An equipment replacement problem with dynamic production planning and capacity considerations

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In this paper, we propose a 0–1 integer programming formulation for an equipment replacement problem in which the demand of a product varies from period to period and different types of equipment with different production capacities are available. Since all the variables involved in the model are restricted to have binary values, the model is fairly efficient and hence can be used for large-sized problems. The use of the model is demonstrated with the help of some numerical examples.

1. Introduction

All equipment eventually wears out with time due to deterioration and usage and, as a result, it begins to function with decreasing efficiency. For example, with the passage of time, a milling machine's operating cost and downtime increases, and a transportation vehicle such as a car or industrial truck requires increasingly more repair and maintenance. With the ever-rising repair and maintenance costs, a stage may come when these costs become so high that it is more economic to replace the used piece of equipment by a new one. A natural question then arises about the optimal time of its replacement. If these repair and maintenance costs decrease or remain constant with time, the best policy, perhaps, then is never to replace the item. However, in real situations such a condition is hardly ever met. Technological developments may also force the user to consider the replacement because better designed machinery may result in improved product quality, better efficiency and reduced maintenance and operating costs. Generally, all costs that depend upon the choice or age of the equipment must be taken into account when analysing the replacement decision.

In addition to rising costs as an incentive for replacing old equipment, there may be other situations. In many production situations, fluctuations in demand cannot be

met by the equipment of fixed capacity. In that case, the problem becomes that of duplication of equipment or purchase of equipment of appropriate capacity, if a choice is available.

A discussion on the body of literature on this topic is provided by Bector *et al.* (1993) and Gill (1992). Evidence of the practical use of such a replacement model can be found in Waddell (1983). Bector *et al.* (1993) formulated a deterministic equipment replacement problem, which is normally solved through the use of dynamic programming, as a 0–1 linear programming problem. The advantage of such a formulation is that linear programming does not require a high degree of expertise, insight and 'art' (thus, it is rather easily understood by practitioners), does not suffer from the so-called 'curse of dimensionality' (Budnick *et al.*, 1988)—a shortcoming of dynamic programming—and can also yield a global optimal solution. The applicability of Bector *et al.* (1993) model is restricted to smaller sized problems. Adil and Gill (1993) gave an alternative formulation wherein the linear programming relaxation of the formulation always yields an integer solution, and hence could be applied to very large sized problems. However, none of these papers consider the dynamic nature of the product demand and hence do not consider capacity restrictions on the equipment used to manufacture that product. Capacity restrictions can be of critical importance in certain situations such as in case of fluctuating production requirements.

In the present paper we develop a more general integer programming model which takes care of dynamic production requirements by considering equipment of different capacities. The remainder of the paper is organized as follows. Section 2 presents the model formulation considering capacity restrictions on equipment of different

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Figure 1.

types and dynamic demand. The model is illustrated by considering numerical examples in § 3. Finally, conclusions are presented in § 4.

2. The model formulation

It is assumed that purchasing, maintenance, operating, and resale values of the equipment under consideration are all deterministically known. We also assume that the age (not the mileage) is used to determine the present state of the equipment. Furthermore, it is assumed that there exists a good second-hand market for many different types of equipment, such that the equipment will have a resale value that may, however, be affected by weather, time, release and/or place of release of new models. The structure of the present model consists of low initial maintenance and operating costs that increase with age of the equipment, while its resale value declines. Due to obsolescence or other reasons (e.g. corporate policies or union contracts), the equipment age is not allowed to exceed the specified maximum limit. However, in order to make the model more general, this assumption can be relaxed by giving very high costs and zero resale values for those years that lie beyond this limit. Since the industrial equipment normally has a certain finite lifetime, we assume that the planning horizon is finite. Since it is illogical to acquire new equipment and sell it immediately without using it, it is generally assumed in such models that the equipment is in use for some minimum time period before it could be replaced. Therefore, in this paper, we assume that the first opportunity for equipment replacement exists at least one time period after its purchase. A time period considered could be a 'day', 'month' or 'year'. We consider one year as the unit time period because, in the case of industrial equipment, such decisions—influenced by annual budget considerations—are made on a yearly basis.

This model considers the case when demand is dynamic, i.e. the production planning for each period requires a different number of machine hours or capacity. Demand can be met by duplicating the existing equipment or purchasing a new type of equipment of different capacity, if a choice is available. The terms 'equipment' and 'machine' are used interchangeably in this paper.

