there are many ways in which the model can be extended. Only a few will be suggested below. The natural extension is to solve the model for closed-loop or feedback Nash equilibrium; this might be of interest for its value-added realism. This type of equilibrium allows each country to condition its strategies on the current and past states. Thus, the feedback model employs a more realistic information pattern. This would, however, require an altogether new genetic game algorithm or the use of another approximation algorithm by simplifying the original structure. Otherwise, the model in this article is already complex enough to find feedback equilibrium of the game with the existing solution techniques in the literature.

The model can be extended to allow stochastic shocks. For example, suppose hostility varies due to political shocks. Changing governments may affect the existing level of hostility, and these changes can be

Figure 3. Sensitivity Analysis



periodically captured by stochastic shocks. The price of imported arms and preferences can also be subject to shocks.

One empirical study implication of the model is that growth rate of the economy reacts in an asymmetric way to hostility. The relation can be testable among historically hostile nations. However, definition of growth in real life is more complex than in modeling. Although these extensions are beyond the scope of this article, they illustrate the broad range of questions that could be addressed based on variations of the model developed here.

Appendix

Table IIIa. Optimal Trajectories for Country i (ψ_j = 1.25)

	$\psi_i = 0, \theta = 0.25$			$\psi_i = 0.25, \theta = 0.25$			$\psi_i = 0.75, \theta = 0.25$		
t	K_i	A_i	Z_i	K_i	A_i	Z_i	K_i	A_i	Z_i
0	500.00	5.00	0.0566	500.00	5.00	0.0535	500.00	5.00	0.0479
1	595.31	34.26	0.0785	598.24	29.01	0.0646	607.04	20.26	0.0437
2	684.75	40.10	0.0812	689.15	33.29	0.0656	703.81	22.59	0.0426
3	771.26	45.35	0.0834	777.13	37.38	0.0664	797.65	24.73	0.0419
4	853.37	50.60	0.0853	860.70	41.27	0.0670	887.10	26.87	0.0411
5	931.09	55.27	0.0866	939.88	44.77	0.0674	970.67	28.43	0.0403
6	1002.93	59.75	0.0879	1013.20	47.88	0.0677	1049.85	29.98	0.0397
7	1068.91	63.83	0.0888	1080.65	50.80	0.0679	1121.70	31.54	0.0392
8	1129.03	67.53	0.0897	1142.23	53.52	0.0682	1187.68	32.71	0.0386
9	1183.28	70.83	0.0906	1197.95	56.05	0.0683	1247.80	33.68	0.0382
10	1233.14	73.75	0.0912	1247.80	58.00	0.0683	1302.05	34.65	0.0378
11	1277.13	76.67	0.0917	1293.26	59.94	0.0684	1350.44	35.63	0.0375
12	1316.72	79.00	0.0922	1332.84	61.69	0.0685	1392.96	36.40	0.0372
13	1351.91	81.14	0.0925	1368.04	63.25	0.0685	1431.09	36.99	0.0370
14	1382.70	83.09	0.0930	1398.83	64.42	0.0686	1464.81	37.57	0.0367
15	1410.56	84.84	0.0933	1426.69	65.78	0.0686	1494.13	38.15	0.0365
16	1434.02	86.59	0.0936	1450.15	66.94	0.0687	1521.99	38.54	0.0364
17	1454.55	87.76	0.0939	1470.67	67.33	0.0686	1545.45	38.93	0.0362
18	1472.14	88.73	0.0939	1488.27	68.11	0.0686	1565.98	39.13	0.0360
19	1491.20	89.51	0.0939	1508.80	68.70	0.0686	1589.44	39.52	0.0360
20	1505.87	92.43	0.0949	1521.99	70.83	0.0692	1589.44	40.49	0.0360

