RETAIL LOCATION COMPETITION UNDER CARBON PENALTY

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By
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March 2016

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

RETAIL LOCATION COMPETITION UNDER CARBON PENALTY

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This thesis examines the retail location problem on a Hotelling line in two different settings: a decentralized system in which two competing retailers simultaneously choose the locations of their own stores, and a centralized system in which a single retail chain chooses the locations of its two stores. In both settings, the stores procure their products from a common warehouse and each consumer purchases from the closest store. The retailers in the decentralized system want to maximize their individual profits determined by the sales revenue minus the transportation costs for replenishment and consumer travels. The retail chain in the centralized system wants to maximize the sum of the two individual profits. Transportation costs depend on not only fuel consumption but also carbon emission. In the decentralized system, we establish that both retailers choose the same location in equilibrium in high margin markets. Numerical experiments provide further insights into the location problem: The retail chain chooses different locations for its stores at optimality in all instances. However, under low transportation costs, the retailers in the decentralized system choose the same location in equilibrium. As the consumer transportation costs increase, the stores are located further away from each other towards their respective consumer segments, converging to the centralized solution. Carbon penalty is more effective for consumer travels than for replenishment in reducing excess emissions due to competition.

Keywords: Retail location, simultaneous game, transportation, carbon emissions, carbon penalty.

ÖZET

KARBON CEZASI ALTINDA REKABETÇİ KONUMLANDIRMA

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Bu tezde, Hotelling doğrusunda bulunan iki perakendeci mağazası için konumlandırma problemi iki farklı senaryo altında çalışılmıştır: rekabetçi sistemde iki mağaza aynı anda mağazaları için konum belirleyecektirler. Merkezi sistemde ise tek bir perakendeci iki mağazası için konum belirleyecektir. İki düzende de mağazalar ürünlerini aynı ambardan satın almakta ve müşteriler en yakın mağazaya gitmektedirler. Rekabetçi sistemdeki mağazalar satış geliri ile müşteri ulaşım ve ambar ikmal maliyetlerinin farkı olan bireysel kârlılıkları artırmak istemektedirler. Merkezi sistemdeki perakendeci ise her iki mağazanın toplam kârlılığını artırmak istemektedir. Ulaşım ve ikmal maliyetleri yakıt tüketiminin yanı sıra karbon emisyonlarına da bağlıdır. Rekabetçi sistemde, yüksek kâr marjı olan marketlerde dengede iki mağaza da aynı noktaya konumlanmaktadır. Sayısal çalışmalar denge noktalarını ve davranışlarını daha iyi gözlemleyebilmemizi sağlamıştır: Merkezi sistemde mağazalar tüm örneklerde eniyilik durumunda farklı noktalara konumlanmaktadırlar; fakat düşük ulaşım ve ikmal maliyetleri altında rekabetçi sistemdeki mağazalar dengede aynı noktaya konumlanmaktadırlar. Müşteri ulaşımı maliyetleri arttığında mağazalar birbirlerinden uzaklaşmakta ve kendi müşteri segmentlerini ortalayacak şekilde konumlanmaktadırlar ve çözüm merkezi sisteme yaklaşmaktadır. İkmal maliyetlerindense müşteri ulaşım maliyetlerini kapsayacak bir vergi politikası üzerinde çalışılması rekabet nedeniyle oluşan fazla emisyonun azaltılmasında daha etkili olacaktır.

Anahtar sözcükler: Konumlandırma, eşzamanlı oyun, ulaşım, karbon emisyonları, karbon cezası.

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Chapter 1

Introduction

Increasing concentrations of greenhouse gases contribute to the change in global climate patterns and the global warming, which can be described as a slow but steady rise in the Earth's surface temperature. Carbon dioxide, methane, ozone, chlorofluorocarbons, nitrous oxide, and water vapor are the main greenhouse gases existing in the atmosphere. Anthropogenic activities such as energy consumption, burning fossil fuels, oil, coal, and natural gas, deforestation, and transportation increase the amount of greenhouse gases (Intergovernmental Panel on Climate Change, IPCC, 2014; and Environmental Protection Agency, EPA, 2015).

Solar radiation passes through the clear atmosphere. Some part of the solar radiation is reflected by the Earth's atmosphere, whereas some other part of the solar radiation is absorbed by the greenhouse gases. This absorption increases the Earth's surface temperature. Without this effect, the Earth's surface temperature would be much colder and less hospitable for life. The more the carbon dioxide levels increase, the more the solar radiation is absorbed, leading to global warming. This process is called the greenhouse effect. Greenhouse gases increased the average global temperature by 0.8^oC over the last 100 years, with 0.6^oC of that increase occurring in the last three decades. Further increases of $2-4.5^oC$ are likely to be observed by the end of the 21st century (Campbell et al. 2009). Human influence on greenhouse gases is the highest in history (IPCC 2014). Since the Industrial Revolution, the atmospheric concentration of carbon dioxide has increased by about 40%, mostly due to the combustion of carbon based fossil fuels, such as coal, oil, and gasoline.

There is a growing conscience about global warming and emission reduction in individual consumers, governments, and the industry. Governments impose carbon taxes, put stringent limits on emissions, and use subsidies to reduce emissions. Companies report their carbon footprint and endeavor to reduce their emissions, in order to meet environmental regulations, benefit from subsidies, and attract green-sensitive customers. "In the United States and Europe, emission markets have been in place for a number of years, for sulphur dioxide in the US and greenhouse gases in Europe" (Field et al. 2011). These markets limit carbon emissions and/or allow trades among companies. Many countries, including Ireland, Australia, Chile, Sweden, Finland, Great Britain, and Canada impose carbon taxes. In British Columbia, for instance, "a carbon tax is usually defined as a tax based on greenhouse gas emissions generated from burning fuels. By reducing fuel consumption, increasing fuel efficiency, using cleaner fuels and adopting new technology, businesses and individuals can reduce the amount they pay in carbon tax, or even offset it altogether" (British Columbia Ministry of Finance 2016). Also, customers prefer environmentally friendly products and services. A survey in 2007 revealed that more than half of the global consumers choose to purchase products and services from a company with a strong environmental reputation (Nastu 2007).

Transportation and energy usage account for a very high percentage of greenhouse gases (EPA 2015). Distances between a retailer and its suppliers greatly influence the total amount of carbon emissions in the transportation domain of a supply chain. In addition to the transportation emissions in the supply chain, retail location influences the patronage to that store and thus the carbon emissions generated by consumers. Consequently, the retail store location is one of the key drivers of environmental performance of the supply chain. The store location is also a critical factor for a retailer to be successful (Anderson et al. 1997). In this thesis, we study the retail location problem under carbon penalty for warehouse transportation and consumer travels to stores. We investigate the impacts of competition on store locations, costs, and emissions. We also analyze the impacts of imposing a carbon tax to retailers on the environmental performance of the supply chain in a competitive environment.

Specifically, this thesis examines the retail location problem on a Hotelling line in two different settings: a decentralized system in which two competing retailers simultaneously choose the locations of their own stores, and a centralized system in which a single retail chain chooses the locations of two of its own stores. In both settings, the stores procure identical products from a common warehouse on the unit line in a full truck-load fashion, consumers are distributed uniformly on the unit line, each consumer travels to the closest store to purchase the product, and both retail stores sell the identical product at the same price.

The retailers in the decentralized system want to maximize their individual profits determined by the difference between the sales revenue and the sum of the transportation costs for replenishment and consumer travels. The retail chain in the centralized system wants to maximize the sum of the two individual profits. The transportation costs vary depending on not only fuel consumption but also carbon emission.

In the decentralized system we characterize the best response of each retailer to the other retailer's location choice (Propositions 4.1–4.2). We prove that both retailers choose the same location in equilibrium when the product price is sufficiently large (Propositions 4.3–4.5). In the centralized system we develop an exact solution algorithm for the optimization problem of the retail chain (Propositions 5.1–5.3). This algorithm also minimizes the total transportation cost in the system (Proposition 5.4).

We then conduct numerical experiments to gain further insights into the location problem: The retail chain chooses different locations for her stores at optimality in all numerical instances. However, when the transportation costs are low, the retailers in the decentralized system choose the same location in equilibrium. As the transportation costs for consumer travels increase, the retailers locate their stores further away from each other towards their respective consumer segments, and the centralized solution converges to the decentralized solution. As the transportation costs for replenishment increase, the retailers locate their stores closer to the warehouse.

The total carbon emission from consumer travels are always higher in the decentralized system than in the centralized system. But the total carbon emission from replenishment is lower under competition in many instances, including all the cases in which consumer travels are too costly. In low margin markets, increasing the consumer transportation costs reduces the "competition carbon penalty" more significantly than increasing the transportation costs for replenishment. Thus imposing a carbon tax for consumer travels proves more effective than that for replenishment in reducing excess emissions due to competition. In addition, when the consumer transportation costs are high, as the warehouse approaches the end-point of the unit line, the carbon penalty tends to decrease. Conversely, when the consumer transportation costs are low, as the warehouse approaches the mid-point of the unit line, the carbon penalty tends to decrease.

We contribute to the literature in several important ways: First, to our knowledge, we are the first to study the location problem by taking into account both supplier- and consumer-related transportation costs under competition. Second, we prove that symmetric equilibria arise in high margin markets. Third, we numerically analyze the behavior of the equilibrium locations with respect to price, transportation costs, and warehouse location, observing asymmetric equilibrium in many instances of low margin markets. Last, our numerical results may have substantial implications for policymakers. Specifically, in low margin markets, imposing carbon tax to a retailer for her consumers' travels has the potential to greatly reduce excess emissions due to competition. Also, incentivizing suppliers to locate their warehouses close to (or away from) the market might be useful in reducing excess emissions under low (or high) consumer transportation costs.

The remainder of the thesis is organized as follows: In Chapter 2, we review the literature dealing with the location problem. In Chapter 3, we define our location problem and formulate the retailers' total profits as functions of the store and warehouse locations. In Chapter 4, we characterize the best responses of the retailers in the decentralized system and establish the equilibrium locations under certain conditions. In Chapter 5, we develop an exact solution algorithm that computes the optimal locations of the two stores in the centralized system. In Chapter 6, we present and interpret our numerical results for the decentralized and centralized systems. In Chapter 7, we offer a summary and concluding remarks.

Chapter 2

Literature Review

We study the retail location problem in the presence of carbon emission and transportation costs. Retail stores sell an identical product at the same price and compete for demand. Consumers are uniformly distributed over the unit line. Consumers travel by car and purchase the product from the nearest store. The stores are supplied with trucks from a single warehouse on the unit line. We consider a decentralized system in which two retailers simultaneously determine their locations on the unit line (similar to Hotelling line) to maximize their individual profits. We also consider a centralized system in which the two stores belong to the same retail chain, and thus the stores are located by a single decision-maker on the unit line. Our work in these aspects is closely related with the location problems (in particular, the Hotelling location model) that include game theoretical settings.

Location problems have been extensively studied in the literature. The economists and geographers significantly contributed to this field. Later, researchers in many fields, such as Marketing, Management Science, Operations Research, and Computational Geometry have dealt with the location problem as well. In his cutting-edge paper, Hotelling (1929) studied a bounded linear city model under transportation costs and price competition. Hotelling's work stimulated the competitive location models and its extensions. For a detailed survey and taxonomy of the location models, see Eiselt et al. (1993) and Eiselt et al. (2004).

The competitive location problem has been first analyzed by Hotelling (1929), who introduced the location-price game where consumers are located on a unit line uniformly and two firms exist in the market. The firms compete for the demand and seek their optimal locations. Customers incur transportation costs and therefore purchase the identical product from the nearest store. Hotelling (1929) establishes the Nash equilibrium for the prices in his linear city and shows the firms want to get closer to each other, eventually ending up in the middle of the unit line. D'Aspremont et al. (1979) modify Hotelling's model and show that no pure price equilibrium exists and the "principle of minimum differentiation" is invalid. Moreover, D'Aspremont et al. (1979) introduce a transportation cost function that is quadratic in distance, in order to reestablish stability of the location game. Balvers and Szerb (1996) study the Hotelling problem under demand uncertainty, observing agglomeration. Puu (2002) studies the Hotelling problem under elastic demand. Brenner (2005) extends the Hotelling model to the case with multiple firms and quadratic transportation costs. Brenner (2005) finds that both firms tend to locate their stores at the center of the line to reach all consumers. Shuai (2014) studies the Hotelling mixed duopoly problem with non-uniform consumer distribution and derive transportation costs.

De Palma et al. (1987) examine the Hotelling's problem with three firms. The authors seek centrally agglomerated (symmetric equilibria in our work) or symmetrically dispersed equilibria (asymmetric equilibrium in our work). An equilibrium can be found depending on the variation in consumer tastes and transportation rate. Bester et al. (1991) analyze the Hotelling's problem by modifying the linear transportation costs to quadratic transportation costs, in the absence of coordination device. They characterize infinitely many equilibria randomized by the firms over locations. Hinloopen et al. (2013) study the Hotelling's problem, introducing the cost of location (such as rent) which increases towards the center of the market in case of linear transportation costs and decreases towards the center of the market in case of quadratic transportation costs.

A variation of the Hotelling's model is the Salop's (1979) circle model: Two nonidentical firms compete in a market in which consumers buy from the firm selling differentiated brands and want to maximize their utilities. Unlike Hotelling's linear city, consumers are located on a circle. This is an important simplification over the Hotelling's model since there is no corner on a circle. Salop (1979) obtains results similar to those in Hotelling (1929).

Several authors study the competitive location problem on a network. Dobson and Karmakar (1987) consider a finite number of customers located at nodes who choose the closest facility to minimize their transportation costs. There is a fixed cost for opening a facility and variable costs for operating the facility. The authors find finite stable sets under competition by formulating a binary integer program that maximizes the profit subject to stability. Hakimi (1983) studies a similar problem in which the numbers of sites to be opened by competitors are fixed a priori and there is no cost of opening a facility. De Palma et al. (1989) consider a network where firms compete over locations and consumers have random utility. The vertices of the graph are weighted by consumers' purchasing power. They then prove the existence of a unique location equilibrium. When the consumer tastes are sufficiently wide, the equilibrium is at the m-median of m facilities of the graph. They also observe that competing firms locate some of their stores on top of each other, hence showing the tendency towards agglomeration.

Labbe and Hakimi (1991) consider a setting in which two competing firms first select location on a network and then determine the quantities to supply to the markets that are located at the vertices of the network (i.e., the Cournot game). Firms incur production and transportation costs. Nash equilibrium exists in the second stage under the assumption that the unit transportation costs plus the unit production costs are never "too large." Melkote and Daskin (2001) study the facility location problem on networks, formulating a mixed integer program with binary and flow decision variables. Buechel and Roehl (2015) study a competitive location problem in a network with heterogeneous consumer perceptions based on distances (edge length). They obtain a strong indication for the principle of minimal clustering.

Godinho and Dias (2010) study the competitive location problem on a discrete location set with firms having different objectives, fixed costs of opening a facility, and budget constraints. If the facilities are located at the same site, they share the demand equally, as in our research. They formulate the problem as a linear program, providing an algorithm to compute the equilibrium solution. They find that worsening one manufacturer in terms of budget or choice of locations benefits the other manufacturer. Godinho and Dias (2013) study a similar problem in the case with overbidding and two decision makers having preferential rights over each other (co-location is not allowed) and level of asymmetry (decreasing location choice or increasing budget).

Küçükaydın et al. (2011) are the first to study the bi-level competitive facility location problem with a discrete set of candidate facility sites and continuous attractiveness of the leader. They model the entrant as a follower who reacts to the leader by adjusting her location and attractiveness levels. Taking the Huff's gravity-based approach, they assume that customers prefer closer and more attractive facilities. They then develop a bi-level mixed-integer nonlinear program, transforming it into one-level mixed-integer nonlinear program to use global optimization methods.

Aboolian et al. (2007) extend the competitive location problem by allowing for market expansion and cannibalization, using the Huff's gravity-based approach. Customer choice rules are probabilistic. They formulate the problem as a non-linear Knapsack problem and solve the problem under piecewise linear approximation schemes of the objective function. They also develop a heuristic algorithm to obtain a tight worst-case error bound of the model.

Dasci and Laporte (2005) identify the location strategies for two leader-follower type competing firms on a linear market who plan to open a number of stores. Consumers are distributed over the unit line according to a probability function. The firms are not always allowed to open their stores at any point. Thus, instead of exact locations, Dasci and Laporte (2005) find location densities. They also consider the follower's problem in a two dimensional market. They conclude that the leader has the first-mover advantage, the leader can make positive profits even if it is cost-disadvantaged, the consumer density and fixed cost play important role in entry rather than location strategies once both present in the market, and finally the consumer density and fixed cost have a small impact on the firms' strategies except for the leader.

Rhim et al. (2003) study the location, capacity, and quantity problems under competition. They also provide a taxonomy for the location problems. Production and logistics costs are heterogeneous. Firms first select their locations (either simultaneously or sequentially) and then determine their capacity and production quantity for each market. After modeling the capacity and quantity problem as a two-stage capacitated Cournot game, they formulate a three-stage game using the Nash equilibrium in each stage. In the sequential entry game (i.e., the Stackelberg leader-follower game), they investigate whether the first-movers may enjoy a higher profit compared to the later entrants. In equilibrium, firms may not produce for all markets and may have limited overlapping market areas, leading to multiple suppliers in any market. In general, the first-mover advantage may not exist and the early entrants may earn lower profits than the later entrants. In a linear market, Shiode et al. (2012) model the sequential competitive facility location problem with three facilities as a Stackelberg game. Diaz-Banez et al. (2011) consider a simultaneous game in a two-dimensional plane where the two firms choose first locations and then their prices with delivery costs. They characterize local and global Nash equilibria, providing an algorithm to generate all Nash equilibria.

Konur and Geunes (2012) consider a Cournot game in which non-identical firms simultaneously decide where to locate their facilities. Firms incur convex transportation, congestion, and location costs. They characterize the market-supply and location decisions of the firms. Likewise, Fernandez et al. (2014) study a location-price game on a plane where firms first select their locations and then set prices in order to maximize their profits. Taking the Huff's gravity-based approach, Saiz et al. (2011) consider two simultaneously competing firms. The problem is formulated as a two-stage game: on the first level quality level is chosen, and on the second level suppliers choose the locations. They use two cost functions (linear and quadratic) and analyze four cases (colocation/no-colocation vs. probabilistic/deterministic) to characterize the equilibrium.

Another stream of research has viewed the competitive location problem as a p-median problem. Drezner and Wesolowsky (1996) consider a setting in which facilities and customers are distributed on a finite line segment and the customers pay some portion of the facility fixed costs. Customers patronizing the same facility share the facility costs. Drezner and Wesolowsky (1996) find that customers never select a farther facility in the solution with p facilities located evenly on the line. Chen et al. (2005) model the facility location problem as a stochastic p-median problem with the objective of minimizing the expected regret.

Several other authors have studied the inventory-location problem. Daskin et al. (2002) study the facility location problem, taking into account inventory costs as well as transportation costs for replenishment. Unlike Daskin et al. (2002), we take into consideration transportation costs for consumer travels. They find that e-commerce technology costs might be reduced to locate additional facilities. Shen et al. (2003) study a similar problem under the assumptions of a single supplier, multiple retailers, and variable demand in each retailer. The objective is to determine which retailers should serve as distribution centers and how the other retailers should be allocated to the distribution centers.

In addition to price and quantity, another important tool used in location competition is product customization processes. Mendelson and Parlaktürk (2008) analyze customization and proliferation under competition. The market is a Hotelling line and the products are represented as locations on the unit line. They derive the equilibrium in a duopoly between the customizing firm and the traditional firm. In another paper, Mendelson and Parlaktürk (2008) investigate a horizontal product differentiation under mass customization adaption when the disutility of the consumer can be eliminated by product customization. They consider a duopoly market with heterogeneous customer tastes on a Hotelling line. They model the competitive pricing problem as a two-stage game. Last, Ulu et al. (2012) consider a firm who modifies its product assortment over time through learning about consumer tastes. In a horizontally differentiated market, the authors study the dynamic assortment decisions, taking consumer tastes as locations on a Hotelling line.

Meng et al. (2009) study competition in a decentralized supply chain that involves manufacturers, retailers, and consumers who can make decisions independently in a free market competition. Together with the costs of shipment, production, and handling of the retailers and manufacturers, they consider a firm entering the existing decentralized supply chain. They formulate a supply chain network equilibrium model with production capacity constraints, and use logarithmic-quadratic proximal prediction-correction method as a solution algorithm. This algorithm finds the optimal Lagrangian multipliers associated with the production capacity constraints, which are used to analyze the competitive facility location problem.

Granot et al. (2010) study the competitive sequential location problem on a linear city, characterizing the equilibrium number of players in the market and the equilibrium locations. They also extend their work to a network. In addition, they analyze the monopolist's choice of the number of facilities to open and their locations. When they compare the results of competition and monopoly, they find that competition leads to more retail locations, i.e., a good service for the consumers, and it reduces consumer transportation costs. Unlike Granot et al. (2010), we find that competition increases consumer transportation costs in a simultaneous game.

Cachon (2014) studies the location problem from a monopolistic retailer's perspective. He assumes that consumers travel to the nearest store by car, and stores are replenished by trucks from the warehouses. Transportation by car and truck incurs fuel costs, carbon emission costs, energy costs, and variable costs. Stores incur variable operating cost (e.g., rent), energy consumption cost (e.g., electricity and natural gas), and carbon emission cost. The objective is to minimize the total cost consisting of storage costs, transportation costs of consumers, and transportation costs for store replenishment. Cachon (2014) finds the size, location, and number of stores to serve a region of customers. Using the tessellation shapes, Cachon (2014) reveals that minimizing operational costs may increase emissions, and a price on carbon is an ineffective mechanism for reducing emissions. Unlike Cachon (2014), we study the location problem on a unit line and allow for competition among retail stores. Also, our analysis reveals that carbon penalty on consumer travels might be an effective mechanism in reducing excess emissions due to competition.

Park et al. (2015) study whether imposing carbon costs and carbon recovery rates changes the supply chain structure and social welfare, based on Cachon's (2014) model settings. Unlike Cachon (2014), they consider the problem of maximizing social welfare from a central policymaker's perspective in three settings (i.e., monopoly, monopolistic competition with symmetric market share, and monopolistic competition with asymmetric market share). Retailers want to maximize their profits, and consumers want to maximize their utilities, both generating carbon emissions. They find that when market competition is intense, the carbon cost can influence the supply chain structure significantly. In the monopoly case, the social welfare may either increase or decrease as the carbon cost increases. They also examine the optimal carbon emission recovery rates from a central policymaker's perspective, showing that these rates are in general larger for the retailers than those for the consumers. Once again, unlike Park et al. (2015), we study the location problem on a unit line and allow for competition among retail stores.

Chapter 3

Problem Formulation

We study the location selection problem for two competing retailers ($i = A, B$). Both retailers sell an identical product in a city represented by a line segment of unit length. A continuum of consumers is uniformly spread over the interval [0,1]. Both retailers source the product from a common warehouse located at point $m \in [0, 1]$. Both retailers purchase the product from the warehouse at the same price p_m and sell the product in their stores at the same price p. The notation we use throughout the thesis is available in Table 3.2 at the end of this chapter.

We denote by a and b the locations of retailers A and B on the unit line, respectively. Total daily demand in the city is λ. The consumers travel straight lines to the nearest retail store to their home by passenger vehicles (e.g., car) to purchase one unit of the product. This is a standard assumption in the literature; see, for instance, Dobson and Karmakar (1987), Küçükaydın et al. (2011), and Cachon (2014). Each retail store visit incurs a transportation cost that is proportional to the distance traveled by the consumer. We denote by $\lambda_i(a, b)$ the total daily demand in retail store i. For example, if $a < b$, then the total daily demand in retail store A is

$$\lambda_A(a, b) = \lambda \left(\frac{a + b}{2} \right)$$

and the total daily demand in retail store B is

$$\lambda_B(a,b) = \lambda\left(1 - \frac{a+b}{2}\right).$$

We assume that there are additional factors that influence a retailer's margin beyond the retail price p and the warehouse wholesale price p_m. Specifically, an additional cost accrues when a consumer's demand is satisfied, and this cost is proportional to the travel distance of the consumer. Governmental tax regulations could easily produce dynamics like this. The governments (such as British Columbia) charge carbon taxes to everyone, including businesses. "Whether you switch to energy-efficient light bulbs, shop locally for produce, or purchase eco-friendly upgrades in your home, your decisions can make a big difference. Simply driving 10 kilometers less per week will help offset the carbon tax for most British Columbians" (British Columbia Ministry of Finance 2016). The more the consumers are attracted considerably far located from the store, the more the consumers pay emission taxes to travel to that store.

We assume that each retailer attracts the far located consumers by compensating their transportation costs, in order to sell her products. In such a setting, the retailer's choice of store location is affected by not only replenishment costs between the retail store and the warehouse, but also transportation costs of consumers traveling to and from the retail store. Note that both types of transportation (truck or car) lead to negative externalities in terms of the carbon emissions.

The same margin structure could also arise if the retailers would implement end-of-season markdown/promotion campaigns (even when taxes are independent from consumer transportation). Then, those consumers located close to a retail store would frequently visit the store and purchase the product at close-to-full price early in the season whereas those others at different locations would visit the store and purchase the same product only when there is a big markdown event.

We also refer the reader to Cachon (2014) and Park et al. (2015) for similar environments. Cachon (2014) states that the carbon emission is an example of a negative production externality, every agent in the supply chain contributes to this negative externality, and too high emission levels lead to a poor supply chain design. Thus Cachon (2014) takes into consideration emission costs for both consumers and retailers in his monopolistic model. Following Cachon's (2014) model settings, Park et al. (2015) consider balancing the retailers' and consumers' self-interest against the negative externalities.

We base our model on Cachon (2014) in quantifying the retailers' revenue and cost trade-offs. Each retailer wants to locate her store close to the warehouse to reduce her replenishment costs, but also close to her consumer base to achieve greater tax benefits. We define c_c as the transportation cost per unit of distance traveled by consumer per unit of product purchased, and c_t as the transportation cost per unit of distance traveled by truck per unit of product delivered. (The subscript 'c' refers to 'cars' and the subscript 't' refers to 'trucks.') Transportation costs are influenced by the fuel efficiency of the vehicles used, the weight of the loads they carry, and the distance they travel. Thus we formulate c_c and c_t in terms of the non-fuel variable cost to transport the vehicle j per unit of distance (v_j), the amount of fuel used to transport the vehicle j per unit of distance (f_j), the per unit cost of fuel (p_j), the amount of carbon emission released by consumption of one unit of fuel (e_j), the price of carbon or cost of emissions per unit released ($p_{e,j}$), and the load carried by vehicle j (q_j), for $j \in \{c, t\}$. When the government increases the emission taxes, $p_{e,j}$ increases. Note that high values of $p_{e,j}$ motivate the retailers to reduce their carbon emissions. Thus:

$$c_j = \frac{v_j + f_j(p_j + e_j p_{e,j})}{q_j} \text{ for } j \in \{c, t\}.$$

Note that the cost c_j consists of two parts: the fuel cost $\frac{f_j(p_j+e_j p_{e,j})}{q_j}$ and the non-fuel cost $\frac{v_j}{q_j}$. The fuel cost includes the price of carbon $\frac{f_j e_j p_{e,j}}{q_j}$ where $\frac{f_j e_j}{q_j}$ is the amount of carbon emissions. Trucks can carry significantly larger quantities than cars. As a result, the economies of scale effect between the truck-load and the passenger-car-load often dominates the transportation cost coefficients. We thus assume that consumer transportation costs are higher than the replenishment transportation costs. In their numerical experiments Cachon (2014) and Park et.

al (2015) assume that $c_c/c_t = 235$. Specifically, we restrict our analysis to the case with $p > c_c > 2c_t$ in the remainder of the thesis.

Assumption 1. *$p > c_c > 2c_t$.*

Last, we define $d_{ic}(a, b)$ as the average round-trip distance a consumer travels to retail store i and $d_{it}(a, b)$ as the length of truck's route from store i to the warehouse. Given the warehouse location $m \in [0, 1]$, each retailer i chooses the location of its store to maximize its expected daily profit $\pi_i(a, b)$:

$$\pi_i(a, b) = (p - c_c d_{ic}(a, b) - c_t d_{it}(a, b))\lambda_i(a, b) \quad \text{for } i \in \{A, B\}.$$

Retailer A's problem is given by

$$\begin{aligned} \underset{a}{\text{maximize}} \quad & \pi_A(a, b) \\ \text{subject to} \quad & 0 \leq a \leq 1 \end{aligned}$$

and retailer B's problem is given by

$$\begin{aligned} \underset{b}{\text{maximize}} \quad & \pi_B(a, b) \\ \text{subject to} \quad & 0 \leq b \leq 1. \end{aligned}$$

The retailers' demand and cost structures depend on their relative locations, with respect to each other and the warehouse. Thus we characterize eight location combinations that yield distinct demand and cost functions for retailers, in Table 3.1. We will formulate the respective demand and profit functions in all these cases. We relegated our derivations of $d_{ic}(a, b)$ and $d_{it}(a, b)$ in terms of our problem parameters and decision variables in cases (1)-(8) to Appendix A.

Table 3.1: Eight distinct location cases in our problem formulation.

i	$Case$
1	$0 \leq a < b \leq m \leq 1$
2	$0 \leq a = b \leq m \leq 1$
3	$0 \leq b < a \leq m \leq 1$
4	$0 \leq b \leq m < a \leq 1$
5	$0 \leq a < m < b \leq 1$
6	$0 \leq m \leq a < b \leq 1$
7	$0 \leq m < a = b \leq 1$
8	$0 \leq m < b < a \leq 1$

Case (1). $\mathbf{0 \leq a < b \leq m \leq 1}$. The average round-trip distance traveled by a consumer to retail store A is given by

$$d_{Ac}(a,b) = \frac{2\left[\int_0^a (a-t)dt + \int_a^{\frac{a+b}{2}} (t-a)dt\right]}{\frac{a+b}{2}} = \frac{5a^2 - 2ab + b^2}{2a + 2b}$$

and the average round-trip distance traveled by a consumer to retail store B is given by

$$d_{Bc}(a,b) = \frac{2\left[\int_b^1 (t-b)dt + \int_{\frac{a+b}{2}}^b (b-t)dt\right]}{1 - \frac{a+b}{2}} = \frac{a^2 - 2ab + 5b^2 - 8b + 4}{4 - 2a - 2b}.$$

The round-trip distance traveled by a truck from the warehouse to retail store A is given by

$$d_{At}(a,b) = 2(m-a)$$

and the round-trip distance traveled by a truck from the warehouse to retail store B is given by

$$d_{Bt}(a,b) = 2(m-b).$$

Hence the expected daily profit of retailer A can be written as

$$\pi_A(a,b) \quad = \quad \frac{\lambda p(a+b)}{2} - \frac{\lambda c_c(5a^2 - 2ab + b^2)}{4} - \lambda c_t(a+b)(m-a).$$

The expected daily profit of retailer B can be written as

$$\pi_B(a,b) = \frac{\lambda p(2-a-b)}{2} - \frac{\lambda c_c(a^2-2ab+5b^2-8b+4)}{4} - \lambda c_t(2-a-b)(m-b).$$

Case (2). $\mathbf{0 \leq a = b \leq m \leq 1}$. For $i \in \{A, B\}$:

$$\pi_i(a,b) = \frac{\lambda p - \lambda c_c(1-2a+2a^2)}{2} - \lambda c_t(m-a).$$

Case (3). $\mathbf{0 \leq b < a \leq m \leq 1}$.

$$\pi_A(a,b) = \frac{\lambda p(2-a-b)}{2} - \frac{\lambda c_c(b^2-2ab+5a^2-8a+4)}{4} - \lambda c_t(2-a-b)(m-a).$$

$$\pi_B(a,b) = \frac{\lambda p(a+b)}{2} - \frac{\lambda c_c(5b^2-2ab+a^2)}{4} - \lambda c_t(a+b)(m-b).$$

Case (4). $\mathbf{0 \leq b \leq m < a \leq 1}$.

$$\pi_A(a,b) = \frac{\lambda p(2-a-b)}{2} - \frac{\lambda c_c(b^2-2ab+5a^2-8a+4)}{4} - \lambda c_t(2-a-b)(a-m).$$

$$\pi_B(a,b) = \frac{\lambda p(a+b)}{2} - \frac{\lambda c_c(5b^2-2ab+a^2)}{4} - \lambda c_t(a+b)(m-b).$$

Case (5). $\mathbf{0 \leq a < m < b \leq 1}$.

$$\pi_A(a,b) = \frac{\lambda p(a+b)}{2} - \frac{\lambda c_c(5a^2-2ab+b^2)}{4} - \lambda c_t(a+b)(m-a).$$

$$\pi_B(a,b) = \frac{\lambda p(2-a-b)}{2} - \frac{\lambda c_c(a^2-2ab+5b^2-8b+4)}{4} - \lambda c_t(2-a-b)(b-m).$$

Case (6). $\mathbf{0 \leq m \leq a < b \leq 1}$.

$$\pi_A(a,b) = \frac{\lambda p(a+b)}{2} - \frac{\lambda c_c(5a^2-2ab+b^2)}{4} - \lambda c_t(a+b)(a-m).$$

$$\pi_B(a,b) = \frac{\lambda p(2-a-b)}{2} - \frac{\lambda c_c(a^2-2ab+5b^2-8b+4)}{4} - \lambda c_t(2-a-b)(b-m).$$

Case (7). $\mathbf{0 \leq m < a = b \leq 1}$. For $i \in \{A, B\}$:

$$\pi_i(a,b) = \frac{\lambda p - \lambda c_c(1 - 2a + 2a^2)}{2} - \lambda c_t(a - m).$$

Case (8). $\mathbf{0 \leq m < b < a \leq 1}$.

$$\pi_A(a,b) = \frac{\lambda p(2 - a - b)}{2} - \frac{\lambda c_c(b^2 - 2ab + 5a^2 - 8a + 4)}{4} - \lambda c_t(2 - a - b)(a - m).$$

$$\pi_B(a,b) = \frac{\lambda p(a + b)}{2} - \frac{\lambda c_c(5b^2 - 2ab + a^2)}{4} - \lambda c_t(a + b)(b - m).$$

In this chapter, we have formulated our problem and have discussed our modeling assumptions. We have identified eight distinct (location) cases that yield different profit functions for the retailers, deriving their respective profits. In the remainder of the thesis, we will use the above functions to analyze the retail location problem.

In Chapter 4, we consider a decentralized system in which the two retailers want to competitively locate their stores on the unit line to maximize their *individual* expected profits. The demand and costs of each store are affected by location decisions of both retailers. In Chapter 5, we consider a centralized system (with the same parameters of the decentralized system) in which a single retail chain wants to locate two of its stores, A and B respectively, on the unit line to maximize its *total* expected profit.

Table 3.2: Summary of the notation.

Parameters	**Definition**
p	In-store price of the product
p_m	Warehouse price of the product
m	Location of the warehouse on the interval [0,1]
v_j	Non-fuel variable cost to transport vehicle $j \in \{c,t\}$ per unit of distance
f_j	Amount of fuel used to transport vehicle $j \in \{c,t\}$ per unit of distance
p_j	Cost of fuel per unit of fuel ($j \in \{c,t\}$)
e_j	Amount of emission released by consumption of one unit of fuel ($j \in \{c,t\}$)
$p_{e,j}$	Per unit price of carbon ($j \in \{c,t,s\}$)
q_j	Load carried by vehicle $j \in \{c,t\}$
λ	Total daily demand
$\lambda_i(a,b)$	Total daily demand in retail store $i \in \{A,B\}$
c_c	Transportation cost of the consumer per unit of item per unit of distance
c_t	Transportation cost of the truck per unit of item per unit of distance
$d_{ic}(a,b)$	Average round trip distance a consumer travels to retail store $i \in \{A,B\}$
$c_c d_{ic}(a,b)$	Average consumer travel cost to retail store $i \in \{A,B\}$ per unit of item
$d_{it}(a,b)$	Length of truck's route to retail store $i \in \{A,B\}$
$c_t d_{it}(a,b)$	Transportation cost of retail store $i \in \{A,B\}$ per unit of item delivered
$\pi_i(a,b)$	Expected daily profit of retailer $i \in \{A,B\}$
Decision variables	**Definition**
a and b	Locations of retail stores A and B on the interval [0,1], respectively

Chapter 4

Decentralized System

In this chapter we consider a decentralized system in which the two retailers simultaneously choose the locations of their stores to maximize their individual profits. We first analytically characterize the best response functions of the retailers. We then establish the Nash equilibrium locations in several special cases based on contraction mapping of the best responses. We will present further results and insights on the decentralized system in Chapter 6.

Recall that the warehouse is located at point $m \in [0, 1]$. Below we characterize the best response function of retailer A in each of the following two scenarios: (i) when the store of retailer B is located at point $b \leq m$ and (ii) when it is located at point $b > m$. We assume that the in-store price p is sufficiently large so that it is always profitable to stay in the market. The proofs of all analytical results are available in Appendix C.

First suppose that $b \leq m$. Retailer A can locate her store relative to store B in one of the following four configurations (cases 1–4 in Chapter 3):

Case (1). $\mathbf{0 \leq a < b \leq m \leq 1}$. The expected daily profits are given by

$$\pi_A^1(a, b) = \lambda \left(\frac{p(a+b)}{2} - \frac{c_c(5a^2 - 2ab + b^2)}{4} - c_t(a+b)(m-a) \right),$$

$$\pi_B^1(a,b) = \lambda\left(\frac{p(2-a-b)}{2} - \frac{c_c(a^2-2ab+5b^2-8b+4)}{4} - c_t(2-a-b)(m-b)\right).$$

Case (2). $\mathbf{0 \le a = b \le m \le 1}$. The expected daily profits are given by

$$\pi_A^2(a,b) = \pi_B^2(a,b) = \lambda\left(\frac{p}{2} - \frac{c_c(1-2a+2a^2)}{2} - c_t(m-a)\right).$$

Case (3). $\mathbf{0 \le b < a \le m \le 1}$. The expected daily profits are given by

$$\pi_A^3(a,b) = \lambda\left(\frac{p(2-a-b)}{2} - \frac{c_c(b^2-2ab+5a^2-8a+4)}{4} - c_t(2-a-b)(m-a)\right),$$

$$\pi_B^3(a,b) = \lambda\left(\frac{p(a+b)}{2} - \frac{c_c(5b^2-2ab+a^2)}{4} - c_t(a+b)(m-b)\right).$$

Case (4). $\mathbf{0 \le b \le m < a \le 1}$. The expected daily profits are given by

$$\pi_A^4(a,b) = \lambda\left(\frac{p(2-a-b)}{2} - \frac{c_c(b^2-2ab+5a^2-8a+4)}{4} - c_t(2-a-b)(a-m)\right),$$

$$\pi_B^4(a,b) = \lambda\left(\frac{p(a+b)}{2} - \frac{c_c(5b^2-2ab+a^2)}{4} - c_t(a+b)(m-b)\right).$$

We next evaluate retailer A's optimal profit and location in each of the above configurations, and identify its optimal location when $b \le m$ as the one that produces the highest profit for its store across all these configurations.

