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A comparative study of cell formation in cellular manufacturing systems

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This paper analyses and evaluates six of the promising cell formation techniques by comparing and contrasting them in relation to the scores of efficiency indices of the exceptional elements and inner-cell densities, work-load balance, and under-utilizations in different scenarios. Accordingly, all six are more or less altered with proper extensions to realize a broader capability. Effectiveness of the suggested efficiency measures in the evaluation is also illustrated. Each technique seems to have a favourite operating area of its own considering a variety of factors.

1. Introduction

Cell formation, i.e. placing machine groups of functionally dissimilar types together to enable the manufacture of a specific range of parts, is acknowledged to be the first and major stage in the design of Cellular Manufacturing systems. The initial decision made at this stage presides over all the other decisions involved in the design process.

Since it was asserted for the first time (Burbidge 1971), the cell formation problem has grown into an area in which much research has been conducted. As the cell formation problem turns out to be NP-complete (Ballakur 1985, Lawler *et al.* 1985), a large number of cell formation heuristics have been designed to obtain feasible solutions. Various taxonomies of the cell formation techniques have also been proposed (Ballakur 1987, Kandiller 1989, King and Nakornchai 1982, and Wemmerlöv and Hyer 1986a).

On the other hand, cell formation techniques in the literature are seldom illustrated by adequate examples. Most of the considered cell formation problems are not only to indicate the effectiveness of a single technique, but also are rather small in size and easy to follow. Although some of the authors report that their proposed cell formation techniques are implemented satisfactorily, the details of these implementations are not disclosed. Even in comparative studies, which are not many, only a few of the techniques are tackled in terms of the example problems. However, it is obvious that such studies should involve a far more comprehensive evaluation in order to produce valuable results.

In this study, six leading cell formation techniques are compared in terms of developed measures in different manufacturing environments so as to help establish guidelines for selecting a proper cell formation procedure for a specific situation. A similar study has been reported recently (Miltenburg and Zhang 1992).

Three 'quick and clean' efficiency indices to evaluate the cell formation solutions are suggested in the next section. The first index is the grouping efficiency which is focused on the combination of the magnitude of exceptional elements and inner-cell densities. The second is related to inner-cell work-load balances. The last index measures the under-utilizations of individual machines.

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Furthermore, a subset of promising cell formation techniques, namely lattice theoretic combinatorial grouping (COMBGR) (Purcheck 1985), modified rank order clustering (MODROC) (Chandrasekharan and Rajagopalan 1986b), machine-component cell formation (MACE) (Waghodekar and Sahu 1984), within-cell utilization based clustering (WUBC) (Ballakur and Steudel 1987), cost analysis algorithm (CAA) (Kusiak and Chow 1987), and zero-one data—ideal-seed clustering (ZODIAC) (Chandrasekharan and Rajagopalan 1987), are selected for detailed analysis. These techniques are also modified to create a common comparative basis.

Finally, to provide a testing ground for evaluating and comparing design techniques, a problem generator is developed. Six of the techniques are evaluated in terms of randomly generated test problems under different scenarios. The effects on the proposed efficiency measures from the number of parts, shop densities, demand and depreciation cost variations, and manufacturing environments on the cell formation problems are investigated statistically for each technique.

2. Efficiency measures

The evaluation of cell formation solutions is a vital issue in the design of CM systems. Although a number of techniques have already been developed, the evaluation of cell formation solutions has remained somewhat qualitative (Ballakur 1985, Chandrasekharan and Rajagopalan1986a, Wemmerlöv and Hyer 1986a, b). Some of the commonly used quantitative efficiency measures are the number of inter- and inner-cell moves, the number and cost of duplicated equipment, the number of parts removed from the system, and machine utilizations (Purcheck 1974, 1985, Waghodekar and Sahu 1984, Chandrasekharan and Rajagopalan 1986a, b, Kumar *et al.* 1986, Askin and Subramanian 1987, Ballakur and Steudel 1987, Kusiak and Chow 1987, Kusiak and İbrahim 1988, Vakaharia and Wemmerlöv 1990). In particular, cell formation techniques have usually been compared in relation to the number of exceptional elements generated in the solutions (Waghodekar and Sahu 1984, Chandrasekharan and Rajagopalan 1986a, Wemmerlöv and Hyer 1986a, Ballakur and Steudel 1987).

A modelling tool for cell formation is the machine-part incidence matrix, the rows of which correspond to machine types and the columns to parts. Each element of the incidence matrix is 'one' if there exists a routeing relation between the associated column and row, otherwise it is 'zero'. The objective of the cell formation problem is to *permute the columns and rows of the incidence matrix so that a block-diagonal structure is obtained.* The resulting diagonal blocks represent the manufacuturing cells. Desirable cell formation solutions are the ones in which all parts complete all their operations in their assigned cell. For such solutions, there is no inter-cell movement of parts. *Exceptional elements* are the entries of the incidence matrix that do not belong to any diagonal block preventing the solution from being a desirable one. Unfortunately, the majority of cell formation solutions contain exceptional elements. The cell formation problem can therefore be defined as *the reordering of the incidence matrix so that a minimum number of exceptional elements are obtained, provided that a block-diagonal structure exists* (Kandiller 1989).

It is also possible to use work-load matrices in solving cell formation problems as they contain considerably more information than incidence matrices. Each entry of a work-load matrix represents the percentage of machine capacity allocated to the corresponding operation. The sum of the elements of a specific row of a work-load matrix indicates the number of machines desired of that type. A work-load matrix enables the user to deal with individual machines instead of machine types. Hence, more than one cell can have the same machine type. Furthermore, if the workload of an exceptional element is high enough, a machine of the corresponding type will be inserted into the corresponding part's cell. Thus, the use of work-load matrices provides an opportunity to eliminate some of the exceptional elements (Kandiller 1989).

The efficiency of each cell formation solution can be measured from its work-load matrix. Three efficiency indices with assumed values between zero and one are suggested for evaluating the solutions. The first measure is the modified grouping efficiency which penalizes exceptional elements and considers inner-cell densities. This measure is an extended version of what Chandrasekharan and Rajagopalan reported (1986a, 1987) whereas the remaining two are developed within the scope of this study. The second measure pertains to the inner-cell load balances. The third measure focuses on under-utilizations of individual machines. Discussion of these efficiency measures requires some notation and definitions:

i = machine type index ($i = 1, \ldots, T$),
j = part index ($j = 1, \ldots, P$),
k = cell index ($k = 1, \ldots, K$),
$CM(k)$ = index set of machine types that are assigned to cell k,
$CP(k)$ = index set of parts that are assigned to cell k,
$AC(j)$ = index of cell to which part j is assigned,
AV_j = annual production volume of part j {units},
$ST_{i,j}$ = total standard times of operations of part j on machine type i {time-units/unit},
C_i = annual availability of machine type i {time-units/machine},
$WL_{i,j}$ = annual work-load on machine type i induced by part j {machine-fraction},

$$WL_{i,j} \doteq \frac{AV_j \times ST_{i,j}}{C_i}$$

$TU_{k,i}$ = total usage of machines of type i in cell k {machine-fraction},

$$TU_{k,i} \doteq \sum_{j \in CP(k)} WL_{i,j}$$

$N_{k,i}$ = number of machines of type i in cell k {machines},

$$N_{k,i} \doteq \lceil TU_{k,i} \rceil$$

S_k = number of different machine types in cell k,
OC_i = annual operating cost (including depreciation) of a machine of type i {\$/machine},
$TWLC_j$ = total work-load cost of part j {\$},

$$TWLC_j \doteq \sum_{i=1}^{T} WL_{i,j} \times OC_i$$

WCC_j = work-load cost of part j in its assigned cell {\$},

$$WCC_j \doteq \sum_{i \in CM(AC(j))} WL_{i,j} \times OC_i$$

$WLCE_j$ = work-load cost of exceptional elements belonging to part j {\$},

$$WLCE_j \doteq TWLC_j - WCC_j$$

$FP_{i,j}$ = field potential value of clustering both part j and machine type i into the same cell $\{\$^2\}$. It is an artificial weight given to each entry of the work-load matrix which represents the solution. The associated weight is the multiplication of column and row weights. Here it is assumed that machine types differ from each other in terms of total annual operating costs whereas parts are differentiated by means of total work-load costs:

$$FP_{i,j} \doteq TWLC_j \times OC_i \times N_{AC(j),i}$$

$AP_{i,j}$ = assignment potential of clustering both part j and machine type i into the same cell. It is the associated field potential value incurred only if both machine type and part are assigned to the same cell.

$$AP_{i,j} \doteq \begin{cases} FP_{i,j} & \text{if } WL_{i,j} > 0, \\ 0 & \text{otherwise.} \end{cases}$$

$MWL_{k,i}$ = mean percent workload on machines of type i assigned to cell k $\{\%\}$,

$$MWL_{k,i} \doteq \frac{100}{N_{k,i}} \times \sum_{j \in CP(k)} WL_{i,j}$$

MCL_k = mean percent cell-load in cell k $\{\%\}$,

$$MCL_k \doteq \frac{1}{S_k} \times \sum_{i \in CM(k)} MWL_{k,i}$$

$UU_{k,i}$ = total under-utilization of machine type i in cell k {machine-fraction},

$$UU_{k,i} \doteq N_{k,i} - TU_{k,i}$$

Each element $WL_{i,j}$ of the work-load matrix is nothing but a machine fraction value used in facilities planning. It has non-negative real value representing the number of machines of type i needed for carrying out the related annual total manufacturing operations of part j.

Machine types are different in terms of costs. It is unrealistic to treat a cheap machine and an expensive one similarly. Specifically, an exceptional element due to an assembly bench can be eliminated simply by duplicating it. However, this is certainly not the case with CNCs. Furthermore, two different machine types of the same initial cost may have different operating expenses. Therefore, rows of the work-load matrix are differentiated from each other in terms of total annual operating cost of all machines of the corresponding type.

Similarly, each part is different in considering product volumes and manufacturing expenses. In this analysis, we solely consider machine operating costs, including depreciation and direct labour, in manufacturing expenses and ignore direct material costs. But these terms can easily be incorporated into cost expressions. Hence, each column of the matrix has different ranks equal to the total work-load cost of the corresponding part.

Modified grouping efficiency is a combined measure made up of two parts. The first part measures the inter-cell work-load created by exceptional elements. Inter-cell flow efficiency, μ_1, is defined as the complementary normalized cost of all exceptional elements:

$$\mu_1 \doteq 1 - \frac{\Sigma_{j=1}^{P} WLCE_j}{\Sigma_{j=1}^{P} TWLC_j}. \tag{1}$$

The second part of the modified grouping efficiency is a weighed estimate of the inner-cell densities. Each block-diagonal entry in the work-load matrix is weighed by multiplying the column and row weights. Consequently, a potential field is defined for each diagonal block representing a manufacturing cell. The estimated density of a cell is the normalized total potential in this field. Therefore, inner-cell efficiency, μ_2, of any cell formation solution is defined as:

$$\mu_2 \doteq \frac{\sum_{j=1}^{P} \sum_{i \in CM(AC(j))} AP_{i,j}}{\sum_{j=1}^{P} \sum_{i \in CM(AC(j))} FP_{i,j}}. \tag{2}$$

There are two extreme cell formation situations. One is the cell system which contains small, dense cells and a lot of exceptional elements. This situation which maximizes inner-cell density measure μ_2 is penalized by inter-cell flow efficiency μ_1. The other is the cell system with large, sparse cells wihout exceptional elements. This situation which maximizes μ_1 is penalized by μ_2. Hence, inter-cell flow efficiency μ_1 and inner-cell density μ_2 are inversely proportional such that each penalizes one extreme whereas the other improves.

Grouping efficiency, μ, is defined as the convex combination of inter-cell flow and inner-cell efficiences:

$$\mu = \alpha \times \mu_1 + (1-\alpha) \times \mu_2, \; \alpha \in [0, 1]. \tag{3}$$

The parameter α can be interpreted as an indication of whether inner-cell efficiences or inter-cell flows are more important to the decision maker. A large value of α gives more weight to exceptional elements. As α approaches unity, there emerges a tendency to eliminate all exceptional elements and the trivial cell formation solution turns out to be a single big cell, that is a job shop system. On the other hand, a very small α value indicates that inner-cell efficiency is more important than inter-cell flow efficiency, which makes a solution with high number of small-sized dense cells acquire the best value among all cell formation alternatives. That is the reason why moderate values of α are suggested to calculate the modified grouping efficiency.

