Express Letters

An Adaptive System Identification Method Based on Bispectrum

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Abstract—This paper presents a new adaptive technique for the identification of a linear system driven by white non-Gaussian noise. The system can be a non-minimum phase system. The adaptive identification technique is a least mean-square (LMS) type algorithm and it is obtained by using the higher order correlations of the system output.

I. INTRODUCTION

Methods based on higher order correlations have recently been applied to many problems in a number of diverse fields [1]–[3]. Adaptive non-minimum phase system identification problems may arise in geophysics and in digital communications. If the input to the system is not accessible, then the classical system identification methods fail to identify non-minimum phase systems [4]. This is because the classical methods are based only on the second-order statistics. By using higher order statistics of the output observations, one can identify a non-minimum phase system since higher order correlations have the ability to preserve the phase information.

This paper presents a new adaptive technique for the identification of a linear system that is driven by white non-Gaussian noise. The system can be a non-minimum phase system. The adaptive identification technique is an LMS type algorithm, and it is obtained by using the higher order correlations of the system output.

Let us describe our problem assumptions and define the bispectrum that is the Fourier transform of the third-order cumulants. The observation sequence is modeled as follows:

$$y(k) = \sum_{n=-N_1}^{N_2} h(n)x(k-n) + w(k) \quad (1)$$

where $h(n)$ is the system impulse response that can be non-minimum phase; $x(k)$ is the input sequence that is non-Gaussian, white, i.i.d, with $E[x(k)] = 0$, $E[x(k)x(k+n)] = \alpha\delta(n)$ and $E[x(k)x(k+n)x(k+m)] = \gamma\delta(n)\delta(m)$; $w(k)$ is a Gaussian additive noise and independent from $x(k)$. In many practical problems the system $H(z)$ is a non-minimum phase system, i.e.,

$$H(z) = \nu P(z)Q(z)z^{-r} \quad (2)$$

where $P(z)$ is a polynomial with zeros outside the unit circle (maximum phase) and $Q(z)$ is a polynomial with zeros inside

the unit circle (minimum phase), r is an integer, and ν is a constant gain factor.

In this paper our aim is to adaptively reconstruct $h(n)$ within a scale factor from the observations $y(k)$ by using higher order correlations.

The two-dimensional (2-D) cumulant sequence of the random process $x(k)$ is defined as follows:

$$r_x(n_1, n_2) = E[x(k)x(k+n_1)x(k+n_2)]. \quad (3)$$

The bispectrum $B_x(\omega_1, \omega_2)$ of the random process $x(k)$ is defined as the Fourier transform of the 2-D cumulant sequence, i.e.,

$$B_x(\omega_1, \omega_2) = \sum_{n_1, n_2} r_x(n_1, n_2) \exp[-j(\omega_1 n_1 + \omega_2 n_2)]. \quad (4)$$

We note that $B_y(\omega_1, \omega_2) = \gamma B_h(\omega_1, \omega_2)$ since $r_y(n_1, n_2) = \gamma r_h(n_1, n_2)$ and $B_w(\omega_1, \omega_2) = 0$ because $w(k)$ is a Gaussian process whose third-order correlations $r_w(n_1, n_2) = 0$.

Researchers have developed system identification methods [1]–[3], [5], [6] by using the bispectrum (see the special issue of IEEE TRANS. ACOUSTICS, SPEECH, AND SIGNAL PROCESSING, July 1990, for an extensive review of these methods). Hatzinakos and Nikias recently presented an adaptive system identification method [7] based on higher order correlations. Their algorithm is an LMS type algorithm. This algorithm first updates the truncated 2-D cepstrum coefficients $\hat{r}_h(n_1, n_2)$ of the cumulants $r_h(n_1, n_2)$. When a satisfactory convergence level is reached, the cepstrum $\hat{h}(n)$ of the desired system response is obtained from $\hat{r}_h(n_1, n_2)$ by using a convolution relation. Finally, $H(z)$ is obtained from estimated cepstral sequence $\hat{h}(n)$.

Our algorithm is also an LMS type algorithm but we update the truncated 1-D cepstrum coefficients corresponding to the diagonal slice $B_y(\omega, \omega)$ of the bispectrum. Because of this the computational cost of our method is lower than the method described in [7]. The convergence rates of both schemes are observed to be comparable, which is due to the fact that both schemes are based on updating cepstral parameters corresponding to the system impulse response $h(n)$.

