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Quasi-Particle Properties of a One-Dimensional Electron System Interacting with a Short-Range Potential

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We study the quasi-particle properties of a one-dimensional electron gas interacting via a short-range electron–electron interaction. The electron self-energy is calculated using the leading-order dynamical-screening approximation with (GWΓ approximation) and without the vertex corrections (GW approximation). We test the reliability of the plasmon-pole approximation against the full self-energy calculations. Energy relaxation rate via longitudinal optical (LO)-phonon emission is also examined to explore the effects of short-range potential. Our results for the quasi-particle properties indicate that an effective short-range interaction can be used as a qualitative model to understand various quantities in a realistic quantum wire with long-range Coulomb interactions.

1. Introduction

Models of one-dimensional (1D) electron systems are of increasing interest because of their applicability to realistic systems such as naturally occurring organic conductors, artificially fabricated semiconductor structures, and certain materials exhibiting superconductivity. Quantum wire structures made out of semiconducting materials using highly developed processing techniques provide a testing ground for the many-body theories describing the dynamics of interacting electrons in restricted geometries. One-dimensional system of electrons interacting via a short-range potential in configuration space is a model being used to understand various properties of many realistic systems.

In the specific model of a 1D electron gas with a repulsive δ-function potential, the ground-state properties have been considered by Yang [1] who presented an exact solution using the Bethe ansatz. Because of the significant role played by the short-range correlations the ladder approximation provides a reasonable account of the ground state energy [2]. Various correlation functions are determined as a function of the coupling strength within mean-field theories [3 to 5] making use of the local-field factor. These calculations show the usefulness of the concept of local-field factor even for a very short-range interaction potential. It was also argued [6] that strongly interacting 1D fermions may be treated using the generalized random phase approximation which includes local-field correlations.

In this work we study the quasi-particle properties of a 1D electron system interacting via a repulsive contact interaction. We calculate the wave vector and frequency dependent self-energy within the GW and GWΓ approximations from which all one-electron properties can be obtained. There are several motivations for our investigation. Firstly, we explore the extent the Fermi liquid theory can be employed in the description of this model system similar to the quasi-one-dimensional electron gas inter-

acting via long-range Coulomb forces. It has been shown by Hu and Das Sarma [7] that disorder and finite temperature effects render the Fermi liquid picture meaningful in the latter system, and we adopt this viewpoint with application to semiconducting quantum wires in mind. Numerous studies [8 to 10] were devoted to the ground-state energy and correlation functions of quasi-one-dimensional electron gas interacting via long-range Coulomb interaction, but relatively less attention is paid to the quasi-particle properties of these models. On the other hand, models of 1D interacting electrons on a lattice are actively being pursued to understand various phenomena, most notable being quantum phase transitions. Secondly, we investigate the effects of vertex corrections. The extension of the random-phase approximation (RPA) GW approach is formulated by the GWΓ approximation where Γ stands for the vertex corrections. In the present model we use the previously obtained [4] local-field factors to describe the vertex corrections and assess their importance in the quasi-particle properties. In the case of long-range Coulomb interaction the incorporation of vertex corrections within the self-energy calculations requires extra computational effort. In the present model the simplicity of the local-field factors allows us to make better comparison to understand these higher-order correlation effects. Lastly, because of the diminished role of the particle–hole excitations and significant contribution from the plasmon modes to the excitation spectrum of 1D electron systems, it has been suggested [11] that the plasmon-pole approximation to the response functions works remarkably well. We also test the plasmon-pole approximation in our comparative study of the self-energy for a system with short-range interactions. As we show in the sequel, a model based on the short-range interactions yields a number of properties of the 1D electron gas which are very similar to that incorporating the long-range Coulomb potential. Thus, the calculation of quasi-particle properties of realistic quantum wire systems may be facilitated by the use of simpler interaction potentials with effective coupling strengths.

The rest of this paper is organized as follows. In the next section we provide the theoretical background for different models in the calculation of the electron self-energy. In Section 3 we present our numerical results of the self-energy and related quantities calculated from it. We conclude in Section 4 with a brief summary of our results.

2. Model and Theory

We consider a system of electrons in 1D interacting with a contact potential $V(r_1, r_2) = V_0\delta(r_1 - r_2)$, where V_0 is the repulsive ($V_0 > 0$) interaction strength. In terms of the electron mass m and the linear density of the particles n, we use the dimensionless parameter $\gamma = mV_0/n$ to characterize the strength of the coupling (we take $\hbar = 1$). The Fermi wave vector is related to the linear density by $n = 2k_F/\pi$, in an unpolarized system. The basic premise of this work is that the long-range Coulomb interaction may be replaced by a very short-range interaction and a number of physical quantities would be accounted for reasonably well. In fact, a class of problems for lattice electrons interacting with a short-range potential have been treated within the 1D Hubbard model [12] to describe more realistic systems. It was argued by Hu and Das Sarma [7] that in some experimental situations the surrounding metallic gates or adjacent quantum wires may screen the long-range Coulomb interaction and render the effective interaction in a given quantum wire a short-ranged one. Another recent example considers photoexcited semiconductor quantum wires and calculates the correlation effects

in an electron–hole system interacting with a contact potential [13]. These examples show that the short-range interactions replacing the long-range Coulomb interactions may be useful in analyzing some experimental results.