Case (1). As we assume $c_c > 2c_t$ (Assumption 1), we are able to prove that the profit function of retailer A is concave in a:

$$\frac{d\pi_A^1(a,b)}{da} = \frac{\lambda}{2}\left(p + c_c(-5a+b) + 2c_t(2a+b-m)\right)$$

and

$$\frac{d^2\pi_A^1(a,b)}{da^2} = \lambda\left(\frac{-5c_c}{2} + 2c_t\right) < 0.$$

The unconstrained maximizer of retailer A's profit function is $a_1^o = \frac{p+c_c b+2c_t(b-m)}{5c_c-4c_t}$. However, with the constraint of $0 \le a < b$, retailer A's optimal location and profit in case (1) are given by

$$(a_1^*, \pi_A^1(a_1^*, b)) = \begin{cases} (a_1^o, \pi_A^1(a_1^o, b)) & \text{if } 0 \le a_1^o < b \ (Con_{1a}), \\ (0, \pi_A^1(0, b)) & \text{if } a_1^o < 0 \ (Con_{1b}), \text{ and} \\ \emptyset & \text{otherwise, i.e., } a_1^o \ge b \ (Con_{1c}), \end{cases}$$

where

$$\pi_A^1(a_1^o, b) = \lambda\left(\frac{(p-2c_t m)^2 + 4b(-3c_c + c_t)(2c_t m - p) + 4b^2(-c_c^2 + 2c_c c_t + c_t^2)}{4(5c_c - 4c_t)}\right)$$

and

$$\pi_A^1(0, b) = \lambda\left(\frac{-b(4c_t m - 2p + c_c b)}{4}\right).$$

We further detail conditions Con_{1a}, Con_{1b}, and Con_{1c} in Appendix B.

Case (2). Note that $a = b$ in this case. Thus:

$$\pi_A^2(b, b) = \lambda\left(\frac{p}{2} - \frac{c_c(1 - 2b + 2b^2)}{2} - c_t(m - b)\right).$$

Case (3). Since $c_c > 2c_t$ (Assumption 1), the profit function of retailer A is concave in a:

$$\frac{d\pi_A^3(a, b)}{da} = \frac{\lambda}{2}\left(-p + c_c(4 - 5a + b) + 2c_t(-2a - b + m + 2)\right)$$

and

$$\frac{d^2\pi_A^3(a, b)}{da^2} = \lambda\left(\frac{-5c_c}{2} - 2c_t\right) < 0.$$

The unconstrained maximizer of retailer A's profit function is $a_3^o = \frac{-p+c_c(4+b)+2c_t(2+m-b)}{5c_c+4c_t}$. However, with the constraint of $b < a \le m$, retailer A's optimal location and profit in case (3) are given by

$$(a_3^*, \pi_A^3(a_3^*, b)) = \begin{cases} (a_3^o, \pi_A^3(a_3^o, b)) & \text{if } b < a_3^o \le m \ (Con_{3a}), \\ (m, \pi_A^3(m, b)) & \text{if } a_3^o > m \ (Con_{3b}), \text{ and} \\ \emptyset & \text{otherwise, i.e., } a_3^o \le b \ (Con_{3c}), \end{cases}$$

where

$$\pi_A^3(a_3^o, b) = \lambda\left(\frac{-4c_c^2(-1+b)^2 - 4c_c(-1+b)(3p+2c_t(2-3m+b))}{4(5c_c+4c_t)}\right)$$
$$+\lambda\left(\frac{(p-2c_t(-2+m+b))^2}{4(5c_c+4c_t)}\right)$$

and

$$\pi_A^3(m,b) = -\lambda\left(\frac{p(-2+m+b)}{2} + \frac{c_c(4+5m^2+b^2-2m(4+b))}{4}\right).$$

We further detail conditions Con_{3a}, Con_{3b}, and Con_{3c} in Appendix B.

Case (4). Since $c_c > 2c_t$ (Assumption 1), the profit function of retailer A is concave in a:

$$\frac{d\pi_A^4(a,b)}{da} = \frac{\lambda}{2}\left(-p + c_c(4-5a+b) + 2c_t(2a+b-m-2)\right)$$

and

$$\frac{d^2\pi_A^4(a,b)}{da^2} = \lambda\left(\frac{-5c_c}{2} + 2c_t\right) < 0.$$

The unconstrained maximizer of retailer A's profit function here is $a_4^o = \frac{-p+c_c(4+b)-2c_t(2+m-b)}{5c_c-4c_t}$. However, with the constraint of $m < a \leq 1$, retailer A's optimal location and profit in case (4) are given by

$$(a_4^*, \pi_A^4(a_4^*, b)) = \begin{cases} (a_4^o, \pi_A^4(a_4^o, b)) & \text{if } m < a_4^o \leq 1 \ (Con_{4a}), \\ (1, \pi_A^4(1,b)) & \text{if } a_4^o > 1 \ (Con_{4b}), \text{ and} \\ \emptyset & \text{otherwise, i.e., } a_4^o \leq m \ (Con_{4c}), \end{cases}$$

where

$$\pi_A^4(a_4^o, b) = \lambda\left(\frac{-4c_c^2(-1+b)^2 + 4c_c(-1+b)(-3p+2c_t(2-3m+b))}{4(5c_c-4c_t)}\right)$$
$$+\lambda\left(\frac{((p+2c_t(-2+m+b))^2}{4(5c_c-4c_t)}\right)$$

and

$$\pi_A^4(1,b) = -\lambda\left(\frac{(4c_t(-1+m)+2p+c_c(-1+b))(-1+b)}{4}\right).$$

We further detail conditions Con_{4a}, Con_{4b}, and Con_{4c} in Appendix B.

In each case, given retailer B's location on the unit line, we find the optimal location of retailer A (the maximizer of retailer A's profit). The unconstrained optimal solution may not belong to the feasible region of the case. If the solution does not belong to the feasible region, since our profit functions are concave in all cases, the best response of retailer A takes the value from the feasible region that is closest to the optimal solution, i.e., one of the end-points. If the solution does not belong to the feasible region of the case and the end-point cannot be achieved since the feasible region is not compact, then retailer A's best response becomes $\emptyset$.

We evaluate retailer A's optimal profit and location in each of the above configurations. The best response of retailer A should give the maximum profit to retailer A among all location options. Proposition 4.1 characterizes the best response of retailer A to retailer B's location choice when $b \leq m$.

Proposition 4.1. *Suppose that retailer B is located at b such that $b \leq m \leq 1$. Retailer A's best response location to b is as follows:*

BestResponse	*Conditions*
$a_1^o = \frac{p+c_c b+2c_t(b-m)}{5c_c-4c_t}$	$b > a_1^o \geq 0,\ \pi_A^1(a_1^o,b) \geq \pi_A^2(b,b)$ *AND* $a_3^o > m,\ 1 \geq a_4^o > m,\ \pi_A^1(a_1^o,b) \geq max\{\pi_A^3(m,b), \pi_A^4(a_4^o,b)\}$ *OR* $m \geq a_3^o > b,\ m \geq a_4^o,\ \pi_A^1(a_1^o,b) \geq \pi_A^3(a_3^o,b)$ *OR* $a_3^o > m,\ m \geq a_4^o,\ \pi_A^1(a_1^o,b) \geq \pi_A^3(m,b)$ *OR* $b \geq a_3^o,\ m \geq a_4^o$;
$a_3^o = \frac{-p+c_c(4+b)+2c_t(2+m-b)}{5c_c+4c_t}$	$m \geq a_3^o > b,\ \pi_A^3(a_3^o,b) \geq \pi_A^2(b,b)$ *AND* $b > a_1^o \geq 0,\ m \geq a_4^o,\ \pi_A^3(a_3^o,b) \geq \pi_A^1(a_1^o,b)$ *OR* $a_1^o \geq b,\ m \geq a_4^o$;
$a_4^o = \frac{-p+c_c(4+b)-2c_t(2+m-b)}{5c_c-4c_t}$	$1 \geq a_4^o > m,\ \pi_A^4(a_4^o,b) \geq \pi_A^2(b,b)$ *AND* $b > a_1^o \geq 0,\ a_3^o > m,\ \pi_A^4(a_4^o,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(m,b)\}$ *OR* $a_1^o \geq b,\ a_3^o > m,\ \pi_A^4(a_4^o,b) \geq \pi_A^3(m,b)$;
m	$a_3^o > m,\ \pi_A^3(m,b) \geq \pi_A^2(b,b)$ *AND* $b > a_1^o \geq 0,\ 1 \geq a_4^o > m,\ \pi_A^3(m,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^4(a_4^o,b)\}$ *OR* $a_1^o \geq b,\ 1 \geq a_4^o > m,\ \pi_A^3(m,b) \geq \pi_A^4(a_4^o,b)$ *OR* $b > a_1^o \geq 0,\ m \geq a_4^o,\ \pi_A^3(m,b) \geq \pi_A^1(a_1^o,b)$ *OR* $a_1^o \geq b,\ m \geq a_4^o$;
b	$b > a_1^o \geq 0,\ a_3^o > m,\ 1 \geq a_4^o > m,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(m,b), \pi_A^4(a_4^o,b)\}$ *OR* $b > a_1^o \geq 0,\ m \geq a_3^o > b,\ m \geq a_4^o,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(a_3^o,b)\}$ *OR* $b > a_1^o \geq 0,\ a_3^o > m,\ m \geq a_4^o,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(m,b)\}$ *OR* $b > a_1^o \geq 0,\ b \geq a_3^o,\ m \geq a_4^o,\ \pi_A^2(b,b) \geq \pi_A^1(a_1^o,b)$ *OR* $a_1^o \geq b,\ a_3^o > m,\ 1 \geq a_4^o > m,\ \pi_A^2(b,b) \geq max\{\pi_A^3(m,b), \pi_A^4(a_4^o,b)\}$ *OR* $a_1^o \geq b,\ m \geq a_3^o > b,\ m \geq a_4^o,\ \pi_A^2(b,b) \geq \pi_A^3(a_3^o,b)$ *OR* $a_1^o \geq b,\ a_3^o > m,\ m \geq a_4^o,\ \pi_A^2(b,b) \geq \pi_A^3(m,b)$ *OR* $a_1^o \geq b,\ b \geq a_3^o,\ m \geq a_4^o$;
$\emptyset$	*otherwise.*

The open forms of expressions a_1^o, a_3^o, a_4^o, $\pi_A^1(a_1^o,b)$, $\pi_A^2(b,b)$, $\pi_A^3(m,b)$, $\pi_A^3(a_3^o,b)$, and $\pi_A^4(a_4^o,b)$ are available in Table 4.1.

Now suppose that retailer B is located at b such that $b > m$. Retailer A's best response to retailer B's location choice can be in one of the following four configurations (cases 5–8 in Chapter 3):

Case (5). $\mathbf{0 \leq a < m < b \leq 1}$. The expected daily profits are given by

$$\pi_A^5(a,b) = \lambda\left(\frac{p(a+b)}{2} - \frac{c_c(5a^2-2ab+b^2)}{4} - c_t(a+b)(m-a)\right),$$

$$\pi_B^5(a,b) = \lambda\left(\frac{p(2-a-b)}{2} - \frac{c_c(a^2-2ab+5b^2-8b+4)}{4} - c_t(2-a-b)(b-m)\right).$$

Case (6). $\mathbf{0 \leq m \leq a < b \leq 1}$. The expected daily profits are given by

$$\pi_A^6(a,b) = \lambda\left(\frac{p(a+b)}{2} - \frac{c_c(5a^2-2ab+b^2)}{4} - c_t(a+b)(a-m)\right),$$

$$\pi_B^6(a,b) = \lambda\left(\frac{p(2-a-b)}{2} - \frac{c_c(a^2-2ab+5b^2-8b+4)}{4} - c_t(2-a-b)(b-m)\right).$$

Case (7). $\mathbf{0 \leq m < a = b \leq 1}$. The expected daily profits are given by

$$\pi_A^7(a,b) = \pi_B^7(a,b) = \lambda\left(\frac{p}{2} - \frac{c_c(1-2a+2a^2)}{2} - c_t(a-m)\right).$$

Case (8). $\mathbf{0 \leq m < b < a \leq 1}$. The expected daily profits are given by

$$\pi_A^8(a,b) = \lambda\left(\frac{p(2-a-b)}{2} - \frac{c_c(b^2-2ab+5a^2-8a+4)}{4} - c_t(2-a-b)(a-m)\right),$$

$$\pi_B^8(a,b) = \lambda\left(\frac{p(a+b)}{2} - \frac{c_c(5b^2-2ab+a^2)}{4} - c_t(a+b)(b-m)\right).$$

We next evaluate retailer A's optimal profit and location in each of the above configurations, and identify its optimal location when $m < b$ as the one that produces the highest profit for its store across all these configurations.

Case (5). Since $c_c > 2c_t$ (Assumption 1), the profit function of retailer A is concave in a:

$$\frac{d\pi_A^5(a,b)}{da} = \frac{\lambda}{2}\left(p + c_c(-5a+b) + c_t(-2m+4a+2b)\right)$$

and

$$\frac{d^2\pi_A^5(a,b)}{da^2} = \lambda\left(\frac{-5c_c}{2} + 2c_t\right) < 0.$$

The unconstrained maximizer of retailer A's profit function is $a_5^o = \frac{p+c_cb+2c_t(b-m)}{5c_c-4c_t}$. However, with the constraint of $0 \leq a < m$, retailer A's optimal location and profit in case (5) are given by

$$(a_5^*, \pi_A^5(a_5^*, b)) = \begin{cases} (a_5^o, \pi_A^5(a_5^o, b)) & \text{if } 0 \leq a_5^o < m \ (Con_{5a}), \\ (0, \pi_A^5(0, b)) & \text{if } a_5^o < 0 \ (Con_{5b}), \text{ and} \\ \emptyset & \text{otherwise, i.e., } a_5^o \geq m \ (Con_{5c}), \end{cases}$$

where

$$\pi_A^5(a_5^o, b) = \lambda\left(\frac{(-2c_tm+p)^2 + 4(-3c_c+c_t)(2c_tm-p)b + 4(-c_c^2+2c_cc_t+c_t^2)b^2}{4(5c_c-4c_t)}\right)$$

and

$$\pi_A^5(0,b) = \lambda\left(\frac{-b(4c_tm - 2p + c_cb)}{4}\right).$$

We further detail conditions Con_{5a}, Con_{5b}, and Con_{5c} in Appendix B.

Case (6). Since $c_c > 2c_t$ (Assumption 1), the profit function of retailer A is concave in a:

$$\frac{d\pi_A^6(a,b)}{da} = \frac{\lambda}{2}\left(p + 2c_t(m-2a-b) + c_c(-5a+b)\right)$$

and

$$\frac{d^2\pi_A^6(a,b)}{da^2} = \lambda\left(\frac{-5c_c}{2} - 2c_t\right) < 0.$$

The unconstrained maximizer of retailer A's profit function is $a_6^o = \frac{p+c_cb+2c_t(m-b)}{5c_c+4c_t}$. However, with the constraint of $m \leq a < b$, retailer A's optimal location and profit in case (6) are given by

$$(a_6^*, \pi_A^6(a_6^*, b)) = \begin{cases} (a_6^o, \pi_A^6(a_6^o, b)) & \text{if } m \leq a_6^o < b \ (Con_{6a}), \\ (m, \pi_A^6(m, b)) & \text{if } a_6^o < m \ (Con_{6b}), \text{ and} \\ \emptyset & \text{otherwise, i.e., } a_6^o \geq b \ (Con_{6c}), \end{cases}$$

where

$$\pi_A^6(a_6^o, b) = \lambda\left(\frac{(2c_t m + p)^2 + 4(3c_c + c_t)(2c_t m + p)b - 4(c_c^2 + 2c_c c_t - c_t^2)b^2}{4(5c_c + 4c_t)}\right)$$

and

$$\pi_A^6(m, b) = \lambda\left(\frac{p(m+b)}{2} - \frac{c_c(5m^2 - 2mb + b^2)}{4}\right).$$

We further detail conditions Con_{6a}, Con_{6b}, and Con_{6c} in Appendix B.

Case (7). Note that $a = b$ in this case. Thus:

$$\pi_A^7(b, b) = \lambda\left(\frac{p}{2} - \frac{c_c(1 - 2b + 2b^2)}{2} - c_t(b - m)\right).$$

Case (8). Since $c_c > 2c_t$ (Assumption 1), the profit function of retailer A is concave in a:

$$\frac{d\pi_A^8(a,b)}{da} = \frac{\lambda}{2}\left((-p - 2c_t(2 + m - 2a - b) + c_c(4 - 5a + b))\right)$$

and

$$\frac{d^2\pi_A^8(a,b)}{da^2} = \lambda\left(\frac{-5c_c}{2} + 2c_t\right) < 0.$$

The unconstrained maximizer of retailer A's profit function is $a_8^o = \frac{-p + c_c(4+b) + 2c_t(b-m-2)}{5c_c - 4c_t}$. However, with the constraint of $b < a \leq 1$, retailer A's optimal location and profit in case (8) are given by

$$(a_8^*, \pi_A^8(a_8^*, b)) = \begin{cases} (a_8^o, \pi_A^8(a_8^o, b)) & \text{if } b < a_8^o \leq 1 \ (Con_{8a}), \\ (1, \pi_A^8(1, b)) & \text{if } a_8^o > 1 \ (Con_{8b}), \text{ and} \\ \emptyset & \text{otherwise, i.e., } a_8^o \leq b \ (Con_{8c}), \end{cases}$$

where

$$\begin{aligned} \pi_A^8(a_8^o, b) &= \lambda\left(\frac{-4c_c^2(-1+b)^2 + 4c_c(-1+b)(-3p + 2c_t(2 - 3m + b))}{4(5c_c - 4c_t)}\right) \\ &\quad + \lambda\left(\frac{(p + 2c_t(-2 + m + b))^2}{4(5c_c - 4c_t)}\right) \end{aligned}$$

and

$$\pi_A^8(1, b) = \lambda\left(-\frac{(4c_t(-1+m) + 2p + c_c(-1+b))(-1+b)}{4}\right).$$

We further detail conditions Con_{8a}, Con_{8b}, and Con_{8c} in Appendix B.

Propositions 4.2 characterizes the best response of retailer A to retailer B's location choice when $b > m$.

Proposition 4.2. *Suppose that retailer B is located at b such that $b > m$. Retailer A's best response location to b is as follows:*

BestResponse	*Conditions*
$a_5^o = \frac{p+c_c b+2c_t(b-m)}{5c_c-4c_t}$	$m > a_5^o \geq 0,\ \pi_A^5(a_5^o,b) \geq \pi_A^7(b,b)$ *AND* $b > a_6^o \geq m,\ 1 \geq a_8^o > b,\ \pi_A^5(a_5^o,b) \geq max\{\pi_A^6(a_6^o,b),\pi_A^8(a_8^o,b)\}$ *OR* $m > a_6^o,\ 1 \geq a_8^o > b,\ \pi_A^5(a_5^o,b) \geq max\{\pi_A^6(m,b),\pi_A^8(a_8^o,b)\}$ *OR* $b > a_6^o \geq m,\ b \geq a_8^o,\ \pi_A^5(a_5^o,b) \geq \pi_A^6(a_6^o,b)$ *OR* $m > a_6^o,\ b \geq a_8^o,\ \pi_A^5(a_5^o,b) \geq \pi_A^6(m,b)$;
$a_6^o = \frac{p+c_c b+2c_t(m-b)}{5c_c+4c_t}$	$b > a_6^o \geq m,\ \pi_A^6(a_6^o,b) \geq \pi_A^7(b,b)$ *AND* $m > a_5^o \geq 0,\ 1 \geq a_8^o > b,\ \pi_A^6(a_6^o,b) \geq max\{\pi_A^5(a_5^o,b),\pi_A^8(a_8^o,b)\}$ *OR* $a_5^o \geq m,\ 1 \geq a_8^o > b,\ \pi_A^6(a_6^o,b) \geq \pi_A^8(a_8^o,b)$ *OR* $m > a_5^o \geq 0,\ b \geq a_8^o,\ \pi_A^6(a_6^o,b) \geq \pi_A^5(a_5^o,b)$ *OR* $a_5^o \geq m,\ b \geq a_8^o$;
$a_8^o = \frac{-p+c_c(4+b)+2c_t(b-m-2)}{5c_c-4c_t}$	$1 \geq a_8^o > b,\ \pi_A^8(a_8^o,b) \geq \pi_A^7(b,b)$ *AND* $m > a_5^o \geq 0,\ b > a_6^o \geq m,\ \pi_A^8(a_8^o,b) \geq max\{\pi_A^5(a_5^o,b),\pi_A^6(a_6^o,b)\}$ *OR* $m > a_5^o \geq 0,\ m > a_6^o,\ \pi_A^8(a_8^o,b) \geq max\{\pi_A^5(a_5^o,b),\pi_A^6(m,b)\}$ *OR* $a_5^o \geq m,\ b > a_6^o \geq m,\ \pi_A^8(a_8^o,b) \geq \pi_A^6(a_6^o,b)$ *OR* $a_5^o \geq m,\ m > a_6^o,\ \pi_A^8(a_8^o,b) \geq \pi_A^6(m,b)$ *OR* $a_5^o \geq m,\ a_6^o \geq b$;
m	$m > a_6^o,\ \pi_A^6(m,b) \geq \pi_A^7(b,b)$ *AND* $m > a_5^o \geq 0,\ 1 \geq a_8^o > b,\ \pi_A^6(m,b) \geq max\{\pi_A^5(a_5^o,b),\pi_A^8(a_8^o,b)\}$ *OR* $a_5^o \geq m,\ 1 \geq a_8^o > b,\ \pi_A^6(m,b) \geq \pi_A^8(a_8^o,b)$ *OR* $m > a_5^o \geq 0,\ b \geq a_8^o,\ \pi_A^6(m,b) \geq \pi_A^5(a_5^o,b)$ *OR* $a_5^o \geq m,\ b \geq a_8^o$;
b	$m > a_5^o \geq 0,\ b > a_6^o \geq m,\ 1 \geq a_8^o > b,\ \pi_A^7(b,b) \geq max\{\pi_A^5(a_5^o,b),\pi_A^6(a_6^o,b),\pi_A^8(a_8^o,b)\}$ *OR* $m > a_5^o \geq 0,\ m > a_6^o,\ 1 \geq a_8^o > b,\ \pi_A^7(b,b) \geq max\{\pi_A^5(a_5^o,b),\pi_A^6(m,b),\pi_A^8(a_8^o,b)\}$ *OR* $m > a_5^o \geq 0,\ b > a_6^o \geq m,\ b \geq a_8^o,\ \pi_A^7(b,b) \geq max\{\pi_A^5(a_5^o,b),\pi_A^6(a_6^o,b)\}$ *OR* $m > a_5^o \geq 0,\ m > a_6^o,\ b \geq a_8^o,\ \pi_A^7(b,b) \geq max\{\pi_A^5(a_5^o,b),\pi_A^6(m,b)\}$ *OR* $a_5^o \geq m,\ b > a_6^o \geq m,\ 1 \geq a_8^o > b,\ \pi_A^7(b,b) \geq max\{\pi_A^6(a_6^o,b),\pi_A^8(a_8^o,b)\}$ *OR* $a_5^o \geq m,\ m > a_6^o,\ 1 \geq a_8^o > b,\ \pi_A^7(b,b) \geq max\{\pi_A^6(m,b),\pi_A^8(a_8^o,b)\}$ *OR* $a_5^o \geq m,\ a_6^o \geq b,\ 1 \geq a_8^o > b,\ \pi_A^7(b,b) \geq \pi_A^8(a_8^o,b)$ *OR* $a_5^o \geq m,\ b > a_6^o \geq m,\ b \geq a_8^o,\ \pi_A^7(b,b) \geq \pi_A^6(a_6^o,b)$ *OR* $a_5^o \geq m,\ m > a_6^o,\ b \geq a_8^o,\ \pi_A^7(b,b) \geq \pi_A^6(m,b)$ *OR* $a_5^o \geq m,\ a_6^o \geq b,\ b \geq a_8^o$;
$\emptyset$	*otherwise.*

The open forms of expressions a_5^o, a_6^o, a_8^o, $\pi_A^5(a_5^o,b)$, $\pi_A^7(b,b)$, $\pi_A^6(m,b)$, $\pi_A^6(a_6^o,b)$, and $\pi_A^8(a_8^o,b)$ are available in Table 4.1.

When the warehouse is exactly in the middle of the unit line, i.e., $m = \frac{1}{2}$, Proposition 4.3 introduces a sufficient condition ensuring that the retailers target the warehouse's location for their stores, in equilibrium. (A retailer does not deviate unilaterally from her location in equilibrium, because if she does so, her profits decrease.)

Proposition 4.3. *Suppose that $m = \frac{1}{2}$ and $p \geq 2c_c + 6c_t$. Then retailers A and B locate their stores at the midpoint of the unit line (i.e., at the same location as the warehouse) in equilibrium.*

When p is sufficiently large, market incentives dominate the transportation and distance concerns in retailers' location decisions. In other words, the retailers have *less* incentive to stay close to the warehouse and consumers, but *more* incentive to capture more demand under competition. As a result, in equilibrium the retailers choose the same location so that the total demand is split equally between the stores. Propositions 4.4 and 4.5 below indicate that when p takes higher values, the same location equilibrium is not limited to the midpoint of the unit line.

Proposition 4.4. *For all* $b \in [0, m]$ *such that*

$$\frac{-p + 4(c_c - c_t) + b(c_c + 2c_t)}{5c_c - 2c_t} \leq m \leq \frac{p + b(4c_c - 2c_t) - 4(c_c + c_t)}{2c_t}, \tag{4.1}$$

(b, b) *is an equilibrium solution.*

Any point $b \leq m$ satisfying the condition introduced in Proposition 4.4 is a symmetric equilibrium location for both retailers. If p is very close to c_c, then the interval in condition (4.1) for m is infeasible, and hence symmetric equilibria do not arise. Thus the stores may choose asymmetric locations on the unit line, in the middle of their respective consumer bases to reduce the consumer transportation costs, or equilibrium may not arise at all. When $b = m = \frac{1}{2}$, then the condition (4.1) simplifies into $2c_c + 6c_t \leq p$, which clearly implies that p should be significantly higher than c_c to have symmetric equilibrium.

Proposition 4.5. *For all* $b \in (m, 1]$ *such that*

$$\frac{b(4c_c + 6c_t) - p}{2c_t} \leq m \leq \frac{p + b(c_c + 2c_t)}{5c_c - 2c_t}, \tag{4.2}$$

(b, b) *is an equilibrium solution.*

Any point $b > m$ satisfying the condition introduced in Proposition 4.5 is a symmetric equilibrium location for both retailers. Again, if p is very close to c_c, then the interval in condition (4.2) for m is infeasible, and hence symmetric equilibria do not arise. Thus the stores may choose asymmetric locations or equilibrium may not arise at all. When $b = m = \frac{1}{2}$, then the condition (4.2) simplifies into $2c_c + 2c_t \leq p$, which again implies that p should be significantly higher than c_c to have symmetric equilibrium.

Propositions 4.4 and 4.5 imply that when p is significantly higher than c_c, the market incentives dominate the transportation related costs and both retailers want to serve a larger demand. Thus both retailers choose the same location so that the total demand is split equally between them: It is more crucial to cover the demand as much as possible than to stay closer to the warehouse or consumers. Last, note that as p increases, the interval of such symmetric equilibria tends to increase.

Table 4.1: Expressions and their open forms.

Expression	Open Form
a_1^o	$\frac{p+c_cb+2c_t(b-m)}{5c_c-4c_t}$
a_3^o	$\frac{-p+c_c(4+b)+2c_t(2+m-b)}{5c_c+4c_t}$
a_4^o	$\frac{-p+c_c(4+b)-2c_t(2+m-b)}{5c_c-4c_t}$
$\pi_A^1(a_1^o, b)$	$\lambda\left(\frac{(p-2c_tm)^2+4b(-3c_c+c_t)(2c_tm-p)+4b^2(-c_c^2+2c_cc_t+c_t^2)}{4(5c_c-4c_t)}\right)$
$\pi_A^2(b, b)$	$\lambda\left(\frac{p}{2}-\frac{c_c(1-2b+2b^2)}{2}-c_t(m-b)\right)$
$\pi_A^3(m, b)$	$-\lambda\left(\frac{p(-2+m+b)}{2}+\frac{c_c(4+5m^2+b^2-2m(4+b))}{4}\right)$
$\pi_A^3(a_3^o, b)$	$\lambda\left(\frac{-4c_c^2(-1+b)^2-4c_c(-1+b)(3p+2c_t(2-3m+b))+(p-2c_t(-2+m+b))^2}{4(5c_c+4c_t)}\right)$
$\pi_A^4(a_4^o, b)$	$\lambda\left(\frac{-4c_c^2(-1+b)^2+4c_c(-1+b)(-3p+2c_t(2-3m+b))+((p+2c_t(-2+m+b))^2}{4(5c_c-4c_t)}\right)$
a_5^o	$\frac{p+c_cb+2c_t(b-m)}{5c_c-4c_t}$
a_6^o	$\frac{p+c_cb+2c_t(m-b)}{5c_c+4c_t}$
a_8^o	$\frac{-p+c_c(4+b)+2c_t(b-m-2)}{5c_c-4c_t}$
$\pi_A^5(a_5^o, b)$	$\lambda\left(\frac{(-2c_tm+p)^2+4(-3c_c+c_t)(2c_tm-p)b+4(-c_c^2+2c_cc_t+c_t^2)b^2}{4(5c_c-4c_t)}\right)$
$\pi_A^6(m, b)$	$\lambda\left(\frac{p(m+b)}{2}-\frac{c_c(5m^2-2mb+b^2)}{4}\right)$
$\pi_A^6(a_6^o, b)$	$\lambda\left(\frac{(2c_tm+p)^2+4(3c_c+c_t)(2c_tm+p)b-4(c_c^2+2c_cc_t-c_t^2)b^2}{4(5c_c+4c_t)}\right)$
$\pi_A^7(b, b)$	$\lambda\left(\frac{p}{2}-\frac{c_c(1-2b+2b^2)}{2}-c_t(b-m)\right)$
$\pi_A^8(a_8^o, b)$	$\lambda\left(\frac{-4c_c^2(-1+b)^2+4c_c(-1+b)(-3p+2c_t(2-3m+b))}{4(5c_c-4c_t)}\right)$

Chapter 5

Centralized System

In this chapter, we consider a single retail chain who wants to locate two of her own stores on the unit line $[0, 1]$ so as to maximize her total profit. Thus the optimization problem of such a retail chain is given by

$$\begin{aligned} \underset{a,b}{\text{maximize}} \quad & \pi_A(a,b) + \pi_B(a,b) \\ \text{subject to} \quad & 0 \leq a, b \leq 1. \end{aligned}$$

We again assume that in-store price p is sufficiently large so that it is always optimal to stay in the market, i.e., the profits are non-negative. The proofs of all analytical results are again available in Appendix C.

Table 5.1 exhibits the total profit function that arises in each of the eight cases described in Chapter 3: $\pi^i_{Total}(a, b)$ is the total profit when the stores are located at points a and b such that case (i) holds, i.e., $\pi^i_{Total}(a, b) = \pi^i_A(a, b) + \pi^i_B(a, b)$. The total profit function, in Table 5.1, is a piecewise function in both a and b. Lemma 5.1 shows that the total profit function is continuous in both a and b. Lemma 5.2 shows that the total profit function in each case (i.e., each piece) is jointly concave in a and b.

Table 5.1: Total profit functions in the centralized system.

i	*Case*	$\pi^i_{Total}(a,b)$
1	$0 \le a < b \le m \le 1$	$\lambda(p + c_c(2b + ab - 1 - \frac{3a^2+3b^2}{2}) + c_t(2b - 2m + a^2 - b^2))$
2	$0 \le a = b \le m \le 1$	$\lambda(p - c_c(a^2 - a + b^2 - b + 1) - c_t(2m - a - b))$
3	$0 \le b < a \le m \le 1$	$\lambda(p + c_c(2a + ab - 1 - \frac{3a^2+3b^2}{2}) + c_t(2a - 2m + b^2 - a^2))$
4	$0 \le b \le m < a \le 1$	$\lambda(p + c_c(ab + 2a - 1 - \frac{3a^2+3b^2}{2}) - c_t(2ma + 2mb - 2ab - a^2 - b^2 + 2a - 2m))$
5	$0 \le a < m < b \le 1$	$\lambda(p + c_c(ab + 2b - 1 - \frac{3a^2+3b^2}{2}) - c_t(2ma + 2mb - 2ab - a^2 - b^2 + 2b - 2m))$
6	$0 \le m \le a < b \le 1$	$\lambda(p + c_c(2b + ab - 1 - \frac{3a^2+3b^2}{2}) + c_t(2m - 2b + b^2 - a^2))$
7	$0 \le m < a = b \le 1$	$\lambda(p - c_c(a^2 - a + b^2 - b + 1) + c_t(2m - a - b))$
8	$0 \le m < b < a \le 1$	$\lambda(p + c_c(2a + ab - 1 - \frac{3a^2+3b^2}{2}) + c_t(2m - 2a + a^2 - b^2))$

Lemma 5.1. *For a given b (or a), the total profit function of a centralized system with stores located at points a and b is continuous in a (or b).*

Lemma 5.2. *The total profit function $\pi^i_{Total}(a,b)$ is jointly concave in a and b in its respective feasible region, $\forall i$.*

Since the total profit functions are continuous and each piece is jointly concave in its feasible region, we are able to develop a solution algorithm for the optimization problem of the single retail chain: We can easily find the optimal solution in each case by solving the first order conditions simultaneously. However, the optimal solution may not be in the interior of the feasible region. In such cases, we calculate the optimal solutions over the end-points of a and b that can be achieved in the feasible region. We keep the optimal end-point solution that yields the maximum profit in these cases. We repeat this procedure and obtain the optimal locations and profit, if any, in each case. We then compare these profits across all cases and select the point that leads to maximum profit.

Algorithm 1 below finds the optimal solutions in the interior of the respective feasible regions in the main problem and compares these solutions, in order to compute the optimal total profit and optimal store locations. Algorithm 2 below shows the pseudo code for implementation of steps 2-4 of Algorithm 1 in case (1), for example. See Appendix D for the pseudo codes in cases (2)–(8). Proposition 5.3 proves that Algorithm 1 always finds an optimal solution across all cases. Proposition 5.4 says that Algorithm 1 also minimizes the total transportation cost in the centralized system.

Algorithm 1 Optimal store locations for the single retail chain.

1: Set $i = 1$.

2: Identify the end-points of the intervals for a and b in case (i).

3: Find the global optima for the unconstrained problem in case (i).

- Calculate the first order conditions of $\pi^i_{Total}(a, b)$.
- Solve these two equations simultaneously to find the global optima (a_i, b_i).

4: IF (a_i, b_i) belongs to the interval of case (i), then (a_i, b_i) is an optimal solution for case (i).
ELSE

- Find the optimal profit over the feasible region of a, at each end-point of the interval of b that can be achieved. If we cannot specify an end-point for b, then there exists no solution.
- Find the optimal profit over the feasible region of b, at each end-point of the interval of a that can be achieved. If we cannot specify an end-point for a, then there exists no solution.
- $(\tilde{a}_i, \tilde{b}_i)$ yielding the maximum profit across all feasible end-point solutions is an optimal solution in case (i). Set $(a_i, b_i) = (\tilde{a}_i, \tilde{b}_i)$.
- If there exists no $(\tilde{a}_i, \tilde{b}_i)$, then there exists no solution in case (i).

5: IF $i < 8$, set $i = i + 1$ and go to step 2.
ELSE let $i^* = \arg\max_{i\in\{1,..,8\}} \pi^i_{Total}(a_i, b_i)$. (a_{i^*}, b_{i^*}) are the optimal locations and $\pi^{i^*}_{Total}(a_{i^*}, b_{i^*})$ is the optimal total profit.

Proposition 5.3. *There always exists an optimal solution in the centralized system. Algorithm 1 always finds the optimal solution in the centralized system.*

Proposition 5.4. *Algorithm 1 minimizes the total transportation cost in the system.*

In Chapter 6 we will numerically compare the centralized solution to the decentralized solution to investigate the impacts of competition on the location decisions and the resulting costs. In Chapter 6 we employ Algorithm 1 to find the centralized solution.

Algorithm 2 Pseudo code for steps 2–4 of Algorithm 1 in case (1)

1: Identify the end-points of the intervals for a and b in case (1). The upper end-points are $(a_1^U, b_1^U) = (\text{undefined}, m)$ and the lower end-points are $(a_1^L, b_1^L) = (0,\text{undefined})$.

2: Find the global optima for the unconstrained problem in case (1).

  Calculate the partial derivatives of $\pi^1_{Total}(a, b)$.

  Solve these two equations simultaneously to find the critical point (a_1, b_1).

3: IF (a_1, b_1) belongs to the interval of case (1), then (a_1, b_1) is an optimal solution in case (1).

ELSE

  Set $b_1 = m$ and take the derivative of $\pi^1_{Total}(a, b_1)$ to find a_1.

  IF a_1 belongs to the interval of case (1), then (a_1, b_1) is a feasible solution in case (1).

  ELSEIF $a_1 \geq b_1$, no solution exists.

  ELSEIF $a_1 \leq 0$, set $a_1 = 0$ to find the value $\pi^1_{Total}(a_1, b_1)$.

  END

  Set $a_1 = 0$ and take the derivative of $\pi^1_{Total}(a_1, b)$ to find b_1.

  IF b_1 belongs to the interval of case (1), then (a_1, b_1) is a feasible solution in case (1).

  ELSEIF $a_1 \geq b_1$, no solution exists.

  ELSEIF $b_1 \geq m$, set $b_1 = m$ to find the value $\pi^1_{Total}(a_1, b_1)$.

  END

  The end-point solution $(\tilde{a}_1, \tilde{b}_1)$ yielding the maximum profit is an optimal solution in case (1). Set $(a_1, b_1) = (\tilde{a}_1, \tilde{b}_1)$.

END

Chapter 6

Numerical Experiments

Demands and profits in the decentralized system depend on the distances between the warehouse and stores, as well as the store locations relative to each other, making our problem difficult to analyze. Although we analytically characterized the best response mappings in the decentralized system, we characterized the equilibrium locations only under certain conditions on our problem parameters (see Chapter 4). In this chapter, we numerically identify the equilibrium locations, if any, in order to provide insights into the location problem in general. Specifically, we conduct numerical experiments to investigate how system parameters affect store location decisions in both the decentralized and centralized systems. We also examine the impacts of competition on store locations, emissions, and profits, comparing the decentralized system equilibrium to the centralized solution.

In our experiments, we consider instances in which p varies between 10 and 15, c_c varies between 5 and 9, c_t varies between 0 and 2 (and thus Assumption 1 holds in all instances), $m \in \{0.3, 0.5, 0.8, 1\}$, and $\lambda = 10$. We find the equilibrium locations by finding the intersection of the best responses. For the decentralized system, in each of our instances, we observed one of the following: symmetric equilibrium, asymmetric equilibrium, or no equilibrium. Without loss of generality, if there are two equilibria such that $(a, b) = (x, y)$ and $(a, b) = (y, x)$, then we only present the pair with $a < b$. We have a tolerance of 0.005 in our computations: If the difference is greater than the tolerance, then the best responses do not intersect, and no equilibrium results. All tables and figures are presented at the end of this chapter.