Work-load balance measure, β, shows the degree of machine load balance in each cell. It is an important measure if other resources in a cell like personnel are shared. On the other hand, balanced cells have better performances especially in JIT systems. If all machines in each cell are evenly loaded, then the work-load balance index takes a value very close to one. This measure considers the squared machine work-load deviations from the mean cell work-load. If there are more than one individual machine of the same type in a cell, the squared deviation is adjusted accordingly. Furthermore, the squared deviation due to a machine type is weighed by the associated operation cost. The weighed squared deviations are first calculated for all machine types in each cell, then they are added and normalized. Hence this efficiency measure is defined as the normalized sum of the square of the weighed deviations between the mean cell load and individual machine loads:

$$\beta \doteq 1 - \left(\frac{\sum_{k=1}^{K} \sum_{i \in CM(k)} (MWL_{k,i} - MCL_k)^2 \times N_{k,i} \times OC_i}{10000 \times \sum_{k=1}^{K} \sum_{i \in CM(k)} N_{k,i} \times OC_i} \right)^{1/2}. \tag{4}$$

The third efficiency measure, $\bar{\gamma}$, shows the relative weighed *under-utilization* levels of the individual machines. Individual machine under-utilizations are multiplied by the associated annual operating costs and normalized to find the case dependent under-utilization index:

$$\gamma \doteq \frac{\Sigma_{k=1}^{K} \Sigma_{i \in CM(k)} UU_{k,i} \times OC_i}{\Sigma_{k=1}^{K} \Sigma_{i \in CM(k)} OC_i}. \tag{5}$$

This index includes the initial under-utilizations of machines before cell formation. The real under utilization index should indicate the amount of relative under-utilizations created due to cell formation. The last efficiency index is calculated as the complement of the relative utilization of the cells scaled by maximum utilization:

$$\bar{\gamma} \doteq 1 - \frac{1-\gamma}{1-\gamma_{JS}}, \tag{6}$$

where γ_{JS} is the initial under-utilizations of the machines in the original job shop. Actually, $\bar{\gamma}$ indicates the ratio between the reduction in average utilization due to Cellular Manufacturing (i.e. $\gamma - \gamma_{JS}$) divided by the original average utilization level (i.e. $1-\gamma_{JS}$).

The solution can be freed of exceptional elements by machine duplication and addition. In the case of machine duplication, the number of individual machines of type i assigned to cell k, $N_{k,i}$, is increased by one. This increases the field potential value of all parts assigned to cell k along row i. Consequently, zero entries in row i are penalized more in the denominator of equation (2), decreasing μ_2. When machine type i is added into the cell k, a new row is created. The zero elements in this row decrease μ_2. On the other hand, the presence of exceptional elements creates inter-cell work loads and clearly decreases μ_1. Both machine duplication or addition and creation of inter-cell flows do affect the mean percent work-loads. However, the influence on β is too complex to investigate analytically. In the case of machine addition due to an exceptional element belonging to part j, the numerator of equation (5) is increased by $OC_i \times (\lceil WL_{i,j} \rceil - WL_{i,j})$ and the denominator by OC_i. It may decrease as well as increase the under-utilization index $\bar{\gamma}$. If bottleneck machine i is duplicated then the under-utilization of that machine type is increased leading to an overall increase in γ, hence in $\bar{\gamma}$. If we keep the exceptional elements and create inter-cell work-loads, the numerator of equation (5) will be increased. Therefore, $\bar{\gamma}$ will be increased as well.

The scores of the three efficiency indices represent the quality of the cell formation solution on a quantitative scale. A number of examples showing the effectiveness of the indices are given in Figs 1–3. The cases in Fig. 1 represent solutions to six different but simple cell formation situations where parts, machines, and operations are assumed to be the same. In the example, it is also presupposed that each machine has a capacity of six operations. In the first two cases, the cell formation solution is a single cell, the original job shop. For the rest, there are two cells in the solutions: cell 1 contains the top two machine types and the leftmost three parts, whereas cell 2 is composed of the bottom three machine types and is dedicated to fabricate the rightmost three parts. In cases a, b, d and e, there are no exceptional elements. However, both cases c and f have two exceptional elements in their solutions. The cells formed in cases a, c and d are full-dense cells, whereas the other cases have solutions with relatively sparse cells. The effect of differences in machine types and parts on grouping efficiency values is investigated in Fig. 2 which contains four cases in which all operations are the same. The only difference is due to part and machine type weights, resulting in different grouping efficiency values. In the last example, the operations are different in the two cases but

$$\left[\begin{array}{cccccc} + & + & + & + & + & + \\ + & + & + & + & + & + \\ + & + & + & + & + & + \\ + & + & + & + & + & + \\ + & + & + & + & + & + \end{array}\right] \quad \left[\begin{array}{cccccc} + & & + & & + & \\ & + & & + & & + \\ + & & + & & + & \\ & + & & + & & + \\ + & & + & & + & \end{array}\right] \quad \left[\begin{array}{ccc|ccc} + & + & + & & + & \\ + & + & + & & & \\ \hline & & & + & + & + \\ & + & & + & + & + \\ & & & + & + & + \end{array}\right]$$

a) $K = 1$
$\mu_1 = 1$, $\mu_2 = 1$,
$\beta = 1$
$\gamma = 1$, $\bar{\gamma} = 0$

b) $K = 1$
$\mu_1 = 1$, $\mu_2 = 1/2$,
$\beta = 1$
$\gamma = 1/2$, $\bar{\gamma} = 0$

c) $K = 2$
$\mu_1 = 15/17$, $\mu_2 = 1$,
$\beta = 1$
$\gamma = 1/2$, $\bar{\gamma} = 2/17$

$$\left[\begin{array}{ccc|ccc} + & + & + & & & \\ + & + & + & & & \\ \hline & & & + & + & + \\ & & & + & + & + \\ & & & + & + & + \end{array}\right] \quad \left[\begin{array}{ccc|ccc} + & + & & & & \\ + & + & + & & & \\ \hline & & & + & + & + \\ & & & + & + & + \\ & & & & + & + \end{array}\right] \quad \left[\begin{array}{ccc|ccc} + & + & & & + & \\ + & + & + & & & \\ \hline & & & + & + & + \\ & + & & + & + & + \\ & & & & + & + \end{array}\right]$$

d) $K = 2$
$\mu_1 = 1$, $\mu_2 = 1$,
$\beta = 1$
$\gamma = 1/2$, $\bar{\gamma} = 0$

e) $K = 2$
$\mu_1 = 1$, $\mu_2 = 13/15$,
$\beta = 0.917$
$\gamma = 17/30$, $\bar{\gamma} = 0$

f) $K = 2$
$\mu_1 = 13/15$, $\mu_2 = 13/15$,
$\beta = 0.921$
$\gamma = 17/30$, $\bar{\gamma} = 2/15$

Figure 1. Some examples of efficiency values.

$$\left[\begin{array}{ccc|ccc} + & + & + & & + & \\ + & + & & & & \\ \hline & & & + & + & \\ & + & & + & + & + \\ & & & + & + & + \end{array}\right]$$

Case 1
$OC = (9, 3, 2, 3, 8)$
$TWLC = (1, 2, 8, 3, 4, 9)$
$\mu_1 = 0.938$
$\mu_2 = 0.873$

Case 2
$OC = (9, 3, 2, 3, 8)$
$TWLC = (1, 8, 2, 3, 9, 4)$
$\mu_1 = 0.818$
$\mu_2 = 0.958$

Case 3
$OC = (2, 9, 9, 3, 4)$
$TWLC = (1, 2, 8, 3, 4, 9)$
$\mu_1 = 0.938$
$\mu_2 = 0.583$

Case 4
$OC = (2, 9, 9, 3, 4)$
$TWLC = (1, 8, 2, 3, 9, 4)$
$\mu_1 = 0.818$
$\mu_2 = 0.853$

Figure 2. Effect of part and machine type weights on grouping efficiency values.

$$\underset{OC}{\left[\begin{array}{c} 30 \\ 20 \\ 40 \\ 60 \\ 50 \end{array}\right]} \quad \left[\begin{array}{ccc|ccc} .25 & .25 & .25 & & .25 & \\ .40 & .40 & & & & \\ \hline & & & .40 & .40 & \\ & .25 & & .25 & .25 & .25 \\ & & & .30 & .30 & .30 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} .05 & .05 & .05 & & .85 & \\ .40 & .40 & & & & \\ \hline & & & .40 & .40 & \\ & .85 & & .05 & .05 & .05 \\ & & & .30 & .30 & .30 \end{array}\right]$$

Case i
$\beta = 0.997$
$\gamma = 0.168$
$\gamma_{JS} = 0.085$
$\bar{\gamma} = 0.090$

Case ii
$\beta = 0.656$
$\gamma = 0.468$
$\gamma_{JS} = 0.085$
$\bar{\gamma} = 0.489$

Figure 3. Effect of work-loads on balance and under-utilization measures.

the machine type weights and the cell formation solution are the same. The effect on work-load balance and under-utilization measures is presented in Fig. 3.

The above explained efficiency indices are quite effective in evaluating the cell formation solutions as can be seen from the examples. All three efficiency indices are more powerful when applied to real life situations. These measures are simple and rough enough to use in the initial phases of the design process. The data for the calculation of the efficiency measures are present when a manufacturing system is to be designed. The standard times and operation sequences, rough estimates of annual demand figures and machine costs constitute the input to the initial phases of designing Cellular Manufacturing systems. Our efficiency indices make use of these data to determine the relative weights of parts, machine types and operations. Although originally developed for a Cellular Manufacturing environment, these indices can well be applied to other manufacturing environments.

3. Techniques evaluated

Some of the cell formation techniques cited in the literature have been developed by academic researchers while others have emerged as a result of practical applications. Moreover, computer implementation of earlier techniques is usually rather painstaking as they are not mathematically orientated. At any rate, most of the techniques proposed for the cell formation problem are of little value for they ignore performance criteria, which are important in achieving satisfactory solutions. What is more, most procedures for manufacturing cells generate different solutions to the same cell formation problem depending on the form of input.

A subset of six analytical cell formation techniques which require routeing information between machine types and parts is selected for a detailed analysis. Although these techniques consider neither possible routeing alternatives nor operation sequences, the resulting solutions are independent of any special block-diagonal structure embedded in the input data. As a result, each technique generates a unique solution to the same problem fed in different input formats. Moreover, they are computationally efficient. If a solution with a perfect block-diagonal structure is found possible, then all of these techniques will generate this ideal cell formation solution with absolutely no exceptional elements.

The techniques selected for further analysis and comparison are lattice–theoretic *comb*inatorial *gr*ouping (COMBGR), *mod*ified *r*ank *o*rder *c*lustering (MODROC), *m*achine-component *ce*ll formation (MACE), and *w*ithin-cell *u*tilization *b*ased *c*lustering (WUBC), *c*ost *a*nalysis *a*lgorithm (CAA), and *z*ero-*o*ne–*i*deal-*s*eed *a*lgorithm for *c*lustering (ZODIAC).

Of these, COMBGR and MACE consider only the machine grouping problem. COMBGR uses set inclusion and joint set union methods whereas MACE employs similarity coefficients. Subsequent to the identification of machine clusters, parts are assigned to the cells in both techniques. However, part and machine assignments are considered simultaneously and/or subsequently in MODROC, WUBC, CAA and ZODIAC. MODROC is a reordering method applied to the machine-part incidence matrix. WUBC and CAA are searching algorithms on the graph generated by parts and machines as vertices and routeing relationships as edges. ZODIAC is the only seed clustering technique reported in the literature.

COMBGR, MACE and MODROC, on the other hand, are hierarchical techniques. Initially candidate cells are generated and subsequently merged into larger cells. One common feature of these techniques is that the candidate cell decision of the first stage directly affects the final solution. If a machine-part pair is clustered initially in a particular cell, the assignment in the final solution will be the same cell. Other techniques are non-hierarchical in that a machine-part previously grouped could possibly be reassigned to a different cell. A summary of the features of these techniques is described next. For more detailed analysis and critique of these techniques, see Kandiller (1989).

COMBGR is a lattice-theoretic hierarchical grouping algorithm developed in Purcheck (1975), Olivia-Lopez and Purcheck (1979) and Purcheck 1984. The basic advantage of COMBGR is that it generates cell formation solutions without any exceptional elements. However, proposed cells are relatively large so they cause more or less the same drawbacks encountered in job shops. This algorithm divides parts into two classes–hosts and guests. Hosts are the parts whose machine set is not contained in the respective machine sets of any other part. Hosts constitute the minimal independent set in terms of routeing relations. Initially, cells are identified in such a way that each host represents a candidate cell. In other words, a candidate cell is constructed by clustering all machines used by the corresponding host. Candidate cells are merged successively until the original job shop is obtained. Each merge iteration creates an alternative cell formation solution.

MACE is another hierarchical, machine-grouping cell formation technique (Waghodekar and Sahu 1984). It uses measures of similarity showing the degree to which a set of parts can be processed on a pair of machines. MACE groups machines only if their similarity coefficient is greater than a prespecified value. During the first stage, the technique generates initial machine clusters, each representing a candidate cell. Consequently, the candidate cells are successively joined by using similarity coefficients. The last stage of MACE involves part assignments to the final cells.