The self-energy of the one-dimensional (1D) electron system within the GW approximation (neglecting the vertex corrections) at $T = 0$ is given by

$$\Sigma(k, \omega) = i \int \frac{\mathrm{d}q\, \mathrm{d}\omega'}{(2\pi)^2} W(q, \omega')\, G_0(k - q,\, \omega - \omega')\,, \tag{1}$$

where $G_0(k, \omega)$ is the Green's function for the noninteracting electron gas,

$$G_0(k, \omega) = \frac{\theta(|k| - k_\mathrm{F})}{\omega - \xi_k + i\eta} + \frac{\theta\,(k_\mathrm{F} - |k|)}{\omega - \xi_k - i\eta}\,, \tag{2}$$

with single-particle energies $\xi_k = k^2/2m - \mu$, (μ is the chemical potential or the Fermi energy $E_\mathrm{F} = k_\mathrm{F}^2/2m$ at $T = 0$). In the above equations $\theta(x)$ is the Heaviside step function, $W(q, \omega)$ is the dynamically screened interaction, which is given by $W(q, \omega) = V_0/\varepsilon(q, \omega)$, and η is a positive infinitesimal quantity. In the effective interaction above V_0 is the bare interaction potential in q-space and $\varepsilon(q, \omega)$ is the dielectric function which describes the dynamical screening properties of the electron gas. We employ the usual practice of separating the dynamically screened interaction $W(q, \omega)$ into a frequency independent term which gives the exchange part of the self-energy and frequency dependent term which gives the correlation part of the self-energy $W(q, \omega) = V_0 + V_0[1/\varepsilon(q, \omega) - 1]$. The exchange part is given by

$$\Sigma_\mathrm{ex}(k) = -\int_{-\infty}^{\infty} \frac{\mathrm{d}q}{2\pi}\, n_\mathrm{F}(k + q)\, V_0\,, \tag{3}$$

where $n_\mathrm{F}(k) = \theta(k_\mathrm{F} - k)$ is the Fermi distribution function at $T = 0$. Performing the integration we obtain $\Sigma_\mathrm{ex}(k) = -4(\gamma/\pi^2)E_\mathrm{F}$, i.e. in units of the Fermi energy. In the GW approximation, the correlation part of the self-energy can be decomposed into two parts [14, 15] $\Sigma_\mathrm{cor}(k, \omega) = \Sigma_\mathrm{line}(k, \omega) + \Sigma_\mathrm{pole}(k, \omega)$. Since $\varepsilon(q, i\omega)$ is a real and even function with respect to ω, the $\Sigma_\mathrm{line}(k, \omega)$ term is completely real.

The self-energy calculation which includes vertex corrections is called the GWΓ approximation [16]. The importance of vertex corrections in strongly correlated systems was recently emphasized in a number of works [17]. In the GWΓ approximation, $\Sigma_\mathrm{cor}(k)$ is again split into two parts, $\Sigma_\mathrm{line}(k, \omega)$ and $\Sigma_\mathrm{pole}(k, \omega)$, which are given, respectively, by

$$\Sigma_\mathrm{line}(k, \omega) = -\int_{-\infty}^{\infty} \frac{\mathrm{d}\omega'}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{d}q}{2\pi}\, V_0 \frac{1}{(\xi_{k+q} - \omega) - i\omega'} \left[\frac{\Gamma(q, i\omega')}{\varepsilon(q, i\omega')} - 1\right], \tag{4a}$$

and

$$\Sigma_\mathrm{pole}(k, \omega) = \int_{-\infty}^{\infty} \frac{\mathrm{d}q}{2\pi}\, V_0 \left[\frac{\Gamma(q, \xi_{k+q} - \omega)}{\varepsilon(q, \xi_{k+q} - \omega)} - 1\right] [\theta(w - \xi_{k+q}) - \theta(-\xi_{k+q})]. \tag{4b}$$

In these expressions $\Gamma(q, \omega)$ is called the vertex function and we use the approximation [15]

$$\Gamma(q, \omega) = \frac{1}{1 + V_0 G(q)\, \chi_0(q, \omega)}. \tag{5}$$

The dielectric function $\varepsilon(q, \omega)$ to be used in the above formulation (generalized RPA framework) is given by

$$\varepsilon(q, \omega) = 1 - V_0 \chi_0(q, \omega)\, \Gamma(q, \omega)\,, \tag{6}$$

where $\chi_0(q, \omega)$ is the density–density correlation function for non-interacting electrons. The function $G(q)$ is called the local-field correction factor. We employ the $G(q)$ calculated within the Singwi-Tosi-Land-Sjölander (STLS) approach [18] by Gold [4]. For the contact potential model we use in this work, the local-field is independent [3, 4] of the wave vector and only depends on the coupling strength γ. This fact greatly facilitates the calculation of the self-energy within the GWΓ approximation. We also note that the above expressions reduce to the familiar GW approximation forms when the vertex corrections are neglected, i.e. $G(q) = 0$ or equivalently $\Gamma(q) = 1$.

For quantum wires with long-range Coulomb interaction, Das Sarma et al. [11] developed a plasmon-pole approximation which turns out to be very accurate in the calculation of quasi-particle properties. Here, we generalize their account to include the local-field effects. We first note that in the GWΓ approximation when a local-field factor $G(q)$ is used in the description of the vertex function $\Gamma(q, \omega)$, the various cancelations render the self-energy expression the same as in the GW approximation, except that the screening function becomes [19, 20] $\varepsilon(q, \omega) = 1 - V_0[1 - G(q)]\, \chi_0(q, \omega)$. Using the plasmon-pole approximation [11] for the density–density correlation function $\chi_0(q, \omega)$, we obtain

$$\varepsilon(q, \omega) = 1 - \frac{[1 - G(q)]\, \omega_0}{\omega^2 - \omega_q + \omega_0^2 + i\eta}\,, \tag{7}$$

where the pole strength $\omega_0^2 = (n/m)V_0 q^2$ is determined by the f-sum rule, and ω_q is the 1D plasmon dispersion. We discuss the explicit form of ω_q in the next section. In the above form of $\varepsilon(q, \omega)$, the RPA is recovered when $G(q) = 0$. With these modifications, the calculation of the self-energy within the plasmon-pole approximation (including the vertex corrections) proceeds along the lines given by Das Sarma et al. [11].