	$\psi_i = 0, \theta = 0.35$			$\psi_i = 0.25, \theta = 0.35$			$\psi_i = 0.75, \theta = 0.35$		
t	K_i	A_i	Z_i	K_i	A_i	Z_i	K_i	A_i	Z_i
0	500.00	5.00	0.0650	500.00	5.00	0.0615	500.00	5.00	0.0550
1	587.98	48.08	0.1187	592.38	41.07	0.0973	602.64	29.01	0.0650
2	675.95	57.41	0.1269	681.82	48.27	0.1018	697.95	33.10	0.0651
3	762.46	65.78	0.1324	768.33	54.88	0.1049	790.32	36.79	0.0649
4	844.57	74.14	0.1373	850.44	61.11	0.1072	878.30	40.10	0.0645
5	922.29	81.73	0.1410	928.15	66.75	0.1089	960.41	43.02	0.0639
6	994.13	88.93	0.1446	1000.00	72.20	0.1105	1036.66	45.74	0.0634
7	1060.12	95.73	0.1476	1065.98	77.06	0.1116	1107.04	48.08	0.0629
8	1120.23	101.76	0.1499	1126.10	81.34	0.1125	1171.55	50.22	0.0625
9	1174.49	107.21	0.1520	1180.35	85.42	0.1134	1231.67	51.97	0.0619
10	1222.87	112.27	0.1538	1228.74	88.73	0.1140	1285.92	53.72	0.0616
11	1265.40	116.74	0.1554	1272.73	92.04	0.1148	1334.31	55.27	0.0612
12	1303.52	120.63	0.1567	1313.78	94.76	0.1152	1376.83	56.63	0.0609
13	1335.78	124.13	0.1580	1348.97	97.48	0.1156	1414.96	57.80	0.0607
14	1365.10	127.05	0.1590	1379.77	99.82	0.1159	1448.68	58.77	0.0604
15	1391.50	129.39	0.1596	1406.16	101.76	0.1165	1479.47	59.55	0.0601
16	1413.49	131.92	0.1603	1428.15	103.52	0.1165	1505.87	60.52	0.0600
17	1431.09	134.44	0.1609	1448.68	104.88	0.1165	1529.33	61.11	0.0597
18	1445.75	135.42	0.1615	1467.74	106.04	0.1168	1548.39	61.50	0.0596
19	1467.74	136.00	0.1615	1485.34	106.82	0.1168	1568.91	62.28	0.0596
20	1485.34	142.03	0.1647	1501.47	110.52	0.1183	1574.78	64.03	0.0600

In the comparative statistics with respect to underlying parameters, we kept one country's ψ constant and analyzed the effects of the changes on the other country's trajectories.

Table IIIb. Optimal Trajectories for Country j ($\psi_j = 1.25$)

	$\psi_i = 0, \theta = 0.25$			$\psi_i = 0.25, \theta = 0.25$			$\psi_i = 0.75, \theta = 0.25$		
t	K_j	A_j	Z_j	K_j	A_j	Z_j	K_j	A_j	Z_j
0	300.00	4.00	0.0453	300.00	4.00	0.0453	300.00	4.00	0.0453
1	394.72	14.81	0.0478	396.38	14.23	0.0457	396.38	13.06	0.0417
2	492.77	16.95	0.0453	494.43	16.17	0.0427	494.43	14.62	0.0383
3	594.13	18.90	0.0425	595.80	17.92	0.0400	595.80	16.17	0.0356
4	695.50	20.65	0.0401	697.17	19.48	0.0377	697.17	17.53	0.0334
5	793.55	22.01	0.0380	795.21	20.84	0.0357	795.21	18.51	0.0314
6	889.93	23.37	0.0363	889.93	22.01	0.0340	889.93	19.48	0.0298
7	981.33	24.34	0.0348	979.67	22.98	0.0326	979.67	20.45	0.0285
8	1067.74	25.32	0.0336	1064.42	23.95	0.0314	1064.42	21.04	0.0274
9	1147.51	26.29	0.0325	1144.18	24.73	0.0304	1142.52	21.62	0.0264
10	1222.29	27.07	0.0315	1217.30	25.32	0.0295	1213.98	22.20	0.0256
11	1288.76	27.65	0.0307	1283.77	25.90	0.0287	1278.79	22.79	0.0249
12	1351.91	28.23	0.0301	1346.92	26.48	0.0281	1338.61	23.18	0.0243
13	1408.41	28.62	0.0295	1401.76	26.87	0.0275	1391.79	23.57	0.0238
14	1459.92	29.21	0.0290	1449.95	27.26	0.0270	1439.98	23.76	0.0234
15	1504.79	29.60	0.0285	1493.16	27.65	0.0266	1481.52	24.15	0.0230
16	1548.00	29.98	0.0282	1531.38	28.04	0.0263	1519.75	24.34	0.0227
17	1584.56	30.37	0.0278	1566.28	28.04	0.0259	1554.64	24.54	0.0224
18	1617.79	30.37	0.0275	1596.19	28.23	0.0257	1584.56	24.54	0.0222
19	1649.36	30.37	0.0272	1627.76	28.43	0.0254	1614.47	24.93	0.0220
20	1644.38	31.74	0.0271	1626.10	29.60	0.0253	1612.81	25.51	0.0219

