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A NEW PERIODIC CONTROLLER FOR DISCRETE TIME CHAOTIC SYSTEMS

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Abstract: In this paper we consider the stabilization problem of unstable periodic orbits of discrete time chaotic systems. For simplicity we consider only one dimensional case. We propose a novel periodic feedback controller law and present some stability results. This scheme may be considered as a novel generalization of the classical delayed feedback scheme, which is also known as Pyragas scheme. The stability results show that all hyperbolic periodic orbits can be stabilized with the proposed method. The stability proofs also give the possible feedback gains which achieve stabilization. We will also present some simulation results.

Keywords: Chaotic Systems, Chaos Control, Delayed Feedback System, Pyragas Controller, Stability.

1. INTRODUCTION

It has been shown that many physical systems may be represented by mathematical models which exhibit chaotic behaviour, see e.g. (Chen and Dong, 1999). Mainly for this reason in recent years the study of chaotic behaviour in dynamical systems has received great attention among scientists from various disciplines, including engineers, mathematicians, physicist, etc. Due to the interdisciplinary nature of the field, various aspects of the chaotic systems have been investigated in the literature. After the seminal work of (Ott, Grebogy and Yorke,1990), where the term “controlling chaos" was introduced, the interest in feedback control of chaotic systems has received great attention among scientist. Due to possible applications, the subject of controlling chaos has also attracted a great deal of attention, see e.g. (Chen and Dong, 1999), (Fradkov and Evans, 2002), and the references therein.

One of the characteristic features of the chaotic systems is that they usually possess attractors which are called "strange", and these attractors usually contain infinitely many unstable periodic orbits, see e.g. (Devaney, 1987). Obviously, as in the classical feedback control theory one may define various control problems for such systems. Among these, one interesting problem is to find some control schemes to achieve the stabilization of some of these periodic orbits. If one can establish such a scheme, then applying such a feedback law will force the chaotic system to exhibit a "regular" behaviour. An interesting and remarkable result first given in (Ott, Grebogy and Yorke,1990) showed that some of these unstable orbits could be stabilized by using small control inputs. Following this seminal work , numerous control schemes have been proposed for the solution of the same problem. Among these, the Delayed Feedback Control (DFC) scheme first proposed in (Pyragas, 1992) has received attention. This scheme is quite simple, has various attractive features, and it has also been used in various applications, see e.g. (Pyragas, 2001), (Morgül, 2003), (Morgül, 2006), and the references therein. As it is shown in (Morgül, 2003), (Ushio, 1996), (Nakajima, 1997), (Morgül, 2005a), the classical DFC has certain inherent limitations, i.e. it cannot stabilize certain periodic orbits. We note that a recent result presented in (Fiedler *et al.*, 2007), showed clearly that under certain cases, odd number limitation property does not hold for autonomous continuous time systems. Although the subject is still open and deserves further investigation, we note that the limitation of DFC stated above holds for discrete time case, see e.g. (Ushio, 1996), (Morgül, 2003), (Morgül, 2005a).

Various modifications of classical DFC scheme have been proposed in the literature to overcome the limitation mentioned above, see e.g. (Pyragas, 2001), (Socolar *et. al.,* 1994), (Pyragas, 1995), (Bleich, and Socolar, 1996), (Vieira, and Lichtenberg, 1996), and the references therein. One of these schemes is the so-called periodic, or oscillating feedback given in (Schuster and Stemmler, 1997), and it eliminates the limitations of classical DFC for period T=1 case. This scheme can be generalized to the case $T > 1$ in various ways, and two such generalizations are given in (Morgül, 2006), (Morgül, 2005b) ; it has been shown in these references that any hyperbolic periodic orbit can be stabilized with these schemes. Another modification is the so-called extended DFC (EDFC), see (Socolar *et. al.,* 1994). It has also been shown that EDFC also has inherent limitations similar to the DFC. In (Vieira, and Lichtenberg, 1996), a nonlinear version of EDFC has been proposed and it was shown that an optimal version of this scheme becomes quite simple. A generalization of this scheme for arbitrary periodic orbits for one dimensional systems has been given in (Morgül, 2009a). Preliminary results of the extension of these ideas to higher dimensional case for the latter approach has been presented in (Morgül, 2009b).