Let

L = maximum number of machine types available
 q = index for equipment or machine type,
 $q = 1, 2, \dots, L$

$$x_{ij}^q = \begin{cases} 1 & \text{if equipment type } q \text{ purchased at time } i \\ & \text{is used until time } j (j > i) \\ 0 & \text{otherwise} \end{cases}$$

P_i^q = acquisition cost of equipment type q at time i

M_{ij}^q = maintenance cost of equipment type q from time i to time j for equipment purchased at time i

r_{ij}^q = resale value at time j of the equipment type q purchased at time i

m^q = maximum life of equipment of type q

n = total number of planning periods

b^q = the capacity (in units of product per period) of machine type q

T_j = the demand (in units of product per period) in period j

Let a piece of equipment of type q be purchased by an industry at time i , where $i = 0, 1, 2, \dots, n-1$ and let the length of the planning horizon be n . Let the origin of the year i be at time $i-1$, ($i \geq 1$), as shown in Fig. 1.

The objective of the model is to minimize the total cost given by the following

$$\text{minimize } Z = \sum_{q=1}^L \sum_{i=0}^{n-1} \sum_{j=i+1}^k C_{ij}^q x_{ij}^q,$$

where

$$k = \min(i + m^q, n) \quad \text{and} \quad C_{ij}^q = P_i^q - r_{ij}^q + M_{ij}^q$$

subject to

$$\sum_{q=1}^L \sum_{i=\max[j-m^q, 0]}^{j-1} \sum_{k=j}^{\min[i+m^q, n]} x_{ik}^q b^q \geq T_j \quad \text{for all } j=1, 2, \dots, n$$

$$x_{ik}^q = 0 \text{ or } 1 \quad \text{for all } i, j, k. \quad (1)$$

It may be noted that by defining x_{ik}^q as a binary variable, we restrict it not to take any other discrete value higher than 1. If for some reason it is economical to buy two machines of type 1 at time 0 and to sell them at time 2, then this definition will not allow that. To some extent this duplication can be modelled by defining another machine type that has the capacity of twice that of type 1 and all the relevant costs are also doubled as well; or equivalently, we can say first, we define this variable as a general integer and then apply the binary transformation to that integer variable. For example, if a machine type 1 can have maximum value of 4, then we can do the following transformation. Original general variable x^q can be

expressed in terms of two other binary variables y^q and z^q as $x^q = 2y^q + z^q$. However, the use of such a transformation should be limited because such a transformation results in a higher number of variables.

We now derive the Adil and Gill (1993) model as a special case of the more general model presented in this paper. If we consider only one type of equipment, as in Adil and Gill (1993), i.e. $L = 1$, then $q = 1$. The other variables are modified as follows.

Set

$$\begin{aligned} x_{ik}^q &= x_{ik}^1 = x_{ik}, P_i^q = P_i^1 = P_i, M_{ij}^q = M_{ij}^1 = M_{ij}, \\ r_{ij}^q &= r_{ij}^1 = r_{ij}, m^q = m^1 = m, b^q = b^1 = b, T_j = T, \\ C_{ij}^q &= C_{ij}^1 = C_{ij} \end{aligned}$$

then

$$C_{ij} = C_{ij}^1 = P_i^1 - r_{ij}^1 + M_{ij}^1 = P_i - r_{ij} + M_{ij}$$

The objective function of the above model becomes

$$\text{minimize } Z = \sum_{q=1}^1 \sum_{i=0}^{n-1} \sum_{j=i+1}^n C_{ij}^q x_{ij}^q$$

This yields

$$\text{minimize } Z = \sum_{i=0}^{n-1} \sum_{j=i+1}^n C_{ij} x_{ij}$$

which is the same as the objective function of the Adil and Gill (1993) model.

Constraint set (1) in the present model has the dual purpose of meeting the demand of the product with enough capacity and having at least one piece of equipment of any type. An implicit assumption made in the Adil and Gill (1993) model is that we have exactly one piece of equipment available at a time with capacity large enough to meet the demand of the product. In light of this assumption, constraint set (1) has only one purpose, to ensure that we have exactly one of piece of equipment available. This can be mathematically shown as below.

Constraint set (1) of the present model for one equipment type becomes

$$\sum_{q=1}^1 \sum_{i=\max\{j-m, 0\}}^{j-1} \sum_{k=j}^{\min\{i+m, n\}} x_{ik} b \geq T \quad \text{for all } j = 1, 2, \dots, n$$

This yields

$$\sum_{i=\max\{j-m, 0\}}^{j-1} \sum_{k=j}^{\min\{i+m, n\}} x_{ik} b \geq T \quad \text{for all } j = 1, 2, \dots, n \quad (2)$$

Since the total capacity should be large enough to meet the demand, therefore, $b \geq T$.