We make some definitions and observations that are used in obtaining the adaptive method. it can be shown that [1], [2]

$$B_y(\omega_1, \omega_2) = \gamma H(\omega_1)H(\omega_2)H^*(\omega_1 + \omega_2) \quad (5)$$

where $H(\omega)$ is the Fourier transform of $\mathbf{h} = \{h(n)\}_{n=-N_1}^{N_2}$ and we assume $\gamma = 1$ without loss of generality. The complex cepstrum $\hat{\mathbf{h}} = \{\hat{h}(n)\}_{n=-\infty}^{\infty}$ of the sequence $\mathbf{h}$ is defined as the inverse z-transform of $\text{Log } H(z)$, i.e.,

$$\hat{\mathbf{h}} = Z^{-1}[\text{Log } H(z)] \quad (6)$$

where Z^{-1} is the inverse z-transform operator and $H(z)$ is the z-transform of $\mathbf{h}$. Let us define a sequence, $\mathbf{g} = \{g(n)\}$, as follows:

$$\mathbf{g} = \mathcal{F}^{-1}[G(\omega)] \quad (7)$$

where $G(\omega) = B_y(\omega_1, \omega_2)|_{\omega_1=\omega_2=\omega}$ and $\mathcal{F}^{-1}$ is the inverse Fourier transform operator. Since $G(\omega)$ is the diagonal slice of

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Fig. 1. Adaptation of the impulse response coefficients using the adaptive algorithm of Section II, with $K = 16$ and $p = q = 6$.

$B_y(\omega_1, \omega_2)$ the sequence $g(n) = \sum_k r_y(k, n-k)$. The complex cepstrum $\hat{g}(n)$ of the sequence $g(n)$ is given by

$$\hat{g} = Z^{-1}[\text{Log } G(z)] \quad (8)$$

where $G(z)$ is the z -transform of g . If $G(z)$ has no zeros on the unit circle, which is equivalent to the condition that $H(z)$ has no zeros on the unit circle then we compute $\hat{g}$ as follows:

$$\hat{g} = \mathcal{F}^{-1}[\text{Log } G(\omega)]. \quad (9)$$

We obtain the following equation from (5):

$$\text{Log } B_y(\omega, \omega) = 2\text{Log } H(\omega) + \text{Log } H^*(2\omega). \quad (10)$$

By taking the inverse Fourier transform of both sides of (10), one can show that the complex cepstrum $\hat{g}$ of the sequence g is given by [5]

$$\hat{g}(n) = \begin{cases} 2\hat{h}(n) + \hat{h}(-n/2), & \text{if } n \text{ even} \\ 2\hat{h}(n), & \text{if } n \text{ odd.} \end{cases} \quad (11)$$

The complex cepstrum of the impulse response sequence $\hat{h}(n)$

can be recursively obtained from $\hat{g}(n)$, [5], [6] i.e.,

$$\begin{aligned} \hat{h}(0) &= \hat{g}(0)/3, \hat{h}(1) = \hat{g}(1)/2, \hat{h}(-1) \\ &= \hat{g}(-1)/2, \hat{h}(2) = (\hat{g}(2) - \hat{h}(-1))/2, \hat{h}(-2) \\ &= (\hat{g}(-2) - \hat{h}(1))/2, \hat{h}(3) \\ &= \hat{g}(3)/2, \hat{h}(-3) = \hat{g}(-3)/2, \dots \end{aligned} \quad (12)$$

The cepstral sequence $\hat{h}(n)$ of a finite extent sequence $h(n)$ is of infinite extent. However, the cepstral sequences decay exponentially with a $1/n$ scaling factor for finite extent sequences that have no zeros on the unit circle; thus we can select two large integers $p, q > 0$ such that the truncated sequence $\{\hat{h}(n)\}_{n=-p}^q$ approximates $\{\hat{h}(n)\}_{n=-\infty}^{\infty}$ arbitrarily closely [7].

By computing the derivative of the both sides of the equation $\hat{G}(z) = \text{Log } G(z)$ with respect to z , we obtain

$$G(z)z \frac{\partial \hat{G}(z)}{\partial z} = z \frac{\partial G(z)}{\partial z} \quad (13)$$

and by computing the inverse z -transform of (13) we get the

following equation:

$$\sum_{k=-\infty}^{\infty} k \hat{g}(k) g(n-k) = n g(n). \quad (14)$$

Since the cepstral sequence $\hat{g}(n)$ decay exponentially with a $1/n$ scaling factor, we can approximate the above infinite sum by a finite sum:

$$g(n) \approx \sum_{k=-p}^q (k/n) g(n-k) \hat{g}(k). \quad (15)$$

The relation (15) is the key result used in the adaptive algorithm. We rewrite (15) in matrix form as follows:

$$d \approx D b \quad (16)$$

where $b = [\hat{g}(-p), \hat{g}(-p+1), \dots, \hat{g}(q)]^T$, $d = [g(-K), g(-K+1), \dots, g(K)]^T$, D is a matrix of size $2K \times (p+q)$, and the entries of the matrix D are determined according to (15).