In this work, we calculate the self-energy in leading order perturbation theory. Our expression for the GW and GWΓ self-energies contain non-interacting Green's functions. We have not attempted to perform a self-consistent calculation [21, 22] but surmise that our results would remain qualitatively the same.

3. Results and Discussion

Before we present our results for the self-energy, we discuss the plasmon-pole approximation as applied to the present model. The collective excitation modes (plasmons) within the present short-range interaction model were discussed by Gold [4]. The dispersion relation for the collective density fluctuations is formally similar to that of 1D electron systems with long-range Coulomb interaction, and is given by [23, 24]

$$\omega_q / E_{\mathrm{F}} = \left[\frac{A(q)\, \omega_+^2 - \omega_-^2}{A(q) - 1}\right]^{1/2}\,, \tag{8}$$

where $\omega_\pm = |q^2/2m \pm q k_{\mathrm{F}}/m|$ are the boundaries of the particle–hole excitation region, and $A(q) = \exp\{\pi^2 q/(2k_{\mathrm{F}}\gamma[1 - G(\gamma)])\}$. Here, $G(\gamma)$ is the local-field factor representing the many-body effects beyond the mean-field approximation (i.e. RPA). The RPA result is recovered when $G(\gamma) = 0$. Within the STLS approximation the local-field factor

is independent [3, 4] of wave vector q, unlike the situation for systems interacting via long-range Coulomb interaction [8 to 10]. The low energy part of the phase space in 1D electron systems largely excludes the particle–hole excitations, thus most of the contribution comes from the collective modes. We follow Das Sarma et al. [11] to calculate the oscillator strength given by

$$F(q) = -\frac{2m}{\pi n q^2} \int_0^\infty \mathrm{d}\omega\, \omega\, S_{\mathrm{PL}}(q, \omega)\,, \tag{9}$$

for a quantitative measure of the plasmon contribution. The dynamic structure factor (spectral weight) for plasmon excitations is given by

$$S_{\mathrm{PL}}(q, \omega) = -\frac{\pi}{V_0} \frac{1}{\left|\dfrac{\partial}{\partial\omega}\ \mathrm{Re}\ [\varepsilon(q, \omega)]\right|}\, \delta(\omega - \omega_q)\,, \tag{10}$$

where the RPA dielectric function is $\varepsilon(q, \omega) = 1 - V_0\chi_0(q, \omega)$. In Fig. 1, we compare the oscillator strengths of plasmons within our short-range interaction model and in the RPA for different interaction strength parameters. The plasmon oscillator strength drops quickly to zero for small values of γ at a critical vector. For large values of γ, on the other hand, the oscillator strength of the plasmon excitations extends well into the range of large wave vectors. When we contrast this result with the situation in quasi-one-dimensional structures interacting via the long-range Coulomb potential, we find that $F(q)$ is close to unity up to rather large values of q and for the range of densities of interest (i.e. $n \approx 10^4$ to 10^7 cm^{-1}), as pointed out by Das Sarma et al. [11]. Thus, the rapid decrease in $F(q)$ for small γ is attributed to the nature of short-range interactions. Furthermore, since the RPA plasmons never enter the particle–hole continuum in 1D, the drop in $F(q)$ cannot be associated with damping as in quantum-well structures. Plasmon dominance of the spectral weight is significantly increased as γ increases. Since the plasmon-pole approximation assumes that the excitation spectrum consists of only a

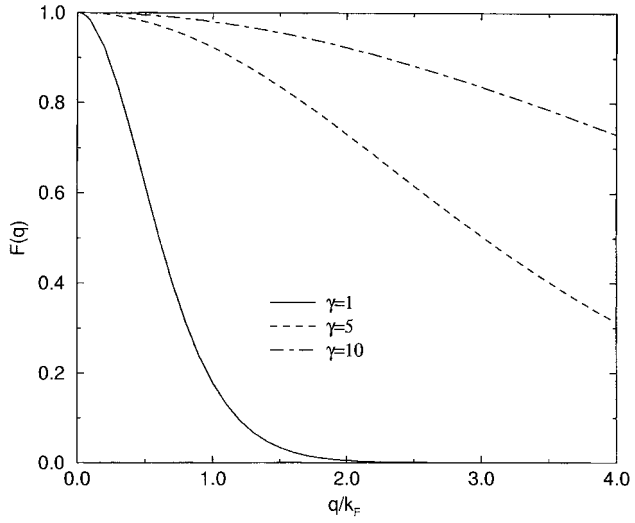

Fig. 1. The oscillator strength $F(q)$ of plasmon excitations within the RPA in a 1D electron system with short-range interactions for different interaction strengths γ