MODROC is based on the original Rank Order Clustering (ROC) technique (King 1980). The ROC algorithm treats each row or column of the machine-part incidence matrix as a binary word. Integer equivalents of binary words are calculated and rows and columns are reordered successively in descending order of integer equivalents. A ROC iteration consists of a row reordering followed by a column reordering. Those iterations are terminated when no change is encountered. This algorithm was altered (King and Nakornchai 1982) by utilizing a new data structure and a new sorting mechanism. Yet another modification was offered later by Chandrasekharan and Rajagopalan (1986b). They used King's iterations twice to obtain an incidence matrix containing a rectangular block of 'ones' at its top-left corner. This rectangular block represents a candidate cell. The corresponding columns of the candidate cell are all eliminated from the incidence matrix and the procedure resumes. After all candidate cells are formed, they are merged successively until the final solution is attained. A similarity coefficient method is applied to assist this merging process.

WUBC is a graph searching cell formation technique (Ballakur and Steudel 1987). WUBC induces a breadth-first search on the graph generated by the routeing relationships between parts and machine types. A *key* machine type is selected as the root in the search and all parts routed through the key machine type are examined. These parts are either admitted to the cell generated by the key machine type or remain in their previously assigned cells. The parts that are not previously assigned are automatically included in the cell when examined for the first time. Consequently, all machine types related to the admitted parts are examined in this process. Machine types are added to the cell if their within-cell work-loads due to the parts already assigned exceed a prespecified level. Upon completion of each search, the required number of machines of each type is allocated to the cell based on the within-cell utilizations. Next, another search is initiated after selecting a new key machine type among the remaining machines. Finally, a *remainder cell* is formed by bringing all left-over machines together. All the previously assigned parts may be reassigned to the remainder cell provided that it is not empty. Conversely, unassigned parts, if any, are included in the remainder cell.

Like WUBC, CAA (Kusiak and Chow 1987) is a graph searching cell formation technique. However, it focuses on parts rather than machines in defining the root during the search process. CAA initiates a breadth-first search on the machine-part graph. Each search identifies a different cell. During the search, an admit/reject decision is made for all parts except for the root. A rejection eliminates the part under consideration from the analysis. A part is admitted to the cell unless it increases the number of machine types above a prespecified value. The search continues until there are no parts to be assigned to the cell. The next cell is constructed by taking another key part as the root. This process ceases when all the parts have been either assigned to a cell rejected from the analysis.

ZODIAC is a seed clustering cell formation technique (Chandrasekharan and Rajagopalan 1986a, 1987). The parts and machine types are treated independently in the initial phase. Rows of the machine-part incidence matrix represent machine types in binary vector format. Similarly, the binary vector for a specific part can be obtained from the corresponding column. Parts and machine types are clustered separately by means of seeds, where a seed is a binary vector. Part and machine clusters are then assigned to each other by the use of similarity coefficients. Consequently, each assignment produces a cell.

In addition to the evaluation of these techniques, various modifications and extensions to the original algorithms are made in order to bring them into the same state so that a sound comparative analysis becomes possible. To illustrate, most of the selected techniques are designed to operate on the incidence matrix, which means they do not recognize individual machines of the same type. These techniques are altered to enjoy the advantage of using work-load and cost information (Kandiller 1989). Other modifications and extensions are as follows:

COMBGR's set partitioning scheme is modified to improve the quality and number of alternative cell formation solutions. COMBGR joins each candidate cell with at least one other cell, and this puts an artificial condition on the candidate cell in each merging iteration. Besides, it is sometimes beneficial to keep a candidate cell untouched. Part assignments are not considered in the original COMBGR. Hence, we propose a part assignment scheme for COMBGR which is optimal in terms of grouping efficiency when all parts, machines and operations are identical. COMBGR performs merging iterations sequentially until the original shop is obtained. In the meantime, the inner-cell efficiency value of the alternative solutions is getting worse since the proposed cells are getting larger. There is no need to iterate further if the machine requirements for all types are met. This can be considered as a sensible stopping criterion. Another stopping criterion we suggest is the minimum machine-difference between the candidate cells to be merged. If this value exceeds a prespecified threshold, the merging of candidate cells is interrupted. This threshold value should be determined by taking the trade-off between extra investment and inner-cell densities into account.

After initial candidate cells are identified, MODROC merges one pair of candidate cells at a time. Therefore, MODROC's cell formations during the final iterations usually contain one large cell and a number of small ones. Naturally, the big cell attracts nearby small cells, which reduces the quality of cell formation since the resulting large cell decreases the inner-cell density. The following merging scheme is proposed to overcome this drawback and to reduce the total number of merging iterations. All pairs of cells with similarity coefficient values greater than a specified value are picked in each merging iteration so that more than one cell-pair merging take place. The threshold for similarity coefficients can be determined as a prespecified percentage of the maximum similarity value of the current pairs. If the number of independent pairs of candidate cells exceeds another prespecified value such as a percentage of number of current candidate cells, the threshold is increased for the current iteration. Each merging iteration generates an alternative cell formation. As in the case of COMBGR, we offer the grouping efficiency measure as the sole criterion for picking the best cell formation solution alternative pertaining to the machine availabilities.

The part assignment scheme suggested by MACE leads to a large number of exceptional elements. Therefore, a superior part assignment scheme using work-load cost fractions is proposed. The work-load cost fraction of a part is the percentage of its total work-load cost that is allocated to its assigned cell. For each part, the cell with the highest work-load cost fraction is selected for assignment. Two threshold values for similarity coefficients are introduced in merging machines. After the pair with maximum similarity is picked, two passes are made to configure the candidate cell. If the similarity values between a candidate machine and the selected pair are higher than the first threshold value, the candidate machine is joined to the cell identified by the selected pair. This operation is carried out for all the machines other than the selected pair. During the second pass, candidates with similarities to all machine types in the current composition of the cell exceeding the second threshold value are joined. The next pair is chosen among the remaining candidate cells and the two passes are carried out again. The MACE technique can alternatively use three different similarity coefficients whose individual effects are explained in the above mentioned study. The original version of MACE refers to the machines of the cells without any part assignments as 'blocking machines'; nevertheless, it lacks a convenient method to handle them. Hence, such a method is also suggested.

When a part is reassigned by WUBC, work-loads of the related machine types in the part's previous cell are affected. A decrease in the work-load of a machine type might give rise to a decrease in the number of machines of that type, even to the removal of the type. This influences the cell-parts still having loads on the removed machines. If such a part has operations in another cell, it will naturally be reassigned. The new part assignment could lead to new work-load decreases. Therefore, WUBC is altered accordingly. Moreover, alternative measures for selecting either the key machine type or part assignments are evaluated. The machine type with the highest total work-load is found to be the best candidate for the key machine type and the cell in which a part has the maximum work-load percentage is found to be the best cell to assign. The same study reveals that 0·60 is the best value for the cell admission factor.

As for CAA, an alternative rule to select the key part is suggested. The part with the highest total work-load cost is offered as the key part. CAA expands cells only by examining the costs of allowable non-key parts which do not increase the cell size above an upper limit. This method usually prevents similar parts from joining the cell. Therefore, the following method is introduced. First, allowable non-key parts are examined for extra machine requirements. If the number of machines needed for a non-key part is less than the cell admission factor, then that part is considered as a candidate. The cell admission factor is defined as the ratio of the maximum number of extra machines needed to the current cell size. Secondly, the set of machine requirements for each candidate part is examined to see whether it contains the machine requirement set of other candidates. The cost of such candidates should be taken into account whenever a non-key part is selected for cell admission. After each cell is formed, the required number of individual machines of each type is calculated. If there are any remaining machines, then their corresponding types might be considered in the formation of future cells to prevent some parts from being rejected totally. Rejected parts are suggested to be reassigned to the already formed cells. For the assignment, the cells with the maximum work-load are proposed. Furthermore, the effects of the cell admission factor and the cell size upper limits are investigated. Results indicate that a cell admission factor of 0·20 is satisfactory, and the upper limit on the cell size should be of the minimum value which is the number of 'ones' in the most dense column of the incidence matrix.

ZODIAC chooses an arbitrary representative seed from each group, which may fail to represent the corresponding cluster. The most dense binary vector in each cluster is offered as the first representative seed. The remaining representative seeds can be determined in such a way that they will be distant from all current seeds. First, the candidates with the maximum distance from all the seeds become representative seeds. The maximum distance is controlled by machine difference factor, and this factor is decreased by a threshold percentage for the next set of representative seeds. Moreover, the steps of ZODIAC are resynchronized to improve the quality of solutions and to accelerate the algorithm.

Furthermore, the selected techniques make use of fine tuning variables such as threshold values. With the help of the efficiency indices described in the previous section, the alternatives are evaluated and the best combinations of fine tuning values are identified. Yet the reader should refer to the original study (Kandiller 1989) for a detailed analysis of the extensions, modifications and the improvements offered. The modified algorithms used in the analysis are listed in the Appendix.

4. Problem generator

In order to provide a critical analysis of the cell formation techniques, the evaluation should be based on common test problems representing real life situations rather than specific, small-sized test problems. A random problem generator is used to evaluate and compare the existing cell formation techniques. The problem generator initially produces a machine-part incidence matrix and then generates operation sequences and standard times for each and every part. Thereafter, annual operating costs, availabilities of machine types, and annual demands of parts, which determine the work-load matrix, are created. The number of individual machines of each type and total work-load cost of each part are computed from the work-load matrix. Finally, the efficiency measures for the generated shop are calculated.

The incidence matrix indicates the size and the density of the original shop, and the place of the generated shop in the *manufacturing spectrum*. The manufacturing spectrum has two boundaries: ideal job shop and ideal CM shop. In an ideal job shop, all parts do use all machines. On the other hand, ideal CM shop involves mutually separated and therefore independent cells. Four parameters are required to construct a machine-part incidence matrix. The first two, the number of parts and the number of machine types in the system, are *size* parameters showing the size of the shop to be generated. The other two are *shape* parameters. One is the density which is actually the ratio of 'ones' in the incidence matrix to the total area. The last parameter is clumpiness which identifies the location of the generated shop on the manufacturing spectrum. The clumpiness parameter shows the degree of possible block-diagonalization in the machine-part incidence matrix. This parameter always takes positive values. An ideal job shop can be realized by specifying the density parameter as one. Ideal CM systems can be generated by assigning low values such as 0·10 to the density parameter, and high values such as 1000 to the clumpiness parameter.

A detailed analysis of the clumpiness parameter requires the definition of inner-cell density and off-diagonal density. These definitions are used only in explaining the random problem generation module, and they are not general definitions that apply to cell formation problems. Let,

T = number of machine types (number of rows of incidence matrix),
P = number of parts (number of columns of incidence matrix),
k = number of imaginary cells (number of possible diagonal blocks in incidence matrix),
c = clumpiness parameter,
d = overall density of the shop (density of incidence matrix),
d_I = inner-cell density of imaginary cells (density of block-diagonal area in incidence matrix),
d_O = density of exceptional elements (density of off-diagonal area in incidence matrix),
$\bar{t}$ = average number of different machine types per cell,
$\bar{p}$ = average number of different parts per cell.