In this paper we will propose a scheme which is a generalization of the ideas presented in (Morgül, 2009a) by using periodic feedback scheme, see also (Morgül, 2006). This paper is organized as follows. In section 2 we will outline the basic problem. In section 3 and section 4 we will provide the basic controller structure as well as the related stability results given in (Morgül, 2009a) (Morgül, 2006). In section 5 we will propose a novel controller for the same problem and provide some stability results. In section 6 we will provide some simulation results and finally we will give some concluding remarks.

2. PROBLEM STATEMENT

Let us consider the following discrete-time system

$$x(k+1) = f(x(k)) \quad , \qquad (1)$$

where $k = 1,2\dots$ is the discrete time index, $x \in \mathbf{R}$, $f : \mathbf{R} \to \mathbf{R}$ is an appropriate function, which is assumed to be differentiable wherever required. We assume that the system given by (1) possesses a period T orbit characterized by the set

$$\Sigma_T = \{x_1^*, x_2^*, \dots, x_T^*\} \quad , \qquad (2)$$

where $x_i^* \in \mathbf{R}$, $i = 1,2,\dots,T$.

Let $x(\cdot)$ be a solution of (1). To characterize the convergence of $x(\cdot)$ to Σ_T, we need a distance measure, which is defined as follows. For x_i^*, we will use circular notation, i.e. $x_i^* = x_j^*$ for $i = j \ (mod\ (T))$. Let us define the following indices ($j = 1,\dots,T$):

$$d_k(j) = \sqrt{\sum_{i=0}^{T-1} (x(k+i) - x_{i+j}^*)^2} \quad . \qquad (3)$$

We then define the following distance measure

$$d(x(k), \Sigma_T) = \min\{d_k(1), \dots, d_k(T)\} \quad . \qquad (4)$$

Clearly, if $x(1) \in \Sigma_T$, then $d(x(k), \Sigma_T) = 0$, $\forall k$. Conversely if $d(x(k), \Sigma_T) = 0$ for some k_0, then it remains 0 and $x(k) \in \Sigma_T$, for $k \geq k_0$. We will use $d(x(k), \Sigma_T)$ as a measure of convergence to the periodic solution given by Σ_T.

Let $x(\cdot)$ be a solution of (1) starting with $x(1) = x_1$. We say that Σ_T is (locally) asymptotically stable if there exists an $\varepsilon > 0$ such that for any $x(1) \in \mathbf{R}^n$ for which $d(x(1), \Sigma_T) < \varepsilon$ holds, we have $\lim_{k\to\infty} d(x(k), \Sigma_T) = 0$. Moreover if this decay is exponential, i.e. the following holds for some $M \geq 1$ and $0 < \rho < 1$, ($k > 1$)

$$d(x(k), \Sigma_T) \leq M\rho^k d(x(1), \Sigma_T) \quad , \qquad (5)$$

then we say that Σ_T is (locally) exponentially stable.

To stabilize the periodic orbits of (1), let us apply the following control law :

$$x(k+1) = f(x(k)) + u(k) \qquad (6)$$

where $u(\cdot) \in \mathbf{R}$ is the control input. In classical DFC, the following feedback law is used ($k > T$):