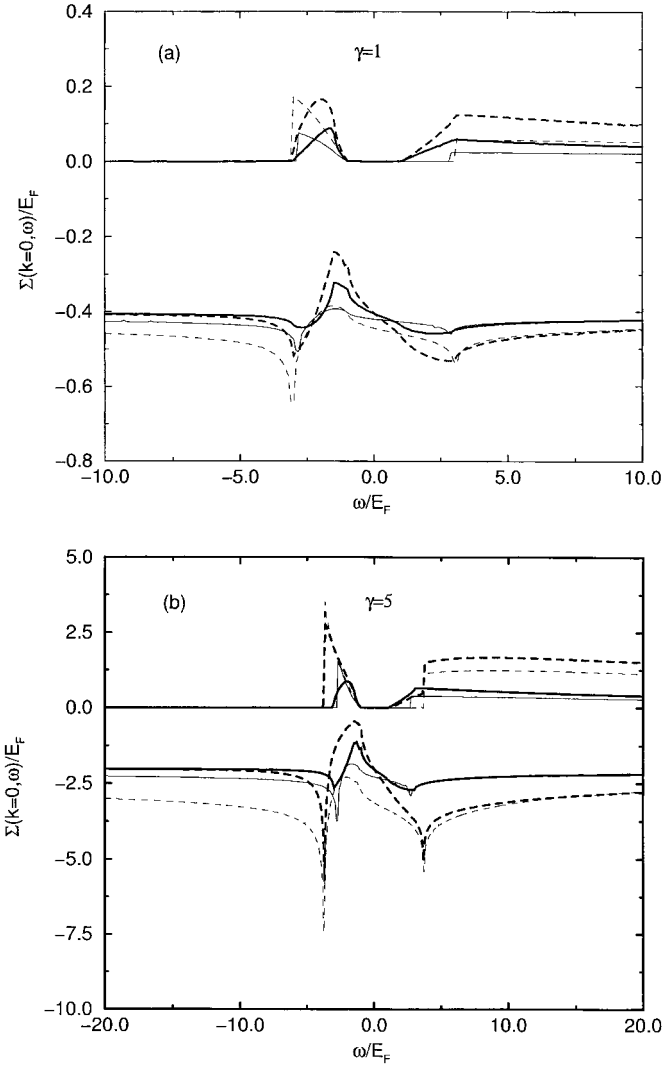

Fig. 2a, b

collective mode which exists for all values of wave vectors and possesses an oscillator strength of unity, the plasmon-pole approximation works quite well at large γ values. We have also found that $F(q)$ becomes even smaller when the local-field corrections are included. The effect of $G(\gamma)$ is to lower the plasmon energies, thus the spectral weight has less contribution in this case.

We now discuss the quasi-particle properties of the present model. In Fig. 2 we show the frequency dependence of the real and imaginary parts of the self-energy calculated in four different approximations; the RPA-GW, the GWΓ, and corresponding plasmon-pole approximations. The self-energies at the band edge ($k = 0$) are depicted in Figs. 2a and b, for $\gamma = 1$ and $\gamma = 5$, respectively. As expected the results of the plasmon-pole approximation are very close to the GW and GWΓ approximation results for large γ. However, for small values of γ there are significant differences between the plasmon-

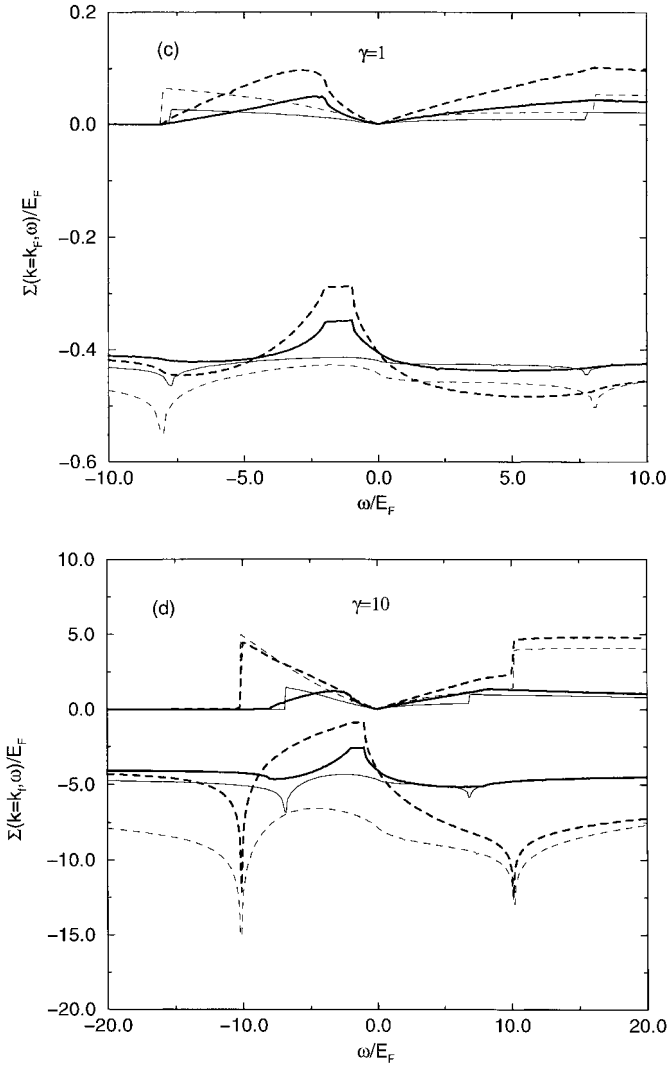

Fig. 2. The self-energy $\Sigma(k,\ \omega)$ as function of the frequency ω for a) and c) $\gamma = 1$ and b) and d) $\gamma = 5$ values, and for two fixed wave vectors $k = 0$ (a) and b)) and $k = k_{\mathrm{F}}$ (c) and d)). The thick solid and dashed lines correspond to the GWΓ and GW (RPA) approximations, respectively. The thin solid and dashed lines are for the corresponding plasmon-pole approximations. The upper and lower set of curves show $|\mathrm{Im}\,\Sigma(k,\ \omega)|$ and $\mathrm{Re}\,\Sigma(k,\ \omega)$, respectively

pole approximation results and those of GW and GWΓ. This is due to the fact that the plasmon contribution of the spectral weight is significantly larger at large interaction strengths. Similar conclusions may be drawn from the self-energy results illustrated in Figs. 2c and d, where we look at $k = k_{\mathrm{F}}$. The imaginary part $|\mathrm{Im}\,\Sigma\,(k,\ \omega)|$ as a function of ω has finite discontinuities at $\omega = \pm\omega_q(k + k_{\mathrm{F}})$ within the plasmon-pole approximation. Since $\mathrm{Re}\,[\Sigma]$ and $\mathrm{Im}\,[\Sigma]$ are related to each other through the Kramers-Kronig relation, a finite discontinuity in $\mathrm{Im}\,[\Sigma]$ gives rise to a logarithmic singularity in $\mathrm{Re}\,[\Sigma]$.