II. THE ADAPTIVE ALGORITHM

In this section we present the adaptive system identification algorithm.

i) At each discrete-time instant l estimate the third-order cumulant sequence $r_y^{(3)}(n_1, n_2)$ where $-N \leq n_1, n_2 \leq N$ from a finite window of the observed sequence $y(k)$. In [3] adaptive estimation methods of third-order cumulants from $y(k)$ were described.

ii) Compute $g^{(l)}(n) = \sum_k r_y^{(3)}(k, n-k)$.

iii) Adaptive estimation of the cepstrum $\hat{h}(n)$ of $h(n)$: We update the parameters $\hat{h}^{(l)}(n)$ with a gradient type LMS algorithm. In this algorithm the error term is given by the following equation:

$$e(l) = d(l) - D(l)b(l) \quad (17)$$

where $d(l)$, $D(l)$, and $b(l)$ are defined in (16) and the cepstral coefficient update equation is:

$$b(l+1) = b(l) + \mu D^T(l) e(l) \quad (18)$$

where μ is a constant that satisfies the following inequality:

$$0 < \mu < 2 / \text{trace}[D^T D]. \quad (19)$$

iv) Once a satisfactory convergence level is reached at time instant L , we obtain $\hat{h}^L(n)$ from $b(L) = [\hat{g}^L(-p), \hat{g}^L(-p+1), \dots, \hat{g}^L(q)]^T$ by using the recursive relation (12), and we compute

$$H^L(\omega) = \exp[\mathcal{F}(\hat{h}^L(n))] \quad (20)$$

and

$$h^L(n) = \mathcal{F}^{-1}[H^L(\omega)] \quad (21)$$

where $h^L(n)$ is the estimate of $h(n)$ at time instant L . These computations can be carried out by using an FFT algorithm.

The minimum and maximum phase components of the system $h(n)$ can also be computed very easily since the sequences $\hat{h}_+ = \{\dots, 0, 0, \hat{h}(0)/2, \hat{h}(1), \hat{h}(2), \dots\}$ and $\hat{h}_- = \{\dots, \hat{h}(-2), \hat{h}(-1), \hat{h}(0)/2, 0, 0, \dots\}$ are the cepstra of the minimum and maximum phase components of the sequence $h(n)$, respectively, i.e., $Q(\omega) = \exp[\mathcal{F}(\hat{h}_+(n))]$ and $P(\omega) = \exp[\mathcal{F}(\hat{h}_-(n))]$ where $Q(z)$ and $P(z)$ are defined in (2).

The complex cepstra of $h(n)$ and $c \times h(n-K)$, c is a real number, are equal to each other. Thus the factor ν in (2) cannot be computed. We also assumed that $H(\omega) \neq 0$ for all $\omega \in [-\pi, \pi]$. Any method using cepstral domain methods including [7] suffers from this restriction.

III. SIMULATION EXAMPLE

Let us consider the following non-minimum phase linear time-invariant system having impulse response $h = \{\dots, 0, 0.2866, -0.9745, h(0) = 0.7938, 0.2, 0, \dots\}$. This system is driven by a zero mean, non-Gaussian white noise with skewness 1. The driving sequence is generated using the GGEXN subroutine of the IMSL library. The adaptive algorithm is used with parameters $p = q = 6$ and $K = 16$. The impulse response coefficients of the unknown system is updated after each 128 samples of the output sequence. The result obtained is shown in Fig. 1. As it is seen from Fig. 1, the algorithm converges to the true system parameters after approximately 850 iterations. This shows the ability of the proposed algorithm to track the true values of the impulse response.

An independent Gaussian noise is also added to the filter output. The updated coefficients are found to be almost the same as those of the previous case (the signal-to-noise ratio is chosen to be 10). This is due to the fact that all higher order (than two) correlations of a Gaussian process are zero.

IV. CONCLUSION

In this paper we present a new computationally efficient adaptive identification method for linear systems which are driven by white non-Gaussian noise. Our adaptive method is an LMS type algorithm that uses the third-order cumulants of the system output.

Similar algorithms based on fourth- and higher order correlations can also be developed.

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A Correction to "Adaptive Nonlinear Digital Filter with Canonical Piecewise-Linear Structure"¹

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Equation (20) in the above paper¹ should be read as follows:

$$\nu^-(n) := [p_1^-(n)x^T(n) p_2^-(n)x^T(n) \dots p_\sigma^-(n)x^T(n)]^T. \quad (20)$$

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