$$u(k) = K(x(k) - x(k-T)) \quad , \qquad (7)$$

where $K \in \mathbf{R}$ is a constant gain to be determined. It is known that the scheme given above has certain inherent limitations, see e.g. (Ushio, 1996). For simplicity, let us assume one dimensional case. For Σ_T, let us set $a_i = f'(x_i^*)$. It can be shown that Σ_T cannot be stabilized with this scheme if $a = \prod_{i=1}^{T} a_i > 1$, see e.g. (Morgül, 2003), (Ushio, 1996), and a similar condition can be generalized to the case $n > 1$, (Nakajima, 1997), (Morgül, 2005a). A set of necessary and sufficient conditions to guarantee exponential stabilization can be found in (Morgül, 2003) for $n = 1$ and in (Morgül, 2005a) for $n > 1$. By using these results one can find a suitable gain K when the stabilization is possible. We note that for $a > 1$ case, the stabilization is not possible by classical DFC, but even for $a < 1$ case, stabilization may not be possible for certain periodic orbits, see e.g. (Morgül, 2003).

3. A NONLINEAR CONTROLLER

As mentioned in the introduction, to overcome the basic limitations of the classical DFC various modifications has been proposed in the literature . Our work presented here is related to the EDFC scheme first proposed in (Socolar *et. al.,* 1994) and its nonlinear version proposed in (Vieira, and Lichtenberg, 1996) for one dimensional case (i.e. $n = 1$). In the sequel, first we will consider one dimesional case ($n = 1$) and propose a scheme which is related to the optimal version of the scheme proposed in (Vieira, and Lichtenberg, 1996) for the period 1 case. A generalization of this scheme for higher order periods for one dimensional case has been given in(Morgül, 2009a).

Let us consider a period 1 orbit for (1). For simplicity, let $\Sigma_1 = \{x_1^*\}$ be a period 1 orbit of (1) (i.e. fixed point of $f : \mathbf{R} \to \mathbf{R}$), and consider the controlled system given by (6). Instead of the DFC scheme given by (7), let us use the following law

$$u(k) = \frac{K}{K+1}(x(k) - f(x(k)) \quad , \tag{8}$$

where $K \in \mathbf{R}$ is a constant gain to be determined. Clearly we require $K \neq -1$. By using (8) in (6), we obtain :

$$x(k+1) = \frac{1}{K+1} f(x(k)) + \frac{K}{K+1} x(k) \ . \tag{9}$$

Obviously on Σ_1, we have $u(k) = 0$, see (8). Furthermore if $x(k) \to \Sigma_1$ (i.e. when Σ_1 is asymptotically stable) we have $u(k) \to 0$ as well. Therefore, the scheme proposed in (8) enjoys the similar properties of DFC.

Next, we will consider the stability of Σ_1 as defined in the section 2. For simplicity, set $\Sigma_1 = \{x_1^*\}$, $a = a_1 = f'(x_1^*)$. By using linearization, (9) and the classical Lyapunov stability analysis, we can easily show that Σ_1 is (locally) exponentially stable for (9) if and only if

$$\mid \frac{K+a}{K+1} \mid < 1 \quad , \tag{10}$$

see e.g. (Khalil, 2002). It can easily be shown that if $a \neq 1$, then any Σ_1 can be stabilized by choosing K appropriately to satisfy (10). In fact, for any ρ satisfying $-1 < \rho < 1$, we can choose the stabilizing gain as :

$$K = \frac{\rho - a}{1 - \rho} \quad . \tag{11}$$

Hence the limitations of DFC and EDFC are eliminated greatly by the proposed approach. It appears that the only restriction remains (i.e. $a \neq 1$) is quite inherent and appears in (Morgül, 2006) and (Morgül, 2005b) as well. By using the arguments given in these latter references, we can state that all hyperbolic fixed points can be stabilized with the proposed scheme. We note that the control law given by (8) for period $T = 1$ case could be generalized to period $T = m > 1$ case as

$$u(k) = \frac{K}{K+1}(x(k-m+1) - f(x(k)) \quad . \tag{12}$$

But in this case, as it was shown in (Morgül, 2009a), not all hyperbolic periodic orbits can be stabilized. In fact, when $m > 1$, the inherent limitation of the classical DFC holds for this controller as well. However, an improvement over the classical DFC is possible, for details see (Morgül, 2009a).