In the RPA calculation of the self-energy, the finite discontinuities in Im $[\Sigma]$ occur also at $\omega = \pm\omega_q(k + k_F)$ for large γ values. For small values of γ, Im $[\Sigma]$ is continuous but its derivative is discontinuous at the same points. In the GWΓ approximation calculation of the self-energy the discontinuities in Im $[\Sigma]$ occur at larger $|\omega|$ values than $\omega = |\omega_q(k + k_F)|$ for large γ values. For small γ values the derivative of Im $[\Sigma]$ is discontinuous at the same point with the RPA calculations of self-energy. When we compare our results with the results of a one-dimensional system with long-range Coulomb interaction [7, 11], we observe that they are qualitatively the same. This shows that the quasi-particle properties in 1D systems are largely determined by the short-range part of the interaction. We can make several comments regarding the self-energy calculations. There are considerable differences between the GW and GWΓ approximation results. The main reason for this is that for the short-range interaction model the RPA does not provide a good description of the ground-state energy beyond $\gamma \gtrsim 3$. The local-field corrections restore the quality of approximation in the intermediate coupling region ($1 \lesssim \gamma \lesssim 10$), thus the quasi-particle properties calculated within the GWΓ approximation are expected to give a better account.

Once the self-energy $\Sigma(k, \omega)$ is known, other one-particle properties can be readily calculated. We now examine the single-particle spectral function $A(k, \omega)$ defined as

$$A(k, \omega) = \frac{2|\mathrm{Im}\,\Sigma(k, \omega)|}{[\omega - \xi_k - \mathrm{Re}\,\Sigma(k, \omega)]^2 + [\mathrm{Im}\,\Sigma(k, \omega)]^2} . \tag{11}$$

In Figs. 3a and b we show $A(k, \omega)$ as a function of the frequency at $k = 0$ and $k = k_F$, respectively, for $\gamma = 5$. We again note that our results for the short-range potential are very similar to those for the long-range Coulomb interaction [7, 11], indicating the importance of short-range effects. We note that the difference between the GW and GWΓ approximation results are more prominent at the band-edge $k = 0$ than at $k = k_F$. The observed quasi-particle peaks correspond the solutions of the Dyson's equation $\omega - \xi_k - \mu = \Sigma(k, \omega)$. The single-particle spectral density also satisfies the sum-rule $\int \mathrm{d}\omega/(2\pi)\, A(k, \omega) = 1$, which we verify numerically to a very high accuracy. The first frequency sum-rule yields [22] $\int \mathrm{d}\omega/(2\pi)\, \omega A(k, \omega) = E_k^{\mathrm{HF}}$, where $E_k^{\mathrm{HF}} = \xi_k + \Sigma_{\mathrm{ex}}(k)$ is the quasi-particle energy in the Hartree-Fock approximation, and the spectral function $A(k, \omega)$ is evaluated in the GW approximation. We have selectively checked for various γ values that this sum-rule is also satisfied. We have found that the above sum-rules are not fulfilled satisfactorily when the plasmon-pole approximation was made in the calculation of $\Sigma(k, \omega)$ for small coupling strengths. This is consistent with our earlier observations in connection with the particle–hole excitations. The spectral function $A(k, \omega)$ can be observed experimentally with photoemission spectroscopy [25]. Calculations of $A(k, \omega)$ in the Hubbard model or more general lattice models [26] together with our results can be compared with the experiments. The momentum distribution of particles $n(k) = \int_{-\infty}^{\infty} A(k, \omega)\mathrm{d}\omega/2\pi$, is shown in Fig. 4 for various values of the interaction strength. The expected behavior of a jump discontinuity at $k = k_F$ decreases with increasing γ, as in a normal Fermi liquid. That the short-range interaction model yields qualitatively similar $n(k)$ results to a 1D electron system with long-range Coulomb interaction, once again indicates the importance of the short-range part of the repulsive interaction in determining the physical properties. In a recent paper Schäfer and Schuck [21] applied Green's function methods to calculate the momentum distribution