6.1 Decentralized System

In this section, we examine how the equilibrium responds to a change in the price in each of the following four different configurations: (1) both c_c and c_t are low, (2) c_c is low and c_t is high, (3) c_c is high and c_t is low, and (4) both c_c and c_t are high. We repeat our experiments for various locations of the warehouse. We also investigate (i) how the equilibrium changes with consumer transportation cost c_c, again under four different cases of p and c_t, and (ii) how it changes with the replenishment transportation cost c_t, once again in four different cases of p and c_c. Since our results for (i) and (ii) are similar to those presented in Figures 6.1–6.3, we relegated our results for (i) and (ii) to Appendix E.

Note that the costs c_c and c_t vary depending on the emission and fuel prices. Also, the non-fuel variable cost (v_j), the amount of fuel used (f_j), the per unit cost of fuel (p_j), and the load carried by the vehicle (q_j) may greatly vary from one vehicle type to another; and thus the costs c_c and c_t are significantly affected by vehicle choices of the retailers and consumers. Our numerical experiments thus provide insights into the location problem for various vehicle choices, as well as various product, emission, and fuel prices.

First we examine how the equilibrium responds to a change in the price p when $m = 0.3$ (see Figure 6.1):

Low c_c and low c_t. We observe from Figure 6.1(a) that, for each value of p, symmetric equilibria exist in the market, i.e., both retailers choose the same location in equilibrium. The intuition behind the same location equilibrium is that, given the location choice of retailer B (or A), retailer A (or B) wants to capture a bigger market than retailer B because the market effect dominates the transportation-related costs. The retailers can eventually reach an equilibrium only when both retailers choose the same location. Proposition 4.4 and 4.5 support this argument when transportation-related costs are relatively small with respect to p. This result is also in line with the findings of Hotelling (1929), where firms compete for demand and choose the same location, in the middle of the market. We also observe that as p increases, the number of possible equilibrium locations tends to increase: The retailers form symmetric equilibria since the price of the product further dominates the consumer and replenishment transportation costs, and thus increasing the demand becomes more crucial than being close to the warehouse or consumer base.

Warehouse location m becomes equilibrium point only when p is high enough. For instance, if $m = 0.3$, $p = 11.5$, $c_c = 5$, and $c_t = 0.5$, then $a = b = 0.3$ is not an equilibrium solution because when $b = 0.3$, retailer A's best response is 0.35, in order to have a larger consumer base without increasing her consumer transportation costs too much. The total costs of retailer A increase, and her profit decreases as she gets closer to B. However, when $p = 15$, market incentives are even more dominant, extending the line of possible symmetric equilibria: Retailer A gets her highest profit when she is around 0.31, and thus retailer A wants to approach retail store B for a larger consumer base and to increase her profits due to the market incentives. But the stores eventually end up at the same location, as shown in Chapter 4. Last, we observe that the symmetric equilibrium locations when $p = 15$ can be specified by the interval $[0.3, 0.67]$: The equilibrium locations are widely dispersed in the market.

Low c_c and high c_t. When c_t is high, we observe from Figure 6.1(b) that symmetric equilibria still exist in the market. Both retailers again choose the same location in each of these equilibrium solutions for the same reasons described above. However, Figures 6.1(a) and (b) together indicate that, for many values of p, the number of equilibria is lower when c_t is higher. For instance, let $b = 0.6$. Then retailer A's total costs are very high when retailer A is close to retailer B. Therefore retailer A'a best response is around 0.48 to be close to warehouse and reduce her transportation-related costs. Hence the retailers want to be closer to the warehouse due to higher replenishment costs, leading to a narrower range of symmetric equilibria, which is closer to m. This also explains why the number of equilibria only slightly increases as p increases in Figure 6.1(b).

High c_c and low c_t. When c_c is high, asymmetric equilibrium arises for each value of p (see Figure 6.1(c)), i.e., the retailers choose different points: High transportation costs of consumers induce the retailers to move closer to the center of their respective market segments and further away from each other. Figure 6.1(c) also shows that, for each value of p, one store is located close to the center of the market whereas the other is located on the side that contains the warehouse: Store B has the advantage of being visited by a larger group of consumers, whereas store A has the advantage of being closer to the warehouse. Last, as p increases, we observe that the stores approach each other as a result of the market and margin effects. When the price becomes sufficiently large (i.e., for some $p > 15$), we observe that the retailers form symmetric equilibria (i.e., they choose the same location in equilibrium), as in Figure 6.1(a).

High c_c and high c_t. In this case, we again observe asymmetric equilibrium (see Figure 6.1(d)), but the store locations are now closer to the warehouse. Compared to Figure 6.1(c), both retailers are also closer to each other. For instance, let $p = 10$ and $c_c = 9$. When $c_t = 0.5$ the equilibrium points are (0.33, 0.63), whereas when $c_t = 2$ the equilibrium points are (0.3,0.56). The consumer transportation cost in this case has a weaker effect than in the previous case. Hence, the retailers slightly move from the midpoints of their respective markets towards the warehouse. For large values of p, market incentives dominate the transportation-related costs and both retailers want to choose the same location, leading to symmetric equilibria, as seen from Figure E.1(d) in Appendix E. Equilibrium may not even arise when p is too high. Likewise, equilibrium may also not arise when c_t is too high: The retailers want to be closer to their consumer bases to lower consumer transportation costs, but at the same time they want to be closer to warehouse to lower replenishment transportation costs, thus equilibrium may not exist.

Figures 6.1(a) to 6.1(d) together show that, for each value of p, increasing both c_c and c_t restricts location decisions, leading to fewer equilibrium solutions or no equilibrium. In addition, as c_c and c_t increase, the retailers choose their locations so as to partition the market more effectively. We observe a relatively high c_c is critical in inducing an asymmetric equilibrium. A high c_t by itself is not sufficient to induce asymmetric equilibrium, but is very effective to induce the store locations to approach to the warehouse.

Change in m. We next consider the instances in which $m = 0.5$ (see Figure 6.2) and $m = 1$ (see Figure 6.3). The basic insights that we generated from Figure 6.1 continue to hold in Figures 6.2 and 6.3. Unlike Figure 6.1, equilibrium exists in each of our instances in Figure 6.2. We also note from Figure 6.2 that (i) the intervals identifying the equilibrium solutions in which both retailers choose the same location are *symmetric* with respect to the midpoint of the unit line, and (ii) the retail locations in the asymmetric equilibria are again *symmetric* with respect to the midpoint. We observe from Figures 6.2(c) and (d) that high c_c values induce the retailers to approach the middle of their respective markets. Note the significant shift of the symmetric equilibria range between Figure 6.1(b) and Figure 6.3(b). The equilibrium locations tend to move towards the upper-end of the market since $m = 1$.

Figures 6.4 and 6.5 map out the existence and type of equilibrium with respect to various transportation costs relative to the market price, and various warehouse locations. When c_c/p is less than 0.5, the price dominates the consumer transportation cost, leading to greater demand incentives and thus symmetric equilibria in many cases. When c_c/p is higher than 0.75, one retailer takes advantage of being close to the warehouse whereas the other retailer takes advantage of being close to her consumer base, and an asymmetric equilibrium exists in many cases. When $m = 0.5$, the retailers again choose asymmetric locations on the unit line due to high c_c, but this time both stores want to be close to their consumer bases and to the warehouse to some extent. A change in c_c relative to p is more significant than that in c_t for the equilibrium structure. When p is much greater than c_c, the retailers choose the same location. However, when c_c is close to p, the retailers choose asymmetric locations in the middle of their respective markets in the equilibrium to reduce the consumer transportation costs.

When $m \in \{0.7, 0.8, 1\}$ and $c_c/p \in [0.5, 0.75]$, Figures 6.4 and 6.5 indicate that no equilibrium exists in a significant number of instances. When the costs are high, the stores want to reduce consumer and replenishment transportation cost at the same time. Equilibrium may not exist when c_c and c_t are comparable and the warehouse location favors one side of the market. In these cases, we do not observe any asymmetric equilibrium because the store that is further away from the warehouse wants to deviate to a closer location. We do not observe a symmetric equilibrium either because c_c is high enough (compared to p) for stores to move towards the centers of their respective markets.

Specifically, when $m = 0.3$, $p = 14$, $c_c = 9$, and $c_t = 2$, equilibrium does not exist. As retailer B's location b gets closer to the middle of the unit line, retailer A's location a changes so that $b < a$. However, when $b = 0.48$, retailer A jumps back to 0.33 because retailer A would pay more replenishment costs and a greater value of consumer transportation cost if she kept changing so that $b < a$ or staying at that location: A relatively smaller market but being close to the warehouse is more preferable for retailer A in this case. Therefore, although retailer A loses some of her customer base, she maintains her profit by reducing her costs, locating her store on the left of store B. Thus, the best responses do not match. Last, note that when $m \in \{0.7, 0.8, 1\}$ and $c_c/p \in [0.5, 0.75]$, as c_t/p decreases, the number of instances with no equilibrium tends to decrease since replenishment costs fade in comparison to the consumer transportation costs. And the stores are located in the middle of their respective markets and further away from each other, leading to asymmetric equilibrium.

6.2 Centralized System

From the perspective of a single retail chain, the optimal store locations are independent from the price p because the total revenue of the chain is the total revenue that can be obtained in the whole market. Thus we will focus on the store locations with respect to transportation costs via Figures 6.1–6.3. First we examine the optimal store locations when $m = 0.3$ (see Figure 6.1):

Low c_c and low c_t. Figure 6.1(a) indicates that store A is at the same location as the warehouse $m = 0.3$ and store B is at 0.75 in the optimal solution: Store A enjoys zero replenishment cost whereas store B has the advantage of being close to a very large group of consumers who are too far away from store A.

Low c_c and high c_t. Figure 6.1(b) indicates that store A is at the same location as the warehouse. However, unlike Figure 6.1(a), store B is closer to the warehouse: It is more crucial to reduce replenishment costs when c_t is high and close to the value of c_c.

High c_c and low c_t. The locations of store B are similar in Figures 6.1(a) and 6.1(c). However, in Figure 6.1(c), store A is no longer at $m = 0.3$; it is at 0.276 because high transportation costs of consumers induce the retail chain to get closer to their respective consumer bases. Store B is at 0.748 to maintain her profit. The markets of the stores change as well to balance out the transportation-related costs. The optimal store locations approach 0.25 and 0.75 as c_t decreases to zero. When the store locations are 0.25 and 0.75, the total demand is split equally between the two stores with minimal distances to their respective consumer bases. Last, as expected, the stores are closer to the warehouse in Figure 6.1(b) than in Figure 6.1(c).

High c_c and high c_t. Since the price has no impact on the optimal store locations, increasing both c_c and c_t may have an impact on the optimal locations only if their magnitude relative to each other changes. Store A is at the same location as the warehouse and store B is at 0.726 in the optimal solution. Although store B wants to be in the middle of her consumer base, high c_t forces her to be closer to the warehouse.

Change in m. We next examine the optimal store locations when $m = 0.5$; see Figure 6.2. Unlike Figure 6.1, the store locations are symmetric with respect to the midpoint in each of the four configurations of c_c and c_t. Also, the distance between the warehouse and any of the stores is greatest in Figure 6.2(c), and smallest in Figure 6.2(b). Under high consumer transportation costs, the retailers move away from each other. Under high replenishment transportation costs, the retailers get closer to the warehouse. Our results when $m = 1$ are similar to those when $m = 0.3$.

To sum up, in both decentralized and centralized systems, we observe that (i) as c_c increases the stores move further away from each other, and (ii) as c_t increases the stores approach the warehouse. If the costs are low, the decentralized system leads to a wide range of symmetric equilibria where both retailers choose the same location in equilibrium, as in Hotelling(1929) and De Palma (1989). Unlike Hotelling(1929) and De Palma (1989), we find multiple equilibria instead of one equilibrium location. The centralized system solution is always asymmetric and each store centers its respective consumer base, but the solution shifts in the market towards the side where the warehouse is located.

6.3 Comparison of Decentralized and Centralized Solutions

We will compare the centralized and decentralized solutions in terms of (i) total transportation costs, (ii) total consumer transportation costs, and (iii) total replenishment transportation costs (see Figures 6.6–6.14). See Appendix E for comparison of total profits. We define T_D and T_C as the total costs, TC_D and TC_C as the total consumer transportation costs, and TR_D and TR_C as the total replenishment costs, in the decentralized and centralized systems, respectively. For $k \in \{D, C\}$,

$$\begin{aligned} T_k &= (c_c d_{Ac}(a,b) + c_t d_{At}(a,b))\lambda_A(a,b) + (c_c d_{Bc}(a,b) + c_t d_{Bt}(a,b))\lambda_B(a,b), \\ TC_k &= c_c d_{Ac}(a,b)\lambda_A(a,b) + c_c d_{Bc}(a,b)\lambda_B(a,b), \\ TR_k &= c_t d_{At}(a,b)\lambda_A(a,b) + c_t d_{Bt}(a,b)\lambda_B(a,b). \end{aligned}$$

Total Transportation Cost. We calculate the percentage gap between decentralized and centralized solutions with respect to the total transportation costs, i.e., $100\frac{T_D - T_C}{T_C}$. We label this as the "Competition Carbon Penalty" in each of our instances where equilibrium exists (see Figures 6.6–6.8). We consider the maximum and minimum values of total transportation cost in the decentralized system for the instances with multiple symmetric equilibria. All of the carbon penalty values that are presented in Figures 6.6–6.14 can be found in Table 6.1. Total transportation costs, transportation costs of consumers, and transportation costs for replenishment can be found in Tables 6.2–6.4.

Recall that the centralized solution is not affected by a change in p, and thus the store locations remain unchanged. This implies that transportation-related costs (and carbon emissions) stay the same across different values of p in the centralized solution. Hence, as p increases, the carbon penalty can change only if the total costs in the decentralized system change.

We first examine the competition carbon penalty when $m = 0.3$ (see Figure 6.6). The competition carbon penalty increases with p. When c_c is low, in Figures 6.6(a) and 6.6(b), as p increases the minimum competition carbon penalty stays constant. The maximum competition carbon penalty increases because c_c is low enough to induce a widely dispersed symmetric equilibria. In addition, when c_t is higher, the stores in the decentralized system are closer to the warehouse to lower their replenishment transportation costs, hence reducing the competition carbon penalty much more than in Figure 6.6(a) (see Figures 6.1(b) and 6.6(b)).

When c_c is high, in Figures 6.6(c) and 6.6(d), the stores move further away from each other towards their respective consumer bases to reduce the consumer transportation costs. Thus the competition carbon penalty is lower than in Figures 6.6(a) and 6.6(b). When c_c is high, c_t has a weaker effect but is still useful in reducing the competition carbon penalty.

In both systems the lowest competition carbon penalty occurs when both c_c and c_t are high: High transportation-related costs mitigate the competition effects on retail locations and the decentralized solution approaches the centralized solution.

We next examine the competition carbon penalty when $m = 0.5$ (see Figure

6.7) and $m = 1$ (see Figure 6.8). As p increases, the competition carbon penalty for both warehouse locations has similar patterns to those in Figure 6.6. When c_c is low, the competition carbon penalty is lowest when $m = 0.5$: The average distance from the warehouse to any consumer is longer when $m = 0.3$ or $m = 1$. However, when c_c is high, the competition carbon penalty is lowest if $m = 1$ and highest if $m = 0.5$: The retailers choose asymmetric locations and are much closer to each other when m is located at the mid-point. When $m = 1$, the retailers choose asymmetric locations to partition the market more effectively when compared to the other two warehouse locations.

We observe that low margin markets are favorable for low competition carbon penalty. Increasing c_c is more effective than increasing c_t in pushing the decentralized system towards the centralized system. When c_c is high, m should be located at the end-point for the lowest competition carbon penalty. When c_c is low, c_t has an important effect in reducing the competition carbon penalty, and m should be located at 0.5 for the lowest competition carbon penalty. We below divide the total transportation costs into two groups: The total transportation costs for consumer travels and for replenishment.

Total Transportation Costs for Consumers. We calculate the percentage gap between decentralized and centralized solutions with respect to total consumer transportation costs, i.e., $100\frac{TC_D - TC_C}{TC_C}$. We label this as the "Competition Carbon Penalty for Consumers" in each of our instances for which equilibrium exists (see Figures 6.9–6.11). This also yields the percentage gap between decentralized and centralized solutions with respect to total distances traveled by consumers. We first examine the competition carbon penalty for consumers when $m = 0.3$.

We observe that when c_c is low, the stores form symmetric equilibria in a large range of the unit line, leading to a high competition carbon penalty for consumers. It further increases with p because the retailers may choose very distant locations in equilibrium. Increasing c_t does not decrease or may increase the competition carbon penalty for consumers. This is because the equilibrium points get closer to the warehouse so that the farthest consumers travel greater distances.

When c_c is high, the stores move further away from each other towards their respective consumer bases to reduce the consumer transportation costs. Thus the competition carbon penalty for consumers is lower than in Figure 6.9(a). An increment in c_t increases the competition carbon penalty for consumers slightly: The stores still want to be close to the center points of their respective markets to reduce consumer transportation costs since c_c is still significantly greater than c_t. Although the stores are closer to consumers, they still incur higher consumer transportation costs than in the centralized system. The competition carbon penalty for consumers is always higher in the decentralized system.

Low margin markets are desirable for low carbon penalty for consumers. However, the lowest carbon penalty for consumers occurs when c_c is high and c_t is low. This is because the retailers choose significantly distant locations towards their respective consumer bases in this case.

We also examine the carbon penalty for consumers when $m = 0.5$ (see Figure 6.10) and $m = 1$ (see Figure 6.11). As p increases, the carbon penalty for consumers for both warehouse locations have similar patterns to those in Figure 6.9. When c_c is low, the carbon penalty for consumers is again lowest when $m = 0.5$. However, when c_c is high, the carbon penalty for consumers is highest when $m = 0.5$. A warehouse favoring one side of the market induces the stores to be further away from each other, yielding lower consumer transportation costs than in the case of $m = 0.5$.

Total Transportation Costs for Replenishment. We calculate the percentage gap between decentralized and centralized solutions with respect to total replenishment transportation costs, i.e., $100\frac{TR_D - TR_C}{TR_C}$. We label this as the "Competition Carbon Penalty for Replenishment" in each of our instances for which equilibrium exists (see Figures 6.12–6.14). This also yields the percentage gap between decentralized and centralized solutions with respect to total distances traveled for replenishment. We first examine the competition carbon penalty for replenishment when $m = 0.3$.

Figure 6.12(a) indicates that when p and c_c are low, the maximum competition carbon penalty for replenishment is below zero and thus lower in the decentralized system: The centralized solution has two different store locations whereas symmetric equilibria arise in the decentralized system, in the interval between the locations of the centralized system. In particular, when p is sufficiently low, both stores are located closer to the warehouse in the decentralized system. As p increases, the maximum competition carbon penalty for replenishment increases because market incentives become more dominant, extending the line of possible symmetric equilibria and increasing the replenishment transportation cost.

When c_c is high, the carbon penalty for replenishment does not increase with p since one of the stores incurs low replenishment transportation costs whereas her rival enjoys greater demand. The retailer may locate her store at the same point as the warehouse when c_t is high or p is very low, thus eliminating the replenishment transportation costs totally. This is different from the centralized system because the centralized solution partitions the market with its stores: One of the store is at or around the warehouse whereas the other store is located at the other side of the market, further away from the decentralized solution. We observe when c_c is high, no matter what m and c_t are, the carbon penalty for replenishment is lower, and it decreases with p. The decentralized system is always more efficient in terms of the carbon penalty for replenishment.

We also examine the carbon penalty when $m = 0.5$ (see Figure 6.13) and $m = 1$ (see Figure 6.14). As p increases, the carbon penalty for replenishment for both warehouse locations have similar patterns to those in Figure 6.12. No matter what c_c is, the carbon penalty for replenishment is lowest when $m = 0.5$. This is because the store locations are asymmetric and much closer to the warehouse. The lowest carbon penalty for replenishment occurs when both c_c and c_t are high.

6.4 Summary of Insights

Symmetric equilibria arise in high margin markets, especially when c_c is low. When p is too high, the market incentives dominate the transportation costs, extending the line of possible symmetric equilibria. We observe that Hotelling (1929) results hold in general. If the market incentives dominate the transportation costs, the retailers choose the same locations in equilibrium, as in Hotelling(1929) and De Palma (1989). Unlike Hotelling(1929) and De Palma (1989), we find multiple equilibria instead of one equilibrium location. As p increases, the number of possible equilibrium locations tends to increase. As c_t increases, the retailers want to be closer to the warehouse, leading to a narrower range of symmetric equilibria that is closer to m. A warehouse favoring one side of the market (by being closer to an end-point) extends the line of possible symmetric equilibria. Last, the warehouse location becomes equilibrium point only when p is sufficiently high.

Asymmetric equilibrium arises in low margin markets, especially when c_c is high: The retailers locate their stores further away from each other and want to be closer to their respective market segments, leading to asymmetric equilibrium. As c_t increases, the retailers slightly shift from the midpoints of their respective markets towards the warehouse. For large values of p, the market incentives again dominate the transportation costs and both retailers want to choose the same location, leading to symmetric equilibria. A warehouse favoring one side of the market pulls the stores towards the warehouse.

Equilibrium does not exist when c_c and c_t are comparable in power and the warehouse location favors one side of the market. When the costs are high, stores want to reduce consumer and replenishment transportation costs at the same time. But since both factors are comparable in power, both stores deviate from their current locations and no equilibrium results. We do not observe an asymmetric equilibrium in these cases because the store that is further away from the warehouse wants to deviate to a closer location. We do not observe a symmetric equilibrium either because the stores also want to move towards the centers of their respective markets.

The carbon penalty tends to increase with the price. In low margin markets (especially when c_c is high), high c_c is very effective in reducing the carbon penalty, the carbon penalty for consumers, and the carbon penalty for replenishment. High c_t is only slightly effective in reducing the carbon penalty and the carbon penalty for replenishment, whereas it increases the carbon penalty for consumers. The carbon penalty and the carbon penalty for consumers are lower if m is at the end-point. The competition carbon penalty for replenishment is lower if m is at the mid-point.

In high margin markets (especially when c_c is low), high c_t is very effective in reducing the carbon penalty. But it may increase the carbon penalty for consumers and replenishment. The carbon penalty, and the carbon penalty for consumers, and the carbon penalty for replenishment are lower if m is at the mid-point.

From a central policymaker's perspective, imposing a tax policy for consumer travels to increase c_c may be very effective in reducing the carbon penalty: The decentralized system converges to the centralized system when c_c is high; see Figures 6.6–6.14. Increasing c_t has a minor effect, but it can still be useful in reducing the carbon penalty and the carbon penalty for replenishment. A high c_c with low c_t incurs the lowest carbon penalty for consumers. When c_c is high, the carbon penalty and the carbon penalty for consumers are lowest when m is at the end-point. The carbon penalty for replenishment is lower if m is at the mid-point.

Table 6.1: Competition carbon penalty values for $m \in \{0.3, 0.5, 1\}$.

			Competition Carbon Penalty (%)						Penalty for Consumers (%)						Penalty for Replenishment (%)					
			m = 0.3		m = 0.5		m = 1		m = 0.3		m = 0.5		m = 1		m = 0.3		m = 0.5		m = 1	
p	c_c	c_t	max	min	max	min	max	min	max	min	max	min	max	min	max	min	max	min	max	min
10	5	0.5	82	81	68	68	72	71	98	97	98	98	100	99	-11	-25	-100	-100	1	-4
10.5	5	0.5	82	80	68	68	72	70	101	97	98	98	101	99	-11	-39	-100	-100	1	-10
11	5	0.5	82	80	68	68	72	71	105	97	98	98	105	99	-11	-53	-100	-100	1	-16
11.5	5	0.5	85	80	70	68	73	71	110	97	98	98	110	99	-2	-67	-91	-100	3	-22
12	5	0.5	89	80	72	68	76	71	117	96	99	98	116	99	11	-81	-82	-100	7	-28
12.5	5	0.5	94	80	75	68	79	71	125	97	101	98	123	99	21	-95	-70	-100	11	-34
13	5	0.5	96	80	78	68	84	71	129	97	103	98	132	99	31	-100	-61	-100	15	-40
13.5	5	0.5	96	80	83	68	90	71	129	97	107	98	143	99	40	-100	-51	-100	19	-46
14	5	0.5	100	80	88	68	97	71	129	97	111	98	155	99	50	-100	-42	-100	23	-52
14.5	5	0.5	105	80	93	68	105	71	129	97	116	98	168	99	59	-100	-33	-100	29	-58
15	5	0.5	111	80	99	68	114	71	129	97	121	98	183	99	72	-100	-24	-100	33	-64
10	5	2	52	38	22	22	38	31	120	90	72	72	128	83	-13	-100	-100	-100	2	-45
10.5	5	2	52	38	22	22	39	31	120	90	72	72	154	83	-13	-100	-100	-100	4	-59
11	5	2	54	38	22	22	40	31	120	90	72	72	185	83	-7	-100	-100	-100	4	-72
11.5	5	2	54	38	22	22	49	31	120	90	72	72	221	82	-7	-100	-100	-100	7	-86
12	5	2	55	38	22	22	59	31	120	89	72	72	263	82	-2	-100	-100	-100	9	-100
12.5	5	2	55	38	22	22	59	31	120	89	72	72	263	82	-2	-100	-100	-100	9	-100
13	5	2	55	38	22	22	59	31	120	89	72	72	263	82	-2	-100	-100	-100	11	-100
13.5	5	2	55	38	22	22	59	31	120	89	72	72	263	82	-2	-100	-100	-100	11	-100
14	5	2	55	38	22	22	59	31	120	89	72	72	263	82	-2	-100	-100	-100	11	-100
14.5	5	2	55	38	25	22	59	31	120	89	72	71	263	82	-2	-100	-88	-100	11	-100
15	5	2	58	38	28	22	59	31	120	88	73	72	263	82	7	-100	-80	-100	11	-100
10	9	0.5	12	12	16	16	11	11	15	15	22	22	14	14	-20	-20	-44	-44	-2	-2
10.5	9	0.5	15	15	19	19	14	14	18	18	26	26	17	17	-20	-20	-48	-48	-2	-2
11	9	0.5	18	18	22	22	17	17	22	22	30	30	21	21	-21	-21	-53	-53	-2	-2
11.5	9	0.5	21	21	26	26	20	20	26	26	34	34	24	24	-21	-21	-57	-57	-2	-2
12	9	0.5	25	25	30	28	23	23	30	30	39	37	28	28	-21	-21	-59	-61	-2	-2
12.5	9	0.5	29	29	34	32	26	25	34	34	44	42	32	31	-22	-22	-63	-65	-2	-3
13	9	0.5	33	33	38	36	30	28	38	38	49	47	37	35	-22	-22	-67	-69	-2	-3
13.5	9	0.5	37	37	41	41	32	32	43	43	53	53	40	40	-23	-23	-71	-71	-3	-3
14	9	0.5	42	42	46	46	36	36	48	48	59	59	45	45	-23	-23	-75	-75	-3	-3
14.5	9	0.5	47	47	51	51	41	41	54	54	65	65	50	50	-23	-23	-80	-80	-3	-3
15	9	0.5	52	52	56	56	45	45	59	59	71	71	56	56	-24	-24	-84	-84	-3	-3
10	9	2	10	10	15	15	5	5	24	24	39	39	16	16	-29	-29	-59	-59	-7	-7
10.5	9	2	11	11	18	18	7	7	27	27	45	45	18	18	-31	-31	-64	-64	-8	-8
11	9	2	14	14	21	21	8	8	32	32	50	50	21	21	-35	-35	-69	-69	-8	-8
11.5	9	2	16	16	24	24	10	10	35	35	56	56	25	25	-37	-37	-74	-74	-8	-8
12	9	2	17	17	27	27	12	12	38	38	63	63	29	29	-40	-40	-79	-79	-9	-9
12.5	9	2	21	20	31	31	13	13	45	42	69	69	32	32	-41	-44	-85	-85	-10	-10
13	9	2	24	24	35	35	15	15	49	49	76	76	37	37	-45	-45	-90	-90	-11	-11
13.5	9	2	28	28	39	39	18	18	55	55	83	83	41	41	-46	-46	-95	-95	-11	-11
14	9	2	-	-	43	43	20	20	-	-	91	91	46	46	-	-	-100	-100	-12	-12
14.5	9	2	-	-	43	43	23	23	-	-	91	91	51	51	-	-	-100	-100	-12	-12
15	9	2	-	-	43	43	26	26	-	-	91	91	57	57	-	-	-100	-100	-12	-12

Table 6.2: Total transportation costs for $m \in \{0.3, 0.5, 1\}$.

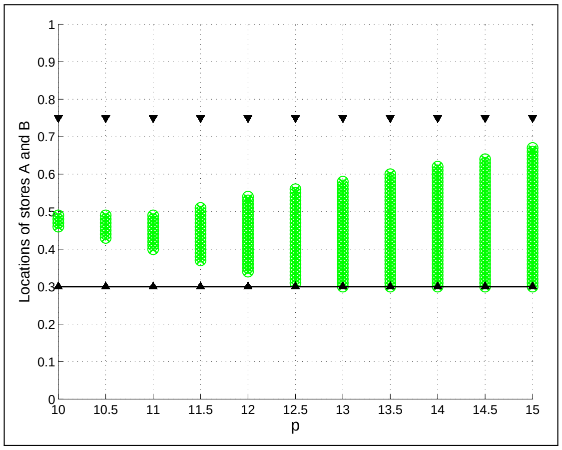
			m = 0.3			m = 0.5			m = 1		
p	c_c	c_t	max T_D	min T_D	T_C	max T_D	min T_D	T_C	max T_D	min T_D	T_C
10	5	0.5	26.91	26.76	14.83	25.00	25.00	14.88	29.91	29.77	17.44
10.5	5	0.5	26.91	26.75	14.83	25.00	25.00	14.88	29.91	29.72	17.44
11	5	0.5	27.00	26.75	14.83	25.00	25.00	14.88	29.91	29.75	17.44
11.5	5	0.5	27.39	26.75	14.83	25.24	25.00	14.88	30.14	29.75	17.44
12	5	0.5	27.96	26.75	14.83	25.56	25.00	14.88	30.62	29.75	17.44
12.5	5	0.5	28.71	26.75	14.83	25.96	25.00	14.88	31.28	29.75	17.44
13	5	0.5	29.00	26.75	14.83	26.54	25.00	14.88	32.12	29.75	17.44
13.5	5	0.5	29.00	26.75	14.83	27.24	25.00	14.88	33.13	29.75	17.44
14	5	0.5	29.64	26.75	14.83	27.99	25.00	14.88	34.32	29.75	17.44
14.5	5	0.5	30.36	26.75	14.83	28.75	25.00	14.88	35.69	29.75	17.44
15	5	0.5	31.29	26.75	14.83	29.59	25.00	14.88	37.23	29.75	17.44
10	5	2	31.89	29.00	20.98	25.00	25.00	20.50	43.25	41.00	31.41
10.5	5	2	31.89	29.00	20.98	25.00	25.00	20.50	43.56	41.00	31.41
11	5	2	32.24	29.00	20.98	25.00	25.00	20.50	44.12	41.00	31.41
11.5	5	2	32.24	29.00	20.98	25.00	25.00	20.50	46.69	41.00	31.41
12	5	2	32.61	29.00	20.98	25.00	25.00	20.50	50.00	41.00	31.41
12.5	5	2	32.61	29.00	20.98	25.00	25.00	20.50	50.00	41.00	31.41
13	5	2	32.61	29.00	20.98	25.00	25.00	20.50	50.00	41.00	31.41
13.5	5	2	32.61	29.00	20.98	25.00	25.00	20.50	50.00	41.00	31.41
14	5	2	32.61	29.00	20.98	25.00	25.00	20.50	50.00	41.00	31.41
14.5	5	2	32.61	29.00	20.98	25.53	25.00	20.50	50.00	41.00	31.41
15	5	2	33.15	29.00	20.98	26.29	25.00	20.50	50.00	41.00	31.41
10	9	0.5	27.90	27.90	24.91	28.89	28.89	24.93	30.60	30.60	27.47
10.5	9	0.5	28.60	28.60	24.91	29.65	29.65	24.93	31.28	31.28	27.47
11	9	0.5	29.37	29.37	24.91	30.48	30.48	24.93	32.03	32.03	27.47
11.5	9	0.5	30.19	30.19	24.91	31.37	31.37	24.93	32.84	32.84	27.47
12	9	0.5	31.08	31.08	24.91	32.34	31.95	24.93	33.72	33.72	27.47
12.5	9	0.5	32.04	32.04	24.91	33.37	32.95	24.93	34.67	34.29	27.47
13	9	0.5	33.06	33.06	24.91	34.47	34.02	24.93	35.69	35.28	27.47
13.5	9	0.5	34.14	34.14	24.91	35.16	35.16	24.93	36.34	36.34	27.47
14	9	0.5	35.29	35.29	24.91	36.36	36.36	24.93	37.46	37.46	27.47
14.5	9	0.5	36.51	36.51	24.91	37.64	37.64	24.93	38.65	38.65	27.47
15	9	0.5	37.78	37.78	24.91	38.97	38.97	24.93	39.90	39.90	27.47
10	9	2	34.49	34.49	31.32	36.10	36.10	31.39	44.15	44.15	41.93
10.5	9	2	34.89	34.89	31.32	36.96	36.96	31.39	44.72	44.72	41.93
11	9	2	35.74	35.74	31.32	37.90	37.90	31.39	45.37	45.37	41.93
11.5	9	2	36.21	36.21	31.32	38.90	38.90	31.39	46.08	46.08	41.93
12	9	2	36.69	36.69	31.32	39.98	39.98	31.39	46.86	46.86	41.93
12.5	9	2	37.97	37.48	31.32	41.12	41.12	31.39	47.47	47.47	41.93
13	9	2	38.97	38.97	31.32	42.34	42.34	31.39	48.37	48.37	41.93
13.5	9	2	40.04	40.04	31.32	43.64	43.64	31.39	49.34	49.34	41.93
14	9	2	-	-	-	45.00	45.00	31.39	50.39	50.39	41.93
14.5	9	2	-	-	-	45.00	45.00	31.39	51.50	51.50	41.93
15	9	2	-	-	-	45.00	45.00	31.39	52.69	52.69	41.93

Table 6.3: Total transportation costs for consumers for $m \in \{0.3, 0.5, 1\}$.

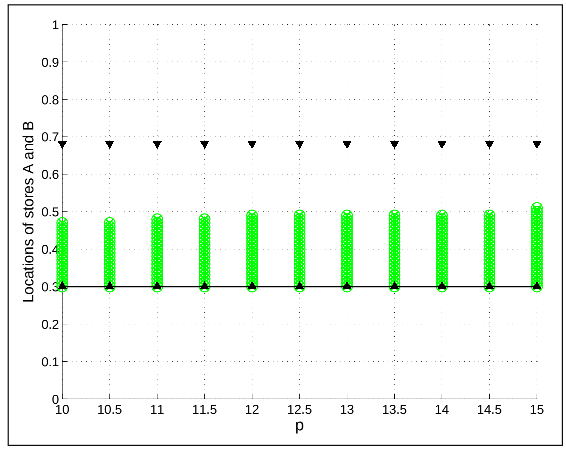
			m = 0.3			m = 0.5			m = 1		
p	c_c	c_t	max TC_D	min TC_D	TC_C	max TC_D	min TC_D	TC_C	max TC_D	min TC_D	TC_C
10	5	0.5	25.16	25.01	12.69	25.00	25.00	12.63	25.07	25.01	12.56
10.5	5	0.5	25.49	25.01	12.69	25.00	25.00	12.63	25.31	25.01	12.56
11	5	0.5	26.00	25.01	12.69	25.00	25.00	12.63	25.73	25.01	12.56
11.5	5	0.5	26.69	25.00	12.69	25.04	25.00	12.63	26.33	25.00	12.56
12	5	0.5	27.56	24.92	12.69	25.16	25.00	12.63	27.11	25.00	12.56
12.5	5	0.5	28.61	25.00	12.69	25.36	25.00	12.63	28.06	25.00	12.56
13	5	0.5	29.00	25.00	12.69	25.66	25.00	12.63	29.19	25.00	12.56
13.5	5	0.5	29.00	25.00	12.69	26.14	25.00	12.63	30.50	25.00	12.56
14	5	0.5	29.00	25.00	12.69	26.69	25.00	12.63	31.99	25.00	12.56
14.5	5	0.5	29.00	25.00	12.69	27.25	25.00	12.63	33.65	25.00	12.56
15	5	0.5	29.00	25.00	12.69	27.89	25.00	12.63	35.49	25.00	12.56
10	5	2	29.00	25.09	13.21	25.00	25.00	14.50	31.40	25.25	13.78
10.5	5	2	29.00	25.09	13.21	25.00	25.00	14.50	34.95	25.16	13.78
11	5	2	29.00	25.04	13.21	25.00	25.00	14.50	39.23	25.16	13.78
11.5	5	2	29.00	25.04	13.21	25.00	25.00	14.50	44.25	25.09	13.78
12	5	2	29.00	25.01	13.21	25.00	25.00	14.50	50.00	25.04	13.78
12.5	5	2	29.00	25.01	13.21	25.00	25.00	14.50	50.00	25.04	13.78
13	5	2	29.00	25.01	13.21	25.00	25.00	14.50	50.00	25.01	13.78
13.5	5	2	29.00	25.01	13.21	25.00	25.00	14.50	50.00	25.01	13.78
14	5	2	29.00	25.01	13.21	25.00	25.00	14.50	50.00	25.01	13.78
14.5	5	2	29.00	25.01	13.21	25.01	24.82	14.50	50.00	25.01	13.78
15	5	2	29.00	24.83	13.21	25.09	25.00	14.50	50.00	25.01	13.78
10	9	0.5	26.06	26.06	22.60	27.57	27.57	22.57	25.76	25.76	22.53
10.5	9	0.5	26.76	26.76	22.60	28.43	28.43	22.57	26.45	26.45	22.53
11	9	0.5	27.53	27.53	22.60	29.36	29.36	22.57	27.19	27.19	22.53
11.5	9	0.5	28.37	28.37	22.60	30.35	30.35	22.57	28.01	28.01	22.53
12	9	0.5	29.27	29.27	22.60	31.41	30.98	22.57	28.89	28.89	22.53
12.5	9	0.5	30.23	30.23	22.60	32.54	32.08	22.57	29.84	29.51	22.53
13	9	0.5	31.26	31.26	22.60	33.73	33.25	22.57	30.86	30.50	22.53
13.5	9	0.5	32.36	32.36	22.60	34.48	34.48	22.57	31.56	31.56	22.53
14	9	0.5	33.51	33.51	22.60	35.78	35.78	22.57	32.68	32.68	22.53
14.5	9	0.5	34.74	34.74	22.60	37.15	37.15	22.57	33.87	33.87	22.53
15	9	0.5	36.02	36.02	22.60	38.59	38.59	22.57	35.13	35.13	22.53
10	9	2	28.57	28.57	23.02	32.90	32.90	23.61	26.69	26.69	23.10
10.5	9	2	29.14	29.14	23.02	34.16	34.16	23.61	27.33	27.33	23.10
11	9	2	30.36	30.36	23.02	35.50	35.50	23.61	28.05	28.05	23.10
11.5	9	2	31.01	31.01	23.02	36.90	36.90	23.61	28.84	28.84	23.10
12	9	2	31.69	31.69	23.02	38.38	38.38	23.61	29.70	29.70	23.10
12.5	9	2	33.31	32.61	23.02	39.92	39.92	23.61	30.56	30.56	23.10
13	9	2	34.42	34.42	23.02	41.54	41.54	23.61	31.54	31.54	23.10
13.5	9	2	35.58	35.58	23.02	43.24	43.24	23.61	32.60	32.60	23.10
14	9	2	-	-	-	45.00	45.00	23.61	33.72	33.72	23.10
14.5	9	2	-	-	-	45.00	45.00	23.61	34.92	34.92	23.10
15	9	2	-	-	-	45.00	45.00	23.61	36.19	36.19	23.10

Table 6.4: Total transportation costs for replenishment for $m \in \{0.3, 0.5, 1\}$.