4. PERIODIC CONTROLLER

To overcome the limitations of DFC scheme, as mentioned above, various modifications have been proposed, see e.g. (Pyragas, 2001), (Fradkov and Evans, 2002). One of these schemes is the so-called periodic, or oscillating feedback, see (Schuster and Stemmler, 1997). For period 1 case, the corresponding feedback law is given by :

$$u(k) = \varepsilon(k)(x(k) - x(k-1)) \tag{13}$$

where $\varepsilon(k)$ is given as :

$$\varepsilon(k) = \begin{cases} K & k \ (mod\ 2) = 0 \\ 0 & k \ (mod\ 2) \neq 0 \end{cases} \tag{14}$$

where K is a constant gain to be determined. Let $\Sigma_1 = \{x_1^*\}$ be the period 1 orbit of (1), and define the error as $e(k) = x(k) - x_1^*$. By using the first two iterations of (6), (13), (14) and $x_1^* = f(x_1^*)$, after linearization and considering only the first order terms, we obtain $e(2) = ae(1)$, $e(3) = (a+K)e(2) - Ke(1) = (a^2 + (a-1)K)e(1)$ where $a = f'(x_1^*)$. Clearly, if $\mid a^2 + (a-1)K \mid < 1$, then Σ_1 is (locally exponentially) stabilizable. If $a \neq 1$, then by using the above inequality one can easily find a range of K for which the (locally exponential) stabilization is possible. This simple analysis shows that for the case $T = 1$, the inherent limitation of the classical DFC can be avoided by using the periodic feedback law given above.

The idea given above can be generalized to the case $T = m > 1$. One particular generalization is given in (Schuster and Stemmler, 1997). However, as noted in (Pyragas, 2001), the stability analysis is not clear. We note that various such generalizations are possible, and two such generalizations are given in (Morgül, 2006), (Morgül, 2005b) ; it has been shown in these references that any hyperbolic periodic orbit can be stabilized with these schemes.

5. A NOVEL PERIODIC CONTROLLER

Following the ideas presented in (Morgül, 2006), (Morgül, 2009a), we propose a periodic controller in this section. To motivate our analysis, let $\Sigma_1 = \{x_1^*\}$ be a period 1 orbit of (1). Following previous section, let us define the following periodic controller :

$$u(k) = \varepsilon(k)(x(k) - f(x(k)) \quad , \tag{15}$$

where $\varepsilon(k)$ is given as :

$$\varepsilon(k) = \begin{cases} \frac{K}{K+1} & k\ (mod\ 2) = 0 \\ 0 & k\ (mod\ 2) \neq 0 \end{cases} \tag{16}$$

where K is a constant gain to be determined. Obviously, we require $K \neq -1$. Note that x_1^* is a fixed point of $f(\cdot)$, i.e. $f(x_1^*) = x_1^*$. Let us define the error as $e(k) = x(k) - x_1^*$, and set $a = f'(x_1^*)$. By using (15)-(16) in (6), and after linearization, we easily obtain

$$e(2) = ae_1 \quad , \tag{17}$$

$$e(3) = \frac{a+K}{K+1}e(2) = a\frac{a+K}{K+1}e(1) \quad . \tag{18}$$

Iterating this idea, after straightforward calculations, we obtain

$$e(2j+1) = (a\frac{a+K}{K+1})^j e(1) \quad . \tag{19}$$

Clearly, we will have exponential decay of the error if and only if

$$|\ a\frac{a+K}{K+1}\ | \ < \ 1 \quad . \tag{20}$$

It is interesting to see the difference between (10) and (20). Clearly, if $a \neq 1$, one can always find a K for which (20) is satisfied. Indeed, if $a \neq 1$, the left hand side of (20) is 0 for $K = -a$. It can also be shown after straightforward calculations that for $a \neq 1$, there exists two finite constants K_{max} and K_{min} , satisfying $K_{min} < K_{max}$ such that (20) is satisfied for $K_{min} < K < K_{max}$.