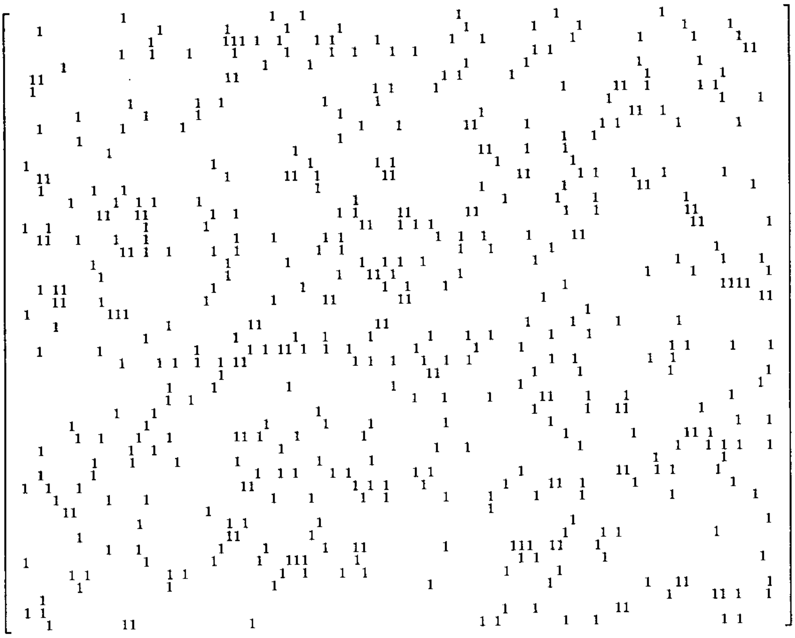
The number of 'ones' in the incidence matrix, E, can be calculated in the following two ways:

$$E = T \times P \times d,$$

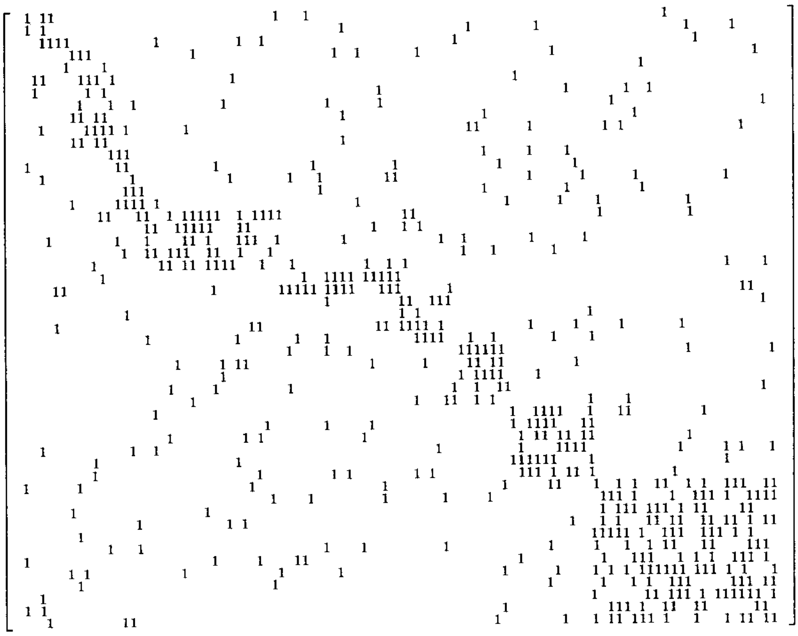
$$E = (k \times \bar{t} \times \bar{p}) \times d_I + ((T \times P) - (k \times \bar{t} \times \bar{p})) \times d_O.$$

c	d	d_O	d_I	c	d	d_O	d_I
	0·10	0·1000	0·1000		0·10	0·0166	0·8500
1	0·15	0·1500	0·1500	6	0·15	0·0250	0·8583
	0·20	0·2000	0·2000		0·20	0·0333	0·8687
	0·10	0·0500	0·5500		0·10	0·0143	0·8714
2	0·15	0·0750	0·5750	7	0·15	0·0214	0·8786
	0·20	0·1000	0·6000		0·20	0·0286	0·8857
	0·10	0·0333	0·7000		0·10	0·0125	0·8875
3	0·15	0·0500	0·7167	8	0·15	0·0188	0·8938
	0·20	0·0667	0·7333		0·20	0·0250	0·9000
	0·10	0·0250	0·7750		0·10	0·0111	0·9000
4	0·15	0·0375	0·7875	9	0·15	0·0167	0·9056
	0·20	0·0500	0·8000		0·20	0·0222	0·9111
	0·10	0·0200	0·8200		0·10	0·0100	0·9100
5	0·15	0·0300	0·8300	10	0·15	0·0150	0·9150
	0·20	0·0400	0·8400		0·20	0·0200	0·9200

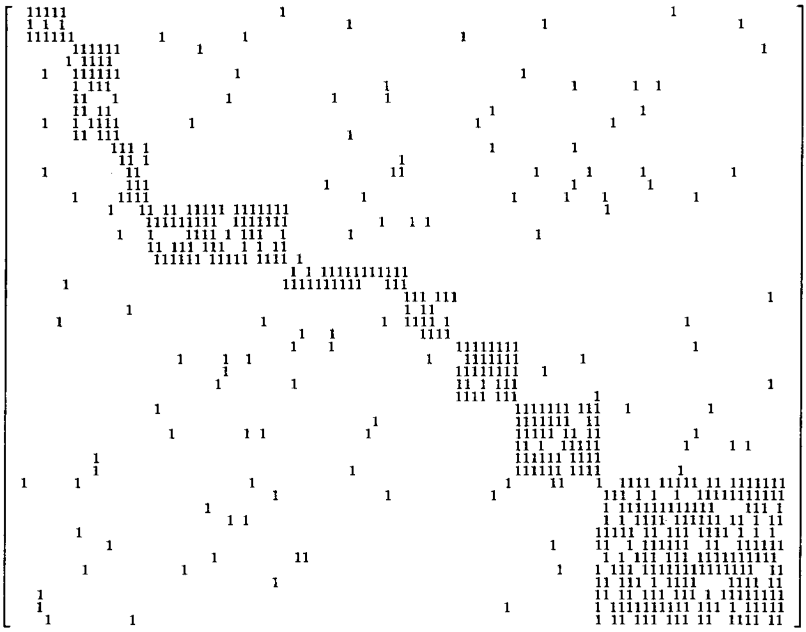
Table 1. Effect of shape parameters on inner-cell and off-diagonal densities.



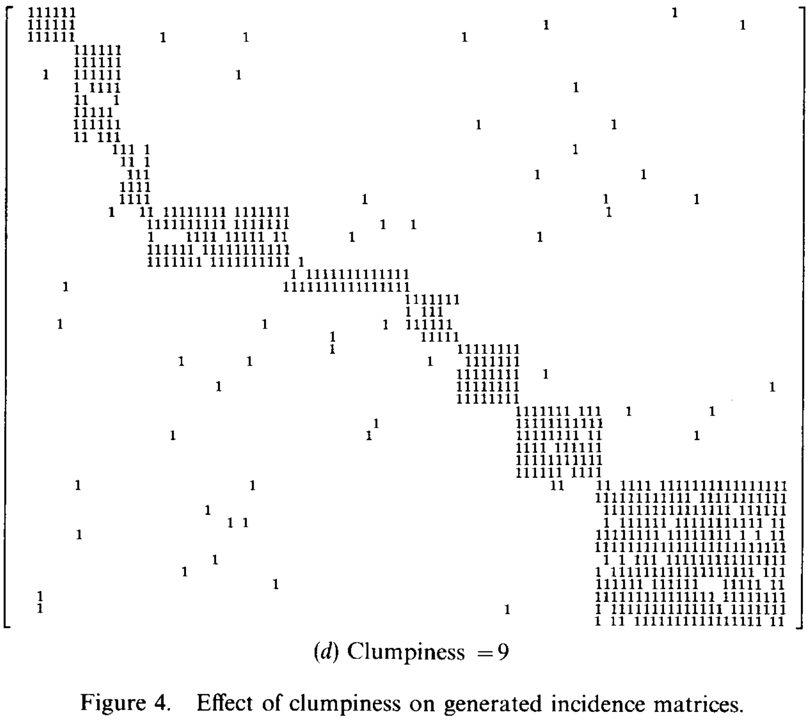
(*a*) Clumpiness = 1



(*b*) Clumpiness = 2



(*c*) Clumpiness = 4



(*d*) Clumpiness = 9

Figure 4. Effect of clumpiness on generated incidence matrices.

If full block diagonal machine part incidence matrix is square, the overall density d is exactly the inverse of the number of diagonal blocks k. With this particular situation in mind, if we assume that the number of imaginary cells is inversely proportional to the overall density, i.e. $k=\frac{1}{d}$, from the above equations we have

$$d=d\times d_I+(1-d)\times d_O,$$

which can be solved for d_O and d_I by introducing a new parameter c as:

$$d_O=\frac{d}{c}, \qquad d_I=1-\frac{1-d}{c}.$$

Thus, the clumpiness parameter and the overall density give rise to inner-cell and off-diagonal densities. The effect of the shape parameters on the densities is shown in Table 1. It can be observed from the table than the effect of the clumpiness parameter on both of the individual densities is more significant than that of the overall density. Some examples of the generated incidence matrices in relation to various clumpiness values are given in Fig. 4.

Once density and clumpiness values and the sizes of the problem are set, the incidence matrix can well be generated according to inner-cell and off-diagonal densities. After determining the number of imaginary cells k, the mean number of different machine types ($\bar{t}=T\cdot d$) and parts ($\bar{p}=P\cdot d$) in each imaginary cell is calculated. Then the imaginary diagonal blocks are generated in random with sizes around these mean values without changing T and P limits. For each entry of the incidence matrix, a random number between 0 and 1 is generated, and if it is less than the inner-cell (or off-diagonal) density then the element will be 1; if not, it will be 0. Finally, the rows and columns of the incidence matrix are permuted randomly to eliminate a trivial cell formation solution.

Subsequent to the incidence matrix, a corresponding work-load matrix is formed. At this stage, standard times and annual demands of parts are generated. In the meantime, annual available capacities of machine types are produced. Afterwards, the work-load matrix is computed. Finally, the number of machines of each type and total work-load costs of each part are calculated from the work-load matrix.

5. Experiments performed

In the present study, the performance of the selected cell formation techniques are tested under randomly generated conditions. In order to reduce the size of the experiment, some input parameters of the generator are fixed. The varying factors in the experiment are the number of parts, density of the shop, clumpiness, demand of parts, and machine operating costs.

A summary of the factors and their levels in the experimentation is presented in Table 2. Working of the techniques under different problem sizes are analysed by setting the number of parts to two levels. Both of the shape parameters which affect the type of the shop being generated are altered at different levels. The selected densities are based on the results of example problems and specific implementations in the literature. Densities between 0·10 and 0·20 are found to represent different scenarios adequately. Small values for the clumpiness parameter change the shape of incidence matrices substantially. However, the greater the clumpiness value, the smaller the change in the shape. Therefore, the greater the clumpiness value, the smaller the change in the shape. Therefore, the levels considered are 1, 2, 4 and 9. The effect of these values on a specific

	Factors	Values		#
Size parameters	Number of machine types	$T=50$		1
	Number of parts	$P=100, 150$		2
Shape parameters	Density	$d=0{\cdot}10, 0{\cdot}15, 0{\cdot}20$		3
	Clumpiness	$c=1, 2, 4, 9$		4
Distribution parameters	Annual part demands	High variance	$U[50; 4050]$	
		Low variance	$U[1050; 3050]$	2
	Annual machine operating cost	High variance	$U[100; 100100]$	
		Low variance	$U[25050; 75050]$	2
Total number of selected PF/MG-F techniques		6		

Table 2. Factors of the experiment.

Algorithm	Factor	Value
COMBGR	Maximum machine difference limit	7
MODROC	Lower limit on similarities	0·75
	Upper limit on independent pairs	5
MACE	Threshold	0·10
	Job shop like	*SCTF*
	Intermediate	PSC
	Ideal CM like	SC
ZODIAC	Weighting factor	0·50
	Threshold value	7
WUBC	Cell admission factor	0·60
	Cell size upper limit	50
	Key machine selection rule	A4
	Part assignment rule	B2
CAA	Cell admission factor	0·20
	Extra factor on cell size limit	0

Table 3. Best fine tuning values of cell formation techniques.

incidence matrix is illustrated in Fig. 4. Finally, both demand and operating cost factors are analysed at two levels, high variance and low variance.

Each of the six selected techniques requires specific parameter settings. The best fine-tuned values of these parameters are shown in Table 3. In the evaluation process, each technique is analysed under 96 different scenarios.

Ten different statistically independent cell formation problems are generated for each combination of the factors. Hence, the number of runs in the experimentation amounts to 5760:

$$10 \times 6 \times 2 \times 3 \times 4 \times 2 \times 2 = 5760.$$

6. Results and discussion

For each combination of the factors, the work-load balance index, the under-utilization index, and the modified grouping efficiency index, which is determined by inter-cell flow efficiency and inner-cell density, are computed. The analyses of variance (ANOVA) for all efficiency measures are listed in Tables 4–7. The significance level for all tests is 0·05. In the tables, only statistically significant factors are shown. ANOVA tables for each measure are analysed as follows.

Source	Sum of square	df.	Mean square	F	$F_{df,\infty}$
x	1137001·922	5	227400·384	13918·841	2·21
y	11700·834	1	11700·834	716·191	3·84
z	279·948	1	279·948	17·135	3·84
u	3042·076	2	1521·038	93·100	3·00
v	261906·236	3	87302·079	5343·631	2·60
xy	14068·955	11	1278·996	78·285	1·79
xz	928·395	11	84·400	5·166	1·79
xu	124615·210	17	7330·306	448·677	1·63
xv	4123291·605	23	179273·548	10973·068	1·53
yu	2746·815	5	549·363	33·626	2·21
yv	1194·537	7	170·648	10·445	2·01
uv	32865·258	11	2987·751	182·876	1·79
xyu	12211·712	35	348·906	21·356	1·42
xyv	9997·868	47	212·721	13·020	1·37
xuv	151836·510	71	2138·542	130·897	1·30
yuv	8223·912	23	357·561	21·886	1·53
$xyuv$	35089·307	143	245·380	15·019	1·20
Error	84677·753	5183	16·338	1·00	1·00

x, Algorithms; y number of parts; z, operating cost; t, demand; u, density; and v, clumpiness.

Table 4. Analysis of variance table for inter-cell flow.