	$\psi_i = 0, \theta = 0.35$			$\psi_i = 0.25, \theta = 0.35$			$\psi_i = 0.75, \theta = 0.35$		
t	K_j	A_j	Z_j	K_j	A_j	Z_j	K_j	A_j	Z_j
0	300.00	4.00	0.0532	300.00	4.00	0.0532	300.00	4.00	0.0532
1	389.74	22.40	0.0771	391.40	21.43	0.0728	393.06	19.29	0.0641
2	484.46	27.07	0.0767	487.78	25.32	0.0710	491.10	22.20	0.0608
3	582.50	30.57	0.0734	587.49	28.62	0.0676	590.81	24.93	0.0573
4	682.21	33.87	0.0705	687.19	31.54	0.0646	690.52	27.26	0.0543
5	780.25	36.60	0.0677	785.24	34.07	0.0618	788.56	29.21	0.0517
6	873.31	39.32	0.0652	879.96	36.40	0.0594	883.28	30.96	0.0494
7	963.05	41.66	0.0632	969.70	38.35	0.0574	973.02	32.51	0.0474
8	1047.80	43.60	0.0613	1054.45	40.10	0.0555	1057.77	33.87	0.0458
9	1125.90	45.35	0.0597	1132.55	41.66	0.0540	1139.20	34.85	0.0443
10	1200.68	46.91	0.0583	1204.01	43.02	0.0526	1213.98	36.01	0.0430
11	1267.16	48.27	0.0571	1268.82	44.38	0.0515	1283.77	36.99	0.0420
12	1330.30	49.44	0.0560	1326.98	45.35	0.0505	1343.60	37.77	0.0410
13	1385.14	50.60	0.0550	1380.16	46.32	0.0498	1396.77	38.54	0.0402
14	1436.66	51.58	0.0542	1426.69	47.10	0.0491	1444.97	39.13	0.0396
15	1481.52	52.16	0.0534	1468.23	48.08	0.0485	1488.17	39.52	0.0390
16	1523.07	52.74	0.0528	1506.45	48.46	0.0479	1528.05	40.10	0.0386
17	1562.95	53.33	0.0523	1539.69	48.66	0.0474	1562.95	40.29	0.0381
18	1602.83	53.91	0.0516	1572.92	49.24	0.0470	1594.53	40.68	0.0377
19	1634.41	53.91	0.0510	1609.48	49.44	0.0466	1627.76	41.07	0.0374
20	1632.75	57.02	0.0511	1607.82	51.58	0.0465	1624.44	42.43	0.0373

Table IVa. Optimal Trajectories for Country i ($\psi_i = 1.25$)

	$\psi_j = 0, \theta = 0.25$			$\psi_j = 0.25, \theta = 0.25$			$\psi_j = 0.75, \theta = 0.25$		
t	K_i	A_i	Z_i	K_i	A_i	Z_i	K_i	A_i	Z_i
0	500.00	5.00	0.0428	500.00	5.00	0.0428	500.00	5.00	0.0428
1	611.44	16.56	0.0355	611.44	15.98	0.0343	611.44	15.01	0.0319
2	712.61	18.31	0.0345	712.61	17.73	0.0330	712.61	16.37	0.0302
3	810.85	20.06	0.0334	812.32	19.09	0.0319	810.85	17.53	0.0290
4	906.16	21.43	0.0325	907.62	20.45	0.0309	906.16	18.51	0.0278
5	995.60	22.79	0.0317	997.07	21.62	0.0300	995.60	19.48	0.0268
6	1079.18	23.95	0.0310	1080.65	22.79	0.0292	1079.18	20.26	0.0260
7	1156.89	24.93	0.0303	1158.36	23.57	0.0285	1156.89	21.04	0.0252
8	1230.21	25.70	0.0297	1230.21	24.34	0.0279	1228.74	21.62	0.0246
9	1296.19	26.68	0.0292	1294.72	25.12	0.0274	1293.26	22.20	0.0241
10	1354.84	27.26	0.0288	1353.37	25.70	0.0270	1351.91	22.59	0.0236
11	1407.62	27.84	0.0284	1406.16	26.29	0.0266	1404.69	23.18	0.0232
12	1454.55	28.43	0.0281	1453.08	26.68	0.0263	1451.61	23.37	0.0228
13	1497.07	29.01	0.0278	1495.60	27.07	0.0260	1494.13	23.76	0.0225
14	1535.19	29.40	0.0276	1533.72	27.46	0.0257	1532.26	23.95	0.0223
15	1567.45	29.60	0.0274	1567.45	27.84	0.0255	1565.98	24.34	0.0221
16	1596.77	29.98	0.0272	1598.24	27.84	0.0253	1595.31	24.54	0.0219
17	1626.10	30.18	0.0270	1624.63	28.23	0.0251	1621.70	24.54	0.0217
18	1651.03	30.37	0.0268	1651.03	28.23	0.0250	1648.09	24.73	0.0216
19	1674.49	30.37	0.0267	1674.49	28.62	0.0248	1671.55	24.93	0.0215
20	1659.82	31.54	0.0267	1665.69	29.40	0.0248	1662.76	25.51	0.0214