Let us consider the case $T = m > 1$, and the period m orbit Σ_m of (1), as given by (2). Let us define the m-iterate map F as $F = f^m$. Similar to (1), let us define the uncontrolled dynamics associated with F as follows:

$$z(j+1) = F(z(j)) \quad , \tag{21}$$

where $j = 1,2\ldots$ is the discrete time index, $z \in \mathbf{R}$. Let Σ_m be given as $\Sigma_m = \{x_1^*, x_2^*, \ldots, x_m^*\}$ and define $\Sigma_{mi} = \{x_i^*\}$, for $i = 1,2,\ldots,m$. Clearly, Σ_m is a period m orbit of (1) if and only if Σ_{mi} is a period 1 orbit of (21), for any $i = 1,2,\ldots,m$. Now consider the following controlled system :

$$z(j+1) = F(z(j)) + u(j) \quad , \tag{22}$$

where the control input $u(j)$ is defined as

$$u(j) = \varepsilon(j)(z(j) - F(z(j)) \quad , \tag{23}$$

and $\varepsilon(\cdot)$ is given by (16). Now consider Σ_{mi} and define $a = F'(x_i^*)$. From the preceding analysis, it is clear that Σ_{mi} is (locally) exponentially stable for the system (22)-(23), if and only if (20) holds, where $a = F'(x_i^*)$. On the other hand, if we define $a_i = f'(x_i^*)$, by using the chain rule, it is clear that $a = \prod_{i=1}^m a_i$.

To transform (6) into (22)-(23), let us choose $u(k)$ as follows

$$u(k) = \varepsilon(k)(x(k-m+1) - F(x(k-m+1)) \tag{24}$$

where $\varepsilon(k)$ is given as :

$$\varepsilon(k) = \begin{cases} \frac{K}{K+1} & k\ (mod\ 2m) = 0 \\ 0 & k\ (mod\ 2m) \neq 0 \end{cases} \tag{25}$$

Clearly, for $m = 1$, both (24) and (25) reduces to (15), and (16), respectively. To establish the transformation mentioned above, let us set

$$z(j) = x((j-1)m+1) \quad , \quad j = 1,2,\ldots \quad . \tag{26}$$

If j is odd by using (24)-(25) in (6), we obtain :

$$x(jm+1) = f(x(jm)) = f^m((j-1)m+1) \quad . \tag{27}$$

which is the same as $z(j+1) = F(z(j))$, i.e. (21). If j is even, similarly we obtain :

$$\begin{aligned} x(jm+1) = f^m(x((j-1)m+1)) & \quad , \\ +\frac{K}{K+1}(x((j-1)m+1) & \\ -F(x((j-1)m+1)) & \end{aligned} \tag{28}$$

which is the same as $z(j+1) = F(z(j) + \frac{K}{K+1}(z(j) - F(z(j))$. From these derivations, it easily follows that (6) with (24)-(25) is equivalent to (22)-(23). Following the stability analysis given above, see (17)-(20), we can state the following stability result.

Theorem 1 : Let a period m orbit of (1) be given as $\Sigma_m = \{x_1^*, \ldots, x_m^*\}$ and set $a_i = f'(x_i^*)$, $i = 1,2,\ldots,m$, $a = \prod_{i=1}^m a_i$. The control law given by (6), (24)-(25) (locally exponentially) stabilizes Σ_m if and only if

$$|\ a\frac{a+K}{K+1}\ | < 1 \quad . \tag{29}$$

Proof : First note that each point in Σ_m becomes a fixed point of (21). Also note that the local exponential stability of an equilibrium point is equivalent to the stability of the linearized system around the equilibrium point, see e.g. (Khalil, 2002). The proof of the theorem then easily follows from the results stated above. □

The following corollary easily follows from (29).