Source	Sum of square	df.	Mean square	F	$F_{df,\infty}$
x	2538027·762	5	507605·552	22837·396	2·21
y	1520·752	1	1520·752	68·419	3·84
z	587·645	1	587·645	26·438	3·84
u	50185·288	2	25092·644	1128·929	3·00
v	388971·227	3	129657·076	5833·329	2·60
xy	1718·845	11	156·259	7·030	1·79
xu	100569·687	17	5915·864	266·157	1·63
xv	5607977·421	23	243825·105	10969·799	1·53
yz	187·819	3	62·606	2·817	2·60
yu	747·498	5	149·500	6·726	2·01
yv	5985·449	7	855·064	38·470	2·21
tv	536·264	7	76·609	3·447	2·01
uv	20299·418	11	1845·402	83·025	1·79
xyu	14227·483	35	406·500	18·289	1·42
xyv	11140·386	47	237·029	10·664	1·37
xuv	106817·481	71	1504·472	67·687	1·30
yuv	4246·403	23	184·626	8·306	1·53
$xyuv$	27962·479	143	195·542	8·798	1·20
Error	115202·254	5183	22·227	1·000	1·00

x, Algorithms; y, number of parts; z, operating cost; t, demand; u, density; and v, clumpiness.

Table 5. Analysis of variance table for inner-cell density.

The ANOVA analysis for inter-cell flow is presented in Table 4. According to the F-test, the main effects of all the factors except for the demand variation are found to be significant. The main effects of the algorithms and clumpiness are the most significant whereas the effect of the operating cost variation is the least. Furthermore, all 2-, 3- and 4-way interaction of the algorithms, the number of parts, density and clumpiness are statistically significant.

The same conclusions can be drawn for inner-cell density as can be seen from Table 5. However, the effect of the operating cost variation is less significant than that of inter-cell flow measure. One should notice the small error sum of squares in the two parts of the modified grouping measure. This indicates that the factors and their interactions explain 96·6% of the variability in inter-cell flow, and 97·2% in inner-cell density measure.

According to the ANOVA analysis given in Table 6, the main effects of all the factors except for the operating cost on the work-load balance measure are considerably significant. The main effects of the algorithms, the number of parts and density are the most significant ones. Another source of variation is due to the cross effect of the algorithms and clumpiness. The other significant factors detected by means of the F-test have relatively small effects on work-load balances. The factors and their indicated interactions explain only 80·5% of the variability in the randomly generated problems.

The ANOVA analysis for under-utilization is presented in Table 7. The majority of the rows are found to be statistically significant. All of the 1-, 2-, 3- and even 4-way interactions of the algorithms, the number of parts, density and clumpiness have a considerable variation effect on the under-utilization measures. In addition, demand × density and demand × density × clumpiness factors are remarkable. The factors and associated interactions explain 92·7% of the variability in under-utilization values.

The results of pairwise comparisons among the factor levels are obtained by using Sheffée-type simultaneous 95% confidence intervals. Since the number of parts, machine operating costs and demand factors have only two levels, the existing sources of variations are clearly due to these two. Table 8 presents the estimated mean

Source	Sum of square	df.	Mean square	F	$F_{df,\infty}$
x	159061·038	5	31812·208	3320·729	2·21
y	4894·913	1	4894·913	510·957	3·84
t	150·324	1	150·324	15·692	3·84
u	3650·322	2	1825·161	190·520	3·00
v	812·572	3	270·857	28·274	2·60
xy	1816·069	11	165·097	17·234	1·79
xu	6400·994	17	376·529	39·304	1·63
xv	735071·386	23	31959·625	3336·117	1·53
yu	164·150	5	32·830	3·427	2·21
uv	655·938	11	59·631	6·225	1·79
xyv	817·853	47	17·401	1·816	1·37
xuv	4340·320	71	61·131	6·381	1·30
yuv	533·465	23	23·194	2·421	1·53
Error	49652·559	5183	9·580	1·000	1·00

x, Algorithms; y, number of parts; z, operating cost; t, demand; u, density and v, clumpiness.

Table 6. Analysis of variance table for workload balance.

Source	Sum of square	df.	Mean square	F	$F_{df,\infty}$
x	156519·064	5	31303·813	3932·899	2·21
y	38542·999	1	38542·999	4842·405	3·84
z	146·945	1	146·945	18·462	3·84
t	100·238	1	100·238	12·593	3·84
u	25083·095	2	12541·548	1575·675	3·00
v	55604·223	3	18534·741	2328·639	2·60
xy	11131·329	11	1011·939	127·136	1·79
xz	602·981	11	54·816	6·887	1·79
xt	687·626	11	62·511	7·854	1·79
xu	9886·806	17	581·577	73·067	1·63
xv	611978·783	23	26607·773	3342·906	1·53
yu	3810·705	5	762·141	95·753	2·21
yv	2973·707	7	424·815	53·372	2·01
zv	209·038	7	29·863	3·752	2·01
tu	1685·847	5	337·169	42·361	2·21
tv	262·909	7	37·558	4·719	2·01
uv	17795·541	11	1617·776	203·252	1·79
xyt	661·150	23	28·746	3·612	1·53
xyu	12371·598	35	353·474	44·409	1·42
xyv	8731·090	47	185·768	23·339	1·37
xtv	1297·532	47	27·607	3·468	1·37
xuv	26812·639	71	377·643	47·446	1·30
ytv	405·593	15	27·040	3·397	1·67
yuv	3380·852	23	146·994	18·468	1·53
zuv	298·341	23	12·971	1·630	1·53
tuv	2498·887	23	108·647	13·650	1·53
$xytu$	970·852	71	13·674	1·718	1·30
$xytv$	1922·754	95	20·240	2·543	1·26
$xyuv$	35720·473	143	249·794	31·383	1·20
$xzuv$	1379·883	143	9·650	1·212	1·20
$xtuv$	2122·083	143	14·840	1·864	1·20
$ytuv$	979·439	47	20·839	2·618	1·37
$xytuv$	3020·234	287	10·523	1·322	1·16
Error	41253·956	5183	7·959	1·000	1·00

x, Algorithm; y, number of parts; z, operating cost; t, demand; u, density; and v clumpiness.

Table 7. Analysis of variance table for under-utilization.

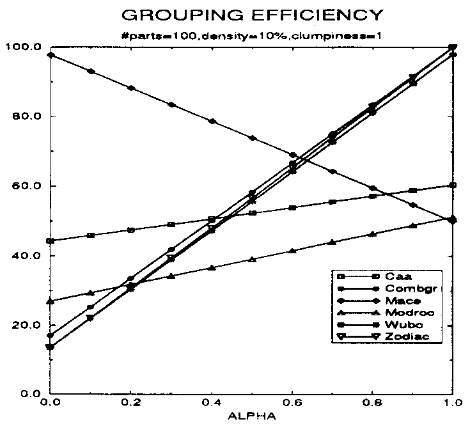
differences $\hat{L}_{i,j}$ for the factor levels i and j. The 95% confidence interval for these mean differences can be obtained from $\hat{L}_{i,j} \pm \varepsilon$, where ε stands for the error term and is given in the parentheses for each factor. As an example, the mean inter-cell flow index difference between COMBGR and MODROC is considered. From the table, the 95% confidence interval for this difference is $26{\cdot}938 \pm 0{\cdot}808$ which is (26·130, 27·746). Since the confidence interval does not contain zero, a statistical difference is observed between the inter-cell flow efficiency means of COMBGR and MODROC. The other factors at each efficiency measure can be interpreted similarly. From this analysis, it is observed that only few pairwise comparisons prove insignificant and these are indicated in boldface letters in the table.

As a summary, the main sources of variation in all of the efficiency measures are due to the algorithms, clumpiness, the number of parts, density and their interactions. The ANOVA tables indicate that the levels of these factors and their interactions do have

Algorithm	Inter-cell flow (0·809)	Inner-cell density (0·945)	Work-load (0·619)	Under-utilization (0·567)
COMBGR versus MODROC	100·000 − 73·062 = 26·938	31·417 − 48·000 = −16·584	72·864 − 86·438 = −13·573	21·729 − 8·125 = 13·604
COMBGR versus WUBC	100·000 − 96·010 = 3·990	31·417 − 22·896 = 8·521	72·864 − 86·417 = −13·552	21·729 − 6·073 = 15·656
COMBGR versus CAA	100·000 − 76·260 = 23·740	31·417 − 58·438 = −27·021	72·864 − 78·458 = −5·594	21·729 − 15·021 = 6·708
COMBGR versus MACE	100·000 − 64·385 = 35·615	31·417 − 73·510 = −42·094	72·864 − 86·646 = −13·781	21·729 − 16·292 = 5·438
COMBGR versus ZODIAC	100·000 − 64·938 = 35·062	31·417 − 81·240 = −49·823	72·864 − 85·458 = −12·594	21·729 − 14·635 = 7·094
MODROC versus WUBC	73·062 − 96·010 = −22·948	48·000 − 22·896 = 25·105	86·438 − 86·417 = 0·021	8·125 − 6·073 = 2·052
MODROC versus CAA	73·062 − 76·260 = −3·198	48·000 − 58·438 = −10·437	86·438 − 78·458 = 7·979	8·125 − 15·021 = −6·896
MODROC versus MACE	73·062 − 64·385 = 8·677	48·000 − 73·510 = −25·510	**86·438 − 86·646 = −0·208**	8·125 − 16·292 = −8·167
MODROC versus ZODIAC	73·062 − 64·938 = 8·125	48·000 − 81·240 = −33·239	86·438 − 85·458 = 0·979	8·125 − 14·635 = −6·510
WUBC versus CAA	96·010 − 76·260 = 19·750	22·896 − 58·438 = −35·542	86·417 − 78·458 = 7·958	6·073 − 15·021 = −8·948
WUBC versus MACE	96·010 − 64·385 = 31·625	22·896 − 73·510 = −50·615	**86·417 − 86·646 = −0·229**	6·073 − 16·292 = −10·219
WUBC versus ZODIAC	96·010 − 64·938 = 31·073	22·896 − 81·240 = −58·344	86·417 − 85·458 = 0·958	6·073 − 14·635 = −8·563
CAA versus MACE	76·260 − 64·385 = 11·875	58·438 − 73·510 = −15·073	78·458 − 86·646 = −8·187	15·021 − 16·292 = −1·271
CAA versus ZODIAC	76·260 − 64·938 = 11·323	58·438 − 81·240 = −22·802	78·458 − 85·458 = −7·000	**15·021 − 14·635 = 0·385**
MACE versus ZODIAC	**64·385 − 64·938 = −0·552**	73·510 − 81·240 = −7·729	86·646 − 85·458 = 1·187	16·292 − 14·635 = 1·656
No. of parts	†	†	†	†
100 versus 150	77·684 − 80·535 = −2·851	52·070 − 53·097 = −1·027	81·792 − 83·635 = −1·843	16·233 − 11·059 = 11·059
Operating cost	†	†	‡	†
Low versus High	79·330 − 78·889 = 0·441	52·903 − 52·264 = 0·639	82·708 − 82·719 = −0·011	13·486 − 13·806 = −0·320
Demand	‡	‡	†	†
Low versus High	79·111 − 79·108 = 0·003	52·625 − 542·264 = 0·083	82·552 − 82·875 = −0·323	13·778 − 13·514 = 0·264
Density	(0·306)	(0·358)	(0·235)	(0·214)
0·10 versus 0·15	80·073 − 78·937 = 1·135	48·672 − 53·276 = −4·604	81·630 − 82·989 = −1·359	16·531 − 12·740 = 3·792
0·10 versus 0·20	80·073 − 78·318 = 1·755	48·672 − 55·802 = −7·130	81·630 − 83·521 = −1·891	16·531 − 11·667 = 4·865
0·15 versus 0·20	78·937 − 78·318 = 0·620	53·276 − 55·802 = −2·526	82·989 − 83·521 = −0·531	12·740 − 11·667 = 1·073
Clumpiness	(0·475)	(0·555)	(0·363)	(0·331)
1 versus 2	70·840 − 71·840 = −1·000	42·354 − 43·528 = −1·174	83·306 − 82·569 = 0·736	18·486 − 14·979 = 3·507
1 versus 4	70·840 − 83·007 = −12·167	42·354 − 56·576 = −14·222	83·306 − 82·486 = 0·820	18·486 − 11·708 = 6·778
1 versus 9	70·840 − 90·750 = −19·910	42·354 − 67·875 = −25·521	83·306 − 82·493 = 0·813	18·486 − 9·410 = 9·077
2 versus 4	71·840 − 83·007 = −11·167	43·528 − 56·576 = −13·049	**82·569 − 82·486 = 0·084**	14·979 − 11·708 = 3·271
2 versus 9	71·840 − 90·750 = −18·910	43·528 − 67·875 = −24·347	**82·569 − 82·493 = 0·077**	14·979 − 9·410 = 5·569
4 versus 9	83·007 − 90·750 = −7·743	56·576 − 67·875 = −11·299	**82·486 − 82·493 = −0·007**	11·708 − 9·410 = 2·299

† Mean difference of the two factor levels is significant; and ‡ mean difference of the two factor levels is insignificant.