	$\psi_j = 0, \theta = 0.35$			$\psi_j = 0.25, \theta = 0.35$			$\psi_j = 0.75, \theta = 0.35$		
t	K_i	A_i	Z_i	K_i	A_i	Z_i	K_i	A_i	Z_i
0	500.00	5.00	0.0492	500.00	5.00	0.0492	500.00	5.00	0.0492
1	605.57	24.54	0.0552	607.04	23.76	0.0529	609.97	21.81	0.0479
2	705.28	28.62	0.0565	706.74	27.26	0.0532	711.14	24.54	0.0469
3	803.52	31.93	0.0561	804.99	30.18	0.0524	810.85	26.68	0.0456
4	897.36	34.85	0.0556	898.83	32.71	0.0516	906.16	28.82	0.0444
5	985.34	37.57	0.0550	988.27	35.04	0.0509	995.60	30.57	0.0433
6	1067.45	39.91	0.0544	1071.85	37.18	0.0501	1079.18	32.12	0.0423
7	1143.70	42.04	0.0539	1149.56	38.93	0.0494	1156.89	33.49	0.0414
8	1214.08	43.99	0.0534	1221.41	40.68	0.0487	1228.74	34.65	0.0406
9	1278.59	45.55	0.0530	1285.92	42.04	0.0481	1293.26	35.63	0.0399
10	1338.71	47.10	0.0526	1344.57	43.41	0.0476	1353.37	36.60	0.0394
11	1391.50	48.46	0.0522	1397.36	44.38	0.0472	1407.62	37.38	0.0388
12	1439.88	49.83	0.0519	1444.28	45.55	0.0468	1456.01	38.15	0.0384
13	1480.94	50.80	0.0516	1483.87	46.52	0.0465	1498.53	38.74	0.0380
14	1519.06	51.77	0.0514	1521.99	47.30	0.0463	1536.66	39.32	0.0377
15	1551.32	52.55	0.0511	1555.72	47.88	0.0460	1570.38	39.71	0.0373
16	1580.65	53.33	0.0510	1585.04	48.46	0.0458	1601.17	40.29	0.0371
17	1607.04	53.72	0.0508	1612.90	48.85	0.0457	1630.50	40.49	0.0368
18	1630.50	54.11	0.0506	1637.83	49.44	0.0454	1655.43	40.88	0.0366
19	1658.36	54.11	0.0502	1662.76	49.63	0.0452	1680.35	41.07	0.0364
20	1652.49	56.63	0.0506	1653.96	51.58	0.0455	1670.09	42.43	0.0365

Table IVb. Optimal Trajectories for Country j ($\psi_i = 1.25$)