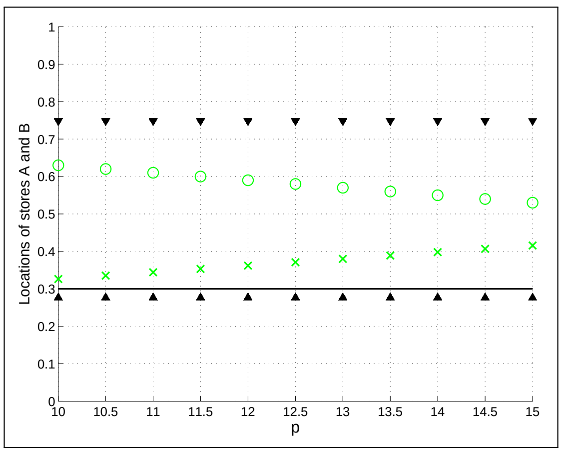
			m = 0.3			m = 0.5			m = 1		
p	c_c	c_t	max TR_D	min TR_D	TR_C	max TR_D	min TR_D	TR_C	max TR_D	min TR_D	TR_C
10	5	0.5	1.90	1.60	2.14	0.00	0.00	2.25	4.90	4.70	4.87
10.5	5	0.5	1.90	1.30	2.14	0.00	0.00	2.25	4.90	4.40	4.87
11	5	0.5	1.90	1.00	2.14	0.00	0.00	2.25	4.90	4.11	4.87
11.5	5	0.5	2.10	0.70	2.14	0.20	0.00	2.25	5.00	3.81	4.87
12	5	0.5	2.38	0.40	2.14	0.40	0.00	2.25	5.20	3.51	4.87
12.5	5	0.5	2.58	0.10	2.14	0.68	0.00	2.25	5.40	3.22	4.87
13	5	0.5	2.79	0.00	2.14	0.88	0.00	2.25	5.60	2.92	4.87
13.5	5	0.5	3.00	0.00	2.14	1.09	0.00	2.25	5.80	2.63	4.87
14	5	0.5	3.20	0.00	2.14	1.30	0.00	2.25	6.00	2.33	4.87
14.5	5	0.5	3.40	0.00	2.14	1.50	0.00	2.25	6.30	2.03	4.87
15	5	0.5	3.67	0.00	2.14	1.70	0.00	2.25	6.50	1.74	4.87
10	5	2	6.80	0.00	7.78	0.00	0.00	6.00	18.00	9.74	17.64
10.5	5	2	6.80	0.00	7.78	0.00	0.00	6.00	18.40	7.32	17.64
11	5	2	7.20	0.00	7.78	0.00	0.00	6.00	18.40	4.88	17.64
11.5	5	2	7.20	0.00	7.78	0.00	0.00	6.00	18.80	2.44	17.64
12	5	2	7.60	0.00	7.78	0.00	0.00	6.00	19.20	0.00	17.64
12.5	5	2	7.60	0.00	7.78	0.00	0.00	6.00	19.20	0.00	17.64
13	5	2	7.60	0.00	7.78	0.00	0.00	6.00	19.60	0.00	17.64
13.5	5	2	7.60	0.00	7.78	0.00	0.00	6.00	19.60	0.00	17.64
14	5	2	7.60	0.00	7.78	0.00	0.00	6.00	19.60	0.00	17.64
14.5	5	2	7.60	0.00	7.78	0.71	0.00	6.00	19.60	0.00	17.64
15	5	2	8.33	0.00	7.78	1.20	0.00	6.00	19.60	0.00	17.64
10	9	0.5	1.85	1.85	2.31	1.31	1.31	2.36	4.84	4.84	4.93
10.5	9	0.5	1.84	1.84	2.31	1.22	1.22	2.36	4.83	4.83	4.93
11	9	0.5	1.83	1.83	2.31	1.12	1.12	2.36	4.83	4.83	4.93
11.5	9	0.5	1.82	1.82	2.31	1.02	1.02	2.36	4.83	4.83	4.93
12	9	0.5	1.82	1.82	2.31	0.96	0.93	2.36	4.83	4.83	4.93
12.5	9	0.5	1.81	1.81	2.31	0.87	0.83	2.36	4.83	4.78	4.93
13	9	0.5	1.80	1.80	2.31	0.77	0.73	2.36	4.83	4.78	4.93
13.5	9	0.5	1.79	1.79	2.31	0.68	0.68	2.36	4.78	4.78	4.93
14	9	0.5	1.78	1.78	2.31	0.58	0.58	2.36	4.77	4.77	4.93
14.5	9	0.5	1.77	1.77	2.31	0.48	0.48	2.36	4.77	4.77	4.93
15	9	0.5	1.76	1.76	2.31	0.39	0.39	2.36	4.77	4.77	4.93
10	9	2	5.93	5.93	8.30	3.20	3.20	7.78	17.47	17.47	18.83
10.5	9	2	5.75	5.75	8.30	2.80	2.80	7.78	17.39	17.39	18.83
11	9	2	5.38	5.38	8.30	2.40	2.40	7.78	17.32	17.32	18.83
11.5	9	2	5.19	5.19	8.30	2.00	2.00	7.78	17.24	17.24	18.83
12	9	2	5.00	5.00	8.30	1.60	1.60	7.78	17.17	17.17	18.83
12.5	9	2	4.87	4.66	8.30	1.20	1.20	7.78	16.91	16.91	18.83
13	9	2	4.55	4.55	8.30	0.80	0.80	7.78	16.83	16.83	18.83
13.5	9	2	4.45	4.45	8.30	0.40	0.40	7.78	16.75	16.75	18.83
14	9	2	-	-	-	0.00	0.00	7.78	16.66	16.66	18.83
14.5	9	2	-	-	-	0.00	0.00	7.78	16.58	16.58	18.83
15	9	2	-	-	-	0.00	0.00	7.78	16.50	16.50	18.83



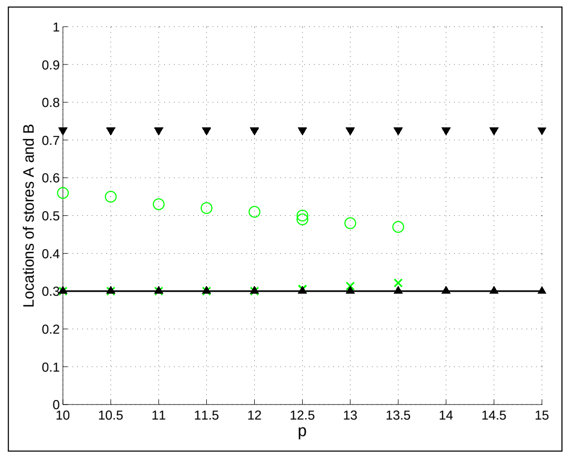
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.

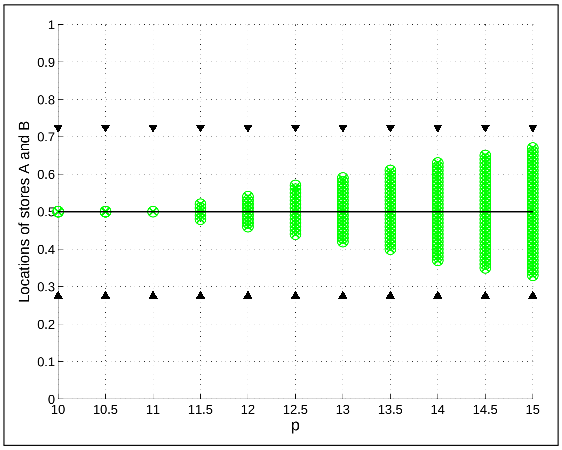


(c) $c_c = 9$ and $c_t = 0.5$.

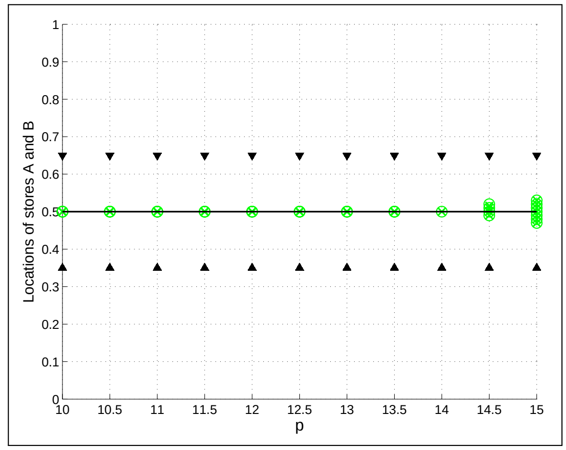


(d) $c_c = 9$ and $c_t = 2$.

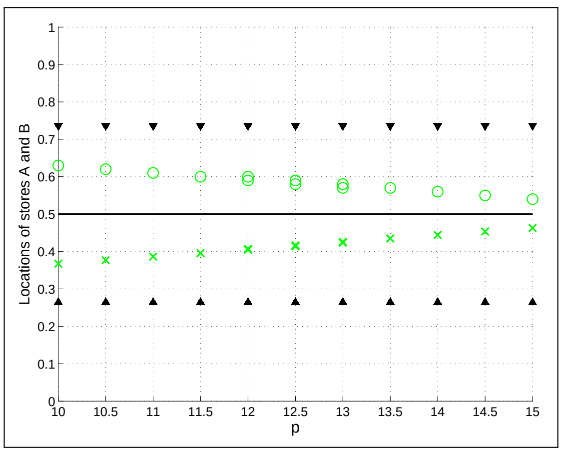
Figure 6.1: Locations in the centralized and decentralized systems when $m = 0.3$ and $\lambda = 10$. Locations of stores A and B are denoted by "▲" and "▼" in the centralized system, and by "x" and "o" in the decentralized system, respectively.



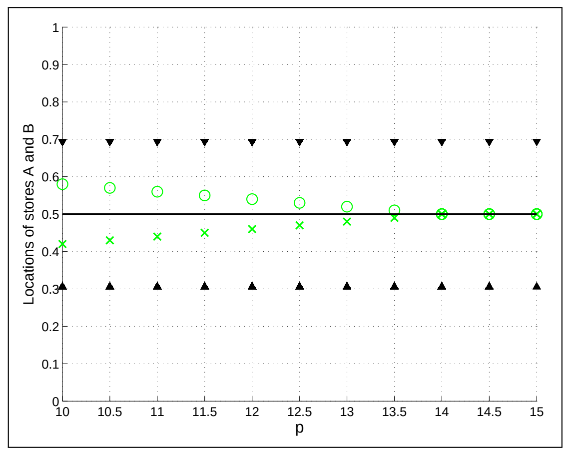
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.

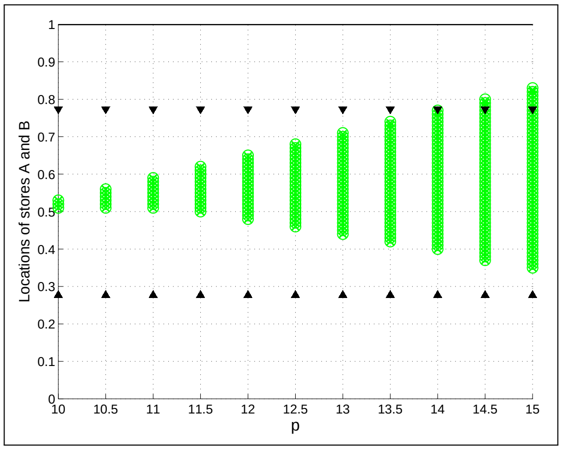


(c) $c_c = 9$ and $c_t = 0.5$.

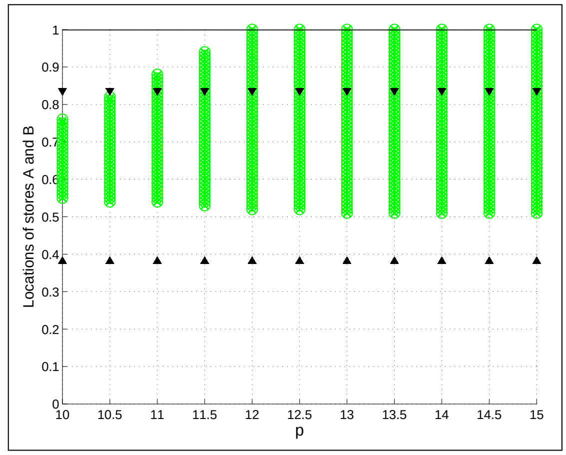


(d) $c_c = 9$ and $c_t = 2$.

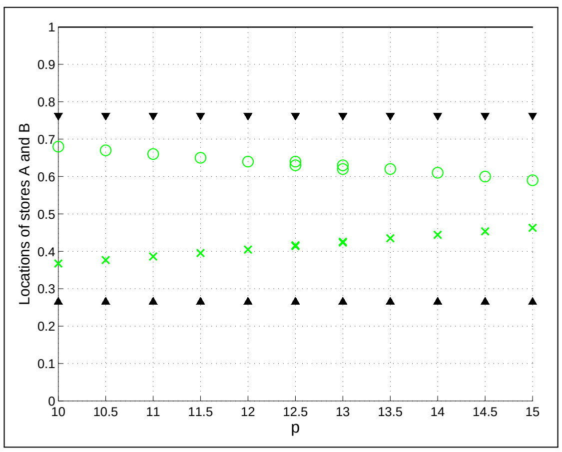
Figure 6.2: Locations in the centralized and decentralized systems when $m = 0.5$ and $\lambda = 10$. Locations of stores A and B are denoted by "▲" and "▼" in the centralized system, and by "x" and "o" in the decentralized system, respectively.



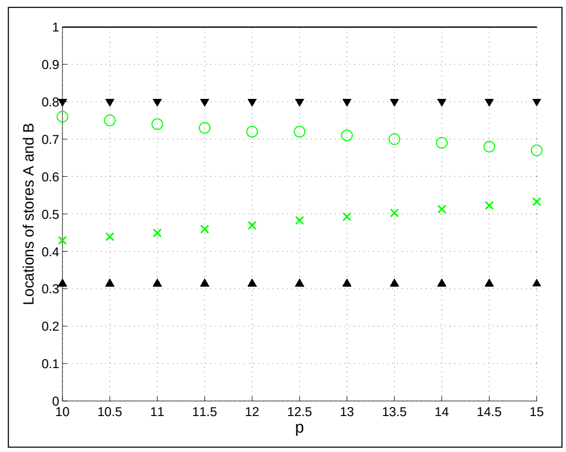
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.

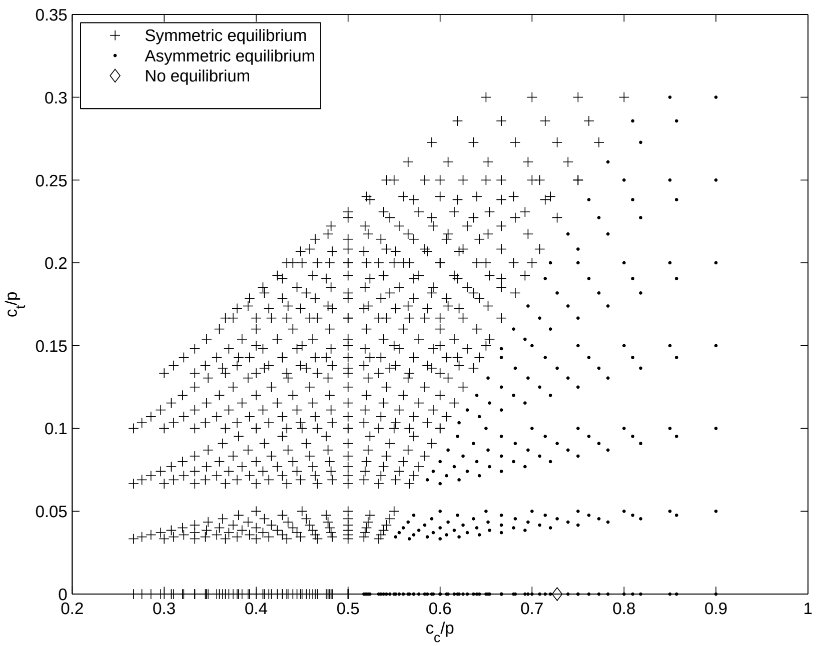


(c) $c_c = 9$ and $c_t = 0.5$.

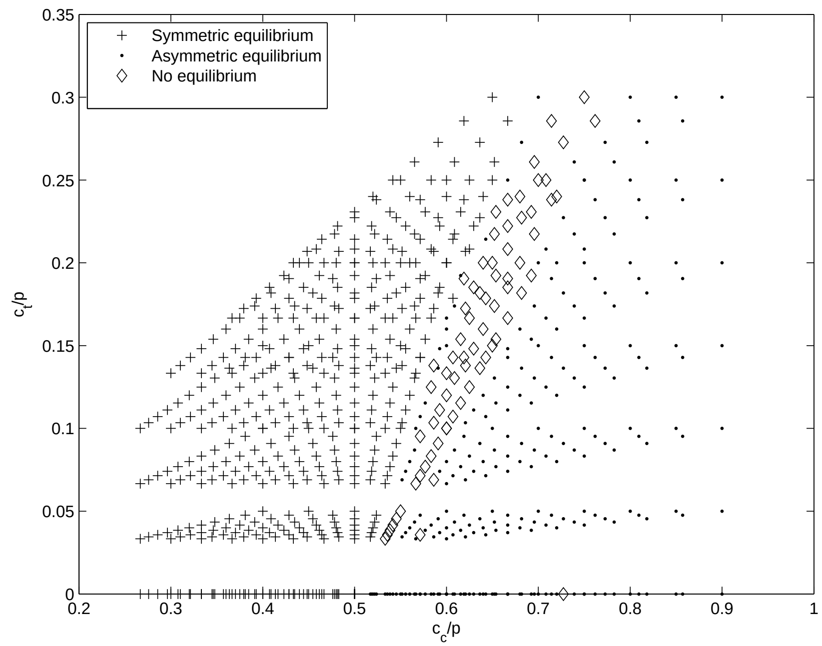


(d) $c_c = 9$ and $c_t = 2$.

Figure 6.3: Locations in the centralized and decentralized systems when $m = 1$ and $\lambda = 10$. Locations of stores A and B are denoted by "▲" and "▼" in the centralized system, and by "x" and "o" in the decentralized system, respectively.

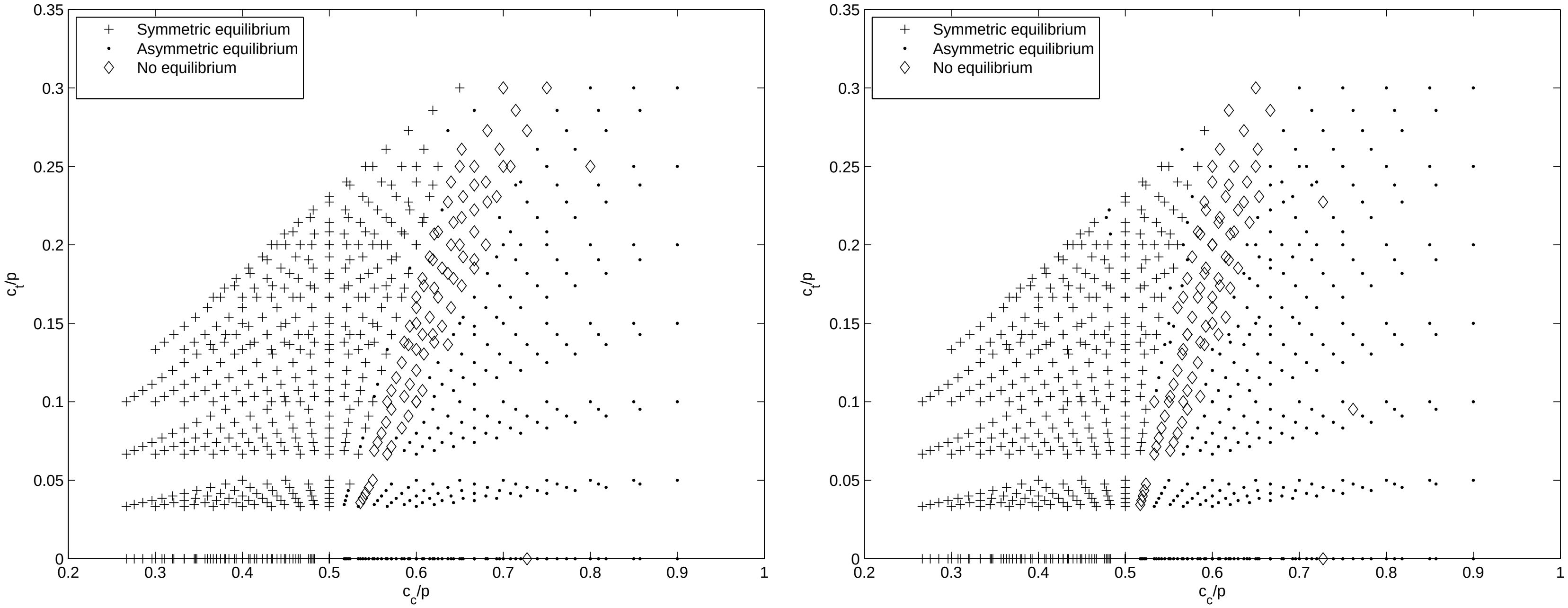


(a) $m = 0.5$.



(b) $m = 0.7$.

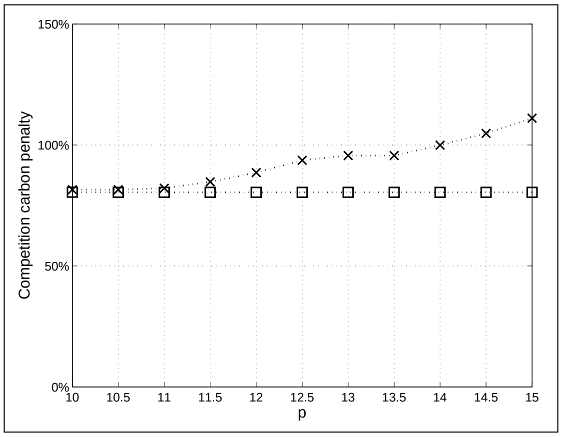
Figure 6.4: Types of equilibria for various parameter ratios when $m = 0.5$ and $m = 0.7$, respectively.



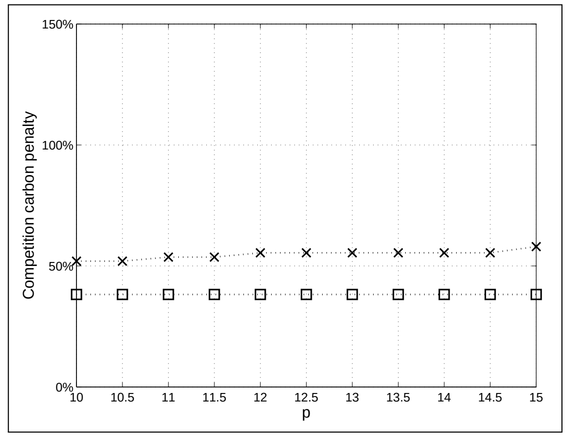
(a) $m = 0.8$.

(b) $m = 1$.

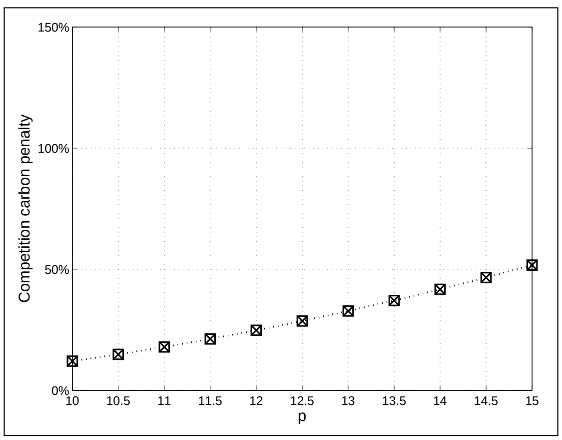
Figure 6.5: Types of equilibria for various parameter ratios when $m = 0.8$ and $m = 1$, respectively.



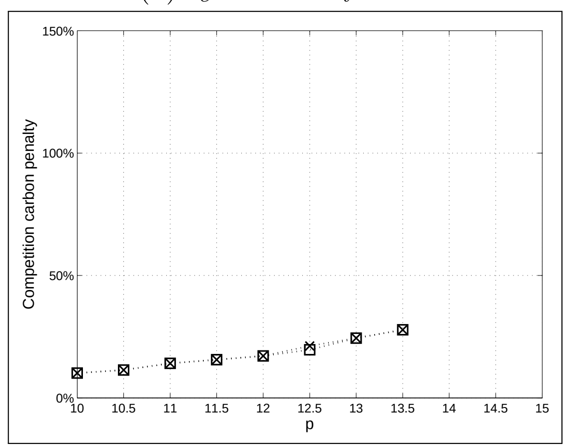
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.

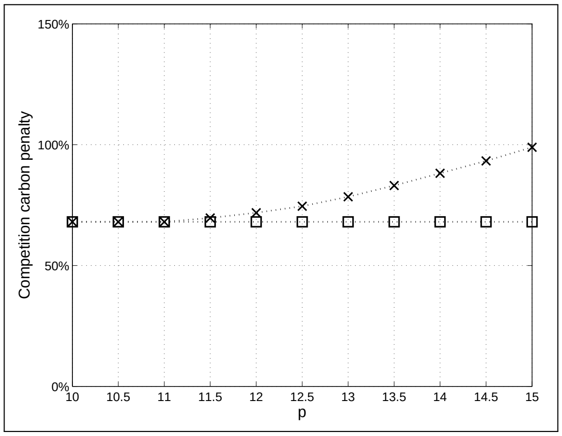


(c) $c_c = 9$ and $c_t = 0.5$.

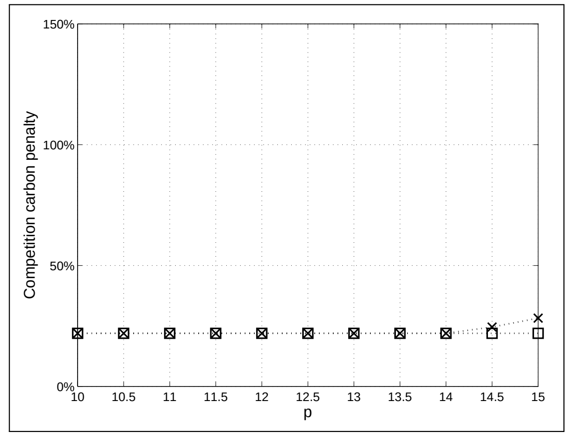


(d) $c_c = 9$ and $c_t = 2$.

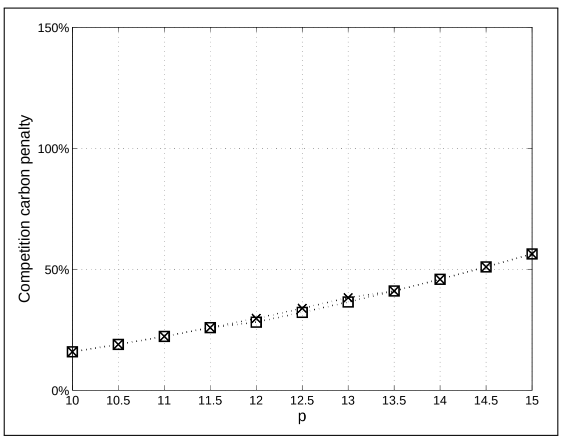
Figure 6.6: $100(T_D - T_C)/T_C$ when $m = 0.3$ and $\lambda = 10$. "x" and "□" indicate the maximum and minimum gaps between total transportation costs, respectively.



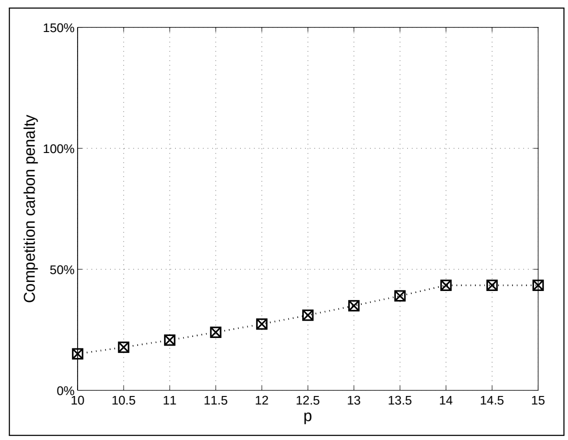
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.

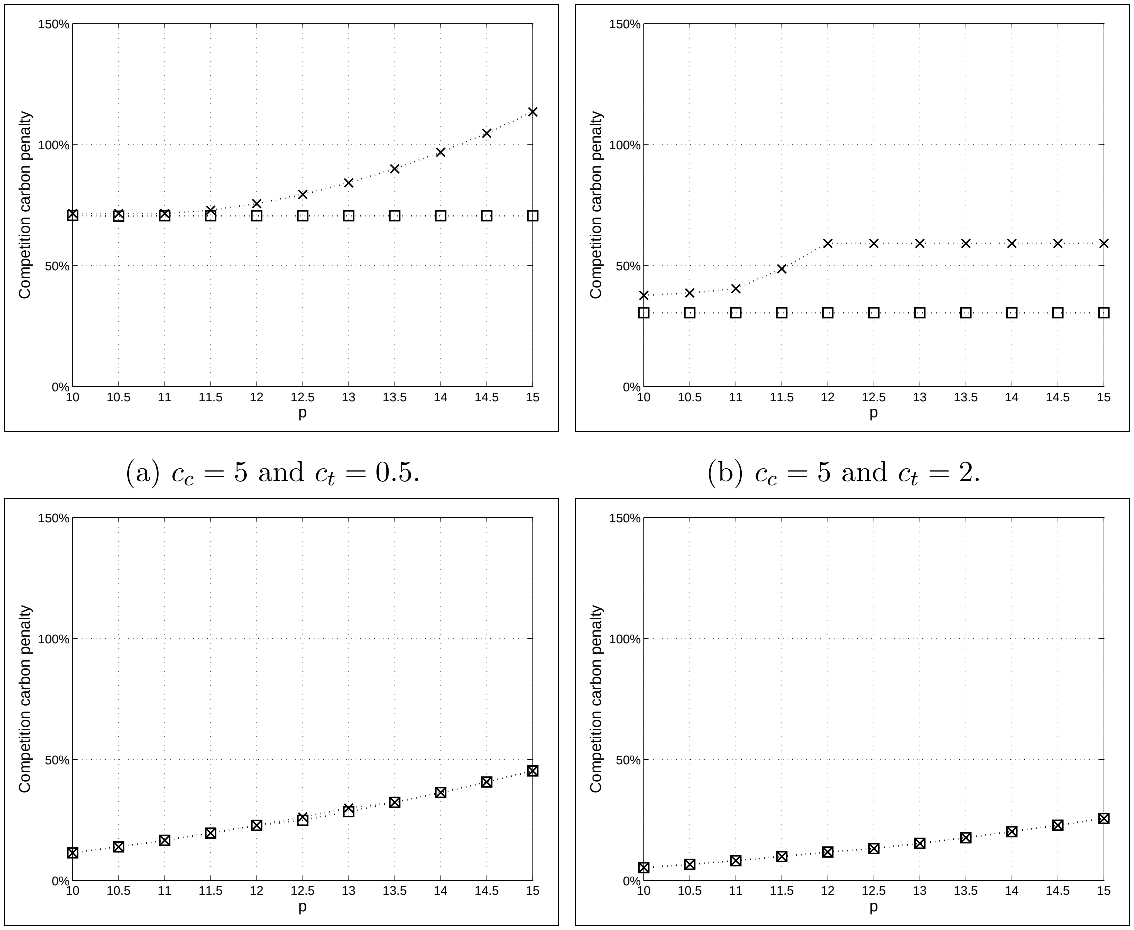


(c) $c_c = 9$ and $c_t = 0.5$.



(d) $c_c = 9$ and $c_t = 2$.

Figure 6.7: $100(T_D - T_C)/T_C$ when $m = 0.5$ and $\lambda = 10$. "x" and "□" indicate the maximum and minimum gaps between total transportation costs, respectively.



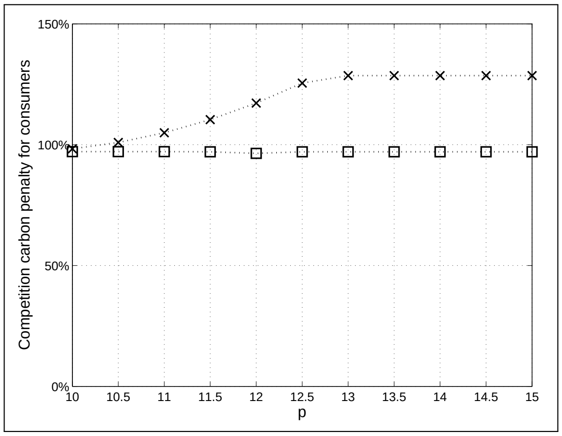
(a) $c_c = 5$ and $c_t = 0.5$.

(b) $c_c = 5$ and $c_t = 2$.

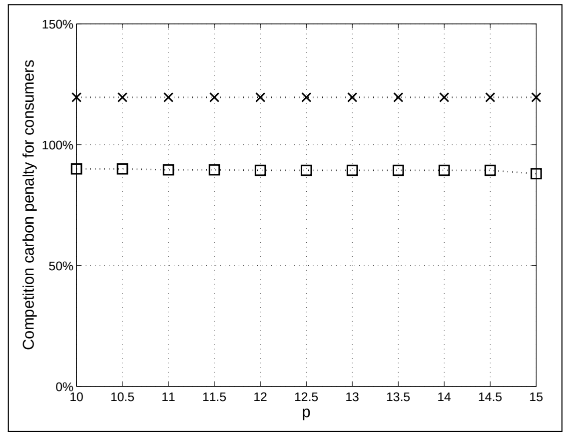
(c) $c_c = 9$ and $c_t = 0.5$.

(d) $c_c = 9$ and $c_t = 2$.

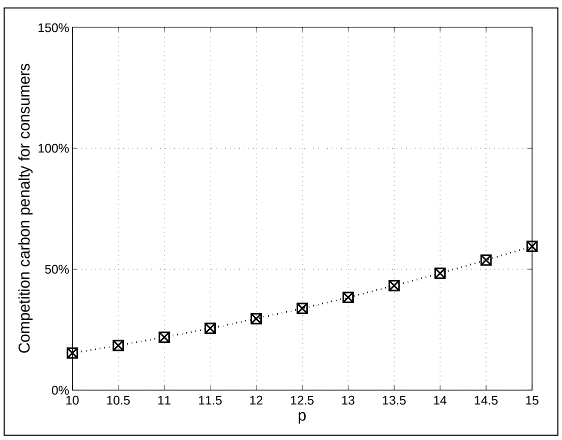
Figure 6.8: $100(T_D - T_C)/T_C$ when $m = 1$ and $\lambda = 10$. “x” and “□” indicate the maximum and minimum gaps between total transportation costs, respectively.



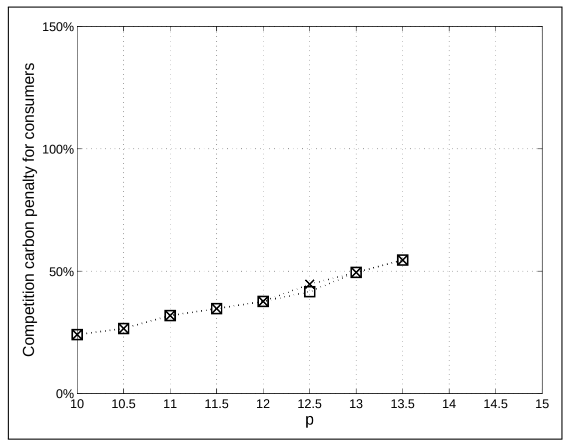
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.

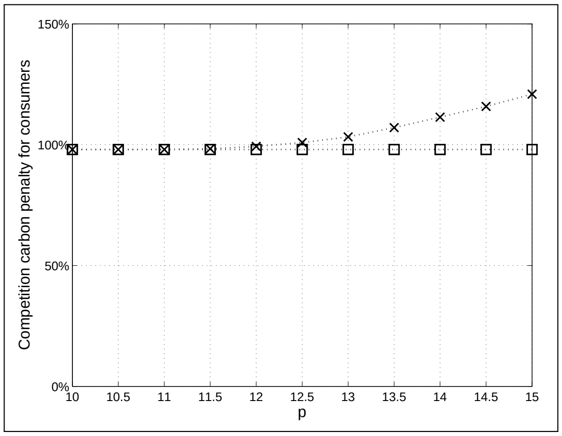


(c) $c_c = 9$ and $c_t = 0.5$.

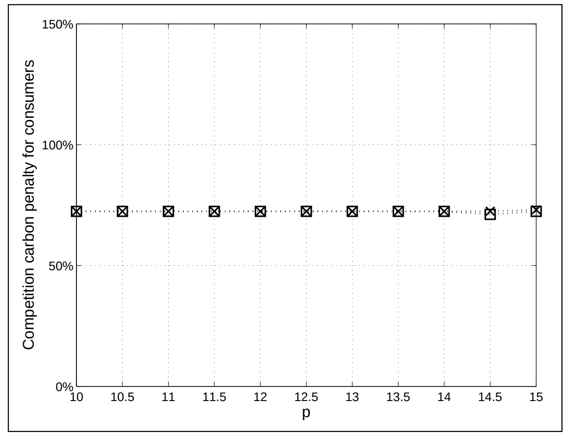


(d) $c_c = 9$ and $c_t = 2$.

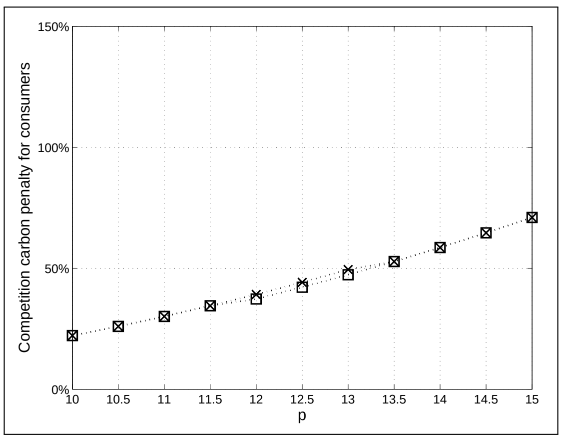
Figure 6.9: $100(TC_D - TC_C)/TC_C$ when $m = 0.3$ and $\lambda = 10$. "x" and "□" indicate the maximum and minimum gaps between total distances traveled by consumers, respectively.