Case 1. $b = T$

The above constraint set (2) becomes

$$\sum_{i=\max\{j-m, 0\}}^{j-1} \sum_{k=j}^{\min\{i+m, n\}} x_{ik} = 1, \quad \text{for all } j = 1, 2, \dots, n$$

Case 2. $b > T$

The constraint set (2) becomes either

$$\sum_{i=\max\{j-m, 0\}}^{j-1} \sum_{k=j}^{\min\{i+m, n\}} x_{ik} \geq 1, \quad \text{for all } j = 1, 2, \dots, n$$

or

$$\sum_{i=\max\{j-m, 0\}}^{j-1} \sum_{k=j}^{\min\{i+m, n\}} x_{ik} < 1, \quad \text{for all } j = 1, 2, \dots, n$$

Since variables x_{ik} can only take a value of '0' or '1',

$$\sum_{i=\max\{j-m, 0\}}^{j-1} \sum_{k=j}^{\min\{i+m, n\}} x_{ik} < 1 \quad \text{for all } j = 1, 2, \dots, n$$

can hold only as

$$\sum_{i=\max\{j-m, 0\}}^{j-1} \sum_{k=j}^{\min\{i+m, n\}} x_{ik} = 0, \quad \text{for all } j = 1, 2, \dots, n$$

which contradicts the fact the demand is positive and that there is at least one piece of equipment of any type available.

Therefore, we have

$$\sum_{i=\max\{j-m, 0\}}^{j-1} \sum_{k=j}^{\min\{i+m, n\}} x_{ik} \geq 1, \quad \text{for all } j = 1, 2, \dots, n$$

Since, in the Adil and Gill (1993) model, it is also assumed that we have exactly one piece of equipment available at a time, the above constraint set then becomes

$$\sum_{i=\max\{j-m, 0\}}^{j-1} \sum_{k=j}^{\min\{i+m, n\}} x_{ik} = 1, \quad \text{for all } j = 1, 2, \dots, n$$

From Case 1 and Case 2 above, we may conclude that, in light of the implicit assumptions made in the Adil and Gill (1993) model, the constraint set in the present model becomes

$$\sum_{i=\max\{j-m, 0\}}^{j-1} \sum_{k=j}^{\min\{i+m, n\}} x_{ik} = 1, \quad \text{for all } j = 1, 2, \dots, n$$

which is exactly the same constraint set as in the Adil and Gill (1993) model. Therefore, the Adil and Gill (1993) model is a special case of this more general model.

3. Numerical results

Data to illustrate this model are shown in Table 1. Three examples were considered, in each of which the choice available for equipment type is different. The choices available are equipment type 1 only in example 1, equipment type 1 and 3 in example 2, and equipment type 1 and 2 in example 3.

We formulate the problems using the data in Table 1 and the model described in § 2. The solutions obtained are as follows.

Table 1. $m^q = 3, n = 5$

Equipment type p	Capacity (units/period)	Cost, $C_{ij}^q = P_i^q - r_{ij}^q + M_{ij}^q$ (in dollars)		
		$C_{01}^q = C_{12}^q = C_{23}^q = C_{34}^q = C_{45}^q$	$C_{02}^q = C_{13}^q = C_{24}^q = C_{35}^q$	$C_{03}^q = C_{14}^q = C_{25}^q$
1	10	260	540	760
2	15	390	810	1140
3	20	520	1080	1520

Demand Data

Period	1	2	3	4	5
Demand (units)	30	45	25	20	10

Example 1—Equipment type 1 only:

Objective value = \$3640

 $x_{01}^1 = x_{02}^1 = x_{03}^1 = x_{12}^1 = x_{13}^1 = x_{14}^1 = x_{34}^1 = x_{45}^1 = 1$ and all other variables = 0. $\square$
Example 2—Equipment types 1 and 3 available:

Objective value = \$3580

 $x_{01}^1 = x_{03}^3 = x_{12}^1 = x_{25}^3 = x_{34}^1 = 1$ and all other variables = 0. $\square$
Example 3—Equipment types 1 and 2 available:

Objective value = \$3360

 $x_{02}^2 = x_{03}^3 = x_{12}^2 = x_{25}^3 = x_{34}^1 = 1$ and all other variables = 0. $\square$

Comparisons of these three results are shown in Table 2.

We observe from Table 2 that the object value, i.e. cost, is highest in Example 1 and lowest in Example 3. The reason for this is that in Example 1, we have the highest under-utilization of the machinery. In other words, the machine capacity available is such that we cannot exactly

meet the required demand and we need to have machines of higher capacity

In Example 3, we have two types of machines available with capacity 10 and 15. In this case, we find that the demand is exactly met and hence we have minimum cost.

4. Conclusions and extensions

A 0-1 integer programming model is given for the equipment replacement problem with the dynamic production demand consideration. Availability of different types of equipment with different production capacities has been modelled in the formulation. It has been shown that the Adil and Gill (1993) model is a special case of this more general model. Since it is a pure binary variable formulation, it is very efficient in solving large-sized problems.

The model can be extended further for multistage processing and for planning periods of different durations. Another extension could be the development of a more general model, which considers production costs and inventory costs to couple the equipment replacement and production planning decisions.

Table 2.

Example	M/C hours available in different periods (from the solutions)					Total cost (\$)
	1	2	3	4	5	
1	30	50	30	20	10	3640
2	30	50	30	20	10	3580
3	30	45	25	20	10	3360
Required machine hours	30	45	25	20	10	

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