	$\psi_j = 0, \theta = 0.25$			$\psi_j = 0.25, \theta = 0.25$			$\psi_j = 0.75, \theta = 0.25$		
t	K_j	A_j	Z_j	K_j	A_j	Z_j	K_j	A_j	Z_j
0	300.00	4.00	0.0598	300.00	4.00	0.0566	300.00	4.00	0.0506
1	388.07	23.95	0.0807	389.74	21.04	0.0693	394.72	15.98	0.0513
2	477.81	29.40	0.0827	481.13	25.32	0.0696	489.44	18.51	0.0489
3	569.21	34.85	0.0847	574.19	29.60	0.0697	585.83	21.04	0.0471
4	658.94	40.29	0.0861	665.59	33.87	0.0698	682.21	23.18	0.0455
5	745.36	45.55	0.0874	753.67	37.77	0.0698	775.27	25.32	0.0441
6	828.45	50.60	0.0885	838.42	41.46	0.0698	866.67	27.07	0.0429
7	906.55	55.27	0.0894	918.18	44.77	0.0697	953.08	28.82	0.0419
8	979.67	59.55	0.0901	992.96	48.08	0.0696	1032.84	30.37	0.0409
9	1046.14	63.64	0.0909	1061.09	50.99	0.0696	1109.29	31.74	0.0402
10	1107.62	67.33	0.0914	1124.24	53.52	0.0695	1177.42	32.90	0.0394
11	1164.13	70.64	0.0919	1180.74	55.86	0.0694	1238.91	34.07	0.0389
12	1215.64	73.75	0.0924	1232.26	58.00	0.0693	1297.07	34.85	0.0383
13	1260.51	76.48	0.0928	1278.79	59.94	0.0692	1350.24	35.63	0.0379
14	1300.39	78.81	0.0931	1320.33	61.50	0.0692	1396.77	36.40	0.0375
15	1335.29	80.95	0.0933	1356.89	63.05	0.0692	1438.32	37.18	0.0372
16	1365.20	82.90	0.0936	1388.47	64.42	0.0690	1478.20	37.77	0.0369
17	1390.13	84.45	0.0938	1416.72	65.58	0.0690	1516.42	38.15	0.0366
18	1415.05	85.81	0.0939	1443.30	66.36	0.0689	1548.00	38.54	0.0363
19	1439.98	86.59	0.0939	1468.23	67.33	0.0690	1579.57	39.32	0.0361
20	1464.91	90.09	0.0948	1486.51	69.86	0.0693	1584.56	40.29	0.0361

	$\psi_j = 0, \theta = 0.35$			$\psi_j = 0.25, \theta = 0.35$			$\psi_j = 0.75, \theta = 0.35$		
t	K_j	A_j	Z_j	K_j	A_j	Z_j	K_j	A_j	Z_j
0	300.00	4.00	0.0703	300.00	4.00	0.0664	300.00	4.00	0.0594
1	384.75	33.49	0.1226	386.41	29.79	0.1052	389.74	22.79	0.0768
2	474.49	42.24	0.1294	476.15	36.79	0.1082	482.80	27.07	0.0752
3	565.88	50.80	0.1344	567.55	43.41	0.1102	577.52	30.96	0.0733
4	655.62	59.36	0.1386	657.28	50.02	0.1116	672.24	34.65	0.0718
5	742.03	67.72	0.1423	743.70	56.44	0.1127	763.64	38.15	0.0702
6	826.78	75.31	0.1454	826.78	62.28	0.1137	851.71	41.07	0.0687
7	906.55	82.90	0.1480	904.89	67.72	0.1143	936.46	43.80	0.0675
8	981.33	90.09	0.1504	978.01	72.78	0.1150	1014.57	46.32	0.0663
9	1051.12	96.51	0.1522	1046.14	77.45	0.1154	1089.35	48.66	0.0653
10	1115.93	102.74	0.1540	1109.29	81.73	0.1159	1157.48	50.80	0.0644
11	1174.10	108.38	0.1556	1167.45	85.62	0.1160	1218.96	52.55	0.0636
12	1223.95	113.44	0.1571	1220.63	89.12	0.1164	1275.46	54.30	0.0630
13	1272.14	118.10	0.1582	1270.48	92.43	0.1167	1325.32	55.66	0.0623
14	1315.35	122.00	0.1592	1312.02	95.54	0.1169	1371.85	57.02	0.0619
15	1351.91	125.50	0.1601	1348.58	97.87	0.1169	1413.39	58.00	0.0613
16	1383.48	128.80	0.1609	1383.48	100.21	0.1171	1448.29	59.16	0.0610
17	1411.73	131.53	0.1613	1411.73	102.35	0.1171	1479.86	59.94	0.0606
18	1438.32	134.06	0.1617	1438.32	103.90	0.1173	1509.78	60.52	0.0604
19	1461.58	134.06	0.1617	1463.25	105.46	0.1172	1538.03	61.30	0.0601
20	1479.86	141.45	0.1643	1481.52	109.93	0.1185	1548.00	63.44	0.0604

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