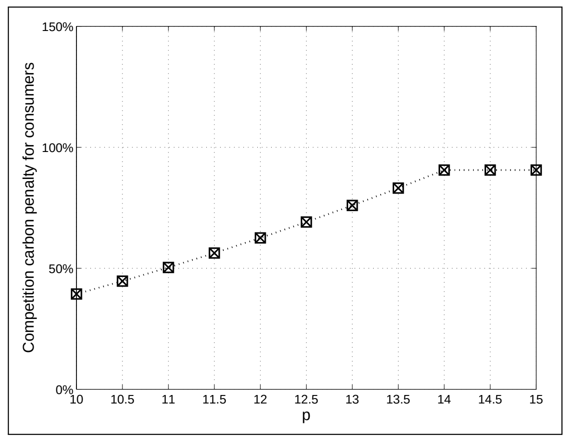
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.

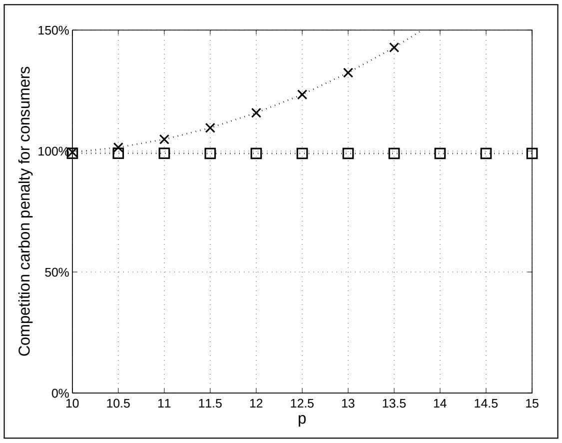


(c) $c_c = 9$ and $c_t = 0.5$.

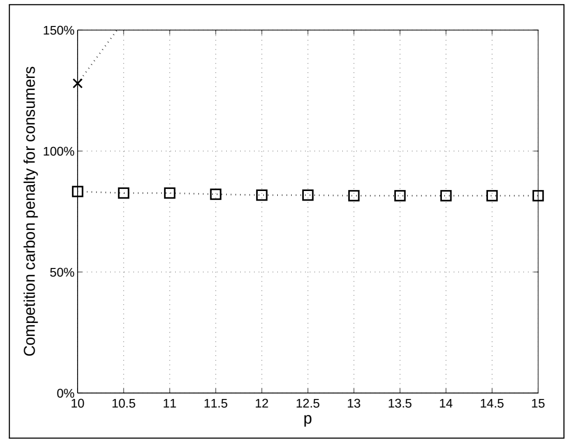


(d) $c_c = 9$ and $c_t = 2$.

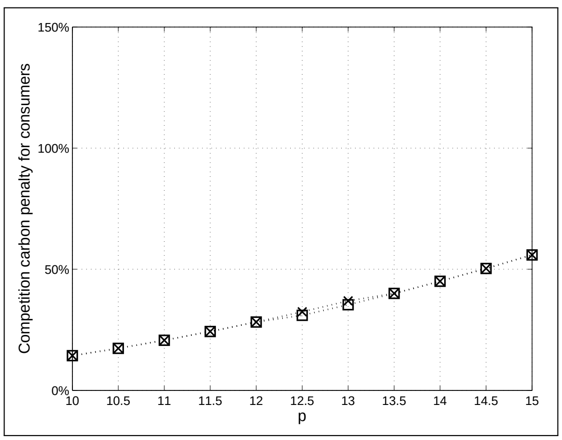
Figure 6.10: $100(TC_D - TC_C)/TC_C$ when $m = 0.5$ and $\lambda = 10$. "x" and "□" indicate the maximum and minimum gaps between total distances traveled by consumers, respectively.



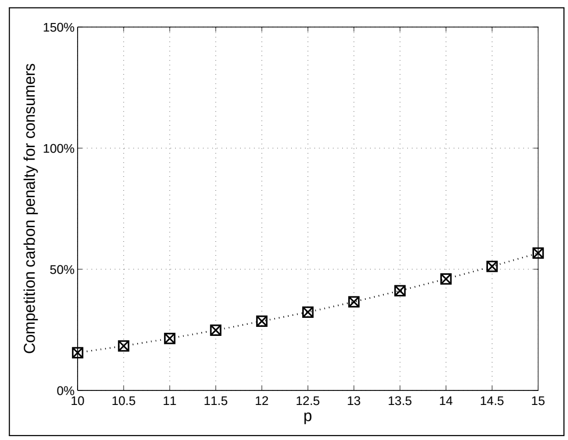
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.

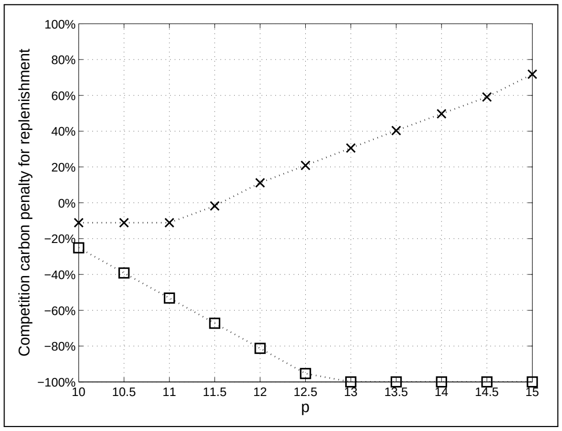


(c) $c_c = 9$ and $c_t = 0.5$.

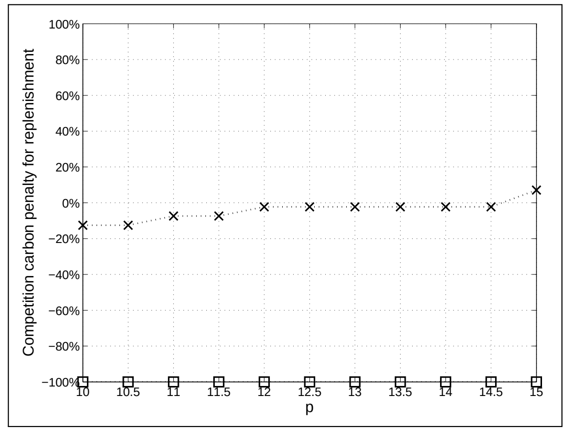


(d) $c_c = 9$ and $c_t = 2$.

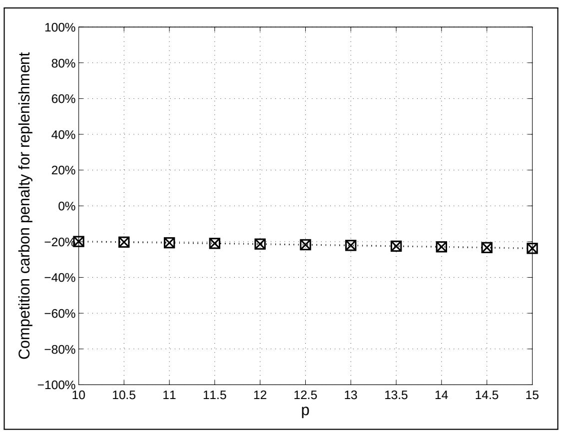
Figure 6.11: $100(TC_D - TC_C)/TC_C$ when $m = 1$ and $\lambda = 10$. "x" and "□" indicate the maximum and minimum gaps between total distances traveled by consumers, respectively.



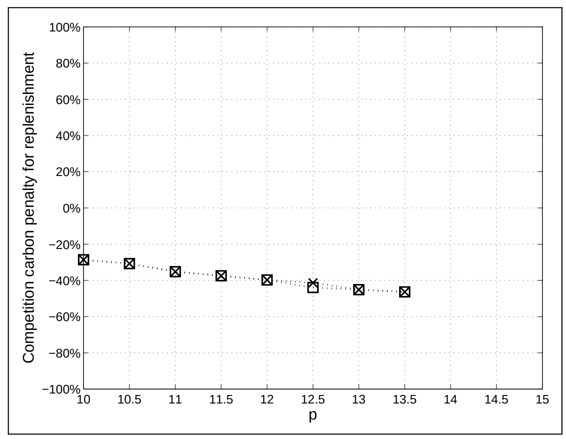
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.

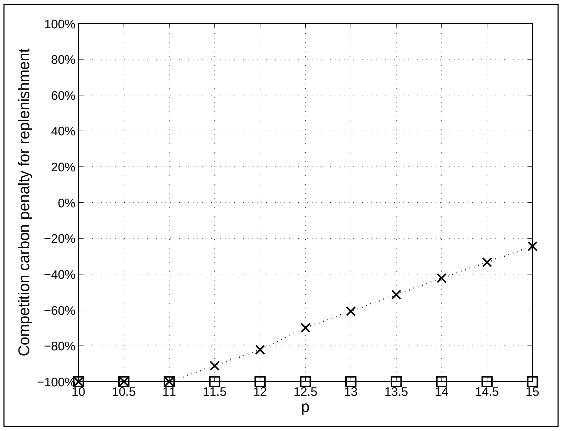


(c) $c_c = 9$ and $c_t = 0.5$.

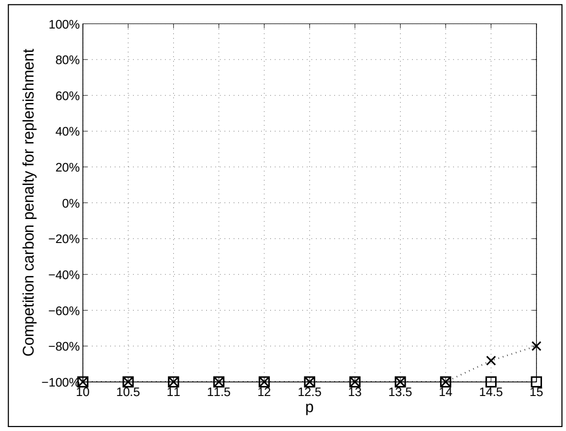


(d) $c_c = 9$ and $c_t = 2$.

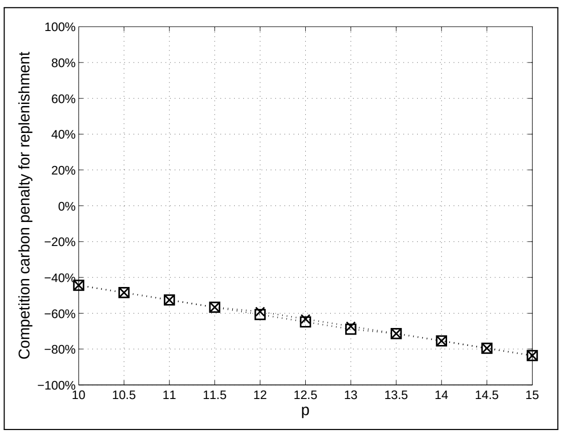
Figure 6.12: $100(TR_D - TR_C)/TR_C$ when $m = 0.3$ and $\lambda = 10$. "x" and "□" indicate the maximum and minimum gaps between total distances traveled for replenishment, respectively.



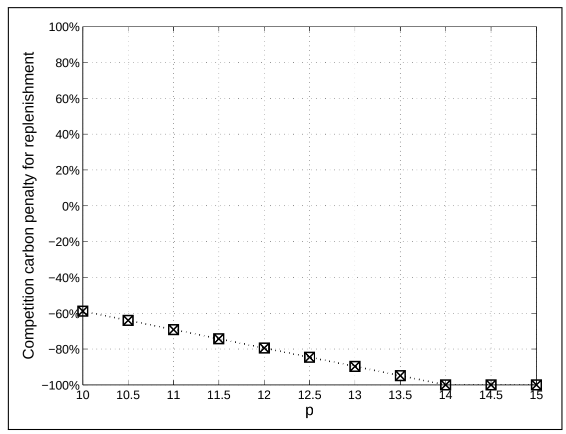
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.

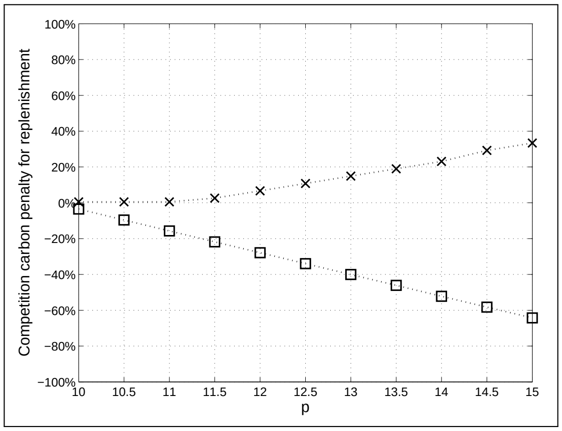


(c) $c_c = 9$ and $c_t = 0.5$.

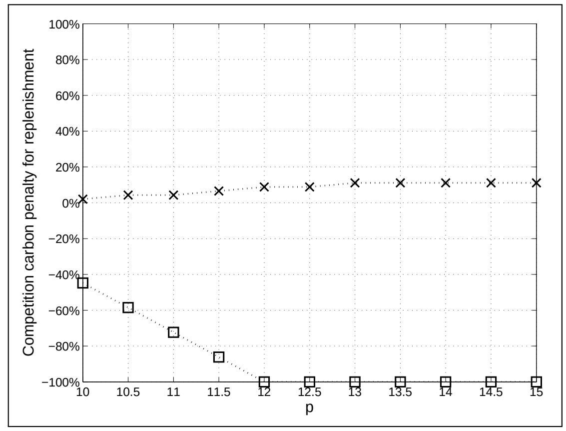


(d) $c_c = 9$ and $c_t = 2$.

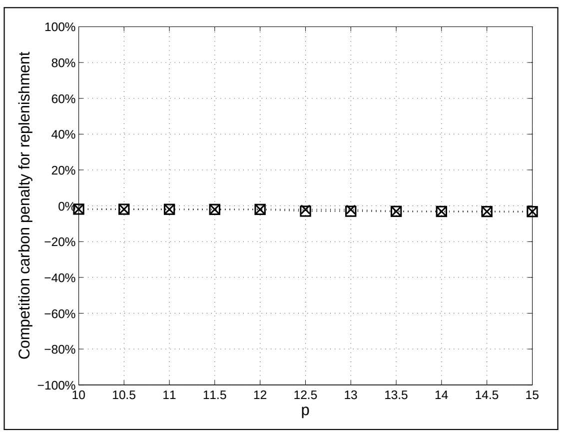
Figure 6.13: $100(TR_D - TR_C)/TR_C$ when $m = 0.5$ and $\lambda = 10$. "x" and "□" indicate the maximum and minimum gaps between total distances traveled for replenishment, respectively.



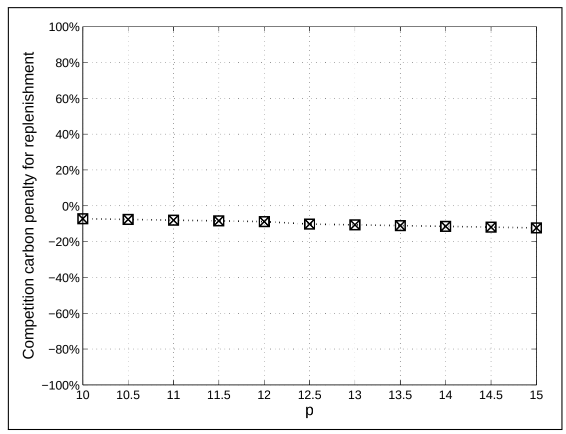
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.



(c) $c_c = 9$ and $c_t = 0.5$.



(d) $c_c = 9$ and $c_t = 2$.

Figure 6.14: $100(TR_D - TR_C)/TR_C$ when $m = 1$ and $\lambda = 10$. "x" and "□" indicate the maximum and minimum gaps between total distances traveled for replenishment, respectively.

Chapter 7

Conclusion

We study the simultaneous location problem on a Hotelling line for two retail stores in the decentralized and centralized systems. The decentralized system has two stores managed by competitive retailers, whereas both stores in the centralized system belong to the same retail chain. We characterize the best response of each retailer to the location of her rival and establish the Nash equilibrium, under certain conditions, in the decentralized system. We develop an algorithm to calculate the optimal locations in the centralized system. Furthermore, we conduct a numerical study to gain better insights into the behavior of equilibrium. We compare and contrast the centralized and decentralized system solutions to investigate the impacts of competition on store locations, emissions, and profits.

Our numerical experiments indicate that when the consumer transportation costs are high, the retailers locate their stores away from each other and want to get closer to their respective consumer bases, leading to asymmetric equilibrium. As the warehouse replenishment costs increase, the stores tend to approach the warehouse. Symmetric equilibria only arise in high margin markets, especially when the consumer transportation cost is low compared to the price. Thus Hotelling's (1929) results hold in such cases. The decentralized solution is closer to the centralized solution when transportation costs for consumer travels are higher. The competition carbon penalty is lowest when both transportation costs are high. The competition carbon penalty from consumer transportation is always higher. However, when the consumer transportation costs are high, the total amount of emissions from replenishment is lower in the decentralized system than in the centralized system.

When the consumer transportation cost is low, the competition carbon penalty is lowest when the warehouse is at the mid-point of the unit line. Increasing the replenishment cost in this case is very effective in reducing the carbon penalty. When the consumer transportation cost is high, the competition carbon penalty is lowest when the warehouse is at the end-point. Increasing the replenishment cost in this case reduces the carbon penalty slightly. Regardless of the warehouse location, increasing consumer transportation cost is more effective than increasing replenishment cost in mitigating the impacts of competition. Hence imposing a tax policy for the consumer travels can be more effective in cutting excess emissions due to competition.

Our research can be extended to allow for random demand and include positive storage costs; see Appendix A for profit function derivations under random demand. Intuitively, we would expect the role of the storage costs to be similar to that of the transportation cost for replenishment in this extension. This is because the storage costs would enforce the retailers to shorten their replenishment lead-times, in order to hold less inventory. And the lead-times can only be reduced by being closer to the warehouse. Note that a retailer does not keep inventory at all if it is at the same location as the warehouse. Thus the presence of the positive storage cost could be interpreted as an increase in the replenishment transportation cost.

Future extensions of our research could also allow consumers to be non-uniformly distributed over the unit line. Another direction for future research is to extend our model to include the joint replenishment problem for stores. The retailers might prefer to procure the items from the warehouse via a common truck in order to share transportation costs. The full-truck load assumption could also be relaxed and compared with the no-full-truck load assumption. Furthermore, we could issue a bus that picks a number of consumers from several specific bus stops on the unit line. Last, it would be more realistic to take the price as a decision variable and/or focus on a two-dimensional region instead of the unit line.

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Appendix A

Profit Function Derivations

For analytical convenience, we assumed in Chapter 3 that consumer demand is deterministic. Below we show our profit function derivations by allowing for stochastic demand. When demand is stochastic, the inventory holding cost should be incorporated into profit calculations.

Let $X(t)$ denote the random demand at location $t \in [0, 1]$ such that $X(t)$ is normally distributed with (λ, σ^2). We denote by $\lambda_i(a, b)$ the expected daily demand in retail store i, and $\sigma_i^2(a, b)$ the variance of the daily demand in retail store i. For example, if $a < b$, then the expected daily demand in retail store A is

$$\lambda_A(a, b) = E\left[\int_0^{\frac{a+b}{2}} X(t)dt\right] = \int_0^{\frac{a+b}{2}} E[X(t)]dt = \int_0^{\frac{a+b}{2}} \lambda dt = \lambda\left(\frac{a+b}{2}\right)$$

and the variance of the daily demand in store A is

$$\sigma_A^2(a, b) = \sigma^2\left(\frac{a+b}{2}\right).$$

Likewise, $\lambda_B(a, b)$ and $\sigma_B^2(a, b)$ are calculated as follows:

$$\lambda_B(a, b) = \lambda\left(1 - \frac{a+b}{2}\right)$$

and

$$\sigma_B^2(a, b) = \sigma^2\left(1 - \frac{a+b}{2}\right).$$

We assume that each retailer holds inventory by implementing a base-stock replenishment policy and replenishes its store from the warehouse via trucks in a full truck-load fashion. Replenishment lead time for each retailer is proportional to its distance from the warehouse, and replenishment lead time per unit distance is L days. Any unmet demand is backordered. Each retailer chooses a base-stock level that achieves a sufficiently high service level so that backordering costs are negligible. Both retailers choose the same service level.

We define c_s as the store space cost per unit per day the unit is held in the store's inventory. We formulate c_s in terms of the variable cost per unit of space per day (v_s), the amount of energy needed to maintain one unit of space per day (f_s), the per unit price of energy (p_s), the amount of carbon emission released from each unit of energy (e_s), and the number of units of product stored per unit of space (q_s). Thus:

$$c_s = \frac{v_s + f_s(p_s + e_s p_{e,s})}{q_s}.$$

Inventory held in retail store i at any time period is calculated as follows:

$$I_{i,\tau+1} = I_{i,\tau} + ((S_i - I_{i,\tau})^+ + \omega_{i,\tau+1})^+ - \lambda_{i,\tau+1}, \ \forall \tau \geq 0,$$

where S_i is the base-stock level in store i and $\omega_{i,\tau}$ is an adjustment to store i in order to ensure a full truck-load delivery, i.e., $\omega_{i,\tau+1} = q_t - (S_i - I_{i,\tau})^+$. Similar storage cost formulations also appear in Daskin et al. (2002), Shen et al. (2003), Cachon (2014), and Shuai (2014).

Recall that $d_{ic}(a,b)$ is the average round-trip distance a consumer travels to retail store i, and $d_{it}(a,b)$ is the length of truck's route from store i to the warehouse. In addition, we define $E[I_i(a,b)]$ as the expected inventory in retail store i. Given the warehouse location $m \in [0,1]$, each retailer i chooses the location of its store to maximize its expected daily profit $\pi_i(a,b)$:

$$\pi_i(a,b) = (p - c_c d_{ic}(a,b) - c_t d_{it}(a,b))\lambda_i(a,b) - c_s E[I_i(a,b)] \ \text{ for } i \in \{A,B\}.$$

We below show our derivations of $d_{ic}(a,b)$, $d_{it}(a,b)$, and $E[I_i(a,b)]$ in terms of our problem parameters and decision variables in each of the eight cases in Table 3.1.

Case (1). $\mathbf{0 \le a < b \le m \le 1}$. The average round-trip distance traveled by a consumer to retail store A is given by

$$d_{Ac}(a,b) = \frac{2\left[\int_0^a (a-t)dt + \int_a^{\frac{a+b}{2}} (t-a)dt\right]}{\frac{a+b}{2}} = \frac{5a^2 - 2ab + b^2}{2a + 2b}$$

and the average round-trip distance traveled by a consumer to retail store B is given by

$$d_{Bc}(a,b) = \frac{2\left[\int_b^1 (t-b)dt + \int_{\frac{a+b}{2}}^b (b-t)dt\right]}{1 - \frac{a+b}{2}} = \frac{a^2 - 2ab + 5b^2 - 8b + 4}{4 - 2a - 2b}.$$

The round-trip distance traveled by a truck from the warehouse to retail store A is given by

$$d_{At}(a,b) = 2(m-a)$$

and the round-trip distance traveled by a truck from the warehouse to retail store B is given by

$$d_{Bt}(a,b) = 2(m-b).$$

Under the full truck-load assumption, the inventory system in each retail store i can be approximated by a periodic review inventory system with a period length of $\frac{q_t}{\lambda_i(a,b)}$ days (see Cachon 2014). Because the replenishment lead times for retailers A and B are $L(m-a)$ days and $L(m-b)$ days, respectively, the expected inventories in retail stores A and B can be approximated as follows (see Silver et al. 1998 for details).

$$\begin{aligned} E[I_A(a,b)] &\approx z\sigma_A(a,b)\sqrt{\frac{q_t}{\lambda_A(a,b)} + L(m-a)} \\ &= z\sigma\sqrt{\frac{a+b}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{a+b}{2}\right)} + L(m-a)} \\ &= z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(m-a)(a+b)}{2}} \end{aligned}$$

and

$$\begin{aligned}
E[I_B(a,b)] &\approx z\sigma_B(a,b)\sqrt{\frac{q_t}{\lambda_B(a,b)} + L(m-b)} \\
&= z\sigma\sqrt{1-\frac{a+b}{2}}\sqrt{\frac{q_t}{\lambda\left(1-\frac{a+b}{2}\right)} + L(m-b)} \\
&= z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(m-b)(2-a-b)}{2}},
\end{aligned}$$

where z is a constant chosen by the firm to determine the service level (i.e., the probability of being in stock at the end of a period). Hence the expected daily profit of retailer A can be written as

$$\begin{aligned}
\pi_A(a,b) &= \frac{\lambda p(a+b)}{2} - \frac{\lambda c_c(5a^2-2ab+b^2)}{4} \\
&\quad -\lambda c_t(a+b)(m-a) - c_s z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(m-a)(a+b)}{2}}.
\end{aligned}$$

The expected daily profit of retailer B can be written as

$$\begin{aligned}
\pi_B(a,b) &= \frac{\lambda p(2-a-b)}{2} - \frac{\lambda c_c(a^2-2ab+5b^2-8b+4)}{4} \\
&\quad -\lambda c_t(2-a-b)(m-b) - c_s z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(m-b)(2-a-b)}{2}}.
\end{aligned}$$

Note that when demand is deterministic, i.e., $\sigma = 0$, we obtain the profit functions in Chapter 3.

Case (2). $0 \le a = b \le m \le 1$. The average round-trip distance traveled by a consumer to retail store A is given by

$$\begin{aligned}
d_{Ac}(a,b) &= \frac{2\left[\int_0^a (a-t)dt + \int_a^1 (t-a)dt\right]}{1} \\
&= 1-2a+2a^2.
\end{aligned}$$

Since $a = b$, $d_{Bc}(a,b) = d_{Ac}(a,b)$. The round-trip distance traveled by a truck from the warehouse to retail store A is given by

$$d_{At}(a,b) = 2(m-a).$$

Since $a = b$, $d_{Bt}(a,b) = d_{At}(a,b)$. The expected inventory in retail store A is given by

$$\begin{aligned} E[I_A(a,b)] &\approx z\sigma_A(a,b)\sqrt{\frac{q_t}{\lambda_A(a,b)} + L(m-a)} \\ &= z\sigma\sqrt{\frac{1}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{1}{2}\right)} + L(m-a)} \\ &= z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(m-a)}{2}}. \end{aligned}$$

Since $a = b$, $E[I_A(a,b)] = E[I_B(a,b)]$. Hence the expected daily profit of retailer A can be written as

$$\begin{aligned} \pi_A(a,b) &= \frac{\lambda p - \lambda c_c(1 - 2a + 2a^2)}{2} \\ &\quad -\lambda c_t(m-a) - c_s z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(m-a)}{2}}. \end{aligned}$$

Since $a = b$, $\pi_A(a,b) = \pi_B(a,b)$.

Case (3). $0 \leq b < a \leq m \leq 1$. The average round-trip distances traveled by a consumer to retail stores A and B are given by

$$\begin{aligned} d_{Ac}(a,b) &= \frac{2\left[\int_a^1 (t-a)dt + \int_{\frac{a+b}{2}}^{a} (a-t)dt\right]}{1 - \frac{a+b}{2}} \\ &= \frac{b^2 - 2ab + 5a^2 - 8a + 4}{4 - 2a - 2b} \end{aligned}$$

$$\begin{aligned} d_{Bc}(a,b) &= \frac{2\left[\int_0^b (b-t)dt + \int_b^{\frac{a+b}{2}} (t-b)dt\right]}{\frac{a+b}{2}} \\ &= \frac{5b^2 - 2ab + a^2}{2a + 2b}. \end{aligned}$$

The round-trip distance traveled by a truck from the warehouse to retail store A is given by

$$d_{At}(a,b) = 2(m-a)$$

and the round-trip distance traveled by a truck from the warehouse to retail store B is given by

$$d_{Bt}(a,b) = 2(m-b).$$

The expected inventory in retail store A is given by

$$\begin{aligned} E[I_A(a,b)] &\approx z\sigma_A(a,b)\sqrt{\frac{q_t}{\lambda_A(a,b)}+L(m-a)} \\ &= z\sigma\sqrt{\frac{2-a-b}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{2-a-b}{2}\right)}+L(m-a)} \\ &= z\sigma\sqrt{\frac{q_t}{\lambda}+\frac{L(m-a)(2-a-b)}{2}}. \end{aligned}$$

The expected inventory in retail store B is given by

$$\begin{aligned} E[I_B(a,b)] &\approx z\sigma_B(a,b)\sqrt{\frac{q_t}{\lambda_B(a,b)}+L(m-b)} \\ &= z\sigma\sqrt{\frac{a+b}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{a+b}{2}\right)}+L(m-b)} \\ &= z\sigma\sqrt{\frac{q_t}{\lambda}+\frac{L(m-b)(a+b)}{2}}. \end{aligned}$$

Hence the expected daily profit of retailer A can be written as

$$\begin{aligned} \pi_A(a,b) &= \frac{\lambda p(2-a-b)}{2}-\frac{\lambda c_c(b^2-2ab+5a^2-8a+4)}{4} \\ &\quad -\lambda c_t(2-a-b)(m-a)-c_s z\sigma\sqrt{\frac{q_t}{\lambda}+\frac{L(m-a)(2-a-b)}{2}}. \end{aligned}$$

The expected daily profit of retailer B can be written as

$$\begin{aligned} \pi_B(a,b) &= \frac{\lambda p(a+b)}{2}-\frac{\lambda c_c(5b^2-2ab+a^2)}{4} \\ &\quad -\lambda c_t(a+b)(m-b)-c_s z\sigma\sqrt{\frac{q_t}{\lambda}+\frac{L(m-b)(a+b)}{2}}. \end{aligned}$$

Case (4). $0 \le b \le m < a \le 1$**.** The average round-trip distances traveled by a consumer to retail stores A and B are given by

$$\begin{aligned} d_{Ac}(a,b) &= \frac{2\left[\int_a^1 (t-a)dt+\int_{\frac{a+b}{2}}^a (a-t)dt\right]}{1-\frac{a+b}{2}} \\ &= \frac{b^2-2ab+5a^2-8a+4}{4-2a-2b} \end{aligned}$$

$$
\begin{aligned}
d_{Bc}(a,b) &= \frac{2\left[\int_0^b (b-t)dt + \int_b^{\frac{a+b}{2}} (t-b)dt\right]}{\frac{a+b}{2}} \\
&= \frac{5b^2 - 2ab + a^2}{2a+2b}.
\end{aligned}
$$

The round-trip distance traveled by a truck from the warehouse to retail store A is given by

$$d_{At}(a,b) = 2(a-m)$$

and the round-trip distance traveled by a truck from the warehouse to retail store B is given by

$$d_{Bt}(a,b) = 2(m-b).$$

The expected inventory in retail store A is given by

$$
\begin{aligned}
E[I_A(a,b)] &\approx z\sigma_A(a,b)\sqrt{\frac{q_t}{\lambda_A(a,b)} + L(a-m)} \\
&= z\sigma\sqrt{\frac{2-a-b}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{2-a-b}{2}\right)} + L(a-m)} \\
&= z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(a-m)(2-a-b)}{2}}.
\end{aligned}
$$

The expected inventory in retail store B is given by

$$
\begin{aligned}
E[I_B(a,b)] &\approx z\sigma_B(a,b)\sqrt{\frac{q_t}{\lambda_B(a,b)} + L(m-b)} \\
&= z\sigma\sqrt{\frac{a+b}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{a+b}{2}\right)} + L(m-b)} \\
&= z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(m-b)(a+b)}{2}}.
\end{aligned}
$$

Hence the expected daily profit of retailer A can be written as

$$
\begin{aligned}
\pi_A(a,b) &= \frac{\lambda p(2-a-b)}{2} - \frac{\lambda c_c(b^2 - 2ab + 5a^2 - 8a + 4)}{4} \\
&\quad -\lambda c_t(2-a-b)(a-m) - c_s z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(a-m)(2-a-b)}{2}}.
\end{aligned}
$$

The expected daily profit of retailer B can be written as

$$
\begin{aligned}
\pi_B(a,b) &= \frac{\lambda p(a+b)}{2} - \frac{\lambda c_c(5b^2 - 2ab + a^2)}{4} \\
&\quad -\lambda c_t(a+b)(m-b) - c_s z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(m-b)(a+b)}{2}}.
\end{aligned}
$$

Case (5). $0 \leq a < m < b \leq 1$**.** The average round-trip distances traveled by a consumer to retail stores A and B are given by

$$\begin{aligned} d_{Ac}(a,b) &= \frac{2\left[\int_0^a (a-t)dt + \int_a^{\frac{a+b}{2}} (t-a)dt\right]}{\frac{a+b}{2}} \\ &= \frac{5a^2 - 2ab + b^2}{2a+2b} \end{aligned}$$

$$\begin{aligned} d_{Bc}(a,b) &= \frac{2\left[\int_b^1 (t-b)dt + \int_{\frac{a+b}{2}}^b (b-t)dt\right]}{1-\frac{a+b}{2}} \\ &= \frac{a^2 - 2ab + 5b^2 - 8b + 4}{4-2a-2b}. \end{aligned}$$

The round-trip distance traveled by a truck from the warehouse to retail store A is given by

$$d_{At}(a,b) = 2(m-a)$$

and the round-trip distance traveled by a truck from the warehouse to retail store B is given by

$$d_{Bt}(a,b) = 2(b-m).$$

The expected inventory in retail store A is given by

$$\begin{aligned} E[I_A(a,b)] &\approx z\sigma_A(a,b)\sqrt{\frac{q_t}{\lambda_A(a,b)} + L(m-a)} \\ &= z\sigma\sqrt{\frac{a+b}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{a+b}{2}\right)} + L(m-a)} \\ &= z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(m-a)(a+b)}{2}}. \end{aligned}$$

The expected inventory in retail store B is given by

$$\begin{aligned} E[I_B(a,b)] &\approx z\sigma_B(a,b)\sqrt{\frac{q_t}{\lambda_B(a,b)} + L(b-m)} \\ &= z\sigma\sqrt{\frac{2-a-b}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{2-a-b}{2}\right)} + L(b-m)} \\ &= z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(b-m)(2-a-b)}{2}}. \end{aligned}$$

Hence the expected daily profit of retailer A can be written as

$$\begin{aligned}\pi_A(a,b) &= \frac{\lambda p(a+b)}{2} - \frac{\lambda c_c(5a^2-2ab+b^2)}{4} \\ &\quad -\lambda c_t(a+b)(m-a) - c_s z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(m-a)(a+b)}{2}}.\end{aligned}$$

The expected daily profit of retailer B can be written as

$$\begin{aligned}\pi_B(a,b) &= \frac{\lambda p(2-a-b)}{2} - \frac{\lambda c_c(a^2-2ab+5b^2-8b+4)}{4} \\ &\quad -\lambda c_t(2-a-b)(b-m) - c_s z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(b-m)(2-a-b)}{2}}.\end{aligned}$$

Case (6). $0 \le m \le a < b \le 1$**.** The average round-trip distances traveled by a consumer to retail stores A and B are given by

$$\begin{aligned}d_{Ac}(a,b) &= \frac{2\left[\int_0^a (a-t)dt + \int_a^{\frac{a+b}{2}}(t-a)dt\right]}{\frac{a+b}{2}} \\ &= \frac{5a^2-2ab+b^2}{2a+2b}\end{aligned}$$

$$\begin{aligned}d_{Bc}(a,b) &= \frac{2\left[\int_b^1 (t-b)dt + \int_{\frac{a+b}{2}}^{b}(b-t)dt\right]}{1-\frac{a+b}{2}} \\ &= \frac{a^2-2ab+5b^2-8b+4}{4-2a-2b}.\end{aligned}$$

The round-trip distance traveled by a truck from the warehouse to retail store A is given by

$$d_{At}(a,b) = 2(a-m)$$

and the round-trip distance traveled by a truck from the warehouse to retail store B is given by

$$d_{Bt}(a,b) = 2(b-m).$$

The expected inventory in retail store A is given by

$$\begin{aligned} E[I_A(a,b)] &\approx z\sigma_A(a,b)\sqrt{\frac{q_t}{\lambda_A(a,b)}+L(a-m)} \\ &= z\sigma\sqrt{\frac{a+b}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{a+b}{2}\right)}+L(a-m)} \\ &= z\sigma\sqrt{\frac{q_t}{\lambda}+\frac{L(a-m)(a+b)}{2}}. \end{aligned}$$

The expected inventory in retail store B is given by

$$\begin{aligned} E[I_B(a,b)] &\approx z\sigma_B(a,b)\sqrt{\frac{q_t}{\lambda_B(a,b)}+L(b-m)} \\ &= z\sigma\sqrt{\frac{2-a-b}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{2-a-b}{2}\right)}+L(b-m)} \\ &= z\sigma\sqrt{\frac{q_t}{\lambda}+\frac{L(b-m)(2-a-b)}{2}}. \end{aligned}$$

Hence the expected daily profit of retailer A can be written as

$$\begin{aligned} \pi_A(a,b) &= \frac{\lambda p(a+b)}{2}-\frac{\lambda c_c(5a^2-2ab+b^2)}{4} \\ &\quad -\lambda c_t(a+b)(a-m)-c_s z\sigma\sqrt{\frac{q_t}{\lambda}+\frac{L(a-m)(a+b)}{2}}. \end{aligned}$$

The expected daily profit of retailer B can be written as

$$\begin{aligned} \pi_B(a,b) &= \frac{\lambda p(2-a-b)}{2}-\frac{\lambda c_c(a^2-2ab+5b^2-8b+4)}{4} \\ &\quad -\lambda c_t(2-a-b)(b-m)-c_s z\sigma\sqrt{\frac{q_t}{\lambda}+\frac{L(b-m)(2-a-b)}{2}}. \end{aligned}$$

Case (7). $0 \le m < a = b \le 1$. The average round-trip distance traveled by a consumer to retail store A is given by

$$\begin{aligned} d_{Ac}(a,b) &= \frac{2\left[\int_0^a (a-t)dt+\int_a^1 (t-a)dt\right]}{1} \\ &= 1-2a+2a^2. \end{aligned}$$

Since $a = b$, $d_{Bc}(a,b) = d_{Ac}(a,b)$. The round-trip distance traveled by a truck from the warehouse to retail store A is given by

$$d_{At}(a,b) = 2(a-m).$$

Since $a = b$, $d_{Bt}(a,b) = d_{At}(a,b)$. The expected inventory in retail store A is given by

$$\begin{aligned} E[I_A(a,b)] &\approx z\sigma_A(a,b)\sqrt{\frac{q_t}{\lambda_A(a,b)} + L(a-m)} \\ &= z\sigma\sqrt{\frac{1}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{1}{2}\right)} + L(a-m)} \\ &= z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(a-m)}{2}}. \end{aligned}$$

Since $a = b$, $E[I_A(a,b)] = E[I_B(a,b)]$. Hence the expected daily profit of retailer A can be written as

$$\begin{aligned} \pi_A(a,b) &= \frac{\lambda p - \lambda c_c(1-2a+2a^2)}{2} \\ &\quad -\lambda c_t(a-m) - c_s z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(a-m)}{2}}. \end{aligned}$$

Since $a = b$, $\pi_A(a,b) = \pi_B(a,b)$.