Corollary 1 : Let a period m orbit of (1) be given as $\Sigma_m = \{x_1^*, \ldots, x_m^*\}$ and set $a_i = f'(x_i^*)$, $i = 1,2,\ldots,m$, $a = \prod_{i=1}^m a_i$. Assume that $a \neq 1$, and consider the control law given by (6), (24)-(25). Then there exists two finite constants K_{max} and K_{min} , satisfying $K_{min} < K_{max}$ such that (29) is satisfied for $K_{min} < K < K_{max}$.

Proof : Note that when $a \neq 1$, (29) is satisfied for $K = -a$. The stated result easily follows from the form of (29). □

Remark 1 : Σ_m is called as a hyperbolic periodic orbit of (6) if $\mid a \mid \neq 1$. From Corollary 1, it easily follows that all hyperbolic periodic orbits can be stabilized with the proposed scheme. □

Remark 2 : First note that when $\mid a \mid < 1$, then Σ_m is a stable periodic orbit. Now consider the case $\mid a \mid > 1$. If $a > 1$, then from the structure of (29), it follows that $K_{max} < 0$, i.e. the stabilizing gains are negative. On the other hand, if $a < -1$, then by using a similar argument, it can easily be shown that $K_{min} > 0$, i.e. the stabilizing gains are positive. □.

6. SIMULATION RESULTS

For the simulation results, we consider the logistic map given by $f(x) = 4x(1-x)$. It is well-known that this map has chaotic solutions and periodic orbits of all orders. This map has two period 3 orbits, which are given as $\Sigma_{3-} = \{0.413175, 0.969846, 0.116977\}$, $\Sigma_{3+} = \{0.611260, 0.950484, 0.188255\}$.

For Σ_{3+}, we have $a = 8$, and since $a > 1$, this orbit cannot be stabilized by classical DFC, (Ushio, 1996), (Morgul, 2003). By using (29) it can easily be shown that Σ_{3+} can be stabilized for the system given by (6), (24)-(25) for $-9 < K < -65/9$, see Remark 2. For the simulations we chose $K = -8.2$. Typical simulation results for $x(1) = 0.4$ are given in Figures 1-3. Figure 1 shows $d(x(k), \Sigma_{3+})$ versus k graph, Figure 2 shows $u(k)$ versus k graph and Figure 3 shows $x(k-1)$ vs. $x(k)$ graph. These graphs show that the solutions converge exponentially to Σ_{3+}.

For Σ_{3-}, we have $a = -8$, and it can be shown that this orbit also cannot be stabilized by classical DFC, see (Morgul, 2003). By using (29) it can easily be shown that Σ_{3-} can be stabilized for the system given by (6), (24)-(25) for $7 < K < 65/7$, see Remark 2. For the simulations we chose $K = 8.2$. Typical simulation results for $x(1) = 0.4$ are given in Figures 4-6. Figure 4 shows $d(x(k), \Sigma_{3-})$ versus k graph, Figure 5 shows $u(k)$ versus k graph and Figure 6 shows $x(k-1)$ vs. $x(k)$ graph. These graphs show that the solutions converge exponentially to Σ_{3-}.

7. CONCLUSION

In this paper, we proposed a novel nonlinear periodic feedback law given by (13) to stabilize the unstable periodic orbits for one dimensional discrete time chaotic systems. We showed that under a very mild condition ($a \neq 1$) local exponential stabilization of any periodic orbit is possible. This imply that any hyperbolic periodic orbit can be stabilized with the proposed scheme. Since hyperbolicity is a generic property, we may claim that almost all unstable periodic orbits can be stabilized with the proposed scheme. This scheme can be generalized to higher dimensional systems in a straightforward way by utilizing the ideas given in (Morgül, 2009b).