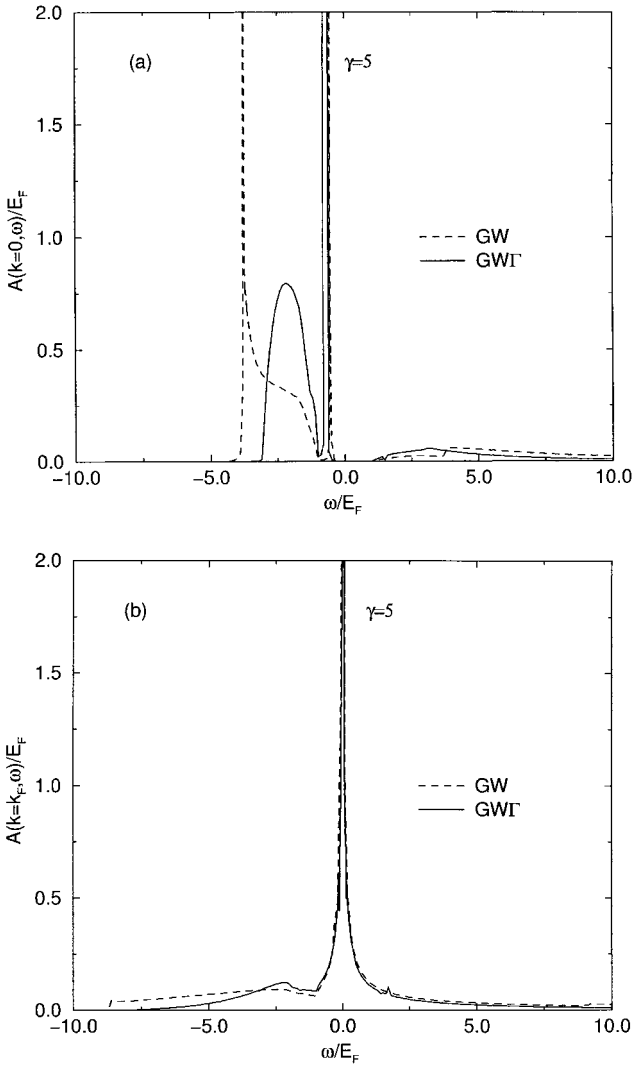

Fig. 3. The single-particle spectral function $A(k, \omega)$ as a function of frequency at a) the band-edge $k = 0$ and b) $k = k_F$, for $\gamma = 5$. The solid and dashed lines indicate the GWΓ and GW approximations, respectively

for a 1D Hubbard model. They also find discontinuity in n_k for small enough on-site interaction strengths. Although the details of both approaches are quite different it is interesting to note their qualitative agreement.

The many-body effects renormalize the bare mass of electrons, and the deviations from the free-particle behavior are embodied in the effective mass. The quasi-particle effective mass m^* is calculated from the knowledge of the wave vector and frequency dependence of the self-energy, through the expression [15]

$$\frac{m^*}{m} = \left| \frac{1 - \partial\Sigma(k, \omega)/\partial\omega}{1 + \partial\Sigma(k, \omega)/\partial\xi_k} \right|_{\omega=\xi_k}. \tag{12}$$

The derivative terms need to be handled with care, since logarithmic divergences appear [27] as $\omega \to 0$ and $k \to k_F$. These are treated by including a phenomenological

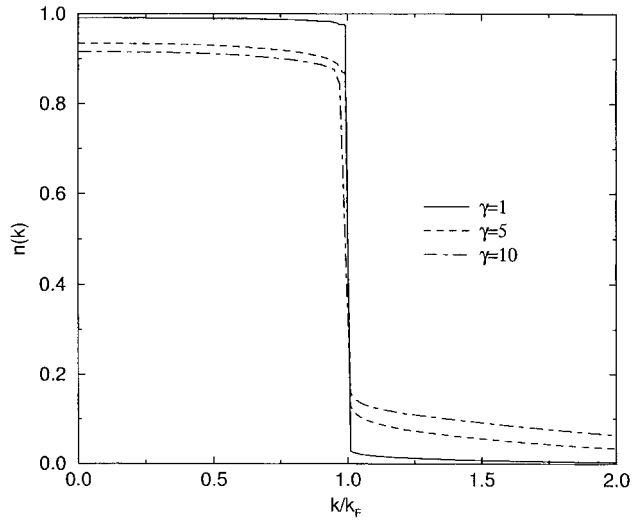

Fig. 4. The momentum distribution $n(k)$ of interacting electrons within the GW scheme at various values of the interaction strength γ

broadening factor in the density–density response function $\chi_0(q, \omega)$ which restores the Fermi liquid behavior in our system. We display in Fig. 5 the effective mass m^*/m as a function of γ, which we obtain by calculating Eq. (12) at $k = k_F$ and $\omega = 0$. The effective mass shows a strong dependence on the coupling strength in the GW approximation, but the inclusion of vertex corrections somewhat diminishes this effect. This may be related to the overestimation of correlation effects in the RPA.

The quasi-particle broadening or the damping rate $\Gamma(k)$ is given by the imaginary part of the self-energy,

$$\Gamma(k) = -\operatorname{Im}\Sigma(k, \xi_k). \tag{13}$$

From the damping rate, we can calculate the quasi-particle scattering rate $2\Gamma(k)$, the inelastic lifetime $\tau(k) = [2\Gamma(k)]^{-1}$, and the inelastic mean free path $l(k) = v(k)\,\tau(k)$,

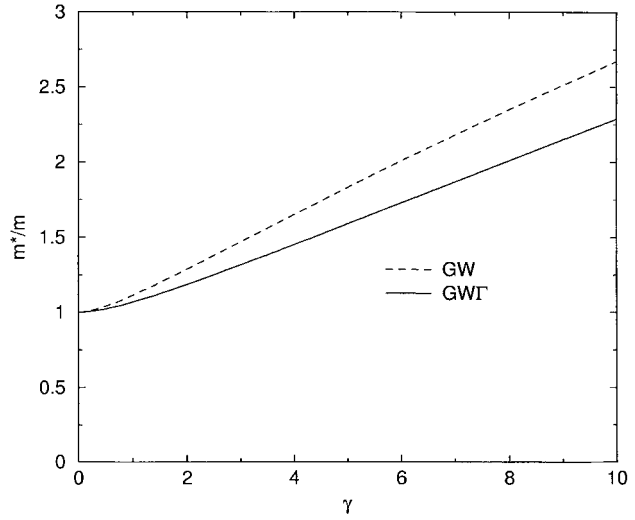

Fig. 5. The effective mass ratio m^*/m as a function of γ with and without the vertex corrections. The solid and dashed lines are for GWΓ and GW approximations, respectively