Case (8). $0 \leq m < b < a \leq 1$**.** The average round-trip distances traveled by a consumer to retail stores A and B are given by

$$\begin{aligned} d_{Ac}(a,b) &= \frac{2\left[\int_a^1 (t-a)dt + \int_{\frac{a+b}{2}}^a (a-t)dt\right]}{1 - \frac{a+b}{2}} \\ &= \frac{b^2 - 2ab + 5a^2 - 8a + 4}{4 - 2a - 2b} \end{aligned}$$

$$\begin{aligned} d_{Bc}(a,b) &= \frac{2\left[\int_0^b (b-t)dt + \int_b^{\frac{a+b}{2}} (t-b)dt\right]}{\frac{a+b}{2}} \\ &= \frac{5b^2 - 2ab + a^2}{2a + 2b}. \end{aligned}$$

The round-trip distance traveled by a truck from the warehouse to retail store A is given by

$$d_{At}(a,b) = 2(a-m)$$

and the round-trip distance traveled by a truck from the warehouse to retail store B is given by

$$d_{Bt}(a,b) = 2(b-m).$$

The expected inventory in retail store A is given by

$$\begin{aligned} E[I_A(a,b)] &\approx z\sigma_A(a,b)\sqrt{\frac{q_t}{\lambda_A(a,b)} + L(a-m)} \\ &= z\sigma\sqrt{\frac{2-a-b}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{2-a-b}{2}\right)} + L(a-m)} \\ &= z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(a-m)(2-a-b)}{2}}. \end{aligned}$$

The expected inventory in retail store B is given by

$$\begin{aligned} E[I_B(a,b)] &\approx z\sigma_B(a,b)\sqrt{\frac{q_t}{\lambda_B(a,b)} + L(b-m)} \\ &= z\sigma\sqrt{\frac{a+b}{2}}\sqrt{\frac{q_t}{\lambda\left(\frac{a+b}{2}\right)} + L(b-m)} \\ &= z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(b-m)(a+b)}{2}}. \end{aligned}$$

Hence the expected daily profit of retailer A can be written as

$$\begin{aligned} \pi_A(a,b) &= \frac{\lambda p(2-a-b)}{2} - \frac{\lambda c_c(b^2 - 2ab + 5a^2 - 8a + 4)}{4} \\ &\quad -\lambda c_t(2-a-b)(a-m) - c_s z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(a-m)(2-a-b)}{2}}. \end{aligned}$$

The expected daily profit of retailer B can be written as

$$\begin{aligned} \pi_B(a,b) &= \frac{\lambda p(a+b)}{2} - \frac{\lambda c_c(5b^2 - 2ab + a^2)}{4} \\ &\quad -\lambda c_t(a+b)(b-m) - c_s z\sigma\sqrt{\frac{q_t}{\lambda} + \frac{L(b-m)(a+b)}{2}}. \end{aligned}$$

Table A.1: Summary of the notation for inventory costs.

Parameters	**Definition**
σ^2	Variance of daily demand at any location
$\sigma_i^2(a,b)$	Variance of total daily demand in retail store $i \in \{A,B\}$
v_s	Variable cost per unit of space per unit of time
f_s	Amount of energy needed per unit of space per unit of time
p_s	Per unit price of energy
e_s	Amount of emission released by consumption of one unit of energy
q_s	Number of units of product stored per unit of space
S_i	Base-stock level in retail store $i \in \{A,B\}$
z	Service level in both retail stores
L	Lead time in days per unit of distance
c_s	Space cost per unit of item per unit of time
$E[I_i(a,b)]$	Expected inventory in retail store $i \in \{A,B\}$
$c_sE[I_i(a,b)]$	Space cost per unit of item sold in retail store $i \in \{A,B\}$
ω_i	Adjustment to retail store $i \in \{A,B\}$ to ensure a full truck-load delivery
$I_{i,\tau}$	Inventory of retail store $i \in \{A,B\}$ at the end of period τ

Appendix B

Best Response Conditions

Suppose that $b \leq m$. We can further detail Con_{1a}, Con_{1b}, and Con_{1c} as below (in addition to $b \leq m$, $c_c > 0$, $c_t \geq 0$, $c_c > 2c_t$, and $p > 0$):

$$Con_{1a} = \begin{cases} 0 < b < \frac{1}{3} \text{ and} & b \leq m \leq 2b,\ p < 2c_t m + b(4c_c - 6c_t) \text{ OR} \\ & 2b < m \leq 1,\ c_t \leq \frac{c_c b}{2m-2b},\ p < 2c_t m + b(4c_c - 6c_t) \text{ OR} \\ & 2b < m \leq 1,\ \frac{c_c b}{2m-2b} < c_t,\ 2c_t m b(c_c + 2c_t) \leq p < 2c_t m + b(4c_c - 6c_t) \\ \text{OR} & \\ \frac{1}{3} \leq b \leq \frac{1}{2} \text{ and} & b \leq m < 2b,\ p < 2c_t m + b(4c_c - 6c_t) \text{ OR} \\ & m = 2b,\ c_t < \frac{c_c b}{2m-2b},\ p < 2c_t m + b(4c_c - 6c_t) \text{ OR} \\ & 2b < m \leq 1,\ c_t \leq \frac{c_c b}{2m-2b},\ p < 2c_t m + b(4c_c - 6c_t) \text{ OR} \\ & 2b < m \leq 1,\ \frac{c_c b}{2m-2b} < c_t,\ 2c_t m - b(c_c + 2c_t) \leq p < 2c_t m + b(4c_c - 6c_t) \\ \text{OR} & \\ \frac{1}{2} < b < 1 \text{ and} & p < 2c_t m + b(4c_c - 6c_t) \\ \text{OR} & \\ b = 1 \text{ and} & m = 1,\ p < 4(c_c - c_t) \end{cases}$$

$$Con_{1c} = \begin{cases} 0 < b < 1 \text{ and} & p \geq 2c_t m + b(4c_c - 6c_t) \\ \text{OR} & \\ b = 1 \text{ and} & m = 1,\ p \geq 4(c_c - c_t) \end{cases}$$

Con_{1b} is infeasible when we impose the condition $p > c_c > 2c_t$ (Assumption 1).

We can further detail Con_{3a}, Con_{3b}, and Con_{3c} as below (in addition to $b \leq m$, $c_c > 0$, $c_t \geq 0$, $c_c > 2c_t$, and $p > 0$):

$$
Con_{3a} = \begin{cases} 0 \leq b < 1 \text{ and} \\ b < m < \frac{4+b}{5},\ c_c(4+b) - m(5c_c + 2c_t) + c_t(4-2b) \leq p < 4c_c(1-b) + c_t(4+2m-6b) \text{ OR} \\ m = \frac{4+b}{5},\ c_t = 0,\ p < 4c_c(1-b) \text{ OR} \\ m = \frac{4+b}{5},\ c_t > 0,\ c_c(4+b) - m(5c_c + 2c_t) + c_t(4-2b) \leq p < 4c_c(1-b) + c_t(4+2m-6b) \text{ OR} \\ \frac{4+b}{5} < m < 1,\ c_t \leq \frac{4c_c - 5c_c m + c_c b}{-4+2m+2b},\ p < 4c_c(1-b) + c_t(4+2m-6b) \text{ OR} \\ \big[\frac{4+b}{5} < m < 1,\ \frac{4c_c - 5c_c m + c_c b}{-4+2m+2b} < c_t,\ c_c(4+b) - m(5c_c + 2c_t) + c_t(4-2b) \leq p, \\ p < 4c_c(1-b) + c_t(4+2m-6b)\big] \text{ OR} \\ m = 1,\ c_t < \frac{c_c(b-1)}{2b-2},\ p < (1-b)(4c_c + 6c_t) \end{cases}
$$

$$
Con_{3b} = \begin{cases} p < c_c(4-5m+b) + c_t(4-2m-2b),\ 0 \leq y < 1 \text{ and} \\ m < \frac{4+b}{5} \text{ OR} \\ m = \frac{4+b}{5},\ c_t > 0 \text{ OR} \\ \frac{4+b}{5} < m < 1,\ \frac{4c_c - 5c_c m + c_c b}{-4+2m+2b} < c_t \end{cases}
$$

$$
Con_{3c} = \begin{cases} 0 \leq b < 1,\ p \geq 4c_c(1-b) + c_t(4+2m-6b) \\ \text{OR} \\ b = m = 1 \end{cases}
$$

We can further detail Con_{4a}, Con_{4b}, and Con_{4c} as below (in addition to $b \leq m$, $c_c > 0$, $c_t \geq 0$, $c_c > 2c_t$, and $p > 0$):

$$
Con_{4a} = \begin{cases} b = 0 \text{ and} & 0 \leq m \leq \frac{1}{2},\ p < 4(c_c - c_t) + m(2c_t - 5c_c) \text{ OR} \\ & \frac{1}{2} < m < \frac{4}{5},\ c_t < \frac{-4cc + 5c_c m}{-4+2m},\ p < 4(c_c - c_t) + m(2c_t - 5c_c) \\ \text{OR} & \\ 0 < b < 1 \text{ and} & b \leq m < \frac{1+b}{2},\ p < c_c(4-5m+b) + c_t(-4+2m+2b) \text{ OR} \\ & \frac{1+b}{2} \leq m < \frac{4+b}{5},\ c_t < \frac{-4c_c + 5c_c m - c_c b}{-4+2m+2b},\ p < c_c(4-5m+b) + c_t(-4+2m+2b) \end{cases}
$$

$$
Con_{4c} = \begin{cases} b = 0 \text{ and} & 0 \leq m \leq \frac{1}{2},\ p \geq 4c_c - 4c_t - 5c_c m + 2c_t m \text{ OR} \\ & \frac{1}{2} < m < \frac{4}{5},\ c_t < \frac{-4cc + 5c_c m}{-4+2m},\ p \geq 4(c_c - c_t) + m(2c_t - 5c_c) \text{ OR} \\ & \frac{1}{2} < m < \frac{4}{5},\ \frac{-4c_c + 5c_c m}{-4+2m} \leq c_t \text{ OR} \\ & \frac{4}{5} \leq m \leq 1 \\ \text{OR} & \\ 0 < b < 1 \text{ and} & b \leq m < \frac{1+b}{2},\ p \geq 4c_c - 4c_t - 5c_c m + 2c_t m + c_c y + 2c_t y) \text{ OR} \\ & m = \frac{1+b}{2},\ c_t < \frac{-4c_c + 5c_c m - c_c b}{-4+2m+2b},\ p \geq c_c(4-5m+b) + c_t(-4+2m+2b) \text{ OR} \\ & \big[\frac{1+b}{2} < m < \frac{4+b}{5},\ c_t < \frac{-4c_c + 5c_c m - c_c b}{-4+2m+2b}, \\ & p \geq c_c(4-5m+b) + c_t(-4+2m+2b)\big] \text{ OR} \\ & \frac{1+b}{2} < m < \frac{4+b}{5},\ \frac{-4c_c + 5c_c m - c_c b}{-4+2m+2b} \leq c_t \text{ OR} \\ & \frac{4+b}{5} \leq m \leq 1 \\ \text{OR} & \\ b = 1,\ m = 1 & \end{cases}
$$

Con_{4b} is infeasible when we impose the condition $p > c_c > 2c_t$ (Assumption 1).

Suppose that $b > m$. We can further detail Con_{5a}, Con_{5b}, and Con_{5c} as below (in addition to $b > m$, $c_c > 0$, $c_t \geq 0$, $c_c > 2c_t$, and $p > 0$):

$$
Con_{5a} = \begin{cases} 0 < b \leq 1,\ p < c_c(5m-b) - 2c_t(m+b) \text{ and} & \frac{b}{5} < m \leq \frac{b}{2},\ c_t < \frac{5c_c m - c_c b}{2m+2b} \text{ OR} \\ & \frac{b}{2} < m < b \end{cases}
$$

$$
Con_{5c} = \begin{cases} 0 < b \leq 1 \quad \text{and} & 0 \leq m < \frac{b}{5} \text{ OR} \\ & \frac{b}{5} < m < \frac{b}{2},\ c_t < \frac{5c_c m - c_c b}{2m+2b},\ p \geq c_c(5m-b) - 2c_t(m+b) \text{ OR} \\ & \frac{b}{5} < m < \frac{b}{2},\ \frac{5c_c m - c_c b}{2m+2b} \leq c_t \text{ OR} \\ & m = \frac{b}{2},\ c_t < \frac{5c_c m - c_c b}{2m+2b},\ p \geq c_c(5m-b) - 2c_t(m+b) \text{ OR} \\ & \frac{b}{2} < m < b,\ p \geq c_c(5m-b) - 2c_t(m+b) \end{cases}
$$

Con_{5b} is infeasible when we impose the condition $p > c_c > 2c_t$ (Assumption 1).

We can further detail Con_{6a}, Con_{6b}, and Con_{6c} as below (in addition to $b > m$, $c_c > 0$, $c_t \geq 0$, $c_c > 2c_t$, and $p > 0$):

$$
\begin{aligned}
Con_{6a} &= \begin{cases} 0 < b \leq 1 \text{ and} & m = 0,\ p < b(4c_c + 6c_t) \text{ OR} \\ & 0 < m < \frac{b}{5},\ c_t \leq \frac{-5c_cm + c_cb}{2m+2b},\ p < -2c_tm + b(4c_c + 6c_t) \text{ OR} \\ & 0 < m < \frac{b}{5},\ \frac{-5c_cm + c_cb}{2m+2b} < c_t,\ c_c(5m - b) + 2ct(m + b) \leq p < -2c_tm + b(4c_c + 6c_t) \text{ OR} \\ & m = \frac{b}{5},\ c_t = 0,\ p < 4bc_c \text{ OR} \\ & m = \frac{b}{5},\ c_t > 0,\ c_c(5m - b) + 2ct(m + b) \leq p < -2c_tm + b(4c_c + 6c_t) \text{ OR} \\ & \frac{b}{5} < m < y,\ c_c(5m - b) + 2ct(m + b) \leq p < -2c_tm + b(4c_c + 6c_t) \end{cases} \\
Con_{6b} &= \begin{cases} 0 < b \leq 1 \text{ and} & 0 < m < \frac{b}{5},\ \frac{-5c_cm + c_cb}{2m+2b} < c_t,\ p < c_c(5m - b) + 2ct(m + b) \text{ OR} \\ & m = \frac{b}{5},\ c_t > 0,\ p < c_c(5m - b) + 2ct(m + b) \text{ OR} \\ & \frac{b}{5} < m < b,\ p < c_c(5m - b) + 2ct(m + b) \end{cases} \\
Con_{6c} &= \begin{cases} 0 < b \leq 1,\ p \geq -2c_tm + b(4c_c + 6c_t) \end{cases}
\end{aligned}
$$

We can further detail Con_{8a}, Con_{8b}, and Con_{8c} as below (in addition to $b > m$, $c_c > 0$, $c_t \geq 0$, $c_c > 2c_t$, and $p > 0$):

$$
\begin{aligned}
Con_{8a} &= \begin{cases} p < c_c(4 - 4b) - c_t(4 + 2m - 6b) \text{ and} & \\ \left[0 < b \leq \frac{1}{2}\right], & \\ \text{OR} & \\ \left[\frac{1}{2} < b < 1 \text{ and}\right. & m < -1 + 2b,\ c_t \leq \frac{c_c - c_cb}{2b - 2m} \text{ OR} \\ & m < -1 + 2b,\ \frac{c_c - c_cb}{2b - 2m} < c_t,\ c_c(b - 1) + 2c_t(b - m) \leq p \text{ OR} \\ & m = -1 + 2b,\ c_t < \frac{c_c - c_cb}{2b - 2m} \text{ OR} \\ & \left. -1 + 2b < m < b\right] \end{cases} \\
Con_{8c} &= \begin{cases} 0 < b < 1,\ p \geq 4c_c(1 - b) + c_t(-4 - 2m + 6b) & \\ \text{OR} & \\ b = 1 \text{ and} & m < 1,\ c_t = 0 \text{ OR} \\ & c_t > 0,\ p \geq 2ct(1 - m) \end{cases}
\end{aligned}
$$

Con_{8b} is infeasible when we impose the condition $p > c_c > 2c_t$ (Assumption 1).

Appendix C

Proofs of Analytical Results

Proof of Proposition 4.1. After a detailed analysis of the four possible configurations when $b \leq m$, we enumerate all possible solutions.

$$
R_A(b|b \leq m) = \begin{cases}
a_1^o & \text{if } 0 \leq a_1^o < b \text{ and } \pi_A^1(a_1^o, b) \geq \pi_A^2(b, b) \text{ AND} \\
& \quad b < a_3^o \leq m < a_4^o \leq 1,\ \pi_A^1(a_1^o, b) \geq max\{\pi_A^3(a_3^o, b), \pi_A^4(a_4^o, b)\} \text{ OR} \\
& \quad b < a_3^o \leq m,\ a_4^o > 1,\ \pi_A^1(a_1^o, b) \geq max\{\pi_A^3(a_3^o, b), \pi_A^4(1, b)\} \text{ OR} \\
& \quad a_3^o > m,\ m < a_4^o \leq 1,\ \pi_A^1(a_1^o, b) \geq max\{\pi_A^3(m, b), \pi_A^4(a_4^o, b)\} \text{ OR} \\
& \quad a_3^o > m,\ a_4^o > 1,\ \pi_A^1(a_1^o, b) \geq max\{\pi_A^3(m, b), \pi_A^4(1, b)\} \text{ OR} \\
& \quad a_3^o \leq b,\ m < a_4^o \leq 1,\ \pi_A^1(a_1^o, b) \geq \pi_A^4(a_4^o, b) \text{ OR} \\
& \quad a_3^o \leq b,\ a_4^o > 1,\ \pi_A^1(a_1^o, b) \geq \pi_A^4(1, b) \text{ OR} \\
& \quad b < a_3^o \leq m,\ a_4^o \leq m,\ \pi_A^1(a_1^o, b) \geq \pi_A^3(a_3^o, b) \text{ OR} \\
& \quad a_3^o > m,\ a_4^o \leq m,\ \pi_A^1(a_1^o, b) \geq \pi_A^3(m, b) \text{ OR} \\
& \quad a_3^o \leq b,\ a_4^o \leq m \\
0 & \text{if } a_1^o < 0 \text{ and } \pi_A^1(0, b) \geq \pi_A^2(b, b) \text{ AND} \\
& \quad b < a_3^o \leq m < a_4^o \leq 1,\ \pi_A^1(0, b) \geq max\{\pi_A^3(a_3^o, b), \pi_A^4(a_4^o, b)\} \text{ OR} \\
& \quad b < a_3^o \leq m,\ a_4^o > 1,\ \pi_A^1(0, b) \geq max\{\pi_A^3(a_3^o, b), \pi_A^4(1, b)\} \text{ OR} \\
& \quad a_3^o > m,\ m < a_4^o \leq 1,\ \pi_A^1(0, b) \geq max\{\pi_A^3(m, b), \pi_A^4(a_4^o, b)\} \text{ OR} \\
& \quad a_3^o > m,\ a_4^o > 1,\ \pi_A^1(0, b) \geq max\{\pi_A^3(m, b), \pi_A^4(1, b)\} \text{ OR} \\
& \quad a_3^o \leq b,\ m < a_4^o \leq 1,\ \pi_A^1(0, b) \geq \pi_A^4(a_4^o, b) \text{ OR} \\
& \quad a_3^o \leq b,\ a_4^o > 1,\ \pi_A^1(0, b) \geq \pi_A^4(1, b) \text{ OR} \\
& \quad b < a_3^o \leq m,\ a_4^o \leq m,\ \pi_A^1(0, b) \geq \pi_A^3(a_3^o, b) \text{ OR} \\
& \quad a_3^o > m,\ a_4^o \leq m,\ \pi_A^1(0, b) \geq \pi_A^3(m, b) \text{ OR} \\
& \quad a_3^o \leq b,\ a_4^o \leq m \\
\ldots
\end{cases}
$$

$$
R_A(b|b \leq m) = \begin{cases}
\ldots \\
b \quad \text{if} \\
\quad 0 \leq a_1^o < b < a_3^o \leq m < a_4^o \leq 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(a_3^o,b), \pi_A^4(a_4^o,b)\} \text{ OR} \\
\quad 0 \leq a_1^o < b < a_3^o \leq m,\ a_4^o > 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(a_3^o,b), \pi_A^4(1,b)\} \text{ OR} \\
\quad 0 \leq a_1^o < b,\ a_3^o > m,\ m < a_4^o \leq 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(m,b), \pi_A^4(a_4^o,b)\} \text{ OR} \\
\quad 0 \leq a_1^o < b,\ a_3^o > m,\ a_4^o > 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(m,b), \pi_A^4(1,b)\} \text{ OR} \\
\quad 0 \leq a_1^o < b,\ a_3^o \leq b,\ m < a_4^o \leq 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^4(a_4^o,b)\} \text{ OR} \\
\quad 0 \leq a_1^o < b,\ a_3^o \leq b,\ a_4^o > 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^4(1,b)\} \text{ OR} \\
\quad 0 \leq a_1^o < b < a_3^o \leq m,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(a_3^o,b)\} \text{ OR} \\
\quad 0 \leq a_1^o < b,\ a_3^o > m,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(m,b)\} \text{ OR} \\
\quad 0 \leq a_1^o < b,\ a_3^o \leq b,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq \pi_A^1(a_1^o,b) \text{ OR} \\
\quad a_1^o < 0,\ b < a_3^o \leq m < a_4^o \leq 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(0,b), \pi_A^3(a_3^o,b), \pi_A^4(a_4^o,b)\} \text{ OR} \\
\quad a_1^o < 0,\ b < a_3^o \leq m,\ a_4^o > 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(0,b), \pi_A^3(a_3^o,b), \pi_A^4(1,b)\} \text{ OR} \\
\quad a_1^o < 0,\ a_3^o > m,\ m < a_4^o \leq 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(0,b), \pi_A^3(m,b), \pi_A^4(a_4^o,b)\} \text{ OR} \\
\quad a_1^o < 0,\ a_3^o > m,\ a_4^o > 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(0,b), \pi_A^3(m,b), \pi_A^4(1,b)\} \text{ OR} \\
\quad a_1^o < 0,\ a_3^o \leq b,\ m < a_4^o \leq 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(0,b), \pi_A^4(a_4^o,b)\} \text{ OR} \\
\quad a_1^o < 0,\ a_3^o \leq b,\ a_4^o > 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(0,b), \pi_A^4(1,b)\} \text{ OR} \\
\quad a_1^o < 0,\ b < a_3^o \leq m,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq max\{\pi_A^1(0,b), \pi_A^3(a_3^o,b)\} \text{ OR} \\
\quad a_1^o < 0,\ a_3^o > m,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq max\{\pi_A^1(0,b), \pi_A^3(m,b)\} \text{ OR} \\
\quad a_1^o < 0,\ a_3^o \leq b,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq \pi_A^1(0,b) \text{ OR} \\
\quad a_1^o \geq b,\ b < a_3^o \leq m,\ m < a_4^o \leq 1,\ \pi_A^2(b,b) \geq max\{\pi_A^3(a_3^o,b), \pi_A^4(a_4^o,b)\} \text{ OR} \\
\quad a_1^o \geq b,\ b < a_3^o \leq m,\ a_4^o > 1,\ \pi_A^2(b,b) \geq max\{\pi_A^3(a_3^o,b), \pi_A^4(1,b)\} \text{ OR} \\
\quad a_1^o \geq b,\ a_3^o > m,\ m < a_4^o \leq 1,\ \pi_A^2(b,b) \geq max\{\pi_A^3(m,b), \pi_A^4(a_4^o,b)\} \text{ OR} \\
\quad a_1^o \geq b,\ a_3^o > m,\ a_4^o > 1,\ \pi_A^2(b,b) \geq max\{\pi_A^3(m,b), \pi_A^4(1,b)\} \text{ OR} \\
\quad a_1^o \geq b,\ a_3^o \leq b,\ m < a_4^o \leq 1,\ \pi_A^2(b,b) \geq \pi_A^4(a_4^o,b) \text{ OR} \\
\quad a_1^o \geq b,\ a_3^o \leq b,\ a_4^o > 1,\ \pi_A^2(b,b) \geq \pi_A^4(1,b) \text{ OR} \\
\quad a_1^o \geq b,\ b < a_3^o \leq m,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq \pi_A^3(a_3^o,b) \text{ OR} \\
\quad a_1^o \geq b,\ a_3^o > m,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq \pi_A^3(m,b) \text{ OR} \\
\quad a_1^o \geq b,\ a_3^o \leq b,\ a_4^o \leq m \\
a_3^o \quad \text{if } b < a_3^o \leq m \text{ and } \pi_A^3(a_3^o,b) \geq \pi_A^2(b,b) \text{ AND} \\
\quad 0 \leq a_1^o < b,\ m < a_4^o \leq 1,\ \pi_A^3(a_3^o,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^4(a_4^o,b)\} \text{ OR} \\
\quad 0 \leq a_1^o < b,\ a_4^o > 1,\ \pi_A^3(a_3^o,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^4(1,b)\} \text{ OR} \\
\quad a_1^o < 0,\ m < a_4^o \leq 1,\ \pi_A^3(a_3^o,b) \geq max\{\pi_A^1(0,b), \pi_A^4(a_4^o,b)\} \text{ OR} \\
\quad a_1^o < 0,\ a_4^o > 1,\ \pi_A^3(a_3^o,b) \geq max\{\pi_A^1(0,b), \pi_A^4(1,b)\} \text{ OR} \\
\quad a_1^o \geq b,\ m < a_4^o \leq 1,\ \pi_A^3(a_3^o,b) \geq \pi_A^4(a_4^o,b) \text{ OR} \\
\quad a_1^o \geq b,\ a_4^o > 1,\ \pi_A^3(a_3^o,b) \geq \pi_A^4(1,b) \text{ OR} \\
\quad 0 \leq a_1^o < b,\ a_4^o \leq m,\ \pi_A^3(a_3^o,b) \geq \pi_A^1(a_1^o,b) \text{ OR} \\
\quad a_1^o < 0,\ a_4^o \leq m,\ \pi_A^3(a_3^o,b) \geq \pi_A^1(0,b) \text{ OR} \\
\quad a_1^o \geq b,\ a_4^o \leq m \\
\ldots
\end{cases}
$$

$$R_A(b|b \leq m) = \begin{cases} \dots & \\ m & \text{if } a_3^o > m \text{ and } \pi_A^3(m,b) \geq \pi_A^2(b,b) \text{ AND} \\ & \quad 0 \leq a_1^o < b,\ m < a_4^o \leq 1,\ \pi_A^3(m,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^4(a_4^o,b)\} \text{ OR} \\ & \quad 0 \leq a_1^o < b,\ a_4^o > 1,\ \pi_A^3(m,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^4(1,b)\} \text{ OR} \\ & \quad a_1^o < 0,\ m < a_4^o \leq 1,\ \pi_A^3(m,b) \geq max\{\pi_A^1(0,b), \pi_A^4(a_4^o,b)\} \text{ OR} \\ & \quad a_1^o < 0,\ a_4^o > 1,\ \pi_A^3(m,b) \geq max\{\pi_A^1(0,b), \pi_A^4(1,b)\} \text{ OR} \\ & \quad a_1^o \geq b,\ m < a_4^o \leq 1,\ \pi_A^3(m,b) \geq \pi_A^4(a_4^o,b) \text{ OR} \\ & \quad a_1^o \geq b,\ a_4^o > 1,\ \pi_A^3(m,b) \geq \pi_A^4(1,b) \text{ OR} \\ & \quad 0 \leq a_1^o < b,\ a_4^o \leq m,\ \pi_A^3(m,b) \geq \pi_A^1(a_1^o,b) \text{ OR} \\ & \quad a_1^o < 0,\ a_4^o \leq m,\ \pi_A^3(m,b) \geq \pi_A^1(0,b) \text{ OR} \\ & \quad a_1^o \geq b,\ a_4^o \leq m \\ a_4^o & \text{if } m < a_4^o \leq 1 \text{ and } \pi_A^4(a_4^o,b) \geq \pi_A^2(b,b) \text{ AND} \\ & \quad 0 \leq a_1^o < b < a_3^o \leq m,\ \pi_A^4(a_4^o,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(a_3^o,b)\} \text{ OR} \\ & \quad 0 \leq a_1^o < b,\ a_3^o > m,\ \pi_A^4(a_4^o,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(m,b)\} \text{ OR} \\ & \quad a_1^o < 0,\ b < a_3^o \leq m,\ \pi_A^4(a_4^o,b) \geq max\{\pi_A^1(0,b), \pi_A^3(a_3^o,b)\} \text{ OR} \\ & \quad a_1^o < 0,\ a_3^o > m,\ \pi_A^4(a_4^o,b) \geq max\{\pi_A^1(0,b), \pi_A^3(m,b)\} \text{ OR} \\ & \quad a_1^o \geq b,\ b < a_3^o \leq m,\ \pi_A^4(a_4^o,b) \geq \pi_A^3(a_3^o,b) \text{ OR} \\ & \quad a_1^o \geq b,\ a_3^o > m,\ \pi_A^4(a_4^o,b) \geq \pi_A^3(m,b) \text{ OR} \\ & \quad 0 \leq a_1^o < b,\ a_3^o \leq b,\ \pi_A^4(a_4^o,b) \geq \pi_A^1(a_1^o,b) \text{ OR} \\ & \quad a_1^o < 0,\ a_3^o \leq b,\ \pi_A^4(a_4^o,b) \geq \pi_A^1(0,b) \text{ OR} \\ & \quad a_1^o \geq b,\ a_3^o \leq b \\ 1 & \text{if } a_4^o > 1 \text{ and } \pi_A^4(1,b) \geq \pi_A^2(b,b) \text{ AND} \\ & \quad 0 \leq a_1^o < b < a_3^o \leq m,\ \pi_A^4(1,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(a_3^o,b)\} \text{ OR} \\ & \quad 0 \leq a_1^o < b,\ a_3^o > m,\ \pi_A^4(1,b) \geq max\{\pi_A^1(a_1^o,b), \pi_A^3(m,b)\} \text{ OR} \\ & \quad a_1^o < 0,\ b < a_3^o \leq m,\ \pi_A^4(1,b) \geq max\{\pi_A^1(0,b), \pi_A^3(a_3^o,b)\} \text{ OR} \\ & \quad a_1^o < 0,\ a_3^o > m,\ \pi_A^4(1,b) \geq max\{\pi_A^1(0,b), \pi_A^3(m,b)\} \text{ OR} \\ & \quad a_1^o \geq b,\ b < a_3^o \leq m,\ \pi_A^4(1,b) \geq \pi_A^3(a_3^o,b) \text{ OR} \\ & \quad a_1^o \geq b,\ a_3^o > m,\ \pi_A^4(1,b) \geq \pi_A^3(m,b) \text{ OR} \\ & \quad 0 \leq a_1^o < b,\ a_3^o \leq b,\ \pi_A^4(1,b) \geq \pi_A^1(a_1^o,b) \text{ OR} \\ & \quad a_1^o < 0,\ a_3^o \leq b,\ \pi_A^4(1,b) \geq \pi_A^1(0,b) \text{ OR} \\ & \quad a_1^o \geq b,\ a_3^o \leq b \\ \emptyset & \text{otherwise.} \end{cases}$$

In the above list of conditions, we observe that under Assumption 1, $a_1^o < 0$ (Con_{1b}) never holds. This is because if

$$\begin{aligned} a_1^o &= \frac{p + c_c b + 2c_t(b-m)}{5c_c - 4c_t} < 0 \\ &\Rightarrow\ p + c_c b + 2c_t(b-m) < 0 \\ &\Rightarrow\ 0 < -p - c_c b, \end{aligned}$$

which violates $p > 0$ and $c_c > 0$. We also observe that under Assumption 1, $a_4^o > 1$ (Con_{4b}) never holds. This is because if

$$\begin{aligned} a_4^o &= \frac{-p + c_c(4+b) - 2c_t(2+m-b)}{5c_c - 4c_t} \geq 1 \\ &\Rightarrow\ -p + 4c_c + c_c b - 4c_t - 2c_t m + 2c_t b \geq 5c_c - 4c_t \\ &\Rightarrow\ c_c(b-1) + 2c_t(b-m) \geq p \\ &\Rightarrow\ 0 > p, \end{aligned}$$

which violates $p > 0$.

Each of the following inequalities can never hold: $a_1^o \leq 0$, $a_3^o \geq 1$, $a_4^o \geq 1$. Similarly, conditions $0 \leq a_1^o < b$, $b < a_3^o \leq m$, $m < a_4^o \leq 1$ $(Con_{1a}, Con_{3a}, Con_{4a})$ do not hold together; $a_3^o \leq b$, $m < a_4^o \leq 1$ (Con_{3c}, Con_{4a}) do not hold together; $a_1^o \geq b$, $b < a_3^o \leq m$, $m < a_4^o \leq 1$ $(Con_{1c}, Con_{3a}, Con_{4a})$ do not hold together; and finally $a_1^o \geq b$, $a_3^o \leq b$, $m < a_4^o \leq 1$ $(Con_{1c}, Con_{3c}, Con_{4a})$ do not hold together. Thus the best response of retailer A can be rewritten as

$$R_A(b|b \leq m) = \begin{cases} a_1^o & \text{if } 0 \leq a_1^o < b \text{ and } \pi_A^1(a_1^o, b) \geq \pi_A^2(b,b) \text{ AND} \\ & \quad a_3^o > m,\ m < a_4^o \leq 1,\ \pi_A^1(a_1^o, b) \geq max\{\pi_A^3(m,b), \pi_A^4(a_4^o, b)\} \text{ OR} \\ & \quad b < a_3^o \leq m,\ a_4^o \leq m,\ \pi_A^1(a_1^o, b) \geq \pi_A^3(a_3^o, b) \text{ OR} \\ & \quad a_3^o > m,\ a_4^o \leq m,\ \pi_A^1(a_1^o, b) \geq \pi_A^3(m, b) \text{ OR} \\ & \quad a_3^o \leq b,\ a_4^o \leq m \\ b & \text{if} \\ & \quad 0 \leq a_1^o < b,\ a_3^o > m,\ m < a_4^o \leq 1,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o, b), \pi_A^3(m,b), \pi_A^4(a_4^o, b)\} \text{ OR} \\ & \quad 0 \leq a_1^o < b,\ b < a_3^o \leq m,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o, b), \pi_A^3(a_3^o, b)\} \text{ OR} \\ & \quad 0 \leq a_1^o < b,\ a_3^o > m,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq max\{\pi_A^1(a_1^o, b), \pi_A^3(m,b)\} \text{ OR} \\ & \quad 0 \leq a_1^o < b,\ a_3^o \leq b,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq \pi_A^1(a_1^o, b) \text{ OR} \\ & \quad a_1^o \geq b,\ a_3^o > m,\ m < a_4^o \leq 1,\ \pi_A^2(b,b) \geq max\{\pi_A^3(m,b), \pi_A^4(a_4^o, b)\} \text{ OR} \\ & \quad a_1^o \geq b,\ b < a_3^o \leq m,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq \pi_A^3(a_3^o, b) \text{ OR} \\ & \quad a_1^o \geq b,\ a_3^o > m,\ a_4^o \leq m,\ \pi_A^2(b,b) \geq \pi_A^3(m,b) \text{ OR} \\ & \quad a_1^o \geq b,\ a_3^o \leq b,\ a_4^o \leq m \\ a_3^o & \text{if } b < a_3^o \leq m \text{ and } \pi_A^3(a_3^o, b) \geq \pi_A^2(b,b) \text{ AND} \\ & \quad 0 \leq a_1^o < b,\ a_4^o \leq m,\ \pi_A^3(a_3^o, b) \geq \pi_A^1(a_1^o, b) \text{ OR} \\ & \quad a_1^o \geq b,\ a_4^o \leq m \\ m & \text{if } a_3^o > m \text{ and } \pi_A^3(m,b) \geq \pi_A^2(b,b) \text{ AND} \\ & \quad 0 \leq a_1^o < b,\ m < a_4^o \leq 1,\ \pi_A^3(m,b) \geq max\{\pi_A^1(a_1^o, b), \pi_A^4(a_4^o, b)\} \text{ OR} \\ & \quad a_1^o \geq b,\ m < a_4^o \leq 1,\ \pi_A^3(m,b) \geq \pi_A^4(a_4^o, b) \text{ OR} \\ & \quad 0 \leq a_1^o < b,\ a_4^o \leq m,\ \pi_A^3(m,b) \geq \pi_A^1(a_1^o, b) \text{ OR} \\ & \quad a_1^o \geq b,\ a_4^o \leq m \\ a_4^o & \text{if } m < a_4^o \leq 1 \text{ and } \pi_A^4(a_4^o, b) \geq \pi_A^2(b,b) \text{ AND} \\ & \quad 0 \leq a_1^o < b,\ a_3^o > m,\ \pi_A^4(a_4^o, b) \geq max\{\pi_A^1(a_1^o, b), \pi_A^3(m,b)\} \text{ OR} \\ & \quad a_1^o \geq b,\ a_3^o > m,\ \pi_A^4(a_4^o, b) \geq \pi_A^3(m,b) \text{ OR} \\ \emptyset & \text{otherwise.} \end{cases}$$

$\square$

Proof of Proposition 4.2. After a detailed analysis of the four possible configurations when $b > m$, we enumerate all possible solutions.