Table 8 Estimated mean differences for the factor levels.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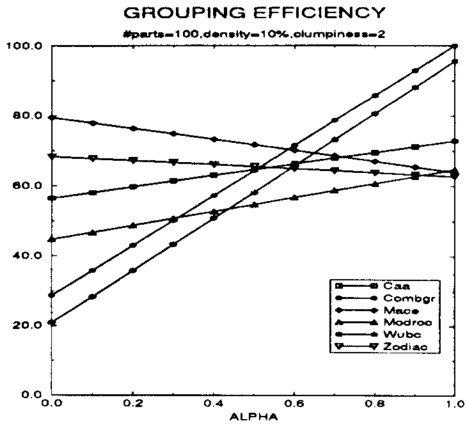
ALPHA
Caa
Combgr
Mace
Modroc
Wubc
Zodiac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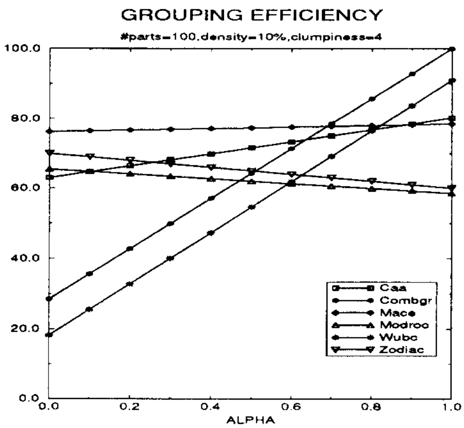
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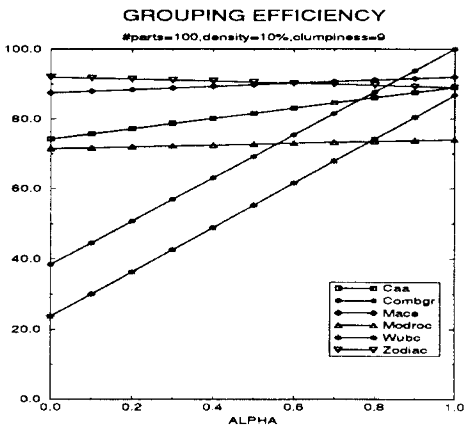
Zodiac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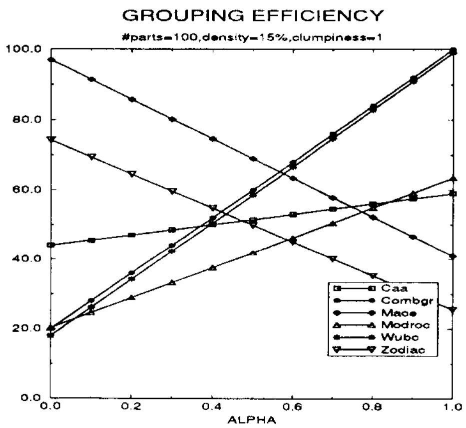
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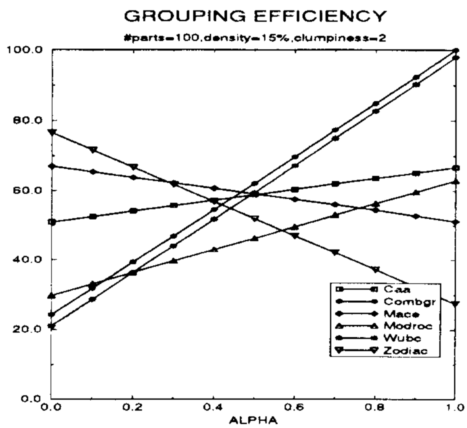
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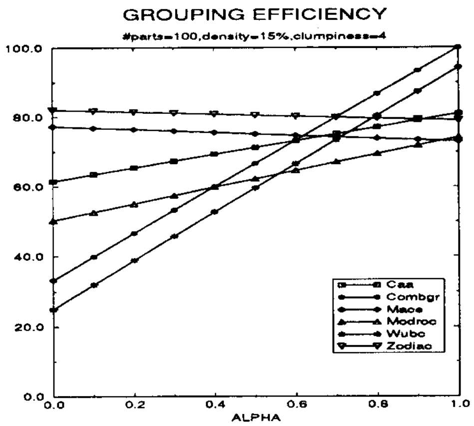
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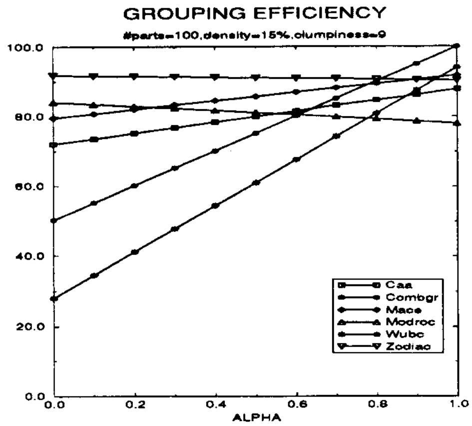
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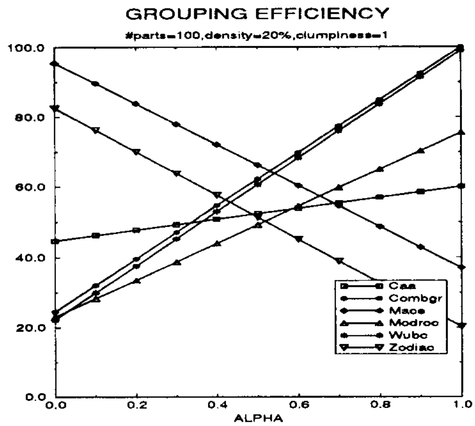
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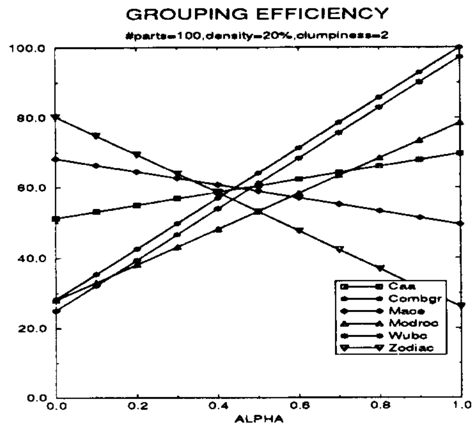
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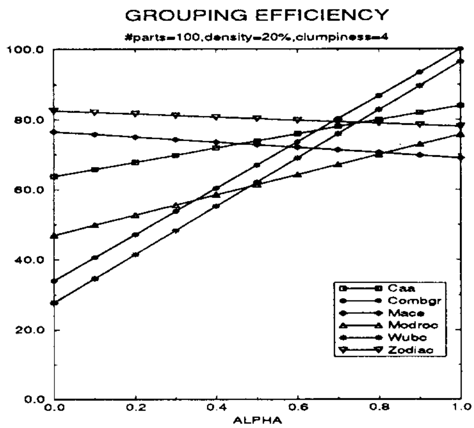
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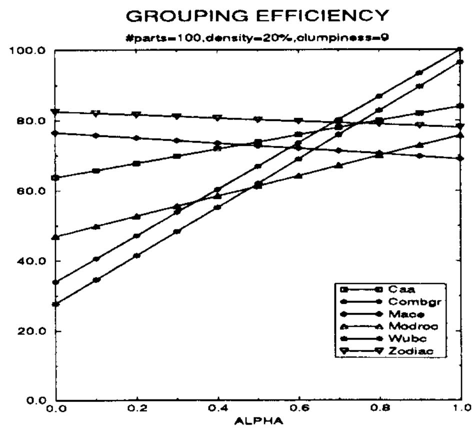
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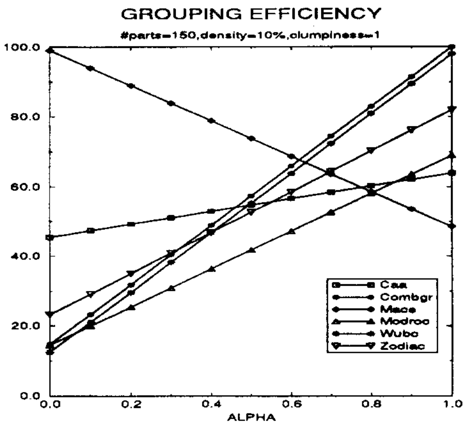
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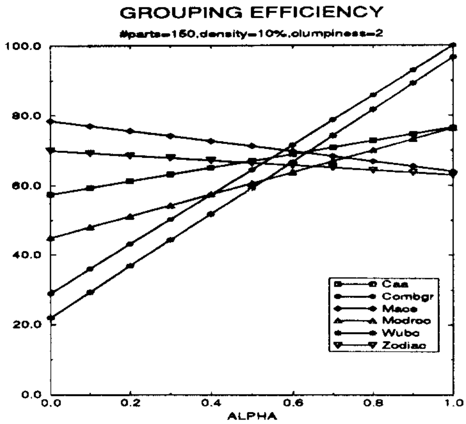
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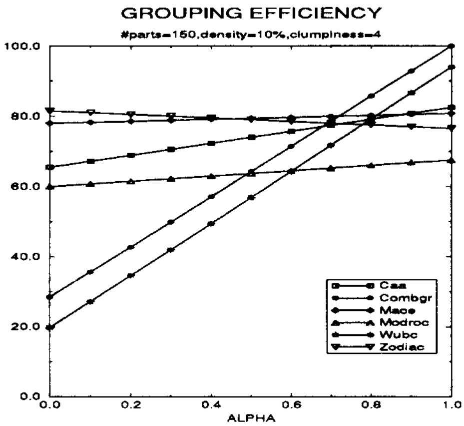
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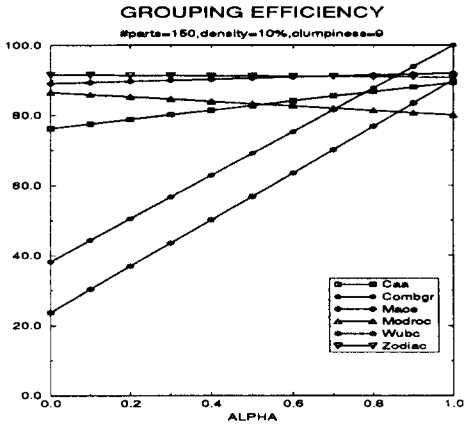
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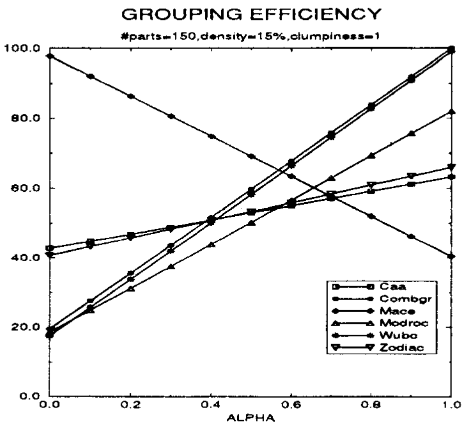
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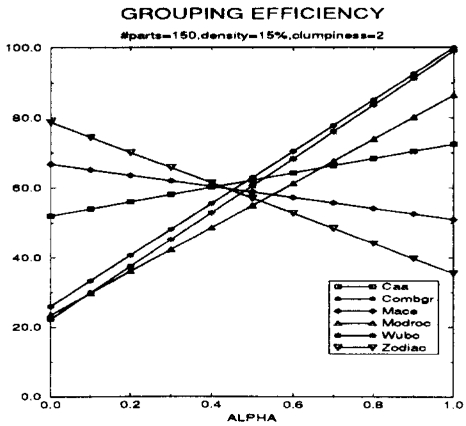
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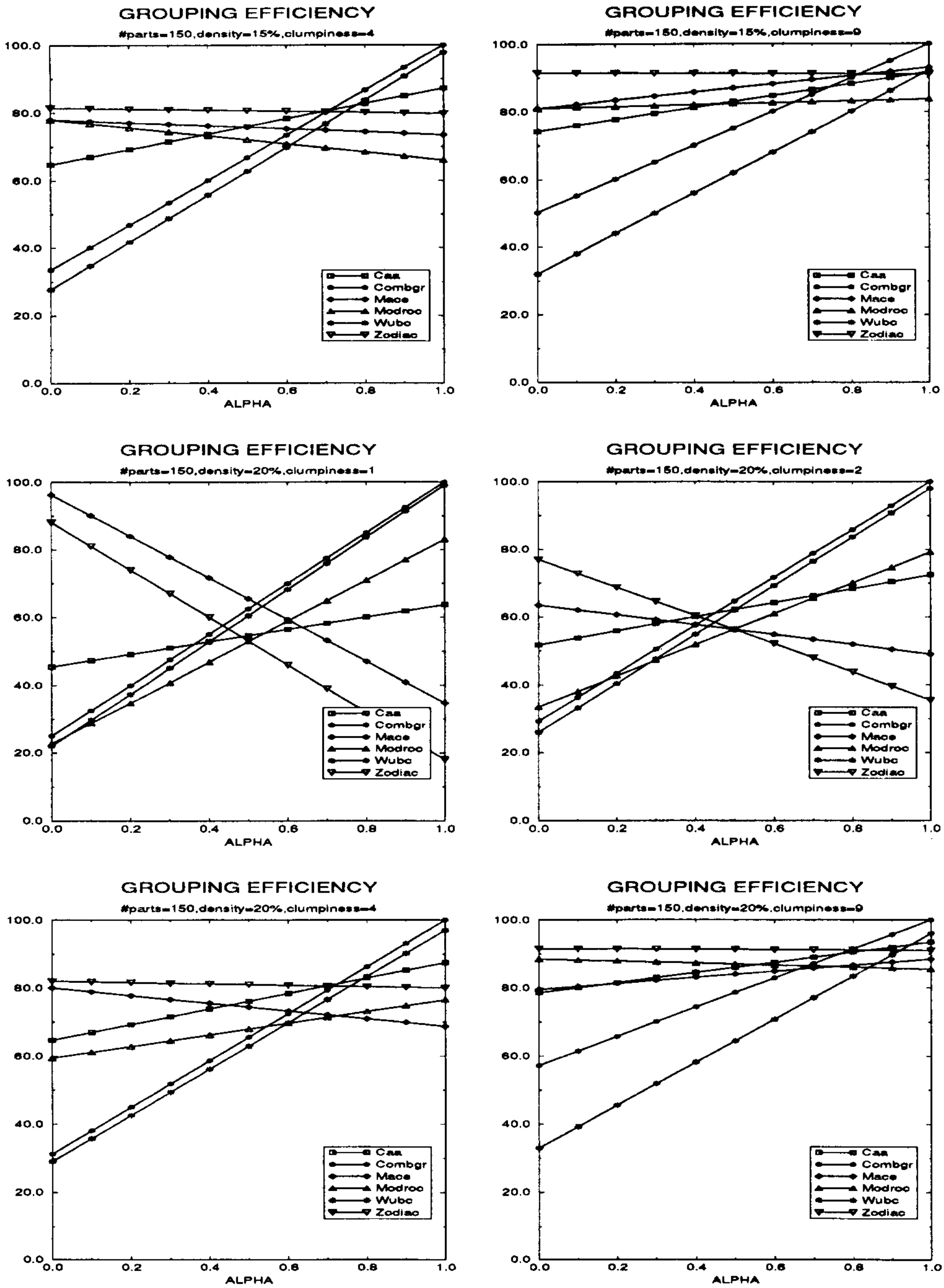