where $v(k)$ is the electron velocity. Figure 6 shows $\Gamma(k)$ for $\gamma = 5$ in the RPA-GW, GWΓ and their corresponding plasmon-pole approximations. We find that the plasmon-pole approximations provide a reasonable agreement at this intermediate coupling. The damping rate calculated within the GWΓ approximation is not drastically different from the one calculated within the GW (RPA). Firstly, $\Gamma(k)$ within the GWΓ is about three times smaller in magnitude than that in the RPA-GW. This is consistent with the earlier results shown in Fig. 2. A similar comparison for 2D electron systems we made by Marmorkos and Das Sarma [28]. There, it was found that the vertex corrections were only 10 to 30% smaller than the RPA results. We are not aware of any calculations of the damping rate in 1D electron systems with long-range interaction which includes the vertex corrections. The threefold decrease in $\Gamma(k)$ may be partly due to the dimensionality and partly due to the short-range nature of the interaction. In any case, our results indicate the significance of vertex corrections or correlation effects beyond the RPA. For wave vectors away from k_F, but less than some threshold wave vector k_c ($k_c \approx 1.6\, k_F$ for $\gamma = 5$ and $k_c \approx 2.1\, k_F$ for $\gamma = 10$ in the RPA), damping rates including the vertex corrections are higher than those in the RPA. In the large wave vector regime, indicated by the sharp increase in $\Gamma(k)$ in Fig. 6, plasmon excitation mechanism becomes important. Here the effects of vertex corrections are observed to decrease the damping rate. Furthermore, the threshold wave vector k_c for the onset of plasmons decreases compared to the RPA values ($k_c \approx 2\, k_F$ for $\gamma = 5$ and $k_c \approx 3\, k_F$ for $\gamma = 10$ in the GWΓ approximation). As it is known, the vertex corrections (local-field corrections) in general lower the plasmon energies. As argued before, for larger values of γ the RPA breaks down, and the results which include the local-field corrections (i.e. GWΓ approximation) should be more trustworthy.

The above examples of quasi-particle properties, when compared with the results for quantum-wires with long-range interaction [7], suggest that as far as the Fermi liquid concept is applicable the short-range Coulomb interaction plays an important role. Similar to the recent attempts [13, 17] one can model the realistic quantum-wire structures with an effective contact interaction to calculate various quantities, provided that

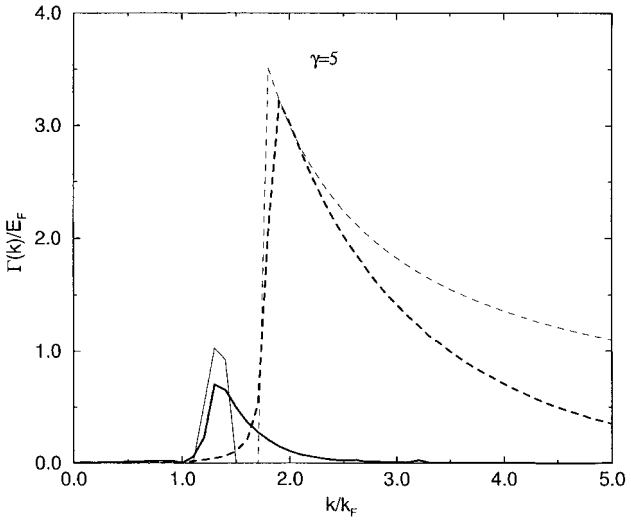

Fig. 6. The damping rate $\Gamma(k)$ as a function of k for $\gamma = 5$. The thick solid and dashed lines correspond to the GWΓ and GW approximations, respectively. The thin solid and dashed lines indicate the corresponding plasmon-pole approximations

dynamical effects are accounted for through the dielectric function $\varepsilon(q, \omega)$. An example was given by Hu and Das Sarma [7] of a situation under which a 1D system of electrons (a quantum wire) may be regarded as interacting with a contact potential (cf. Section 2). Thus, our results may be useful in understanding the quasi-particle properties of similar systems within the Fermi liquid theory. The idea is basically to relate the density and lateral width of a quantum wire to a single interaction parameter γ in our model, as employed by Tassone and Piermarocchi [13] in their study. Our calculations indicate that in contrast to the long-range interaction case, the plasmon-pole approximation may not be very reliable for small couplings. Nevertheless, the plasmon-pole approximation yields qualitatively correct description of the 1D electrons interacting via a short-range potential.

To further test the similarity between the results of the short-range interaction model and that of the long-range Coulomb potential, we investigate the energy relaxation in a 1D structure due to electron–phonon interaction [29]. Since we shall be interested in the qualitative effects rather than a precise comparison, we employ the plasmon-pole approximation in this part of the calculations to study the hot-electron energy relaxation through longitudinal optical (LO)-phonon emission within the Fröhlich coupling. When excess energy is supplied to the electron gas, either by a strong applied electric field or by optical excitation, the electrons go out of equilibrium with the lattice and attain a higher electron temperature T than the ambient lattice temperature T_{L}. The hot-electron gas loses energy to its surroundings in order to achieve equilibrium with the lattice. For lattice temperature $T_{\mathrm{L}} = 0$, the energy relaxation rate is given by [30]

$$P = \frac{2}{\pi} \sum_{q} \int_{0}^{\infty} \mathrm{d}\omega \, \omega n_T(\omega) \, \mathrm{Im} \, \chi_0(q, \omega) \, \mathrm{Im} \, V_{\mathrm{sc-ph}}(q, \omega) \,, \tag{14}$$