$$R_A(b|b>m) = \begin{cases} a_5^o & \text{if } 0 \leq a_5^o < m \text{ and } \pi_A^5(a_5^o,b) \geq \pi_A^7(b,b) \text{ AND} \\ & \quad m \leq a_6^o < b < a_8^o \leq 1,\ \pi_A^5(a_5^o,b) \geq max\{\pi_A^6(a_6^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\ & \quad m \leq a_6^o < b,\ a_8^o > 1,\ \pi_A^5(a_5^o,b) \geq max\{\pi_A^6(a_6^o,b), \pi_A^8(1,b)\} \text{ OR} \\ & \quad a_6^o < m,\ b < a_8^o \leq 1,\ \pi_A^5(a_5^o,b) \geq max\{\pi_A^6(m,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\ & \quad a_6^o < m,\ a_8^o > 1,\ \pi_A^5(a_5^o,b) \geq max\{\pi_A^6(m,b), \pi_A^8(1,b)\} \text{ OR} \\ & \quad a_6^o \geq b,\ b < a_8^o \leq 1,\ \pi_A^5(a_5^o,b) \geq \pi_A^8(a_8^o,b) \text{ OR} \\ & \quad a_6^o \geq b,\ a_8^o > 1,\ \pi_A^5(a_5^o,b) \geq \pi_A^8(1,b) \text{ OR} \\ & \quad m \leq a_6^o < b,\ a_8^o \leq b,\ \pi_A^5(a_5^o,b) \geq \pi_A^6(a_6^o,b) \text{ OR} \\ & \quad a_6^o < m,\ a_8^o \leq b,\ \pi_A^5(a_5^o,b) \geq \pi_A^6(m,b) \text{ OR} \\ & \quad a_6^o \geq b,\ a_8^o \leq b \\ 0 & \text{if } a_5^o < 0 \text{ and } \pi_A^5(0,b) \geq \pi_A^7(b,b) \text{ AND} \\ & \quad m \leq a_6^o < b < a_8^o \leq 1,\ \pi_A^5(0,b) \geq max\{\pi_A^6(a_6^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\ & \quad m \leq a_6^o < b,\ a_8^o > 1,\ \pi_A^5(0,b) \geq max\{\pi_A^6(a_6^o,b), \pi_A^8(1,b)\} \text{ OR} \\ & \quad a_6^o < m,\ b < a_8^o \leq 1,\ \pi_A^5(0,b) \geq max\{\pi_A^6(m,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\ & \quad a_6^o < m,\ a_8^o > 1,\ \pi_A^5(0,b) \geq max\{\pi_A^6(m,b), \pi_A^8(1,b)\} \text{ OR} \\ & \quad a_6^o \geq b,\ b < a_8^o \leq 1,\ \pi_A^5(0,b) \geq \pi_A^8(a_8^o,b) \text{ OR} \\ & \quad a_6^o \geq b,\ a_8^o > 1,\ \pi_A^5(0,b) \geq \pi_A^8(1,b) \text{ OR} \\ & \quad m \leq a_6^o < b,\ a_8^o \leq b,\ \pi_A^5(0,b) \geq \pi_A^6(a_6^o,b) \text{ OR} \\ & \quad a_6^o < m,\ a_8^o \leq b,\ \pi_A^5(0,b) \geq \pi_A^6(m,b) \text{ OR} \\ & \quad a_6^o \geq b,\ a_8^o \leq b \\ b & \text{if} \\ & \quad 0 \leq a_5^o < m \leq a_6^o < b,\ b < a_8^o \leq 1,\ \pi_A^7(b,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^6(a_6^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\ & \quad 0 \leq a_5^o < m \leq a_6^o < b,\ a_8^o > 1,\ \pi_A^7(b,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^6(a_6^o,b), \pi_A^8(1,b)\} \text{ OR} \\ & \quad 0 \leq a_5^o < m,\ a_6^o < m,\ b < a_8^o \leq 1,\ \pi_A^7(b,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^6(m,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\ & \quad 0 \leq a_5^o < m,\ a_6^o < m,\ a_8^o > 1,\ \pi_A^7(b,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^6(m,b), \pi_A^8(1,b)\} \text{ OR} \\ & \quad 0 \leq a_5^o < m,\ a_6^o \geq b,\ b < a_8^o \leq 1,\ \pi_A^7(b,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\ & \quad 0 \leq a_5^o < m,\ a_6^o \geq b,\ a_8^o > 1,\ \pi_A^7(b,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^8(1,b)\} \text{ OR} \\ & \quad 0 \leq a_5^o < m \leq a_6^o < b,\ a_8^o \leq b,\ \pi_A^7(b,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^6(a_6^o,b)\} \text{ OR} \\ & \quad 0 \leq a_5^o < m,\ a_6^o < m,\ a_8^o \leq b,\ \pi_A^7(b,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^6(m,b)\} \text{ OR} \\ & \quad 0 \leq a_5^o < m,\ a_6^o \geq b,\ a_8^o \leq b,\ \pi_A^7(b,b) \geq \pi_A^5(a_5^o,b) \text{ OR} \\ & \quad a_5^o < 0,\ m \leq a_6^o < b,\ b < a_8^o \leq 1,\ \pi_A^7(b,b) \geq max\{\pi_A^5(0,b), \pi_A^6(a_6^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\ & \quad a_5^o < 0,\ m \leq a_6^o < b,\ a_8^o > 1,\ \pi_A^7(b,b) \geq max\{\pi_A^5(0,b), \pi_A^6(a_6^o,b), \pi_A^8(1,b)\} \text{ OR} \\ & \quad a_5^o < 0,\ a_6^o < m,\ b < a_8^o \leq 1,\ \pi_A^7(b,b) \geq max\{\pi_A^5(0,b), \pi_A^6(m,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\ & \quad a_5^o < 0,\ a_6^o < m,\ a_8^o > 1,\ \pi_A^7(b,b) \geq max\{\pi_A^5(0,b), \pi_A^6(m,b), \pi_A^8(1,b)\} \text{ OR} \\ & \quad a_5^o < 0,\ a_6^o \geq b,\ b < a_8^o \leq 1,\ \pi_A^7(b,b) \geq max\{\pi_A^5(0,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\ & \quad a_5^o < 0,\ a_6^o \geq b,\ a_8^o > 1,\ \pi_A^7(b,b) \geq max\{\pi_A^5(0,b), \pi_A^8(1,b)\} \text{ OR} \\ & \quad a_5^o < 0,\ m \leq a_6^o < b,\ a_8^o \leq b,\ \pi_A^7(b,b) \geq max\{\pi_A^5(0,b), \pi_A^6(a_6^o,b)\} \text{ OR} \\ & \quad a_5^o < 0,\ a_6^o < m,\ a_8^o \leq b,\ \pi_A^7(b,b) \geq max\{\pi_A^5(0,b), \pi_A^6(m,b)\} \text{ OR} \\ & \quad a_5^o < 0,\ a_6^o \geq b,\ a_8^o \leq b,\ \pi_A^7(b,b) \geq \pi_A^5(0,b) \text{ OR} \\ & \quad a_5^o \geq m,\ m \leq a_6^o < b < a_8^o \leq 1,\ \pi_A^7(b,b) \geq max\{\pi_A^6(a_6^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\ & \quad a_5^o \geq m,\ m \leq a_6^o < b,\ a_8^o > 1,\ \pi_A^7(b,b) \geq max\{\pi_A^6(a_6^o,b), \pi_A^8(1,b)\} \text{ OR} \\ \ldots & \end{cases}$$

$$
R_A(b|b>m) = \begin{cases}
\cdots \\
b & \text{if} \\
& a_5^o \geq m,\ a_6^o < m,\ b < a_8^o \leq 1,\ \pi_A^7(b,b) \geq max\{\pi_A^6(m,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& a_5^o \geq m,\ a_6^o < m,\ a_8^o > 1,\ \pi_A^7(b,b) \geq max\{\pi_A^6(m,b), \pi_A^8(1,b)\} \text{ OR} \\
& a_5^o \geq m,\ a_6^o \geq b,\ b < a_8^o \leq 1,\ \pi_A^7(b,b) \geq \pi_A^8(a_8^o,b) \text{ OR} \\
& a_5^o \geq m,\ a_6^o \geq b,\ a_8^o > 1,\ \pi_A^7(b,b) \geq \pi_A^8(1,b) \text{ OR} \\
& a_5^o \geq m,\ m \leq a_6^o < b,\ a_8^o \leq b,\ \pi_A^7(b,b) \geq \pi_A^6(a_6^o,b) \text{ OR} \\
& a_5^o \geq m,\ a_6^o < m,\ a_8^o \leq b,\ \pi_A^7(b,b) \geq \pi_A^6(m,b) \text{ OR} \\
& a_5^o \geq m,\ a_6^o \geq b,\ a_8^o \leq b \\
a_6^o & \text{if } m \leq a_6^o < b \text{ and } \pi_A^6(a_6^o,b) \geq \pi_A^7(b,b) \text{ AND} \\
& 0 \leq a_5^o < m,\ b < a_8^o \leq 1,\ \pi_A^6(a_6^o,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& 0 \leq a_5^o < m,\ a_8^o > 1,\ \pi_A^6(a_6^o,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^8(1,b)\} \text{ OR} \\
& a_5^o < 0,\ b < a_8^o \leq 1,\ \pi_A^6(a_6^o,b) \geq max\{\pi_A^5(0,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& a_5^o < 0,\ a_8^o > 1,\ \pi_A^6(a_6^o,b) \geq max\{\pi_A^5(0,b), \pi_A^8(1,b)\} \text{ OR} \\
& a_5^o \geq m,\ b < a_8^o \leq 1,\ \pi_A^6(a_6^o,b) \geq \pi_A^8(a_8^o,b) \text{ OR} \\
& a_5^o \geq m,\ a_8^o > 1,\ \pi_A^6(a_6^o,b) \geq \pi_A^8(1,b) \text{ OR} \\
& 0 \leq a_5^o < m,\ a_8^o \leq b,\ \pi_A^6(a_6^o,b) \geq \pi_A^5(a_5^o,b) \text{ OR} \\
& a_5^o < 0,\ a_8^o \leq b,\ \pi_A^6(a_6^o,b) \geq \pi_A^5(0,b) \text{ OR} \\
& a_5^o \geq m,\ a_8^o \leq b \\
m & \text{if } a_6^o < m \text{ and } \pi_A^6(m,b) \geq \pi_A^7(b,b) \text{ AND} \\
& 0 \leq a_5^o < m,\ b < a_8^o \leq 1,\ \pi_A^6(m,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& 0 \leq a_5^o < m,\ a_8^o > 1,\ \pi_A^6(m,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^8(1,b)\} \text{ OR} \\
& a_5^o < 0,\ b < a_8^o \leq 1,\ \pi_A^6(m,b) \geq max\{\pi_A^5(0,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& a_5^o < 0,\ a_8^o > 1,\ \pi_A^6(m,b) \geq max\{\pi_A^5(0,b), \pi_A^8(1,b)\} \text{ OR} \\
& a_5^o \geq m,\ b < a_8^o \leq 1,\ \pi_A^6(m,b) \geq \pi_A^8(a_8^o,b) \text{ OR} \\
& a_5^o \geq m,\ a_8^o > 1,\ \pi_A^6(m,b) \geq \pi_A^8(1,b) \text{ OR} \\
& 0 \leq a_5^o < m,\ a_8^o \leq b,\ \pi_A^6(m,b) \geq \pi_A^5(a_5^o,b) \text{ OR} \\
& a_5^o < 0,\ a_8^o \leq b,\ \pi_A^6(m,b) \geq \pi_A^5(0,b) \text{ OR} \\
& a_5^o \geq m,\ a_8^o \leq b \\
a_8^o & \text{if } b < a_8^o \leq 1 \text{ and } \pi_A^8(a_8^o,b) \geq \pi_A^7(b,b) \text{ AND} \\
& 0 \leq a_5^o < m,\ m \leq a_6^o < b,\ \pi_A^8(a_8^o,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^6(a_6^o,b)\} \text{ OR} \\
& 0 \leq a_5^o < m,\ a_6^o < m,\ \pi_A^8(a_8^o,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^6(m,b)\} \text{ OR} \\
& a_5^o < 0,\ m \leq a_6^o < b,\ \pi_A^8(a_8^o,b) \geq max\{\pi_A^5(0,b), \pi_A^6(a_6^o,b)\} \text{ OR} \\
& a_5^o < 0,\ a_6^o < m,\ \pi_A^8(a_8^o,b) \geq max\{\pi_A^5(0,b), \pi_A^6(m,b)\} \text{ OR} \\
& a_5^o \geq m,\ m \leq a_6^o < b,\ \pi_A^8(a_8^o,b) \geq \pi_A^6(a_6^o,b) \text{ OR} \\
& a_5^o \geq m,\ a_6^o < m,\ \pi_A^8(a_8^o,b) \geq \pi_A^6(m,b) \text{ OR} \\
& 0 \leq a_5^o < m,\ a_6^o \geq b,\ \pi_A^8(a_8^o,b) \geq \pi_A^5(a_5^o,b) \text{ OR} \\
& a_5^o < 0,\ a_6^o \geq b,\ \pi_A^8(a_8^o,b) \geq \pi_A^5(0,b) \text{ OR} \\
& a_5^o \geq m,\ a_6^o \geq b \\
1 & \text{if } a_8^o > 1 \text{ and } \pi_A^8(1,b) \geq \pi_A^7(b,b) \text{ AND} \\
& 0 \leq a_5^o < m \leq a_6^o < b,\ \pi_A^8(1,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^6(a_6^o,b)\} \text{ OR} \\
& 0 \leq a_5^o < m,\ a_6^o < m,\ \pi_A^8(1,b) \geq max\{\pi_A^5(a_5^o,b), \pi_A^6(m,b)\} \text{ OR} \\
& a_5^o < 0,\ m \leq a_6^o < b,\ \pi_A^8(1,b) \geq max\{\pi_A^5(0,b), \pi_A^6(a_6^o,b)\} \text{ OR} \\
& a_5^o < 0,\ a_6^o < m,\ \pi_A^8(1,b) \geq max\{\pi_A^5(0,b), \pi_A^6(m,b)\} \text{ OR} \\
& a_5^o \geq m,\ m \leq a_6^o < b,\ \pi_A^8(1,b) \geq \pi_A^6(a_6^o,b) \text{ OR} \\
& a_5^o \geq m,\ a_6^o < m,\ \pi_A^8(1,b) \geq \pi_A^6(m,b) \text{ OR} \\
& 0 \leq a_5^o < m,\ a_6^o \geq b,\ \pi_A^8(1,b) \geq \pi_A^5(a_5^o,b) \text{ OR} \\
& a_5^o < 0,\ a_6^o \geq b,\ \pi_A^8(1,b) \geq \pi_A^5(0,b) \text{ OR} \\
& a_5^o \geq m,\ a_6^o \geq b \\
\emptyset & \text{otherwise.}
\end{cases}
$$

In the above list of conditions, we observe that under Assumption 1, $a_5^o < 0$

(Con_{5b}) never holds. This is because if

$$\begin{aligned} a_5^o &= \frac{p + c_c b + 2c_t(b-m)}{5c_c - 4c_t} \leq 0 \\ &\Rightarrow p + c_c b + 2c_t(b-m) \leq 0 \\ &\Rightarrow p \leq 2c_t(m-b) - c_c b \\ &\Rightarrow p < 0, \end{aligned}$$

which violates $p > 0$. We also observe that under Assumption 1, $a_8^o > 1$ (Con_{8b}) never holds. This is because if

$$\begin{aligned} a_8^o &= \frac{-p + c_c(4+b) + 2c_t(b-m-2)}{5c_c - 4c_t} > 1 \\ &\Rightarrow 2c_t < p < bc_c + 2c_t(b-m) - c_c \\ &\Rightarrow 2c_t(1-b+m) < c_c(b-1) \\ &\Rightarrow 2c_t(1-b+m) < 0 \\ &\Rightarrow 1 + m < b, \end{aligned}$$

which violates $b \leq 1$.

Each of the following inequalities can never hold: $a_5^o \leq 0$ and $a_8^o \geq 1$. Similarly, $0 \leq a_5^o < m$, $a_6^o \geq b$, $b < a_8^o \leq 1$ ($Con_{5a}, Con_{6c}, Con_{8a}$) do not hold together; and $0 \leq a_5^o < m$, $a_6^o \geq b$, $a_8^o \leq b$ ($Con_{5a}, Con_{6c}, Con_{8c}$) do not hold together. Thus the best response of retailer A can be rewritten as

$$
R_A(b|b>m) = \begin{cases}
a_5^o & \text{if } 0 \le a_5^o < m \text{ and } \pi_A^5(a_5^o,b) \ge \pi_A^7(b,b) \text{ AND} \\
& \quad m \le a_6^o < b < a_8^o \le 1,\ \pi_A^5(a_5^o,b) \ge max\{\pi_A^6(a_6^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& \quad a_6^o < m,\ b < a_8^o \le 1,\ \pi_A^5(a_5^o,b) \ge max\{\pi_A^6(m,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& \quad m \le a_6^o < b,\ a_8^o \le b,\ \pi_A^5(a_5^o,b) \ge \pi_A^6(a_6^o,b) \text{ OR} \\
& \quad a_6^o < m,\ a_8^o \le b,\ \pi_A^5(a_5^o,b) \ge \pi_A^6(m,b) \\
b & \text{if} \\
& \quad 0 \le a_5^o < m \le a_6^o < b,\ b < a_8^o \le 1,\ \pi_A^7(b,b) \ge max\{\pi_A^5(a_5^o,b), \pi_A^6(a_6^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& \quad 0 \le a_5^o < m,\ a_6^o < m,\ b < a_8^o \le 1,\ \pi_A^7(b,b) \ge max\{\pi_A^5(a_5^o,b), \pi_A^6(m,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& \quad 0 \le a_5^o < m \le a_6^o < b,\ a_8^o \le b,\ \pi_A^7(b,b) \ge max\{\pi_A^5(a_5^o,b), \pi_A^6(a_6^o,b)\} \text{ OR} \\
& \quad 0 \le a_5^o < m,\ a_6^o < m,\ a_8^o \le b,\ \pi_A^7(b,b) \ge max\{\pi_A^5(a_5^o,b), \pi_A^6(m,b)\} \text{ OR} \\
& \quad a_5^o \ge m,\ m \le a_6^o < b,\ b < a_8^o \le 1,\ \pi_A^7(b,b) \ge max\{\pi_A^6(a_6^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& \quad a_5^o \ge m,\ a_6^o < m,\ b < a_8^o \le 1,\ \pi_A^7(b,b) \ge max\{\pi_A^6(m,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& \quad a_5^o \ge m,\ a_6^o \ge b,\ b < a_8^o \le 1,\ \pi_A^7(b,b) \ge \pi_A^8(a_8^o,b) \text{ OR} \\
& \quad a_5^o \ge m,\ m \le a_6^o < b,\ a_8^o \le b,\ \pi_A^7(b,b) \ge \pi_A^6(a_6^o,b) \text{ OR} \\
& \quad a_5^o \ge m,\ a_6^o < m,\ a_8^o \le b,\ \pi_A^7(b,b) \ge \pi_A^6(m,b) \text{ OR} \\
& \quad a_5^o \ge m,\ a_6^o \ge b,\ a_8^o \le b \\
a_6^o & \text{if } m \le a_6^o < b \text{ and } \pi_A^6(a_6^o,b) \ge \pi_A^7(b,b) \text{ AND} \\
& \quad 0 \le a_5^o < m,\ b < a_8^o \le 1,\ \pi_A^6(a_6^o,b) \ge max\{\pi_A^5(a_5^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& \quad a_5^o \ge m,\ b < a_8^o \le 1,\ \pi_A^6(a_6^o,b) \ge \pi_A^8(a_8^o,b) \text{ OR} \\
& \quad 0 \le a_5^o < m,\ a_8^o \le b,\ \pi_A^6(a_6^o,b) \ge \pi_A^5(a_5^o,b) \text{ OR} \\
& \quad a_5^o \ge m,\ a_8^o \le b \\
m & \text{if } a_6^o < m \text{ and } \pi_A^6(m,b) \ge \pi_A^7(b,b) \text{ AND} \\
& \quad 0 \le a_5^o < m,\ b < a_8^o \le 1,\ \pi_A^6(m,b) \ge max\{\pi_A^5(a_5^o,b), \pi_A^8(a_8^o,b)\} \text{ OR} \\
& \quad a_5^o \ge m,\ b < a_8^o \le 1,\ \pi_A^6(m,b) \ge \pi_A^8(a_8^o,b) \text{ OR} \\
& \quad 0 \le a_5^o < m,\ a_8^o \le b,\ \pi_A^6(m,b) \ge \pi_A^5(a_5^o,b) \text{ OR} \\
& \quad a_5^o \ge m,\ a_8^o \le b \\
a_8^o & \text{if } b < a_8^o \le 1 \text{ and } \pi_A^8(a_8^o,b) \ge \pi_A^7(b,b) \text{ AND} \\
& \quad 0 \le a_5^o < m,\ m \le a_6^o < b,\ \pi_A^8(a_8^o,b) \ge max\{\pi_A^5(a_5^o,b), \pi_A^6(a_6^o,b)\} \text{ OR} \\
& \quad 0 \le a_5^o < m,\ a_6^o < m,\ \pi_A^8(a_8^o,b) \ge max\{\pi_A^5(a_5^o,b), \pi_A^6(m,b)\} \text{ OR} \\
& \quad a_5^o \ge m,\ m \le a_6^o < b,\ \pi_A^8(a_8^o,b) \ge \pi_A^6(a_6^o,b) \text{ OR} \\
& \quad a_5^o \ge m,\ a_6^o < m,\ \pi_A^8(a_8^o,b) \ge \pi_A^6(m,b) \text{ OR} \\
& \quad a_5^o \ge m,\ a_6^o \ge b \\
\emptyset & \text{otherwise.}
\end{cases}
$$

$\square$

Proof of Proposition 4.3. Follows directly from conditions $a_1^o \geq b$, $a_3^o \leq b$, and $a_4^o \leq m$ in Proposition 4.1 (the last row for b as the best response in the table of Proposition 4.1). If these conditions are satisfied, then there is a symmetric equilibrium location for both retailers. Let $b = m = \frac{1}{2}$:

$$\begin{aligned} a_1^o &= \frac{p + c_c b + 2c_t(b-m)}{5c_c - 4c_t} \geq b \\ &\Rightarrow\ p \geq b(4c_c - 6c_t) + c_t \\ &\Rightarrow\ p \geq 2c_c - 2c_t \end{aligned} \tag{C.1}$$

$$\begin{aligned} a_3^o &= \frac{-p + c_c(4+b) + 2c_t(2+m-b)}{5c_c - 4c_t} \leq b \\ &\Rightarrow\ p \geq 4c_c + 5c_t + b(2c_t - 4c_c) \\ &\Rightarrow\ p \geq 2c_c + 6c_t \end{aligned} \tag{C.2}$$

$$\begin{aligned} a_4^o &= \frac{-p + c_c(4+b) - 2c_t(2+m-b)}{5c_c - 4c_t} \leq m \\ &\Rightarrow\ p \geq \frac{3c_c}{2} - 3c_t + b(c_c + 2c_t) \\ &\Rightarrow\ p \geq 2c_c - 2c_t \end{aligned} \tag{C.3}$$

Inequality (C.2) is stronger than inequalities (C.1) and (C.3).

□

Proof of Proposition 4.4. Follows directly from conditions $a_1^o \geq b$, $a_3^o \leq b$, $a_4^o \leq m$, $c_c > 2c_t \geq 0$, $p > 0$, and $b \leq m$ in Proposition 4.1 (the last row for b as the best response in the table of Proposition 4.1). If these conditions are satisfied, then there is a symmetric equilibrium for both retailers.

$$\begin{aligned} a_1^o &= \frac{p + c_c b + 2c_t(b-m)}{5c_c - 4c_t} \geq b \\ &\Rightarrow\ p - 4c_c b + 6c_t b \geq 2c_t m \\ &\Rightarrow\ \frac{p + b(6c_t - 4c_c)}{2c_t} \geq m \end{aligned} \tag{C.4}$$

$$\begin{aligned} a_3^o &= \frac{-p + c_c(4+b) + 2c_t(2+m-b)}{5c_c - 4c_t} \leq b \\ &\Rightarrow \; b(4c_c - 2c_t) - 4(c_c + c_t) + p \leq 2c_t m \\ &\Rightarrow \; \frac{b(4c_c - 2c_t) - 4(c_c + c_t) + p}{2c_t} \geq m \end{aligned} \tag{C.5}$$

$$\begin{aligned} a_4^o &= \frac{-p + c_c(4+b) - 2c_t(2+m-b)}{5c_c - 4c_t} \leq m \\ &\Rightarrow \; -p + 4c_c - 4c_t + b(c_c + 2c_t) \leq 5c_c m - 2c_t m \\ &\Rightarrow \; \frac{-p + 4c_c - 4c_t + b(c_c + 2c_t)}{5c_c - 2c_t} \leq m \end{aligned} \tag{C.6}$$

Inequality (C.5) is stronger than inequality (C.4).

□

Proof of Proposition 4.5. Follows directly from conditions $a_5^o \geq m$, $a_6^o \geq b$, $a_8^o \leq b$, $c_c > 2c_t \geq 0$, $p > 0$, and $m < b$ in Proposition 4.2 (the last row for b as the best response in the table of Proposition 4.2). If these conditions are satisfied, then there is a symmetric equilibrium for both retailers.

$$\begin{aligned} a_5^o &= \frac{p + c_c b + 2c_t(b-m)}{5c_c - 4c_t} \geq m \\ &\Rightarrow \; p + b(c_c + 2c_t) \geq 5c_c m - 2c_t m \\ &\Rightarrow \; \frac{p + b(c_c + 2c_t)}{5c_c - 2c_t} \geq m \end{aligned} \tag{C.7}$$

$$\begin{aligned} a_6^o &= \frac{p + c_c b + 2c_t(m-b)}{5c_c + 4c_t} \geq b \\ &\Rightarrow \; -p + 4c_c b + 6c_t b \leq 2c_t m \\ &\Rightarrow \; \frac{b(4c_c + 6c_t) - p}{2c_t} \leq m \end{aligned} \tag{C.8}$$

$$\begin{aligned} a_8^o &= \frac{-p + c_c(4+b) + 2c_t(b-m-2)}{5c_c - 4c_t} \leq b \\ &\Rightarrow \; -p + 6c_t b + 4c_c - 4c_t - 4c_c b \leq 2c_t m \\ &\Rightarrow \; \frac{-p + b(6c_t - 4c_c) + 4(c_c - c_t)}{2c_t} \leq m \end{aligned} \tag{C.9}$$

Inequality (C.8) is stronger than inequality (C.9).

$\square$

Proof of Lemma 5.1. Define, for $a, b \in [0,1]$,

$$\pi_{Total}(a,b) = \begin{cases} \lambda(p + c_c(2b + ab - 1 - \frac{3a^2+3b^2}{2}) + c_t(2b - 2m + a^2 - b^2)) & \text{if } a < b \leq m \leq 1 \text{ (Case 1),} \\ \lambda(p - c_c(a^2 - a + b^2 - b + 1) - c_t(2m - a - b)) & \text{if } a = b \leq m \leq 1 \text{ (Case 2),} \\ \lambda(p + c_c(2a + ab - 1 - \frac{3a^2+3b^2}{2}) + c_t(2a - 2m + b^2 - a^2)) & \text{if } b < a \leq m \leq 1 \text{ (Case 3),} \\ \lambda(p + c_c(ab + 2a - 1 - \frac{3a^2+3b^2}{2}) - c_t(2ma + 2mb - 2ab - a^2 - b^2 + 2a - 2m)) & \text{if } b \leq m < a \leq 1 \text{ (Case 4),} \\ \lambda(p + c_c(ab + 2b - 1 - \frac{3a^2+3b^2}{2}) - c_t(2ma + 2mb - 2ab - a^2 - b^2 + 2b - 2m)) & \text{if } a < m < b \leq 1 \text{ (Case 5),} \\ \lambda(p + c_c(2b + ab - 1 - \frac{3a^2+3b^2}{2}) + c_t(2m - 2b + b^2 - a^2)) & \text{if } m \leq a < b \leq 1 \text{ (Case 6),} \\ \lambda(p - c_c(a^2 - a + b^2 - b + 1) + c_t(2m - a - b)) & \text{if } m < a = b \leq 1 \text{ (Case 7), and} \\ \lambda(p + c_c(2a + ab - 1 - \frac{3a^2+3b^2}{2}) + c_t(2m - 2a + a^2 - b^2)) & \text{if } m < b < a \leq 1 \text{ (Case 8).} \end{cases}$$

Notice that $\pi^i_{Total}(a,b)$ is a bivariate polynomial function, and thus a continuous function in each case (i). Suppose that $b \leq m$. Note that

$$\begin{aligned} \lim_{a \to b^-} \pi^1_{Total}(a,b) &= \lambda\left(p + c_c\left(2b + b^2 - 1 - \frac{3b^2+3b^2}{2}\right)\right) \\ &+ \lambda\left(c_t(2b - 2m + b^2 - b^2)\right) \\ &= \lambda\left(p + c_c(2b - 2b^2 - 1) + c_t(2b - 2m)\right) \\ &= \pi^2_{Total}(b,b), \\ \lim_{a \to b^+} \pi^3_{Total}(a,b) &= \lambda\left(p + c_c(2b - 1 - 2b^2) + c_t(2b - 2m)\right) \\ &= \pi^2_{Total}(b,b), \\ \lim_{a \to m^+} \pi^4_{Total}(a,b) &= \lambda\left(p + c_c\left(2m + mb - 1 - \frac{3m^2+3b^2}{2}\right) + c_t(b^2 - m^2)\right) \\ &= \pi^3_{Total}(m,b). \end{aligned}$$

Hence $\pi^i_{Total}(a,b)$ is continuous in a when $b \leq m$. Now suppose that $b > m$:

$$\begin{aligned}
\lim_{a \to m^-} \pi^5_{Total}(a,b) &= \lambda\left(p - c_c\left(-mb - 2b + 1 + \frac{3m^2 + 3b^2}{2}\right)\right) \\
&- \lambda\left(c_t(m^2 - 2m - b^2 + 2b)\right) \\
&= \pi^6_{Total}(m,b), \\
\lim_{a \to b^-} \pi^6_{Total}(a,b) &= \lambda\left(p + c_c(2b - 2b^2 - 1) + c_t(2m - 2b)\right) \\
&= \pi^7_{Total}(b,b), \\
\lim_{a \to b^+} \pi^8_{Total}(a,b) &= \lambda\left(p + c_c(2b - 1 - 2b^2) + c_t(2m - 2b)\right) \\
&= \pi^7_{Total}(b,b).
\end{aligned}$$

Likewise, for a given a, we can show that the total profit function is continuous in b.

□

Proof of Lemma 5.2. As we want to maximize the total profit function, we need to check the concavity of the total profit function in each case. To this end, we calculate the Hessian matrices of the total profit functions and show that each of these matrices is negative semi-definite. Let us consider the following 2×2 matrix:

$$C = \begin{bmatrix} \alpha & \beta \\ \gamma & \nu \end{bmatrix}$$

The above matrix is negative semi-definite if and only if $\alpha \leq 0$, $\nu \leq 0$, and $det(C) \geq 0$. If the Hessian matrix of our total profit function (i) is negative semi-definite, then our total profit function (i) is jointly concave in a and b. Below we calculate the Hessian matrix for the function $\pi^i_{Total}(a,b)$ in each case (i).

Case (1). The Hessian matrix is given by

$$\begin{bmatrix} 2c_t - 3c_c & c_c \\ c_c & -3c_c - 2c_t \end{bmatrix}$$

The function $\pi^1_{Total}(a, b)$ is jointly concave since $2c_t - 3c_c < 0$ and $8c_c^2 - 4c_t^2 > 0$ (recall that $c_c > 2c_t$).

Case (2). The Hessian matrix is given by

$$\begin{bmatrix} -2c_c & 0 \\ 0 & -2c_c \end{bmatrix}$$

The function $\pi^2_{Total}(a, b)$ is jointly concave since $-2c_c < 0$ and $4c_c^2 > 0$.

Case (3). The Hessian matrix is given by

$$\begin{bmatrix} -3c_c - 2c_t & c_c \\ c_c & 2c_t - 3c_c \end{bmatrix}$$

The function $\pi^3_{Total}(a, b)$ is jointly concave since $-3c_c - 2c_t < 0$ and $8c_c^2 - 4c_t^2 > 0$ (recall that $c_c > 2c_t$).

Case (4). The Hessian matrix is given by

$$\begin{bmatrix} 2c_t - 3c_c & c_c + 2c_t \\ c_c + 2c_t & 2c_t - 3c_c \end{bmatrix}$$

The function $\pi^4_{Total}(a, b)$ is jointly concave since $2c_t - 3c_c < 0$ and $8c_c^2 - 16c_t c_c > 0$ (recall that $c_c > 2c_t$).

Case (5). The Hessian matrix is given by

$$\begin{bmatrix} 2c_t - 3c_c & c_c + 2c_t \\ c_c + 2c_t & 2c_t - 3c_c \end{bmatrix}$$

The function $\pi^5_{Total}(a, b)$ is jointly concave since $2c_t - 3c_c < 0$ and $8c_c^2 - 16c_t c_c > 0$ (recall that $c_c > 2c_t$).

Case (6). The Hessian matrix is given by

$$\begin{bmatrix} -3c_c - 2c_t & c_c \\ c_c & 2c_t - 3c_c \end{bmatrix}$$

The function $\pi^6_{Total}(a, b)$ is jointly concave since $-3c_c - 2c_t < 0$ and $8c_c^2 - 4c_t^2 > 0$ (recall that $c_c > 2c_t$).

Case (7). The Hessian matrix is given by

$$\begin{bmatrix} -2c_c & 0 \\ 0 & -2c_c \end{bmatrix}$$

The function $\pi^7_{Total}(a,b)$ is jointly concave since $-2c_c < 0$ and $4c_c^2 > 0$.

Case (8). The Hessian matrix is given by

$$\begin{bmatrix} 2c_t - 3c_c & c_c \\ c_c & -3c_c - 2c_t \end{bmatrix}$$

The function $\pi^8_{Total}(a,b)$ is jointly concave since $2c_t - 3c_c < 0$ and $8c_c^2 - 4c_t^2 > 0$ (recall that $c_c > 2c_t$).

□

Proof of Proposition 5.3. Let $v(x,y)$ be a differentiable and concave function with domain $a \leq x \leq b$, $m \leq y \leq n$. If a stationary point exists, it is an interior maximizer. If no stationary point exists, v is maximized along the edge of the domain; see Leach (2004).

Notice that our total profit functions are bivariate polynomial functions in all cases. Polynomials are differentiable for all arguments. We also note from Lemma 5.2 that each of our total profit functions is concave.

The maximizers are stationary points of $\pi^i_{Total}(a,b)$ and the Hessian matrices are negative semi-definite in all cases, and thus we find the global optima in each case.

If edges of the domain cannot be achieved and global solution does not belong to the domain, then there is no solution. However, there exists at least one edge that can be achieved (i.e., included in the domain) among eight cases. We know from Lemma 5.1 that the total profit function is continuous: If there is no solution in any case, the edges and the global solution of the next case will be checked. Thus we find at least one feasible solution from eight cases in the centralized system. We label the point that yields the maximum profit as the optimal solution.

Algorithm 1 finds the optimal solution because it calculates the interior maximizers if any, and edge points of the domain that can be achieved across all cases. The point that yields the maximum profit is the optimal solution.

□

Proof of Proposition 5.4. Given the warehouse location $m \in [0,1]$, each retailer i chooses the location of its store to maximize its expected daily profit $\pi_i(a,b)$:

$$\pi_i(a,b) = p\lambda_i(a,b) - (c_c d_{ic}(a,b) - c_t d_{it}(a,b))\lambda_i(a,b).$$

Note that $\sum_{i\in\{A,B\}} p\lambda_i(a,b) = p\lambda$ is total revenue in the centralized system. Thus the optimization problem of the retail chain can be rewritten as

$$\begin{aligned} \underset{a,b}{\text{maximize}} \quad & \text{Total Revenue}-\text{Total Cost} \\ \text{subject to} \quad & 0 \le a, b \le 1, \end{aligned}$$

which is equivalent to

$$\begin{aligned} \underset{a,b}{\text{minimize}} \quad & \text{Total Cost} \\ \text{subject to} \quad & 0 \le a, b \le 1. \end{aligned}$$

□

Appendix D

Centralized System Solution Algorithms

Algorithm 3 Pseudo code for steps 2–4 of Algorithm 1 in case (2).

1: Identify the end-points of the intervals for a and b in case (2). The upper limits are $(a_2^U, b_2^U) = (m, m)$ and the lower limits are $(a_2^L, b_2^L) = (0, 0)$.

2: Find the global optima for the unconstrained problem in case (2).

- Calculate the first order conditions of $\pi^2_{Total}(a, b)$.
- Solve these two equations simultaneously to find the global optima (a_2, b_2).

3: IF (a_2, b_2) belongs to the interval of case (2), then (a_2, b_2) is an optimal solution for case (2).

ELSE

Set $a_2 = m$ and $b_2 = m$ to find the value $\pi^2_{Total}(a_2, b_2)$.

Set $a_2 = 0$ and $b_2 = 0$ to find the value $\pi^2_{Total}(a_2, b_2)$.

The end-point solution $(\tilde{a}_2, \tilde{b}_2)$ yielding the maximum profit is an optimal solution in case (2). Set $(a_2, b_2) = (\tilde{a}_2, \tilde{b}_2)$.

END

Algorithm 4 Pseudo code for steps 2–4 of Algorithm 1 in case (3).

1: Identify the end-points of the intervals for a and b in case (3). The upper limits are $(a_3^U, b_3^U) = (m,\text{undefined})$ and the lower limits are $(a_3^L, b_3^L) = (\text{undefined}, 0)$.

2: Find the global optima for the unconstrained problem in case (3).

- Calculate the first order conditions of $\pi^3_{Total}(a, b)$.
- Solve these two equations simultaneously to find the global optima (a_3, b_3).

3: IF (a_3, b_3) belongs to the interval of case (3), then (a_3, b_3) is an optimal solution for case (3).

ELSE

Set $a_3 = m$ and take the derivative of $\pi^3_{Total}(a_3, b)$ to find b_3.

IF b_3 belongs to the interval of case (3), then (a_3, b_3) is a feasible solution of case (3).

ELSEIF $a_3 \leq b_3$, there exists no solution!

ELSEIF $b_3 \leq 0$, set $b_3 = 0$ to find the value $\pi^3_{Total}(a_3, b_3)$.

END

Set $b_3 = 0$ and take the derivative of $\pi^3_{Total}(a, b_3)$ to find a_3.

IF a_3 belongs to the interval of case (3), then (a_3, b_3) is a feasible solution of case (3).

ELSEIF $a_3 > m$ OR $a_3 \leq b_3$, there exists no solution!

END

The end-point solution $(\tilde{a}_3, \tilde{b}_3)$ yielding the maximum profit is an optimal solution in case (3). Set $(a_3, b_3) = (\tilde{a}_3, \tilde{b}_3)$.

END

Algorithm 5 Pseudo code for steps 2–4 of Algorithm 1 in case (4).

1: Identify the end-points of the intervals for a and b in case (4). The upper limits are $(a_4^U, b_4^U) = (1, m)$ and the lower limits are $(a_1^L, b_1^L) = (\text{undefined}, 0)$.

2: Find the global optima for the unconstrained problem in case (4).

- Calculate the first order conditions of $\pi_{Total}^4(a, b)$.
- Solve these two equations simultaneously to find the global optima (a_4, b_4).

3: IF (a_4, b_4) belongs to the interval of case (4), then (a_4, b_4) is an optimal solution for case (4).

ELSE

Set $a_4 = 1$ and $b_4 = m$ to find the value $\pi_{Total}^4(a_4, b_4)$.

Set $b_4 = 0$ and take the derivative of $\pi_{Total}^4(a, b_4)$ to find a_4.

IF a_4 belongs to the interval of case (4), then (a_4, b_4) is a feasible solution of case (4).

ELSEIF $a_4 \leq m$, there exists no solution!

ELSEIF $a_4 \geq 1$, set $a_4 = 1$ to find the value $\pi_{Total}^4(a_4, b_4)$.

END

The end-point solution $(\tilde{a}_4, \tilde{b}_4)$ yielding the maximum profit is an optimal solution in case (4). Set $(a_4, b_4) = (\tilde{a}_4, \tilde{b}_4)$.

END

Algorithm 6 Pseudo code for steps 2–4 of Algorithm 1 in case (5).

1: Identify the end-points of the intervals for a and b in case (5). The upper limits are $(a_5^U, b_5^U) = (\text{undefined}, 1)$ and the lower limits are $(a_5^L, b_5^L) = (0,\text{undefined})$.

2: Find the global optima for the unconstrained problem in case (5).

- Calculate the first order conditions of $\pi^5_{Total}(a, b)$.
- Solve these two equations simultaneously to find the global optima (a_5, b_5).

3: IF (a_5, b_5) belongs to the interval of case (5), then (a_5, b_5) is an optimal solution for case (5).