Figure 5. Effects clumpiness, density, number of parts and algorithms on grouping efficiencies.

different means. However, they do not identify the levels of the factors which are significantly different than the others. In order to illustrate such differences, the mean values of these levels are plotted in Figs 5–7.

A summary of the important findings for each technique is given below:

(1) CAA is the most insensitive technique regardless of the factors. Only a slight increase in the grouping efficiency value is observed as we increase clumpiness.

(2) Because COMBGR generates solutions with no exceptional elements, machine duplications occur and this leads to:

- 100% inter-cell flow efficiency;
- low inner-cell density measures;
- low work-load balance values;
- high machine under-utilization values.

(3) MACE creates

- large number of small sized cells in low clumpiness values;
- grouping efficiency values that are insensitive to alpha changes in the case of high clumpiness values;
- good work-load balances;
- high under-utilization figures in low clumpiness values.

(4) It has been observed that MODROC solutions depend heavily on the first two ROC iterations. The presence of exceptional elements affects final merging iterations leading to solutions with:

- low grouping efficiency values for low clumpiness and moderate values for high clumpiness values;
- satisfactory work-load balance and under-utilization scores, but not the best;
- sensitivity to an introduction of new parts into manufacturing environment.

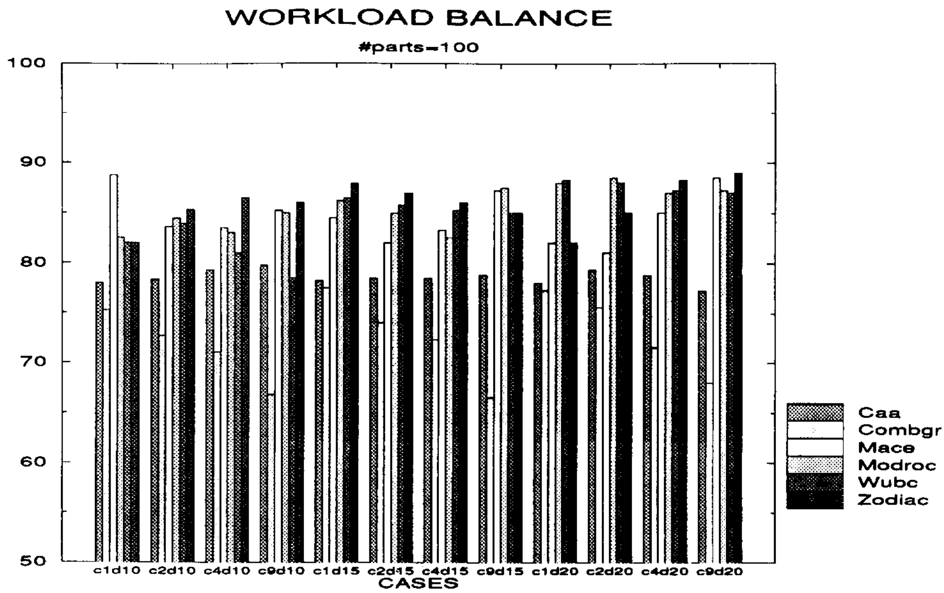
(5) WUBC behaves like COMBGR except that it allows the existence of exceptional elements. WUBC provides solutions with:

- close grouping efficiency values for low clumpiness and worse for high clumpiness values;
- higher work-load balance values due to clustering based on work-loads; better machine utilization measures.

(6) Since ZODIAC is designed to generate a perfect block-diagonal structure, the technique does not perform well for low clumpiness values. For high clumpiness values, the solutions generated by ZODIAC have:

- robust grouping efficiency values in changing α values;
- best grouping efficiency values;
- high work-load balance measures;
- low machine under-utilizations.

The best cell formation technique suggested for each combination of the shape parameters in terms of the grouping efficiency measure are shown in Table 9. The best technique for each clumpiness-density pair is extracted from the associated plot. The techniques with results very close to the best score are also included in the related entry



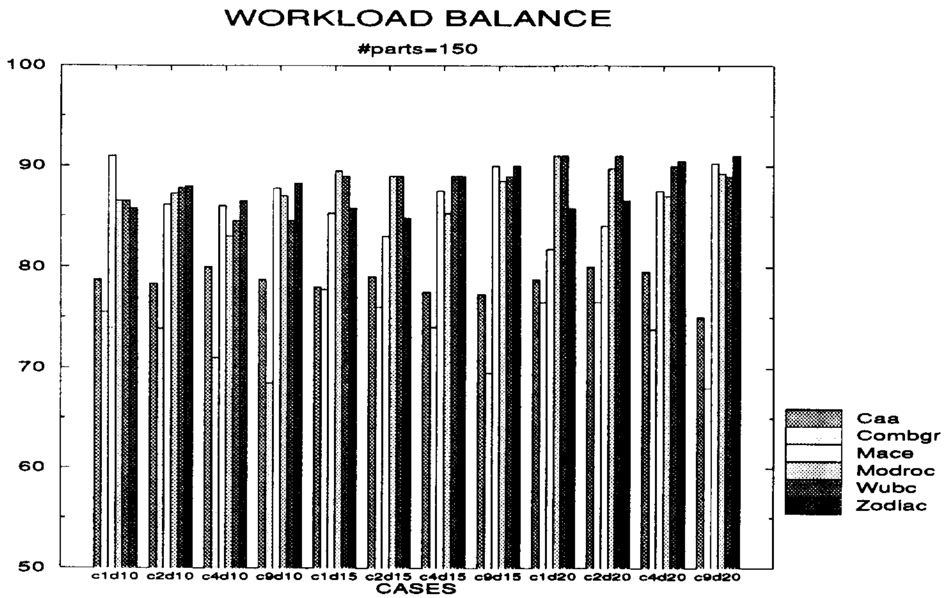


Figure 6. Effects of clumpiness, density, number of parts and algorithms on work-load balances.

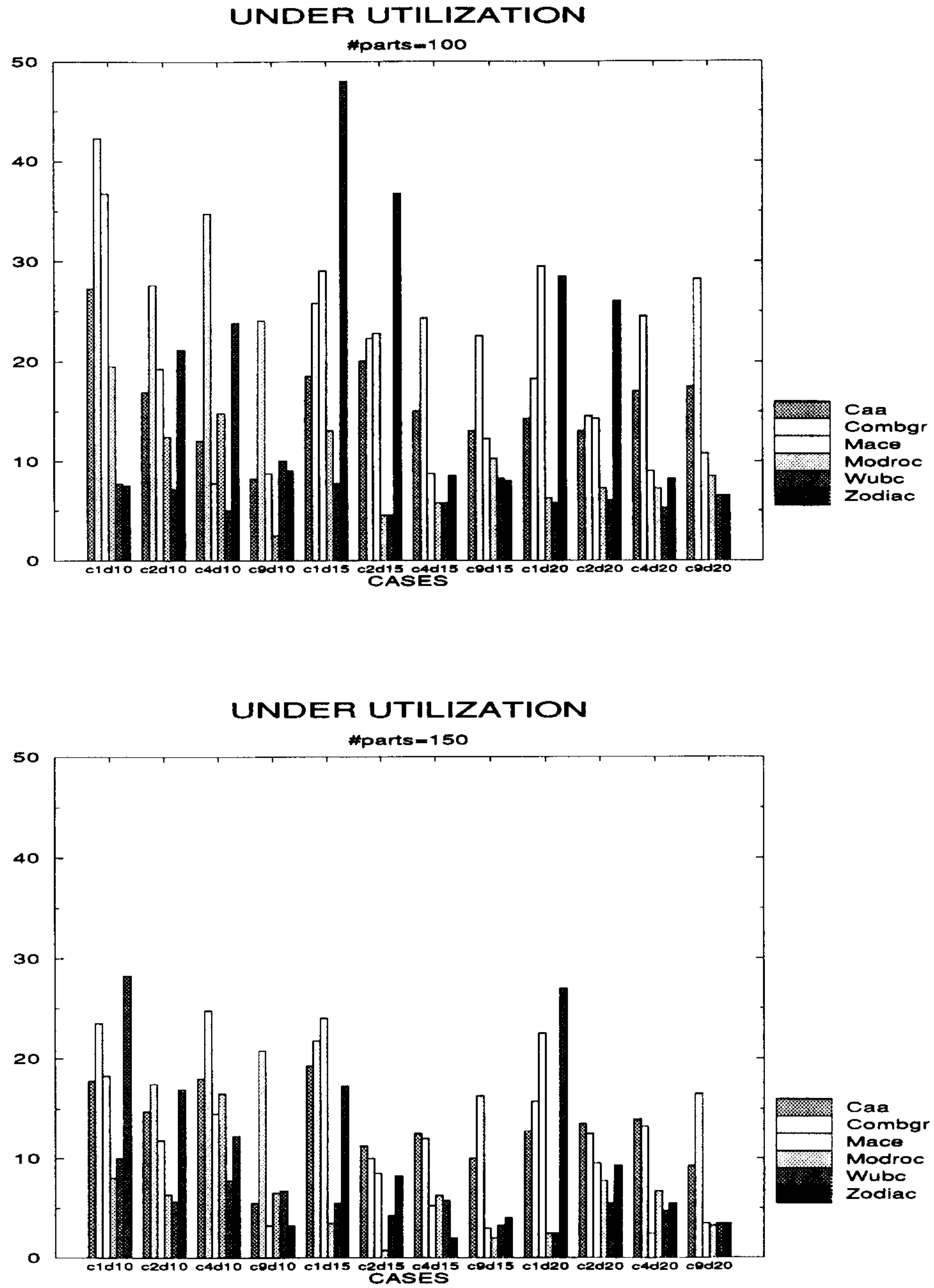


Figure 7. Effects clumpiness, density, number of parts and algorithms on under-utilizations.