where $n_T(\omega)$ is the Bose distribution factor at electron temperature T, and $V_{\mathrm{sc-ph}}(q, \omega) = |M_q|^2 \, D(q, \omega)/\varepsilon^2(q, \omega)$ is the dynamically screened phonon potential in which $|M_q|^2$ is the Fröhlich coupling matrix element squared given by $|M_q|^2 = V_0(1/\epsilon_\infty - 1/\epsilon_0) \, \omega_{\mathrm{LO}}/2$. We use the material parameters appropriate for GaAs: $\epsilon_0 = 12.9$, $\epsilon_\infty = 10.9$, and $\omega_{\mathrm{LO}} = 36.8$ meV. The phonon propagator $D(q, \omega)$ is given by

$$D(q, \omega) = \frac{2\omega_{\mathrm{LO}}}{\omega^2 - \omega_{\mathrm{LO}}^2 - 2\omega_{\mathrm{LO}} |M_q|^2 \, \chi_0(q, \omega)/\varepsilon(q, \omega)} \,. \tag{15}$$

The last term in the denominator (phonon self-energy) is the correction due to many-body electron–phonon coupling, which broadens the phonon spectral function. If the phonon self-energy is ignored, $\mathrm{Im} \, D(q, \omega)$ becomes

$$\mathrm{Im} \, D_0(q, \omega) = \pi[\delta(\omega + \omega_{\mathrm{LO}}) - \delta(\omega - \omega_{\mathrm{LO}})] \,. \tag{16}$$

Inserting this into Eq. (14), we obtain the energy loss rate for bare (without many-body coupling) phonon emission as

$$P_0 = -2\omega_{\mathrm{LO}} n_T(\omega_{\mathrm{LO}}) \sum_{q} |M_q|^2 \, \frac{\mathrm{Im} \, \chi_0(q, \omega_{\mathrm{LO}})}{\varepsilon^2(q, \omega_{\mathrm{LO}})} \,. \tag{17}$$

When we keep the phonon self-energy in the denominator of Eq. (15), $\mathrm{Im} \, D(q, \omega)$ can again be expressed as a sum of a pair of δ-functions at the coupled plasmon–phonon

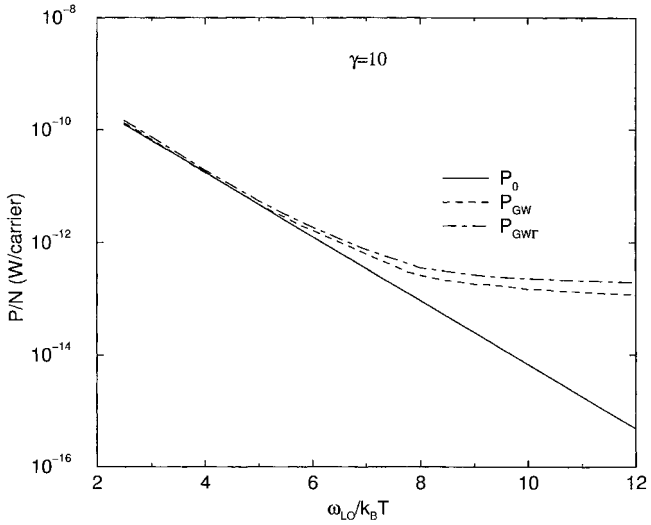

Fig. 7. The energy relaxation rates (per electron) as a function of the electron temperature T

excitation frequencies. In this case, the energy loss rate is given by [11] $P = P_+ + P_-$, where

$$P_\pm = -2\sum_q \omega_\pm n_T(\omega_\pm)\,\frac{\omega_{\mathrm{LO}}|\omega_\pm^2 - \omega_q^2|}{\omega_\pm(\omega_+^2 - \omega_-^2)}\,|M_q|^2\,\frac{\mathrm{Im}\,\chi_0(q,\,\omega_\pm)}{\varepsilon^2(q,\,\omega_\pm)}\,, \tag{18}$$

in which $\omega_\pm$ (not to be confused with the particle–hole excitation region boundaries in this context) are the coupled plasmon–phonon excitation energies [11] and ω_q is the uncoupled plasmon excitation for a 1D electron gas discussed earlier. We show the energy loss rates with and without the phonon renormalization effects in Fig. 7. The effect of the phonon renormalization is seen to be negligible at high temperatures but it is very large at low temperatures. The inclusion of vertex corrections seems to increase the power loss. These results are very similar to those obtained with the long-range Coulomb interaction [11, 31, 32]. We believe that the plasmon-pole approximation gives a qualitatively faithful description of the energy loss rate at this large coupling strength ($\gamma = 10$, and our results may be compared with the experiments [33].

4. Summary

In this work, we have calculated the quasi-particle properties of a 1D electron gas interacting via a short-range repulsive contact potential. We have used the RPA based GW approximation and the GWΓ approximation which includes the local-field corrections to calculate the electron self-energy. We found that a number of quasi-particle properties are very similar to those calculated for 1D electron systems interacting via long-range Coulomb potential. Thus, a number of physical quantities of interest for realistic 1D electron systems may be modeled by an effective short-range Coulomb interaction. Our results demonstrate the importance of short-range correlation effects and also the significance of Fermi-liquid picture in some of the quasi-particle properties. Making the plasmon-pole approximation to the dielectric function in the self-energy calculations is tested against the exact RPA-GW and GWΓ approximation schemes. We found that

although the plasmon-pole approximation captures the qualitative aspects of the full calculations, it works less satisfactorily (compared to the 1D electron gas with long-range interaction) in the present problem for small couplings.

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