ELSE

Set $b_5 = 1$ and take the derivative of $\pi^5_{Total}(a, b_5)$ to find a_5.

IF a_5 belongs to the interval of case (5), then (a_5, b_5) is a feasible solution of case (5).

ELSEIF $a_5 \geq m$, there exists no solution!

ELSEIF $a_5 \leq 0$, set $a_5 = 0$ to find the value $\pi^5_{Total}(a_5, b_5)$.

END

Set $a_5 = 0$ and take the derivative of $\pi^5_{Total}(a_5, b)$ to find b_5.

IF b_5 belongs to the interval of case (5), then (a_5, b_5) is a feasible solution of case (5).

ELSEIF $b_5 \leq m$, there exists no solution!

ELSEIF $b_5 \geq 1$, set $b_5 = 1$ to find the value $\pi^5_{Total}(a_5, b_5)$.

END

The end-point solution $(\tilde{a}_5, \tilde{b}_5)$ yielding the maximum profit is an optimal solution in case (5). Set $(a_5, b_5) = (\tilde{a}_5, \tilde{b}_5)$.

END

Algorithm 7 Pseudo code for steps 2–4 of Algorithm 1 in case (6).

1: Identify the end-points of the intervals for a and b in case (6). The upper limits are $(a_6^U, b_6^U) = (\text{undefined}, 1)$ and the lower limits are $(a_6^L, b_6^L) = (m,\text{undefined})$.

2: Find the global optima for the unconstrained problem in case (6).

- Calculate the first order conditions of $\pi_{Total}^6(a, b)$.
- Solve these two equations simultaneously to find the global optima (a_6, b_6).

3: IF (a_6, b_6) belongs to the interval of case (6), then (a_6, b_6) is an optimal solution for case (6).

ELSE

Set $b_6 = 1$ and take the derivative of $\pi_{Total}^6(a, b_6)$ to find a_6.

IF a_6 belongs to the interval of case (6), then (a_6, b_6) is a feasible solution of case (6).

ELSEIF $a_6 \geq b_6$, there exists no solution!

ELSEIF $a_6 \leq m$, set $a_6 = m$ to find the value $\pi_{Total}^6(a_6, b_6)$.

END

Set $a_6 = m$ and take the derivative of $\pi_{Total}^6(a_6, b)$ to find b_6.

IF b_6 belongs to the interval of case (6), then (a_6, b_6) is a feasible solution of case (6).

ELSEIF $a_6 \geq b_6$, there exists no solution!

ELSEIF $b_6 \geq 1$, set $b_6 = 1$ to find the value $\pi_{Total}^6(a_6, b_6)$.

END

The end-point solution $(\tilde{a}_6, \tilde{b}_6)$ yielding the maximum profit is an optimal solution in case (6). Set $(a_6, b_6) = (\tilde{a}_6, \tilde{b}_6)$.

END

Algorithm 8 Pseudo code for steps 2–4 of Algorithm 1 in case (7).

1: Identify the end-points of the intervals for a and b in case (7). The upper limits are $(a_7^U, b_7^U) = (1, 1)$ and the lower limits are $(a_2^L, b_2^L) =$ (undefined,undefined).

2: Find the global optima for the unconstrained problem in case (7).

- Calculate the first order conditions of $\pi_{Total}^7(a, b)$.
- Solve these two equations simultaneously to find the global optima (a_7, b_7).

3: IF (a_7, b_7) belongs to the interval of case (7), then (a_7, b_7) is an optimal solution for case (7).

ELSE

IF $a_7 \leq m$ and $b_7 \leq m$ there exists no solution!

ELSEIF $a_7 \geq 1$ and $b_7 \geq 1$, set $a_7 = 1$ and $b_7 = 1$ to find the value $\pi_{Total}^7(a_7, b_7)$.

END

The end-point solution $(\tilde{a}_7, \tilde{b}_7)$ yielding the maximum profit is an optimal solution in case (7). Set $(a_7, b_7) = (\tilde{a}_7, \tilde{b}_7)$.

END

Algorithm 9 Pseudo code for steps 2–4 of Algorithm 1 in case (8).

1: Identify the end-points of the intervals for a and b in case (8). The upper limits are $(a_8^U, b_8^U) =$ (1,undefined) and the lower limits are $(a_8^L, b_8^L) =$ (undefined,undefined).

2: Find the global optima for the unconstrained problem in case (8).

- Calculate the first order conditions of $\pi_{Total}^8(a, b)$.
- Solve these two equations simultaneously to find the global optima (a_8, b_8).

3: IF (a_8, b_8) belongs to the interval of case (8), then (a_8, b_8) is an optimal solution for case (8).

ELSE

Set $a_8 = 1$ (to its upper limit) and take the derivative of $\pi_{Total}^8(a_8, b)$ to find b_8.

IF b_8 belongs to the interval of case 8, then (a_8, b_8) is a feasible solution of case (8).

ELSEIF $b_8 \leq m$ OR $b_8 \geq a_8$, there exists no solution!

END

The end-point solution $(\tilde{a}_8, \tilde{b}_8)$ yielding the maximum profit is an optimal solution in case (8). Set $(a_8, b_8) = (\tilde{a}_8, \tilde{b}_8)$.

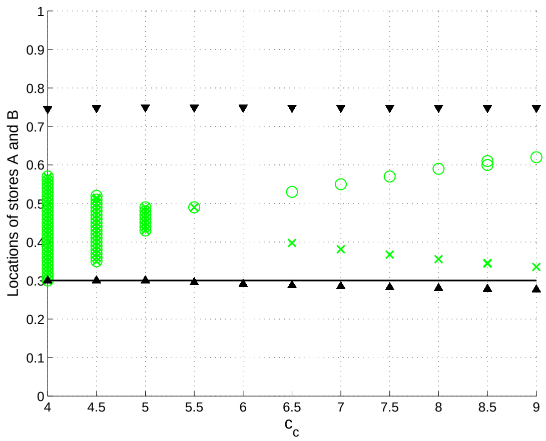
END

Appendix E

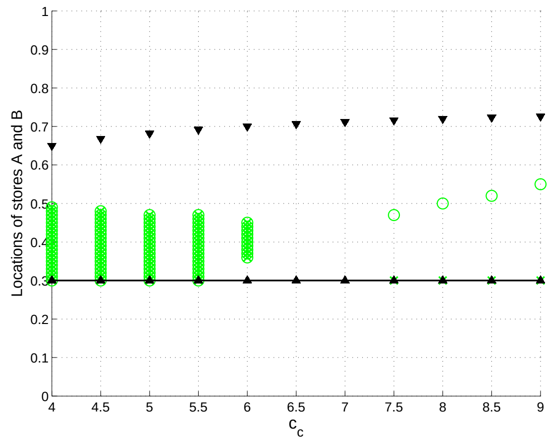
Additional Numerical Results

E.1 Locations in the Centralized and Decentralized Systems

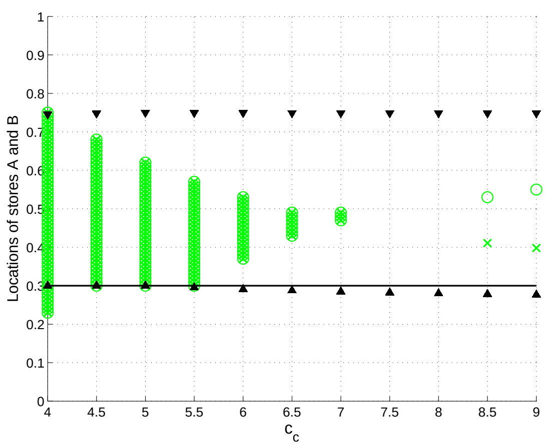
Figures E.1-E.3 exhibit how the equilibrium responds to a change in c_c for $m \in \{0.3, 0.5, 1\}$. Figures E.4-E.6 exhibit how the equilibrium responds to a change in c_t for $m \in \{0.3, 0.5, 1\}$.



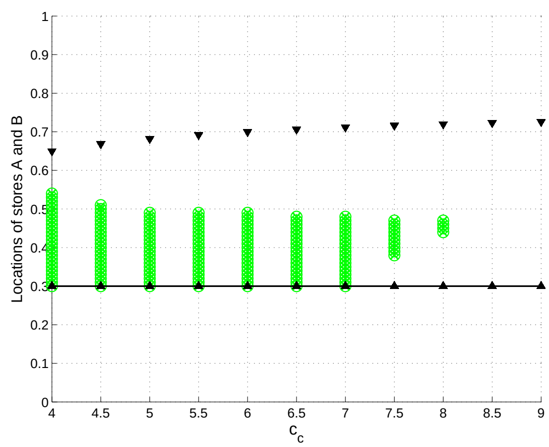
(a) $p = 10.5$ and $c_t = 0.5$.



(b) $p = 10.5$ and $c_t = 2$.

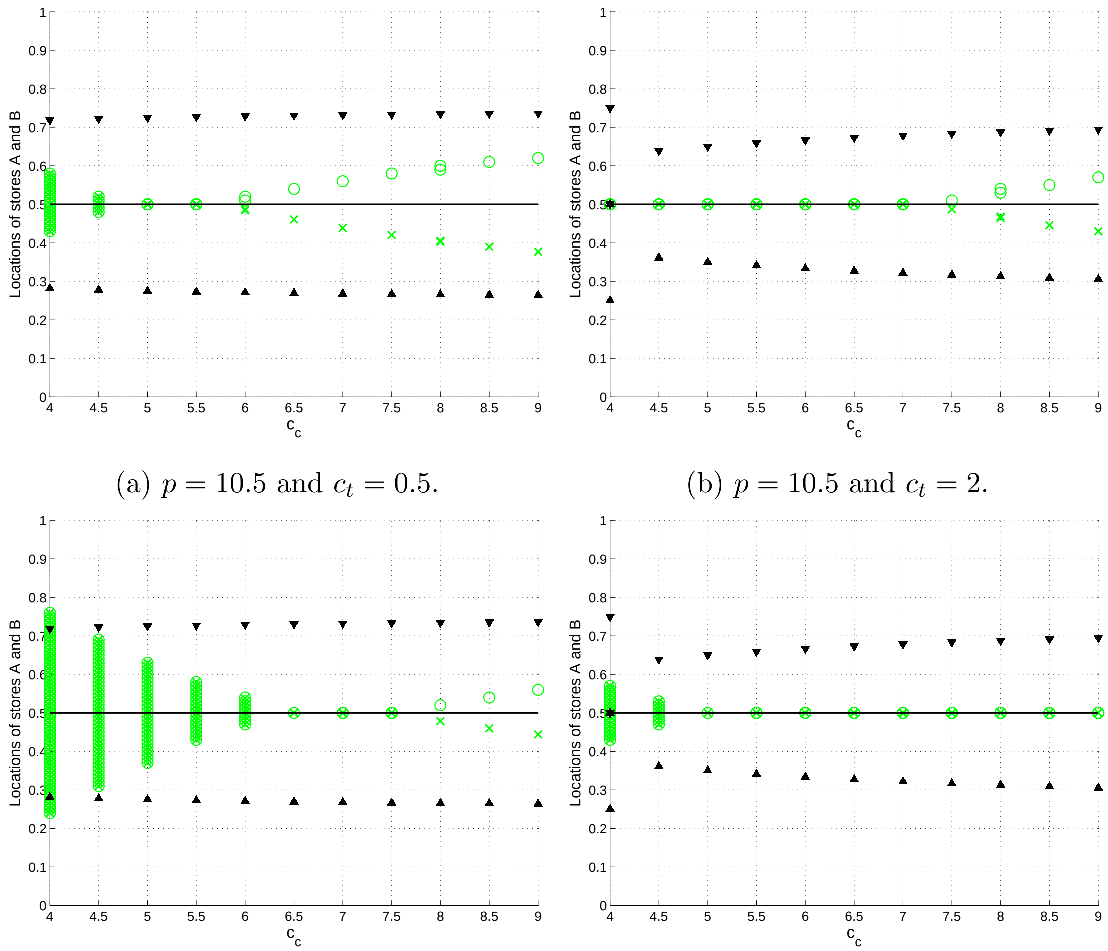


(c) $p = 14$ and $c_t = 0.5$.



(d) $p = 14$ and $c_t = 2$.

Figure E.1: Locations in the centralized and decentralized systems when $m = 0.3$ and $\lambda = 10$. Locations of stores A and B are denoted by "▲" and "▼" in the centralized system, and by "x" and "o" in the decentralized system, respectively.



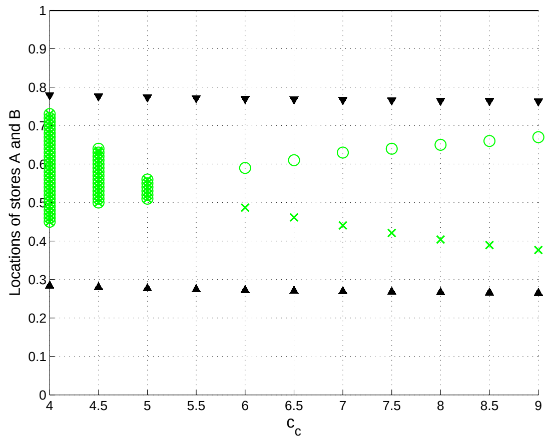
(a) $p = 10.5$ and $c_t = 0.5$.

(b) $p = 10.5$ and $c_t = 2$.

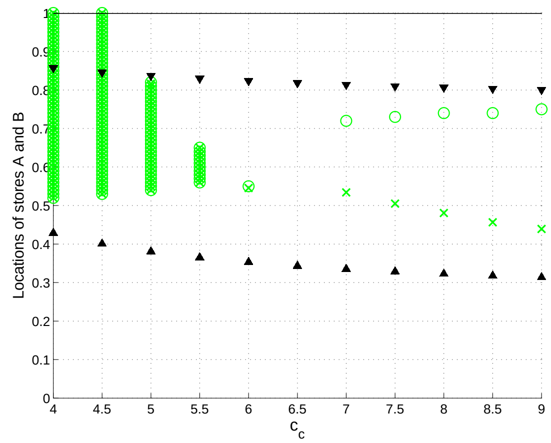
(c) $p = 14$ and $c_t = 0.5$.

(d) $p = 14$ and $c_t = 2$.

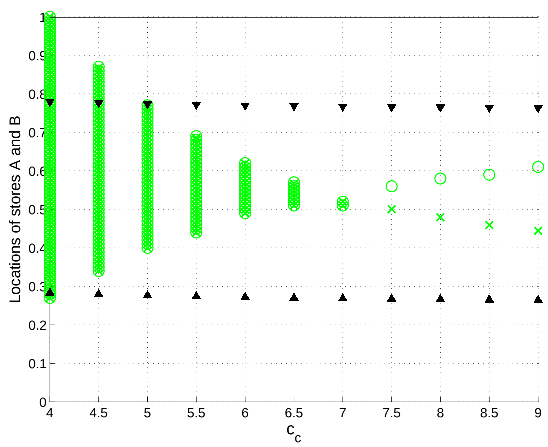
Figure E.2: Locations in the centralized and decentralized systems when $m = 0.5$ and $\lambda = 10$. Locations of stores A and B are denoted by "▲" and "▼" in the centralized system, and by "x" and "o" in the decentralized system, respectively.



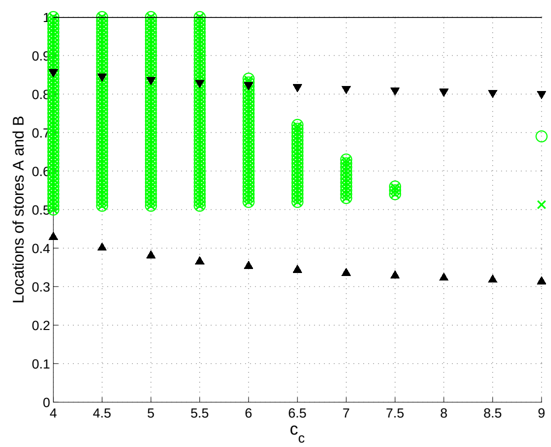
(a) $p = 10.5$ and $c_t = 0.5$.



(b) $p = 10.5$ and $c_t = 2$.

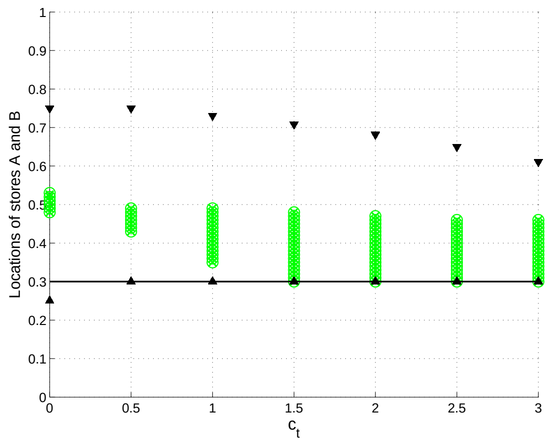


(c) $p = 14$ and $c_t = 0.5$.

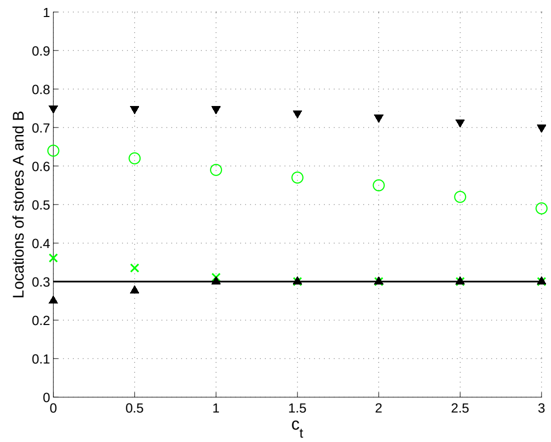


(d) $p = 14$ and $c_t = 2$.

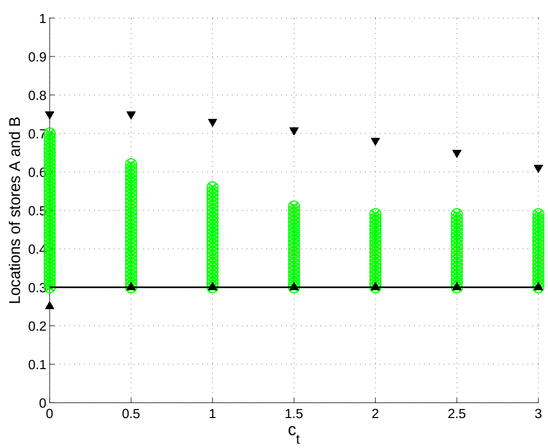
Figure E.3: Locations in the centralized and decentralized systems when $m = 1$ and $\lambda = 10$. Locations of stores A and B are denoted by "▲" and "▼" in the centralized system, and by "x" and "o" in the decentralized system, respectively.



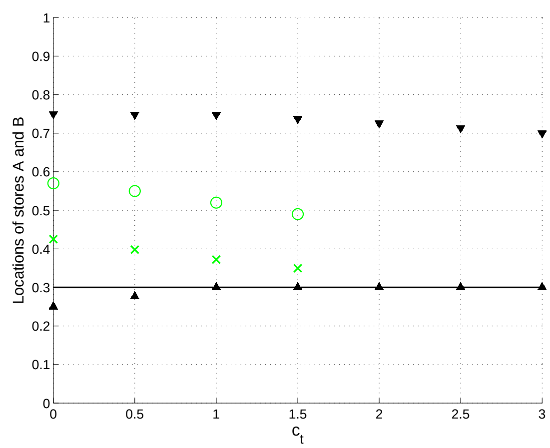
(a) $p = 10.5$ and $c_c = 5$.



(b) $p = 10.5$ and $c_c = 9$.

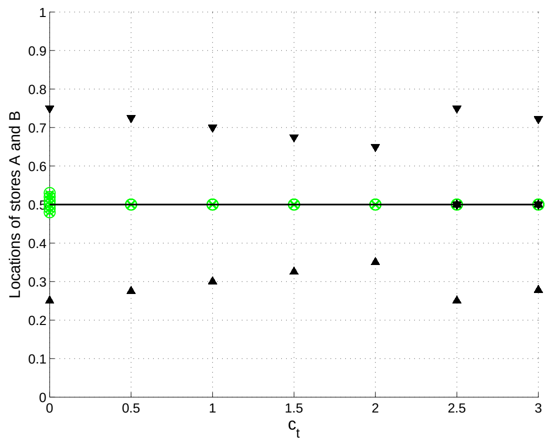


(c) $p = 14$ and $c_c = 5$.

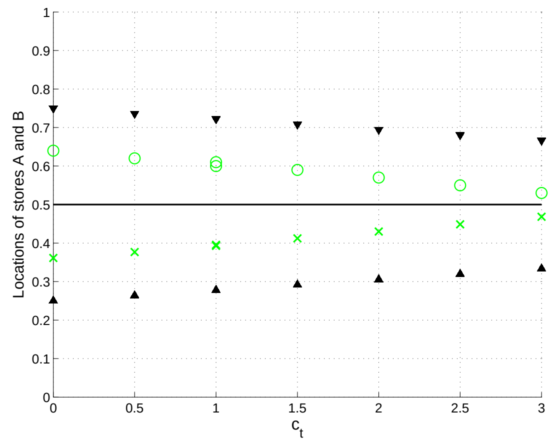


(d) $p = 14$ and $c_c = 9$.

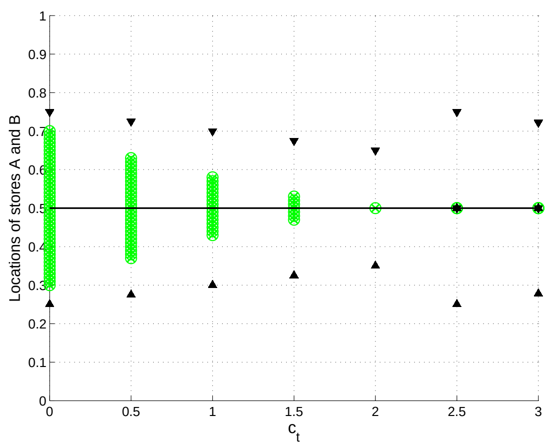
Figure E.4: Locations in the centralized and decentralized systems when $m = 0.3$ and $\lambda = 10$. Locations of stores A and B are denoted by "▲" and "▼" in the centralized system, and by "x" and "o" in the decentralized system, respectively.



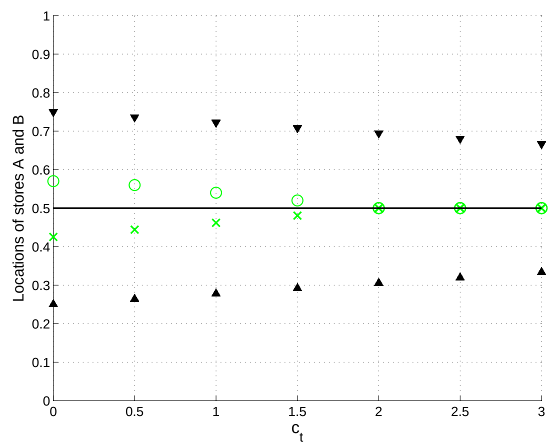
(a) $p = 10.5$ and $c_c = 5$.



(b) $p = 10.5$ and $c_c = 9$.

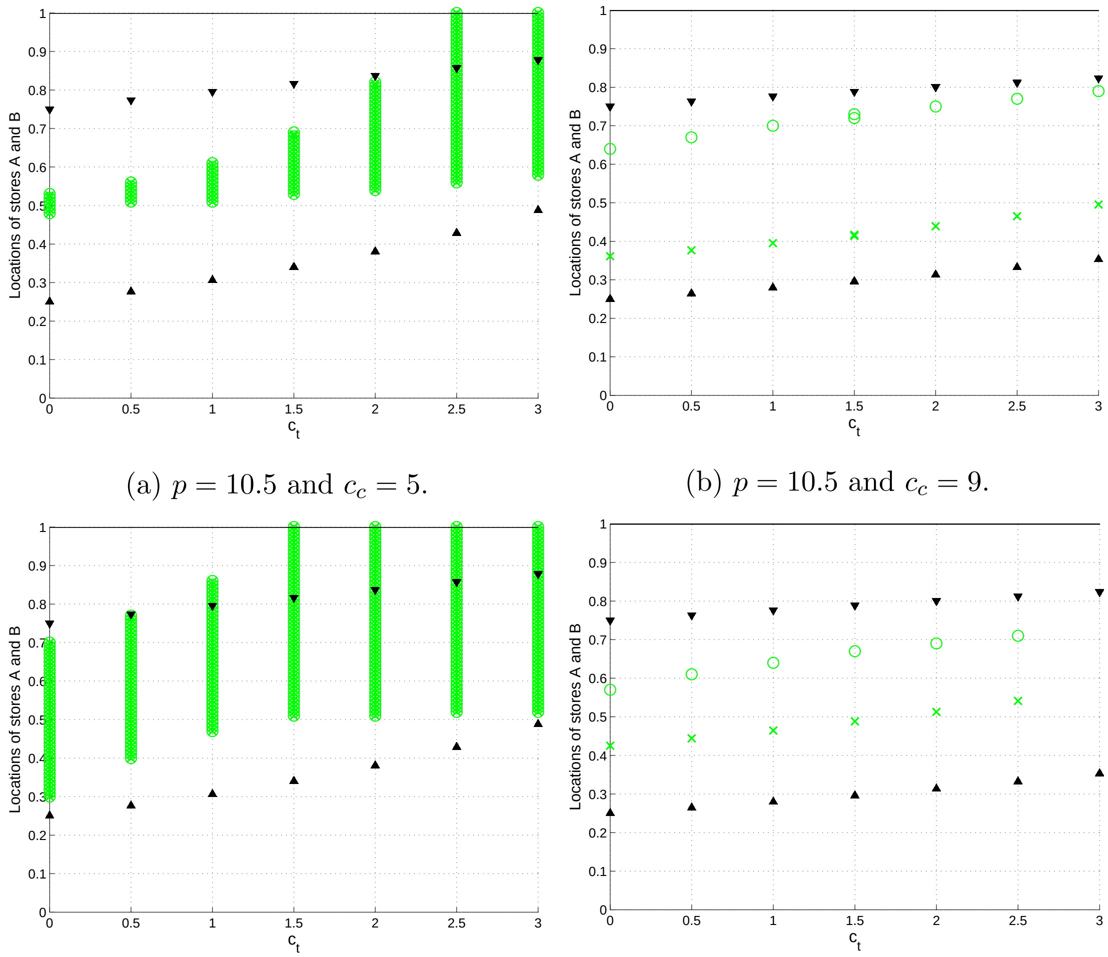


(c) $p = 14$ and $c_c = 5$.



(d) $p = 14$ and $c_c = 9$.

Figure E.5: Locations in the centralized and decentralized systems when $m = 0.5$ and $\lambda = 10$. Locations of stores A and B are denoted by "▲" and "▼" in the centralized system, and by "x" and "o" in the decentralized system, respectively.



(a) $p = 10.5$ and $c_c = 5$.

(b) $p = 10.5$ and $c_c = 9$.

(c) $p = 14$ and $c_c = 5$.

(d) $p = 14$ and $c_c = 9$.

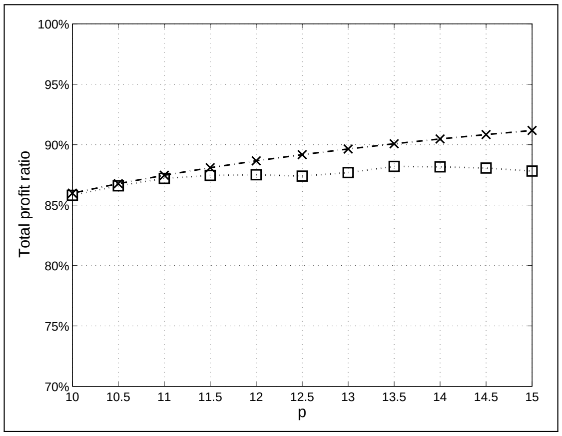
Figure E.6: Locations in the centralized and decentralized systems when $m = 1$ and $\lambda = 10$. Locations of stores A and B are denoted by "▲" and "▼" in the centralized system, and by "x" and "o" in the decentralized system, respectively.

E.2 Total Profit Ratios

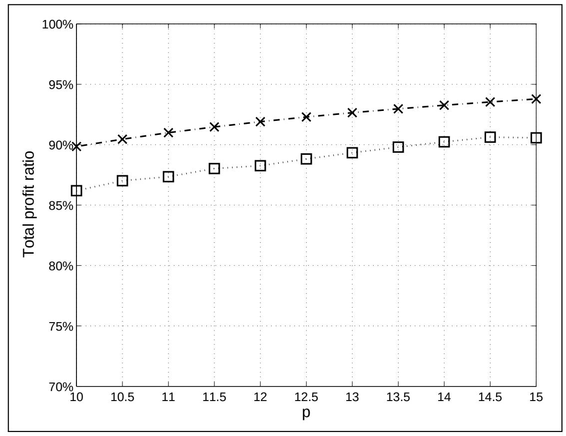
We calculate the ratio of the total profit made by the two stores in the decentralized system to the centralized system in each of our instances for which equilibrium exists (see Figures E.7–E.9). For the instances with symmetric equilibria, we calculate the maximum and minimum ratios of total profits.

We first examine the profit ratios when $m = 0.3$; see Figure E.7. We observe that the total profit in the centralized solution is always greater in each of our instances for which equilibrium exists. When c_c is low, the percentage gap between the centralized and decentralized solutions tends to decrease as p increases. This is because the total sales revenue in the total profit function increases in either case as p increases, outweighing the transportation costs, and the total sales revenue is always the same in both cases. Conversely, when c_c is high, the gap between the centralized and decentralized solutions tends to increase as p increases. In the decentralized system, as p increases, increasing the demand becomes more crucial than being close to the warehouse or consumer bases. Thus the retailers want to get closer to each other, in order to increase their demands under competition. Such a deviation from the optimal locations is too costly when c_c is high, and a larger gap results.

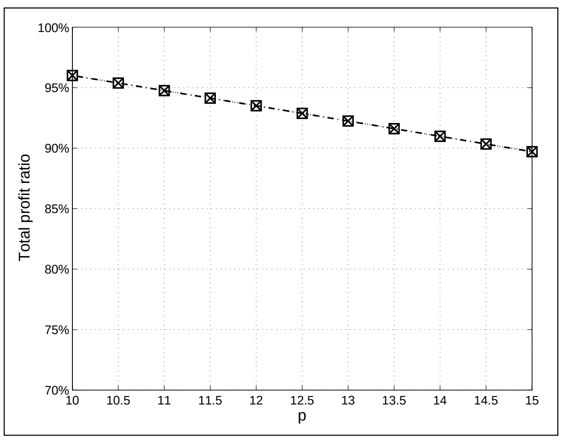
We next examine the profit ratios when $m = 0.5$; see Figure E.8. Unlike Figure E.7, when c_c is low, the minimum profit ratio tends to decrease as p increases from 13 to 15 in Figures E.8(a) and E.8(b), respectively.



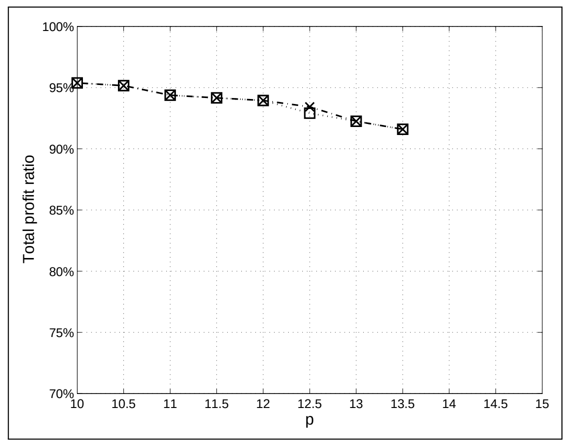
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.

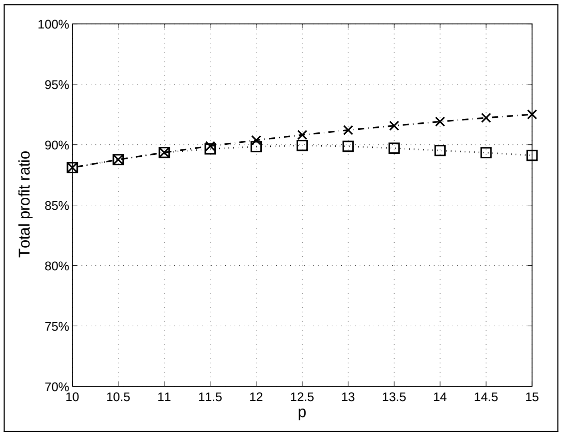


(c) $c_c = 9$ and $c_t = 0.5$.

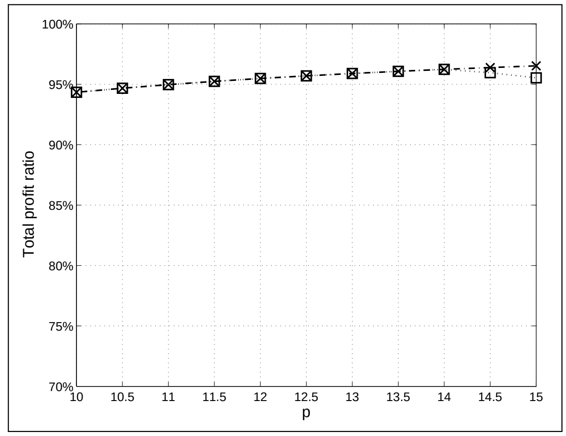


(d) $c_c = 9$ and $c_t = 2$.

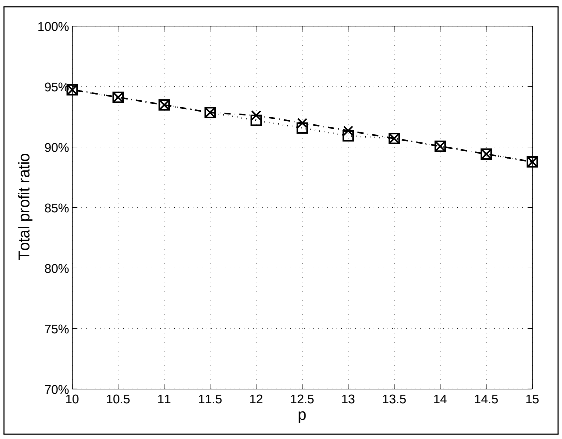
Figure E.7: Total profit ratios of the decentralized solution to the centralized solution when $m = 0.3$ and $\lambda = 10$. "x" and "□" indicate the maximum and minimum ratios, respectively.



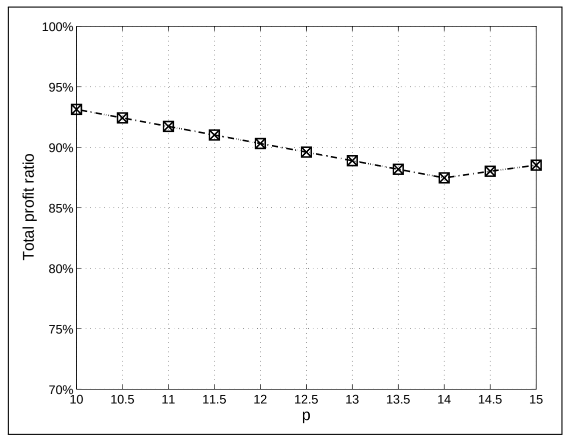
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.

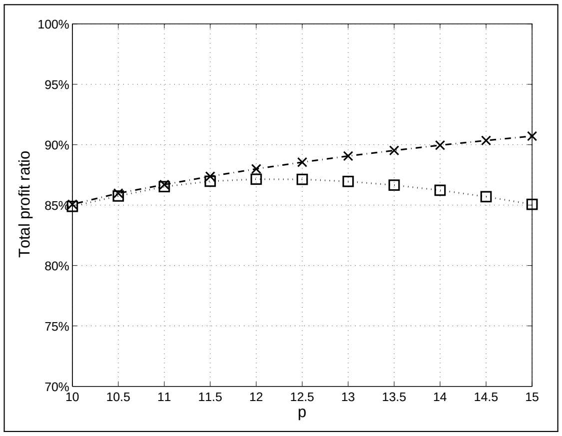


(c) $c_c = 9$ and $c_t = 0.5$.

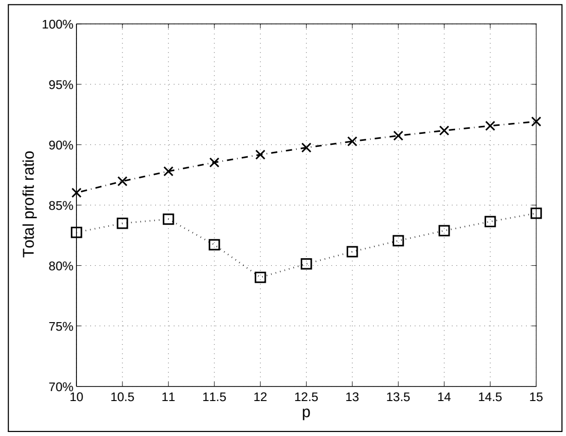


(d) $c_c = 9$ and $c_t = 2$.

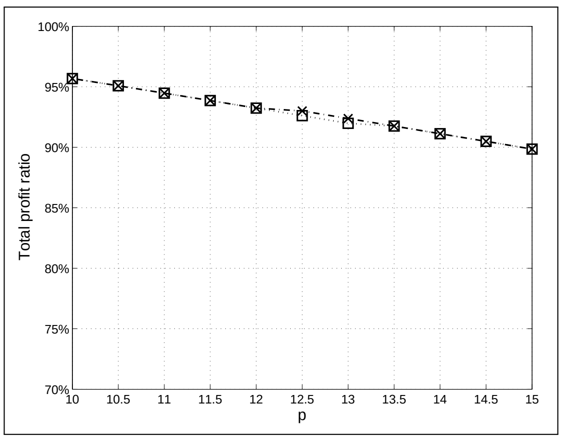
Figure E.8: Total profit ratios of the decentralized solution to the centralized solution when $m = 0.5$ and $\lambda = 10$. "x" and "□" indicate the maximum and minimum ratios, respectively.



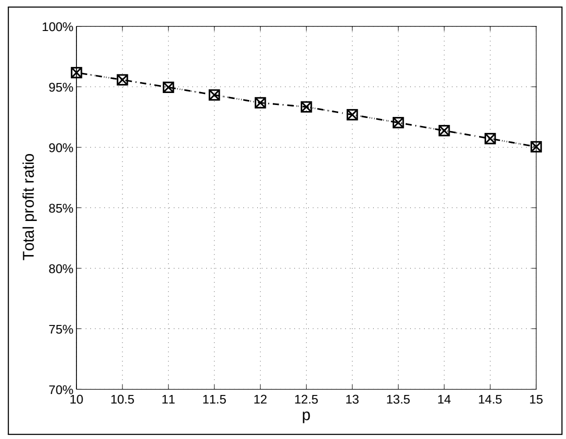
(a) $c_c = 5$ and $c_t = 0.5$.



(b) $c_c = 5$ and $c_t = 2$.



(c) $c_c = 9$ and $c_t = 0.5$.



(d) $c_c = 9$ and $c_t = 2$.

Figure E.9: Total profit ratios of the decentralized solution to the centralized solution when $m = 1$ and $\lambda = 10$. "x" and "□" indicate the maximum and minimum ratios, respectively.