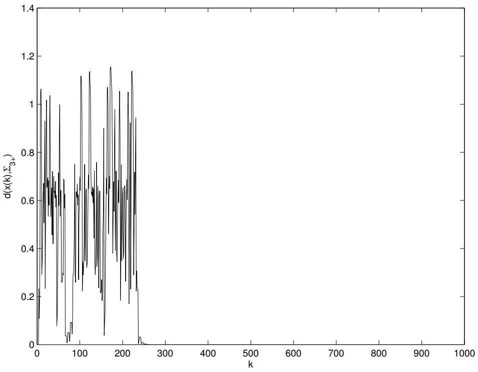

Fig. 1. $d(x(k), \Sigma_{3+})$ vs. k

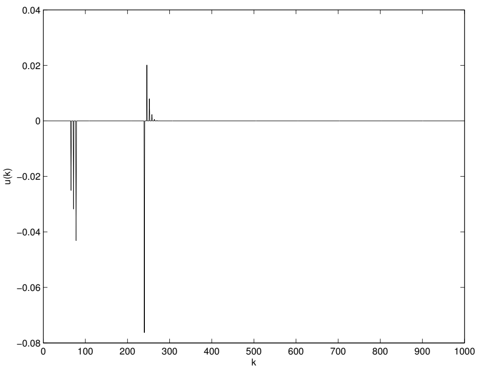

Fig. 2. $u(k)$ vs. k

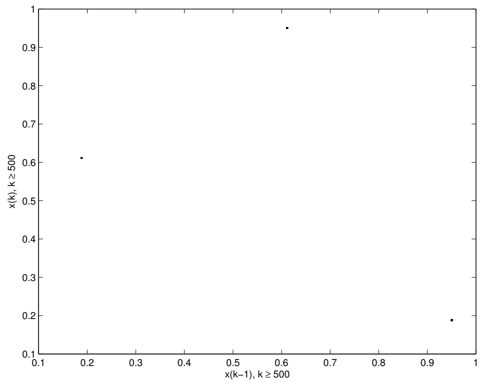

Fig. 3. $x(k-1)$ vs. $x(k)$ for $k \geq 500$

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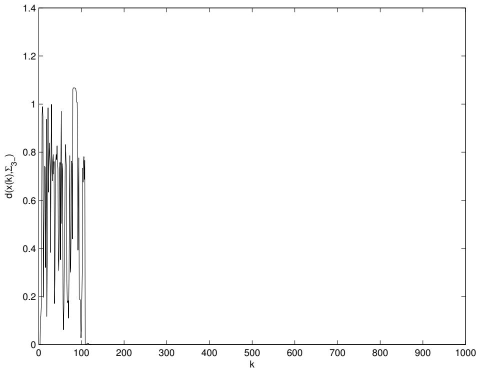

Fig. 4. $d(x(k), \Sigma_{3-})$ vs. k

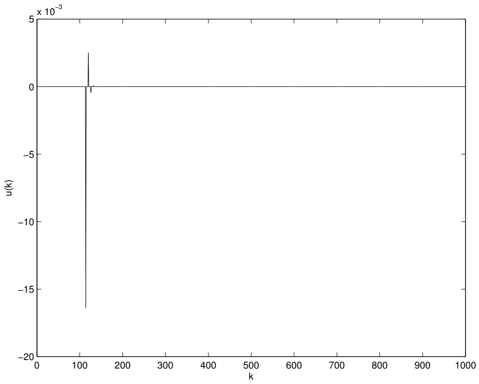

Fig. 5. $u(k)$ vs. k

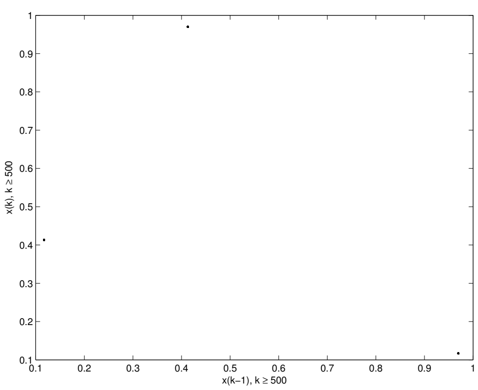

Fig. 6. $x(k-1)$ vs. $x(k)$ for $k \geq 500$

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