Clumpiness (c)	Density (d)	Grouping efficiency $= f(\alpha)$ 0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
Number of parts = 100	0·10	C	C	C	C	C	C	C	B, F, E	B, F, E	B, F	B, F
1	0·15	C	C	C	C	C	C	B, E	B, E	B, E	B, E	B, E
	0·20	C	C	C	C	C	C	B, E	B, E	B, E	B, E	B, E
	0·10	C	C	C	C	C	C	B, C	B	B	B	B
2	0·15	F	F	F	F, C	C	B, E	B, E	B, E	B, E	B, E	B, E
	0·20	F	F	F	F	C, A	B	B	B	B	B	B
	0·10	C	C	C	C	C	C	C	B, C	B	B	B
4	0·15	F	F	F	F	F	F	F	F, B	B	B	B
	0·20	F	F	F	F	F	F	F	B, F, A	B	B	B
	0·10	F	F	F	F, C	F, C	F, C	C, F	C, F	C, F	B	B
9	0·15	F	F	F	F	F	F	F	F	F, C, B	B	B
	0·20	F	F	F	F	F	F	F	B, F, A	B	B	B
Number of parts = 150	0·10	C	C	C	C	C	C	C	B, E	B, E	B, E	B, E
1	0·15	C	C	C	C	C	C	B, E	B, E	B, E	B, E	B, E
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2	0·15	F	F	F	F	F, C, A	B, A	B, E	B, E	B, E	B, E	B, E
	0·20	F	F	F	F	F, A	B	B	B	B	B	B
	0·10	F	F	F, C	C, F	C, F	C, F	C, F	C, F, B, A	B	B	B
4	0·15	F	F	F	F	F	F	F, A	F, A, B	B	B	B
	0·20	C, F	F	F	F	F	F	F	B, F, A	B	B	B
	0·10	F	F	F	F, C	F, C	F, C	C, F	C, F	C, F	B, C	B
9	0·15	F	F	F	F	F	F	F	F	F, C, B	B	B
	0·20	F	F	F	F	F	F	F	B, F, A	B	B	B

A, CAA; B, COMBGR; C, MACE; D, MODROC; E, WUBC; and F, ZODIAC.

Table 9. Best cell formation techniques in terms of grouping efficiencies.

in the table. Similarly, the best cell formation techniques in terms of the work-load balance measure and the under-utilization values are given in Table 10.

In relation to the grouping efficiency, MACE seems to be the best technique for low clumpiness values if the inner-cell densities are considered more critical than inter-cell flow efficiencies. COMBGR and WUBC are the best when inter-cell flows gain importance. As clumpiness increases, ZODIAC dominates the other techniques, even at moderate to high α values. Nevertheless, COMBGR is still the best alternative provided that the existence of exceptional elements are totally undesirable. If the two extremes are in balance i.e., the inner-cell densities and the inter-cell flows are equally important, ZODIAC and MACE should be preferred. Hence, MACE, COMBGR, and WUBC are the best for low clumpiness whereas ZODIAC and MACE are the best for high clumpiness values.

The cell formation solutions generated by COMBGR and CAA are inferior in terms of work-load balances. The other four techniques, however, perform reasonably well. In almost all of the cases, WUBC performs best in terms of the under-utilization. In addition to WUBC, MODROC and ZODIAC generate good cell formation solutions in terms of under-utilization measures. For high clumpiness values, MACE also has a good performance.

7. Conclusion

This article tries to assert some basic guidelines for the evaluation and the selection of a cell formation technique under different situations. Modifications and extension to the considered techniques are intended to eliminate some of their deficiencies. The techniques are implemented on the computer and tested via large problems representing real life situations.

What is more, it is hoped that our results warn the potential user of the weaknesses pertaining to the technique they are using. The three efficiency indices are easy to calculate provided that the work-load matrix is available. Our comparative analysis indicates that these indices are quite powerful in evaluating a cell formation solution.

		No. of parts = 100		No. of parts = 150	
Clumpiness (c)	Density (d)	Work-load balance	Under utilization	Work-load balance	Under utilization
	0·10	C	F, E	C	D
1	0·15	F, E, D	E	D, E	D, E
	0·20	D, E	D, E	D, E	D, E
	0·10	F, D, E, C	E	F, E, D, C	E, D
2	0·15	F, E, D	D, E	D, E	D
	0·20	D, E	E, D	E, D	E
	0·10	F	E, C	F, C	E
4	0·15	F, E	D, E	F, E, C	F
	0·20	F, E, D	E	F, E	C
	0·10	F, D, C	D	F, C, D	F, C
9	0·15	D, C	F, E	F, C, E, D	D, C, E
	0·20	F, C, D, E	F, E	F, C, D, E	D, E, F, C

A, CAA; B, COMBGR; C, MACE; D, MODROC; E, WUBC; and F ZODIAC.

Table 10. Best cell formation techniques in terms of work-load balance and under utilizations.

A computational study due to Miltenberg and Zhang in 1991 considers four factors: the algorithms, the number of machines (25, 50), the number of parts (35, 50) and density (0·10, 0·15, 0·20) and five replications. They claim that the most significant factor is the number of machines, whereas the least is the algorithms. It is reported that none of the algorithms appears to be the best in terms of all the performance measures they consider. They also state that ZODIAC performs best in terms of their grouping measure which is quite similar to our modified grouping efficiency.

Research on the cell formation problem is far from complete. The definition of the problem, the efficiency measures suggested to evaluate the techniques, and the problem generation require even more exquisite analysis so as to:

- study the cell formation from the widest possible perspective implementing a multi-criteria approach;
- measure the sensitivity of a technique for different exogenous variables and provide feasible boundaries accordingly;
- identify the most promising approaches for future developments;
- establish a comprehensive testing ground for new cell formation algorithms;
- pick up the most appropriate cell formation technique to employ in a particular instance.

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Appendix

1. *Algorithm* {*COMBGR*}

Step (1) Input part routeing codes, number of machines in the job shop, and maximum machine-difference limit;
(2) Compute sizes and code significances of parts,
Sort parts by size in decreasing order
Order parts of the same size by code significance in descending order;
(3) Find hosts and guests, construct hospitality and flexibility relationships,
Identify initial candidate cells characterized by hosts,
Assign parts;
(4) Calculate size of minimal machine-differences between cells,
If minimal machine-difference size > maximum machine-difference limit, jump to step (6),
Compute set combination sizes, forward and inverse relations,
Assign priorities;
(5) Form super-hosts,
Calculate total machine requirements,
If total machine requirements $\leqslant$ total number of machines, jump to step (6),
Replace hosts by superhosts,
Return to step (4)
(6) Output the solution,
Calculate efficiency measures;

end {COMBGR}.

2. *Algorithm* {*MODROC*}

Step (1) Input incidence matrix, machines in the job shop, lower limit on similarity coefficient, upper limit on number of independent parts, and aspiration level α;
(2) Make two ROC iteration on the incidence matrix;
(3) Identify the largest top-left block of 'ones',
Determine the candidate cell,

Slice the corresponding columns in the incidence matrix,
If the resultant incidence matrix is not empty, return back to step (6);
(4) If total machine requirement $\leqslant$total number of machines, save the solution,
If number of cells is equal to one, go to step (6),
Generate similarity coefficient matrix,
Choose independent pairs of cells having higher similarities than the lower limit,
If number of independent pairs is zero, save the solution and go to step (6);
(5) Merge cells,
Join part families,
Return back to step (4);
(6) Based on α, choose the solution with the highest grouping efficiency value among all cell formation proposals,
Output the solution,
Calculate efficiency measures;
end {MODROC}.

3. *Algorithm {WUBC}*

Step (1) Input work-load matrix, cell admission factor (CAF), cell size upper limit (CSUL), number of machines of each type, rule for key machine type selection, rule for part assignments;
(2) Select key machine type according to the inputted rule,
If there is no key machine type, then go to step (10),
Insert key machine type into the FCFS queue,
Add key machine type into cell;
(3) Examine all parts routed through the key machine type,
If the examined part is not already assigned, then assign the part,
If the examined part has a higher part assignment value in this cell, then mark the part;
(4) Evaluate all non-key machine types in the routeings of marked or assigned parts,
If non-key type is neither admitted nor rejected and its work-load fraction, WLF$\geqslant$CAF, then admit the non-key type, otherwise reject;
(5) Insert all single machine admitted non-key types into the FCFS queue,
Delete the top machine type from the FCFS queue,
If FCFS queue is not empty, then set the new key as the top element of the queue and go to step (3);
(6) If there is at least one machine type other than the key in the cell, go to step (7),
Erase marks on parts,
Release machines of the key type in this cell,
Prevent this type from being a key in further iterations,
Go to step (2);
(7) Compute within cell utilization (WCU) of all admitted machine types due to marked parts,
List admitted machine types in decreasing order of WCU values,
Assign admitted types in this order until CSUL is reached;
(8) Examine all marked parts,
Assign a marked part if it has a higher assignment value in this cell;
(9) If there is no part assigned to the cell, then discard the cell,
Otherwise, for assigned marked parts, update the work-loads of the corresponding machine types in previous cells, release them if necessary,
Erase marks on parts,
Go to step (2);
(10) Add all left-over machines into the remainder cell,
Examine all parts for possible reassignment to the remainder cell;
(11) If there is no part assignment to the remainder cell, then go to step (12),
For reassigned parts, update work-loads of the corresponding machine types in previous cells,
If there is a machine release, then go to step (10);

(12) Output solution,
Calculate efficiency measures;

end {WUBC}.

4. *Algorithm {CAA}*

Step (1) Input work-load matrix (WL), cell admission factor (CAF), total work-load costs for parts (TWLC), and number of machines of each type,
Compute cell size upper limit (CSUL);
(2) Select the key part,
Assign the key part to the cell,
Add all machine types related to the key part into the cell;
(3) Find candidate parts,
If there is no candidate part, then go to step (5)
Investigate set inclusion relations between extra machine requirement of candidate parts,
Update cost of candidate parts;
(4) Find the candidate part having the maximum cost,
Expand the cell,
If CSUL is not reached, then go to step (3);
(5) Calculate number of machines for each type in the cell,
If there are left-over machines go to step (2);
(6) Assign rejected parts;
(7) Output solution,
Calculate efficiency measures;

end {CAA}.

5. *Algorithm {MACE}*

Step (1) Input similarity coefficient type, threshold value, number of machines in the job shop;
(2) Compute $NCC_{i,j}$, TNC_i, TFC_i,
Calculate similarity coefficients of the selected type;
(3) Select the machine pair with the maximum similarity,
Examine the closest machines,
Form a candidate cell;
(4) Repeat step (3) until no more machine type is left;
(5) Compute inter-cell flows,
Replace machines by candidate cells,
Calculate similarity coefficients between candidate cells ($SCTF_{k,l}$),
Repeat step (3) until no more candidate cells are left;
(6) Assign parts,
Check the existence of blocking machines,
If there exists blocking machines, relocate them;
(7) Output the solution,
Calculate efficiency measures;

end {MACE}.

6. *Algorithm {ZODIAC}*

Step (1) Input machine-part incidence matrix, weighting factor q, threshold value for representative seeds,
Calculate maximum allowable number of cells, K^*,
Set $K \leftarrow K^*$;
(2) Choose K artificial seeds for columns,
Cluster columns,
Choose representative seeds for columns,
Cluster columns;

(3) Find number of non-null column clusters, K_C,
Modify $K \leftarrow K_C$,
Repeat step (2) for rows,
Find number of non-null row clusters, K_R;
(4) Modify $K \leftarrow min\{K_R, K_C\}$,
If $K_R \neq K_C$, then go to step (2),
Reorder rows and columns in the order of cluster membership;
(5) Compute similarity coefficients,
Allocate part clusters to machine clusters,
Reorder columns according to the new order of clusters;
(6) Compute clustering efficiency ξ, and relative efficiency ξ_R,
If $\xi_R \cong 1$, then go to step (11),
Set $\hat{\xi} \leftarrow \xi$;
(7) Generate ideal seeds for machine clusters,
Cluster rows,
Modify $K \leftarrow K_R$,
Generate ideal seeds for part clusters,
Cluster columns,
Modify $K \leftarrow min\{K_R, K_C\}$,
Generate representative seeds for part clusters,
Cluster columns,
Modify $K \leftarrow min\{K_R, K_C\}$,
Generate ideal seeds for machine clusters,
Cluster rows;
(8) If $K_R \neq K_C$, then go to step (9),
Generate ideal seeds for part clusters,
Cluster columns,
If $K_R \neq K_C$, then go to (7),
Go to step (10);
(9) Replace columns by rows,
Repeat step (7),
If $K_R \neq K_C$, then go to step (7);
(10) Reorder columns according to the new order of clusters,
Compute ξ and ξ_R,
If $\xi_R \cong 1$, then go to step (11),
If $\xi < \hat{\xi}$, then revert to the earlier grouping and go to step (11),
Replace $\hat{\xi} \leftarrow \xi$,
Liquidate the smallest block (optional),
Go to step (7);
(11) Output solution,
Calculate efficiency measures;
end {ZODIAC}.

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